Q1) For the image classifier I recreated the model used in the keras example using the direct tf.reshape, tf.nn.relu, tf.nn.dropout and tf.nn.softmax functions rather than the keras specified ones. The epochs are broken down into batches of size 128 and each run through the model. Once the model is run through, there is a sparse softmax cross entropy loss function to obtain the loss to use to get the gradients of the weights and biases. These gradients alter the weights and biases to train the model. The training goes at a much slower rate than the keras training and often runs into a local minima that prevents further training. To run 50 epochs, it took roughly 666 seconds and about a 1.57 loss which is much higher than the keras training of 0.0075 loss.

Q2) The use of tensorflow 2.0 gpu is required for this question as it uses the gpu to obtain much faster speeds than the cpu. Tensorflow offers a quick switch between devices by telling it the name (which tensorflow can give the name of). It can be determined which device is being used by calling the tensor, which says which device it has been run on. The times can also be an indicator as the gpu runs as a significantly quicker pace as it can use its cuda cores in parallel much quick than the few cores on a cpu.

Q3) The Jacobian can be found by using multiple equations in a 2x2 matrix and finding their derivatives. Once their derivatives are found, the values can be input into the needed positions and solve for the determinant.

Q4) The eigen values are determined by using a lambda matrix to find their values and solve for the determinant. Once the eigen values are found, they can be used to determine the eigen values.

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$y = 3x^2 + 2x + 3, \quad x = \binom{1}{3}$

$\binom{y_1}{y_2} = 3\binom{x_1}{x_2}^2 + 2\binom{x_1}{x_2} + 3$

$$J = \begin{vmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial x_2} \\ \frac{\partial y_2}{\partial x_1} & \frac{\partial y_2}{\partial x_2} \end{vmatrix}$$

$$J = \begin{vmatrix} y_1 = 3x_1^2 + 2x_1 + 3 & y_1 = 3x_2^2 + 2x_2 + 3 \\ y_2 = 3x_1^2 + 2x_1 + 3 & y_2 = 3x_2^2 + 2x_2 + 3 \end{vmatrix}$$

$$J = \begin{vmatrix} 6x_1 + 2 & 6x_2 + 2 \\ 6x_1 + 2 & 6x_2 + 2 \end{vmatrix}$$

$$J = (6x_1 + 2)(6x_2 + 2) - (6x_2 + 2)(6x_1 + 2)$$
$$= 36x_1x_2 + 12x_1 + 12x_2 + 4 - 36x_1x_2 - 12x_2 - 12x_1 - 4$$
$$= 0$$

OR

$$J = \begin{vmatrix} 6(1) + 2 & 6(3) + 2 \\ 6(1) + 2 & 6(3) + 2 \end{vmatrix}$$

$$= \begin{vmatrix} 8 & 20 \\ 8 & 20 \end{vmatrix}$$

$$= (8)(20) - (8)(20) = 0$$

$$A = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$$

1) $\lambda I = \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$

2) $A - \lambda I = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$

$= \begin{bmatrix} 3-\lambda & 2 \\ 2 & 3-\lambda \end{bmatrix}$

3) $\det \begin{bmatrix} 3-\lambda & 2 \\ 2 & 3-\lambda \end{bmatrix} = (3-\lambda)^2 - 4$

$= 9 - 6\lambda + \lambda^2 - 4$

$= \lambda^2 - 6\lambda + 5$

$(\lambda - 5)(\lambda - 1) = 0$

$\lambda = 5 \quad \lambda = 1$

$\lambda = 5, 1$ $\begin{bmatrix} 3-\lambda & 2 \\ 2 & 3-\lambda \end{bmatrix}$

Solve for $\lambda = 1$

$$\begin{bmatrix} 3-1 & 2 \\ 2 & 3-1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = B$$

$$B\bar{x} = 0$$

$$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 2 & 2 & 0 \\ 2 & 2 & 0 \end{array}\right]$$

$$R_1 - R_2 \rightarrow R_2$$

$$= \left[\begin{array}{cc|c} 2 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$$2x_1 = -2x_2$$

$$\boxed{\begin{aligned} x_1 &= 1 \\ x_2 &= -1 \end{aligned}}$$

$\therefore$ when $\lambda = 1$, the eigenvector is $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\lambda = 5, 1$ $\begin{bmatrix} 3-\lambda & 2 \\ 2 & 3-\lambda \end{bmatrix}$

Solve for $\lambda = 5$

$$\begin{bmatrix} 3-5 & 2 \\ 2 & 3-5 \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} = B$$

$$B\underset{\sim}{x} = 0$$

$$\begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$= \left[\begin{array}{cc|c} -2 & 2 & 0 \\ 2 & -2 & 0 \end{array}\right]$$

$R_1 + R_2 \to R_2$

$$= \left[\begin{array}{cc|c} -2 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$$-2x_1 + 2x_2 = 0$$

$$-2x_1 = -2x_2$$

$$x_1 = 1$$

$$x_2 = 1$$

$\therefore$ when $\lambda = 5$, the eigenvector is $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$