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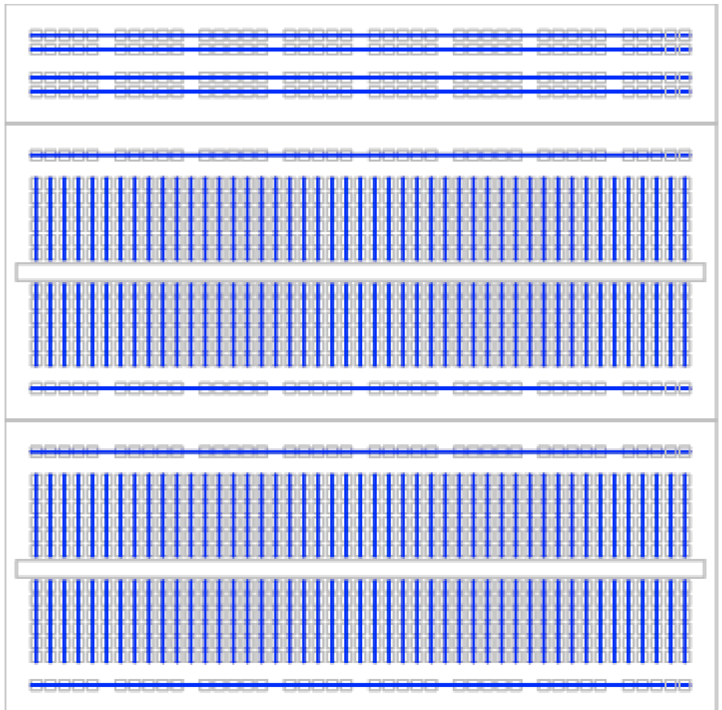
Lab 1 : Linear Circuits

Introduction

The purpose of the first part of the lab is to teach about equipment such as the wave generator, oscilloscope, and DMM, as well as breadboards and linear circuit components. To understand the terminology surrounding the equipment and linear circuits, the lab focused on Thevenin equivalence, the voltage divider, and input and output impedances. Furthermore, the lab introduced circuits with wavelike behavior such as a long coaxial cable and presented the effects of impedance matching signals. The experiment of the long coaxial cable included different boundary conditions for the cable and visualization of the resultant signal on the oscilloscope. The lab stressed mastery of the oscilloscope and wave generator in understanding their interface and behavior. Aside from studying equipment, voltage dividers, and Thevenin equivalence, the second part of the lab introduced RC filters as subset of the knowledge of voltage dividers. A typical exercise in the lab is comprised of predicting or deriving the behavior of a circuit based on the voltage divider model and confirming the prediction with measurements. This was done first for a purely resistive circuit, and then for a circuit containing capacitors. The utility of the black box model of linear circuits was a recurrent topic as well, meant to simplify the complexity of a linear circuit with full knowledge of its behavior. The completion of the lab is meant as a gateway to the following labs which require all the knowledge introduced herein.

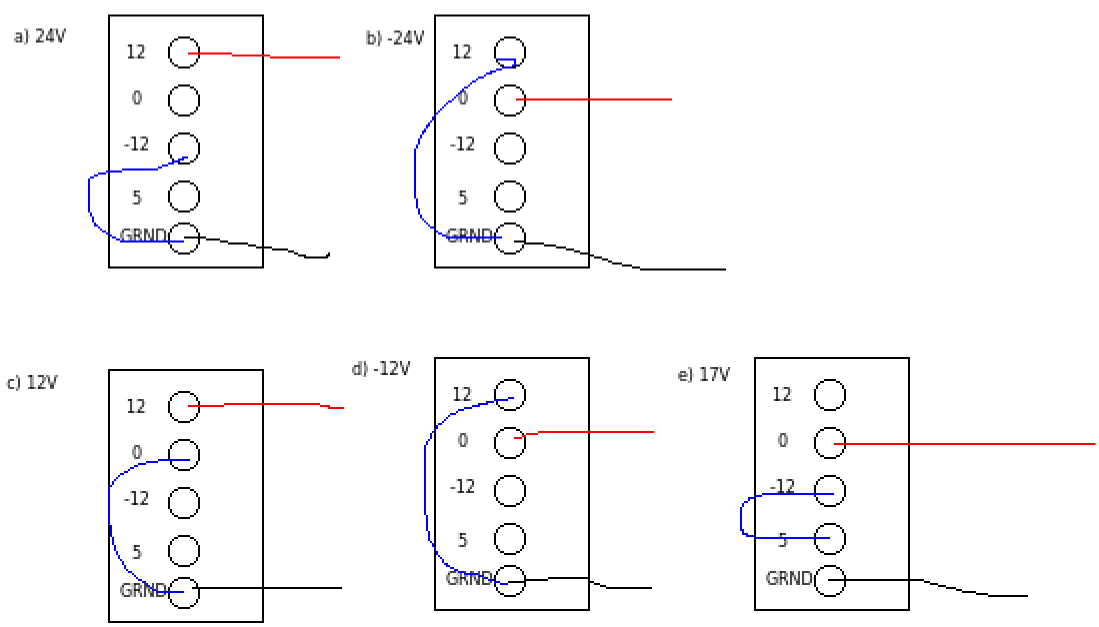
Problem 1.1.1
We used the DMM in resistance measuring mode (omega^2) to measure the resistance between various sockets in the breadboard. A measurement of finite resistance meant the sockets were connected by a wire, and infinite or overloaded resistance meant they were not connected. Our results can be summarized in the following graphic (lines represent connections):



From the Lab 1 writeup

Figure 1.

Problem 1.1.2
The 5V socket is always 5V wrt GRND. The other three all have constant voltage wrt each other, but their voltages wrt GRND can be fixed, causing the triplet to float together. Using these principles, the following graphic illustrates how to generate certain voltages wrt ground: (Figure 2.)



The bottom wire is ground (black) and top wire is red

Problem 1.1.3
Next we constructed the setups shown in the previous problem and measured their voltages with the DMM. The measurement of voltage was done simply by connecting the ground to ground and hot to hot wires from the power supply to the DMM on voltage mode. Here are the results for the measured voltages:
a. 24.5V, b. -24.5V, c. 12.2V, d. -12.2V, e. 17.6V
When measuring the voltage between +12V and GRND the DMM reads 0V. This is because 12,0,-12 are fixed wrt each other but can otherwise float. In the process +12 set itself to be the same voltage as GRND, and the result is a measurement of 0V.

Problem 1.1.4
The following circuit is a voltage divider:

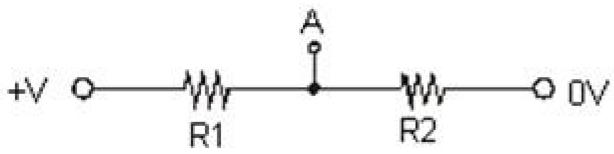


Figure 3. From Lab1 writeup

The current in the bottom wire is I = V/(R1+R2) using Ohm's law. Therefore the voltage drop across R2 = I*R2 = R2/(R1+R2) * V. Since the voltage at the rightmost wire is 0V and the drop across R2 is R2/(R1+R2)*V, the voltage in the middle wire must be V_A = R2/(R1+R2)*V.
In the lab we built the following voltage divider circuit using the breadboard, power supply, and the DMM for measurements.

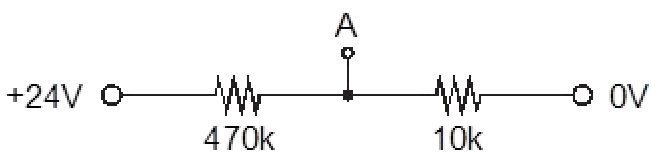
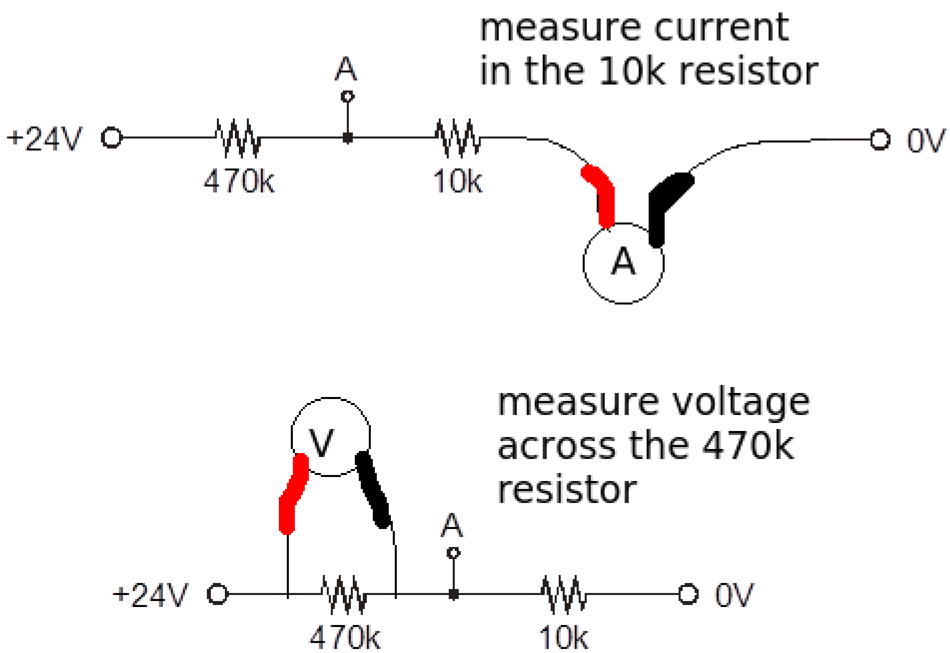


Figure 4. From Lab1 writeup

Problem 1.1.5
Using the nominal values in the circuit of Figure 4, the current in the wire, given by the equivalent resistance of R1 and R2, namely R_eq = R1+R2, and Ohm's law, namely I =V/R, we get that I = V/(R1+R2) = 24V / (470kohm + 10kohm) = 5e-5A
Problem 1.1.6
Using the formula derived in 1.1.4, V_A = R2/(R1+R2)*V, we can calculate V_A using the nominal values. V_A = 10kohm / (470kohm + 10kohm) * 24V = 0.5V

Problem 1.1.7
To measure the current through the 10k resistor, connect the DMM in ammeter setting to the circuit in series with the 10k resistor. Before or after it does not matter since it is on the same wire.
To measure the voltage drop across the 470k resistor connect it in voltmeter setting with one plug to the left and one to the right of the resistor. This connection is in parallel to the resistor. See the diagram: (Figure 5.)



lighter lead on the left is red, darker on the right is black
Figure 5.

Problem 1.1.8
Next we measured the actual resistances of the 10k and 470k resistors.
“470k” was actually 474k
“10k” was actually 9.92k
Both have a tolerance of 5% and are within that tolerance.
The “24V” power supply was actually 24.5V and is within 5% tolerance as well.
Problem 1.1.9
Actual current: I = V/(R1+R2) = 24.5V / (474kohm + 9.92kohm) = 5.06e-5A
The difference between nominally calculated and recalculated current = 6.28e-7A

Actual voltage at A: V_A = R2/(R1+R2)*V = 9.92kohm/(474kohm+9.92kohm) * 24.5V = 0.502V
The difference between nominally calculated and recalculated voltage = 0.002V
Problem 1.1.10
We then measured the voltage at A using the DMM at a range of 1V.
V_A = 0.5054V, error = 1e-4V
The measurement is very close to the recalculated value with a difference of 3e-3V. This is beyond the error of the DMM.
Problem 1.1.11
We then measured the current as 5.1e-5A, error = 4.06e-7A. The error is 0.012% of measured value + 0.004% of the range on the DMM. This only holds for DMM measurements.
The difference from the calculated value is 4.4e-6A, which is beyond the allowed error.

Problem 1.1.12
Using the nominal values, the power dissipation of each resistor can be calculated:
P = I^2*R P1 = (5e-5A)^2*470kohm = 1.175mW , P2 = (5e-5A)^2*10kohm = 0.025mW.
The resistors are rated for this power. They are rated for 0.25W.
In order to approach the maximum power rating, assuming the ratios of the resistors are kept constant, one would have to decrease the resistances to get more power. The formula P=I^2*R is unsafe to use for this purpose since I depends on R. The voltage is assumed to be kept at a constant 24V, so the formula P = V^2/R is safe to use. One can see how decreasing R would increase the power dissipated. If this is done, one may want to know which resistor would reach the maximum power first. For any given resistor with voltage drop V, resistance R, and maximum power W_m, pick a fraction f such that at a fraction of that resistance the power maxes out. Namely, V^2/(f*R) = W_m.
With some algebra we find that f = V^2/(W_m*R). The resistor with the higher f-value would max out first as we proportionally minimize the resistances. Calculating f-values for the resistors in Figure 4:
f1 = (V – V_A)^2/(W_m * R1) = (23.5V)^2/(0.25W*470kohm) = 0.0047
f2 = V_A ^2/(W_m*R2) = (0.5V)^2 / (0.25W*10kohm) = 0.0001
Since f1 > f2, R1 = 470k would max out first. To find how much smaller the resistors have to get for R1 to max out, simply take the reciprocal of the largest f-value, namely 1/0.0047 = 212.77 ~= 213 times smaller. To make such a circuit one would need a 2.2k resistor for R1 and a 47 ohm resistor for R2, which is entirely possible to do.

Problem 1.1.13 – No need to write anything down (learning to use scope)
Problem 1.1.14
It is very bad to put a voltage into the function generator, which is why connections must be checked before using it. If a voltage is accidentally inputted into it, my guess would be that at best it would sum up the voltages from the faulty connectors with the signal although this is doubtful since the function generator is a black box to us and we have no knowledge of its inner workings, and at worse it can destroy the wave generator.
Problem 1.1.15 – No need to write anything down (learning to use wave generator)
Problem 1.1.16
The equipment has been properly calibrated and the wave generator and scope agree.
Problem 1.1.17
In this part we measured the 5V power supply by connecting it to the scope and the DMM in parallel.
DMM measurement: 5.1863V, error = 1e-3V (calculated using the formula mentioned in 1.1.11)
On the setting of 1ms/div, the voltage was measured at different vertical scales from channel 1.
Scope measurement: 4.94V on 5V/div, 5.15V on 2V/div, 5.16V on 1V/div, 5.01V on 5V/div where the signal is brought close to 0V (with an error of up to 200 mV).
The results on less V/div are closer to DMM measurements.
On channel 2, 5V/div → 5.30V, 2V/div → 5.24V, 1V/div → 5.21V.
The results are somewhat consistent with an error of about 0.1V on the scope, determined by inspection. The measurement where the 5V is set to the lowest position on the scope at 1V/div yielded the most accurate result.
Problem 1.1.18
Setting the scope input switch to the AC setting makes it pass the signal through a high pass filter and removes low frequencies such as DC signals. It just shows 0V and noise. Scrolling through the timescale options one can see many sinusoids added together. In the finer scale the sinusoids of the higher frequencies can be seen. It mostly looks noisy but there is some level of periodicity and beat effects. The measured V_pp ~= 20mV

Problem 1.1.19

It is often helpful to be able to convert between the peak voltage (amplitude), peak to peak voltage, and RMS voltage. For sine waves, triangular waves, and square waves, the peak to peak voltage is simply double the amplitude. What follows is calculations of RMS voltages.

V_RMS = sqrt (mean (square(signal))) When the signal is periodical, the mean of the square can be calculated as the average value over a period. Using an integral:

$V_{RMS}=\sqrt{\frac{\int_0^T V(t)^2\,dt}{T}}$ Now given that integral, and with the knowledge that $T=\frac{2\pi}{\omega}$, one can plug in different periodic functions for the signal.

a) sinusoid

$$V(t)=V_0\sin(\omega t)$$

$$V_{RMS}=\sqrt{\frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} V_0^2\sin^2(\omega t)\,dt}=\sqrt{V_0^2\frac{1}{2}}=\frac{V_0}{\sqrt{2}}$$

b) triangular

$$V(t)=V_0\left(4\left|\frac{t}{T}\right|-1\right)\qquad t\in\left(\frac{-T}{2},\frac{T}{2}\right)$$

$$V_{RMS}=\sqrt{\frac{\int_{\frac{-T}{2}}^{\frac{T}{2}} V_0^2\left(4\left|\frac{t}{T}\right|-1\right)^2 dt}{T}}=\frac{V_0}{\sqrt{3}}$$

c) square: The values are always either at V0 or -V0, so the square is always V0^2, and the mean is V0^2, now take the square root, V0 and that is the V_RMS. So V_RMS = V0.

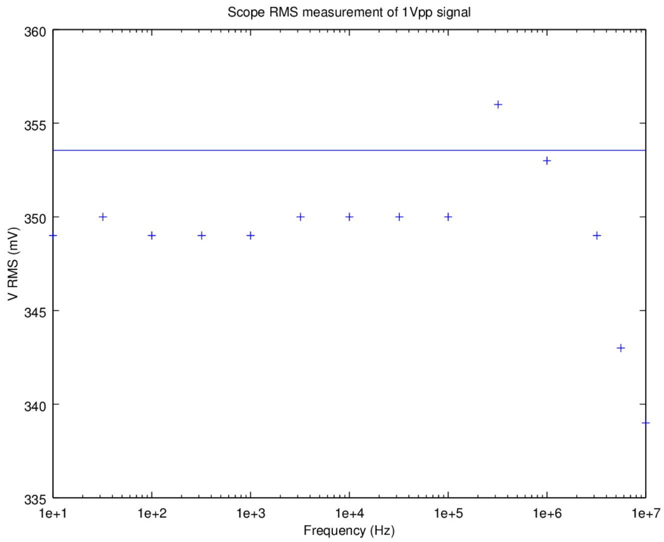
Calculated conversion	amplitude	Peak to peak voltage	V_RMS
sinusoid	0.5	1	0.35355339
triangular	0.5	1	0.28867513
square	0.5	1	0.5

To test that conversion table we fed a 1kHz 1Vpp wave of each type (sinusoid, triangular, square) to the scope, and used the scope to measure amplitude, peak to peak voltage and RMS. We used the automatic measurement options but checked to see that the results made sense before using any automatic measurement. We also used to DMM to measure RMS values.

Measured conversion	amplitude	Peak to peak voltage	V_RMS (scope)	V_RMS (DMM)
sinusoid	0.500 V	1.0V	0.351 V	0.347 V
triangular	0.500 V	1.0V	0.286V	0.284V
square	0.500V	1.0V	0.498V	0.491V

The error on the scope is around 0.003V by inspection, and 0.0001V from the DMM using the error formula. The measurements conform to the calculations to within 3mV from the scope, and 6-9 mV from the DMM. The maximal percent error is in the RMS measurement of the square wave of the DMM where the DMM is off by 9mV which is 1.8%.

Problem 1.1.20



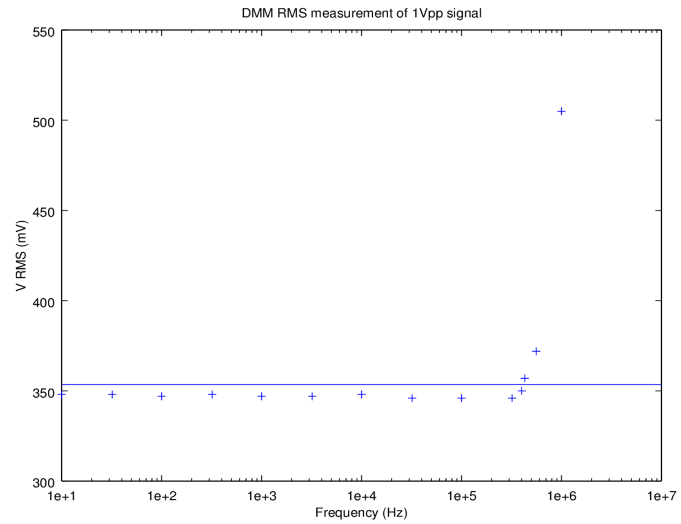


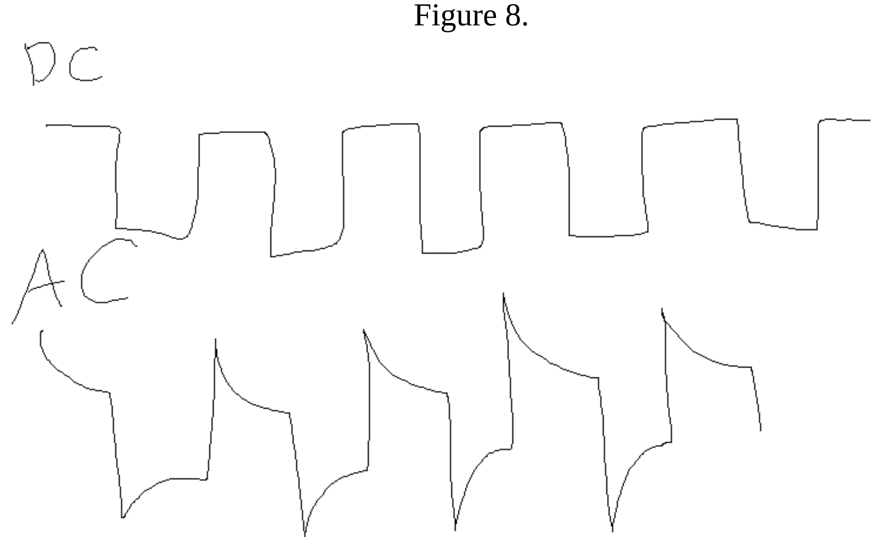
Figure 6. Figure 7.

Figures 6 and 7 show the RMS measurement of a 1Vpp signal in the scope and the DMM. The DMM is accurate up to 4e5 Hz, it performs to within specifications. When changing the scale at levels of accuracy, if the instrument is accurate at a wide range it is much faster to perform a "decimal search", and the result is more meaningful when collected geometrically in this case. It allows us to close in to where the measurement becomes inaccurate. The error on all DMM measurements was 0.08 mV.

Problem 1.1.21

When inputting a square wave into the scope at the AC setting, the AC setting acts as a high pass filter and attenuates low frequency components. Since the square wave is not a pure sinusoid, it is composed of lower and higher frequencies. The low frequencies give the wave its baseline shape, and the higher frequencies are more like finer details. Another way to think of the high pass filter's effect on the square wave is like that of an RC circuit. The shape of the attenuated wave looks like the exponentials associated with the discharging of a capacitor with a characteristic time jointly determined by the resistor and the capacitor (see Figure 8 for a visualization of the shape). The higher frequency square waves give the capacitor less time to discharge so the exponential shapes become less and less prominent as the frequency of the square wave increases.

Figure 8.



Problem 1.2.1

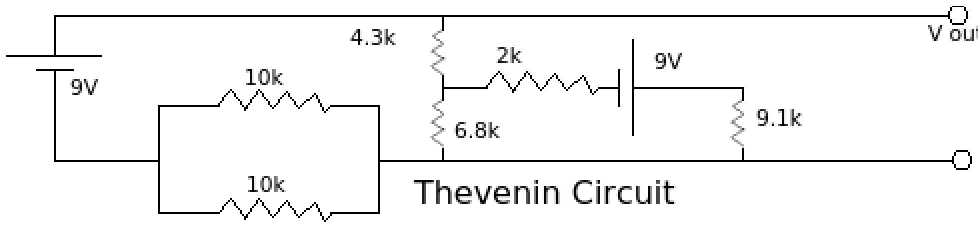


Figure 9.

The circuit in Figure 9 can be turned into a Thevenin equivalent circuit. This is done by measuring the open circuit voltage and the short circuit current. These measurements were done using the DMM.
Open circuit voltage = 4.237 V , uncertainty = 1mV
Short circuit current = 1.323 mA, uncertainty = 6e-4 mA
The Thevenin equivalent circuit is as shown in Figure 10.

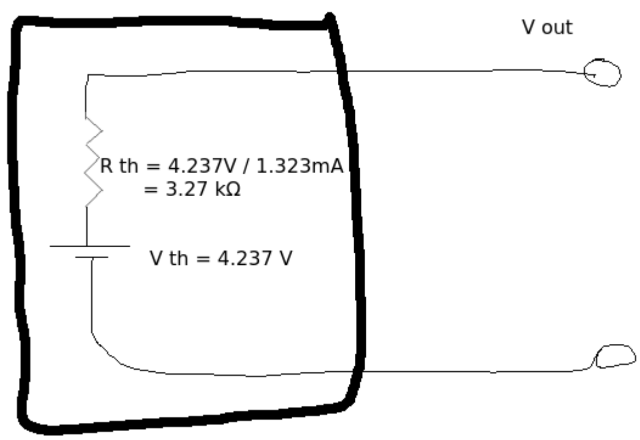


Figure 10.

Then a resistor was added to the output wires and output voltage and current were measured. The current was measured by adding the DMM in series and the voltage measured with the DMM in parallel. The resistor values used were 100Ω, 1kΩ, 10kΩ, and the following is a graph of the associated current and voltage values.

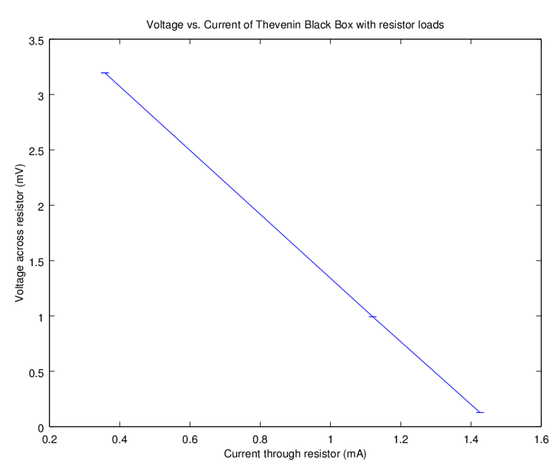


Figure 11.

According to the graph the resistance of the circuit is 2.8756 k (analyzed by the slope of the line). Another way to find the Thevenin resistance is to short all power sources in the circuit and then measure its total resistance. When this was done for the Thevenin circuit, the result was 3.195 kΩ. All the values of resistances found agree to within 400Ω.

Problem 1.2.2

When a minigrabber clip attached to a BNC cable is connected to the oscilloscope and its red lead touched, a sinusoidal AC current of 60Hz can be seen on the scope. The signal is picked up from radiation of power lines. The body of the experimenter acts as an antenna for the signal, and the signal can still be picked up when pinching the insulated wire, but to a much lesser extent. Pinching the BNC cable however, does not result in the signal. When connecting the minigrabbers to a four foot long insulated wire, the scope picks up higher frequencies of radiation probably from radio signals. See Figure 12 for a rough sketch of some of the waveforms.

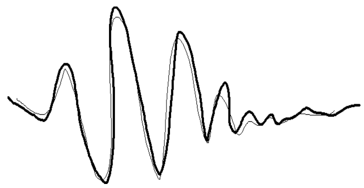


Figure 12.

Problem 1.2.3

The input impedance of the scope can be measured by treating it as a black box or an unknown resistor. A frequency of 100Hz and amplitude 1Vpp as well as several resistors are used to measure the input impedance. See Figure 13 for the circuit model used for this measurement.

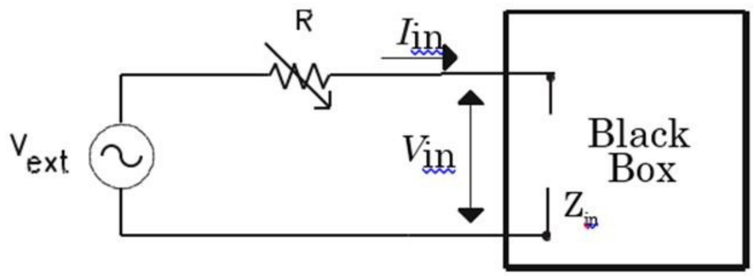


Figure 13.

Using Ohm's Law, input current is $I_{in} = (V_{ext} - V_{in}) / R$ calculated from the V_in

measurements as well as the different R values in the following graph (on the next page):

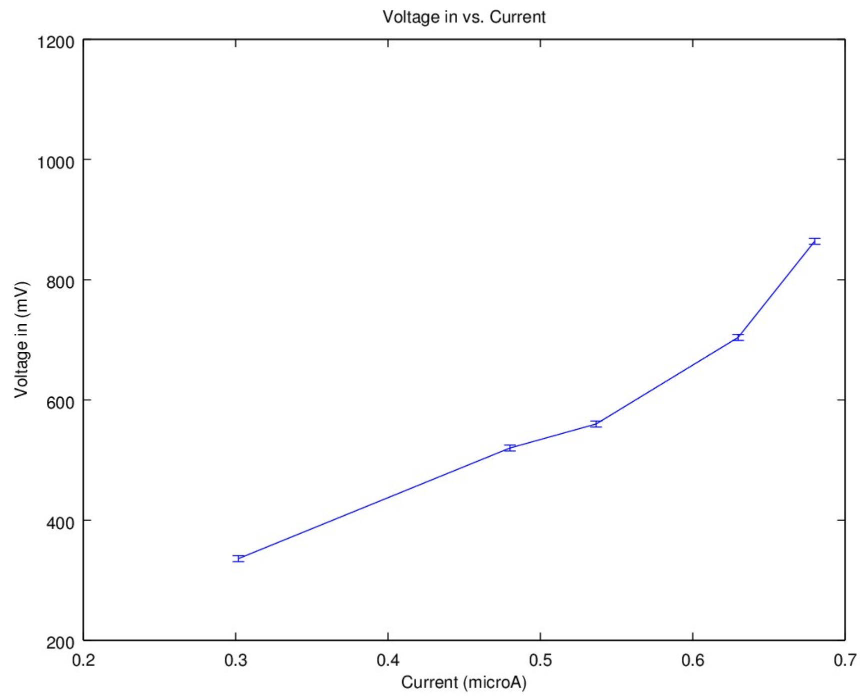


Figure 14.

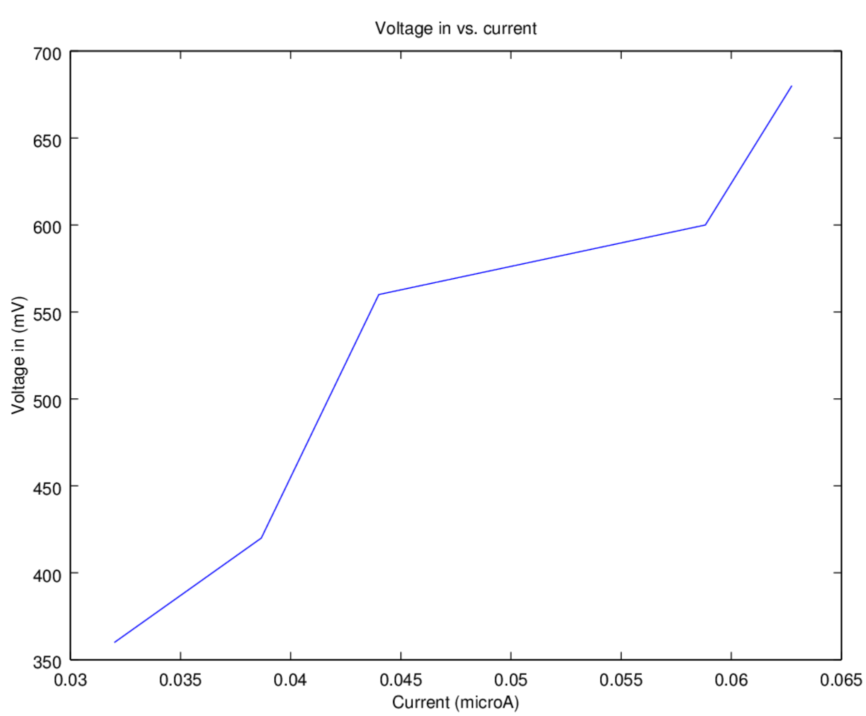
The input impedance is found by $Z_{\text{in}}=\frac{V_{\text{in}}}{I_{\text{in}}}=\frac{V_{\text{in}}}{V_{ext}-V_{\text{in}}}R$, and that equation describes the relationships between these quantities and the desired input impedance. Finding the average slope, the input impedance is around 1.1 megaohms, with a standard deviation of 86k. Clearly, the data fits the theory.

Problem 1.2.4

The same procedure of 1.2.3 is now repeated with a scope probe. The channel gain must be increased by 10 when using the scope probe. Figure 15 summarizes the results:

The average input impedance is 12.2 megaohms, with an error of 946k estimated from standard deviation.

Figure 15.



Problem 1.2.5
Next the impedance of the signal generator is determined using a very similar circuit to Figure 12.
A 1 kHz wave and several load resistors on the order of 10^2 ohms are used, as well as no load.
A potentiometer is used as well, as an alternative method to using many resistors. The resistance of the potentiometer is recorded when it attenuates the signal by ½.

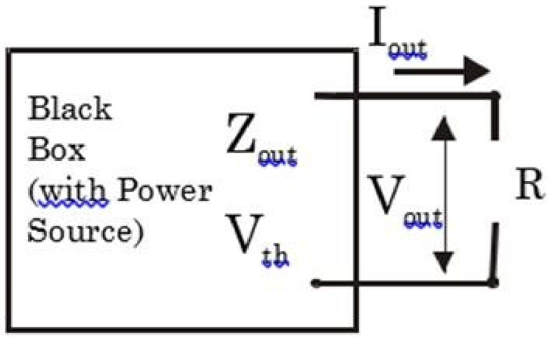


Figure 16.

V_out = I_out*R
V_out + I_out*Z_out – V_th = 0
V_out = V_th – I_out*Z_out
Z_out is simply the absolute value of the slope of the VI graph. See Figure 17 for a summary of the results.

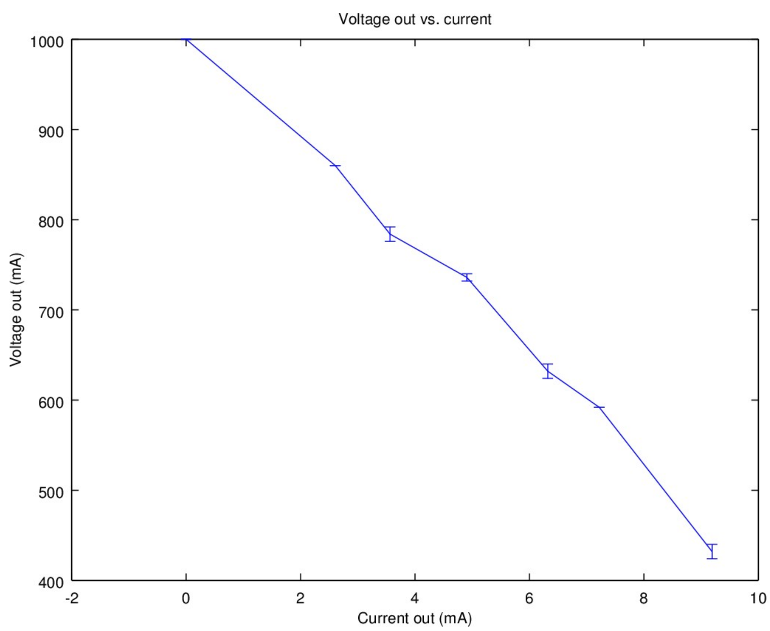


Figure 17.
The calculated absolute value of the slope is 60.5 ohms. Therefore the output impedance of the wave generator is approximately 60.5 ohms. Using the potentiometer, when the voltage Vin drops in half, that means that the two resistors are equal, as the voltage drop across Zout and R must be the same. Using the potentiometer, we measured that a resistance of 59 ohm drops the voltage in half (504mV an approximate value to 500mV), meaning that the output impedance is 59 ohms, a remarkably close value to 60.5 ohms, found using a much faster method.

Problem 1.2.6
We built the following high pass circuit for the purpose of measuring its performance. The measured values of the resistor and the capacitor were 9.91k and 9.46 nF respectively.

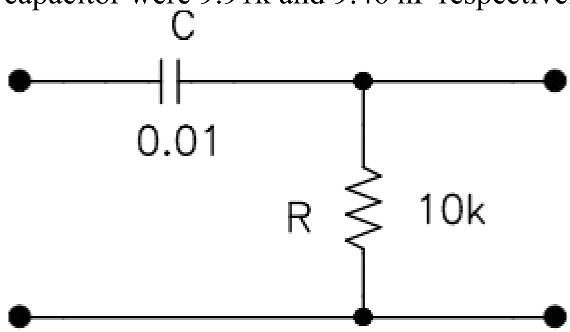
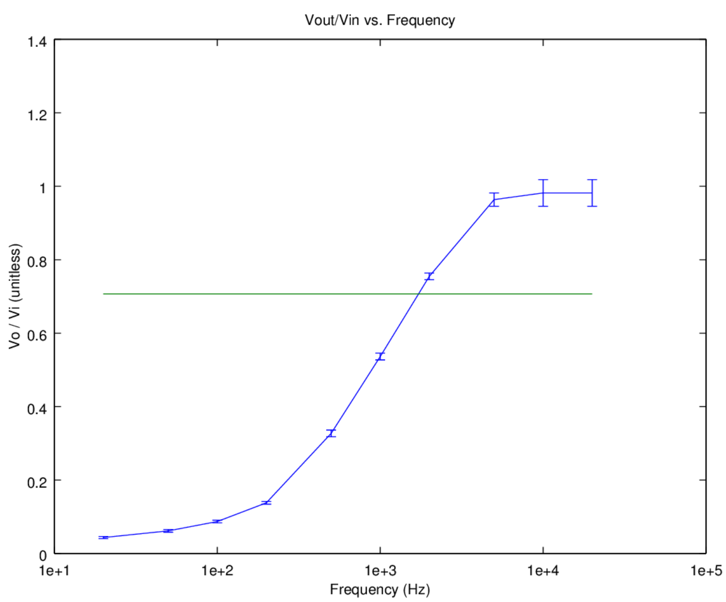


Figure 18.

A sine wave of 1.1Vpp was used with various frequencies in the range 20Hz-20kHz to measure the output and compare it to the input. These measurements were done on the scope with two input signals. The following is a plot of Vout/Vin vs. frequency using a log scale for frequency (the frequencies measured were chosen geometrically):



The straight line shows the rolloff voltage (1/sqrt(2))

Figure 19.

The roll-off point on the graph is around 1.7kHz. The graph shows how it functions as a high pass filter, attenuating the low frequencies.

Problem 1.2.7
If two signals are graphed on the scope in XY or parametric mode, a parametric plot of Vx(t) and Vy(t) is shown, and can be used to measure the phase shift such as when Vx = a*cos(wt), Vy = b*cos(wt+phi). Assuming the ellipse shown is centered, the arcsine of the maximal y divided by the y intercept yields the phase shift.

Problem 1.2.8 – this was a demonstration performed in lab of the arcsine method
We sent the original signal and the output of the RC filter (known to be phase shifted) into the scope on the XY setting. Then measured the maximal voltage on the Y input and the voltage at the y intercept. The arcsine of the quotient was the phase shift as explained in 1.2.7.

Problem 1.2.9

Just as it was possible to measure Vout/Vin for the circuit in 1.2.6 using the peak to peak voltages and plot versus frequency, it is possible to plot the phase shift versus frequency. The following is a graph showing both Vo/Vi and the phase shift between the input and output.

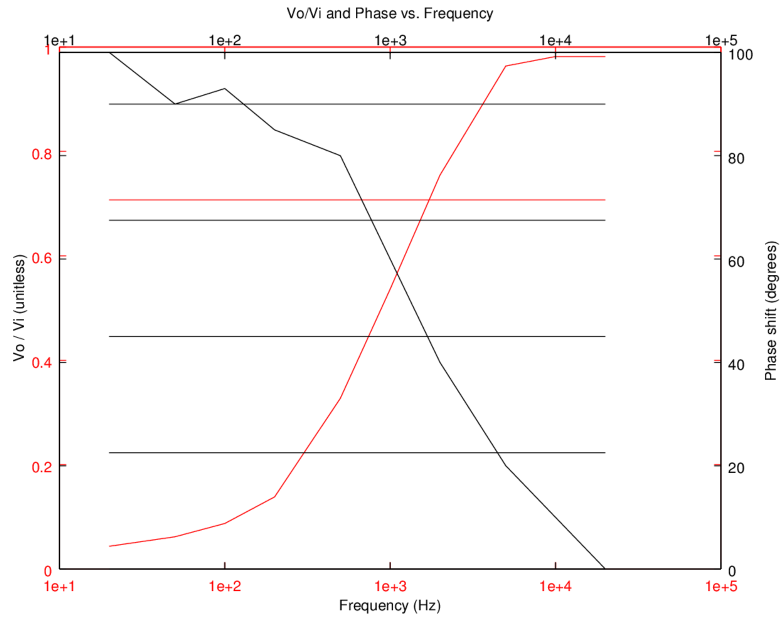


Figure 20.

In Figure 20 the black curve represents the phase, with the black lines showing intersections with 0, 22.5, 45, 77.5, and 90 degrees. The approximate values (up to measurement error) that generate these frequencies are 20kHz, 5kHz, 2kHz, 500Hz, and 50Hz respectively. The red curve is a repetition of the graph under 1.2.6 of Vo/Vi.

Problem 1.2.10

With no load the output impedance of the black box is equal to Vth. Given that Vout/Vth = 0.8 (it decreases by 20%), the circuit is like a voltage divider with R1 = Zth and R2 = 1kΩ.
So 0.8 = 1kΩ/(1kΩ + Zth) => 1kΩ/0.8 – 1kΩ = 0.25 kΩ. Therefore the output impedance is equal to the Thevenin impedance and is 0.25kΩ.

Problem 1.2.11

The resistance of a 100W bulb measured with the DMM is 9 Ohms, assuming it follows ohms law, it should use (110V)^2 / 9ohms power with household power, that is, 1344W, despite being 100W bulbs. Therefore, the light bulb does not follow Ohm's law. It is not a linear circuit component. The filament heats up and radiates light, changing its physical properties and resistivity. The 100W power is correct since 1344W incorrectly assumes that the light bulb is ohmic.

Problem 1.2.12

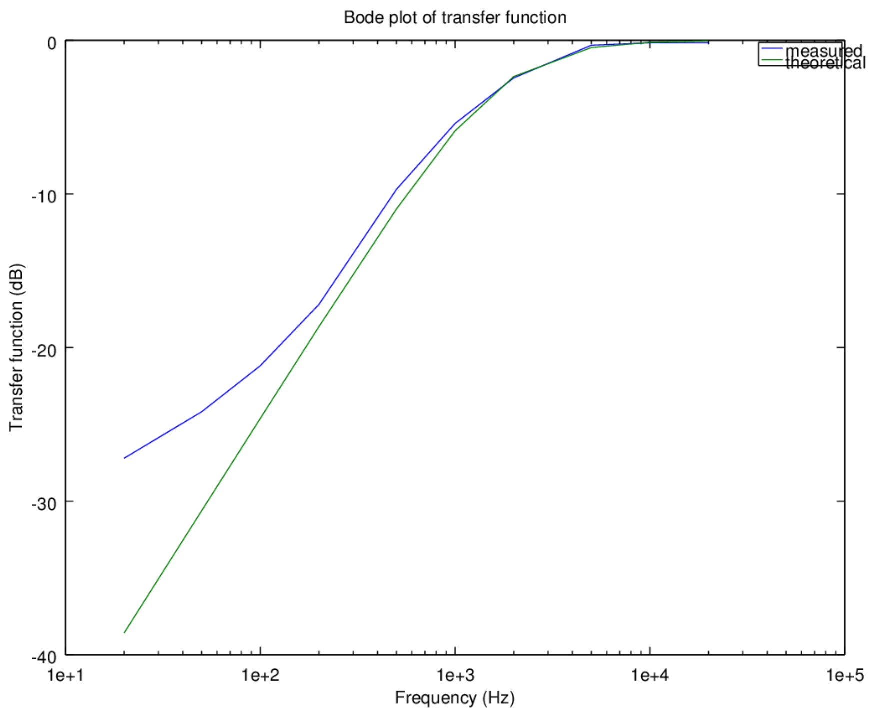
The transfer function described below is a complex valued function and can be converted to polar form.

$$\frac{V_{out}}{V_{in}} = \frac{Z_R}{Z_C+Z_R} = \frac{R}{\frac{1}{j\omega C}+R} = \frac{u}{\sqrt{u^2+1}} e^{j\,acot(u)} \; where \;\; u = \frac{\omega}{\omega_0} \; where \;\; \omega_0 = \frac{1}{RC}$$

$$\frac{V_{out}}{V_{in}} = \cos\varphi\, e^{j\varphi} \quad where \quad \varphi = acot(\omega RC)$$

$$so \;\; \left|\frac{V_{out}}{V_{in}}\right| = \cos\varphi \;\; \text{and} \;\; phase = \varphi = acot(\omega RC)$$

The following is a Bode plot of the transfer function for the RC circuit showing the measured and theoretical transfer functions:

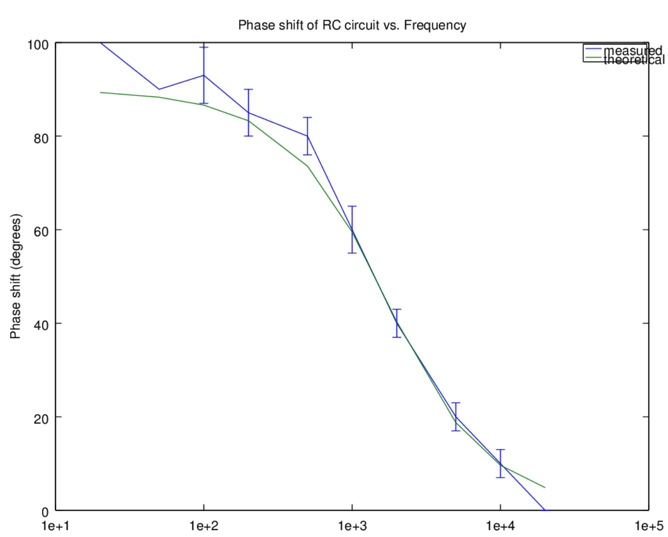


the lower curve is the theoretical prediction

Figure 21.

The following is a semilog plot of the phase from the RC circuit and the theoretical expectation versus frequency:

Figure 22.



The theoretical and experimental roll-off points agree, but the two curves diverge a bit towards the lower side of the Bode plot.

Problem 1.2.13

We generated a pulsating signal shown as follows:

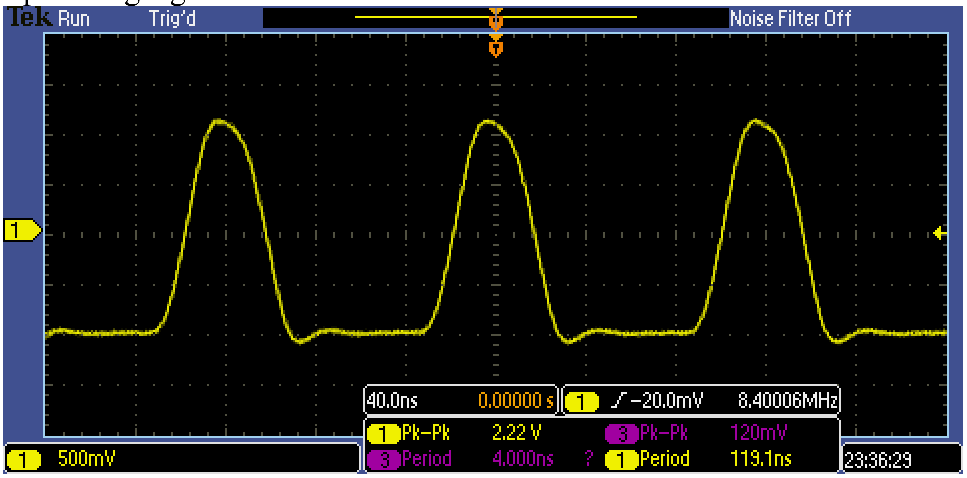


Figure 23.

After connecting a 50 ohm terminator the signal is halved and looks somewhat smoother:

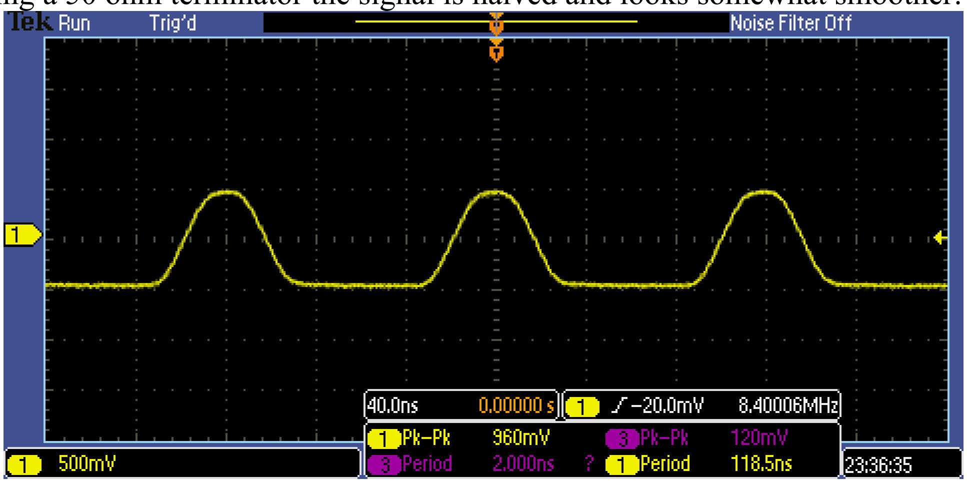


Figure 24.

Shorting the tee with a bare wire, we see some sort of shifted and flipped interference (lower impedance than the wire apparently flips the signal):

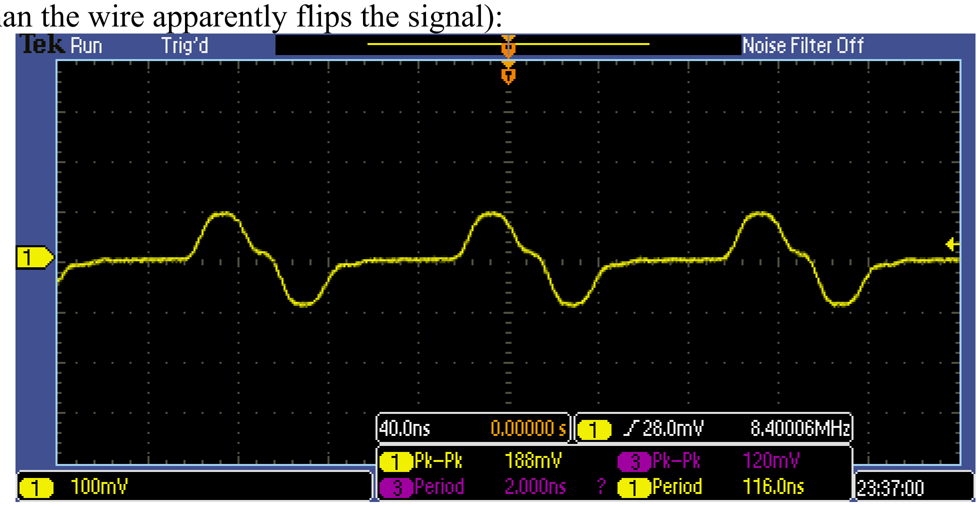


Figure 25.

Connecting a 100ft BNC cable to the tee and feeding it to another channel on the scope, we see a reflection of the signal. The reflection is shifted because of the electrical path length associated with the long cable. Changing the frequency changes the level of phase shift and the two waveforms can be made to interfere in different ways:

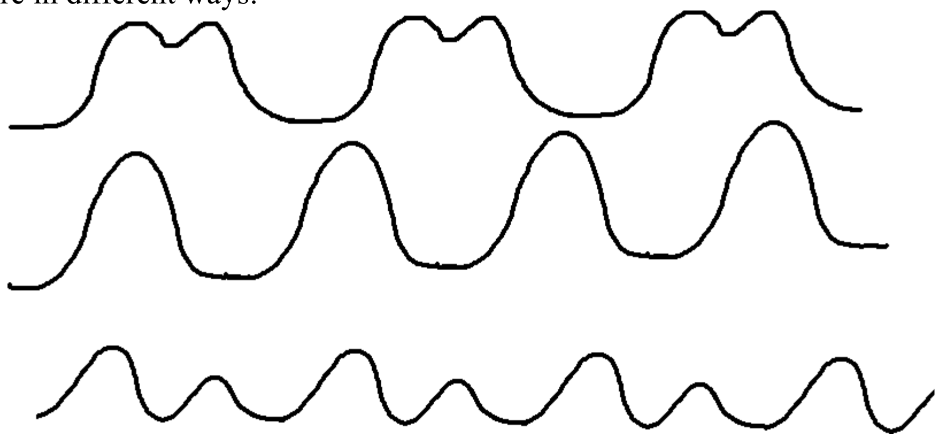


Figure 26.

Connecting a 50 ohm terminator to the second channel (B channel), the following signal emerges, denoting no reflections. Matching the impedance prevents the reflection from occurring:

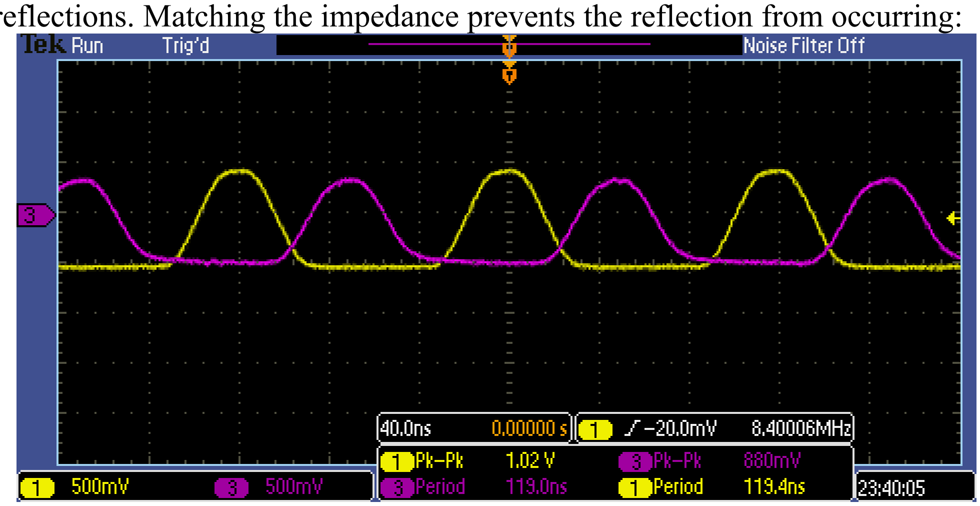


Figure 27.

Replacing it with a shorting wire causes reflections again, but this time the reflections are reversed in sign and can cause destructive interference:

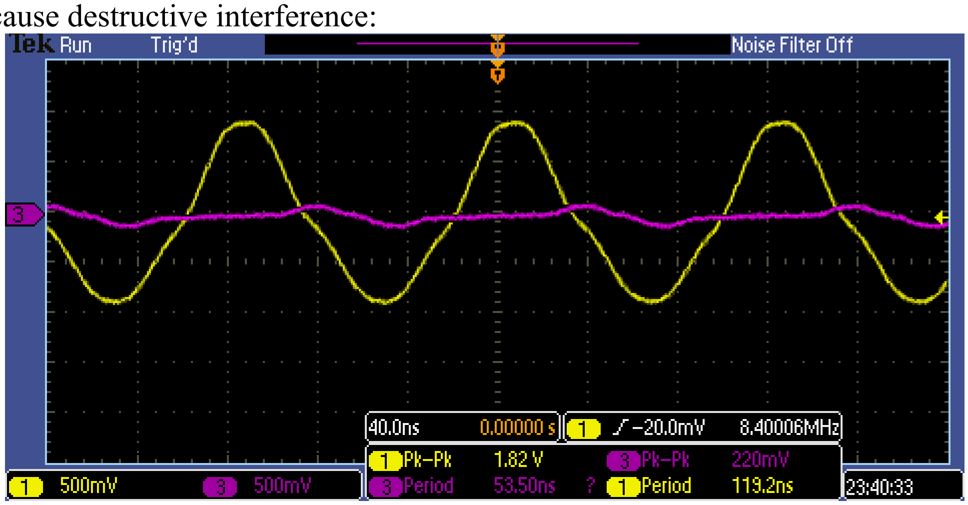


Figure 28.

Connecting a 200ohm resistor on the side of the wave generator, we see the following signal for a normal connection:

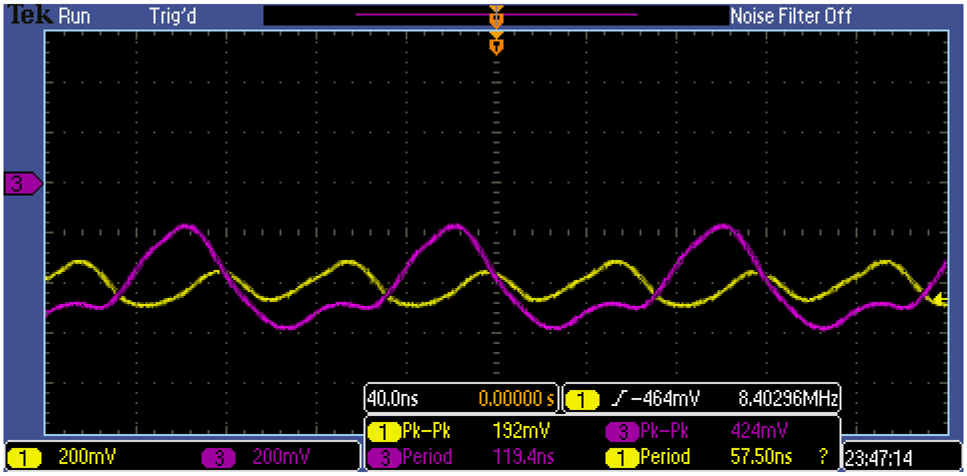


Figure 29.

The following signal for the 200ohm resistor and a 50ohm terminator on the other end:

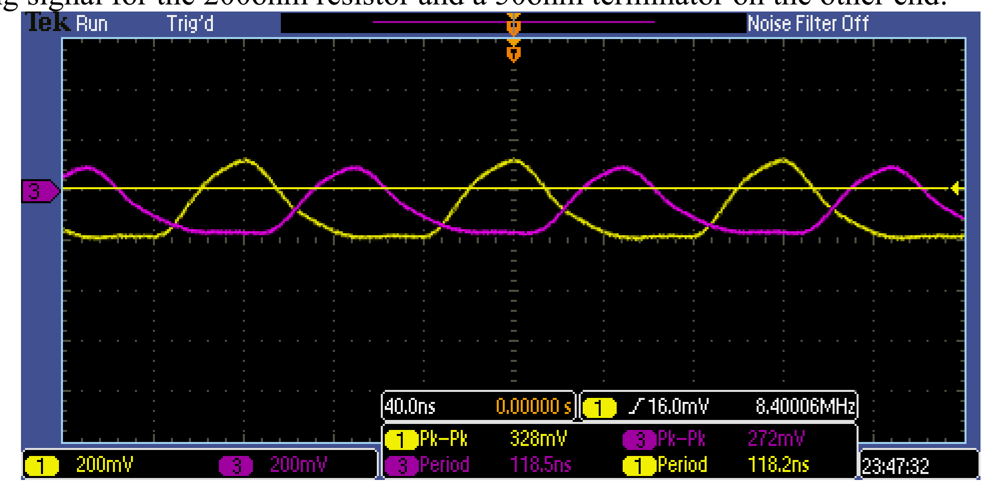


Figure 30.

And the following signal for the 200ohm resistor and the far end of the long cable shorted:

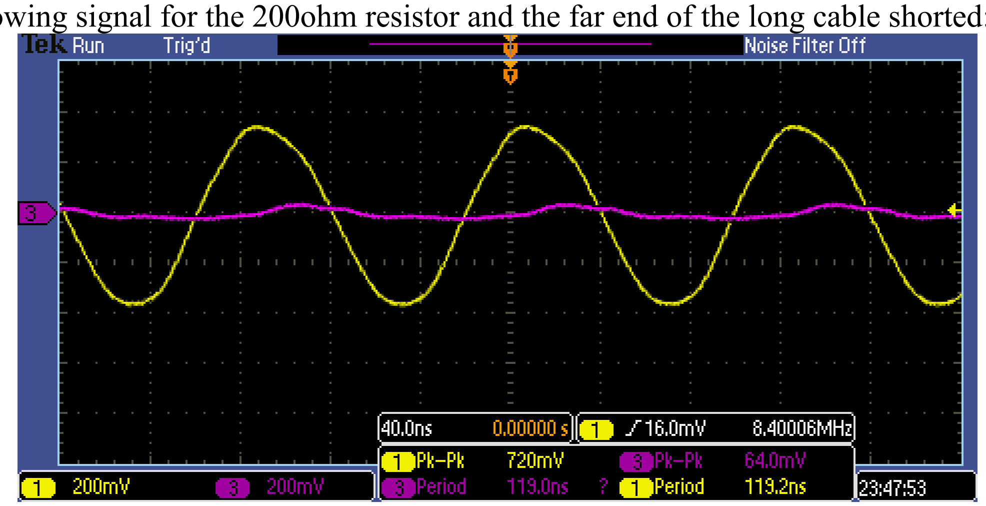
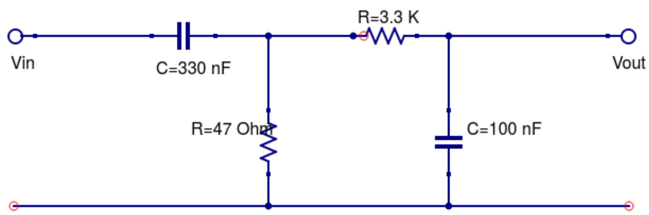


Figure 31.

Problem 1.2.14
The time delay of the signal looks like 6/5*40ns = 48ns. Since d = v*t then t = d / v = 100 ft / (2/3*c) = 152ns. This result actually makes sense since three times half a period would look like half a period, or about 50ns. Extra pulses are seen on A since the reflection travels back all the way to A. The extra pulses are caused by reflections and much like how light waves reflect, reflect upside-down, or not at all depending on the relative indices of refraction, the same happens here with the differing impedances. A larger index of refraction on the second material causes an upside-down reflection (pi phase shift). A smaller n causes an upright reflection, and equal n causes no reflection at all. The same is true with impedances.
Problem 1.2.15
We built a bandpass filter by having a high pass circuit feed into a low pass circuit. The resistor on the connecting wire between the filters was made large in comparison to the other resistor so that the connecting wire, that is, the output of the first filter does not draw a large current and the voltage divider equation would still apply. The design is as follows:

Figure 32.



The bandpass filter has cutoff frequencies of 500Hz and 10kHz. The measured values for each individual component is 320 +- 2nF, 102.08 +- 0.1 nF, 48.7 +- 0.1 Ohm, and 3.276 +- 0.001 kOhm. The following is a graph of |Vout/Vin| of the circuit:

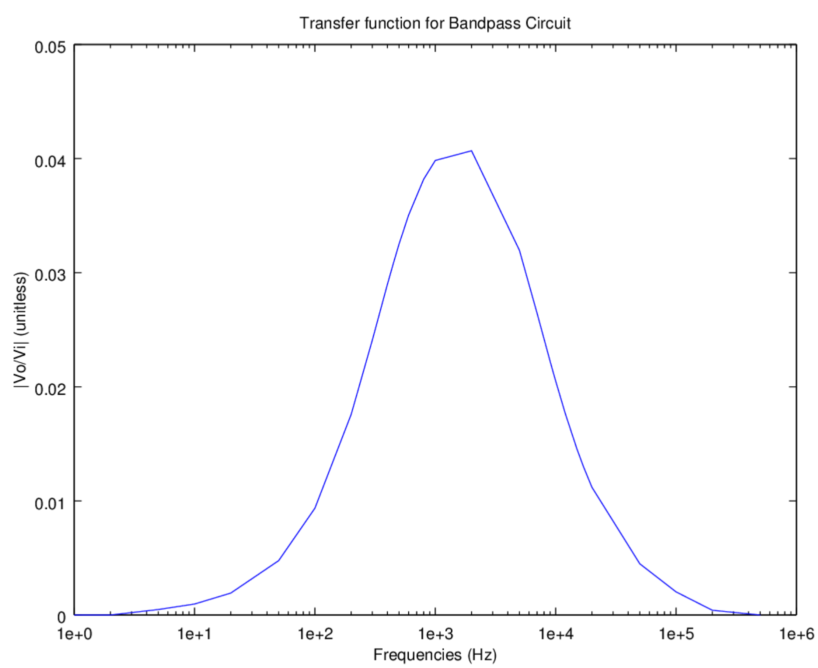


Figure 33.

The circuit has poor transfer since at best it allows 0.04 of the signal, but it does have the proper shape for a bandpass filter.

Conclusion

The major results from this lab are the applications of a voltage divider along with an analysis of its performance, the output of a high pass filter, the inadequacy of a bandpass filter composed of high and low pass RC filters, and the wave dynamics on a coaxial cable. Some useful procedures introduced in the lab and documented herein were the collection of voltage and current from various circuits to measure either their input or output impedances, and the usage of electrical equipment such as the wave generator and the oscilloscope. The results from the lab include unexpected observations such as the pickup of 60Hz AC voltage and radio stations from a disconnected scope and a scope connected to a rudimentary antenna, respectively. Yet another unexpected observation was that the bandpass circuit, designed to be an approximate composition of a high pass filter and a low pass filter, was ultimately inadequate in its performance not due to a failure in the discrimination of frequencies according to the predetermined high and low rolloff points, but due to a significantly large universal loss on all frequencies such that all frequencies of voltage were attenuated by at least a factor of 25. One's experience and familiarity with equipment and the level of uncertainty in measurements gained as a result of completing the lab can be carried forward and may be helpful in the completion of successive labs.