Key Logical Equivalences

Identity:
$$p \wedge T \equiv p$$
$$p \vee F \equiv p$$

Absorption:
$$p \vee (p \wedge q) \equiv p$$
$$p \wedge (p \vee q) \equiv p$$

Domination:
$$p \vee T \equiv T$$
$$p \wedge F \equiv F$$

Negation:
$$p \vee \neg p \equiv T$$
$$p \wedge \neg p \equiv F$$

Idempotent:
$$p \vee p \equiv p$$
$$p \wedge p \equiv p$$

Double negation:
$$p \equiv \neg\neg p$$

More Key Logical Equivalences

Commutativity:
$$p \wedge q \equiv q \wedge p$$
$$p \vee q \equiv q \vee p$$

Distributivity:
$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$
$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

Associativity:
$$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$$
$$p \vee (q \vee r) \equiv (p \vee q) \vee r$$

De Morgan's laws:
$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$
$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

Implication:
$$p \rightarrow q \equiv \neg p \vee q$$

Key Logical Implications ("Valid Argument Forms")

Direct Implication

p
$p \rightarrow q$ ("Modus Ponens")
$\therefore q$

Transitivity of $\rightarrow$

$p \rightarrow q$
$q \rightarrow r$ ("Hypothetical Syllogism")
$\therefore p \rightarrow r$

Contrapositive Implication

$p \rightarrow q$
$\neg q$ ("Modus Tollens")
$\therefore \neg p$

Two Separate Cases

$p \vee q$
$\neg p \vee r$ ("Resolution")
$\therefore q \vee r$

Some More Logical Implications

Eliminating a Possibility

$p \vee q$
$\neg p$ ("Disjunctive Syllogism")
$\therefore q$

In Particular

$p \wedge q$ ("Simplification")
$\therefore p$

Obtaining And

p
q ("Conjunction")
$\therefore p \wedge q$

Obtaining Or

p
$\therefore p \vee q$ ("Addition")

Substitution of Equivalent

$p \leftrightarrow q$
p ("Equivalence")
$\therefore q$