Top-Down Parsing

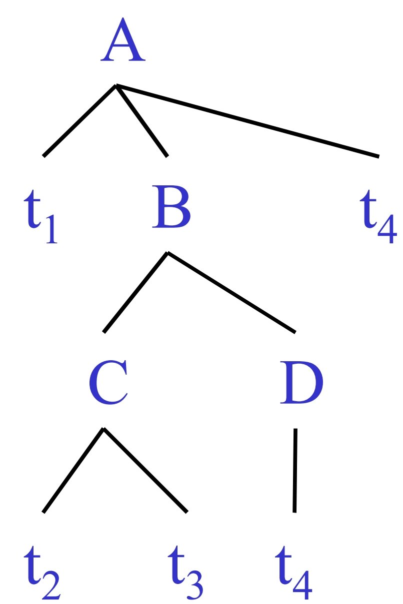
CS164
Lecture 6

Intro to Top-Down Parsing

- Terminals are seen in order of appearance in the token stream:

  $t_1 \; t_2 \; t_3 \; t_4 \; t_5$

- The parse tree is constructed
  - From the top
  - From left to right



Recursive Descent Parsing

- Consider the grammar

  $E \rightarrow T + E \mid T$

  $T \rightarrow (E) \mid int \mid int * T$
- Token stream is: int * int
- Start with top-level non-terminal E
- Try the rules for E in order

- Consider the grammar

  $E \rightarrow T + E \mid T$

  $T \rightarrow (E) \mid int \mid int * T$

- Token stream is: int * int

Recursive-Descent Parsing

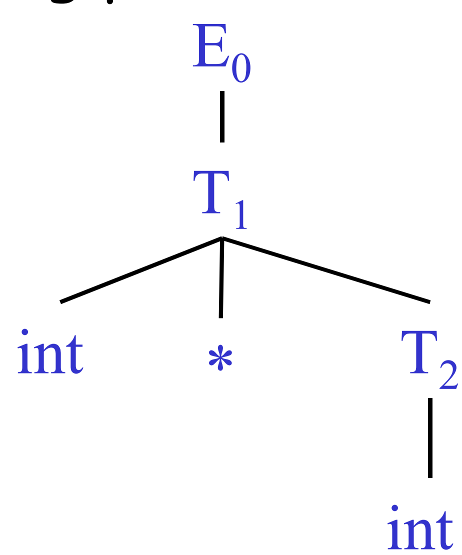
```
void match-A() {
        choose an A-production A → X1 X2 … Xn
        for (i = 1 to k)
                if (Xi is non-terminal)
                        …
                else if (Xi is a terminal and Xi is …               )
                        …
                else
        }
}
```

Recursive Descent Parsing. Example (Cont.)

- Try $E_0 \rightarrow T_1 + E_2$
- Then try a rule for $T_1 \rightarrow (E_3)$
  - But (does not match input token int
- Try $T_1 \rightarrow$ int . Token matches.
  - But + after T_1 does not match input token *
- Try $T_1 \rightarrow$ int * T_2
  - This will match but + after T_1 will be unmatched
- Have exhausted the choices for T_1
  - Backtrack to choice for E_0

Recursive Descent Parsing. Example (Cont.)

- Try $E_0 \to T_1$
- Follow same steps as before for T_1
  - And succeed with $T_1 \to$ int * T_2 and $T_2 \to$ int
  - With the following parse tree



Recursive-Descent Parsing

- Parsing: given a string of tokens $t_1\ t_2\ \dots\ t_n$, find its parse tree
- Recursive-descent parsing: Try all the productions exhaustively
  - At a given moment the fringe of the parse tree is: $t_1\ t_2\ \dots\ t_k\ A\ \dots$
  - Try all the productions for A: if $A \rightarrow BC$ is a production, the new fringe is $t_1\ t_2\ \dots\ t_k\ B\ C\ \dots$
  - Backtrack when the fringe doesn't match the string
  - Stop when there are no more non-terminals

Another Example

- $S \rightarrow S\ 0 \mid 1$ and match 10

When Recursive Descent Does Not Work

- Consider a production $S \to S\ a$:
  - In the process of parsing S we try the above rule
  - What goes wrong?

- A left-recursive grammar has a non-terminal S

  $S \to^{+} S\alpha$ for some α

- Recursive descent does not work in such cases
  - It goes into an infinite loop

Elimination of Left Recursion

- Consider the left-recursive grammar

$$S \to S\,\alpha \mid \beta$$

- S generates all strings starting with a β and followed by a number of α

- Can rewrite using right-recursion

$$S \to \beta\, S'$$

$$S' \to \alpha\, S' \mid \varepsilon$$

Elimination of Left-Recursion. Example

- Consider the grammar

  $S \rightarrow 1 \mid S\ 0$ ($\beta = 1$ and $\alpha = 0$)

  can be rewritten as

  $S \rightarrow 1\ S'$

  $S' \rightarrow 0\ S' \mid \varepsilon$

More Elimination of Left-Recursion

- In general

$$S \to S\,\alpha_1 \mid \dots \mid S\,\alpha_n \mid \beta_1 \mid \dots \mid \beta_m$$

- All strings derived from S start with one of $\beta_1,\dots,\beta_m$ and continue with several instances of $\alpha_1,\dots,\alpha_n$
- Rewrite as

$$S \to \beta_1\,S' \mid \dots \mid \beta_m\,S'$$
$$S' \to \alpha_1\,S' \mid \dots \mid \alpha_n\,S' \mid \varepsilon$$

General Left Recursion

- The grammar

$$S \to A\,\alpha \mid \delta$$
$$A \to S\,\beta$$

is also left-recursive because

$$S \to^{+} S\,\beta\,\alpha$$

- This left-recursion can also be eliminated
- See book, Section 4.3 for general algorithm

Summary of Recursive Descent

- Simple and general parsing strategy
  - Left-recursion must be eliminated first
  - … but that can be done automatically
- Unpopular because of backtracking
  - Thought to be too inefficient

- Often, we can avoid backtracking …

Predictive Parsers

- Like recursive-descent but parser can "predict" which production to use
  - By looking at the next few tokens
  - No backtracking
- Predictive parsers accept LL(k) grammars
  - L means "left-to-right" scan of input
  - L means "leftmost derivation"
  - k means "predict based on k tokens of lookahead"
- In practice, LL(1) is used

Predictive Parsing

```
void match-A() {
        choose an A-production A → X_1 X_2 … X_n
        for (i = 1 to k)
                if (X_i is non-terminal)
                        match-X_i()
                else if (X_i is a terminal and X_i is current input)
                        advance input pointer
                else …
        }
}
```

LL(1) Languages

- In recursive-descent, for each non-terminal and input token there may be a choice of production
- LL(1) means that for each non-terminal and token there is only one production that could lead to success
- Can be specified as a 2D table
  - One dimension for current non-terminal to expand
  - One dimension for next token
  - A table entry contains one production

Predictive Parsing and Left Factoring

- Recall the grammar

$$E \rightarrow T + E \mid T$$
$$T \rightarrow \text{int} \mid \text{int} * T \mid (E)$$

- Impossible to predict because
  - For T two productions start with int
  - For E it is not clear how to predict

- A grammar must be left-factored before use for predictive parsing

Left-Factoring Example

- Recall the grammar

$$E \rightarrow T + E \mid T$$
$$T \rightarrow int \mid int * T \mid (E)$$

- Factor out common prefixes of productions

$$E \rightarrow T\,X$$
$$X \rightarrow + E \mid \varepsilon$$
$$T \rightarrow (E) \mid int\ Y$$
$$Y \rightarrow * T \mid \varepsilon$$

LL(1) Parsing Table Example

- Left-factored grammar

  $E \to T\,X$ $\qquad\qquad$ $X \to +\,E \mid \varepsilon$

  $T \to (\,E\,) \mid \text{int}\;Y$ $\qquad\qquad$ $Y \to *\;T \mid \varepsilon$

- The LL(1) parsing table ($ is a special end marker):

	int	*	+	(	)	$
T	int Y			(E)		
E	T X			T X		
X			+ E		ε	ε
Y		* T	ε		ε	ε

Predictive Parsing

```
void match-A() {
  choose the production Table[A,a], say A → X1 X2 … Xn
    for (i = 1 to k)
        if (Xi is non-terminal)
          match-Xi()
        else if (Xi is a terminal and Xi is current symbol a)
          advance input to the next symbol
        else
          error
```

	int	*	+	(	)	$
T	int Y			(E)		
E	T X			T X		
X			+ E		ε	ε
Y		* T	ε		ε	ε

LL(1) Parsing Table Example (Cont.)

- Consider the [E, int] entry
  - "When current non-terminal is E and next input is int, use production $E \rightarrow TX$
  - This production can generate an int in the first place
- Consider the [Y,+] entry
  - "When current non-terminal is Y and current token is +, get rid of Y"
  - We'll see later why this is so

LL(1) Parsing Tables. Errors

- Blank entries indicate error situations
  - Consider the [E,*] entry
  - "There is no way to derive a string starting with * from non-terminal E"

Using Parsing Tables

- Method similar to recursive descent, except
  - For each non-terminal X
  - We look at the next token a
  - And choose the production shown at [X,a]
- We use a stack to keep track of pending non-terminals
- We reject when we encounter an error state
- We accept when we encounter end-of-input

Predictive Parsing (non-recursive)

	int	*	+	(	)	$
T	int Y			(E)		
E	T X			T X		
X			+ E		ε	ε
Y		* T	ε		ε	ε

LL(1) Parsing Algorithm

```
initialize stack = <S,$> and next (pointer to tokens)
repeat
   case stack of
      <X, rest>  : if T[X,*next] = Y_1...Y_n
                         then stack ← <Y_1... Y_n rest>;
                         else  error ();
      <t, rest>   : if t == *next ++
                         then  stack ← <rest>;
                         else error ();
until stack == < >
```

LL(1) Parsing Example

Stack	Input	Action
E $	int * int $	T X
T X $	int * int $	int Y
int Y X $	int * int $	terminal
Y X $	* int $	* T
* T X $	* int $	terminal
T X $	int $	int Y
int Y X $	int $	terminal
Y X $	$	ε
X $	$	ε
$	$	ACCEPT

Constructing Parsing Tables

- LL(1) languages are those defined by a parsing table for the LL(1) algorithm
- No table entry can be multiply defined

- Once we have the table
  - The parsing algorithm is simple and fast
  - No backtracking is necessary

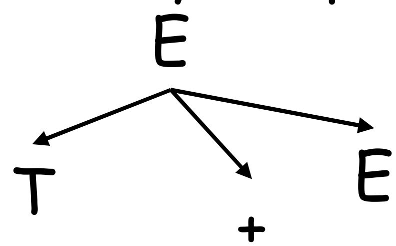
- We want to generate parsing tables from CFG

Predictive Parsing (non-recursive)

	int	*	+	(	)	$
T	int Y			(E)		
E	T X			T X		
X			+ E		ε	ε
Y		* T	ε		ε	ε

Top-Down Parsing. Review

- Top-down parsing expands a parse tree from the start symbol to the leaves
  - Always expand the leftmost non-terminal



int * int + int

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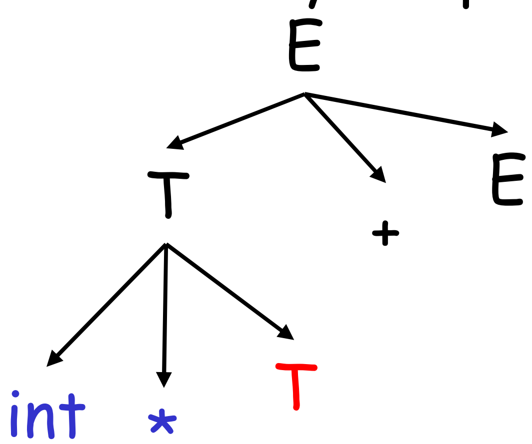

int * int + int

- The leaves at any point form a string $\beta A \gamma$
  - β contains only terminals
  - The input string is $\beta b \delta$
  - The prefix β matches
  - The next token is b

Top-Down Parsing. Review

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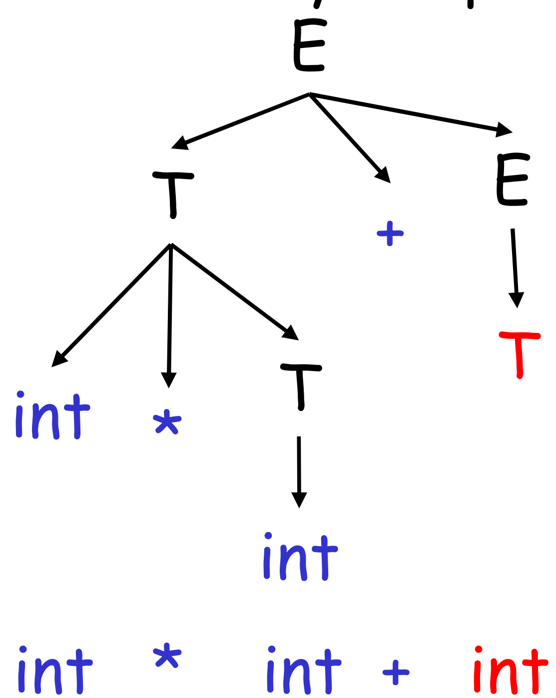

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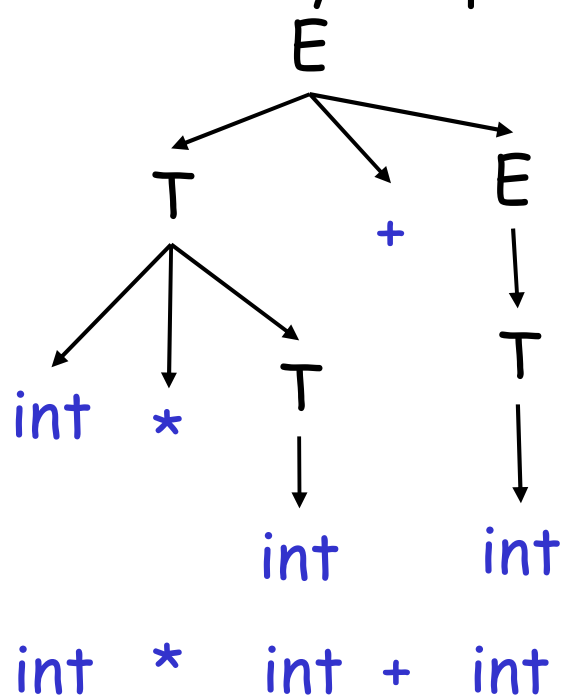

- The leaves at any point form a string $\beta A \gamma$
  - β contains only terminals
  - The input string is $\beta b \delta$
  - The prefix β matches
  - The next token is b

Constructing Predictive Parsing Tables

- Consider the state $S \to^* \beta A \gamma$
  - With b the next token
  - Trying to match $\beta b \delta$

There are two possibilities:

1. b belongs to an expansion of A
   - Any $A \to \alpha$ can be used if b can start a string derived from α

     In this case we say that $b \in First(\alpha)$

Or…

Constructing Predictive Parsing Tables (Cont.)

2. b does not belong to an expansion of A
   - The expansion of A is empty and b belongs to an expansion of γ (e.g., bω)
   - Means that b can appear after A in a derivation of the form $S \rightarrow^* \beta A b \omega$
   - We say that $b \in Follow(A)$ in this case

   - What productions can we use in this case?
     - Any $A \rightarrow \alpha$ can be used if α can expand to ε
     - We say that $\varepsilon \in First(A)$ in this case

Computing First Sets

Definition $\quad First(X) = \{ b \mid X \to^* b\alpha\} \cup \{\varepsilon \mid X \to^* \varepsilon\}$

1. $First(b) = \{ b \}$

2. For all productions $X \to A_1 \ldots A_n$
   - Add $First(A_1) - \{\varepsilon\}$ to $First(X)$. Stop if $\varepsilon \notin First(A_1)$
   - Add $First(A_2) - \{\varepsilon\}$ to $First(X)$. Stop if $\varepsilon \notin First(A_2)$
   - …
   - Add $First(A_n) - \{\varepsilon\}$ to $First(X)$. Stop if $\varepsilon \notin First(A_n)$
   - Add ε to $First(X)$

First Sets. Example

- Recall the grammar

$$E \to T\,X \qquad\qquad X \to +\,E \mid \varepsilon$$
$$T \to (\,E\,) \mid \text{int}\;Y \qquad\qquad Y \to *\,T \mid \varepsilon$$

- First sets

First(( ) = { (} First(T) = {int, (}

First( )) = {) } First(E) = {int, (}

First(int) = { int } First(X) = {+, ε }

First(+) = { + } First(Y) = {*, ε }

First(*) = { * }

Computing Follow Sets

Definition $\quad Follow(X) = \{ b \mid S \to^* \beta X b \omega \}$

1. Compute the First sets for all non-terminals first
2. Add \$ to Follow(S) (if S is the start non-terminal)

3. For all productions $Y \to \ldots X A_1 \ldots A_n$
   - Add $First(A_1) - \{\varepsilon\}$ to $Follow(X)$. Stop if $\varepsilon \notin First(A_1)$
   - Add $First(A_2) - \{\varepsilon\}$ to $Follow(X)$. Stop if $\varepsilon \notin First(A_2)$
   - …
   - Add $First(A_n) - \{\varepsilon\}$ to $Follow(X)$. Stop if $\varepsilon \notin First(A_n)$
   - Add $Follow(Y)$ to $Follow(X)$

First Sets. Example

- Recall the grammar

$E \rightarrow T\,X$ $\qquad$ $X \rightarrow +\,E \mid \varepsilon$

$T \rightarrow (\,E\,) \mid \text{int}\ Y$ $\qquad$ $Y \rightarrow *\,T \mid \varepsilon$

- First sets

First(T) = {int, (}

First(E) = {int, (}

First(X) = {+, ε }

First(Y) = {*, ε }

Follow Sets. Example

- Recall the grammar

$E \to T\,X \qquad\qquad X \to + E \mid \varepsilon$

$T \to (E) \mid \text{int } Y \qquad\qquad Y \to * T \mid \varepsilon$

- Follow sets

Follow(E) = {), $}

Follow(X) = {$,) }

Follow(Y) = {+,) , $}

Follow(T) = {+,) , $}

Constructing LL(1) Parsing Tables

- Construct a parsing table T for CFG G

- For each production $A \to \alpha$ in G do:
  - For each terminal $b \in First(\alpha)$ do
    - $T[A, b] = \alpha$
  - If $\alpha \to^* \varepsilon$, for each $b \in Follow(A)$ do
    - $T[A, b] = \alpha$

Constructing LL(1) Tables. Example

- Recall the grammar

$E \to T X$ $\qquad$ $X \to + E \mid \varepsilon$

$T \to (E) \mid \text{int } Y$ $\qquad$ $Y \to * T \mid \varepsilon$

- Where in the line of Y we put $Y \to * T$?
  - In the lines of First(*T) = { * }

- Where in the line of Y we put $Y \to \varepsilon$?
  - In the lines of Follow(Y) = { \$, +,) }

Notes on LL(1) Parsing Tables

- If any entry is multiply defined then G is not LL(1)
  - If G is ambiguous
  - If G is left recursive
  - If G is not left-factored
  - And in other cases as well
- Most programming language grammars are not LL(1)
- There are tools that build LL(1) tables

Review

- For some grammars there is a simple parsing strategy
  - Predictive parsing (LL(1))
  - Once you build the LL(1) table, you can write the parser by hand

- Next: a more powerful parsing strategy for grammars that are not LL(1)