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CSCI 3104, Algorithms **Profs. Chen & Grochow**

Problem Set 10 – Due Wed April 22 11:55pm **Spring 2020, CU-Boulder**

Advice 1: For every problem in this class, you must justify your answer: show how you arrived at it and why it is correct. If there are assumptions you need to make along the way, state those clearly.

Advice 2: Informal reasoning is typically insufficient for full credit. Instead, write a logical argument, in the style of a mathematical proof.

Instructions for submitting your solutions:

- All submissions must be typed.
- You should submit your work through the **class Canvas page** only.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please allot at least as many pages per problem (or subproblem) as are allotted in this template.

Quicklinks: 1a 1b 1c 2

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1. Consider the following DP table for the Knapsack problem for the list

$$A = [(4,3),(1,2),(3,1),(5,4),(6,3)]$$

of (weight, value) pairs. The weight threshold $W = 10$.

- Fill in the values of the table.
- Draw the backward path consisting of backward edges and do not draw (or erase them) the edges that are not part of the optimal backward paths.

(a) Fill the table with the above requirements (You can also re-create this table in excel/sheet).

Weight	Value	i \ w	0	1	2	3	4	5	6	7	8	9	10
4	3	A[0..0]	0	0	0	0	3	3	3	3	3	3	3
1	2	A[0..1]	0	2	2	2	3	5	5	5	5	5	5
3	1	A[0..2]	0	2	2	2	3	5	5	5	6	6	6
5	4	A[0..3]	0	2	2	2	3	5	6	6	6	7	9
6	3	A[0..4]	0	2	2	2	3	5	6	6	6	7	9

Note that the dark blue arrows indicate the optimal backwards path and the light blue cell represents the cell with the optimal value.

(b) Which cell has the optimal value and what is the optimal value for the given problem?

The cell with the optimal value is the bottom right cell indicated with a light blue cell background on the table. The optimal value for the given problem is 9. While there are two cells with a value of 9 (the one on the bottom right corner and the one above it), the cell on the bottom right corner would be the best answer for most optimal because ir guarantees that we have considered all of possible items in A.

(c) List out the optimal subset and provide it's weight and value.

Optimal Subset is the set $\{(4,3),(1,2),(5,4)\}$
Overall Weight of Optimal Subset: 10
Overall Value of Optimal Subset: 9

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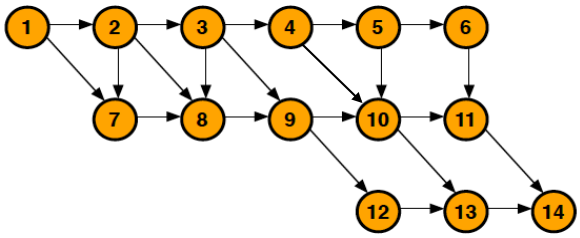
2. Given the following directed acyclic graph. Use dynamic programming to fill in a table that counts number of paths from each node j to 14, for $j \geq 1$. Note that a single vertex is considered a path of length 0.



Node j	$X_{j,14}$
14	0
13	1
12	1
11	1
10	2
9	3
8	3
7	3
6	1
5	3
4	5
3	11
2	17
1	20

Let $X_{j,14}$ denote the number of paths between nodes j and 14.
Let s_j be a subset of nodes that are (1) adjacent to j (ie. each $s \in s_j$ share an edge with j) and (2) the edge between each $s \in s_j$ and j must be directed $j \to s$. The table was filled using the following equation:

$$X_{j,14} = \begin{cases} 0 & \textit{if } j = 14 \\ 1 & \textit{if } j = 11 \textit{ or } j = 13 \\ \sum X_{a,14}, a \in s_j & \textit{else} \end{cases}$$