Implementing Integer Programming in Python

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Overview

Model the use of leverage in a financial portfolio

Specify a minimum long-bias that must be satisfied by the portfolio

Formulate integer programming problems to optimize these portfolios

Use Python to solve these optimization problems

Building Is Hard, Using Is Easy

Builder

Building an integer programming solver is **hard**

User

Using an integer programming solver is **easy**

Demo

Apply integer programming to optimization using Python

Recap - Linear Programming

Assemble financial data

Use data from Yahoo finance

Prices of correlated stocks

Convert prices into returns

Download prices data and convert into returns

Simple step, use Pandas

Estimate risk, return

Use historical data

Risk = max % 1-period drop

Linear Programming

Minimize max loss risk

Threshold on expected return

Quadratic Programming

Minimize portfolio variance

Risk = variance

Long-only Constraint

Minimize portfolio variance

Forced to accept lower return

Moving On - Integer Programming

Model use of leverage

Investment can exceed invested amount

Leverage magnifies risk and return

Impose long-bias

Net long position = 5X

Express as K-of-N constraint

Integer Programming

Minimize portfolio risk

Risk = max loss of portfolio

Buy-sell-hold decision variables

Model each decision as +1, 0 or -1

Integer constraint with 3 acceptable values

Dis-allow short positions

Only allow buy or hold

Decision variables can now be only 0 or 1, not -1

Estimate Portfolio Return and Risk

$$P = w_1Y_1 + w_2Y_2 + w_3Y_3 \ldots + w_kY_k$$

Expected Return

Simple - use average of historical returns

Forecast Risk

Conservative - define as sum of max loss in each stock

Max Loss refers to largest % fall experienced by a stock in any period in our data

Estimate Portfolio Return and Risk

$$P = w_1Y_1 + w_2Y_2 + w_3Y_3 \dots + w_kY_k$$

Expected Return = Mean(y)

Simple - mean of sum is sum of means

Forecast Risk = MaxLoss(y)

Conservative - define as sum of max loss in each stock

Max Loss refers to largest % fall experienced by a stock in any period in our data

Estimating Return

$$P = w_1Y_1 + w_2Y_2 + w_3Y_3 \dots + w_kY_k$$

$$\begin{aligned} \text{Mean}(P) = \; & w_1 \times \text{Mean}(Y_1) + \\ & w_2 \times \text{Mean}(Y_2) + \\ & w_3 \times \text{Mean}(Y_3) + \\ & \dots \\ & w_k \times \text{Mean}(Y_k) \end{aligned}$$

k terms, all linear

Mean of sum = sum of means

Estimating Return

$$P = w_1Y_1 + w_2Y_2 + w_3Y_3 \ldots + w_kY_k$$

$$\begin{aligned} \text{Mean}(P) = \; & w_1\bar{Y}_1 + \\ & w_2\bar{Y}_2 + \\ & w_3\bar{Y}_3 + \\ & \ldots \\ & w_k\bar{Y}_k \end{aligned}$$

k terms, all linear

Mean of sum = sum of means

Estimate Portfolio Return and Risk

$$P = w_1Y_1 + w_2Y_2 + w_3Y_3 \ldots + w_kY_k$$

Expected Return = Mean(y)

Simple - mean of sum is sum of means

Forecast Risk = MaxLoss(y)

Conservative - define as sum of max loss in each stock

Estimating Risk

$$P = w_1Y_1 + w_2Y_2 + w_3Y_3 \ldots + w_kY_k$$

$$\left.\begin{aligned} Risk(P) = \; & w_1 \times MaxLoss(Y_1) + \\ & w_2 \times MaxLoss(Y_2) + \\ & w_3 \times MaxLoss(Y_3) + \\ & \ldots \\ & w_k \times MaxLoss(Y_k) \end{aligned}\right\updownarrow \text{k terms, all linear}$$

Portfolio Risk = Sum of individual asset risks

Portfolio Allocation as an Optimization Problem

Objective Function

Minimize Risk(P)

Risk(P) = MaxLoss(P)

Constraints

$\bar{P} >= R_{threshold}$

$\bar{P} = w_1\bar{Y}_1 + w_2\bar{Y}_2 + \dots \; w_k\bar{Y}_k$

Decision Variables

W

$W = [\, w_1 \quad w_2 \quad w_3 \quad \dots \quad w_k \,]$

Financial Leverage

Invest $100
in Stock G

Gain $1

G rises 1%

G falls 1%

Lose $1

No Leverage

Small losses, small gains

Invest $100
in Stock X

Gain $5

G rises 1%

G falls 1%

Lose $5

5X Leverage

Big losses, big gains

Leverage amplifies both gains and losses

"Blow-up" Risk with Leverage

Invest $100 in Stock G

G rises 21% → Gain $21

G falls 21% → Lose $21

No Leverage

Losses never exceed invested amount

Invest $100 in Stock X

G rises 21% → Gain $105

G falls 21% → Lose $105

5X Leverage

Losses can wipe out capital and lead to bankruptcy

"Blow-up" Risk with Leverage

Gain $21

G rises 21%

Invest $100
in Stock G

G falls 21%

Lose $21

Gain $105

G rises 21%

Invest $100
in Stock X

G falls 21%

Lose $105

No Leverage

Losses never exceed invested amount

5X Leverage

Losses can wipe out capital and lead to bankruptcy

Excessive leverage can easily lead to bankruptcy

Bankruptcy

A legal status of a person or other entity that cannot repay the debts it owes to creditors.

— *Wikipedia*

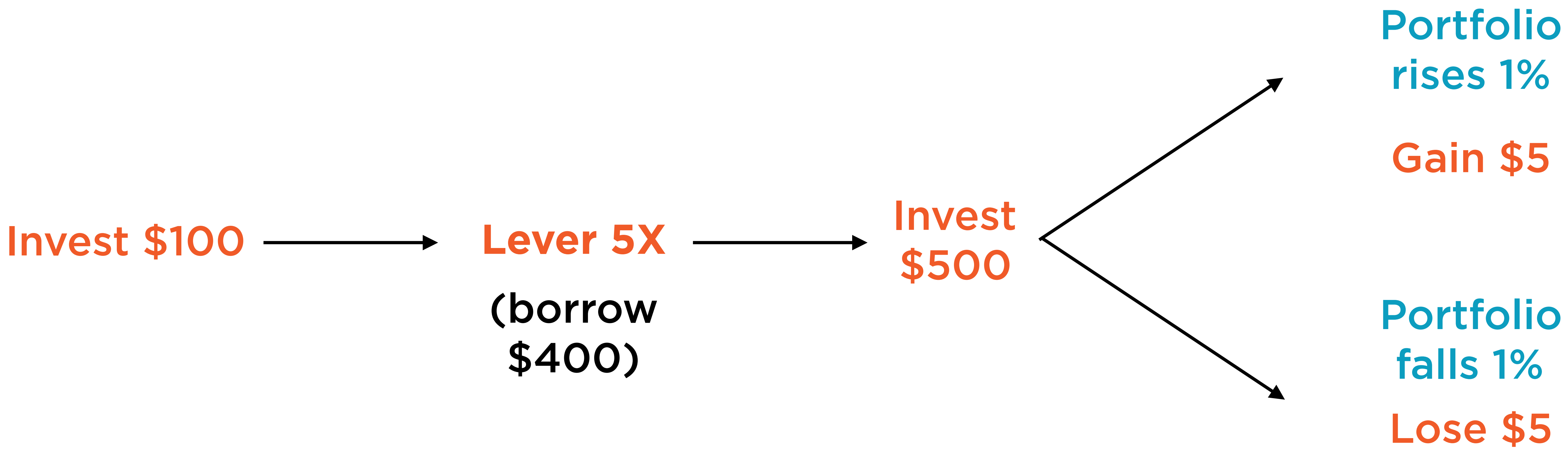
Using Leverage

Borrow to invest

Use derivatives

The mechanics of using leverage are complicated, but the basic idea is fairly simple

Financial Leverage



Leverage amplifies both gains and losses

Moving On - Integer Programming

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Constraints

Impose an integer constraint on W

$$w_1, w_2 \ldots w_k \in \{-1,0,1\}$$

Constraints

Model 5X leverage using a constraint

$$w_1 + w_2 + w_3 + \ldots + w_k = 500\% = 5$$

Unusual Integer Programming Formulations

K-of-N Constraints

Specific Allowable Values

Start-up Costs

K-of-N constraints are a general case of either-or constraints, which we studied

Unusual Integer Programming Formulations

K-of-N Constraints

Specific Allowable Values

Start-up Costs

Unusual Integer Programming Formulations

Either-or Constraints

Specific Allowable Values

Start-up Costs

Constraints

Impose a long-only constraint on W

$$w_1, w_2 \dots w_k \in \{0,1\}$$

Summary

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