

**COMP 4030/6030: Assignment 1**  
**Due date:** 09/08/2015

1. Algorithm *SlowSort* sorts an array  $A$  from smallest to largest. Give/describe the invariant that remains true after each outer loop (with index  $i$ ). This invariant should show why the algorithm is correct at the end.

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```
1: SLOWSORT(A)
2:   for  $i = 0$  to  $\text{len}(A)-1$  do
3:      $m \leftarrow i$ 
4:     for  $j = i + 1$  to  $\text{len}(A)-1$  do
5:       if  $A[j] < m$  then
6:          $m \leftarrow j$ 
7:     SWAP( $A[i], A[m]$ )
```

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2. Algorithm *Process* takes a list  $A$  and returns another list. A few notations:

- An empty list is represented by  $[]$ .
- $A.\text{pop}(0)$  removes and returns the first element of  $A$ .
- As in Python, adding two lists (e.g.  $x + y$ ) results in the concatenation of the two lists.

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```
1: PROCESS(A)
2:   if  $\text{len}(A) == 0$  then
3:     return  $[]$ 
4:   else
5:      $first = A.\text{pop}(0)$ 
6:     return  $\text{PROCESS}(A) + [first]$ 
```

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Questions/problems:

- (a) Trace algorithm *Process* on input  $A = [1, 2, 3, 4]$ . Show the intermediate steps.
- (b) Given any given list  $A$ , what does algorithm *Process* return?
- (c) Use mathematical induction to explain why algorithm *Process* correctly does what it is supposed to do.

## Notes

The abstract framework of mathematical induction consists of two main steps:

1. Show the algorithm works correctly for the smallest input sizes.
2. Assuming the algorithm works correctly for input sizes  $0, 1, 2, \dots, k$ , show the algorithm works correctly for input size  $k + 1$ . This means you can assume recursive calls are correct, if the recursive calls have smaller input sizes.