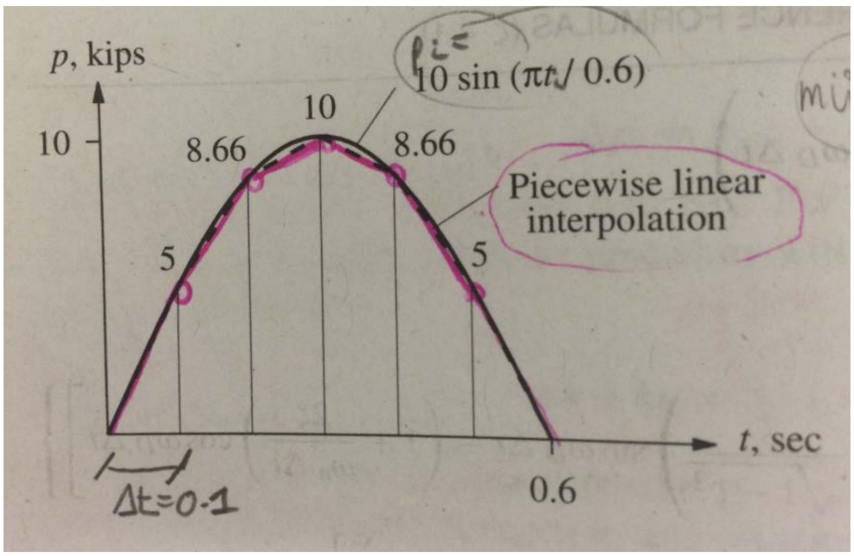
Verification Problem:

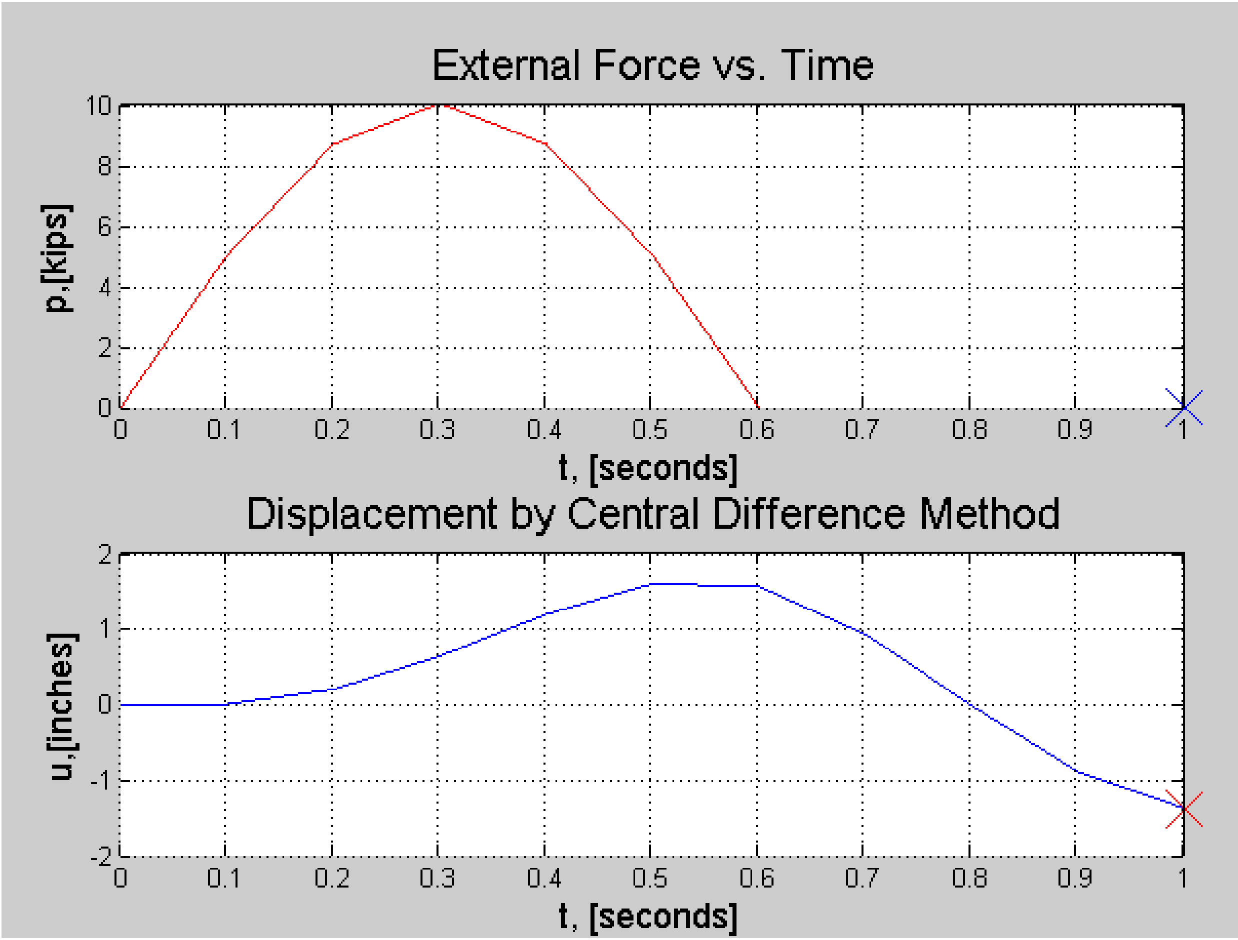
Chopra’s Example 5.1 was used to verify the accuracy of the Single-Degree-of-Freedom Programs for the Central Difference Method, and Newmark’s Method.

Example 5.1 — assume for now -

An SDF system has the following properties: $m = 0.2533$ kip-sec^2/in., $k = 10$ kips/in., $T_n = 1$ sec ($\omega_n = 6.283$ rad/sec), and $\zeta = 0.05$. Determine the response $u(t)$ of this system to $p(t)$ defined by the half-cycle sine pulse force shown in Fig. E5.1 by (**a**) using piecewise linear interpolation of $p(t)$ with $\Delta t = 0.1$ sec, and (**b**) evaluating the theoretical solution.



Central Difference Method Results:



Program Results:

time (s)	disp u (in)
0	0
0.1	0
0.2	0.1914
0.3	0.6293
0.4	1.1825
0.5	1.5808
0.6	1.5412
0.7	0.9140
0.8	-0.0247
0.9	-0.8969
1	-1.3726

Book Results:

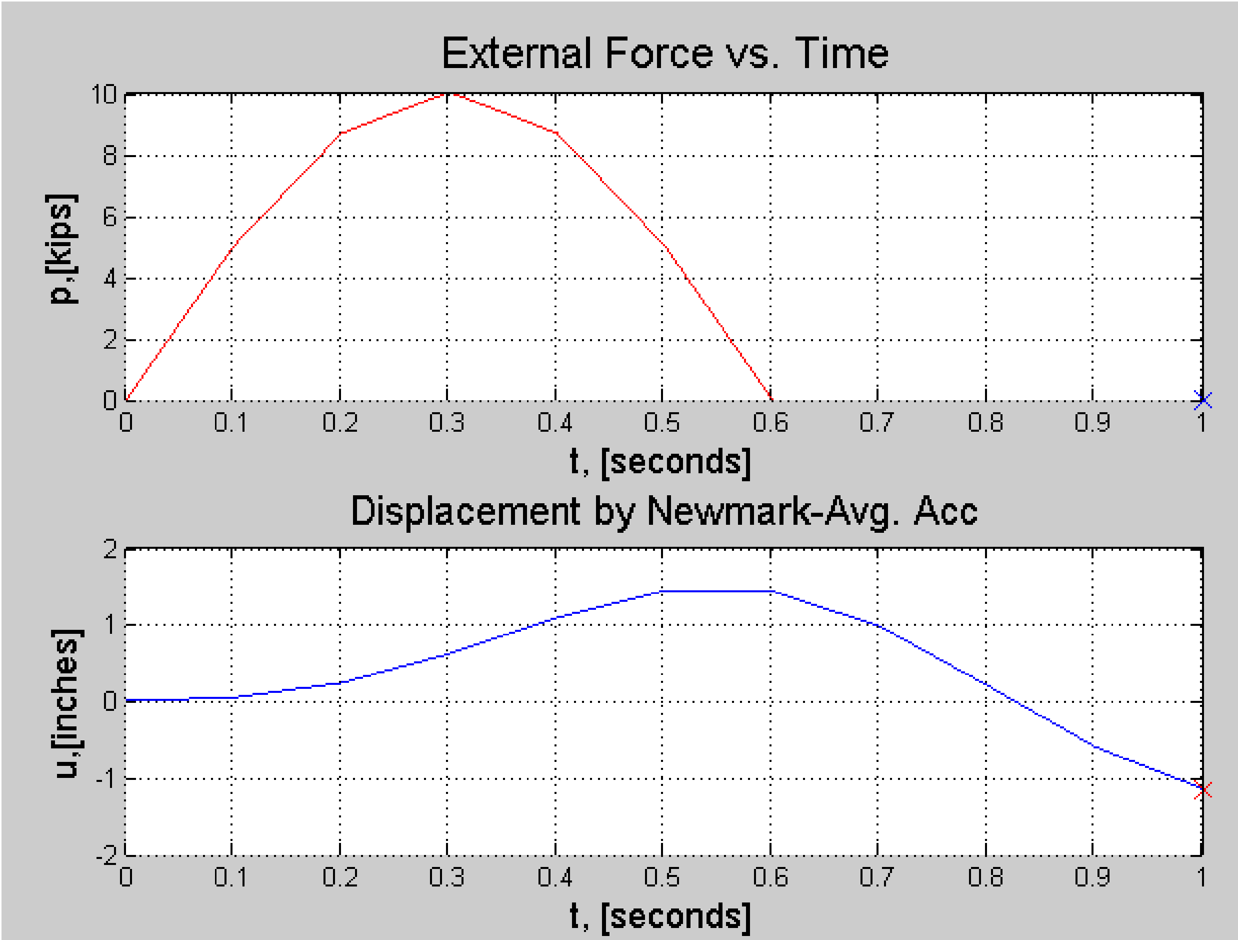
TABLE E5.2 NUMERICAL SOLUTION BY CENTRAL DIFFERENCE METHOD

t_i	p_i	u_{i-1}	u_i	$\hat{p}_i$ [Eq. (2.1)]	u_{i+1} [Eq. (2.2)]	Theoretical u_{i+1}
0.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0328
0.1	5.0000	0.0000	0.0000	5.0000	0.1914	0.2332
0.2	8.6602	0.0000	0.1914	16.4419	0.6293	0.6487
0.3	10.0000	0.1914	0.6293	30.8934	1.1825	1.1605
0.4	8.6603	0.6293	1.1825	41.3001	1.5808	1.5241
0.5	5.0000	1.1825	1.5808	40.2649	1.5412	1.4814
0.6	0.0000	1.5808	1.5412	23.8809	0.9141	0.9245
0.7	0.0000	1.5412	0.9141	−0.6456	−0.0247	0.0593
0.8	0.0000	0.9141	−0.0247	−23.4309	−0.8968	−0.7751
0.9	0.0000	−0.0247	−0.8968	−35.8598	−1.3726	−1.2718
1.0	0.0000	−0.8968	−1.3726	−33.8058	−1.2940	−1.2674

The program exactly outputs the book results for the Central Difference Method.

Newmark's Method:

Program Results:



time (s)	disp u (in)
0	0
0.1	0.0437
0.2	0.2326
0.3	0.6121
0.4	1.0825
0.5	1.4310
0.6	1.4231
0.7	0.9622
0.8	0.1908
0.9	-0.6044
1	-1.1442

Book Results:

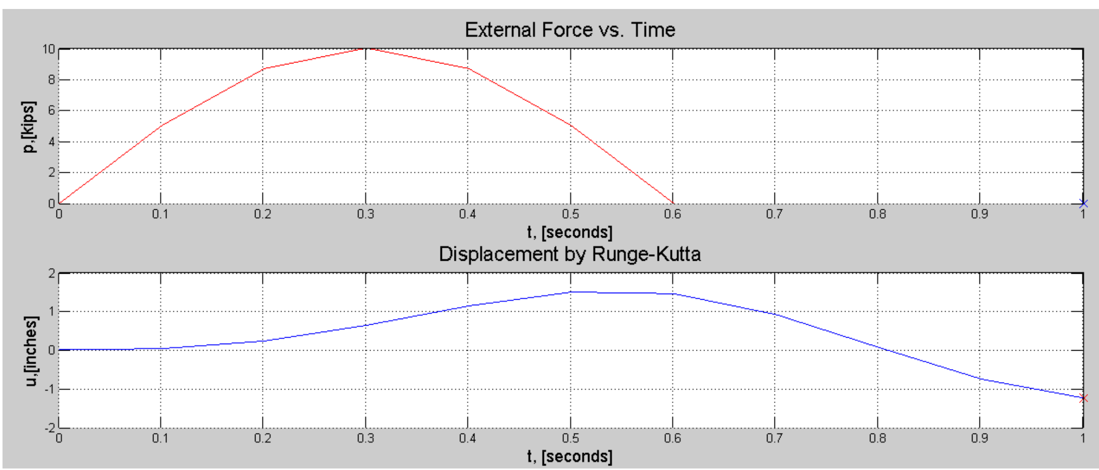
TABLE E5.3 NUMERICAL SOLUTION BY AVERAGE ACCELERATION METHOD

t_i	p_i	$\ddot{u}_i$ (Step 2.5)	Δp_i	$\Delta \hat{p}_i$ (Step 2.1)	Δu_i (Step 2.2)	$\Delta \dot{u}_i$ (Step 2.3)	$\Delta \ddot{u}_i$ (Step 2.4)	$\dot{u}_i$ (Step 2.5)	u_i (Step 2.5)	Theoretical u_i
0.0	0.0000	0.0000	5.0000	5.0000	0.0437	0.8733	17.4666	0.0000	0.0000	0.0000
0.1	5.0000	17.4666	3.6603	21.6356	0.1890	2.0323	5.7137	0.8733	0.0437	0.0328
0.2	8.6602	23.1803	1.3398	43.4485	0.3794	1.7776	−10.8078	2.9057	0.2326	0.2332
0.3	10.0000	12.3724	−1.3397	53.8708	0.4705	0.0428	−23.8893	4.6833	0.6121	0.6487
0.4	8.6603	−11.5169	−3.6602	39.8948	0.3484	−2.4839	−26.6442	4.7261	1.0825	1.1605
0.5	5.0000	−38.1611	−5.0000	−0.9009	−0.0079	−4.6417	−16.5122	2.2422	1.4309	1.5241
0.6	0.0000	−54.6733	0.0000	−52.7740	−0.4609	−4.4187	20.9716	−2.3995	1.4231	1.4814
0.7	0.0000	−33.7017	0.0000	−88.3275	−0.7714	−1.7912	31.5787	−6.8183	0.9622	0.9245
0.8	0.0000	−2.1229	0.0000	−91.0486	−0.7952	1.3159	30.5646	−8.6095	0.1908	0.0593
0.9	0.0000	28.4417	0.0000	−61.8123	−0.5398	3.7907	18.9297	−7.2936	−0.6044	−0.7751
1.0	0.0000	47.3714						−3.5029	−1.1442	−1.2718

Again, the book results are exactly matched.

Runge-Kutta Method:

Program Results:



time (s)	disp u (in)	Theoretic disp. (in)
0	0.0000	0
0.1	0.0324	0.0328
0.2	0.2280	0.2332
0.3	0.6332	0.6487
0.4	1.1319	1.1605
0.5	1.4865	1.5241
0.6	1.4455	1.4814
0.7	0.9044	0.9245
0.8	0.0625	0.0593
0.9	-0.7508	-0.7751
1	-1.2370	-1.2718

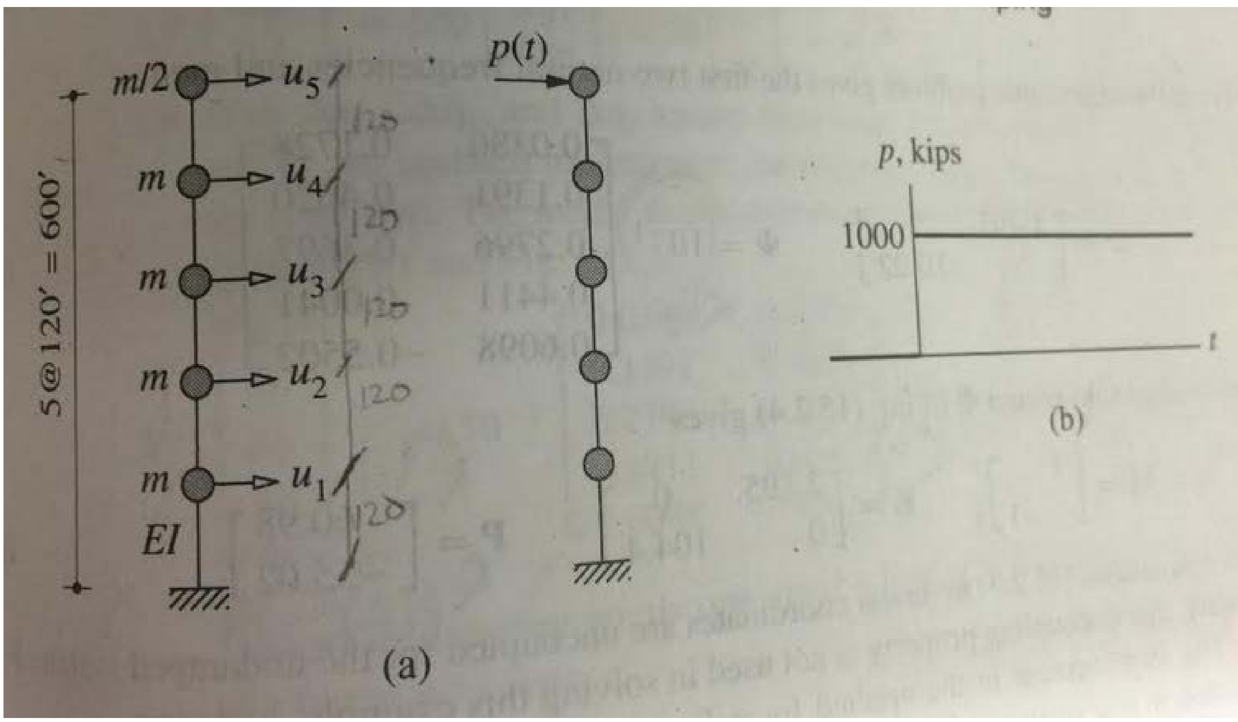
As the Runge-Kutta Method is not included in the book, the method was compared to the theoretical value for displacement as shown in examples for Newmarks' Method. The Runge-Kutta Method is definitely within reason to the theoretical displacement.

MDOF Verification Problem

Example 15.1 from Chopra is used to verify the program.

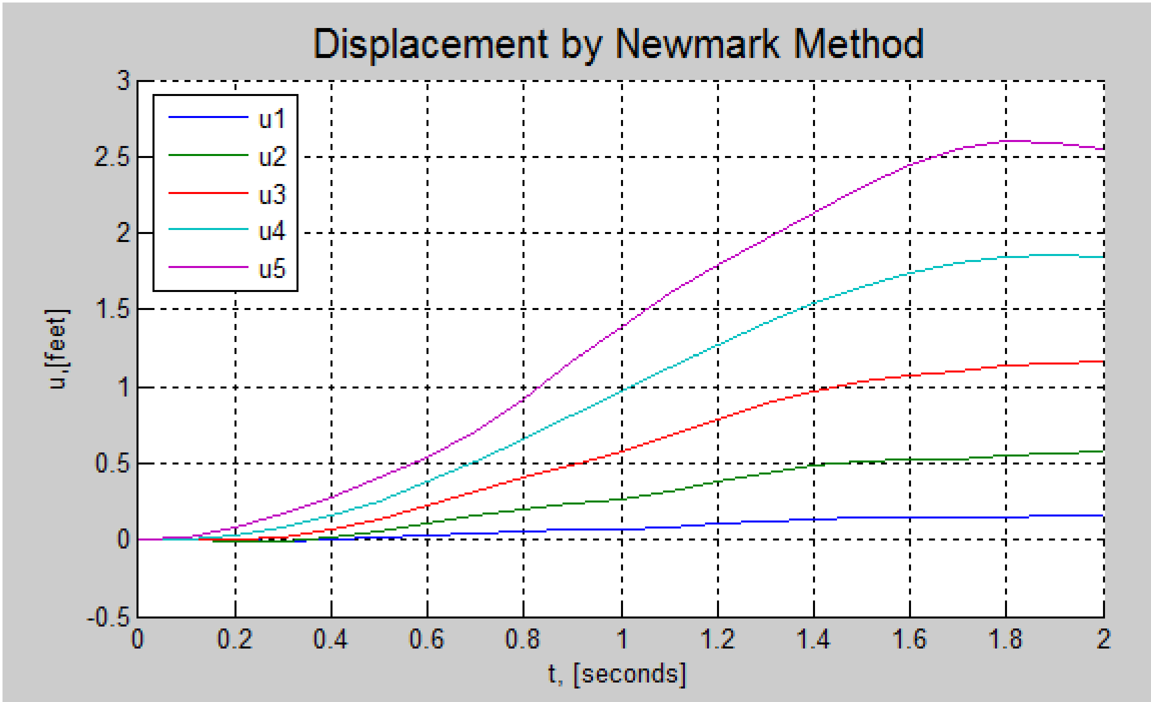
Example 15.1

A reinforced-concrete chimney idealized as the lumped-mass cantilever (Fig. E15.1a) is subjected at the top to a step force $p(t)$ of 1000 kips (Fig. E15.1b); $m = 208.6$ kip-sec^2/ft and $EI = 5.469 \times 10^{10}$ kip-ft^2. Solve the equations of motion after transforming them to the first two modes by the linear acceleration method with $\Delta t = 0.1$ sec.



As all five modes were included in the analysis, linear acceleration method will make the solution "blow up" with the given time step of dt=0.1s. Therefore, average acceleration method was used and the results are compared to the book results, since the average acceleration method is unconditionally stable.

Program Results



time (s)	u1	u2	u3	u4	u5
0.1	-0.0003	-0.0009	-0.0003	0.0051	0.0172
0.2	-0.0024	-0.0046	0.0028	0.0296	0.0753
0.3	-0.0057	-0.0056	0.0214	0.0830	0.1682
0.4	-0.0025	0.0122	0.0660	0.1602	0.2798
0.5	0.0123	0.0553	0.1366	0.2573	0.4043
0.6	0.0301	0.1095	0.2270	0.3754	0.5435
0.7	0.0440	0.1598	0.3221	0.5104	0.7125
0.8	0.0557	0.2013	0.4084	0.6552	0.9226
0.9	0.0631	0.2343	0.4900	0.8073	1.1586
1	0.0689	0.2683	0.5779	0.9655	1.3919
1.1	0.0831	0.3176	0.6767	1.1214	1.6065
1.2	0.1037	0.3810	0.7839	1.2698	1.7972
1.3	0.1219	0.4402	0.8883	1.4114	1.9674
1.4	0.1341	0.4838	0.9735	1.5388	2.1339
1.5	0.1406	0.5095	1.0319	1.6456	2.3011
1.6	0.1413	0.5200	1.0718	1.7338	2.4461
1.7	0.1408	0.5291	1.1053	1.8001	2.5469
1.8	0.1479	0.5486	1.1352	1.8389	2.5954
1.9	0.1575	0.5705	1.1582	1.8519	2.5929
2	0.1600	0.5781	1.1637	1.8404	2.5544

Book Results

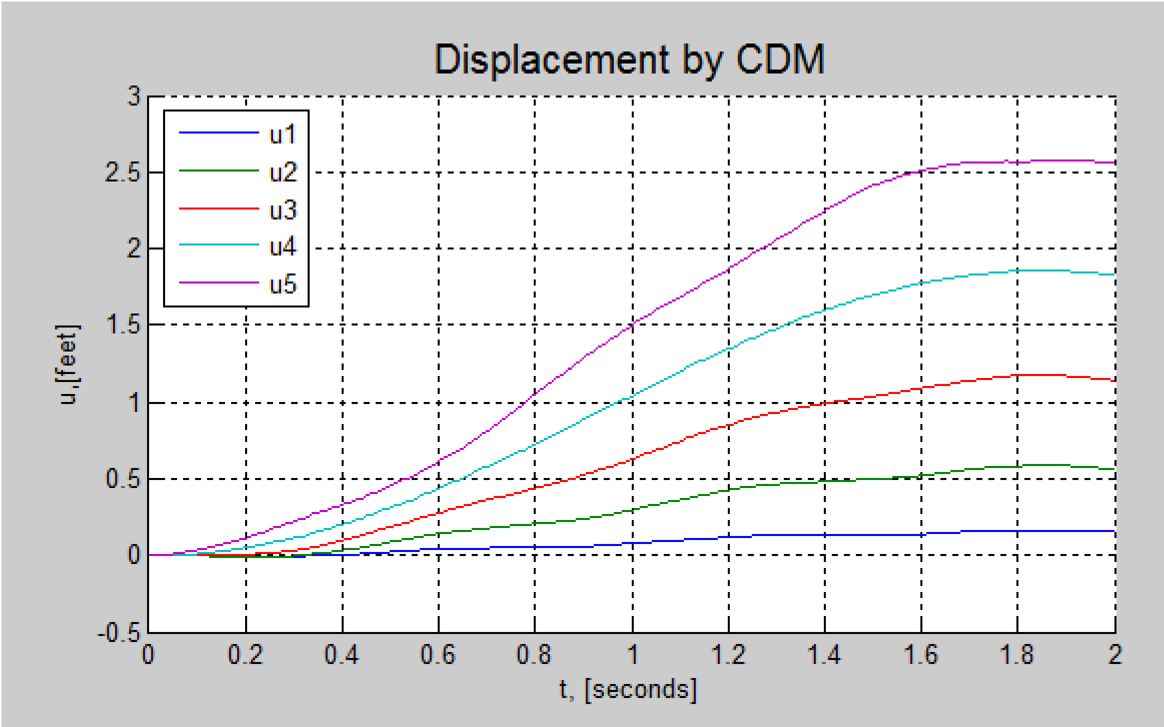
TABLE E15.1 NUMERICAL SOLUTION OF MODAL EQUATIONS BY THE LINEAR ACCELERATION METHOD

Newmark,

Time	q_1	q_2	u_1	u_2	u_3	u_4	u_5
0.1	0.1011	−0.0781	−0.0010	−0.0017	−0.0001	0.0044	0.0105
0.2	0.7051	−0.4773	−0.0055	−0.0094	0.0021	0.0309	0.0693
0.3	1.8956	−0.9207	−0.0086	−0.0107	0.0190	0.0832	0.1663
0.4	3.6384	−1.0141	−0.0035	0.0098	0.0642	0.1601	0.2777
0.5	5.8832	−0.6744	0.0110	0.0547	0.1396	0.2592	0.3959
0.6	8.5654	−0.2036	0.0295	0.1109	0.2320	0.3777	0.5335
0.7	11.6080	−0.0205	0.0444	0.1606	0.3238	0.5120	0.7090
0.8	14.9230	−0.2878	0.0526	0.1960	0.4066	0.6581	0.9258
0.9	18.4140	−0.7678	0.0577	0.2252	0.4865	0.8119	1.1651
1.0	21.9820	−1.0338	0.0669	0.2641	0.5764	0.9692	1.3974
1.1	25.5240	−0.8491	0.0838	0.3208	0.6823	1.1255	1.6032
1.2	28.9370	−0.3781	0.1050	0.3872	0.7951	1.2763	1.7854
1.3	32.1240	−0.0395	0.1232	0.4451	0.8968	1.4170	1.9611
1.4	34.9920	−0.1344	0.1326	0.4812	0.9735	1.5434	2.1412
1.5	37.4580	−0.5785	0.1345	0.4976	1.0260	1.6520	2.3160
1.6	39.4530	−0.9768	0.1353	0.5094	1.0670	1.7398	2.4596
1.7	40.9170	−0.9752	0.1409	0.5298	1.1081	1.8044	2.5488
1.8	41.8100	−0.5751	0.1513	0.5583	1.1478	1.8440	2.5812
1.9	42.1050	−0.1323	0.1601	0.5802	1.1724	1.8572	2.5748
2.0	41.7940	−0.0405	0.1605	0.5796	1.1671	1.8435	2.5508

The book results are quite close with the program results. The minimal differences in values can be attributed to the fact that the program includes all modes instead of just the first two as the book does, and also the fact that the program uses average acceleration method, instead of linear acceleration method.

CDM Program Results



Discussion

As the Central Difference Method requires the time step to be below 0.008 for stability to hold, it is not the most efficient method compared to Newmark's; however the output does indeed display a near identical graph to that of Newmark's Method and therefore, I believe that the program provides accurate results.

Animations

Animation is drawn by using the pause function in Matlab as opposed to the drawnow function that was used during the presentation. Having the program pause at every time increment in in every iteration, it gives a close approximation of animation *in real time*, as opposed to the drawnow function where the frame rate is arbitrarily chosen by the built-in function.

However, the animation speed tends to lag when several degrees of freedom are added (more than 3), the time step is too miniscule or the processing power/graphic card of the computer running the program is not up to high standards.

Generally, keeping the time increment dt to above 0.1 seconds—is enough to show animation close to real time usually off by a second or so in the Ghausi Computer Labs.

Crosshair animation has been eliminated as it slowed down the animations too significantly.

There are two sample outputs recorded using third party software that shows an improved animation timing attached in the file presented.