

SUPPORTING INFORMATION

SI Appendix for:

Population-based 3D genome structure analysis reveals driving forces in spatial genome organization

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A. SI MATERIALS AND METHODS

A.1. Population-based 3D genome modeling approach

Chromosomes are segmented into chromatin domains, which are represented by spherical volumes following our previously published approach (1). The population-based structural modeling approach is a probabilistic framework to generate a large number of 3D genome structures (i.e. the structure population) whose chromatin domain contacts are statistically consistent with the input experimental TCC data. Our structure population represents a deconvolution of the ensemble-averaged TCC data into a population of individual structures and represents the most likely approximation of the true structure population given all the available data. We formulated the structure optimization problem as a maximum likelihood estimation problem and designed an iterative optimization algorithm with a series of optimization strategies for efficient and scalable model estimation.

A.1.1. Defining sphere volumes

We define two types of sphere radius for each domain, the hard- and soft-core radii. The excluded volume (or hard core) radius of domain I , R_I^x , is proportional to the cubic root of the DNA sequence length it contains, l_I , and can be approximated as

$$R_I^x = \rho l_I^{1/3}$$

where

$$\rho^3 = \frac{O_{\text{nuc}} R_{\text{nuc}}^3}{2 \sum_{I=1}^N l_I}$$

ρ is a coefficient that is adjusted to reproduce the nuclear volume occupancy O_{nuc} , which is the fraction of the nuclear volume with radius R_{nuc} occupied by the genome. Published data rank the level of nuclear occupancy between 10-40% (2). Thus we have chosen to model the human genome using spheres that occupy a total volume about 20% of the nuclear volume. Small variations of the volume occupancy will not change the resulting conclusions. **Table S1** contains detail information of all domain spheres. As described below spheres defined by their excluded volume radius cannot penetrate each other due to an excluded volume constraint, which ensures the minimal occupancy of the chromatin in the nucleus. To allow for interactions of chromatin regions we also define a contact radius of a domain I , R_I^c , which is 2 times the hard-

core radius, $R_i^c = 2R_i^x$. This tolerance allows for the possibility that chromatin regions can partially loop out of their bulk domain regions to form contacts.

A.1.2. Model of sphere contacts in 3D structure

We assume a pair of spheres (i, j) have a contact in a structure m , if and only if their surface distance $d_{ijm} = \|\vec{x}_{im} - \vec{x}_{jm}\|_2 - R_i^c - R_j^c$ is equal to or smaller than zero, i.e., $d_{ijm} \leq 0$, where \vec{x}_{im} and R_i^c (\vec{x}_{jm} and R_j^c) are center coordinates in a structure m and contact radius of sphere i (j) respectively. The surface distance d_{ijm} could allow a “soft core” overlap between two spheres and the maximal overlap cannot violate the excluded volume constraint, i.e., $d_{ijm} \geq -R_i^x - R_j^x$, where R_i^x and R_j^x are the excluded volume radius of sphere i and j respectively.

We model a sphere contact by a function of the sphere-sphere surface distance, which is essentially a mixture of a constant function and one-sided truncated Gaussian function. This function implies two features: (i) the contact does concretely take place when the surface distance between two domain spheres d_{ijm} is equal to or smaller than 0, which is modeled as a constant function; (ii) the function sharply decreases with the sphere-sphere surface distance greater than 0, which is modeled as a one-sided truncated Gaussian function. Therefore, we have defined

$$P(w_{ijm} = 1 | d_{ijm}) = \begin{cases} 1, & d_{ijm} \leq 0 \\ \exp\left(\frac{-d_{ijm}^2}{2\sigma^2}\right), & d_{ijm} > 0 \end{cases} \quad [1]$$

where w_{ijm} is the latent variable (it will be introduced in the later section) indicating whether or not two spheres i and j have an assigned contact ($w_{ijm}=1$ represents sphere i and j have a contact in structure m , otherwise $w_{ijm}=0$). **Fig. S1A** shows the curve of the sphere contact function. For those sphere pairs that do not have an assigned contact (i.e. $w_{ijm}=0$), we have $P(w_{ijm} = 0 | d_{ijm}) = 1 - P(w_{ijm} = 1 | d_{ijm})$ (**Fig. S1B**). Please note that (i) d_{ijm} is the surface distance between two spheres and therefore is independent of sphere sizes; and (ii) the case

$d_{ijm} < -R_i^x - R_j^x$ is not allowed, because we require the “excluded volume” constraint to be satisfied in our model (shall be introduced in a later section).

Using above functions, we have $P(w_{ijm} | \vec{x}_{im}, \vec{x}_{jm})$ of two spheres i and j in a structure as below:

$$P(w_{ijm} | \vec{x}_{im}, \vec{x}_{jm}) = P(w_{ijm} = 1 | d_{ijm})^{w_{ijm}} P(w_{ijm} = 0 | d_{ijm})^{1-w_{ijm}} \quad [2]$$

A.1.3. Problem formulation

The chromosome conformation capture data is processed to be a contact probability matrix $\mathbf{A} = (a_{IJ})_{N \times N}$ of N domains in the genome, where $0 \leq a_{IJ} \leq 1$ is the contact probability of two chromosome domains I and J (will be described in section A.3.5). Note that in the human diploid genome, each domain has two homologous copies. Upper case letters (e.g., I or J) are used to denote a domain (as a chromosome region), and lower cases are used when we distinguish between the homolog copies of the domain in the diploid genome (i and i' for I , j and j' for J).

Our model, the structure population, is defined as a set of M diploid genome structures $\mathbf{X} = \{\mathbf{X}_1, \mathbf{X}_2, \dots, \mathbf{X}_M\}$, where the m -th structure \mathbf{X}_m is a set of 3-dimensional vectors representing the center coordinates of $2N$ spheres $\mathbf{X}_m = \{\vec{x}_{im} : \vec{x}_{im} \in \Re^3, i = 1, 2, \dots, 2N\}$.

For reconstructing a structure population \mathbf{X} , in principle we need the detailed information about which domain pairs are in contact in which structure of the population. The domain contact probability matrix (\mathbf{A}) derived from TCC data is incomplete and does not provide this information, because (i) the experimental data provides contact frequencies averaged over a population of cells without contact information at a single cell level, and (ii) the data does not distinguish between contacts from two homologous chromosome copies.

As aforementioned, we have introduced a latent variable, the “*contact indicator tensor*” $\mathbf{W} = (w_{ijm})_{2N \times 2N \times M}$, for complementing every single cell’s contact information (Fig. 1A in the main paper). It is a binary-valued 3rd-order tensor specifying the contacts of chromatin domains for each homologous copy in each structure of the population: i.e., $w_{ijm} = 1$ indicates that the contact between domains i and j in structure m ; $w_{ijm} = 0$ otherwise.

Given $\mathbf{A} = (a_{IJ})_{N \times N}$, we aim to estimate the structure population model \mathbf{X} such that the likelihood $P(\mathbf{A}, \mathbf{W} | \mathbf{X})$ is maximized. The dependence relationships between these variables in an optimized structure population is: $\mathbf{X} \rightarrow \mathbf{W} \rightarrow \mathbf{A}$, because \mathbf{W} is a detailed expansion of \mathbf{A} at the diploid representation and single cell level and \mathbf{X} is the structure population that is consistent to \mathbf{W} . Therefore, the likelihood $P(\mathbf{A}, \mathbf{W} | \mathbf{X})$ can be expanded to $P(\mathbf{A} | \mathbf{W})P(\mathbf{W} | \mathbf{X})$.

According to the model of sphere contacts described in the previous section, $P(\mathbf{W} | \mathbf{X})$ can be expanded as $P(\mathbf{W} | \mathbf{X}) = \prod_{m=1}^M \prod_{\substack{i,j=1 \\ i \neq j}}^{2N} P(w_{ijm} | \vec{x}_{im}, \vec{x}_{jm})$. Also, $P(\mathbf{A} | \mathbf{W})$ can be expanded as $P(\mathbf{A} | \mathbf{W}) = \prod_{I,J} P(a_{IJ} | a'_{IJ})$, where a'_{IJ} is the contact probability of the domain pair I or J computed from \mathbf{W} . We then model each a_{IJ} as $a_{IJ} = a'_{IJ} + \varepsilon_{IJ}$, where ε_{IJ} are independent and identical normally distributed random variables with mean zero $\varepsilon_{IJ} \sim N(0, \sigma'^2)$ (ε_{IJ} is effectively set to 0). a'_{IJ} is calculated as

$$a'_{IJ} = \frac{1}{2M} \sum_{m=1}^M \bar{w}_{IJm} \quad [3]$$

where $\bar{\mathbf{W}} = (\bar{w}_{IJm})_{N \times N \times M}$ is the “projected contact tensor” (Fig. 1A in the main paper), which is derived from \mathbf{W} by projecting its representation (with $2N$ homologous domains) to its counterpart without homologous domain distinction (with N domains) for domain pair I and J and is defined as below. For instance, in the projected tensor $\bar{\mathbf{W}}$, each element $\bar{w}_{IJm} = 1$ indicates that any one of two homologues copies of two domains I and J have a contact in structure m , $\bar{w}_{IJm} = 2$ indicates that two out of 4 possible pairs made by homologues copies of two domains I and J have contacts in structure m .

With these probabilistic models, we can maximize the log-likelihood below,

$$\log P(\mathbf{A}, \mathbf{W} | \mathbf{X}) = \log P(\mathbf{A} | \mathbf{W}) + \log P(\mathbf{W} | \mathbf{X}) = \sum_{\substack{I,J=1 \\ I \neq J}}^N \log P(a_{IJ} | a'_{IJ}) + \sum_{m=1}^M \sum_{\substack{i,j=1 \\ i \neq j}}^{2N} \log P(w_{ijm} | \vec{x}_{im}, \vec{x}_{jm}) \quad [4]$$

This is essentially a maximum likelihood estimation problem.

Additional constraints. In addition to the TCC data, we also include a set of spatial constraints based on additional information about the genome organization. These data are included in

form of general spatial constraints acting on the $2N$ domain spheres: (i) a nuclear volume restraint that forces all spheres to lie inside the nuclear volume, i.e. $\|\vec{x}_{im}\|_2 < R_{\text{nuc}}$ (where R_{nuc} is the nuclear radius); (ii) excluded volume restraints that prevent the “hard core” overlap between any 2 spheres i and j , i.e., $d_{ijm} \geq -R_i^x - R_j^x$; (iii) information from 3D FISH experiment, which showed that the q-arm of chromosome 4 is tethered to the nuclear envelope (NE) (3). Accordingly we add a constraint to for the q-arm telomere domain ($\vec{x}_{4\text{tel}}$) of chromosome 4 to be located close to the nuclear envelope ($\|\vec{x}_{4\text{tel}}\|_2 > 0.75R_{\text{nuc}}$). Note that, without loosing generalization, we use the origin (0,0,0) as the nuclear center, thus $\|\vec{x}\|_2$ is equivalent to the distance from the nuclear center. In summary, the maximum likelihood problem is formally expressed as follows,

$$\begin{aligned} \hat{\mathbf{X}} = \arg \max_{\mathbf{X}} \max_{\mathbf{W}} \{ \log P(\mathbf{A}, \mathbf{W} | \mathbf{X}) \} \\ \text{subject to } \begin{cases} \text{spatial constraint I: nuclear volume} \\ \text{spatial constraint II: excluded volume} \\ \text{spatial constraint III: 4tel-NE proximity} \end{cases} \end{aligned} \quad [5]$$

Note that, in principal we could add more knowledge-based constraints into this formulation.

A.1.4. Optimization procedure

We designed an iterative optimization procedure to solve this maximum likelihood estimation problem. Since our problem does not have a closed-form solution, numerical routines and heuristic strategies are needed to efficiently approximate the solution. This is an efficient iterative solver to alternately optimize \mathbf{W} and \mathbf{X} while holding the other fixed. We refer to this iterative cycle as the *A/M (Assignment/Modeling)* steps (**Fig. 1**) and this procedure as the *A/M* algorithm, which are described as follows.

- Initialization step: an initial model estimate $\mathbf{X}^{(0)}$ is needed to start the iterative procedure. We can generate $\mathbf{X}^{(0)}$ using random domain positions, which satisfy three spatial constraints in Eq. [5] (**Fig. 1B**).
- Assignment step (A-step): Given the current estimated model $\mathbf{X}^{(k)}$, estimate the latent variable \mathbf{W} by maximizing the log-likelihood over all possible values of \mathbf{W} .

$$\mathbf{W}^{(k+1)} = \arg \max_{\mathbf{W}} \{ \log P(\mathbf{A}, \mathbf{W} | \mathbf{X}) \}, \quad \text{given } \mathbf{X} = \mathbf{X}^{(k)} \quad [6]$$

- Modeling step (*M*-step): Given the current estimated latent variable $\mathbf{W}^{(k+1)}$, find the model $\mathbf{X}^{(k+1)}$ that maximizes the log-likelihood of the data \mathbf{A} . A new structure population will be generated in which all assigned contacts in \mathbf{W} will be physically present in the structure population \mathbf{X} .

$$\mathbf{X}^{(k+1)} = \arg \max_{\mathbf{X}} \left\{ \log P(\mathbf{A}, \mathbf{W} | \mathbf{X}) \right\}, \text{ given } \mathbf{W} = \mathbf{W}^{(k+1)} \quad [7]$$

- Iterative *A/M* steps until convergence (detailed convergence criterion will be covered in section A.1.7).

We exploited the parallelism and algorithmic heuristics underlying the *A/M* steps, which can largely speed up the procedure and make the implementation scalable for the large-scale TCC data. In the next sections, we present the procedure in detail.

A.1.5. Step-wise optimization strategy

We developed a stepwise optimization strategy for the structure optimization process, based on the following considerations: (i) an initial model that already fits a portion of domain contacts in \mathbf{A} can guide a more efficient search of the optimum \mathbf{W} than a random structure; (ii) gradually fitting an increasing number of domain contacts (from the highest to the lowest contact probabilities \mathbf{A}) can effectively guide the search to the best solution. The idea of this strategy is to gradually allocate the contacts in \mathbf{A} by using the optimized structure populations \mathbf{X} from the previous steps to determine the contact tensor \mathbf{W} for the following steps. We start the first optimization step by using only the most frequent contacts \mathbf{A}^{θ_1} (using only $a_{IJ} \geq \theta_1$ and $\theta_1=1.0$) as input to obtain $\hat{\mathbf{X}}^{\theta_1}$, which reproduces \mathbf{A}^{θ_1} (i.e., the structure population contains all physical domain contacts according to the experimental contact probability). Then $\hat{\mathbf{X}}^{\theta_1}$ is used as the initial model of the next round of optimization for \mathbf{A}^{θ_2} which includes all domain contacts with lower contact probabilities (i.e., using only $a_{IJ} \geq \theta_2$ and $\theta_2 < \theta_1$). This in turn leads to the refined structured population $\hat{\mathbf{X}}^{\theta_2}$, which serves as input for the next step, and so on. In this way, the contacts in \mathbf{A} are gradually allocated to the optimized structure population \mathbf{X} and contact tensor \mathbf{W} . When θ is close to zero, \mathbf{X}^θ reproduces most elements of \mathbf{A} and represents the best approximation of the true structure population given the available data. Our experience indicates that when a population starts to accumulate restraints violations then we have achieved the lowest θ we can pick. In this work, with θ below 0.01 we observed that the population started to

have restraint violations, thus we decided the final θ was 0.01 where all the given restraints were still satisfied.

The detailed steps of the procedure are as follows (illustrated in **Fig. 1B** of the main paper).

1. Organize a list of contact probability thresholds in decreasing order of contact probability, $\Theta = \{\theta_1, \theta_2, \theta_3, \dots, \theta_{\text{final}}\}$, e.g., $\Theta = \{1, 0, 0.8, 0.6, \dots, 0.01\}$.
2. For each $\theta \in \Theta$ in decreasing order, we apply the following steps:
 - 1) From the TCC data \mathbf{A} , we generate a truncated data $\mathbf{A}^\theta = (a_{IJ}^\theta)_{N \times N}$, where

$$a_{IJ}^\theta = \begin{cases} a_{IJ} & \text{if } a_{IJ} \geq \theta \\ 0 & \text{otherwise} \end{cases}$$
 - 2) Using \mathbf{A}^θ as input to perform the iterative A/M optimization algorithm and generate the solution $\hat{\mathbf{X}}^\theta = \arg \max_{\mathbf{X}} \max_{\mathbf{W}} \{ \log P(\mathbf{A}^\theta, \mathbf{W} | \mathbf{X}) \}$.
 - 3) Only if the optimization in step 2 succeeds and all assigned contacts in \mathbf{W} are physically present in \mathbf{X} , we move to the next round with a new level of θ . If it fails, we retry step 2 until successful.

The probability of observing a given contact in a structure depends on the presence of contacts in the same structure. For example, a certain chromosome contact also brings other chromosome regions into spatial proximity to each other, which in turn enhances their chances of contacting each other in the same structure rather than in a structure where the corresponding domains are far apart from each other and cannot be brought into spatial proximity. Our step-wise optimization approach naturally considers such cooperativity between domain contacts in individual structures. We assume that the more steps could lead to better “cooperativity” effects come into play. However, to speed up the whole process, we usually have several theta values. One could adopt the following recipe. After $\theta_1 = 1$ is done, find θ_2 so that \mathbf{A}^{θ_2} contains pairwise contacts as many as roughly 3 times the number of domain ($3N$). The next ones can be $10N$, $15N$, etc. number of contacts, until the population is hard to optimize. We try to keep minimal number of tunable parameters. Our experience indicates that our recipe and parameters are applicable to different data set; also our structure populations are not sensitive to the different parameter sets.

A.1.6. A-step: Parallel and efficient heuristic optimization for the contact assignment step

The A-step optimization problem is to “find the contact indicator tensor \mathbf{W} whose derived contact probability a'_{IJ} best matches the observed a_{IJ} for every domain pair I and J ”. Equation [4] can be expanded as

$$\log P(\mathbf{A}, \mathbf{W} | \mathbf{X}) = \sum_{\substack{I, J=1 \\ I \neq J}}^N \log P(a_{IJ} | a'_{IJ}) + \sum_{m=1}^M \sum_{\substack{i, j=1 \\ i \neq j}}^{2N} \left[w_{ijm} \log P(w_{ijm} = 1 | d_{ijm}) + (1 - w_{ijm}) \log P(w_{ijm} = 0 | d_{ijm}) \right] \quad [8]$$

To estimate \mathbf{W} for a given structure population \mathbf{X} , a natural and intuitive strategy of maximizing Eq. [8] is to assign $w_{ijm} = 1$ for which the corresponding domains are already closest in 3D space and therefore have the highest likelihood of forming a contact. That is, assignments of a given chromatin contact across the contact indicator tensor \mathbf{W} are more likely realized in those genome structures in which the corresponding chromatin domains are already closer in 3D space. For a given pair of domains i and j , we utilize two mutual exclusive items (i.e. $P(w_{ijm} = 1 | d_{ijm})$ and $P(w_{ijm} = 0 | d_{ijm})$ in Eq. [8]) based on their 3D surface distance in each of the M structures: when d_{ijm} is larger than a distance threshold value (termed as d_{IJ}^{act}), let $w_{ijm} = 0$ for accepting the larger log-likelihood $\log P(w_{ijm} = 0 | d_{ijm})$ out of two mutual exclusive items; when d_{ijm} is smaller than a distance threshold d_{IJ}^{act} , let $w_{ijm} = 1$ for accepting the larger log-likelihood $\log P(w_{ijm} = 1 | d_{ijm})$. When I and J are domains from the same chromosome we select pairs from each homologue copy, namely $\{(i,j), (i',j')\}$. When I and J are domains from the different chromosomes, we select 2 pairs whose distances are the smallest two among $\{d_{ijm}, d_{i'jm}, d_{ij'm}, d_{i'jm}\}$ computed from \mathbf{X}_m . This process is easily implemented in parallel, because the distance threshold of each domain pair can be independently calculated.

To define the distance threshold d_{IJ}^{act} , we designed a heuristic optimization procedure (i.e., distance threshold method), which is a process of determining the distance threshold d_{IJ}^{act} for each domain pair (I,J) , based on the empirical distribution of all distances between their homologous copies across all structures of the population (the pairs are selected based on their distances with a procedure described below).

Distance threshold method:

Let (I, J) be a domain pair (with homologues domain copies i, i' and j, j') and $a_{IJ} > 0$:

- 1) The empirical distribution of domain distances between homologous copies of the domain pair (I, J) is constructed as follows. When I and J are domains from the same chromosome, we collect distances d_{ijm} and $d_{i'j'm}$ in all models ($m=1, 2, \dots, M$) which form a total set of $2M$ distances. When I and J are domains from different chromosomes, we collect the smallest 2 distances from the set of all possible distances $\{d_{ijm}, d_{i'jm}, d_{ij'm}, d_{i'j'm}\}$ for a total set of $2M$ distances.
- 2) The $2M$ distances are ranked in increasing order, and the distance threshold, d_{IJ}^{act} , is determined as the distance value at the $2a_{IJ}M^{\text{th}}$ -quantile of all the $2M$ sorted distances. An illustration is shown in **Fig. S1C**.

This procedure maximizes $\log P(\mathbf{A}, \mathbf{W} | \mathbf{X})$ which have two items $\log P(\mathbf{W} | \mathbf{X})$ and $\log P(\mathbf{A} | \mathbf{W})$, because (i) it assigns contacts to those domain pairs with shortest distances, which maximizes $\log P(\mathbf{W} | \mathbf{X})$ and (ii) it uses the $2a_{IJ}M^{\text{th}}$ -quantile of all $2M$ distances as the distance threshold to determine w_{ijm} , so this heuristically maximizes the first term

$$\log P(\mathbf{A} | \mathbf{W}) = \sum_{\substack{I, J=1 \\ I \neq J}}^N \log P(a_{IJ} | a_{IJ}^*) \text{ by making } a_{IJ} \text{ exactly equal to } a_{IJ}^*.$$

Please note that in practice the predefined parameters σ and σ' in the formulation do not affect the results, if almost all probability items in the objective function are fully maximized to their extreme values (i.e., ones), which is required by our practical optimization heuristics and implementation.

A.1.7. Parallel and efficient numerical approximation for the modeling step

Given the current estimated contacts of \mathbf{W} , the M -step reconstructs the structure population \mathbf{X} that matches \mathbf{W} . Because \mathbf{A} and \mathbf{W} are known in the M -step, the maximization problem in Eq. [4] can be reduced to $\max \log P(\mathbf{W} | \mathbf{X})$, which can be further decomposed to the sub-problem

$$\max \log P(\mathbf{W}_m | \mathbf{X}_m) \quad \text{for every structure } m \text{ in the population, where}$$

$P(\mathbf{W}_m | \mathbf{X}_m) = \prod_{i,j} P(w_{ijm} | \vec{x}_{im}, \vec{x}_{jm})$ and \mathbf{W}_m is the contact indicator matrix of structure m .

Therefore, each individual structure can be independently optimized in parallel. To efficiently optimize an individual structure, we employed simulated annealing dynamics and conjugate gradient optimizations. The former is a structure modeling approach that can efficiently arrive at a stable state minimizing constraints violations; while the latter can adjust local structures in order to reach the optimum with zero constraint violations. Both are implemented in the Integrated Modeling Platform (IMP, <http://www.integrativemodeling.org/>)^(4, 5). **Table S2** lists the parameters used in the modeling step.

According to Eq. [2], the item $\log P(w_{ijm} | \vec{x}_{im}, \vec{x}_{jm})$ in Eq. [4] can be expanded as the summation of two mutually exclusive items $w_{ijm} \log P(w_{ijm} = 1 | d_{ijm})$ and $(1 - w_{ijm}) \log P(w_{ijm} = 0 | d_{ijm})$ which describe the contact and non-contact of two spheres i and j in structure m , respectively. Our practical experiences showed that when we maximize only the contact items in the objective function, the majority of non-contact items are approximately maximized as well, due to their inherently dependent relationships. In all the 10,000 optimized structures that completely satisfied all contact items, on average $(97.11 \pm 0.18)\%$ of non-contact items are also satisfied. We therefore maximize only the contact items and ignore the non-contact items for dramatically speeding up the optimization process while keeping reasonably good optimization performance. Please note that for each structure m , the M -step checks whether or not all contacts in \mathbf{W}_m are physically present in \mathbf{X}_m . If not, the M -step will be repeatedly performed until the check is passed.

Convergence criteria. The A/M optimization steps are iteratively performed until each contact in \mathbf{W} is physically present in \mathbf{X} and the following convergence criteria is satisfied:

$$\left| a_{IJ} - \frac{1}{2M} \sum_{m=1}^M \bar{w}_{IJm}^* \right| \sim 0 \text{ for every domain pair } I \text{ and } J,$$

where a_{IJ} is the matrix element of \mathbf{A} , and $\mathbf{W}^* = (w_{ijm}^*)_{2N \times 2N \times M}$ is defined as $w_{ijm}^* = c_{ijm} w_{ijm}$ for any $1 \leq i, j \leq 2N$ and any $1 \leq m \leq M$. $\mathbf{C} = (c_{ijm})_{2N \times 2N \times M}$ denotes the full contact tensor derived from \mathbf{X} , and $\bar{\mathbf{W}}^* = (\bar{w}_{IJm}^*)_{N \times N \times M}$ is the projected contact tensor of \mathbf{W}^* where contacts at the homologous domain level are projected to the domain level.

A.1.8. Software availability

The population-based genome modeling software and the input data used in this work are available upon request.

A.2. Tethered Conformation Capture (TCC)

The previously described HindIII-TCC library of GM12878 cells was further sequenced for this study and the new sequencing results were combined with the previously sequenced data (1) (**Table S3**). The TCC experimental procedure were described elsewhere (1). Briefly, approximately 25 million GM12878 cells were crosslinked with 1% formaldehyde, lysed and treated with Iodoacetyl-PEG2-Biotin to biotinylate chromatin. This biotinylated chromatin was then digested with HindIII and immobilized on MyOne Streptavidin T1 beads (Invitrogen) with about 100 cm² surface area. With chromatin immobilized, the DNA overhangs were blunted with a mixture of dATP, dTTP, dGTP α S and Biotin-14-dCTP and subjected to ligation. Afterwards, DNA was purified and treated with *E. coli* exonuclease III to remove the biotinylated residues from non-ligated DNA fragments. Ligated DNA fragments were then pulled down with streptavidin coated magnetic beads, attached to Illumina paired-end sequencing adaptors, and amplified to obtain the TCC library.

A.2.1. Assembling contact catalogue

The contact catalogue was assembled as described previously (1) with minor modifications in adjusting for ligation junctions: To increase alignment efficiency, all sequencing reads were scanned for the existence of potential ligation junction sequences (for HindIII libraries the junction sequence is “AAGCTAGCTT”). In reads with ligation junctions, all bases after the 3’ of the midpoint of the junction and the corresponding quality scores were removed. Furthermore, to adjust for star-activity of the restriction enzyme and other factors that result in ligation junctions with a sequence that slightly deviates from the expected consensus (6), we adjusted the scanning algorithm to allow for one mismatch or deletion in the entire expected junction sequence. In other words, any sequence that differed from the ligation junction by one mismatch or deletion was also considered to be a junction, and the sequence after the junction’s midpoint was removed from the corresponding read. This filtering strategy significantly improved the percentage of alignable reads.

A.2.2. Alignment to the human genome

The filtered reads were aligned against the GRCh37/hg19 reference sequence of the human genome using Bowtie-0.12.7 with a maximum of three mismatches allowed. The total number of

aligned reads in each end is shown in **Table S3**. After alignment, the genomic positions of the read pairs that corresponded to the same sequencing cluster were combined to generate a catalogue of binary contacts. All pairs for which at least one of the reads could not be unambiguously aligned were removed from the contact catalogue leading to ~150 millions aligned paired reads (**Table S3**).

A.2.3. Removing non-informative pairs

Three types of pairs do not contain any information about the spatial organization of the genome and can be removed: PCR multiplications, non-ligated DNA fragments (flakes), and self-loops(1). To filter PCR multiplications, groups of read pairs that aligned to identical positions on both ends were removed, leaving only one instance per group in the catalogue. To filter flakes, pairs that aligned less than 1000 base pairs (bp) apart to opposite strands of the reference sequence were removed from the catalogue. To filter the self-loops, all pairs that aligned closer than 30,000 bp were removed.

The total number of read pairs after the above filtration steps for the binary contact catalogue of the library is about 98 Million (**Table S3**).

A.3. TCC Data Processing

Our structural modeling approach is outlined in a flow-chart (**Fig. S1D**).

A.3.1. Raw matrix

Interaction frequency counts were binned every 138 hindIII restriction fragment sites and a matrix registering pairwise interaction frequency between bins was constructed, we denote it as

$$\mathbf{C}_K \equiv \left(c_{ij} \right)_{K \times K}, \text{ where } K=6002 \text{ segments or bins.}$$

A.3.2. Correction on the raw matrix

At the resolution of consecutive 138 hindIII sites bin matrix, we safely corrected some regions that were unusual. One type of correction was filling the 0 consecutive contacts within a chromosome. When such situation occurred for a consecutive bin pair ($i, i+1$) we took an average from the maximum of each bin i and $i+1$ to replace the 0 value. Another type of correction was to replace outstandingly high contact frequency to the EBV genome. For every bin we counted the frequency of contact to the EBV genome and calculated the average, i.e. 29.

We applied a high cutoff, i.e. 75 (about mean + 4 times standard deviations) to indicate which bins possessed high contact frequencies with EBV, and those bins usually have very high contact across genome as well. For those bins, we calculated the average of their direct neighbors contact frequencies for correcting them:

$$c_{i,j} = \min\left\{c_{i,j}, (c_{i+1,j} + c_{i-1,j})/2\right\}, j=1..K, j \neq \{i-1, i+1\}. \quad [9]$$

These corrections are optional since it is not generally applicable for other Hi-C data.

A.3.3. Removing contact frequency biases by iterative correction method (ICE)

We performed normalization on the smoothed matrix with an approach known as iterative correction and eigenvector decomposition (ICE) (7). Before using the ICE normalization method, we removed 1% bins with the fewest contact frequencies by merging them with their direct neighbors, following a suggested contact frequency cutoff value of 1%-2% (7), resulting a matrix with 5941 bins (**Fig. S1E**). We then applied a fast decaying power-law smoothing function on each individual chromosome submatrix:

$$\tilde{c}_{i,j} = \frac{\sum_{k=-\omega}^{\omega} \sum_{l=-\omega}^{\omega} \frac{c_{i+k,j+l}}{|sk|^p + |sl|^p + 1}}{\sum_{k=-\omega}^{\omega} \sum_{l=-\omega}^{\omega} \frac{1}{|sk|^p + |sl|^p + 1}} \quad [10]$$

where ω , s , p we used were 3, 3, and 3, respectively.

The ICE method was applied according to Imakaev et al. (7) with the number of iterations set to 10 (the matrix had achieved convergence before 10th iteration). Interactions of consecutive bins within a chromosome were included to allow the definition of reference frequencies (f^{\max} , see below). The resulting matrix is $\mathbf{F}_K \equiv (f_{ij})_{K \times K}$.

A.3.4. Bin level contact probability

The contact probability is defined as the probability for observing a given contact in the structure population (1). For each chromatin segment (i.e. a bin in the normalized contact frequency matrix) we define a threshold value f^{\max} , which defines the frequency at which a contact is formed in 100% of the structure population.

For a chromosome segment (matrix bin) f^{\max} is chosen based on the contact frequencies to its two adjacent bins within a chromosome.

For each bin i , f^{\max} is set as

$$f_i^{\max} = \min\{f_{i,i-1}, f_{i,i+1}\}. \quad [11]$$

Where f_{ij} , is the normalized contact frequency between bins i and j .

For the first and last bins in a chromosome (subtelomeric regions), we set $f_i^{\max} = f_{i+1}$ and $f_i^{\max} = f_{i-1}$ respectively.

f^{\max} serves as a common reference point to calibrate contact probabilities between chromatin segments. For a given pair of bins i and j the contact probability at bin level is defined as

$$p_{ij} = \min\left\{\frac{f_{ij}}{\min\{f_i^{\max}, f_j^{\max}\}}, 1\right\} \quad [12]$$

With this formulation, two consecutive bins on the same chromosome will have $p_{ij}=1$ to guarantee the structural integrity of the chromosome. An example of this matrix is shown for chromosome 1 (**Fig. S1E**).

A.3.5. Domain level contact probability

A “coarse-grained” domain contact probability matrix, \mathbf{A}_N , defines the fraction of models in the population in which a given domain contact is present. It is defined as $\mathbf{A}_N = (a_{IJ})_{N \times N}$, where a_{IJ} is the contact probability between domains I and J , and N is the total number of domains in the genome. a_{IJ} is calculated from the corresponding contact probabilities at the bin level.

If $b(I)$ is the set of all bins in domain I , $b(J)$ is the set of all bins in domain J , and $p_{\alpha,\beta}$ is the contact probability set of all pairwise combinations between bins in $b(I)$ and $b(J)$, then

$$a_{IJ} \equiv \left\langle \text{top5\%} \left\{ p_{\alpha,\beta} \mid \alpha \in b(I), \beta \in b(J) \right\} \right\rangle \quad [13]$$

is the average value of the top 5% ranked contact probabilities in $p_{\alpha\beta}$ (**Fig. S1E**). If I and J are two consecutive domains in the chromosome chain the maximum $p_{\alpha\beta}$ value is used, which naturally ensures that the two consecutive chromatin domains will always be in contact ($a_{IJ} = 1$) to guarantee the necessary structural integrity of the chromosome.

A.4. Structural Representation of the Genome

Chromosomes are segmented into chromatin domains, which are represented by spherical volumes following our previously published approach (1). The plaid pattern in the chromosome contact frequency heat maps suggests a partition of chromosomes into domains of consecutive regions with similar long-range contact behavior. We have previously determined the domain boundaries of these macro-domains using a constrained clustering algorithm combined with an automatic cluster cutoff detection (1). Regions in a domain form the vast majority of their contacts to regions within the same domain, therefore we can assume that these regions on average are closer to each other than to regions in other blocks. Chromatin regions within a domain share similar functional properties, such as similar replications timing. The long-range contacts of regions in a domain are highly correlated, indicating that the domain acts as a structural unit. Therefore, it is expected that a large fraction of the chromatin regions in a domain preferentially occupy a similar nuclear sub-territory whose spherical volume can be approximated by the domain sequence length and the total occupancy of the genome in the nucleus (1). This model does not exclude mixing of regions between blocks, but assumes that the majority of domain regions localizes in the same sphere.

To balance short and long range interaction profiles in the raw TCC matrix domain segmentation is performed on sequence distance-normalized TCC maps. We apply a distance-scaling normalization similar to our previous works (1, 9),

$$\begin{aligned}
\overline{\mathbf{F}}_{K'} &= \left(\frac{c_{ij}}{\tilde{v}_{d=|i-j|}} \right)_{K' \times K'} \\
\tilde{v} &= \lambda(v) \\
v &\equiv (v_d)_{d \in \{1, \dots, (K'-1)\}} \\
v_d &= \left\langle \frac{\sum_{i=1}^{K'-d} c_{i,i+d}}{K'-d} \right\rangle_{\text{all chromosomes}}
\end{aligned} \tag{14}$$

where the angle brackets denote an average over all chromosomes, d is bin distance of a $K' \times K'$ matrix, K' is the number of bins in an intrachromosome matrix, and λ is a locally weighted polynomial regression smoothing function ("lowess", implemented in R software). Although this normalization is carried out to individual chromosome, the scaling vector, v , is computed by averaging all chromosomes.

Following our previously published procedure (1), we used hierarchical clustering of intra-chromosomal contact frequencies using the following distance matrix: $\mathbf{D}_{K'} = (1 - \text{PCC}(i, j))_{K' \times K'}$ where $\text{PCC}(i, j)$ is the Pearson's correlation coefficient between row vector i and j in the distance-normalized contact frequency matrix $\overline{\mathbf{F}}_{K'}$. To *automatically determine the number of clusters* (i.e. domains), we minimized an objective function to balance the number of clusters and the spread of distances inside the clusters (10). From the ~ 500 kb resolution matrix ($K = 5941$), the clustering method segmented the genome into domains with median size of ~ 3.5 Mb (**Fig. S1F**). Our hierarchical clustering method is applicable for multiple granularities of domains (at different resolution of domains).

A.5. Analyses of the structure population

A.5.1. Centromere cluster detection

To identify centromere clusters in a structure, we ran a density-based spatial clustering algorithm implemented in R (<http://cran.us.r-project.org/>), *DBScan* package, by calling "dbSCAN(d , method='dist', eps=200, MinPts=3)", where variable d is a distance matrix. The smallest cluster size and the 'reachability distance' were set to 3 and 200 nm, respectively. The surface-to-surface distance between 2 centromeric representative beads were used as the 'reachability distance'. A centromeric representative bead was called when it overlaps (covers) a

centromere gap defined in hg19 reference genome. All the 46 beads were included for the non-overlapping centromeric clusters detection (see **Fig. 4C** for some cluster examples in the main paper; the analyses results are plotted in **Figs. S4A-D**). The results did not change when the distance cutoff was varied between 100 to 300 nm.

A.5.2. Correlation coefficients

A correlation of two matrices can be assessed in several ways. In most cases, we presented the element-wise or the row-based correlations. For the element-wise correlation, we construct a vector from each matrix in the same indexing order, and then assess their correlation (usually with Pearson's and Spearman's method). For the row-based assessment, a Pearson's correlation coefficient (PCC) is computed between corresponding rows from both matrices, and then the average PCC over all rows in the matrix is reported.

A.5.3. ICP

On the normalized (iterative corrected) matrix, an ICP index of bin i is calculated as described previously (1) as the fraction of interchromosomal counts across other bins,

$$\text{ICP}_i = \frac{\sum_{j \in \beta(i)} f_{ij}}{\sum_{j \neq i}^N f_{ij}},$$

where $\beta(i)$ are all interchromosomal bin partners of i .

A.5.4. Epigenetic analyses

The data of histone modifications, DNase hypersensitivity, RNA polymerase II binding, and gene expression for GM12878 cell line were obtained from the Encode project (11). We downloaded the bigwig files from the following sites:

- <http://hgdownload.cse.ucsc.edu/goldenPath/hg19/encodeDCC/wgEncodeBroadHistone/>
- <http://hgdownload.cse.ucsc.edu/goldenPath/hg19/encodeDCC/wgEncodeSydhTfbs/>
- <http://hgdownload.cse.ucsc.edu/goldenPath/hg19/encodeDCC/wgEncodeCaltechRnaSeq/>

The data of DNA methylation was obtained from (12, 13). The lincRNA transcripts data (14) was downloaded from http://www.broadinstitute.org/genome_bio/human_lincrnas/?q=lincRNA_catalog.

BigwigSummary program (from USC Genome Browser) was called to extract all the data for requested regions (or the defined 639 domains) in the analyses. The epigenetic signal was computed as an average per domain, thus it did not depend on the genomic length of a domain. As for the recurrent pattern analyses of centromeric regions, we extracted the data for each chromosome from a region defined by the following. The start position of a centromeric region was chosen from either at *domain start position* of the centromere representative bead or position at *5 Mb upstream from the left border* of hg19 centromere gap, which ever was the leftmost. Likewise, the end position of a centromeric region was chosen from either at *domain end position* of the centromere representative bead or position at *5 Mb downstream from the right border* of hg19 centromere gap, which ever was the rightmost. The reason of taking this way was the recurrent structural patterns were mined by using the 46 centromere-representative beads, however some of them were completely representing the gap regions only that yield no epigenetic signals. For a detail list of the centromeric gaps and our defined centromeric regions please refer to **Table S4**.

A.6. Convergence of our results with respect to the size of the structure population

To test the convergence of our results with respect to population size, we generated 5 different populations in size of 100, 1000 5,000, 10,000, and 20,000 structures and also performed replica calculations (repeated independent calculations) for each population totaling independently calculated 10 structure populations. We first calculate the convergence of the domain-domain contact frequencies in the structure populations. At populations larger than 1,000 structures, we observe excellent convergence of contact frequencies with a good correlation value (**Fig. S7A**). Also, when analyzing the 3D structures we observe a similar behavior (**Fig. S7B**). The average radial position of each domain fully converges at population sizes larger than 1000 structures, a size much smaller than our population size reported in the paper. The average radial domain positions from a population of 20,000 structures are essentially identical to those generated with a population at 10,000 structures (PCC = 0.9998, p-value < 2.2e-16).

We also tested the reproducibility of our results in replicate calculations with increasing population sizes. All structural features and the contact frequencies are highly reproducible at population sizes larger than 1000 structures. A population of 10,000 structures has reached high reproducibility and shows fully converged structures.

Table of Pearson's correlation coefficients (PCC) when the features are compared between replica calculations

Feature	100	1k	5k	10k	20k
Average radial position: mean PCC per chromosome	-0.150	0.997	0.999	1.000	1.000
PCC of the average radial position of all domains	0.200	0.998	1.000	1.000	1.000
PCC of the pairwise contact frequencies	0.925	0.999	0.984	1.000	1.000

In summary, our results are well converged at a population size of 10,000 structures and are highly reproducible. Our results remain unchanged when calculating a larger population of 20,000 structures (**Figs. S7C-G**).

A.7. Structure population generated from a reduced data set

To assess the impact of sub-centromeric interactions on the genome structures, we generated a population of 10,000 structures from Hi-C data where interchromosomal interactions were only considered for sub-centromeric spheres (see the right most heat map shown in **Fig. S2A**). The sub-centromeric spheres were defined as those spheres containing the centromere gap (defined in the hg19 reference genome) and the closest 2 direct neighbor spheres from each side (left and right on the chromosome bead chain). The average radial positions of chromosomes in this population correlated very well with those from the “regular” structure population that was generated using the complete Hi-C data (PCC = 0.959, p-value = 6.11e-13; **Fig. 3A bottom left panel**). When these radial positions were compared to those from FISH experiments (15), the PCC is 0.650 (p-value = 0.000786).

A.8. Structure population generated with uniform centromere-centromere interaction probabilities

To test the impact of non-specific centromeric interactions on genome structures, we tested a model, in which the interactions between subcentromeric regions of all chromosomes are equally probable and therefore are not specific with respect to the chromosome identity. In this model each pair of centromeres has equal probability to interact and the total number of subcentromeric interactions is kept equal to those of the original Hi-C data. Specifically, for every pair of centromere spheres, we assigned a probability of $a_{IJ}=0.030$ to be in contact. No other domains were restrained to form interchromosomal contacts. This model does not produce the correct radial positioning of chromosomes as known from FISH experiments (**Fig.**

S2B; in comparison with the FISH data (15), PCC = -0.269 with p-value = 0.2147), ruling out the notion that centromere clustering based on non-specific centromere-centromere interactions lead to the correct positioning of chromosomes. Our calculations indicate that chromosome specific interactions between subcentromeric regions and not non-specific phase separation of pericentromeric heterochromatin are crucial for reproducing experimentally known chromosome positing in our model.

A.9. Structure population generated with *in-situ* Hi-C data

To test if our conclusions remain unchanged when using the recently published higher resolution *in-situ* Hi-C data set from the Lieberman Aiden group (16), we also generated a structure population using the *in situ* Hi-C data set. The raw matrix (C_K , with $K=5941$) is constructed from mapped reads (MAPQ > 30) using 92 "...merged_nodups.txt.gz" files (GSM1551nnn) After excluding intra-chromosome pairs with distance less than 30 kilobase pairs (section A.2.3) the matrix contained more than 3.5 billion contacts. Matrix normalization and generation of the probability matrix were performed as described in sections A.3.3 to A.3.5. The contact probability matrixes from both data sets were highly correlated (0.993, p-value < 2.2e-16). We then generated a structure population of 10,000 structures with the contact probabilities from *in situ* Hi-C. The resulting structure population produces consistent results with those generated from the TCC data. In the following we compare the two structure populations.

Domain-domain contact frequencies in the model. The pairwise domain contact frequencies are highly correlated between the two structure populations (Pearson's correlation coefficient 0.9847 (p-value < 2.2e-6).

Chromosome radial positions. The average radial chromosome positions are very similar in both structure populations with a high Pearson's correlation coefficient of 0.866 (p-value = 2.67e-8; **Fig. S8B**). The radial chromosome position from both structure populations agree well with the FISH data (15) (**Fig. S8A**). Both structure populations reproduce equally well the average radial chromosome positions from the FISH experiments (Pearson's correlation coefficient of 0.762 (p-value = 2.36e-5) for the structure population from *in situ* Hi-C data and PCC = 0.747 for the structure population from the TCC data).

Domain radial positions. Also the radial positions of all the chromatin domains are very similar in both populations. The PCC of the average radial domain position in the two populations is 0.797 (p-value < 2.2e-16).

Domain-domain distances Also distances between domains are remarkably similar in both populations, (**Figs. 3B-C**). The colocalization frequencies of four interchromosomal loci pairs (H0 with H1, H2, L1, and L2) are highly correlated in the two populations (PCC = 0.976, p-value= 0.02393). Also the cumulative distances between domains in 8 inter-chromosomal pairs of genes are reproduced remarkably well (all PCCs > 0.999 with p-values < 2.2e-16) (**Figs. S8C-D**).

Centromere clustering and positions. The structure population from *in situ* Hi-C data confirms our observations from the structure population generated with the Hi-C data. The radial position of a centromere generally decreases with increasing number of surrounding centromeres (see (**Figs. S8E** and **6A**). Also the abundance of centromere clusters per structure and the cluster size distribution is very similar between structure populations generated with *in-situ* Hi-C and TCC data (**Figs. S8F**, **Fig. S4A**, and **S4B**).

A.10. 3D DNA FISH

Based on the finding from our structural models, we designed multiple sets of 3D *fluorescence in situ hybridization* (FISH) experiments to see variation of colocalization formed by different groups of centromeres. Each group consists of 3 centromeres, they are on chromosomes {1, 9, 21}, {7, 10, 12}, and {2, 3, 6} (**Fig. 5B**). Colocalization event can be defined as all the 3 pairwise distances between centromeric domains are within a threshold. The results of these experiments are plotted in **Fig. 5C**.

To verify that centromeric regions are bridging the interaction inter-chromosomally, we performed a comparison between probes on centromeric of chromosome 1, 9, and 21 against the probes on distal regions from centromeres of the same chromosomes (the distal regions were about 56.8 Mb, 61.5 Mb, and 18.3 Mb away from centromere on chromosomes 1, 9, and 21, respectively). The contrast between the two groups is plotted in **Fig. S9B**.

A.10.1. DNA FISH probe design

For any particular chromosome domains (regions), multiple BAC clones were chosen and synthesized by Empire Genomics and tested individually for their specificity. The probe of a pericentromeric region on chromosome 1 lies at region 1q11, a standard designed by the company. Pericentromeric probes are RP11-831B17 (chr2: 96,977,476-97,209,012), RP11-1082I19 (chr3: 87,569,564-87,784,200), RP11-973P24 (chr6: 65,112,008-65,298,120), RP11-144H20 (chr7: 61,968,709-62,155,949), RP11-912B9 (chr9: 40,475,859-40,773,473), RP11-300L24 (chr10: 42,965,138-43,178,326), RP11-349I21 (chr12: 38,143,496-38,364,968), RP11-1089A22 (chr21: 10,949,929-11,145,665). The probes that are far from centromeric regions are RP11-57D16 (chr1: 179,836,951-179,993,708), RP11-760E14 (chr9: 110,383,457-110, 589, 787), and RP11-655P6 (chr21: 31,137,227-31, 348, 263). **Fig. 5B** illustrates the probes locations on the chromosomes.

A.10.2. FISH experimental procedure

The experiment was performed following the previous protocols (1, 17, 18) with slight modification. GM12878 cells were cultured in DMEM medium supplemented with 15% FBS, glutamine and penicillin/streptomycin as suggested by ENCODE. Two days before the experiment, 22mm x 22mm coverslip were cleaned and coated with L-poly-lysine (1mg/ml) at room temperature for 1-2 hours, and dried in tissue culture hood after brief rinse with sterile MilliQ water. On the day of the experiment, 10 million GM12878 cells were harvested by centrifugation at 100g for 10 minutes, resuspended in fresh culture medium (3×10^6 cells/ml), and seeded evenly on the coverslip in a 6-well tissue culture plate. After incubating at 37°C for 1 hour and briefly washing with PBS, the cells (on coverslips) were fixed with 4% freshly made paraformaldehyde (in 0.4x PBS) at room temperature for 10 minutes. The cell membrane was permeabilized firstly with 0.5% triton X100/1xPBS at room temperature for 20 minutes, and then through 4-5 freeze-and-thaw cycles (by dipping in liquid nitrogen and then thaw in room temperature) in the next day after pretreatment overnight with 20% glycerol/1xPBS and also before each dip. To facilitate access of the FISH probe to the chromatin DNA, the samples were washed with 0.05% triton X100/1xPBS 5 minutes each for two times, and then treated with 0.1N HCl at room temperature for 5-10 minutes to remove basic nuclear proteins. The HCl was removed from the sample followed by two washes with 0.05% triton X100/PBS and one wash with 2x SSC (diluted from 20xSSC: 3M NaCl, 0.3M sodium citrate, pH 7.0) 5-10 minutes each wash. The coverslips were then stored in 50% formamide/2x SSC at 4°C and were ready for the next step (good for 2 days to 2 months).

The denaturation and hybridization steps were performed according to the protocols suggested by the manufacturer (https://www.empiregenomics.com/files/store/products/FISH_probes/FISH_Protocol.pdf). The coverslips were brought to room temperature for 24 hours in advance before denaturing. On the day of experiment, fresh 70% formamide/2x SSC was prepared and pre-warmed at 73°C for 30 minutes. Cells on the coverslips were denatured in this solution (73°C) for 5 minutes, and dried through sequentially dipping into 70%, 85% and 100% ethanol 1 minute each at room temperature, and finally through evaporation at 45°C for 20 minutes. FISH probes were denatured similarly in 70% formamide/2x SSC for 5 minutes at 73°C and then quickly cooled down on ice. After incubating at 37°C for 10-20 minutes, three probes (150 ng each) for either targeted regions or for the three control regions were mixed thoroughly with 18 µl hybridization buffer (provided by the manufacturer), and applied evenly with the sample on a microscope slide. Hybridization of FISH probes with the samples occurred in a humidified chamber containing a paper towel soaked with 50% fomamide/ 2x SSC in dark at 37°C for 18-20 hours. Unbound FISH probes were removed by a series of washes, three times with 0.3% NP-40/0.4x SSC at 73°C for 2 minutes, each followed by a wash with 0.1% NP40/2x SSC at room temperature for 1 minute. After air-drying for 5 minutes in dark, the coverslips were mounted on microscope slide with 10 µl DAPI mounting solution and kept in dark at 4°C (ready for imaging).

A.10.3. FISH image acquisition

The FISH images were acquired with Zeiss Laser Scanning Confocal microscope (LSC780) with 63x magnification oil immersion objective lenses. Cells are randomly chosen (each vision field contains 6-15 cells). Signals from four different fluorophores were obtained with two alternative frame scans for best separation: first scan with two laser beams of 488 nm and 594 nm, followed by the second scan of 405 nm (for DAPI) and 532 nm laser beams (for the yellow probe used in targeted group) or 405 nm and 555 nm laser beams (for orange probe used in control group). The minimal laser power was used in combination with appropriate filter settings (MBS 488/594 and MBS 458/514/561/633) to greatly reduce the signal bleed through between channels. Images of cells with optical Z sections from the bottom to the top with 0.25 µm or 0.3 µm intervals were acquired one section after another (frame scanning) with the software Zen provided by the manufacturer. Signals of each probe were stored in separate channels (4 channels for 3 chromosomal regions plus DAPI staining of the whole chromosomal DNA).

A.10.4. Image data analysis procedures

The nucleus detection and distance measurements between probes were performed using the Nemo software for FISH image analyses (19). Automated nucleus detection mode was used as the standard procedure, and additional manual selection followed when needed. Each of the cells was subject to manual inspection and validated for containing at least the expected six bright spots corresponding to the location of the three FISH probes in the diploid nucleus (2 FISH signals per probe/marker are expected). The pairwise distances between probes were extracted to get the average distance among the 3 loci on different chromosomes or to determine whether the 3 loci were colocalized. We refer to the three loci as a triplet. Each of the three probes A, B, C is located on two homologues chromosome copies and the corresponding probe copies are labeled A and A'; B and B'; C and C'. Given 3 probes, there are a total of 8 possible triplett probe combinations per cell that can define a co-localization cluster: {A,B,C}, {A,B,C}, {A,B',C}, {A,B',C}, {A',B,C}, {A',B,C}, {A',B',C}, {A',B',C}. For each possible triplet we calculate the distance between the corresponding loci.

For each triplet, we calculate the “*triplet distance*”, which is defined as the average distances between probes in a triplet (**Fig. 5B**). To determine the cumulative frequency distribution we select the triplet with smallest triplet distance among all the possible triplet combinations per cell (**Figs. 5B and S9B**).

A.10.5. Image data analysis results

To test the chromosome-specific nature of our predicted centromere clusters, we specifically performed 3D FISH experiments for three centromere clusters that are predicted with largely different frequencies in the population (**Fig. 5B**). We analyzed a total of more than 1500 3D FISH images. Since the chromatin domains used in our structural model are of a median size ~3.5 Mb, and the FISH probe is of the size ~200 kb, we expect that the cluster occurrence frequencies observed in FISH images shall be generally much lower than those from our structural population. Therefore, we are not using FISH to validate the absolute frequencies of individual clusters, but the relative frequency order among the clusters.

In our models, the centromere cluster of chromosomes 7, 10, and 12 (cluster 7-10-12) occurs substantially more frequently than the cluster of chromosomes 2, 3, and 6 (cluster 2-3-6), but less frequently than the cluster formed by chromosomes 1, 9, and 21 (cluster 1-9-21) (**Fig. 5C**). Because the cluster 1-9-21 is observed most frequently, its frequency serves as the reference. In our model the frequency of cluster 1-9-21 is about 1.2 fold larger than the frequency of cluster

7-10-12, and about 24 fold larger than the frequency of cluster 2-3-6. The ranked order of cumulative cluster occurrences and also the relative frequencies of the clusters in the FISH experiments were determined and compared with our model as follows.

1) Cumulative percentage of cells with respect to probe distances. To compare the colocalization propensity of centromeres in the three clusters without the use of a specific distance cutoff, we calculated for each cluster the cumulative percentage of cells with respect to the average distance among the triplet probes. In a diploid genome, there are 8 possible triplet combinations of the three probes and we determined for each cell the triplet with the smallest average distance. The cumulative percentage of cells for the smallest triplet distance for all three clusters demonstrates that centromeres 1, 9, and 21 are indeed consistently more frequently in proximity to each other than centromeres 7, 10, and 12, while the centromeres of chromosomes 2-3-6 are the least frequent to be in proximity (**Fig. 5B** lower right panel).

2) Relative cluster frequencies. We then compared the relative frequencies of clusters in the population. Given a specific distance cutoff, we define the three centromeres to form a cluster if all three pairwise probe distances are simultaneously smaller than a cutoff in the same cell image.

Selecting a distance cutoff: Human centromeres contain extensive tandem repeat arrays (1,500 to >30,000 copies) and their actual size can span up to 5Mb of DNA (20). It is fair to assume an average size of ~3Mb of centromeric DNA per chromosome. Because the FISH probes are adjacent to the centromeres, we need to consider the centromeric DNA when choosing the probe distance cutoff. We chose a distance cutoff of 1.5 micron, to allow for a consideration of the total expected ~9 Mb of centromeric DNA for three chromosomes.

Using a distance cutoff of 1.5 micron for both experiments and models, the absolute frequencies of the clusters 1-9-21, 7-10-12 and 2-3-6 are 6.5%, 4.4%, and 1.5% from the 3D FISH experiments, versus 34%, 29.4%, and 1.4% from our structure population, respectively. As expected, the cluster frequencies observed in FISH are lower than those from our structural population, because of the much smaller sizes of the FISH probes (~200kb) compared to those of the domain (~3.5Mb) used in the structure modeling. Therefore, FISH cannot validate the absolute frequencies of individual clusters, but the relative frequency order among the clusters. Indeed, **Fig. 5C** shows that our model predicts very well the relative cluster frequencies in FISH experiments. Also in FISH experiments, the centromere cluster 1-9-21 shows a substantially

higher relative frequency than all the other clusters. For example, in FISH experiments the observed frequency value of the cluster 7-10-12 is only 67% of the frequency of the cluster 1-9-21. The frequency of the cluster 2-3-6 is only 23% of the frequency of the cluster 1-9-21. In the model, the rank order of frequencies is identical. The highest frequency is observed for cluster 1-9-21. The frequency of cluster 7-10-12 is only 86%, and the frequency of cluster 2-3-6 is only 4% of the frequency observed for cluster 1-9-21, respectively. The results are essentially unchanged when using a distance cutoff of 1.25 micron.

3) Centromere as contact points for chromosome cluster formation. Additionally, we tested if the centromeres are the main points of interactions for the chromosome cluster 1-9-21. We found that the three markers located in the pericentromeric regions of chromosomes 1, 9, and 21 showed substantially higher co-localization frequency (~3 fold at distance threshold 1.5 micron; **Fig. 5D**) than a control group of markers located at more distal regions from centromeres on the same chromosomes (56.8 Mb, 61.5 Mb, and 18.3 Mb away from centromere on chromosomes 1, 9, and 21, respectively). The cumulative percentage of the average probe triplet distances is consistently smaller for the subcentromeric probe cluster than for the control probes at more distant locations from the centromere (**Fig. S9**).

In summary, we experimentally validated the finding from our structural modeling that individual chromosomes differ substantially in their propensity to form centromere clusters. The observed rank order of centromere cluster frequencies is consistent with the predictions: cluster 1-9-21 is more frequent than cluster 7-10-12, while cluster 2-3-6 is the least frequent among the 3 clusters. We conclude that centromere cluster formation is highly chromosome specific in nature.

A.11. Cryo-X-ray Tomography

Lymphoblastoid cells (GM12878) were obtained from the Coriell Cell Repositories at the Coriell Institute for Medical Research and cultured in RPMI 1640 Basal Media (Life Technologies # 12633-012; No HEPES, No L-Glutamine, + Non-Essential Amino Acids, + 110 mg/L Sodium Pyruvate) plus 2mM L-Glutamine (1:100, Life Technologies # 25030-149), 0.4g/100mL (0.4% w/v), Pen Strep (1:100, Life Technologies #15140-148), and 15% Fetal Bovine Serum (ATCC # 30-2020). Cells were grown in 10 mL of complete growth media in an upright T25 flask. For soft x-ray tomography, cells were loaded into thin walled (200 nm) glass capillaries (in growth

medium), and rapidly frozen by mechanically plunging, at 2 m/sec, into liquid nitrogen cooled propane. Projection images were collected at 517 eV using XM-2, the National Center for X-ray Tomography soft X-ray microscope at the Advanced Light Source of Lawrence Berkeley National Laboratory; the microscope was equipped with a resolution defining 50-nm objective lens. During data collection, the cells were maintained in a stream of helium gas that had been cooled to liquid nitrogen temperatures (21, 22). Cooling the specimen allows collection of projection images while mitigating the effects of exposure to radiation. For each dataset, 180 projection images were collected sequentially around a rotation axis in 1° increments to give a total rotation of 180°. An exposure time of between 150 and 300 ms was used (depending on synchrotron ring current). Projection images were manually aligned using IMOD software by tracking gold fiducial markers on adjacent images (23) and tomographic reconstructions were calculated using the iterative reconstruction method (24, 25). LAC values were determined as described previously (26).

B. SI FIGURES

B.1. Figure S1

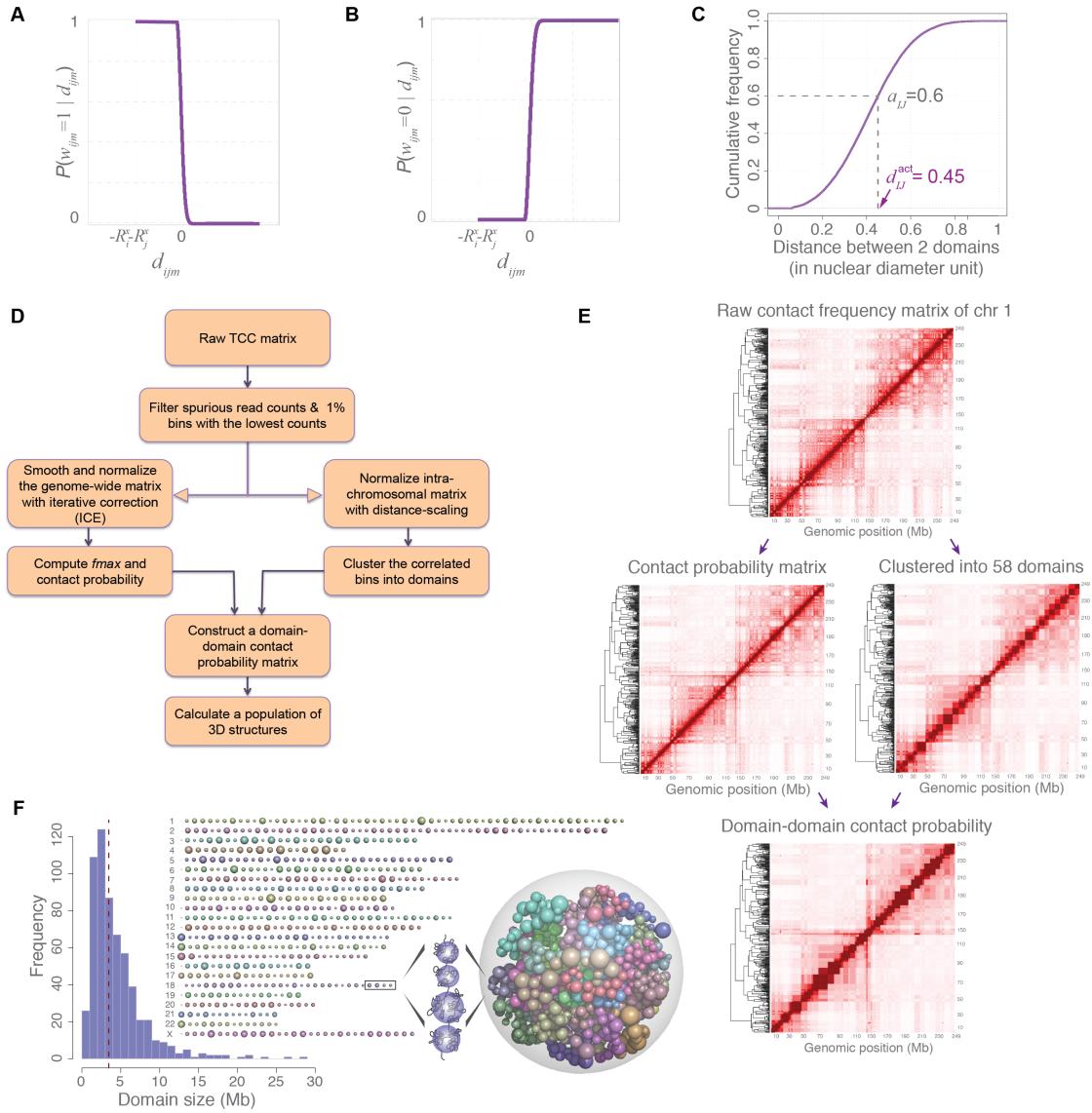


Figure S1 Descriptions of methods. **(A)** The function for spheres i and j in structure m that have a contact (defined in Eq. [1]). **(B)** The function for spheres i and j in structure m that do not have a contact. **(C)** Illustration of obtaining activation distance d_{IJ}^{act} for a given probability of interaction a_{IJ} that is posited to a pair of domains I and J . The curve is cumulative frequency of the pair in the optimized structure population. As an example, sphere pairs whose current distances are within the smallest 60% distances (e.g. ~0.45 nuclear diameter unit in the figure) will be restrained to be in contact in the next stage of optimization. **(D)** Flow chart of our input data processing for 3D modeling. **(E)** The flow of matrix transformations from the raw TC matrix to coarse contact probability matrix. As described in **D**, a contact probability and domain matrix for 3D representation are generated (see sections A.3-A.4). Shown here is chromosome 1, the hierarchical clustering approach resulted in 58 domains. The total number of genome-wide domains is 639. **(F)** The histogram of size of the 639 domains determined for our structural models (see **Table S1**). The vertical dash line marks the median size of domain (3.5 Mb). Spheres representations are illustrated where the volume of each sphere is proportional to the genomic length of represented domain. The spheres also include 27 “dummy beads”, filling the genomic gaps and serve as excluded volumes, totaling 666 beads. In total, there are 1332 beads within each nucleus representing female human lymphoblastoid genome model.

B.2. Figure S2

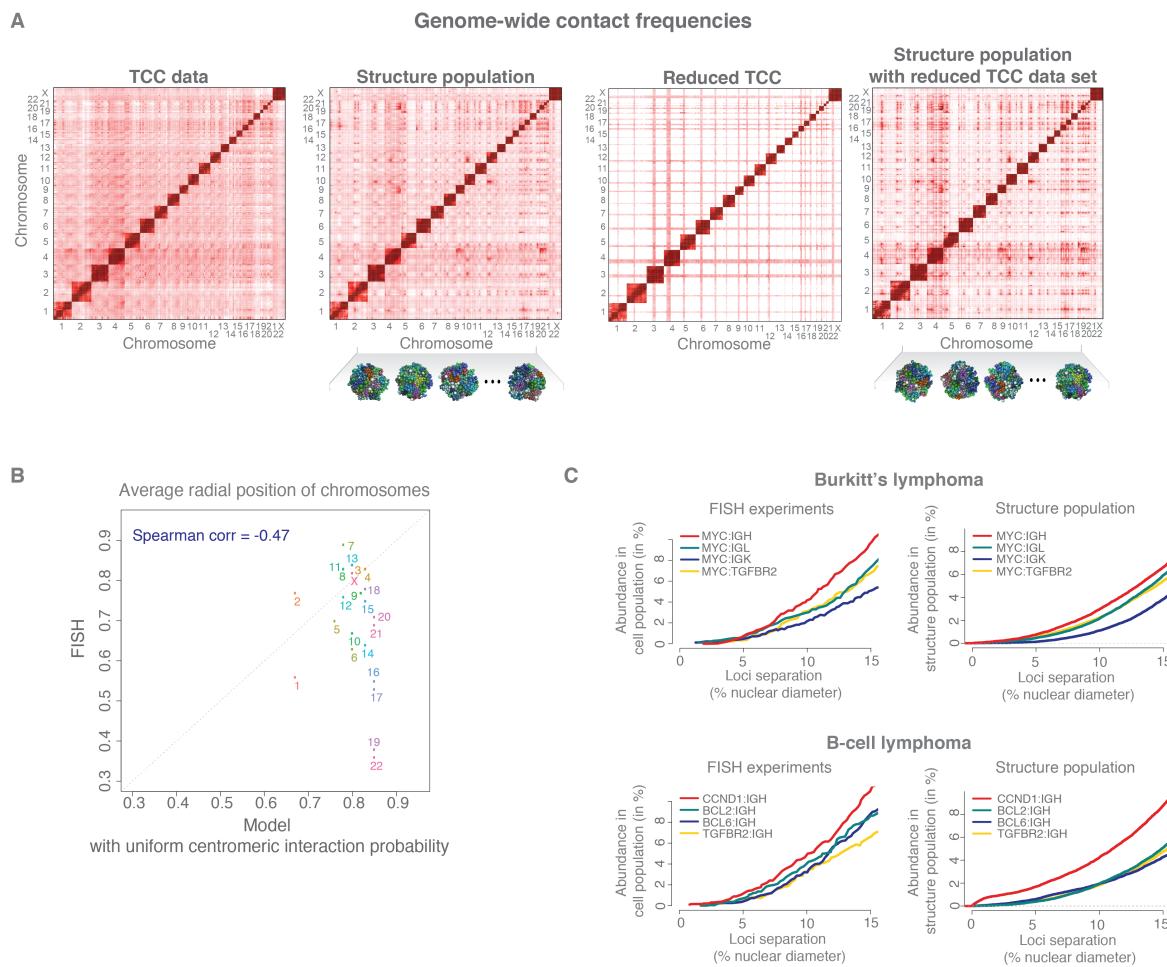


Figure S2 Comparison between models and experiments. **(A)** Genome-wide contact probability heat maps. From left to right: TCC data, structure population, reduced TCC data, structure population from the reduced TCC data. The reduced TCC data only includes intrachromosomal and subcentromeric interchromosomal interactions. To visualize the heat maps, the relative bin size reflects the corresponding domain size. The color scale ranges from white to dark red, for low to high contact frequencies respectively. **(B)** Comparison of the chromosomes' average radial positions from FISH data and a structure population with uniform centromere-centromere interaction probabilities. This model does not reflect the correct chromosomes positions in nucleus. **(C)** Cumulative frequency distribution of gene pair's distances replotted in the same way as in the original Roix et al. paper (27), to compare results from FISH experiments (left panels) and structure population (right panels). Four pairs of genes associated with Burkitt's lymphoma (top panels), and B-cell lymphoma (bottom panels).

B.3. Figure S3

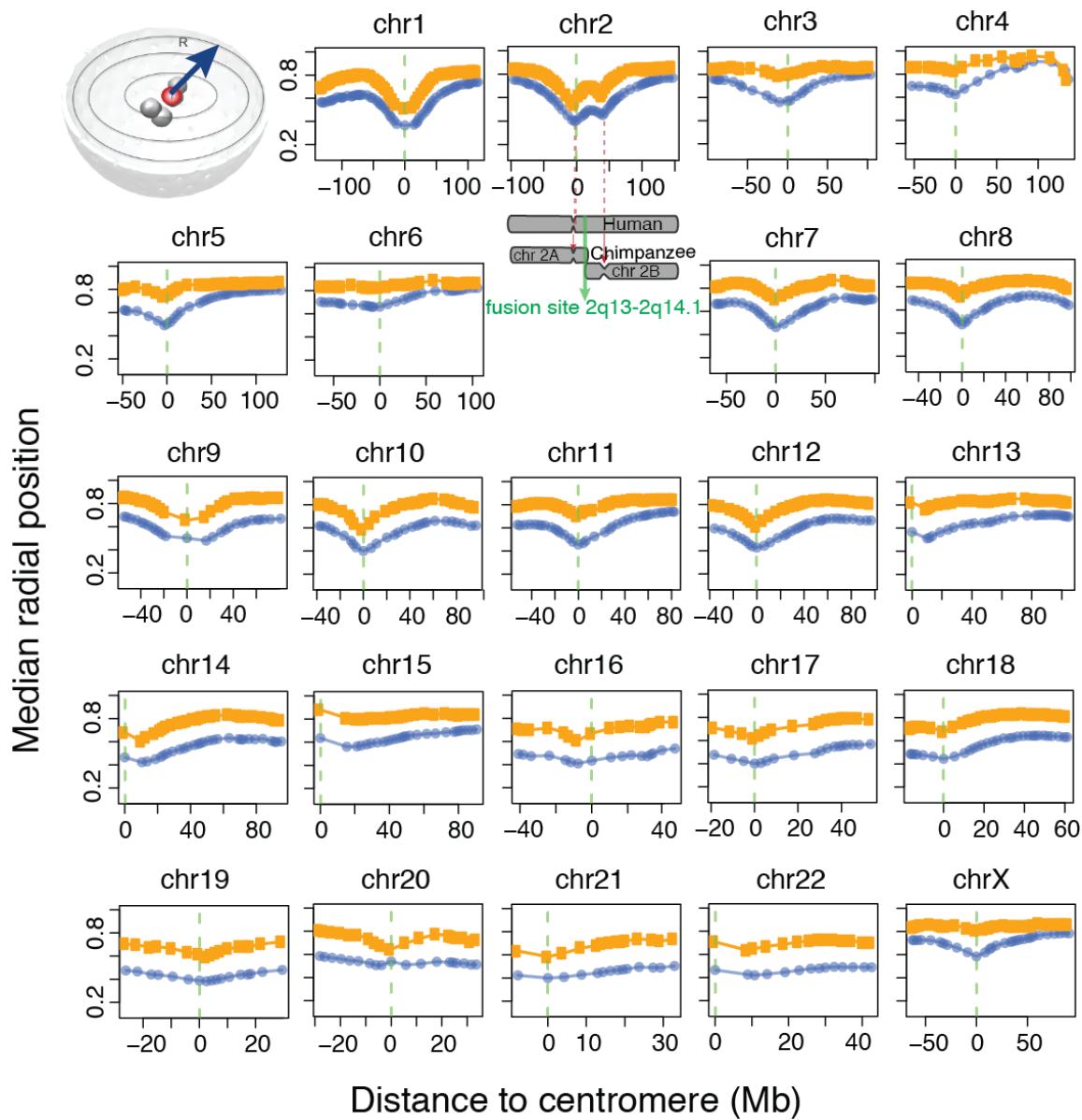


Figure S3 Radial positions. The median radial position of each chromosome domain is plotted against its sequence position (related to Fig. 4A). Blue and orange curves correspond to the domain positions calculated from the radially inner- and outer-most chromosome copy in each structure, respectively. Centromere is at position 0, marked with the vertical green dashed line. For most chromosomes, a dip is apparent near centromere. Below the box of chromosome 2, we illustrate the hypothetical head-to-head fusion of ape's chromosomes 2A and 2B. The second dip in the plot for chromosome 2 could be related to the vestigial centromere. The q-telomere of chromosome 4 is known to locate nearby the nuclear envelope(3), thus we include this information in the model

B.4. Figure S4

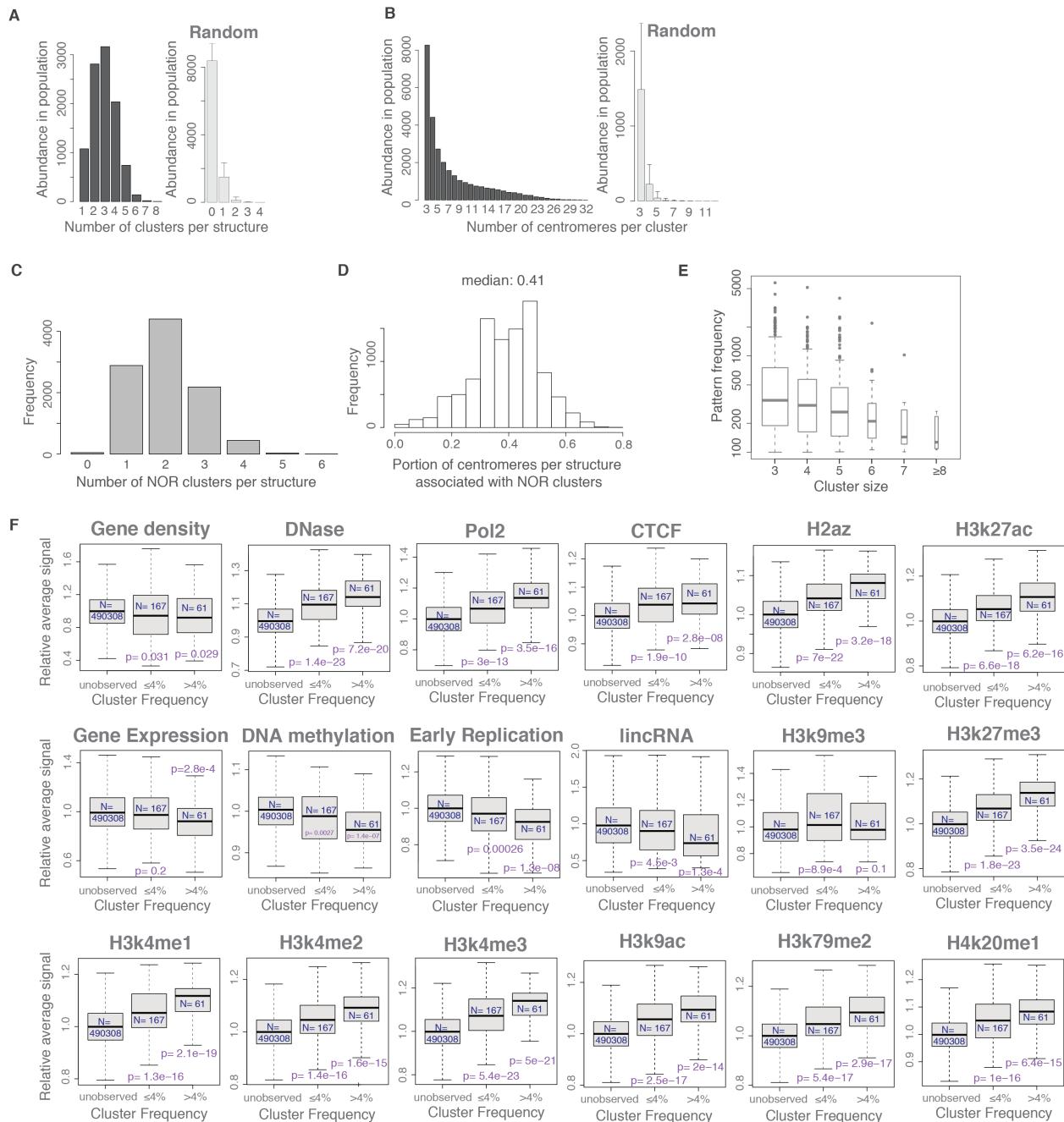


Figure S4 Centromere clusters. **(A)** (left panel) Histogram showing the abundance of centromere clusters per structure in the population. The median and most abundant number is 3. (Right panel) Random control. Histogram showing the abundance of clusters when one domain per chromosome is randomly picked in the structure population; error bars are standard deviations from 1,000 randomizations. The random control shows a dramatically reduced interchromosomal clustering and the majority of structures show no clusters. **(B)** (left panel) Histogram of the size distribution of centromere clusters observed in the structure population. The number of observed clusters decreases sharply with the increase of cluster size. (right panel) random control defined as in **A**. **(C)** The histogram of the number of NOR (nucleolus organizing region) clusters if defined as clustered centromeres that contain at least one acrocentric centromere. With this definition, roughly 66.7% of centromere clusters contain NOR. **(D)** Histogram of fraction out of 46 centromeres that participate in NOR clusters. About 40% of centromeres are associated with NOR clusters. The fraction is computed for each structure, thus the total of data point in the histogram is 10,000. Data

shown in panels A-D were generated with the method described in section A.5.1. (E) Box-and-whisker plots showing the relationship of frequency and size of the 798 centromeric frequent pattern clusters (above 1% abundance in structure population). The more centromeres involve the less frequent such patterns occur in structure population. (F) Epigenetic signals of pericentromeric regions from the frequent pattern clusters grouped in different frequencies: unobserved (below 1% cutoff; see **Material and Methods** section “Detection of centromere cluster recurrent patterns”), infrequent (1%-4%), and frequent (>4%). The p-values shown are calculated with one-sided Wilcoxon test against the ‘unobserved’ group. As an example, clusters formed by more than 4 centromeres are selected; the conclusion still holds as well using different cluster size.

B.5. Figure S5

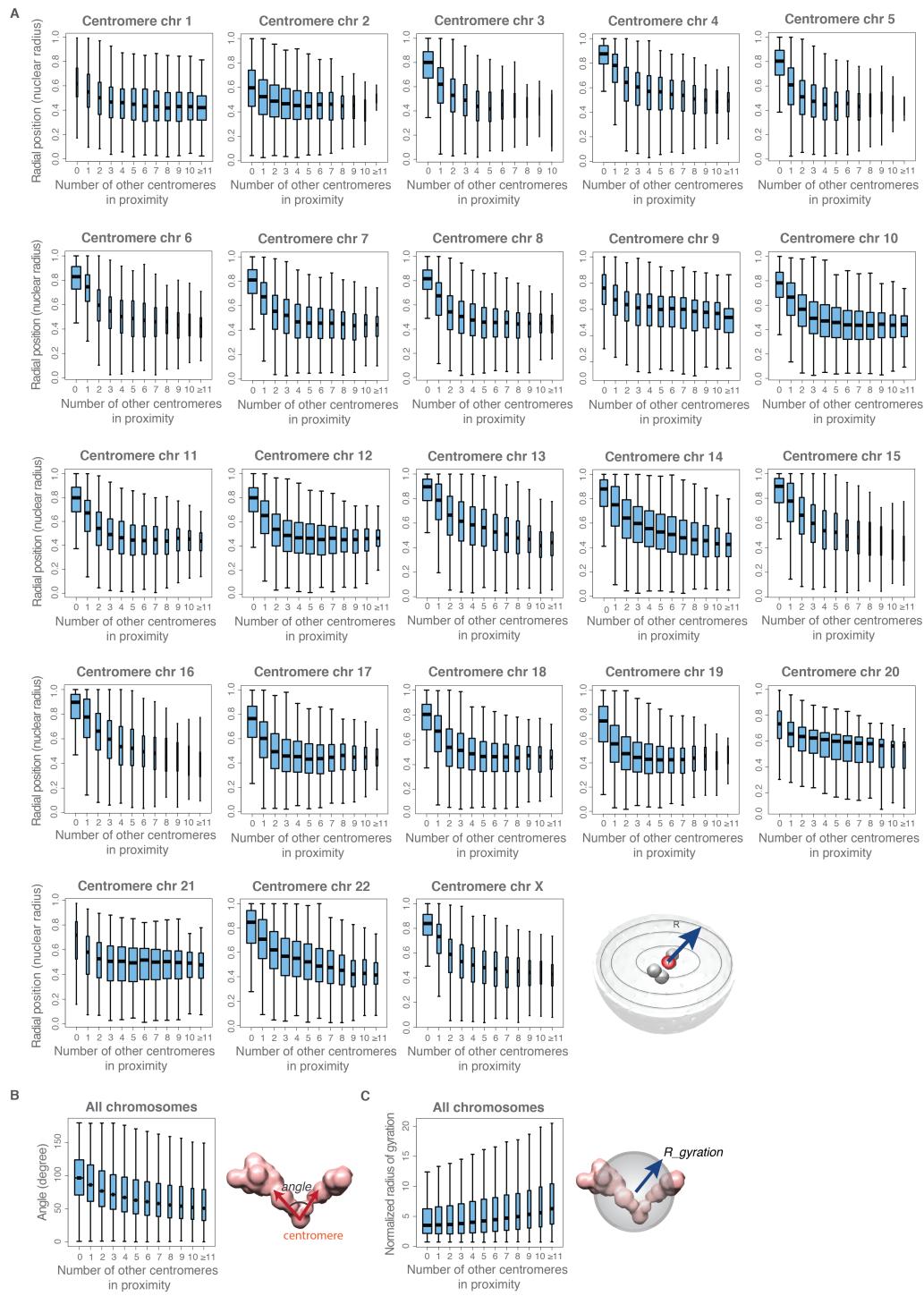


Figure S5 (A) Box-and-whisker plots of radial position of centromeres for each chromosome as a function of cluster size (related to Fig. 6A). The whiskers mark the range of data group within each box. The outliers are not shown. **(B)** Box-and-whisker plots of angle formed between chromosome arms within 30 Mb from centromeres as a function of cluster size. **(C)** Box-and-whisker plots showing radius of gyration normalized by the chromosome size (genomic length) as a function of other centromeres in contact. In general, there is a trend of chromosomes to be more elongated when the centromeres have more other centromeres nearby.

B.6. Figure S6

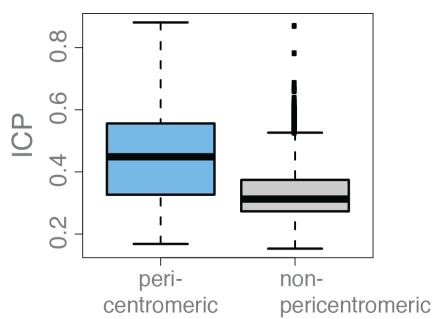


Figure S6 Comparison of Inter-chromosomal interaction fraction (ICP) between pericentromeric regions and the rest of the genome. The ICP was calculated from a genome-wide matrix binned every 138-hindIII sites (the pericentromeric regions were taken up to 2 bins from the right and left centromeric gaps). See section A.5.3 for detail on ICP.

B.7. Figure S7

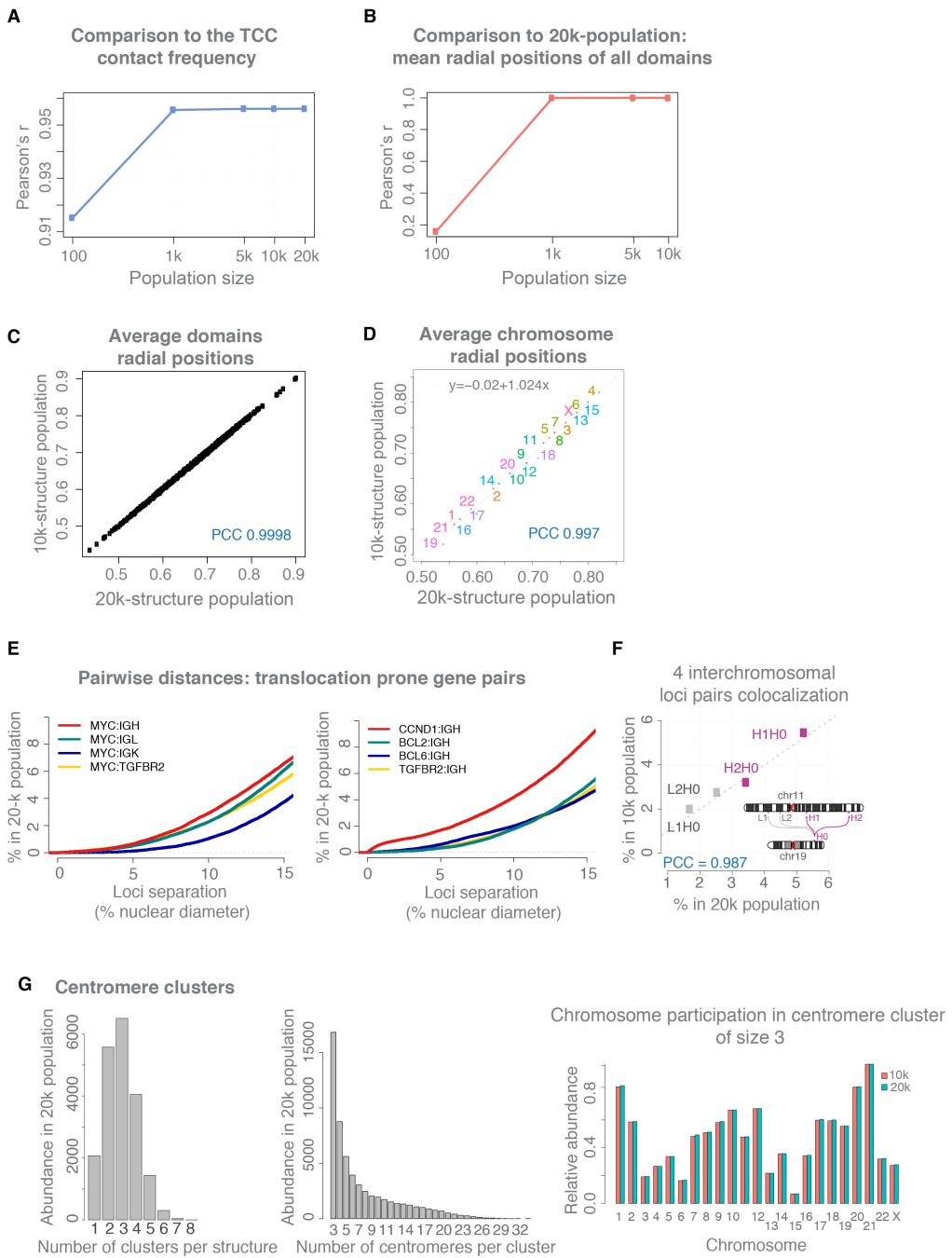


Figure S7 Convergence tests. **(A)** Pearson's correlation coefficients between normalized contact frequency matrices from the experimental TCC data and structure populations with increasing population sizes (section A.5.2). **(B)** Pearson's correlation coefficients of the mean radial positions of all domains in a population of 20,000 structures compared with those from populations ranging in size between 100 and 10,000 structures. **(C)** Comparison of the average radial positions of each domain in populations of 10,000 and 20,000 structures. **(D)** Comparison of the average radial position of chromosomes from populations of 10,000 and 20,000 structures. **(E)** Cumulative frequency plots of distances between pairs of domains in the population of 20,000 structures (related to Fig 3C or Fig. S2C). **(F)** Comparison of the colocalization frequency for 4 interchromosomal loci pairs in a population of 10,000 structures (presented in the main text) and a population of 20,000 structures. **(G)** (Left and middle panels) The statistic of centromere clusters found in the population of 20,000 structures (related to Figs. S4A and S4B). (Right panel)

Chromosome propensity to participate in centromere clusters containing 3 centromeres in the populations with 10,000 and 20,000 structures, respectively. For each structure, a colocalization event of a triplet (3 centromeres form different chromosomes) is called if all pairwise distances connecting the triplet are less than a threshold (1.5 micron) (see **Fig. 5B** bottom mid panel). The total number of detected triplet combinations is 1771. The propensity is calculated as the total frequency of the chromosome to be found in all clusters. The maximum height of the bar is normalized to 1. The bar plots show that the results from both population are essentially identical.

B.8. Figure S8

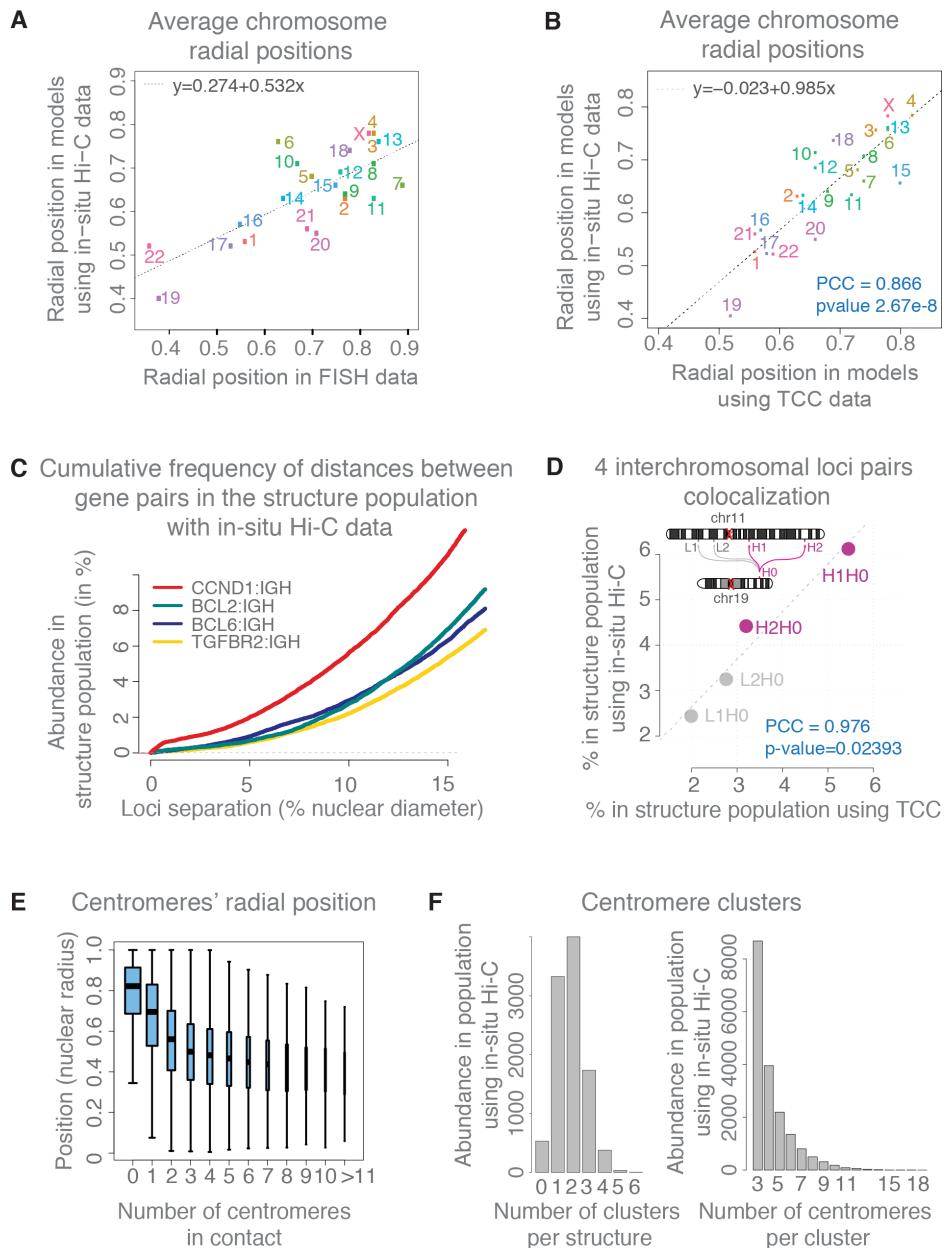


Figure S8 Results from the structure population generated with high-resolution in-situ Hi-C data (16). **(A)** Comparison of chromosomes' average radial positions between the structure population generated with the in-situ Hi-C data and FISH data (15). **(B)** Comparison of chromosomes' average radial positions in structure populations generated with in-situ Hi-C and TCC. **(C)** Cumulative frequency of distances between gene pairs in the structure population generated with the in-situ Hi-C data. The trend is very similar to that in the structure population generated with TCC data (**Fig. 3C**). **(D)** Comparison of colocalization counts for the 4 interchromosomal loci pairs between structure populations generated with in-situ Hi-C and TCC data (related to **Fig. 3B**). **(E)** Radial position of any centromere as a function of the number of other centromeres around it in the structure population generated with in-situ Hi-C data (related to **Fig. 6A**). **(F)** Histogram showing the abundance of centromere clusters per structure (left panel), and the cluster size distribution in the population generated with in-situ Hi-C data (related to **Figs. S4A** and **S4B**).

B.9. Figure S9

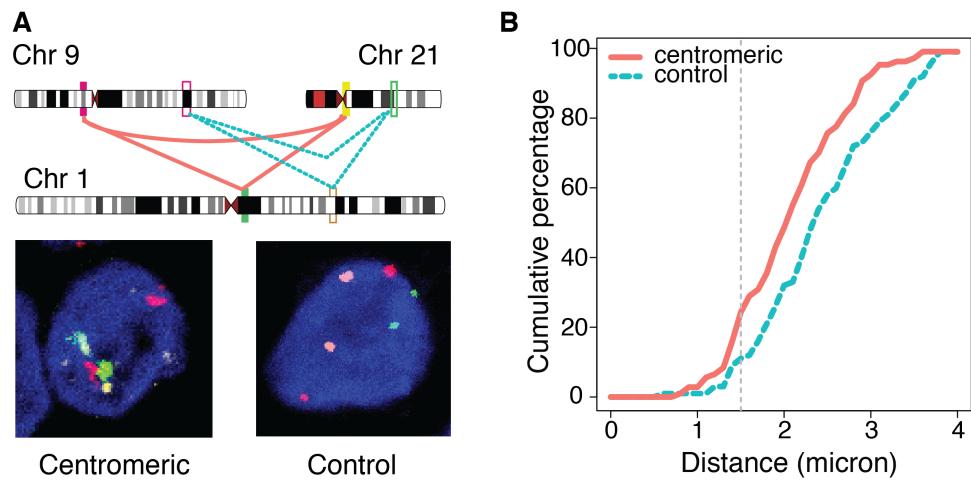


Figure S9 Centromeres as contact points for interchromosomal interactions (**A**). FISH experimental set up to verify that centromeric regions are bridging the interaction inter-chromosomally. A group of pericentromeric and distal regions from the respective centromere locations are referred to as the “centromeric” and “control” domains, respectively. (**B**) The cumulative frequency plots of the shortest triplet distance in each cell for the centromeric and control experiments. The triplet distance formed by pericentromeric regions is on average smaller than that formed by the control regions.

172	chr5	94801807	96487064	385	chr12	38939961	45442952	598	chr22	16050000	17779305
173	chr5	96487065	102956932	386	chr12	45442953	48849895	599	chr22	17779306	19819434
174	chr5	102956933	106586805	387	chr12	48849896	58107253	600	chr22	19819435	24548561
175	chr5	106586806	112774600	388	chr12	58107254	60236681	601	chr22	24548562	29546412
176	chr5	112774601	114319751	389	chr12	60236682	62355762	602	chr22	29546413	32261721
177	chr5	114319752	116517298	390	chr12	62355763	67847079	603	chr22	32261722	35829153
178	chr5	116517299	121317771	391	chr12	67847080	70171457	604	chr22	35829154	36889692
179	chr5	121317772	127610500	392	chr12	70171458	72525591	605	chr22	36889693	39148469
180	chr5	127610501	130646898	393	chr12	72525592	75913586	606	chr22	39148470	41692258
181	chr5	130646899	134337915	394	chr12	75913587	77257973	607	chr22	41692259	43741167
182	chr5	134337916	137126622	395	chr12	77257974	81328268	608	chr22	43741168	47287066
183	chr5	137126623	143379965	396	chr12	81328269	89756495	609	chr22	47287067	49927446
184	chr5	143379966	148580161	397	chr12	89756496	90569009	610	chr22	49927447	51244565
185	chr5	148580162	151211499	398	chr12	90569010	92183896	611	chrX	60000	2834822
186	chr5	151211500	156063324	399	chr12	92183897	97181864	612	chrX	2834823	6614453
187	chr5	156063325	159655616	400	chr12	97181865	101887789	613	chrX	6614454	10007596
188	chr5	159655617	166994349	401	chr12	101887790	105727373	614	chrX	10007597	25413047
189	chr5	166994350	170666438	402	chr12	105727374	108900562	615	chrX	25413048	28860834
190	chr5	170666439	180905259	403	chr12	108900563	114227242	616	chrX	28860835	31457611
191	chr6	60000	8069471	404	chr12	114227243	120409337	617	chrX	31457612	36730613
192	chr6	8069472	10210449	405	chr12	120409338	125962433	618	chrX	36730614	50185810
193	chr6	10210450	18423275	406	chr12	125962434	131371004	619	chrX	50185811	57807014
194	chr6	18423276	20159475	407	chr12	131371005	133841894	620	chrX	57807015	67500977
195	chr6	20159476	21548362	408	chr13	19020000	20293983	621	chrX	67500978	74620720
196	chr6	21548363	24230912	409	chr13	20293984	22047884	622	chrX	74620721	76466498
197	chr6	24230913	44458000	410	chr13	22047885	23991532	623	chrX	76466499	78556126
198	chr6	44458001	47975922	411	chr13	23991533	32406542	624	chrX	78556127	85514782
199	chr6	47975923	51622205	412	chr13	32406543	34674298	625	chrX	85514783	95616814
200	chr6	51622206	53819516	413	chr13	34674299	40150597	626	chrX	95616815	97343516
201	chr6	53819517	56331401	414	chr13	40150598	47614184	627	chrX	97343517	99918171
202	chr6	56331402	57126069	415	chr13	47614185	48459863	628	chrX	99918172	103464238
203	chr6	57126070	70111157	416	chr13	48459864	53477849	629	chrX	103464239	105726332
204	chr6	70111158	74655914	417	chr13	53477850	60296281	630	chrX	105726333	112341341
205	chr6	74655915	87677350	418	chr13	60296282	61103033	631	chrX	112341342	115150654
206	chr6	87677351	91323208	419	chr13	61103034	64599469	632	chrX	115150655	117035759
207	chr6	91323209	97041479	420	chr13	64599470	68601651	633	chrX	117035760	119946726
208	chr6	97041480	101316527	421	chr13	68601652	72957380	634	chrX	119946727	128504492
209	chr6	101316528	105182394	422	chr13	72957381	81351881	635	chrX	128504493	136300266
210	chr6	105182395	112572391	423	chr13	81351882	91452495	636	chrX	136300267	142251119
211	chr6	112572392	134195186	424	chr13	91452496	92347230	637	chrX	142251120	146863673
212	chr6	134195187	139963608	425	chr13	92347231	94916114	638	chrX	146863674	151979450
213	chr6	139963609	142711202	426	chr13	94916115	96864393	639	chrX	151979451	155260559

Table S1 List of 639 domains from the constrained hierarchical clustering method (see section A.4).

C.2. Table S2

Description	Symbol	Value	Unit
Number of spheres (domains)	$2N$	2x666	N/A
Number of structures	M	10,000	N/A
Stepwise optimization in probability threshold	Θ	{1, 0.4, 0.1, 0.01}	N/A
Nuclear occupancy	O_{nuc}	0.2	N/A
Radius of nucleus	R_{nuc}	5,000	IMP length
Harmonic constant	k	1	IMP unit
Sphere mass	$mass$	1	IMP mass
Temperatures, simulated annealing	T	Vary 300-500,000	IMP unit

Table S2 Modeling parameters used in this paper.

C.3. Table S3

Library	HindIII Tethered
Total clusters	211,592,642
Unique alignments:	
First end	175,086,554
Second end	170,949,684
-----	-----
Total pairs	147,262,098
Non-informative:	
PCR multiplications	<i>10,337,451</i> (7%)
Flaking	<i>26,404,870</i> (18%)
Self-looping	<i>11,886,208</i> (8%)
Filtered pairs (final catalogue)	98,633,569 (67%)

Table S3 The sequencing, alignment, pairing, and filtering statistics for the library. The italicized numbers for PCR multiplication, flaking, and self-looping mark the pairs that were filtered out of the initial catalogue in order to obtain the final catalogue. Numbers in parentheses are percentage values of each category compared to the “Total pairs” row. The last row (“Filtered pairs”) represents the catalogues that were used for all later analyses.

C.4. Table S4

Centromere Position	Gap in hg19 genome		Centromere bead domain		Centromere region for epigenetic	
	Start	End	Start	End	Start	End
chr1	121535434	124535433	120471507	144357570	116535421	144357571
chr2	92326171	95326170	92326171	95326170	87326158	100326183
chr3	90504854	93504853	90504854	93504853	85504841	98504866
chr4	49660117	52660116	47099935	58329053	44660104	58329054
chr5	46405641	49405640	46405641	49405640	41405628	54405653
chr6	58830167	61830165	57126070	70111157	53830154	70111158
chr7	58054331	61054330	56446576	64149744	53054318	66054343
chr8	43838887	46838886	43214215	48204267	38838874	51838899
chr9	47367680	50367678	38640740	69684409	38640739	69684410
chr10	39254936	42254934	32640254	43087186	32640253	47254947
chr11	51644206	54644204	48410902	55662028	46644193	59644217
chr12	34856694	37856693	32949875	38939960	29856681	42856706
chr13	16000000	18999999	0	19019999	15999999	24000012
chr14	16000000	18999999	0	18999999	15999999	24000012
chr15	17000000	19999999	0	19999999	16999999	25000012
chr16	35335801	38335800	35285801	46385800	30335788	46385801
chr17	22263006	25263005	20942909	25799526	17262993	30263018
chr18	15460898	18460897	13580868	19065138	10460885	23460910
chr19	24681782	27681781	24631782	27731781	19681769	32681794
chr20	26369569	29369568	26104675	30158120	21369556	34369581
chr21	11288129	14288128	9411193	15606100	6288116	19288141
chr22	13000000	15999999	0	16049999	12999999	21000012
chrX	58632012	61632011	57807015	67500977	53631999	67500978

Table S4 Positions in hg19 genome that flank centromere gaps, domain borders represented by centromeric beads in our model, and centromeric regions that were used for epigenetic data extraction of the recurrent structural patterns.

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