

# Implementing Integer Programming in Excel

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# Overview

**Model the use of leverage in a financial portfolio**

**Specify a minimum long-bias that must be satisfied by the portfolio**

**Formulate integer programming problems to optimize these portfolios**

**Use Excel's Solver to solve these optimization problems**

# Building Is Hard, Using Is Easy



**Builder**

Building an integer programming  
solver is **hard**



**User**

Using an integer programming  
solver is **easy**

# Demo

**Apply integer programming to  
optimization using Excel's Solver  
add-in**

# Recap - Linear Programming

## Assemble financial data

Use data from Yahoo finance

Prices of correlated stocks

## Estimate risk, return

Use historical data

Risk = max % 1-period drop

## Quadratic Programming

Minimize portfolio variance

Risk = variance

## Convert prices into returns

Download prices data and convert into returns

Simple step, use excel formulae

## Linear Programming

Minimize max loss risk

Threshold on expected return

## Long-only Constraint

Minimize portfolio variance

Forced to accept lower return

# Moving On - Integer Programming

## Model use of leverage

Investment can exceed invested amount  
Leverage magnifies risk and return

## Impose long-bias

Net long position = 5X  
Express as K-of-N constraint

## Integer Programming

Minimize portfolio risk  
Risk = max loss of portfolio

## Buy-sell-hold decision variables

Model each decision as +1, 0 or -1  
Integer constraint with 3 acceptable values

## Dis-allow short positions

Only allow buy or hold  
Decision variables can now be only 0 or 1, not -1

# Estimate Portfolio Return and Risk

$$P = w_1Y_1 + w_2Y_2 + w_3Y_3 \dots + w_kY_k$$

## Expected Return

Simple - use average of historical returns

## Forecast Risk

Conservative - define as sum of max loss in each stock

**Max Loss refers to largest % fall experienced by a stock in any period in our data**

# Estimate Portfolio Return and Risk

$$P = w_1Y_1 + w_2Y_2 + w_3Y_3 \dots + w_kY_k$$

**Expected Return =  
Mean(y)**

Simple - mean of sum is sum of means

**Forecast Risk =  
MaxLoss(y)**

Conservative - define as sum of max loss in each stock

**Max Loss refers to largest % fall experienced by a stock in any period in our data**



# Estimating Return

$$P = w_1Y_1 + w_2Y_2 + w_3Y_3 \dots + w_kY_k$$

$$\begin{aligned} \text{Mean}(P) = & w_1 \times \text{Mean}(Y_1) + \\ & w_2 \times \text{Mean}(Y_2) + \\ & w_3 \times \text{Mean}(Y_3) + \\ & \dots \\ & w_k \times \text{Mean}(Y_k) \end{aligned}$$

k terms, all linear

Mean of sum = sum of means

# Estimating Return

$$P = w_1Y_1 + w_2Y_2 + w_3Y_3 \dots + w_kY_k$$

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# Estimate Portfolio Return and Risk

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Conservative - define as sum of max loss in each stock

# Estimating Risk

$$P = w_1 Y_1 + w_2 Y_2 + w_3 Y_3 \dots + w_k Y_k$$

$$\begin{aligned} \text{Risk}(P) = & w_1 \times \text{MaxLoss}(Y_1) + \\ & w_2 \times \text{MaxLoss}(Y_2) + \\ & w_3 \times \text{MaxLoss}(Y_3) + \\ & \dots \\ & w_k \times \text{MaxLoss}(Y_k) \end{aligned}$$

k terms, all linear

**Portfolio Risk = Sum of individual asset risks**

# Portfolio Allocation as an Optimization Problem



## Objective Function

Minimize Risk(P)

$$\text{Risk}(P) = \text{MaxLoss}(P)$$



## Constraints

$$\bar{P} \geq R_{\text{threshold}}$$

$$\bar{P} = w_1 \bar{Y}_1 + w_2 \bar{Y}_2 + \dots + w_k \bar{Y}_k$$

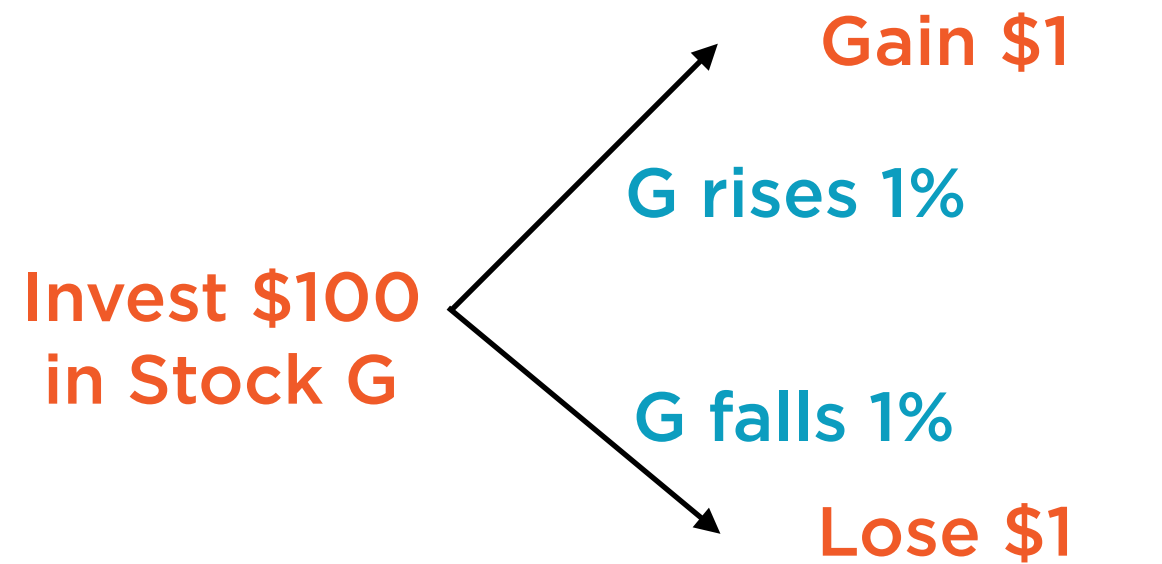


## Decision Variables

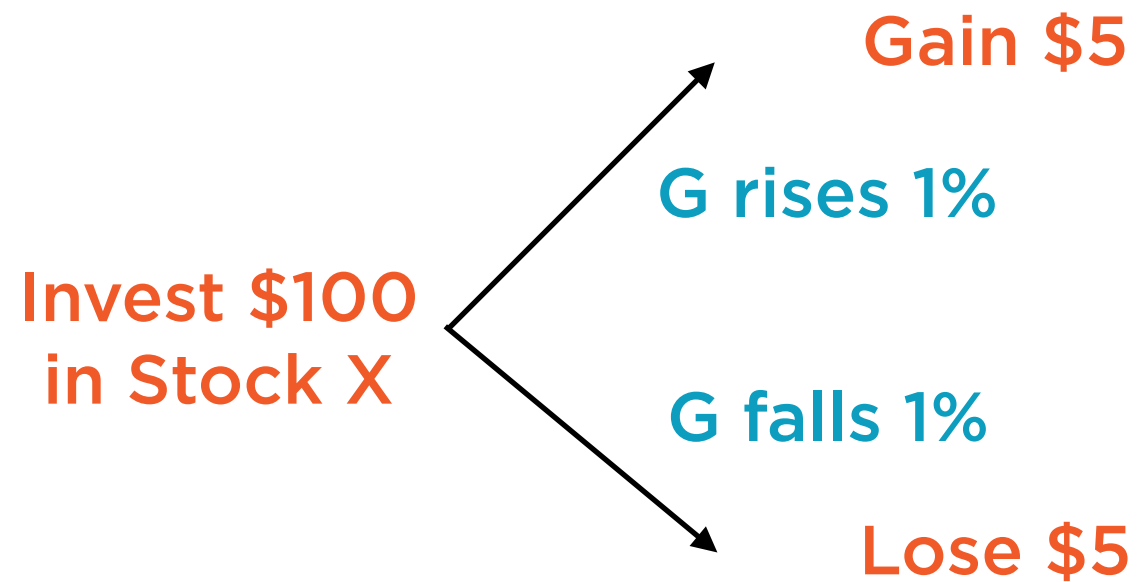
$W$

$$W = [w_1 \ w_2 \ w_3 \ \dots \ w_k]$$

# Financial Leverage



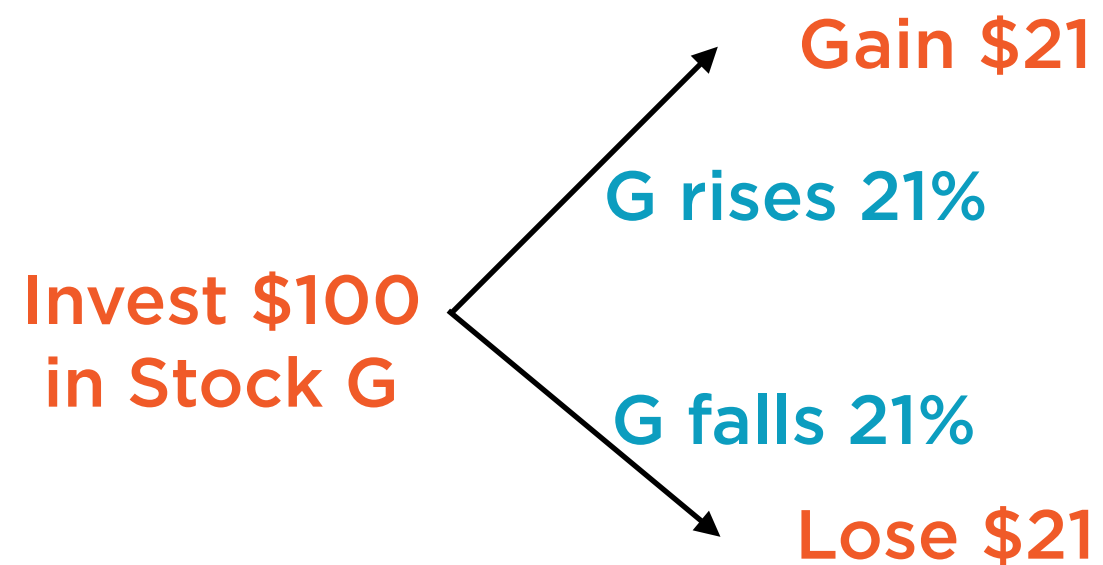
**No Leverage**  
Small losses, small gains



**5X Leverage**  
Big losses, big gains

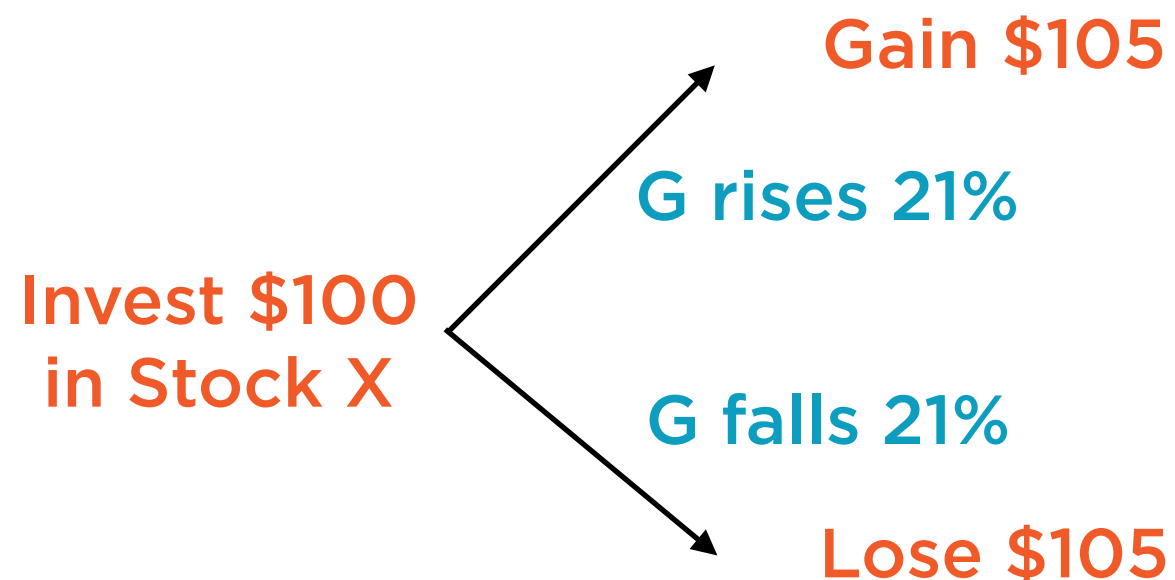
**Leverage amplifies both gains and losses**

# “Blow-up” Risk with Leverage



**No Leverage**

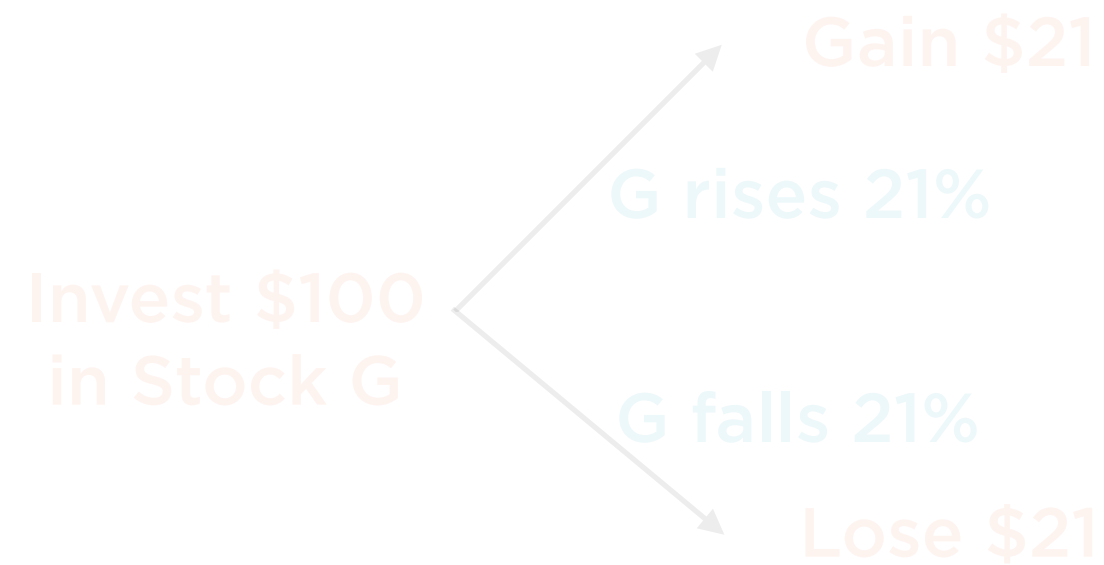
Losses never exceed invested  
amount



**5X Leverage**

Losses can wipe out capital and  
lead to bankruptcy

# “Blow-up” Risk with Leverage



## No Leverage

Losses never exceed invested amount



## 5X Leverage

Losses can wipe out capital and lead to bankruptcy

**Excessive leverage can easily lead to bankruptcy**



# Bankruptcy

A legal status of a person or other entity that cannot repay the debts it owes to creditors.

— *Wikipedia*

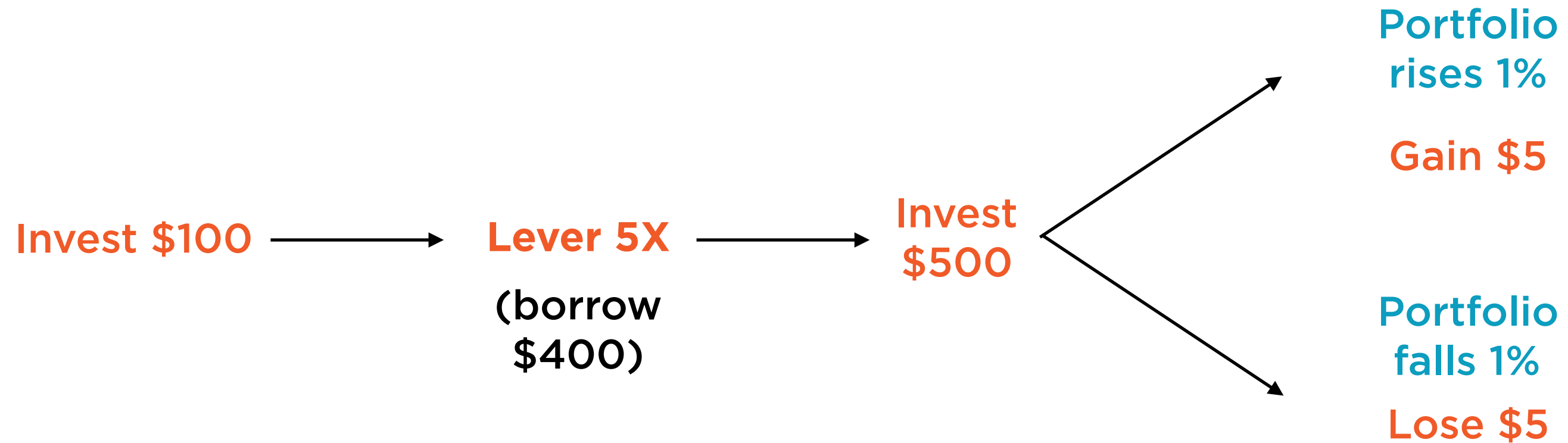
# Using Leverage

**Borrow to invest**

**Use derivatives**

**The mechanics of using leverage are complicated, but  
the basic idea is fairly simple**

# Financial Leverage



**Leverage amplifies both gains and losses**

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**Constraints**

**Impose an integer constraint on  $W$**

$$w_1, w_2 \dots w_k \in \{-1, 0, 1\}$$



**Constraints**

**Model 5X leverage using a constraint**

$$w_1 + w_2 + w_3 + \dots + w_k = 500\% = 5$$

# Unusual Integer Programming Formulations

**K-of-N Constraints**

**Specific Allowable  
Values**

**Start-up Costs**

**K-of-N constraints are a general case of either-or constraints, which we studied**

# Unusual Integer Programming Formulations

**K-of-N Constraints**

Specific Allowable  
Values

**Start-up Costs**



# Unusual Integer Programming Formulations

Either-or  
Constraints

Specific Allowable  
Values

Start-up Costs



**Constraints**

**Impose a long-only constraint on  $W$**

$$w_1, w_2 \dots w_k \in \{0,1\}$$

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