

Implementing Integer Programming in R



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Overview

Model the use of leverage in a financial portfolio

Specify a minimum long-bias that must be satisfied by the portfolio

Formulate integer programming problems to optimize these portfolios

Use R to solve these optimization problems

Building Is Hard, Using Is Easy



Builder

Building an integer programming
solver is **hard**



User

Using an integer programming
solver is **easy**

Demo

**Apply integer programming to
optimization using R**

Recap - Linear Programming

Assemble financial data

Use data from Yahoo finance

Prices of correlated stocks

Estimate risk, return

Use historical data

Risk = max % 1-period drop

Quadratic Programming

Minimize portfolio variance

Risk = variance

Convert prices into returns

Download prices data and convert into returns

Simple step, use R data frames

Linear Programming

Minimize max loss risk

Threshold on expected return

Long-only Constraint

Minimize portfolio variance

Forced to accept lower return

Moving On - Integer Programming

Model use of leverage

Investment can exceed invested amount
Leverage magnifies risk and return

Impose long-bias

Net long position = 5X
Express as K-of-N constraint

Integer Programming

Minimize portfolio risk
Risk = max loss of portfolio

Buy-sell-hold decision variables

Model each decision as +1, 0 or -1
Integer constraint with 3 acceptable values

Dis-allow short positions

Only allow buy or hold
Decision variables can now be only 0 or 1, not -1

Estimate Portfolio Return and Risk

$$P = w_1Y_1 + w_2Y_2 + w_3Y_3 \dots + w_kY_k$$

Expected Return

Simple - use average of historical returns

Forecast Risk

Conservative - define as sum of max loss in each stock

Max Loss refers to largest % fall experienced by a stock in any period in our data

Estimate Portfolio Return and Risk

$$P = w_1Y_1 + w_2Y_2 + w_3Y_3 \dots + w_kY_k$$

**Expected Return =
Mean(y)**

Simple - mean of sum is sum of means

**Forecast Risk =
MaxLoss(y)**

Conservative - define as sum of max loss in each stock

Max Loss refers to largest % fall experienced by a stock in any period in our data

Estimating Return

$$P = w_1Y_1 + w_2Y_2 + w_3Y_3 \dots + w_kY_k$$

$$\begin{aligned} \text{Mean}(P) = & w_1 \times \text{Mean}(Y_1) + \\ & w_2 \times \text{Mean}(Y_2) + \\ & w_3 \times \text{Mean}(Y_3) + \\ & \dots \\ & w_k \times \text{Mean}(Y_k) \end{aligned}$$

k terms, all linear

Mean of sum = sum of means

Estimating Return

$$P = w_1Y_1 + w_2Y_2 + w_3Y_3 \dots + w_kY_k$$

$$\begin{aligned} \text{Mean}(P) = & w_1\bar{Y}_1 + \\ & w_2\bar{Y}_2 + \\ & w_3\bar{Y}_3 + \\ & \dots \\ & w_k\bar{Y}_k \end{aligned}$$

k terms, all linear

Mean of sum = sum of means

Estimate Portfolio Return and Risk

$$P = w_1Y_1 + w_2Y_2 + w_3Y_3 \dots + w_kY_k$$

**Expected Return =
Mean(y)**

Simple - mean of sum is sum of means

**Forecast Risk =
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Conservative - define as sum of max loss in each stock

Estimating Risk

$$P = w_1 Y_1 + w_2 Y_2 + w_3 Y_3 \dots + w_k Y_k$$

$$\begin{aligned} \text{Risk}(P) = & w_1 \times \text{MaxLoss}(Y_1) + \\ & w_2 \times \text{MaxLoss}(Y_2) + \\ & w_3 \times \text{MaxLoss}(Y_3) + \\ & \dots \\ & w_k \times \text{MaxLoss}(Y_k) \end{aligned}$$

k terms, all linear

Portfolio Risk = Sum of individual asset risks

Portfolio Allocation as an Optimization Problem



Objective Function

Minimize Risk(P)

$$\text{Risk}(P) = \text{MaxLoss}(P)$$



Constraints

$$\bar{P} \geq R_{\text{threshold}}$$

$$\bar{P} = w_1 \bar{Y}_1 + w_2 \bar{Y}_2 + \dots + w_k \bar{Y}_k$$

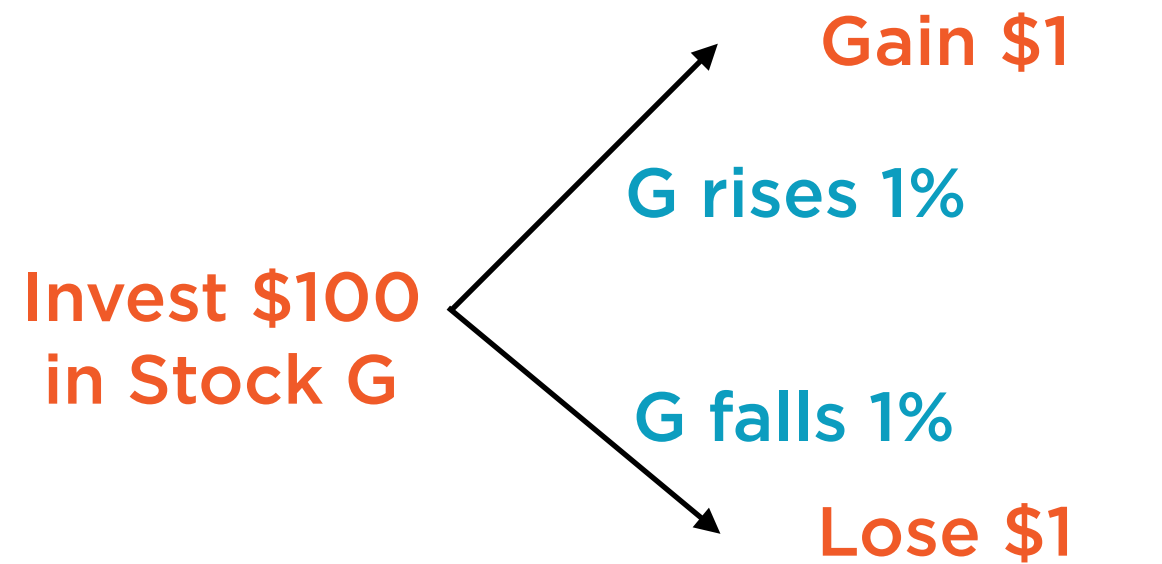


Decision Variables

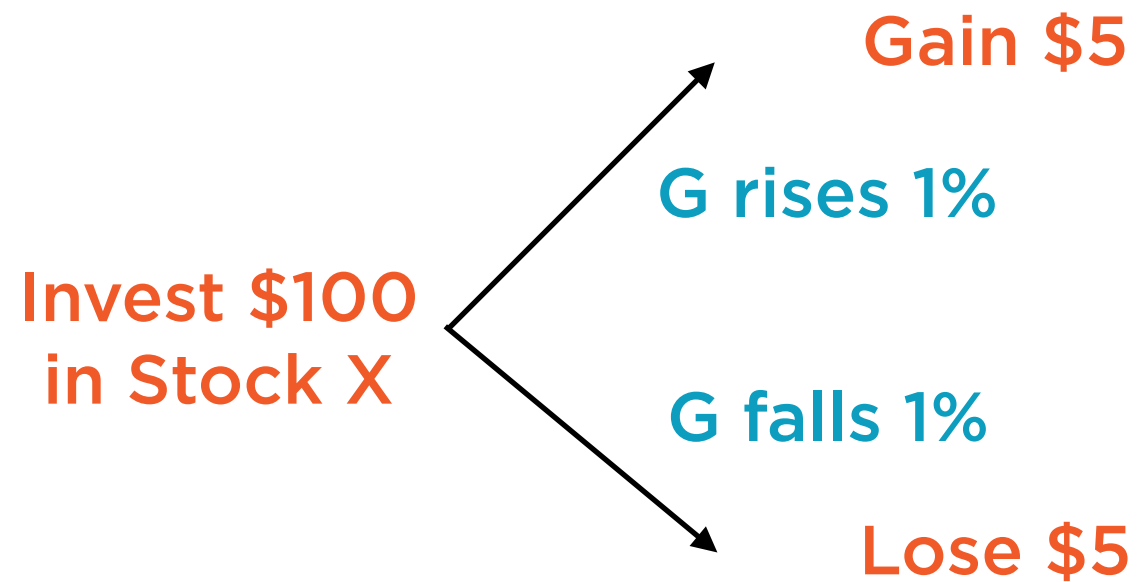
W

$$W = [w_1 \ w_2 \ w_3 \ \dots \ w_k]$$

Financial Leverage



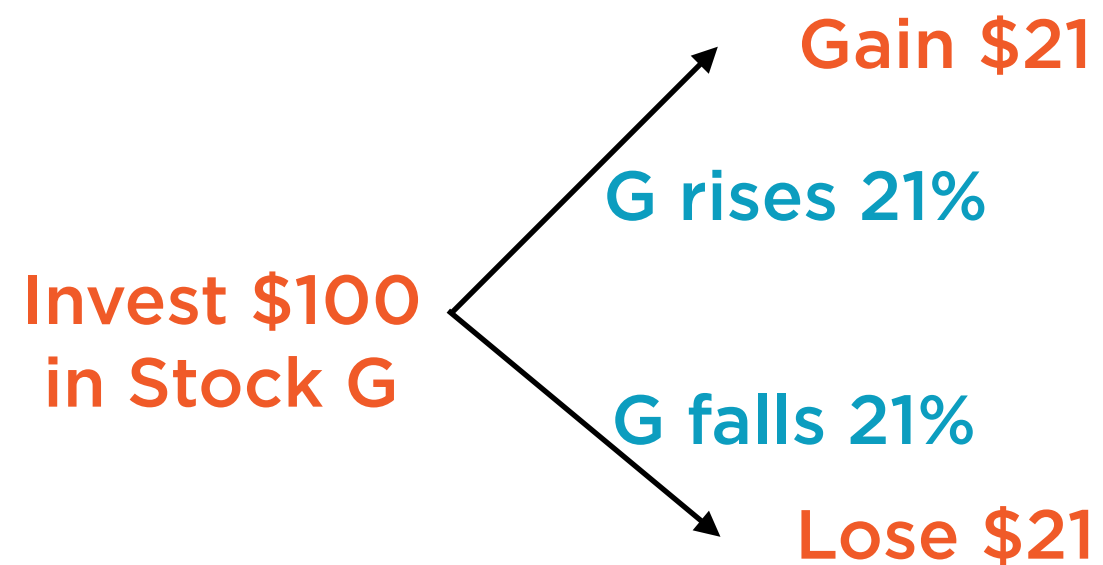
No Leverage
Small losses, small gains



5X Leverage
Big losses, big gains

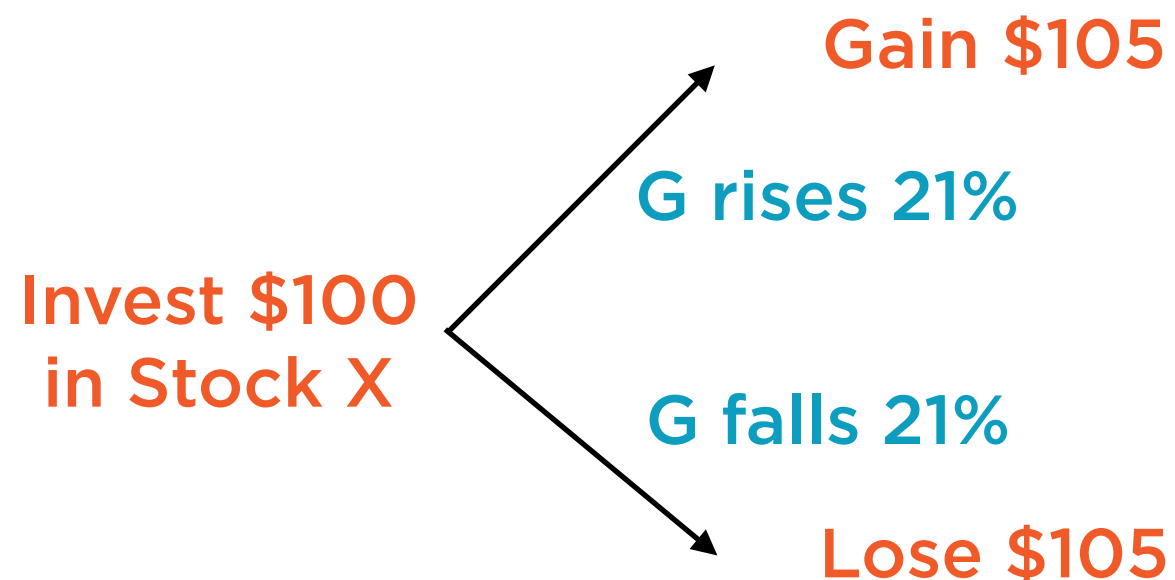
Leverage amplifies both gains and losses

“Blow-up” Risk with Leverage



No Leverage

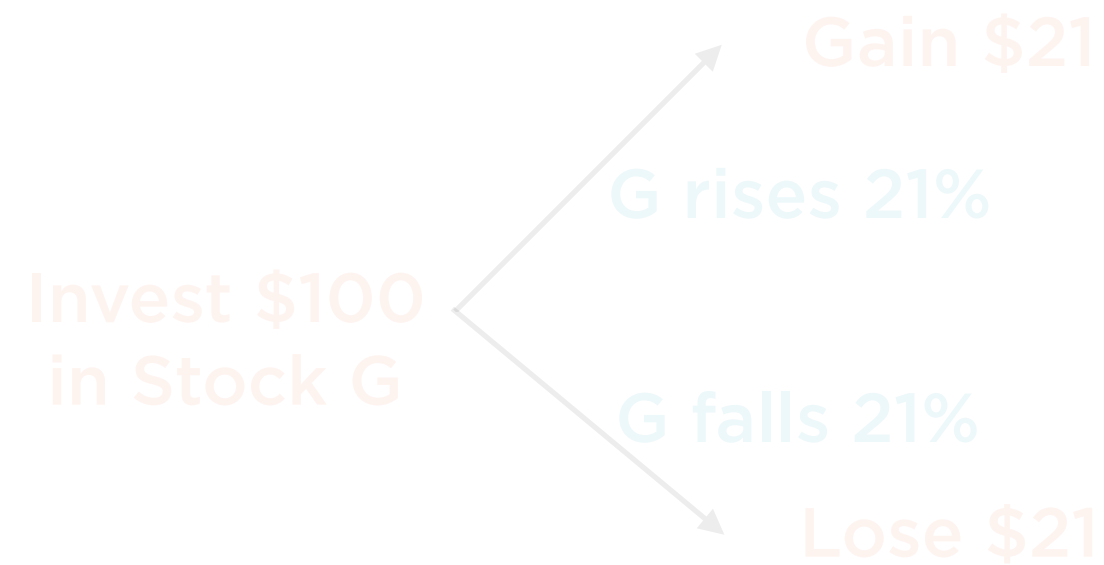
Losses never exceed invested
amount



5X Leverage

Losses can wipe out capital and
lead to bankruptcy

“Blow-up” Risk with Leverage



No Leverage

Losses never exceed invested amount



5X Leverage

Losses can wipe out capital and lead to bankruptcy

Excessive leverage can easily lead to bankruptcy

Bankruptcy

A legal status of a person or other entity that cannot repay the debts it owes to creditors.

— *Wikipedia*

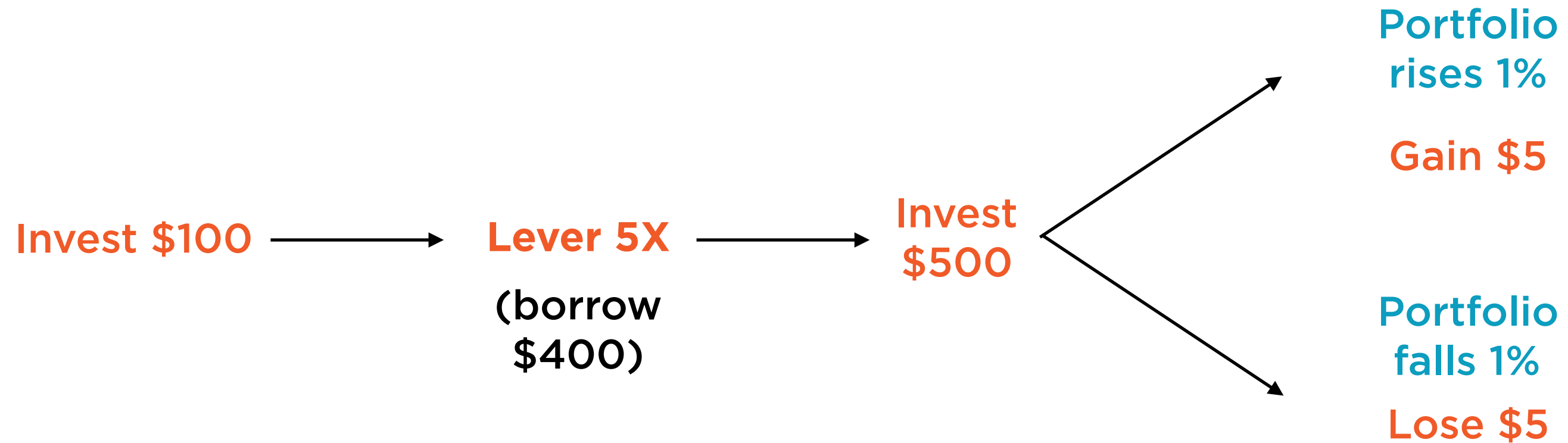
Using Leverage

Borrow to invest

Use derivatives

**The mechanics of using leverage are complicated, but
the basic idea is fairly simple**

Financial Leverage



Leverage amplifies both gains and losses

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Constraints

Impose an integer constraint on W

$$w_1, w_2 \dots w_k \in \{-1, 0, 1\}$$



Constraints

Model 5X leverage using a constraint

$$w_1 + w_2 + w_3 + \dots + w_k = 500\% = 5$$

Unusual Integer Programming Formulations

K-of-N Constraints

**Specific Allowable
Values**

Start-up Costs

K-of-N constraints are a general case of either-or constraints, which we studied

Unusual Integer Programming Formulations

K-of-N Constraints

Specific Allowable
Values

Start-up Costs

Unusual Integer Programming Formulations

Either-or
Constraints

Specific Allowable
Values

Start-up Costs



Constraints

Impose a long-only constraint on W

$$w_1, w_2 \dots w_k \in \{0,1\}$$

Summary

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