Semantic Analysis
Typechecking in COOL

Lectures 10-13

Administration

- WA 1
  - check announcement to get back graded WA
- Midterm I
  - Monday, October 5, in class
  - Be here on time (we start at 1:10 pm sharp)
  - Syllabus includes lexing and parsing, no semantic analysis
  - 1 cheat sheet, front and back, handwritten, by you!

Outline

- The role of semantic analysis in a compiler
  - A laundry list of tasks
- Scope
- Types

The Compiler So Far

- Lexical analysis
  - Detects inputs with illegal tokens
- Parsing
  - Detects inputs with ill-formed parse trees
- Semantic analysis
  - Last “front end” phase
  - Catches more errors

Errors

- Example 1

  let y: Int in x + 3

- Example 2

  let y: String ← “abc” in y + 3

Why a Separate Semantic Analysis?

- Parsing cannot catch some errors
- Some language constructs are not context-free
  - Example: All used variables must have been declared (i.e. scoping)
  - Example: A method must be invoked with arguments of proper type (i.e. typing)

What Does Semantic Analysis Do?

- Checks of many kinds . . . **coolc** checks:
  1. All identifiers are declared
  2. Types
  3. Inheritance relationships
  4. Classes defined only once
  5. Methods in a class defined only once
  6. Reserved identifiers are not misused

  And others . . .

- The requirements depend on the language

Scope

- Matching identifier declarations with uses
  - Important semantic analysis step in most languages
  - Including COOL !

Scope (Cont.)

- The <u>scope</u> of an identifier is the portion of a program in which that identifier is accessible

- The same identifier may refer to different things in different parts of the program
  - Different scopes for same name don't overlap

- An identifier may have restricted scope

Static vs. Dynamic Scope

- Most languages have static scope
  - Scope depends only on the program text, not run-time behavior
  - Cool has static scope
- A few languages are dynamically scoped
  - Lisp, Perl
  - Lisp has changed to mostly static scoping
  - Scope depends on execution of the program

Static Scoping Example

```
let x: Int <- 0 in
  {
      x;
      let x: Int <- 1 in
          x;
      x;
  }
```

Static Scoping Example (Cont.)

```
let x: Int <- 0 in
  {
      x;
      let x: Int <- 1 in
            x;
      x;
  }
```

Uses of x refer to closest enclosing definition

Scope in Cool

- Cool identifier names are introduced by
  - Class declarations (introduce class names)
  - Method definitions (introduce method names)
  - Let expressions (introduce object id's)
  - Formal parameters (introduce object id's)
  - Attribute definitions in a class (introduce object id's)
  - Case expressions (introduce object id's)

Implementing the Most-Closely Nested Rule

- Much of semantic analysis can be expressed as a recursive descent of an AST
  - Process an AST node *n*
  - Process the children of *n*
  - Finish processing the AST node *n*

Implementing . . . (Cont.)

- Example: the scope of let bindings is one subtree

let x: Int ← 0 in e

- x can be used in subtree e

Symbol Tables

- Consider again: let x: Int ← 0 in e
- Idea:
  - Before processing e, add definition of x to current definitions, overriding any other definition of x
  - After processing e, remove definition of x and restore old definition of x
- A *symbol table* is a data structure that tracks the current bindings of identifiers
  - We'll give you an implementation for the project

Scope in Cool (Cont.)

- Not all kinds of identifiers follow the most-closely nested rule

- For example, class definitions in Cool
  - Cannot be nested
  - Are *globally visible* throughout the program

- In other words, a class name can be used before it is defined

Example: Use Before Definition

```
Class Foo {
   . . . let y: Bar in . . .
};

Class Bar {
   . . .
};
```

More Scope in Cool

Attribute names are global within the class in which they are defined

```
Class Foo {
  f(): Int { a };
  a: Int ← 0;
}
```

More Scope (Cont.)

- Method and attribute names have complex rules

- A method need not be defined in the class in which it is used, but in some parent class

- Methods may also be redefined (overridden)

Class Definitions

- Class names can be used before being defined
- We can't check this property
  - using a symbol table
  - or even in one pass
- Solution
  - Pass 1: Gather all class names
  - Pass 2: Do the checking
- Semantic analysis requires multiple passes
  - Probably more than two

Scopes - Summary

- Scoping rules match uses of identifiers with their declarations
  - Static scoping is the most common form
- Scoping rules can be implemented using symbol tables
  - In one or more passes over the AST

Types

- What is a type?
  - The notion varies from language to language
- Consensus
  - A set of values
  - A set of operations on those values
- Classes are one instantiation of the modern notion of type

Types and Operations

- Most operations are legal only for values of some types
  - It doesn't make sense to add a function pointer and an integer in C
  - It does make sense to add two integers
  - But both have the same assembly language implementation!

Type Systems

- A language's type system specifies which operations are valid for which types
- The goal of type checking is to ensure that operations are used with the correct types
  - Enforces intended interpretation of values, because nothing else will!
- Type systems provide a concise formalization of the semantic checking rules

What Can Types do For Us?

- Can detect certain kinds of errors
- Memory errors:
  - Reading from an invalid pointer, etc.
- Violation of abstraction boundaries:

```
class FileSystem {
   open(x : String) : File {
        …
   }
…
}
```

```
class Client {
   f(fs : FileSystem) {
       File fdesc <- fs.open("foo")
       …
   } -- f cannot see inside fdesc !
}
```

Type Checking Overview

- Three kinds of languages:
  - *Statically typed*: All or almost all checking of types is done as part of compilation (C, Java, Cool)
  - *Dynamically typed*: Almost all checking of types is done as part of program execution (Scheme)
  - *Untyped*: No type checking (machine code)

The Type Wars

- Competing views on static vs. dynamic typing
- Static typing proponents say:
  - Static checking catches many programming errors at compile time
  - Avoids overhead of runtime type checks
- Dynamic typing proponents say:
  - Static type systems are restrictive
  - Rapid prototyping easier in a dynamic type system

The Type Wars (Cont.)

- In practice, most code is written in statically typed languages with an "escape" mechanism
  - Unsafe casts in C, native methods in Java, unsafe modules in Modula-3

Type Checking in Cool

Outline

- Type concepts in COOL
- Notation for type rules
  - Logical rules of inference
- COOL type rules
- General properties of type systems

Cool Types

- The types are:
  - Class names
  - SELF_TYPE
  - Note: there are no base types (as int in Java)

- The user declares types for all identifiers

- The compiler infers types for expressions
  - Infers a type for *every* sub-expression

Type Inference

- Type Checking is the process of checking that the program obeys the type system

- Often involves inferring types for parts of the program
  - Some people call the process type inference when inference is necessary

Rules of Inference

- We have seen two examples of formal notation specifying parts of a compiler
  - Regular expressions (for the lexer)
  - Context-free grammars (for the parser)

- The appropriate formalism for type checking is logical rules of inference

Why Rules of Inference?

- Inference rules have the form

  If Hypothesis is true, then Conclusion is true

- Type checking computes via reasoning

  If E_1 and E_2 have certain types, then E_3 has a certain type

- Rules of inference are a compact notation for "If-Then" statements

From English to an Inference Rule

- The notation is easy to read (with practice)

- Start with a simplified system and gradually add features

- Building blocks
  - Symbol $\wedge$ is "and"
  - Symbol $\Rightarrow$ is "if-then"
  - $x:T$ is "x has type T"

From English to an Inference Rule (2)

If e_1 has type Int and e_2 has type Int,
then $e_1 + e_2$ has type Int

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(e_1 has type Int $\wedge$ e_2 has type Int) $\Rightarrow$ $e_1 + e_2$ has type Int

From English to an Inference Rule (2)

If e_1 has type Int and e_2 has type Int, then $e_1 + e_2$ has type Int

(e_1 has type Int $\wedge$ e_2 has type Int) $\Rightarrow$ $e_1 + e_2$ has type Int

$$(e_1 : \mathsf{Int} \wedge e_2 : \mathsf{Int}) \Rightarrow e_1 + e_2 : \mathsf{Int}$$

From English to an Inference Rule (3)

The statement

$$(e_1: \text{Int} \wedge e_2: \text{Int}) \Rightarrow e_1 + e_2: \text{Int}$$

is a special case of

$$(\text{Hypothesis}_1 \wedge \ldots \wedge \text{Hypothesis}_n) \Rightarrow \text{Conclusion}$$

This is an inference rule

Notation for Inference Rules

- By tradition inference rules are written

$$\frac{\vdash \text{Hypothesis}_1 \quad \ldots \quad \vdash \text{Hypothesis}_n}{\vdash \text{Conclusion}}$$

- Cool type rules have hypotheses and conclusions of the form:

$$\vdash e : T$$

- $\vdash$ means "we can prove that . . ."

Two Rules

$$\frac{}{\vdash i : \text{Int}} \quad [\text{Int}] \text{ (i is an integer constant)}$$

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$$\frac{}{\vdash i : \text{Int}} \quad [\text{Int}] \quad \text{(i is an integer constant)}$$

$$\frac{\begin{array}{c}\vdash e_1 : \text{Int} \\ \vdash e_2 : \text{Int}\end{array}}{\vdash e_1 + e_2 : \text{Int}} \quad [\text{Add}]$$

Two Rules (Cont.)

- These rules give templates describing how to type integers and + expressions

- By filling in the templates, we can produce complete typings for expressions

- Example: 1+2

Example: 1 + 2

$$\cfrac{\cfrac{}{\vdash 1 : \text{Int}} \qquad \cfrac{}{\vdash 2 : \text{Int}}}{\vdash 1 + 2 : \text{Int}}$$

Soundness

- A type system is <u>sound</u> if
  - Whenever $\vdash e : T$
  - Then e evaluates to a value of type T

- We only want sound rules
  - But some sound rules are better than others:

$$\frac{}{\vdash i : \text{Object}} \quad (i \text{ is an integer constant})$$

Type Checking Proofs

- Type checking proves facts $e : T$
  - One type rule is used for each kind of expression
- In the type rule used for a node e:
  - The hypotheses are the proofs of types of e's subexpressions
  - The conclusion is the proof of type of e

Rules for Constants

$$\frac{}{\vdash \text{false} : \text{Bool}} \quad [\text{Bool}]$$

$$\frac{}{\vdash s : \text{String}} \quad [\text{String}]$$

(s is a string constant)

Rule for New

new T produces an object of type T

- Ignore SELF_TYPE for now . . .

$$\frac{}{\vdash \text{new } T : T} \quad [\text{New}]$$

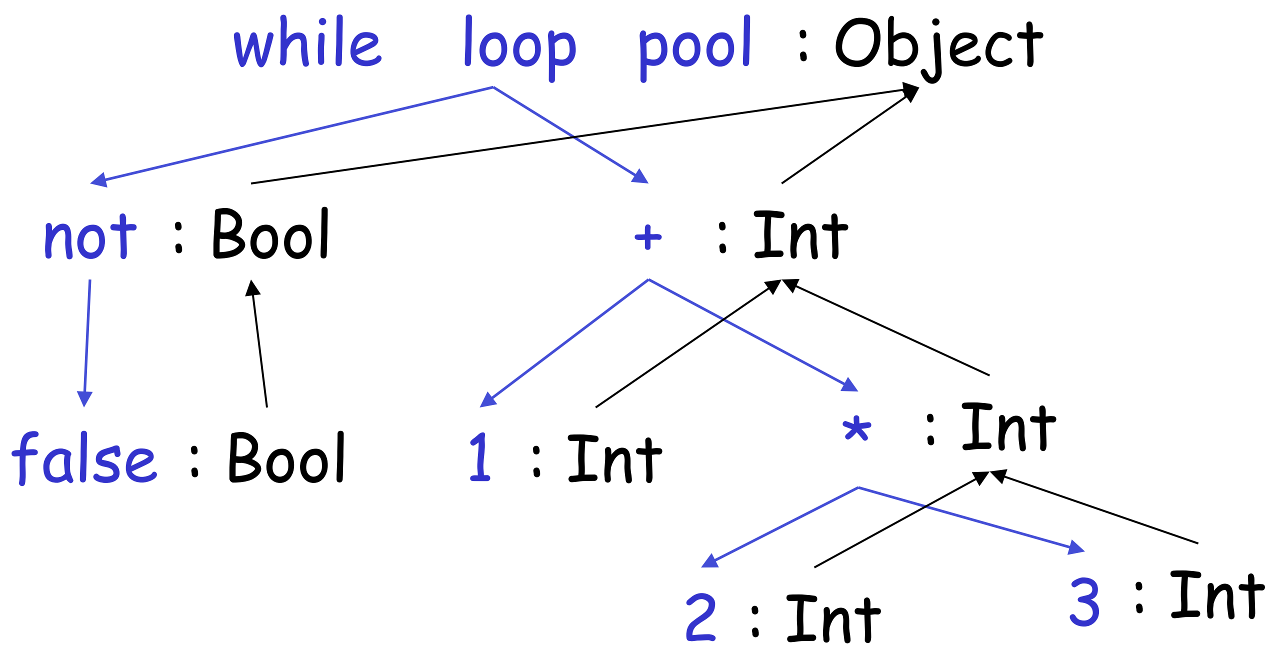
Two More Rules

$$\frac{\vdash e : \text{Bool}}{\vdash \text{not } e : \text{Bool}} \quad [\text{Not}]$$

$$\frac{\begin{array}{c}\vdash e_1 : \text{Bool} \\ \vdash e_2 : T\end{array}}{\vdash \text{while } e_1 \text{ loop } e_2 \text{ pool} : \text{Object}} \quad [\text{Loop}]$$

Typing: Example

- Typing for while not false loop 1 + 2 * 3 pool



Typing Derivations

- The typing reasoning can be expressed as an inverted tree:

$$\frac{\dfrac{\vdash \text{false} : \text{Bool}}{\vdash \text{not false} : \text{Bool}} \qquad \dfrac{\vdash 1 : \text{Int} \qquad \dfrac{\vdash 2 : \text{Int} \qquad \vdash 3 : \text{Int}}{\vdash 2 * 3 : \text{Int}}}{\vdash 1 + 2 * 3 : \text{Int}}}{\vdash \text{while not false loop } 1 + 2 * 3 : \text{Object}}$$

- The root of the tree is the whole expression
- Each node is an instance of a typing rule
- Leaves are the rules with no hypotheses

A Problem

- What is the type of a variable reference?

$$\frac{}{\vdash x : ?} \quad [\text{Var}] \qquad (x \text{ is an identifier})$$

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- What is the type of a variable reference?

$$\frac{}{\vdash x : ?} \quad [\text{Var}] \quad (x \text{ is an identifier})$$

- This rules does not have enough information to give a type.
  - We need a hypothesis of the form "*we are in the scope of a declaration of x with type T*")

A Solution: Put more information in the rules!

- A *type environment* gives types for *free* variables
  - A type environment is a mapping from ObjectIdentifiers to Types
  - A variable is free in an expression if:
    - The expression contains an occurrence of the variable that refers to a declaration outside the expression
  - E.g. in the expression "x", the variable "x" is free
  - E.g. in "let x : Int in x + y" only "y" is free
  - E.g. in "x + let x : Int in x + y" both "x" and "y" are free

Type Environments

Let O be a function from ObjectIdentifiers to Types

The sentence $O \vdash e : T$

is read: Under the assumption that variables in the current scope have the types given by O, it is provable that the expression e has the type T

Modified Rules

The type environment is added to the earlier rules:

$$\frac{}{O \vdash i : \text{Int}} \quad [\text{Int}] \quad (\text{i is an integer})$$

$$\frac{\begin{array}{c} O \vdash e_1 : \text{Int} \\ O \vdash e_2 : \text{Int} \end{array}}{O \vdash e_1 + e_2 : \text{Int}} \quad [\text{Add}]$$

New Rules

And we can write new rules:

$$\frac{(O(x) = T)}{O \vdash x : T} \quad [\text{Var}]$$

Let

$$\frac{O[T_0/x] \vdash e_1 : T_1}{O \vdash \text{let } x : T_0 \text{ in } e_1 : T_1} \quad \text{[Let-No-Init]}$$

$O[T_0/x]$ means "O modified to map x to T_0 and behaving as O on all other arguments":

$$O[T_0/x]\,(x) = T_0$$

$$O[T_0/x]\,(y) = O(y)$$

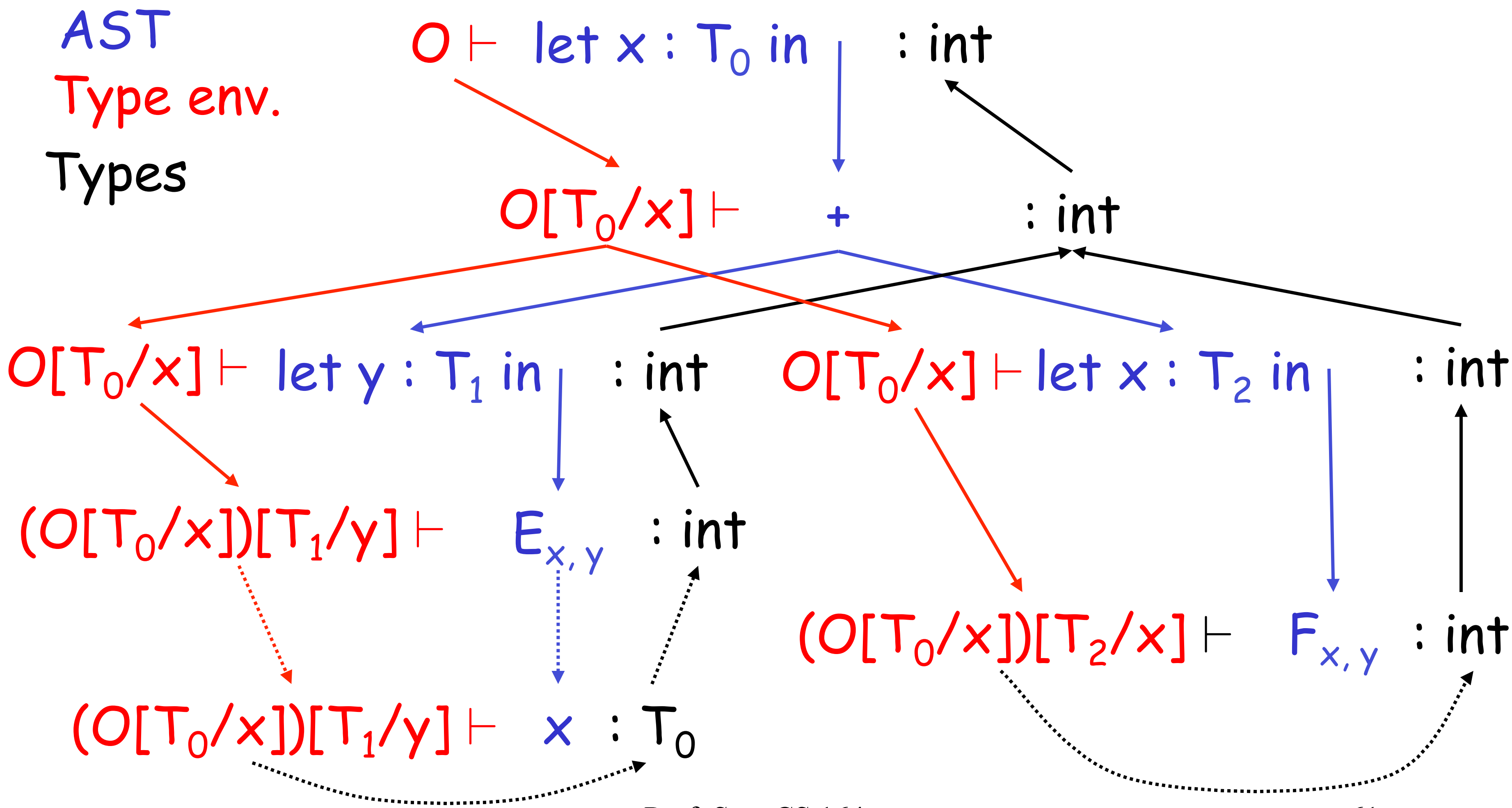
Let. Example.

- Consider the Cool expression

  $\text{let } x : T_0 \text{ in } (\text{let } y : T_1 \text{ in } E_{x,\,y}) + (\text{let } x : T_2 \text{ in } F_{x,\,y})$

  (where $E_{x,\,y}$ and $F_{x,\,y}$ are some Cool expression that contain occurrences of "x" and "y")
- Scope
  - of "y" is $E_{x,\,y}$
  - of outer "x" is $E_{x,\,y}$
  - of inner "x" is $F_{x,\,y}$
- This is captured precisely in the typing rule.

Let. Example.



AST
Type env.
Types
$O \vdash$ let $x : T_0$ in $: int$
$O[T_0/x] \vdash$ + $: int$
$O[T_0/x] \vdash$ let $y : T_1$ in $: int$
$O[T_0/x] \vdash$ let $x : T_2$ in $: int$
$(O[T_0/x])[T_1/y] \vdash E_{x,y} : int$
$(O[T_0/x])[T_2/x] \vdash F_{x,y} : int$
$(O[T_0/x])[T_1/y] \vdash x : T_0$

Notes

- The type environment gives types to the free identifiers in the current scope
- The type environment is passed down the AST from the root towards the leaves
- Types are computed up the AST from the leaves towards the root

Let with Initialization

Now consider let with initialization:

$$\frac{\begin{array}{c} O \vdash e_0 : T_0 \\ O[T_0/x] \vdash e_1 : T_1 \end{array}}{O \vdash \text{let } x : T_0 \leftarrow e_0 \text{ in } e_1 : T_1} \quad \text{[Let-Init]}$$

This rule is weak. Why?

Let with Initialization

- Consider the example:

```
class C inherits P { … }
…
let x : P ← new C in …
 …
```

- The previous let rule does not allow this code
  - We say that the rule is too weak

Subtyping

- Define a relation $X \leq Y$ on classes to say that:
  - An object of type X could be used when one of type Y is acceptable, or equivalently
  - X conforms with Y
  - In Cool this means that X is a subclass of Y
- Define a relation $\leq$ on classes

  $X \leq X$

  $X \leq Y$ if X inherits from Y

  $X \leq Z$ if $X \leq Y$ and $Y \leq Z$

Let with Initialization (Again)

$$\frac{\begin{array}{c} O \vdash e_0 : T \\ T \leq T_0 \\ O[T_0/x] \vdash e_1 : T_1 \end{array}}{O \vdash \text{let } x : T_0 \leftarrow e_0 \text{ in } e_1 : T_1} \text{[Let-Init]}$$

- Both rules for let are sound
- But more programs type check with the latter

Let with Subtyping. Notes.

- There is a tension between
  - Flexible rules that do not constrain programming
  - Restrictive rules that ensure safety of execution

Expressiveness of Static Type Systems

- A static type system enables a compiler to detect many common programming errors
- The cost is that some correct programs are disallowed
  - Some argue for dynamic type checking instead
  - Others argue for more expressive static type checking

- But more expressive type systems are also more complex

Dynamic And Static Types

- The <u>dynamic type</u> of an object is the class *C* that is used in the "*new C*" expression that creates the object
  - A run-time notion
  - Even languages that are not statically typed have the notion of dynamic type
- The <u>static type</u> of an expression is a notation that captures all possible dynamic types the expression could take
  - A compile-time notion

Dynamic and Static Types. (Cont.)

- In early type systems the set of static types correspond directly with the dynamic types
- Soundness theorem: for all expressions E

  dynamic_type(E) = static_type(E)

  (in **all** executions, E evaluates to values of the type inferred by the compiler)

- This gets more complicated in advanced type systems

Dynamic and Static Types in COOL

```
class A { … }
class B inherits A {…}
class Main {
   A x ← new A;
   …
   x ← new B;
   …
}
```

x has static type A

Here, x's value has dynamic type A

Here, x's value has dynamic type B

- *A* variable of static type *A* can hold values of static type B, if B ≤ *A*

Dynamic and Static Types

Soundness theorem for the Cool type system:

$$\forall E.\quad \text{dynamic\_type}(E) \le \text{static\_type}(E)$$

Dynamic and Static Types

Soundness theorem for the Cool type system:

$$\forall E. \quad dynamic\_type(E) \leq static\_type(E)$$

Why is this Ok?

- For E, compiler uses $static\_type(E)$ (call it C)
- All operations that can be used on an object of type C can also be used on an object of type $C' \leq C$
  - Such as fetching the value of an attribute
  - Or invoking a method on the object
- Subclasses can <u>only add</u> attributes or methods
- Methods can be redefined but with same type !

Let. Examples.

- Consider the following Cool class definitions

```
Class A { a() : Int { 0 }; }
Class B inherits A { b() : Int { 1 }; }
```

- An instance of B has methods "a" and "b"
- An instance of A has method "a"
  - A type error occurs if we try to invoke method "b" on an instance of A

Example of Wrong Let Rule (1)

- Now consider a hypothetical let rule:

$$\frac{O \vdash e_0 : T \qquad T \leq T_0 \qquad O \vdash e_1 : T_1}{O \vdash \text{let } x : T_0 \leftarrow e_0 \text{ in } e_1 : T_1}$$

- How is it different from the correct rule?

Example of Wrong Let Rule (1)

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- How is it different from the correct rule?
- The following good program does not typecheck

$$\text{let } x : \text{Int} \leftarrow 0 \text{ in } x + 1$$

- And some bad programs do typecheck

$$\text{foo}(x : B) : \text{Int} \{ \text{ let } x : A \leftarrow \text{new } A \text{ in } A.b() \}$$

Example of Wrong Let Rule (2)

- Now consider another hypothetical let rule:

$$\frac{O \vdash e_0 : T \qquad T_0 \leq T \qquad O[T_0/x] \vdash e_1 : T_1}{O \vdash \text{let } x : T_0 \leftarrow e_0 \text{ in } e_1 : T_1}$$

- How is it different from the correct rule?

Example of Wrong Let Rule (2)

- Now consider another hypothetical let rule:

$$\frac{O \vdash e_0 : T \qquad T_0 \leq T \qquad O[T_0/x] \vdash e_1 : T_1}{O \vdash \text{let } x : T_0 \leftarrow e_0 \text{ in } e_1 : T_1}$$

- How is it different from the correct rule?
- The following bad program is well typed

$$\text{let } x : B \leftarrow \text{new } A \text{ in } x.b()$$

- Why is this program bad?

Example of Wrong Let Rule (3)

- Now consider another hypothetical let rule:

$$\frac{O \vdash e_0 : T \qquad T \leq T_0 \qquad O[T/x] \vdash e_1 : T_1}{O \vdash \text{let } x : T_0 \leftarrow e_0 \text{ in } e_1 : T_1}$$

- How is it different from the correct rule?

Example of Wrong Let Rule (3)

- Now consider another hypothetical let rule:

$$\frac{O \vdash e_0 : T \qquad T \leq T_0 \qquad O[T/x] \vdash e_1 : T_1}{O \vdash \text{let } x : T_0 \leftarrow e_0 \text{ in } e_1 : T_1}$$

- How is it different from the correct rule?
- The following good program is not well typed

  let x : A ← new B in {… x ← new A; x.a(); }
- Why is this program not well typed?

Comments

- The typing rules use very concise notation
- They are very carefully constructed
- Virtually any change in a rule either:
  - Makes the type system unsound
    (bad programs are accepted as well typed)
  - Or, makes the type system less usable
    (good programs are rejected)

- But some good programs will be rejected anyway
  - The notion of a good program is undecidable

Notation for Inference Rules

- By tradition inference rules are written

$$\frac{\vdash \text{Hypothesis}_1 \quad \ldots \quad \vdash \text{Hypothesis}_n}{\vdash \text{Conclusion}}$$

- Cool type rules have hypotheses and conclusions of the form:

$$\vdash e : T$$

- $\vdash$ means "we can prove that . . ."

Assignment

More uses of subtyping:

$$\frac{\begin{array}{c} O(id) = T_0 \\ O \vdash e_1 : T_1 \\ T_1 \leq T_0 \end{array}}{O \vdash id \leftarrow e_1 : T_1} \quad [Assign]$$

Initialized Attributes

- Let $O_C(x) = T$ for all attributes $x{:}T$ in class C
  - O_C represents the class-wide scope
- Attribute initialization is similar to let, except for the scope of names

$$\frac{\begin{array}{c} O_C(id) = T_0 \\ O_C \vdash e_1 : T_1 \\ T_1 \leq T_0 \end{array}}{O_C \vdash id : T_0 \leftarrow e_1 \,;} \quad \text{[Attr-Init]}$$

If-Then-Else

- Consider:
  $\text{if } e_0 \text{ then } e_1 \text{ else } e_2 \text{ fi}$

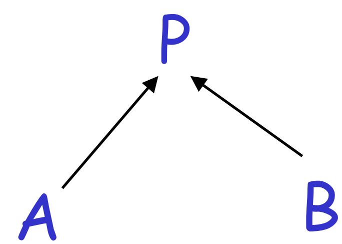
- The result can be either e_1 or e_2

- The dynamic type is either e_1's or e_2's type

- The best we can do statically is the smallest supertype larger than the type of e_1 and e_2

If-Then-Else example

- Consider the class hierarchy



- … and the expression

    if … then new A else new B fi

- Its type should allow for the dynamic type to be both A or B
    - Smallest supertype is P

Least Upper Bounds

- $lub(X,Y)$, the least upper bound of X and Y, is Z if
  - $X \leq Z \wedge Y \leq Z$

    Z is an upper bound
  - $X \leq Z' \wedge Y \leq Z' \Rightarrow Z \leq Z'$

    Z is least among upper bounds
- In COOL, the least upper bound of two types is their least common ancestor in the inheritance tree

If-Then-Else Revisited

$$\frac{\begin{array}{c} O \vdash e_0 : Bool \\ O \vdash e_1 : T_1 \\ O \vdash e_2 : T_2 \end{array}}{O \vdash \text{if } e_0 \text{ then } e_1 \text{ else } e_2 \text{ fi} : lub(T_1, T_2)} \quad \text{[If-Then-Else]}$$

Case

- The rule for case expressions takes a lub over all branches

$$
\frac{\begin{array}{c} O \vdash e_0 : T_0 \\ O[T_1/x_1] \vdash e_1 : T_1' \\ \ldots \\ O[T_n/x_n] \vdash e_n : T_n' \end{array}}{O \vdash \text{case } e_0 \text{ of } x_1{:}T_1 \Rightarrow e_1; \ldots; x_n : T_n \Rightarrow e_n; \text{ esac} : \text{lub}(T_1', \ldots, T_n')} \quad [\text{Case}]
$$

Next

- Type checking method dispatch
- Type checking with SELF_TYPE in COOL

Method Dispatch

- There is a problem with type checking method calls:

$$\frac{\begin{array}{c} O \vdash e_0 : T_0 \\ O \vdash e_1 : T_1 \\ \ldots \\ O \vdash e_n : T_n \end{array}}{O \vdash e_0.f(e_1,\ldots,e_n) : ?} \quad \text{[Dispatch]}$$

- We need information about the formal parameters and return type of f

Notes on Dispatch

- In Cool, method and object identifiers live in different name spaces
  - A method foo and an object foo can coexist in the same scope
- In the type rules, this is reflected by a separate mapping M for method signatures

$$M(C,f) = (T_1, \ldots T_n, T_{n+1})$$

means in class C there is a method f

$$f(x_1{:}T_1, \ldots, x_n{:}T_n){:}\ T_{n+1}$$

An Extended Typing Judgment

- Now we have two environments O and M

- The form of the typing judgment is

$$O, M \vdash e : T$$

read as: "with the assumption that the object identifiers have types as given by O and the method identifiers have signatures as given by M, the expression e has type T"

The Method Environment

- The method environment must be added to all rules
- In most cases, M is passed down but not actually used
  - Example of a rule that does not use M:

$$\frac{O, M \vdash e_1 : T_1 \qquad O, M \vdash e_2 : T_2}{O, M \vdash e_1 + e_2 : Int} \quad [Add]$$

  - Only the dispatch rules use M

The Dispatch Rule Revisited

$$\frac{\begin{array}{c} O, M \vdash e_0 : T_0 \\ O, M \vdash e_1 : T_1 \\ \ldots \\ O, M \vdash e_n : T_n \\ M(T_0, f) = (T_1', \ldots, T_n', T_{n+1}') \\ T_i \leq T_i' \quad (\text{for } 1 \leq i \leq n) \end{array}}{O, M \vdash e_0.f(e_1, \ldots, e_n) : T_{n+1}'} \quad [\text{Dispatch}]$$

Static Dispatch

- Static dispatch is a variation on normal dispatch
- The method is found in the class explicitly named by the programmer
- The inferred type of the dispatch expression must conform to the specified type

Static Dispatch (Cont.)

$$\frac{\begin{array}{c} O, M \vdash e_0 : T_0 \\ O, M \vdash e_1 : T_1 \\ \ldots \\ O, M \vdash e_n : T_n \\ T_0 \leq T \\ M(T, f) = (T_1', \ldots, T_n', T_{n+1}') \\ T_i \leq T_i' \quad \text{(for } 1 \leq i \leq n) \end{array}}{O, M \vdash e_0@T.f(e_1, \ldots, e_n) : T_{n+1}'} \quad \text{[StaticDispatch]}$$

Handling the SELF_TYPE

Flexibility vs. Soundness

- Recall that type systems have two conflicting goals:
  - Give flexibility to the programmer
  - Prevent valid programs to "go wrong"
    - Milner, 1981: "Well-typed programs do not *go wrong*"
- An active line of research is in the area of inventing more flexible type systems while preserving soundness

Dynamic And Static Types. Review.

- The dynamic type of an object is the class *C* that is used in the "new *C*" expression that created it
  - A run-time notion
  - Even languages that are not statically typed have the notion of dynamic type
- The static type of an expression is a notation that captures all possible dynamic types the expression could take
  - A compile-time notion

Dynamic and Static Types. Review

Soundness theorem for the Cool type system:

$$\forall E. \quad \text{dynamic\_type}(E) \leq \text{static\_type}(E)$$

Why is this Ok?

- All operations that can be used on an object of type C can also be used on an object of type $C' \leq C$
  - Such as fetching the value of an attribute
  - Or invoking a method on the object
- Subclasses can <u>only add</u> attributes or methods
- Methods can be redefined but with same type !

Attributes and Methods

$$\frac{\begin{array}{c} O_C(id) = T_0 \\ O_C \vdash e_1 : T_1 \\ T_1 \leq T_0 \end{array}}{O_C \vdash id : T_0 \leftarrow e_1 ;}$$

[Attr-Init]

$$\frac{\begin{array}{c} M(C, f) = (T_1,\ldots,T_n, T_0) \\ O_c[SELF\_TYPE/self][T_1/x_1]\ldots[T_n/x_n], M, C \vdash e : T_0 \end{array}}{O_c,M,C \vdash f(x_1:T_1,\ldots, x_n:T_n) : T_0 \{ e \}}$$

[Method]

An Example

```
class Count {
   i : int ← 0;
   inc () : Count {
        {
           i ← i + 1;
           self;
        }
    };
};
```

- Class Count incorporates a counter
- The inc method works for any subclass
- But there is disaster lurking in the type system

An Example (Cont.)

- Consider a subclass Stock of Count

```
class Stock inherits Count {
   name() : String { ...}; -- name of item
};
```

- And the following use of Stock:

```
class Main {
  a : Stock ← (new Stock).inc ();   Type checking error !
  ...  a.name() ...
};
```

What Went Wrong?

- (new Stock).inc() has dynamic type Stock
- So it is legitimate to write

  a : Stock ← (new Stock).inc ()
- But this is not well-typed

  (new Stock).inc() has static type Count
- The type checker "looses" type information
- This makes inheriting inc useless
  - So, we must redefine inc for each of the subclasses, with a specialized return type

SELF_TYPE to the Rescue

- We will extend the type system
- Insight:
  - inc returns "self"
  - Therefore the return value has same type as "self"
  - Which could be Count or any subtype of Count !
  - In the case of (new Stock).inc () the type is Stock
- We introduce the keyword SELF_TYPE to use for the return value of such functions
  - We will also need to modify the typing rules to handle SELF_TYPE

SELF_TYPE to the Rescue (Cont.)

- SELF_TYPE allows the return type of inc to change when inc is inherited
- Modify the declaration of inc to read

  inc() : SELF_TYPE { … }

- The type checker can now prove:

  $O, M \vdash$ (new Count).inc() : Count

  $O, M \vdash$ (new Stock).inc() : Stock

- The program from before is now well typed

Notes About SELF_TYPE

- SELF_TYPE is not a dynamic type
- It is a static type
- It helps the type checker to keep better track of types
- It enables the type checker to accept more correct programs
- In short, having SELF_TYPE increases the expressive power of the type system

SELF_TYPE and Dynamic Types (Example)

- What can be the dynamic type of the object returned by inc?
  - Answer: whatever could be the type of "self"

```
class A inherits Count { } ;
class B inherits Count { } ;
class C inherits Count { } ;
```

(inc could be invoked through any of these classes)

  - Answer: Count or any subtype of Count

SELF_TYPE and Dynamic Types (Example)

- In general, if SELF_TYPE appears textually in the class C as the declared type of E then it denotes the dynamic type of the "self" expression:

  dynamic_type(E) = dynamic_type(self) $\leq$ C

- Note: The meaning of SELF_TYPE depends on where it appears
  - We write SELF_TYPE_C to refer to an occurrence of SELF_TYPE in the body of C

Type Checking

- This suggests a typing rule:

$$SELF\_TYPE_C \leq C$$

- This rule has an important consequence:
  - In type checking it is always safe to replace $SELF\_TYPE_C$ by C
- This suggests one way to handle SELF_TYPE :
  - Replace all occurrences of $SELF\_TYPE_C$ by C

- This would be correct but it is like not having SELF_TYPE at all

Operations on SELF_TYPE

- Recall the operations on types
  - $T_1 \leq T_2$ T_1 is a subtype of T_2
  - $lub(T_1,T_2)$ the least-upper bound of T_1 and T_2

- We must extend these operations to handle SELF_TYPE

Extending $\leq$

Let T and T' be any types but SELF_TYPE

There are four cases in the definition of $\leq$

1. $\text{SELF\_TYPE}_C \leq T$ if $C \leq T$
   - SELF_TYPE_C can be any subtype of C
   - This includes C itself
   - Thus this is the most flexible rule we can allow
2. $\text{SELF\_TYPE}_C \leq \text{SELF\_TYPE}_C$
   - SELF_TYPE_C is the type of the "self" expression
   - In Cool we never need to compare SELF_TYPEs coming from different classes

Extending ≤ (Cont.)

3. $T \leq SELF\_TYPE_C$ always false

   Note: $SELF\_TYPE_C$ can denote any subtype of C.

4. $T \leq T'$ (according to the rules from before)

Based on these rules we can extend lub …

Extending lub(T,T')

Let T and T' be any types but SELF_TYPE

Again there are four cases:

1. $\text{lub}(\text{SELF\_TYPE}_C, \text{SELF\_TYPE}_C) = \text{SELF\_TYPE}_C$

2. $\text{lub}(\text{SELF\_TYPE}_C, T) = \text{lub}(C, T)$

   This is the best we can do because $\text{SELF\_TYPE}_C \leq C$

3. $\text{lub}(T, \text{SELF\_TYPE}_C) = \text{lub}(C, T)$

4. lub(T, T') defined as before

Where Can SELF_TYPE Appear in COOL?

- The parser checks that SELF_TYPE appears only where a type is expected
- But SELF_TYPE is not allowed everywhere a type can appear:

1. class T inherits T' {...}
   - T, T' cannot be SELF_TYPE
   - Because SELF_TYPE is never a dynamic type
2. x : T
   - T can be SELF_TYPE
   - An attribute whose type is $SELF\_TYPE_C$

Where Can SELF_TYPE Appear in COOL?

3. let x : T in E
    - T can be SELF_TYPE
    - x has type SELF_TYPE_C
4. new T
    - T can be SELF_TYPE
    - Creates an object of the same type as self
5. $m@T(E_1,\ldots,E_n)$
    - T cannot be SELF_TYPE

Typing Rules for SELF_TYPE

- Since occurrences of SELF_TYPE depend on the enclosing class we need to carry more context during type checking
- New form of the typing judgment:

$$O,M,C \vdash e : T$$

(An expression e occurring in the body of C has static type T given a variable type environment O and method signatures M)

Type Checking Rules

- The next step is to design type rules using SELF_TYPE for each language construct
- Most of the rules remain the same except that $\leq$ and lub are the new ones
- Example:

$$\frac{\begin{array}{c} O(id) = T_0 \\ O, M, C \vdash e_1 : T_1 \\ T_1 \leq T_0 \end{array}}{O, M, C \vdash id \leftarrow e_1 : T_1}$$

What's Different?

- Recall the old rule for dispatch

$$\frac{\begin{array}{c} O,M,C \vdash e_0 : T_0 \\ \ldots \\ O,M,C \vdash e_n : T_n \\ M(T_0, f) = (T_1',\ldots,T_n',T_{n+1}') \\ T_{n+1}' \neq SELF\_TYPE \\ T_i \leq T_i' \qquad 1 \leq i \leq n \end{array}}{O,M,C \vdash e_0.f(e_1,\ldots,e_n) : T_{n+1}'}$$

What's Different?

- If the return type of the method is SELF_TYPE then the type of the dispatch is the type of the dispatch expression:

$$\frac{\begin{array}{c} O,M,C \vdash e_0 : T_0 \\ \ldots \\ O,M,C \vdash e_n : T_n \\ M(T_0, f) = (T_1',\ldots,T_n', \text{SELF\_TYPE}) \\ T_i \leq T_i' \qquad 1 \leq i \leq n \end{array}}{O,M,C \vdash e_0.f(e_1,\ldots,e_n) : T_0}$$

What's Different?

- This rule handles the Stock example
- Formal parameters cannot be SELF_TYPE
- Actual arguments can be SELF_TYPE
  - The extended $\leq$ relation handles this case
- The type T_0 of the dispatch expression could be SELF_TYPE
  - Which class is used to find the declaration of f?
  - Answer: it is safe to use the class where the dispatch appears

Static Dispatch

- Recall the original rule for static dispatch

$$\frac{\begin{array}{c} O,M,C \vdash e_0 : T_0 \\ \ldots \\ O,M,C \vdash e_n : T_n \\ T_0 \le T \\ M(T, f) = (T_1',\ldots,T_n',T_{n+1}') \\ T_{n+1}' \neq \text{SELF\_TYPE} \\ T_i \le T_i' \qquad 1 \le i \le n \end{array}}{O,M,C \vdash e_0@T.f(e_1,\ldots,e_n) : T_{n+1}'}$$

Static Dispatch

- If the return type of the method is SELF_TYPE we have:

$$\frac{\begin{array}{c} O,M,C \vdash e_0 : T_0 \\ \ldots \\ O,M,C \vdash e_n : T_n \\ T_0 \leq T \\ M(T, f) = (T_1',\ldots,T_n',SELF\_TYPE) \\ T_i \leq T_i' \qquad 1 \leq i \leq n \end{array}}{O,M,C \vdash e_0@T.f(e_1,\ldots,e_n) : T_0}$$

Static Dispatch

- Why is this rule correct?
- If we dispatch a method returning SELF_TYPE in class T, don't we get back a T?
- No. SELF_TYPE is the type of the self parameter, which may be a subtype of the class in which the method appears
- The static dispatch class cannot be SELF_TYPE

New Rules

- There are two new rules using SELF_TYPE

$$\frac{}{O,M,C \vdash \text{self} : \text{SELF\_TYPE}_C}$$

$$\frac{}{O,M,C \vdash \text{new SELF\_TYPE} : \text{SELF\_TYPE}_C}$$

- There are a number of other places where SELF_TYPE is used

Where SELF_TYPE Cannot Appear in COOL?

m(x : T) : T' { ... }

- Only T' can be SELF_TYPE !

What could go wrong if T were SELF_TYPE?

```
class A {  comp(x : SELF_TYPE) : Bool  {...};  };
class B inherits A {
    b() : int { ... };
    comp(y : SELF_TYPE) : Bool { ... y.b() ...};  };
...
  let x : A ← new B in  ... x.comp(new A); ...
...
```

Summary of SELF_TYPE

- The extended ≤ and lub operations can do a lot of the work. Implement them to handle SELF_TYPE
- SELF_TYPE can be used only in a few places. Be sure it isn't used anywhere else.
- A use of SELF_TYPE always refers to any subtype in the current class
  - The exception is the type checking of dispatch.
  - SELF_TYPE as the return type in an invoked method might have nothing to do with the current class

Why Cover SELF_TYPE ?

- SELF_TYPE is a research idea
  - It adds more expressiveness to the type system
- SELF_TYPE is itself not so important
  - except for the project
- Rather, SELF_TYPE is meant to illustrate that type checking can be quite subtle
- In practice, there should be a balance between the complexity of the type system and its expressiveness

Type Systems

- The rules in these lecture were COOL-specific
  - Other languages have very different rules

- General themes
  - Type rules are defined on the structure of expressions
  - Types of variables are modeled by an environment

- Types are a play between flexibility and safety