Implementing Integer Programming in R

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Overview

Model the use of leverage in a financial portfolio

Specify a minimum long-bias that must be satisfied by the portfolio

Formulate integer programming problems to optimize these portfolios

Use R to solve these optimization problems

Building Is Hard, Using Is Easy

Builder

Building an integer programming solver is **hard**

User

Using an integer programming solver is **easy**

Demo

Apply integer programming to optimization using R

Recap - Linear Programming

Assemble financial data

Use data from Yahoo finance

Prices of correlated stocks

Convert prices into returns

Download prices data and convert into returns

Simple step, use R data frames

Estimate risk, return

Use historical data

Risk = max % 1-period drop

Linear Programming

Minimize max loss risk

Threshold on expected return

Quadratic Programming

Minimize portfolio variance

Risk = variance

Long-only Constraint

Minimize portfolio variance

Forced to accept lower return

Moving On - Integer Programming

Model use of leverage

Investment can exceed invested amount

Leverage magnifies risk and return

Impose long-bias

Net long position = 5X

Express as K-of-N constraint

Integer Programming

Minimize portfolio risk

Risk = max loss of portfolio

Buy-sell-hold decision variables

Model each decision as +1, 0 or -1

Integer constraint with 3 acceptable values

Dis-allow short positions

Only allow buy or hold

Decision variables can now be only 0 or 1, not -1

Estimate Portfolio Return and Risk

$$P = w_1Y_1 + w_2Y_2 + w_3Y_3 \dots + w_kY_k$$

Expected Return

Simple - use average of historical returns

Forecast Risk

Conservative - define as sum of max loss in each stock

Max Loss refers to largest % fall experienced by a stock in any period in our data

Estimate Portfolio Return and Risk

$$P = w_1Y_1 + w_2Y_2 + w_3Y_3 \ldots + w_kY_k$$

Expected Return = Mean(y)

Simple - mean of sum is sum of means

Forecast Risk = MaxLoss(y)

Conservative - define as sum of max loss in each stock

Max Loss refers to largest % fall experienced by a stock in any period in our data

Estimating Return

$$P = w_1Y_1 + w_2Y_2 + w_3Y_3 \ldots + w_kY_k$$

$$\begin{aligned} \text{Mean}(P) = \; & w_1 \times \text{Mean}(Y_1) + \\ & w_2 \times \text{Mean}(Y_2) + \\ & w_3 \times \text{Mean}(Y_3) + \\ & \ldots \\ & w_k \times \text{Mean}(Y_k) \end{aligned}$$

k terms, all linear

Mean of sum = sum of means

Estimating Return

$$P = w_1Y_1 + w_2Y_2 + w_3Y_3 \ldots + w_kY_k$$

$$\begin{aligned} \text{Mean}(P) = \; & w_1\bar{Y}_1 + \\ & w_2\bar{Y}_2 + \\ & w_3\bar{Y}_3 + \\ & \ldots \\ & w_k\bar{Y}_k \end{aligned}$$

k terms, all linear

Mean of sum = sum of means

Estimate Portfolio Return and Risk

$$P = w_1Y_1 + w_2Y_2 + w_3Y_3 \ldots + w_kY_k$$

Expected Return = Mean(y)

Simple - mean of sum is sum of means

Forecast Risk = MaxLoss(y)

Conservative - define as sum of max loss in each stock

Estimating Risk

$$P = w_1Y_1 + w_2Y_2 + w_3Y_3 \ldots + w_kY_k$$

$$\left.\begin{aligned} Risk(P) = \; & w_1 \text{ x } MaxLoss(Y_1) + \\ & w_2 \text{ x } MaxLoss(Y_2) + \\ & w_3 \text{ x } MaxLoss(Y_3) + \\ & \ldots \\ & w_k \text{ x } MaxLoss(Y_k) \end{aligned}\right\} \text{k terms, all linear}$$

Portfolio Risk = Sum of individual asset risks

Portfolio Allocation as an Optimization Problem

Objective Function

Minimize Risk(P)

Risk(P) = MaxLoss(P)

Constraints

$\bar{P} >= R_{threshold}$

$\bar{P} = w_1\bar{Y}_1 + w_2\bar{Y}_2 + \ldots \; w_k\bar{Y}_k$

Decision Variables

W

$W = [\, w_1 \quad w_2 \quad w_3 \quad \ldots \quad w_k \,]$

Financial Leverage

Gain $1

G rises 1%

Invest $100
in Stock G

G falls 1%

Lose $1

Gain $5

G rises 1%

Invest $100
in Stock X

G falls 1%

Lose $5

No Leverage

Small losses, small gains

5X Leverage

Big losses, big gains

Leverage amplifies both gains and losses

“Blow-up” Risk with Leverage

Invest $100 in Stock G

- G rises 21% → Gain $21
- G falls 21% → Lose $21

No Leverage

Losses never exceed invested amount

Invest $100 in Stock X

- G rises 21% → Gain $105
- G falls 21% → Lose $105

5X Leverage

Losses can wipe out capital and lead to bankruptcy

"Blow-up" Risk with Leverage

Invest $100
in Stock G

G rises 21%
Gain $21

G falls 21%
Lose $21

Invest $100
in Stock X

G rises 21%
Gain $105

G falls 21%
Lose $105

No Leverage

Losses never exceed invested amount

5X Leverage

Losses can wipe out capital and lead to bankruptcy

Excessive leverage can easily lead to bankruptcy

Bankruptcy

A legal status of a person or other entity that cannot repay the debts it owes to creditors.

— *Wikipedia*

Using Leverage

Borrow to invest

Use derivatives

The mechanics of using leverage are complicated, but the basic idea is fairly simple

Financial Leverage

Invest $100 → Lever 5X (borrow $400) → Invest $500

- Portfolio rises 1% — Gain $5
- Portfolio falls 1% — Lose $5

Leverage amplifies both gains and losses

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Impose long-bias

Net long position = 5X

Express as K-of-N constraint

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Integer Programming

Minimize portfolio risk

Risk = max loss of portfolio

Constraints

Impose an integer constraint on W

$$w_1, w_2 \ldots w_k \in \{-1,0,1\}$$

Constraints

Model 5X leverage using a constraint

$$w_1 + w_2 + w_3 + \ldots + w_k = 500\% = 5$$

Unusual Integer Programming Formulations

K-of-N Constraints

Specific Allowable Values

Start-up Costs

K-of-N constraints are a general case of either-or constraints, which we studied

Unusual Integer Programming Formulations

K-of-N Constraints

Specific Allowable Values

Start-up Costs

Unusual Integer Programming Formulations

Either-or Constraints

Specific Allowable Values

Start-up Costs

Constraints

Impose a long-only constraint on W

$w_1, w_2 \ldots w_k \in \{0,1\}$

Summary

Model the use of leverage in a financial portfolio

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