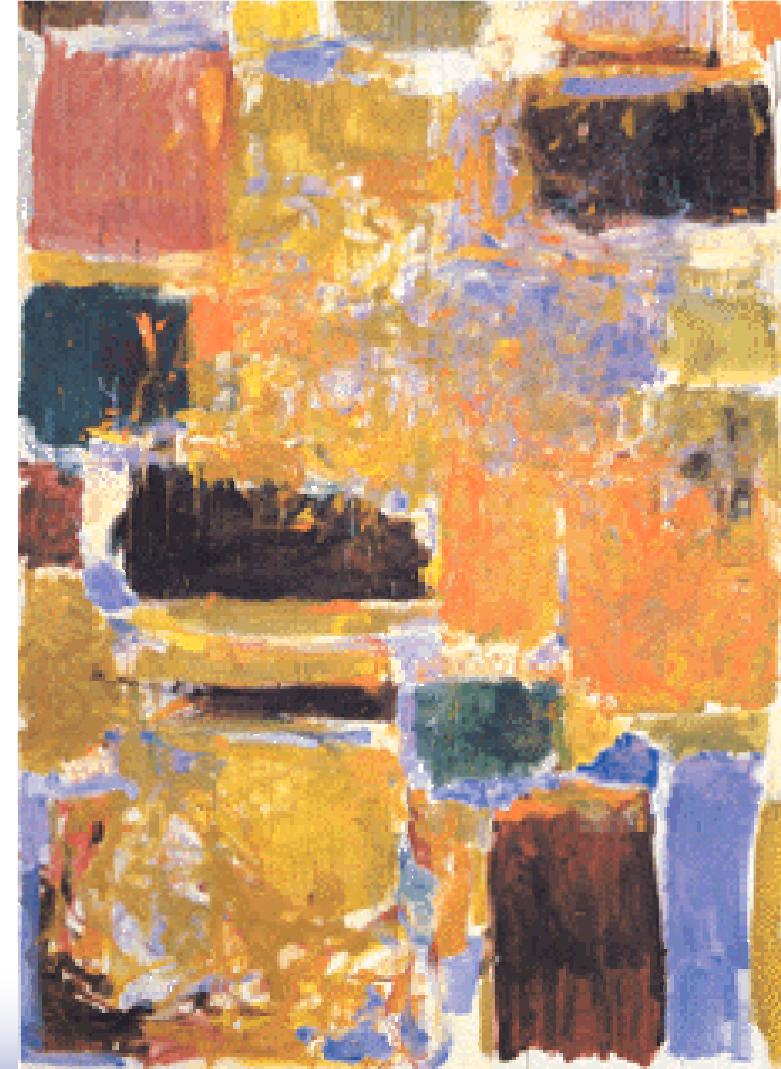


**CS150 - EE141/241A**  
**Fall 2014**  
**Digital Design and**  
**Integrated Circuits**

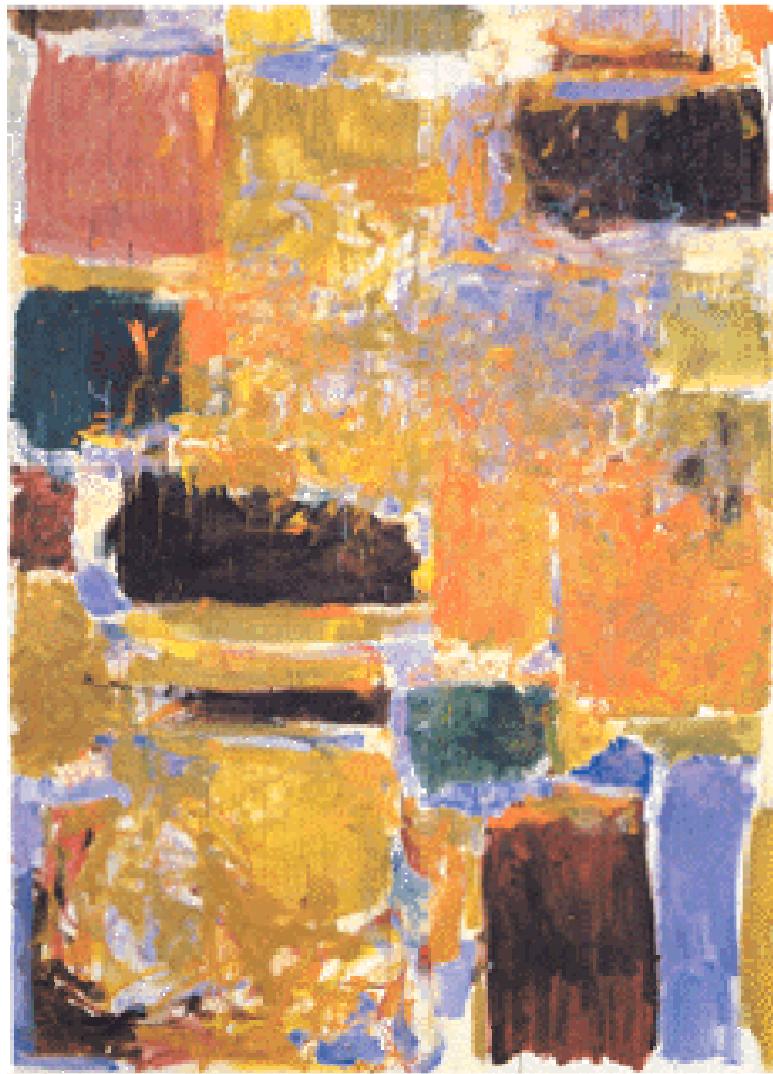
Instructors:  
John Wawrzynek and Vladimir Stojanovic

Lecture 8

# *Outline*



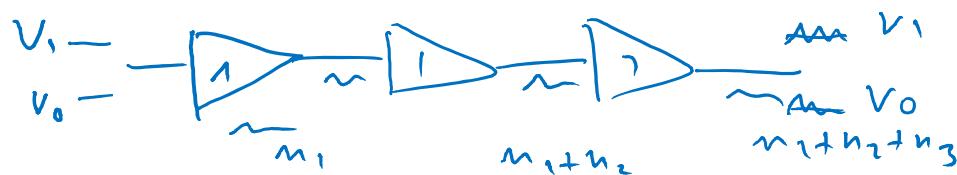
- Digital abstraction
- CMOS abstraction
- Switch logic



Digital abstraction

# Noise and Digital Systems

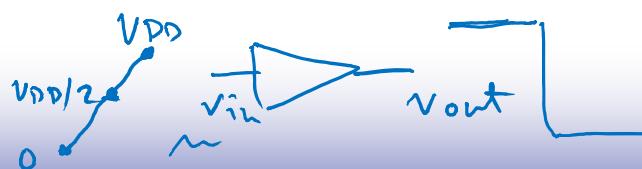
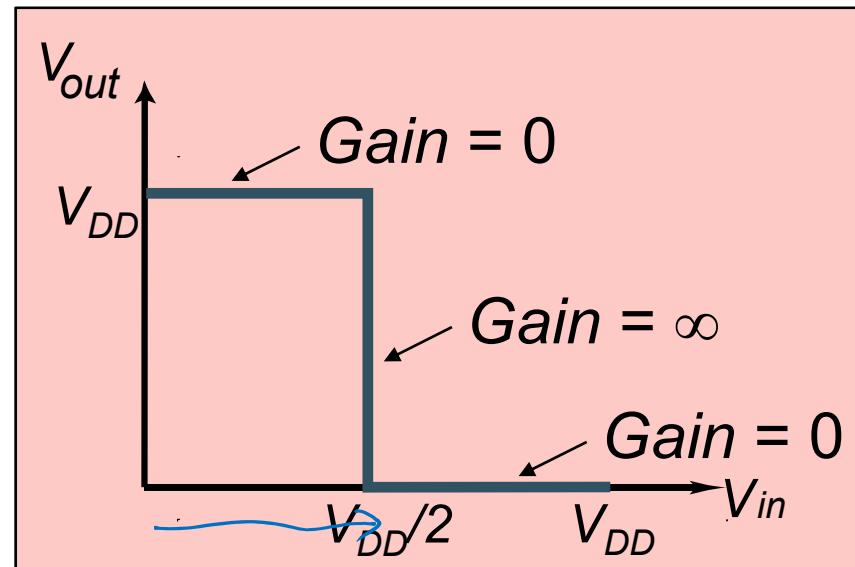
- Circuit needs to work despite “analog” noise
  - Digital gates can reject noise
  - This is actually how digital systems are defined
- Digital system is one where:
  - Discrete values mapped to analog levels and back
  - All the elements (gates) can reject noise
    - For “small” amounts of noise, output noise is less than input noise
  - Thus, for sufficiently “small” noise, the system acts as if it was noiseless



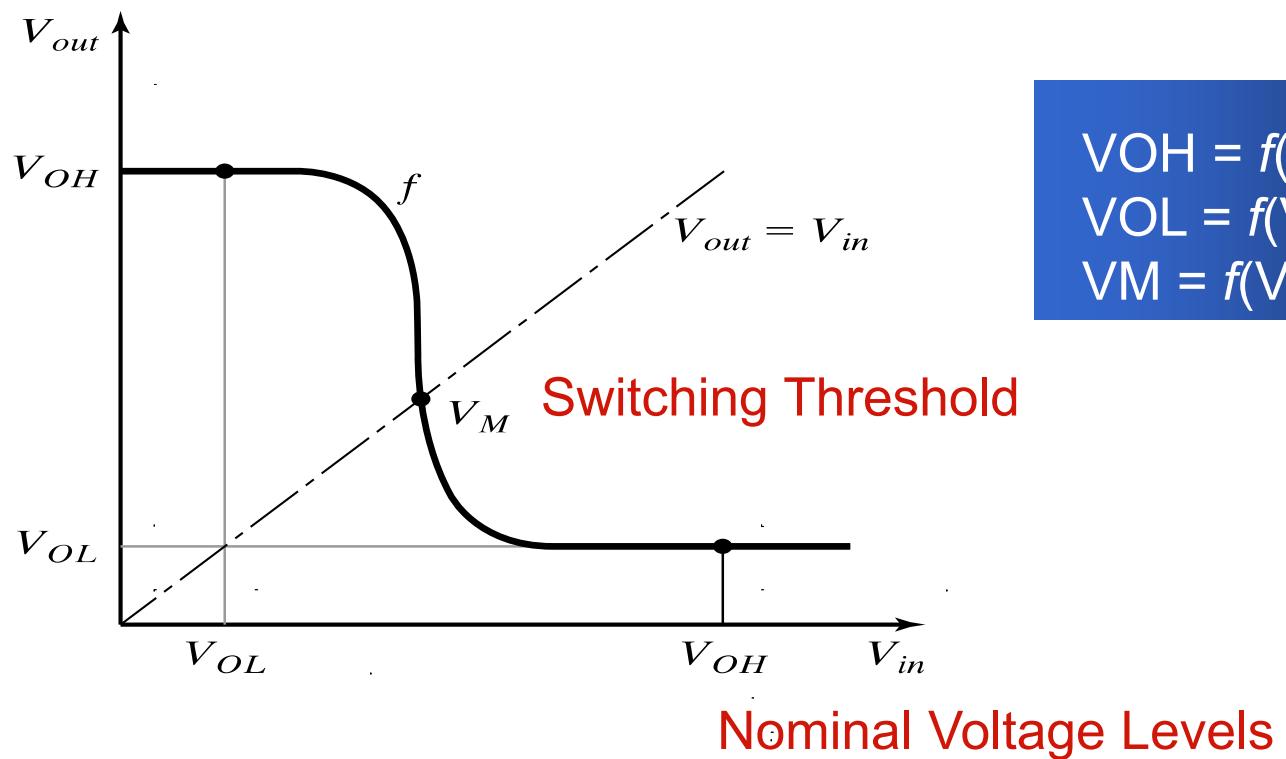
# Noise Rejection

- To see if a gate rejects noise
  - Look at its DC voltage transfer characteristic (VTC)
  - See what happens when input is not exactly 1 or 0

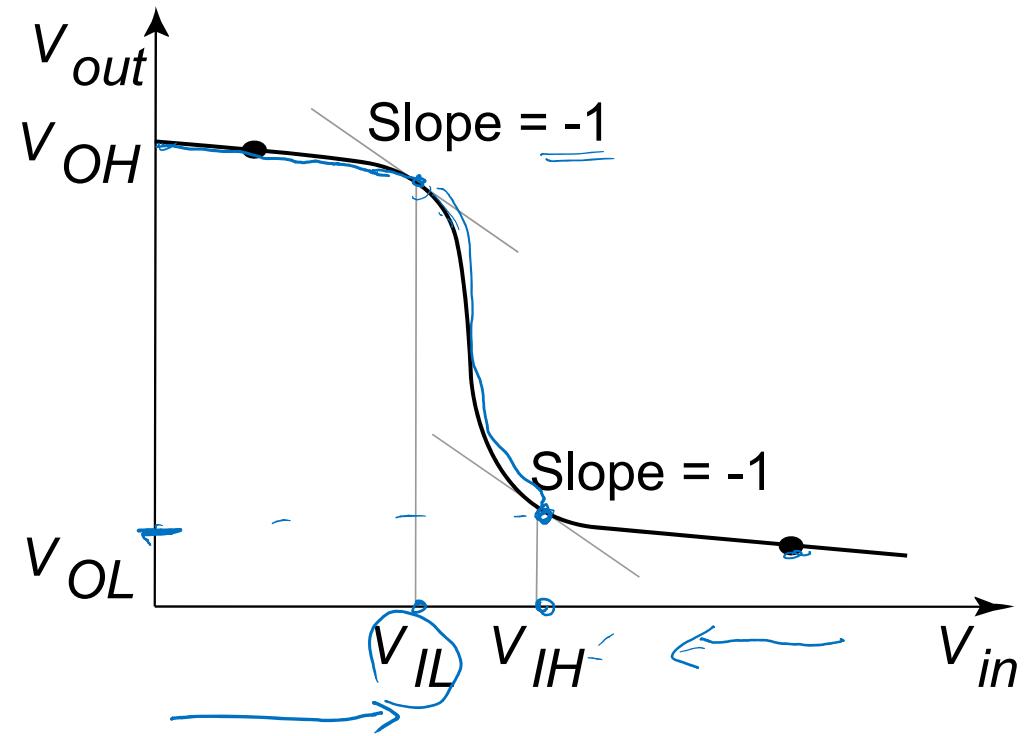
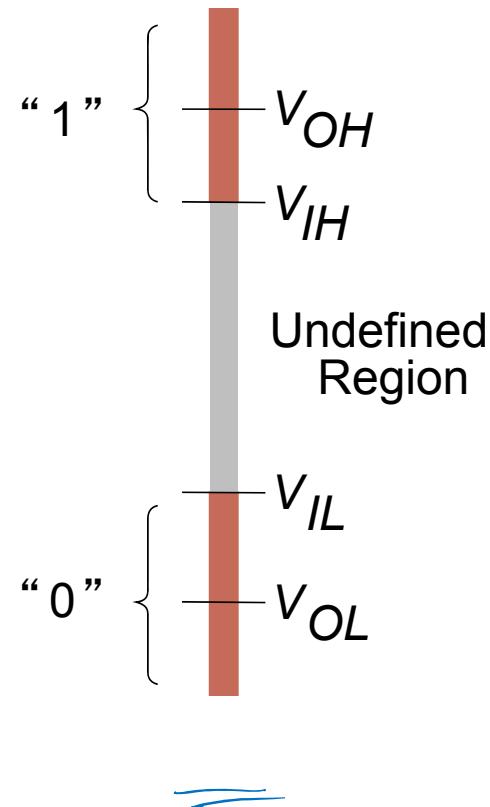
- Ideal digital gate:
  - Noise needs to be larger than  $V_{DD}/2$  to have any effect on gate output



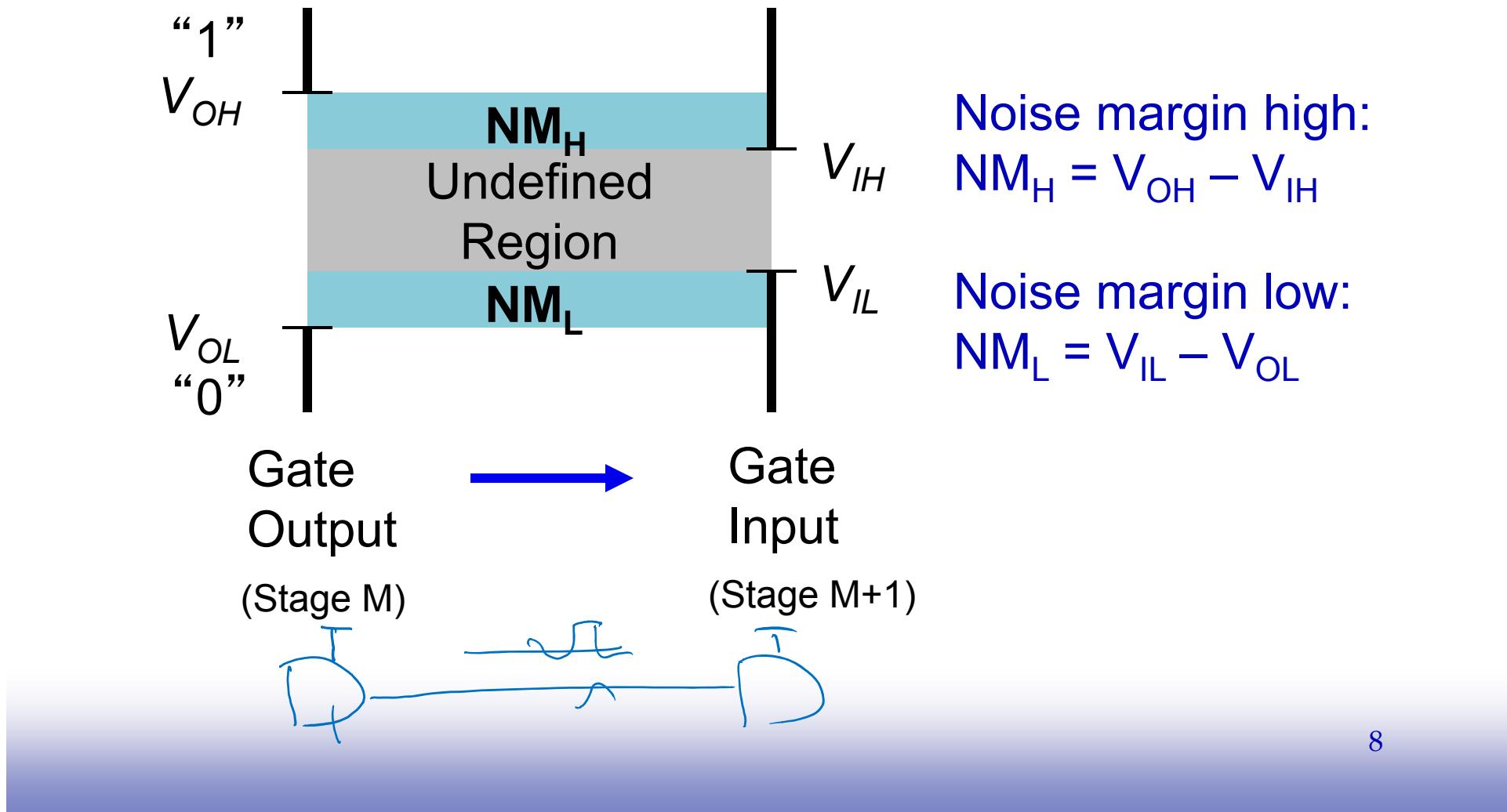
# *The Voltage Transfer Characteristic*

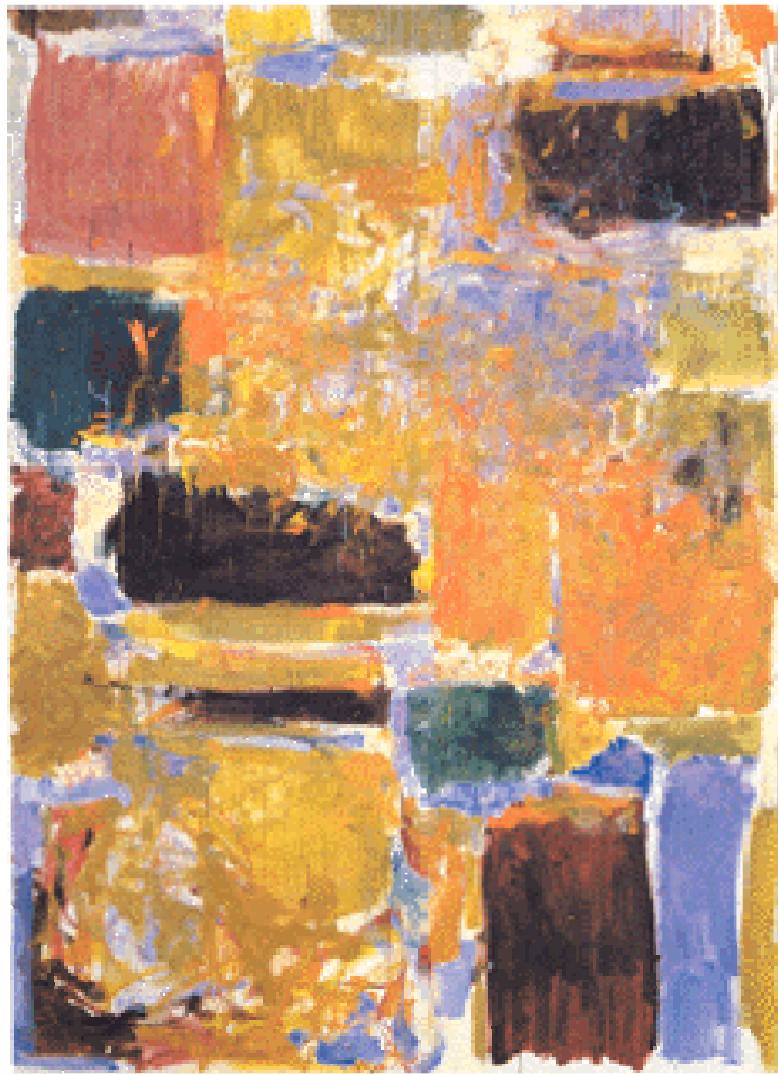


# Voltage Mapping



# *Definition of Noise Margins*

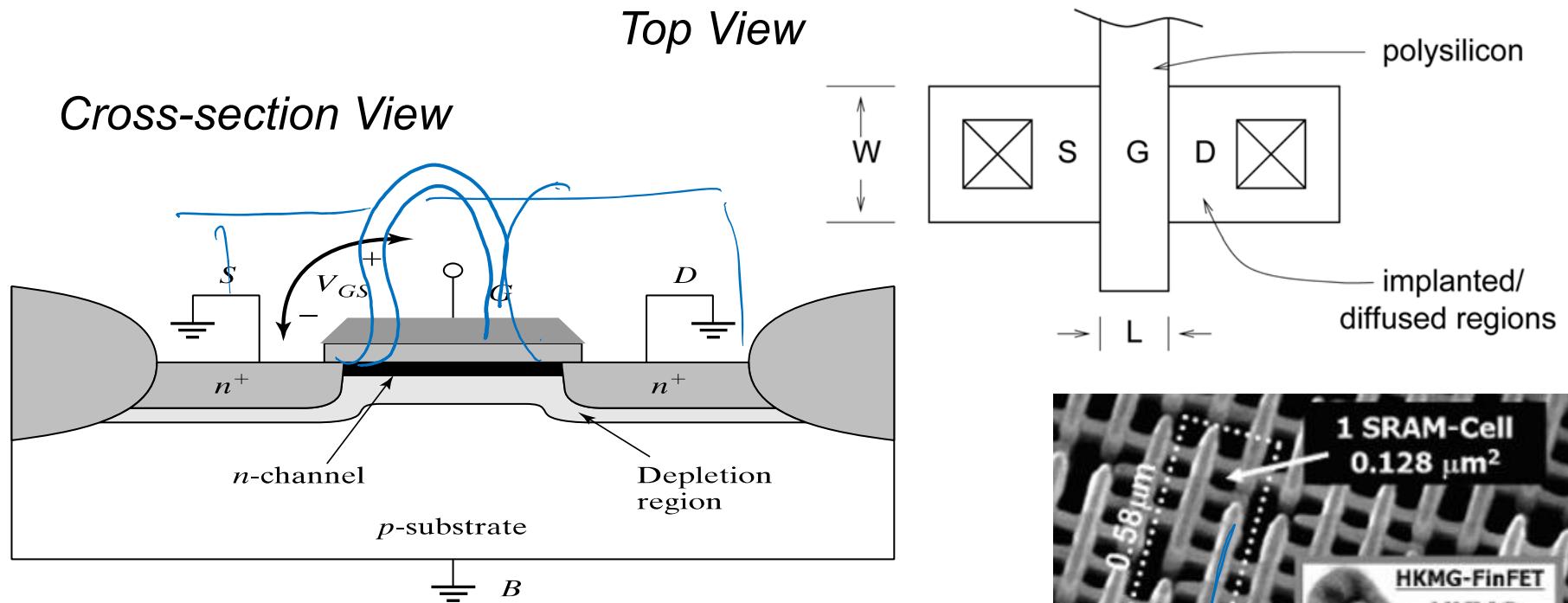




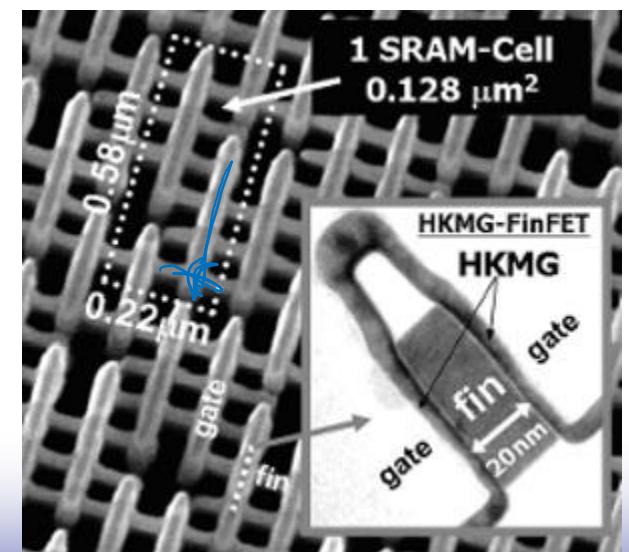
**CMOS abstraction**

# CMOS Devices

- MOSFET (Metal Oxide Semiconductor Field Effect Transistor).

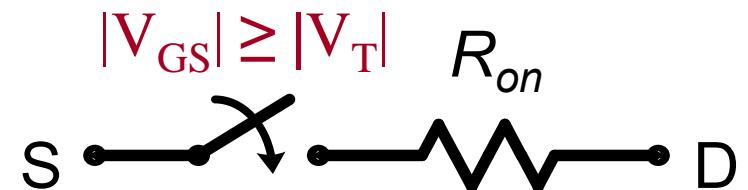
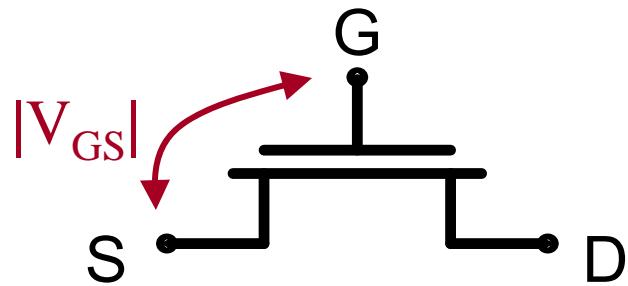


The gate acts like a capacitor. A high voltage on the gate attracts charge into the channel. If a voltage exists between the source and drain a current will flow. In its simplest approximation, the device acts like a switch.

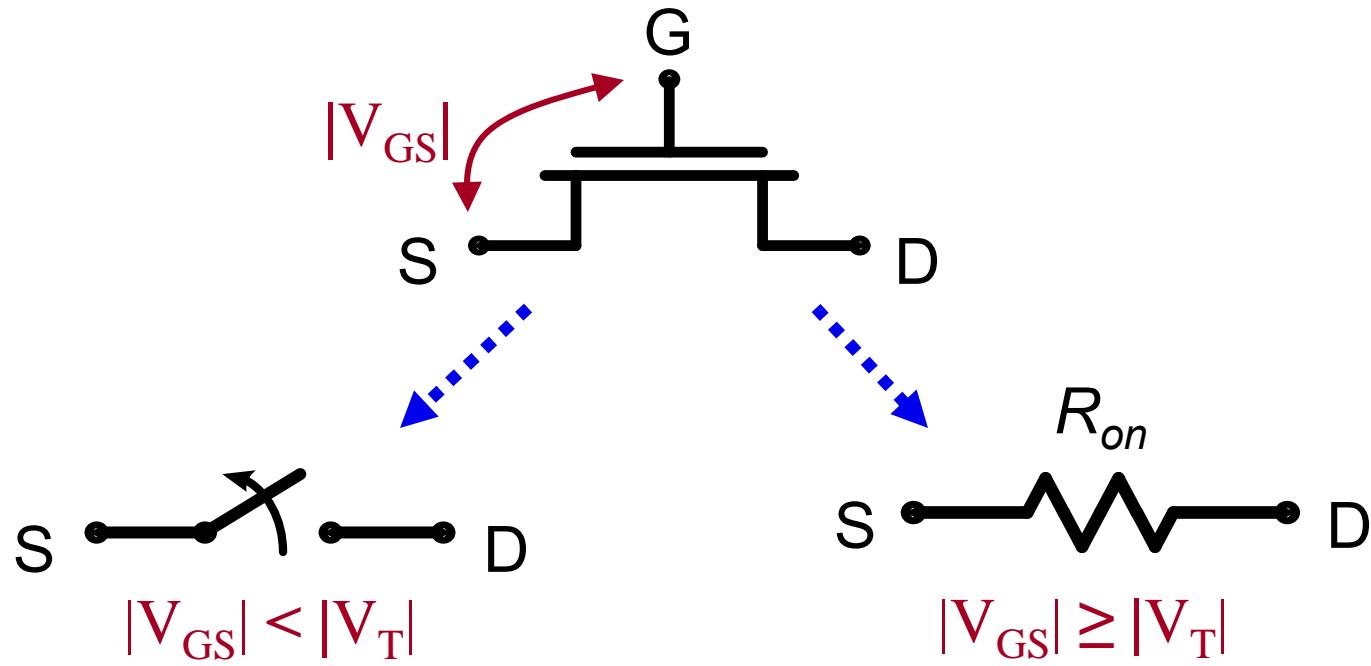


# *MOS Transistor as a Switch*

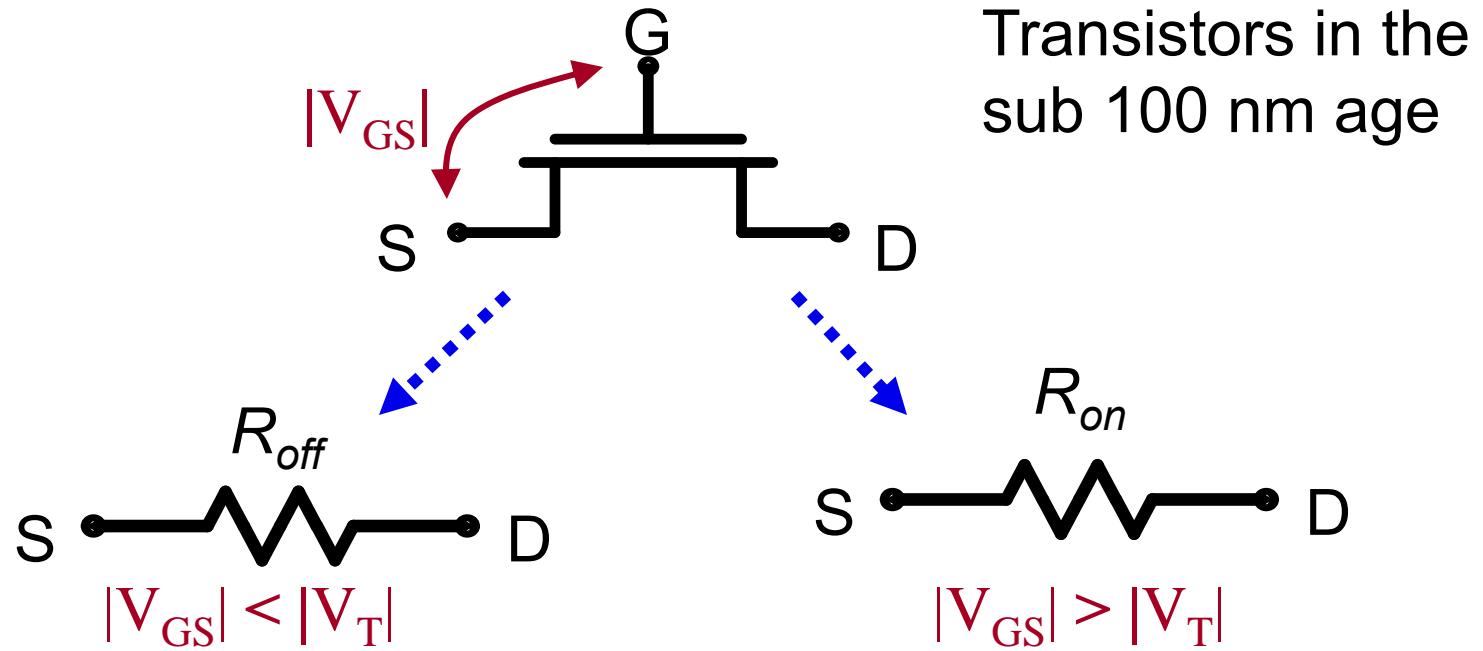
An MOS Transistor  $\longleftrightarrow$  A Switch!



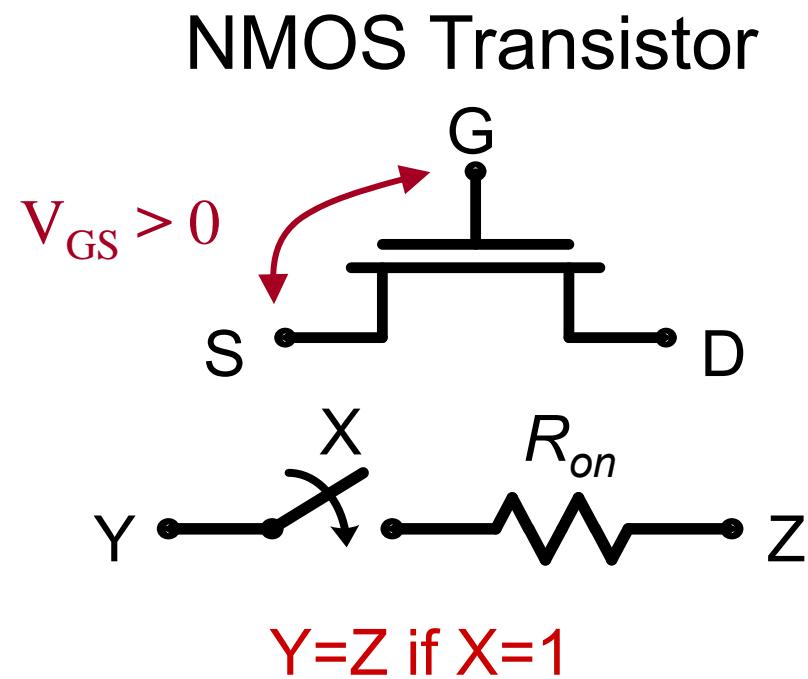
# *ON/OFF Switch Model of MOS Transistor*



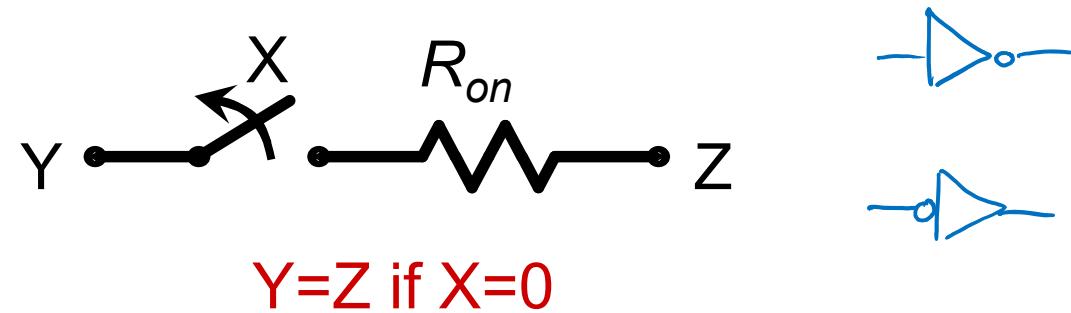
# *A More Realistic Switch*



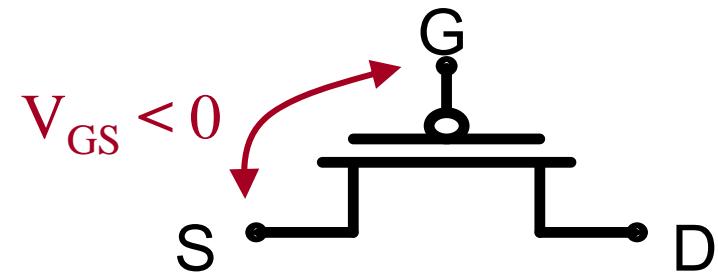
# *A Logic Perspective*



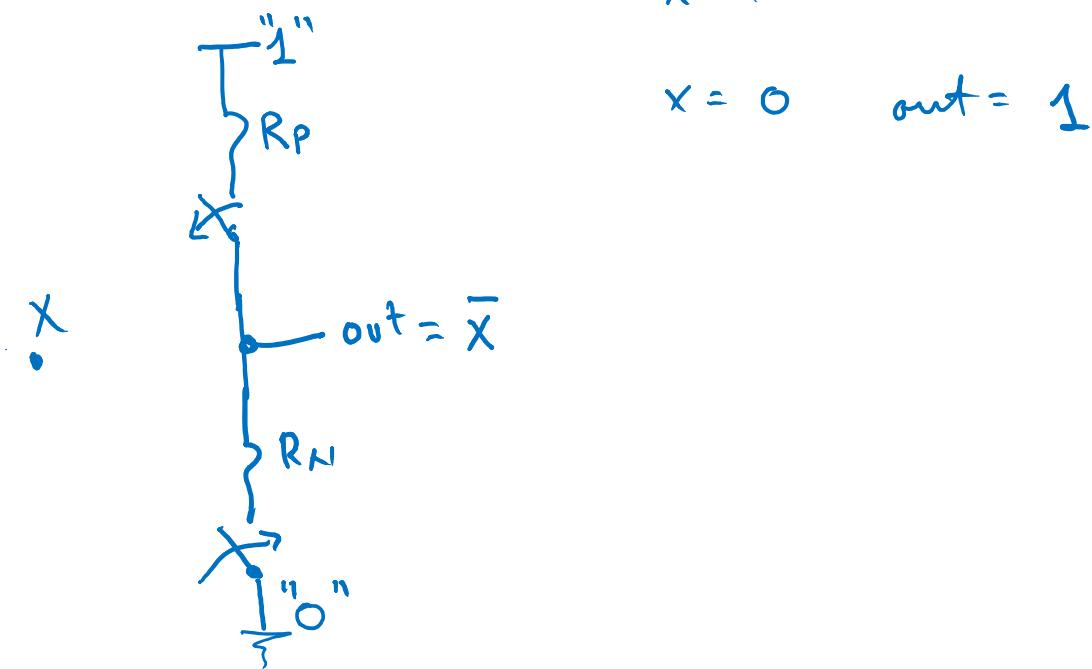
# *A Complementary Switch*



PMOS Transistor

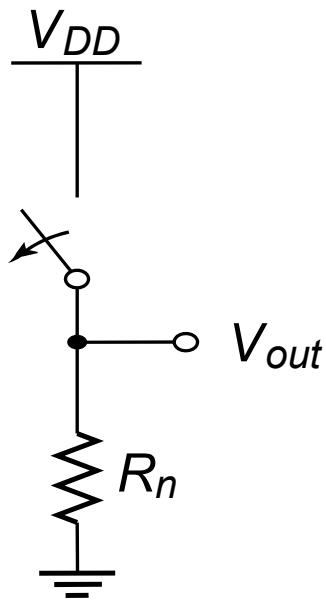


# The Switch Inverter

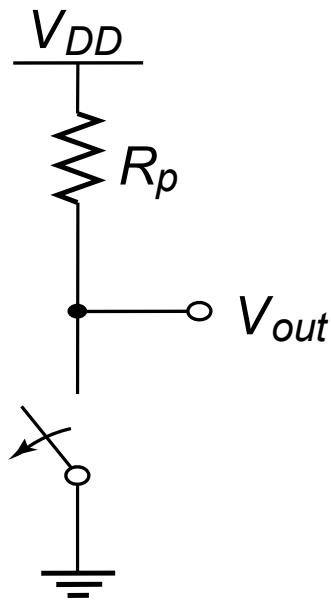


# *The Switch Inverter*

## *First-Order DC Analysis*



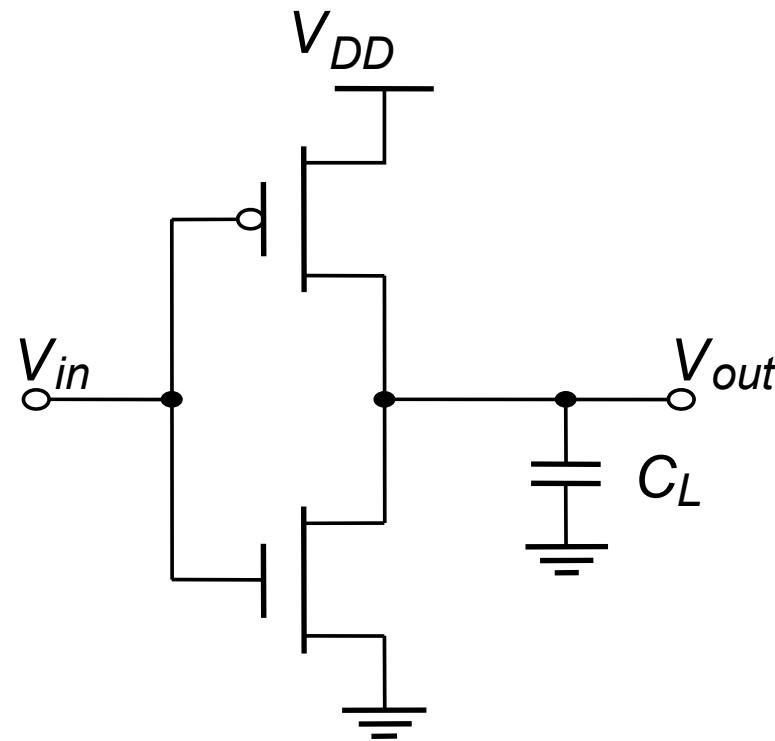
$$V_{in} = V_{DD}$$



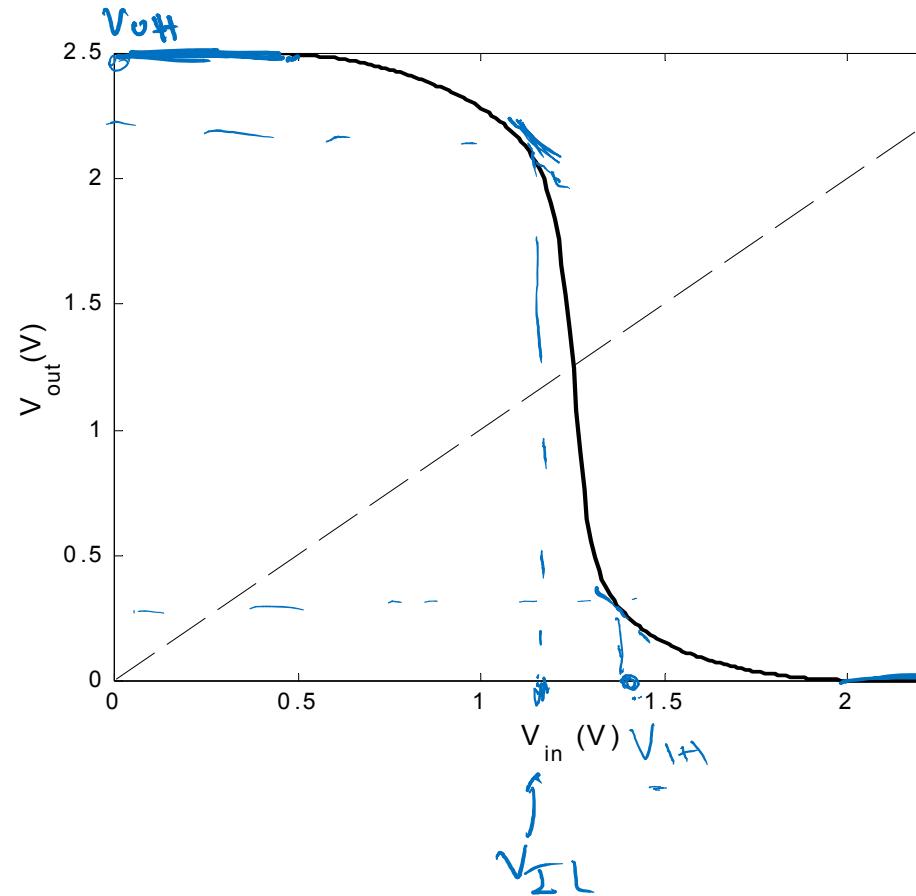
$$V_{in} = 0$$

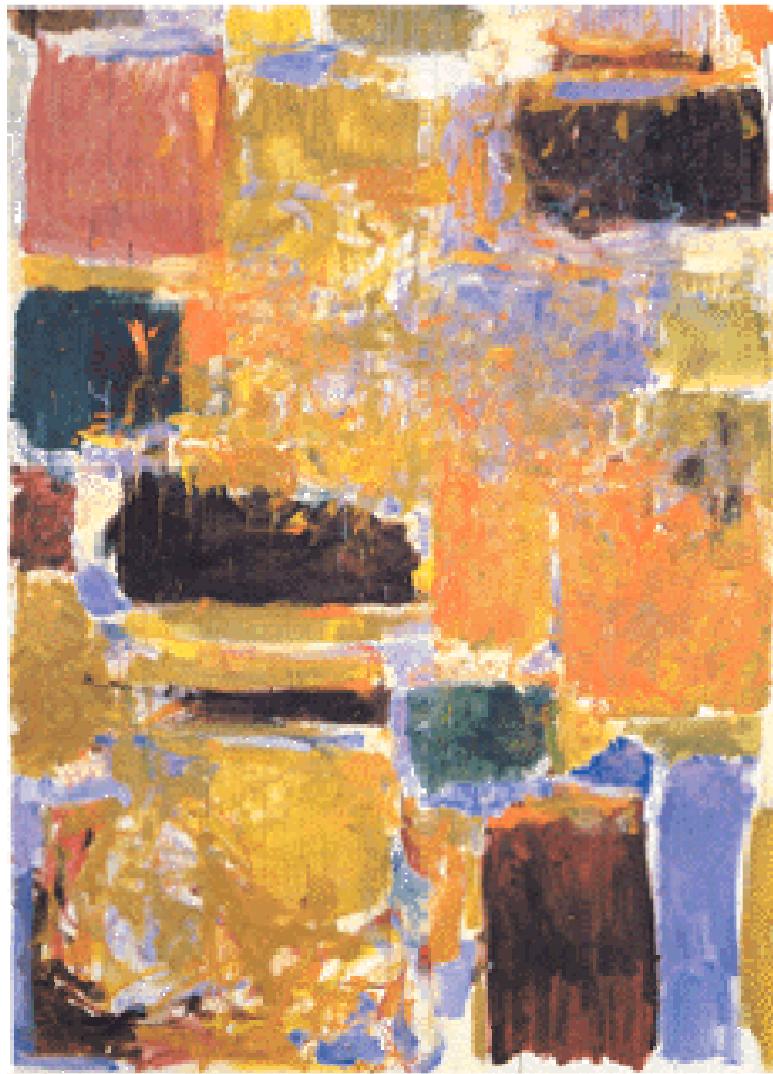
$$\boxed{V_{OL} = 0 \\ V_{OH} = V_{DD} \\ V_M = f(R_n, R_p)}$$

# *The CMOS Inverter: A First Glance*



# Simulated Inverter VTC (Spice)





# Switch logic

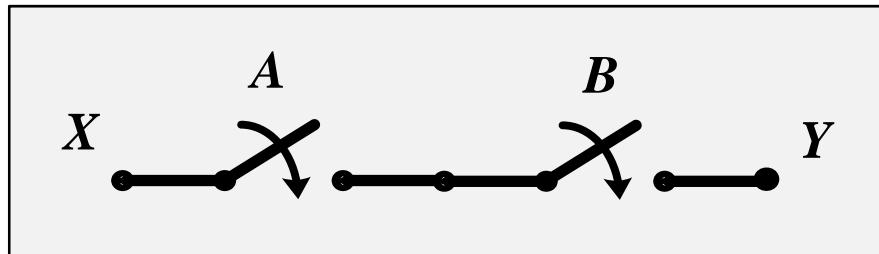
# *Static Logic Gate*

- At every point in time (except during the switching transients) each gate output is connected to either  $V_{DD}$  or  $V_{SS}$  via a low resistive path.
- The outputs of the gate assumes at all times the value of the Boolean function implemented by the circuit (ignoring, once again, the transient effects during switching periods).

Example: CMOS Inverter

# *Building logic from switches*

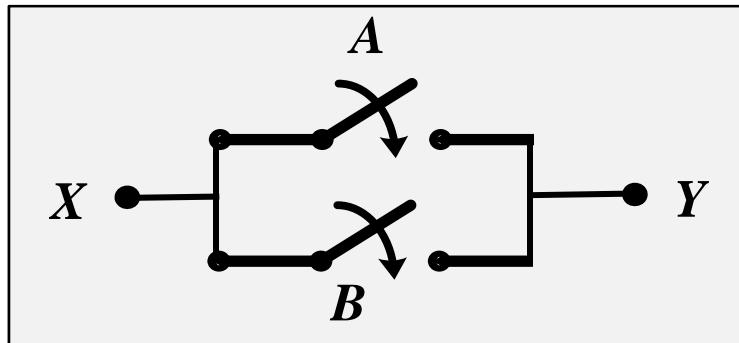
**Series**



**AND**

$Y = X \text{ if } A \text{ AND } B$

**Parallel**



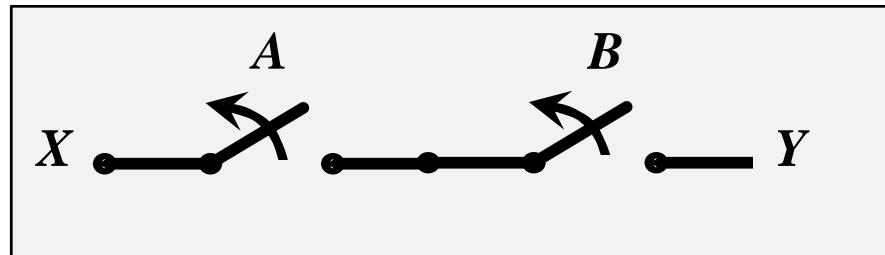
**OR**

$Y = X \text{ if } A \text{ OR } B$

(output undefined if condition not true)

# *Logic using inverting switches*

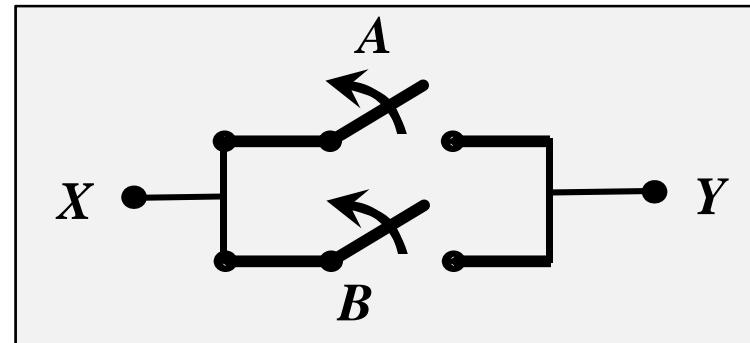
**Series**



**NOR**

$$Y = \overline{X \text{ if } \overline{A} \text{ AND } \overline{B}} \\ = \overline{\overline{A} + \overline{B}}$$

**Parallel**

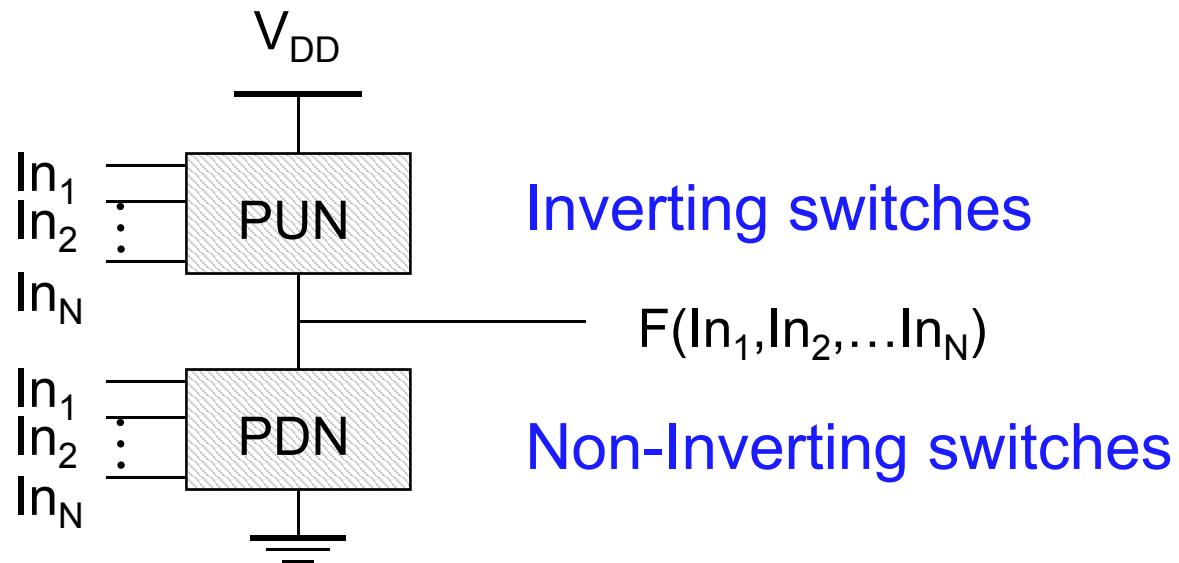


**NAND**

$$Y = \overline{X \text{ if } \overline{A} \text{ OR } \overline{B}} \\ = \overline{\overline{A} \cdot \overline{B}}$$

(output undefined if condition not true)

# *Static Complementary CMOS*



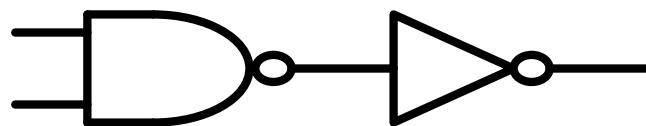
PUN and PDN are **dual** logic networks  
PUN and PDN functions are **complementary**

# *Complementary CMOS Logic Style*

- PUN is the **dual** to PDN  
(can be shown using DeMorgan's Theorems)

$$\begin{aligned}\overline{A + B} &= \overline{AB} \\ \overline{AB} &= \overline{A} + \overline{B}\end{aligned}$$

- Static CMOS gates are always inverting

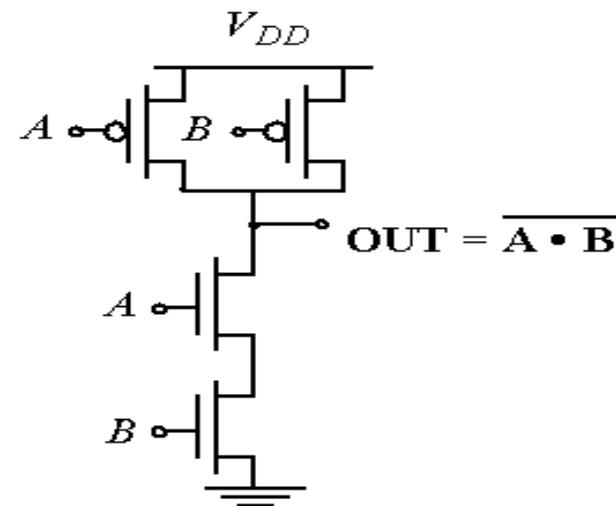


**AND = NAND + INV**

# *Example Gate: NAND*

A	B	Out
0	0	1
0	1	1
1	0	1
1	1	0

**Truth Table of a 2 input NAND gate**

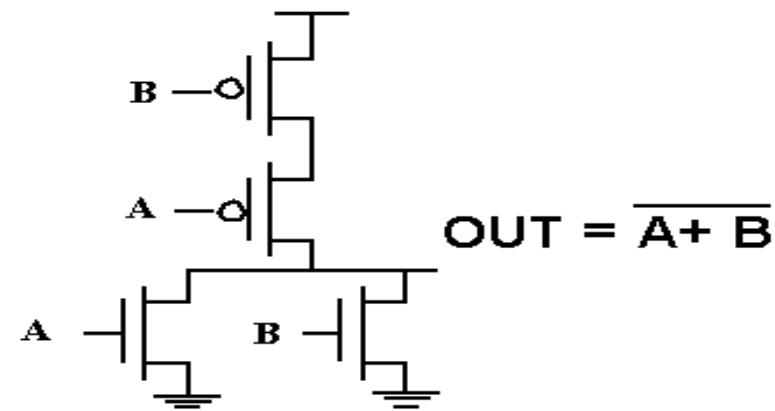


- PDN:  $G = AB \Rightarrow$  Conduction to GND
- PUN:  $F = \overline{A} + \overline{B} = \overline{AB} \Rightarrow$  Conduction to  $V_{DD}$
- $\overline{G(\overline{In}_1, \overline{In}_2, \overline{In}_3, \dots)} \equiv F(\overline{In}_1, \overline{In}_2, \overline{In}_3, \dots)$

# *Example Gate: NOR*

A	B	Out
0	0	1
0	1	0
1	0	0
1	1	0

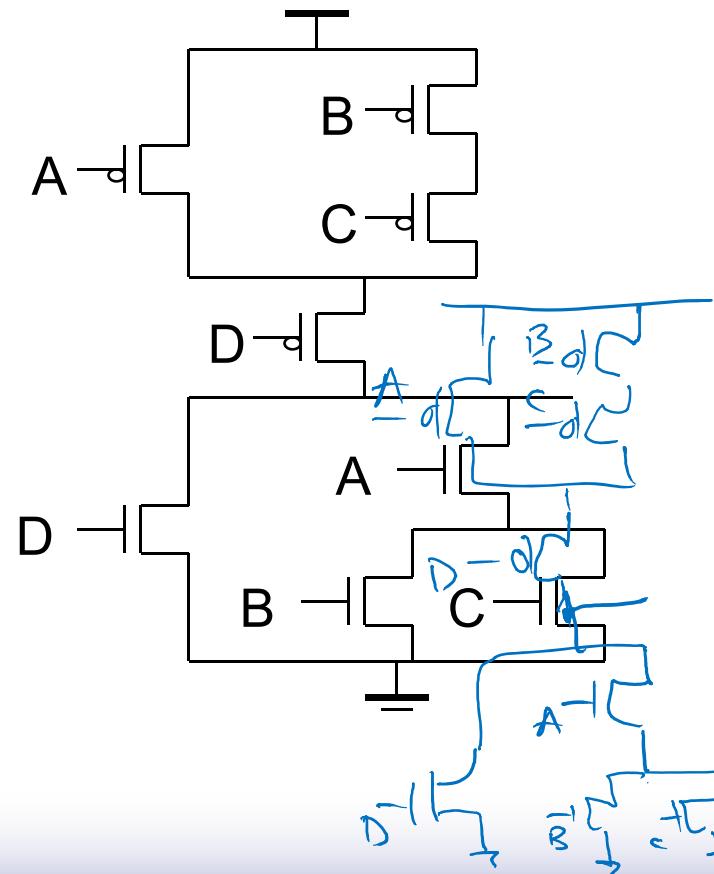
**Truth Table of a 2 input NOR gate**



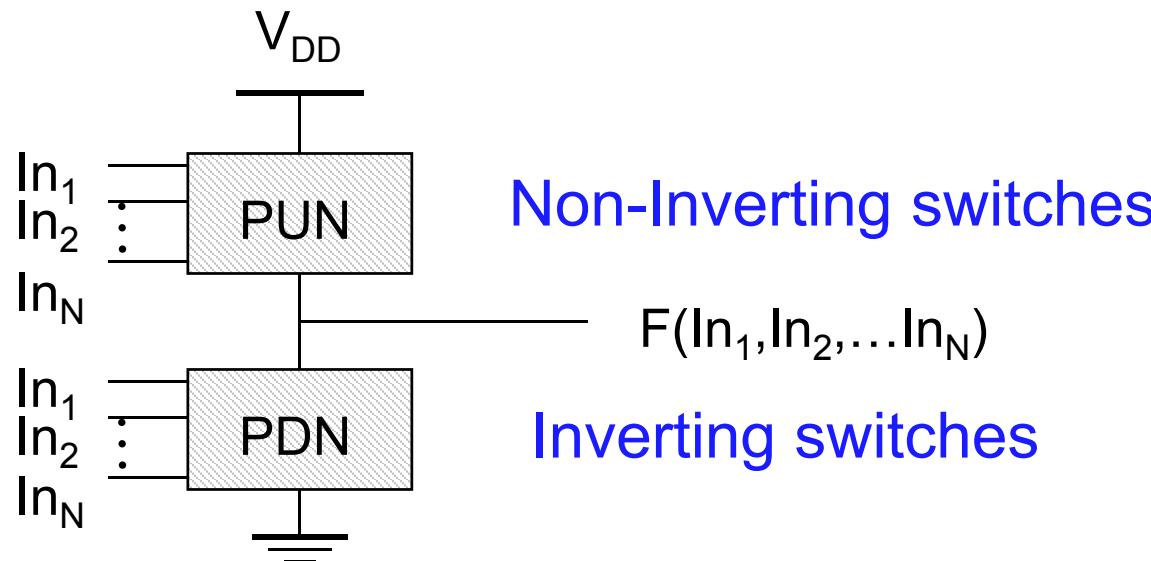
# Complex CMOS Gate

$$\text{OUT} = \overline{\overline{D} + A \cdot (B + C)}$$

$$\text{OUT} = \overline{\overline{D} \cdot A + B \cdot C}$$



# *Non-inverting logic*

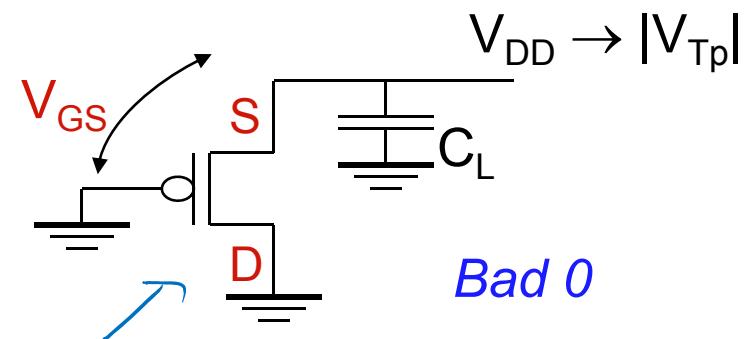
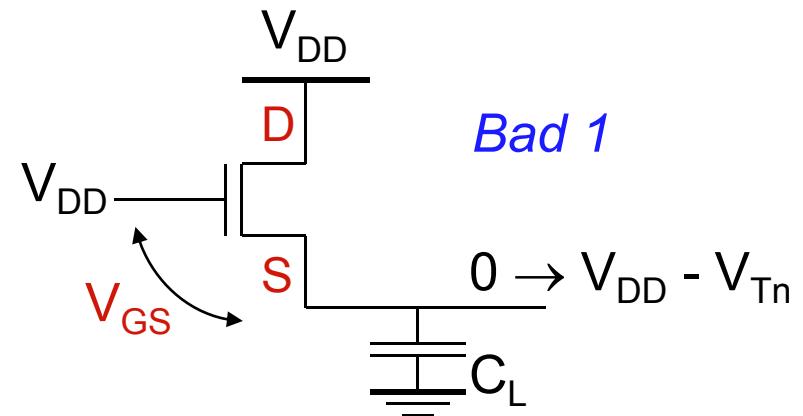
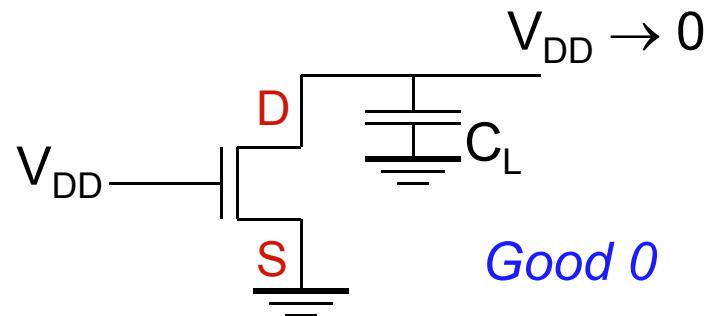
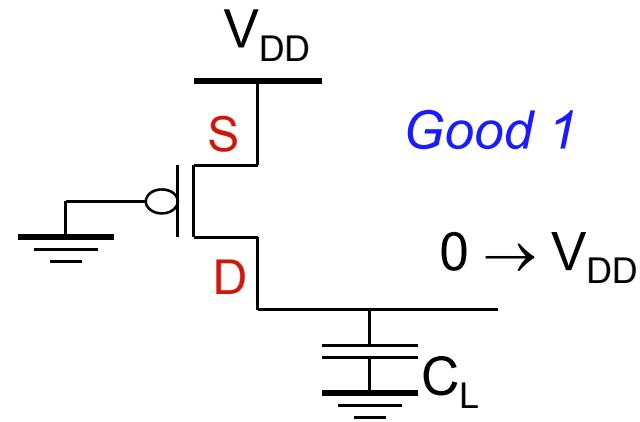


*Why is this  
a bad idea?*

PUN and PDN are **dual** logic networks

PUN and PDN functions are **complementary**

# Switch Limitations

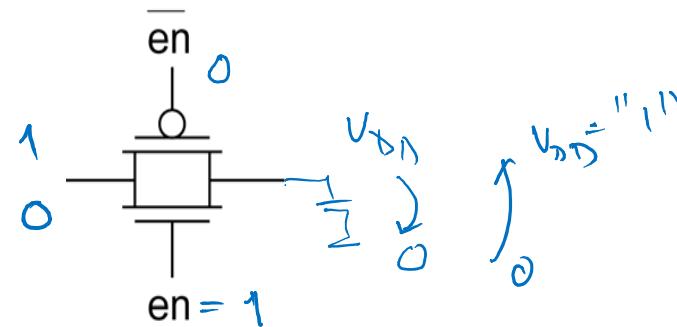


$$V_{OL} = V_{TP}$$

$$V_{OH} = V_{DD} - V_{TN}$$

# Transmission Gate

- Transmission gates are the way to build “switches” in CMOS.
- In general, both transistor types are needed:
  - nFET to pass zeros.
  - pFET to pass ones.
- The transmission gate is bi-directional (unlike logic gates).

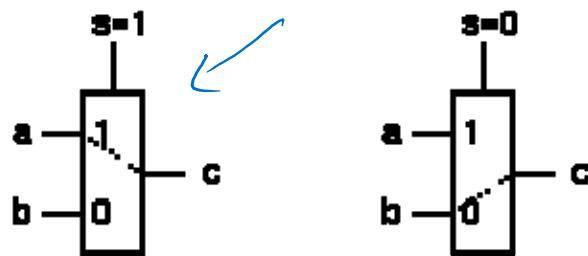


- Does not directly connect to Vdd and GND, but can be combined with logic gates or buffers to simplify many logic structures.

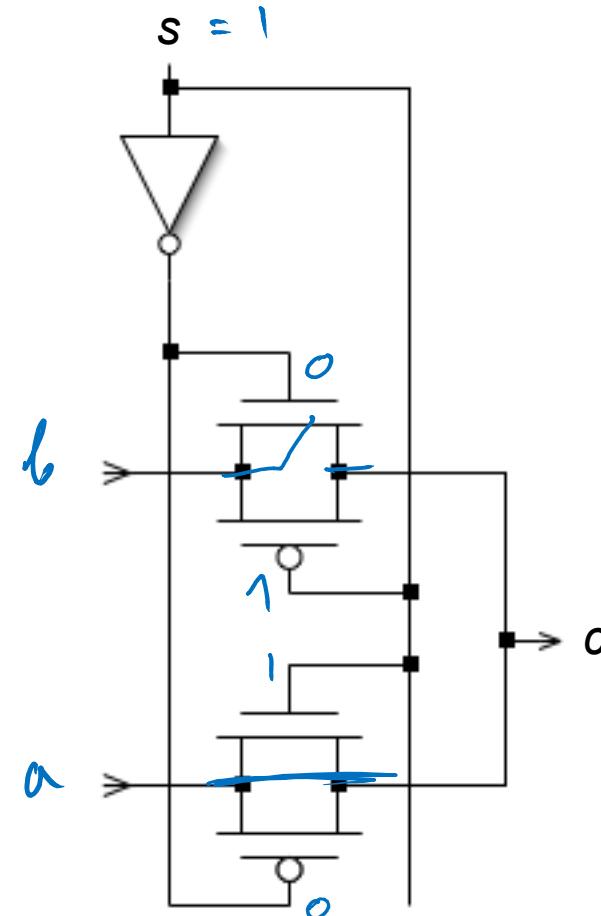
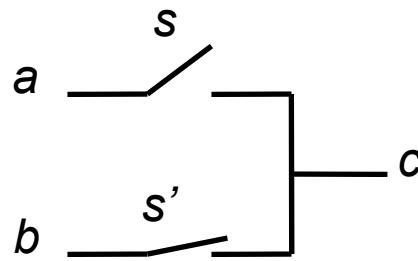
# Transmission-gate Multiplexor

2-to-1 multiplexor:

$$C = sa + s'b$$

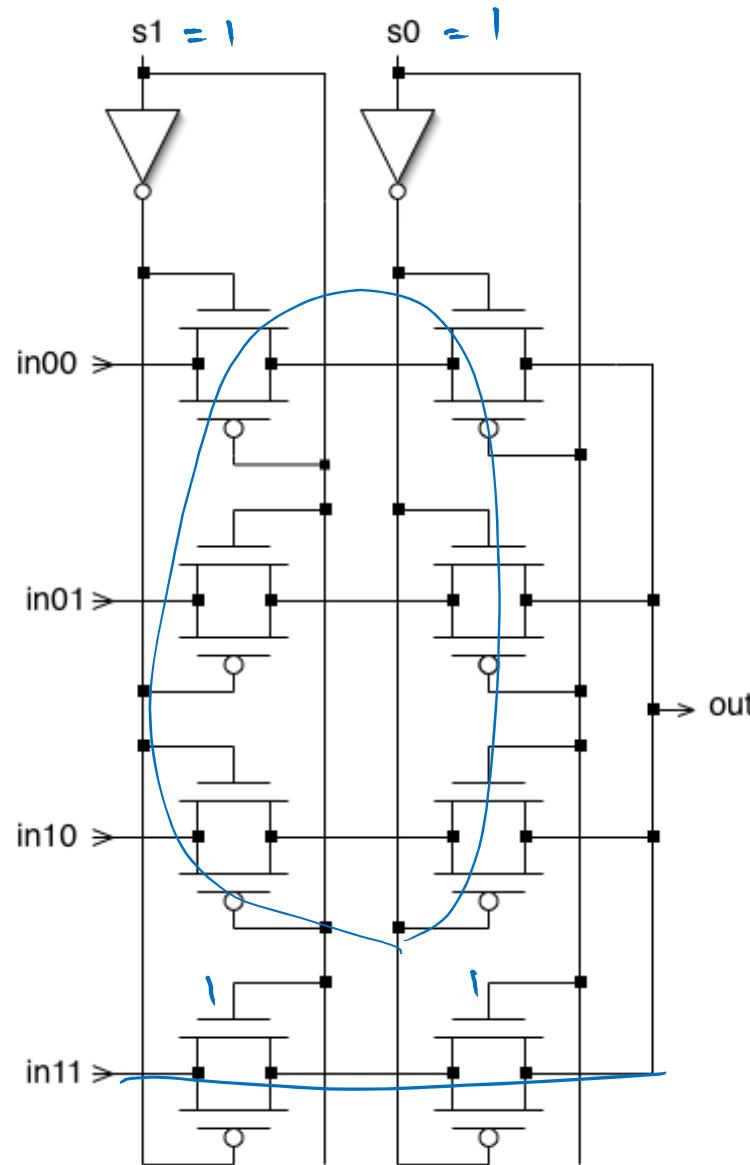


Switches simplify the implementation:



Compare the cost to logic gate implementation.

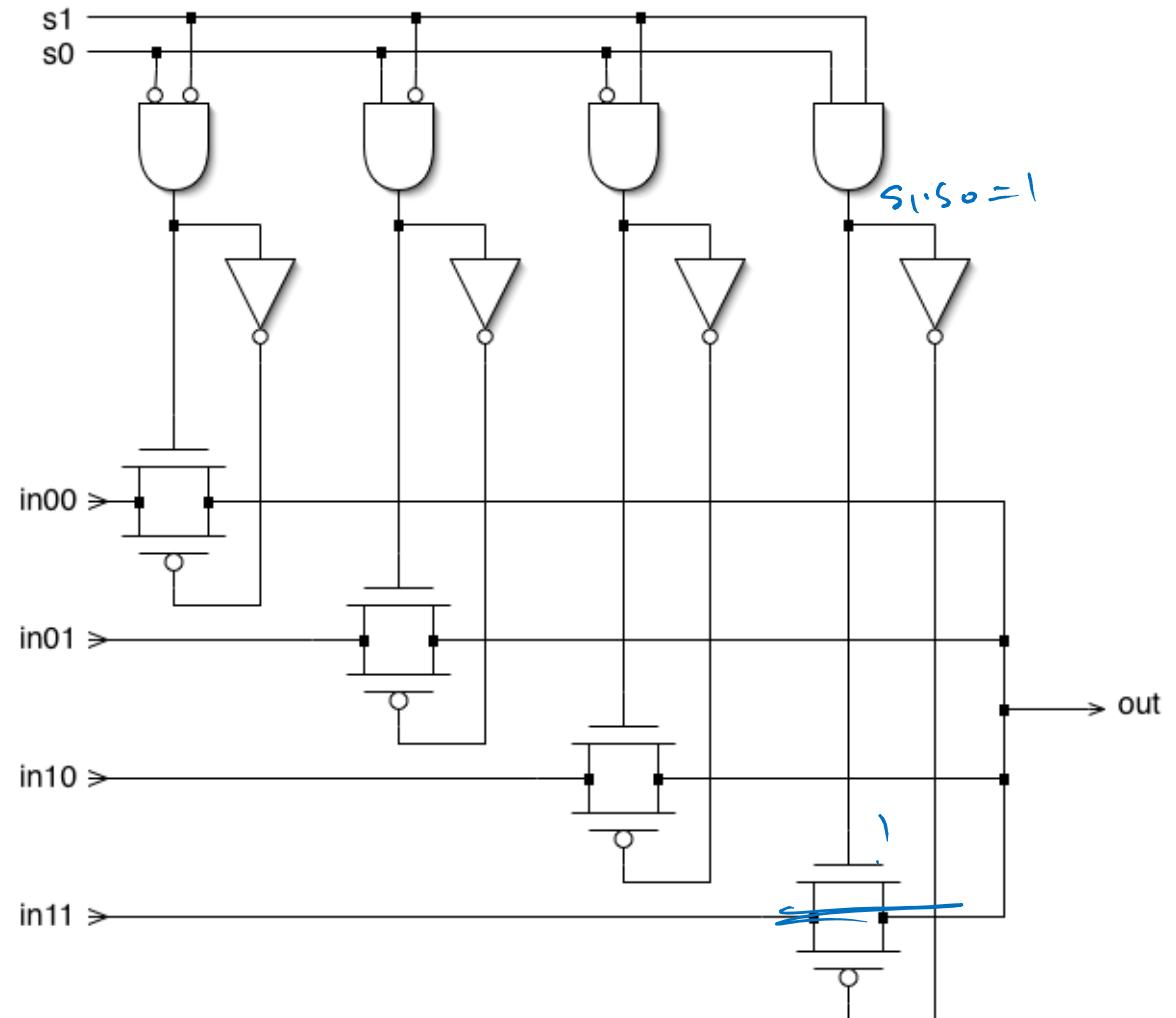
# 4-to-1 Transmission-gate Mux



- The series connection of pass-transistors in each branch effectively forms the AND of  $s_1$  and  $s_0$  (or their complement).
- Compare cost to logic gate implementation

# *Alternative 4-to-1 Multiplexor*

- This version has less delay from in to out.
- In both versions, care must be taken to avoid turning on multiple paths simultaneously (shorting together the inputs).



# Tri-state Buffers

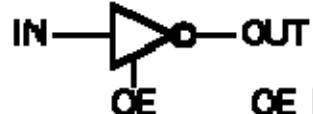
Tri-state Buffer:



OE	IN	OUT
0	0	Z
0	1	Z
1	0	0
1	1	1

"high impedance"  
(output disconnected)

Variations:



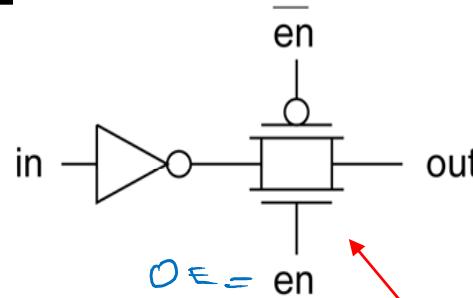
OE	IN	OUT
0	-	Z
1	0	0

Inverting buffer

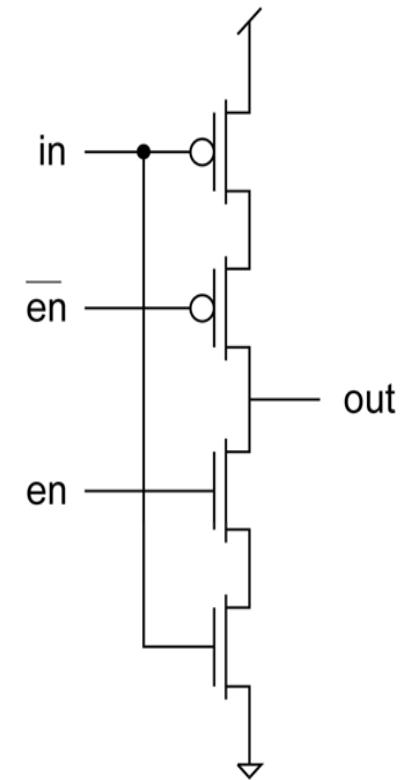


OE	IN	OUT
0	0	0
1	-	Z

Inverted enable

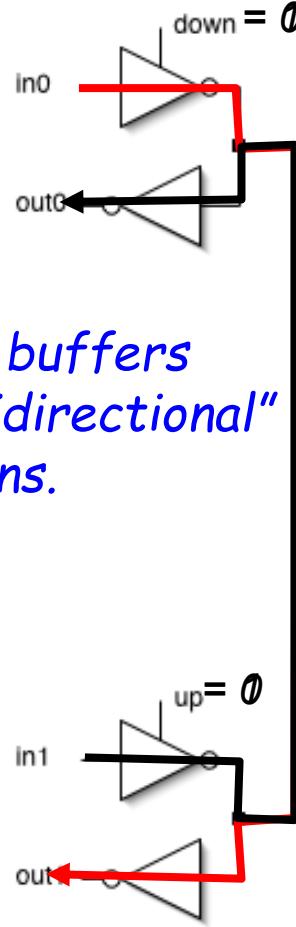


transmission  
gate useful in  
implementation

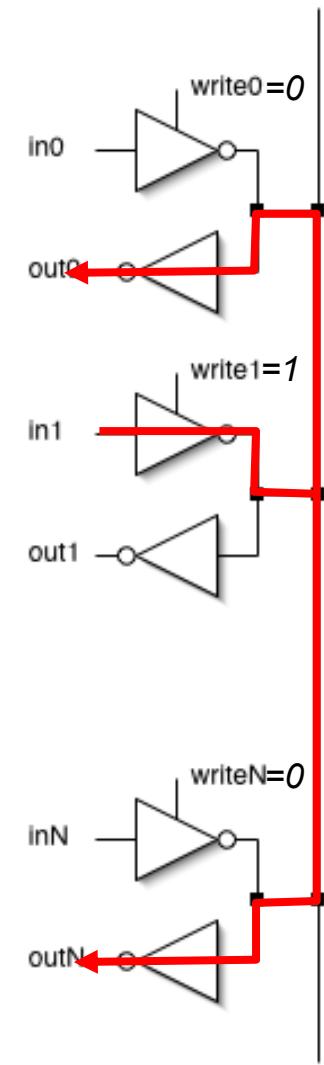


# Tri-state Buffers

Tri-state buffers enable "bidirectional" connections.

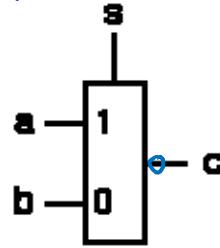


Tri-state buffers are used when multiple circuits all connect to a common wire. Only one circuit at a time is allowed to drive the bus. All others "disconnect" their outputs, but can "listen".

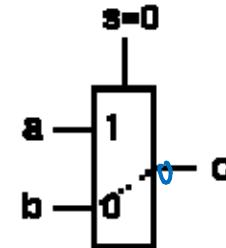
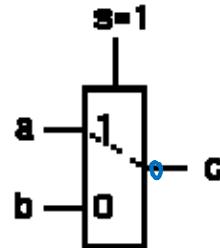


# Tri-state Based Multiplexor

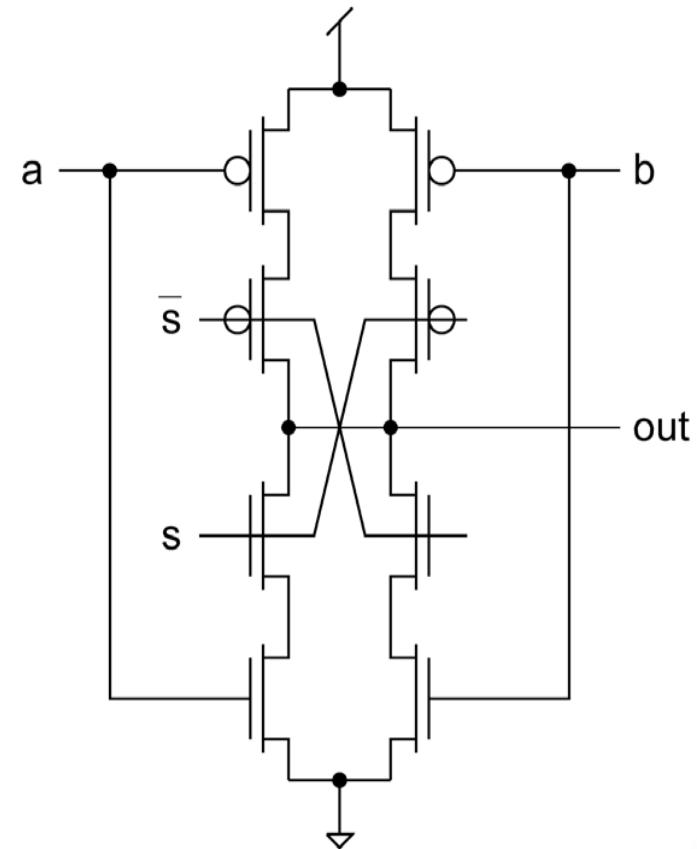
Multiplexor



If  $s=1$  then  $c=a$

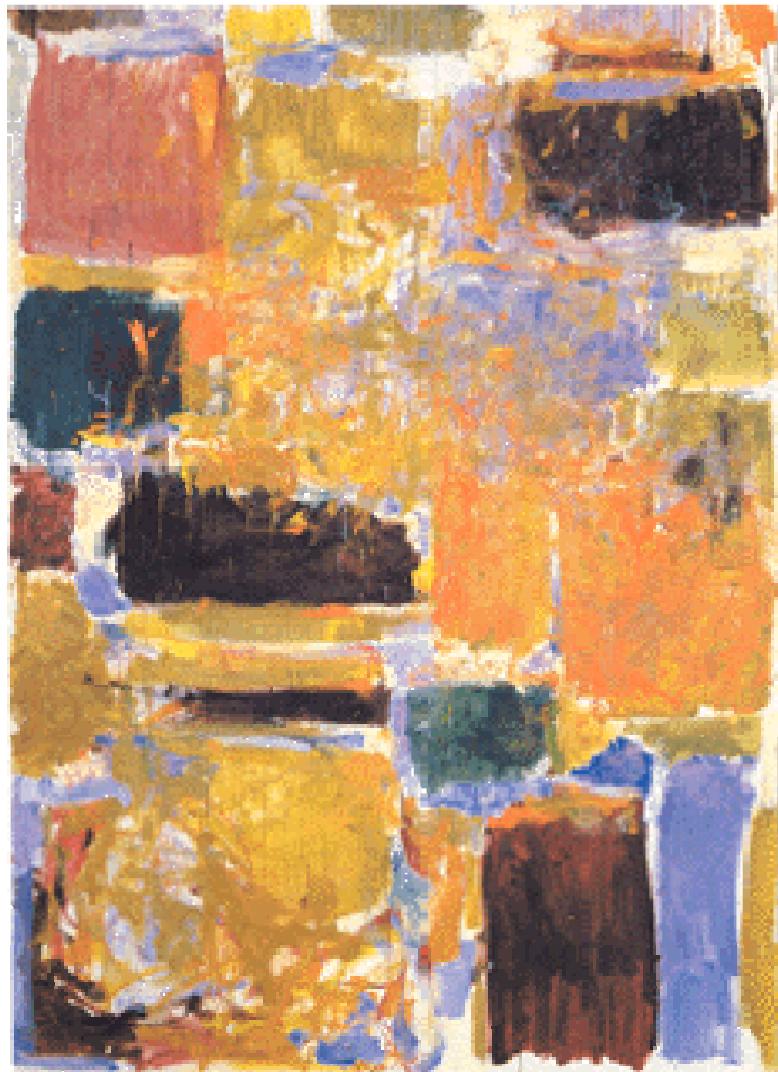


Transistor Circuit for inverting multiplexor:



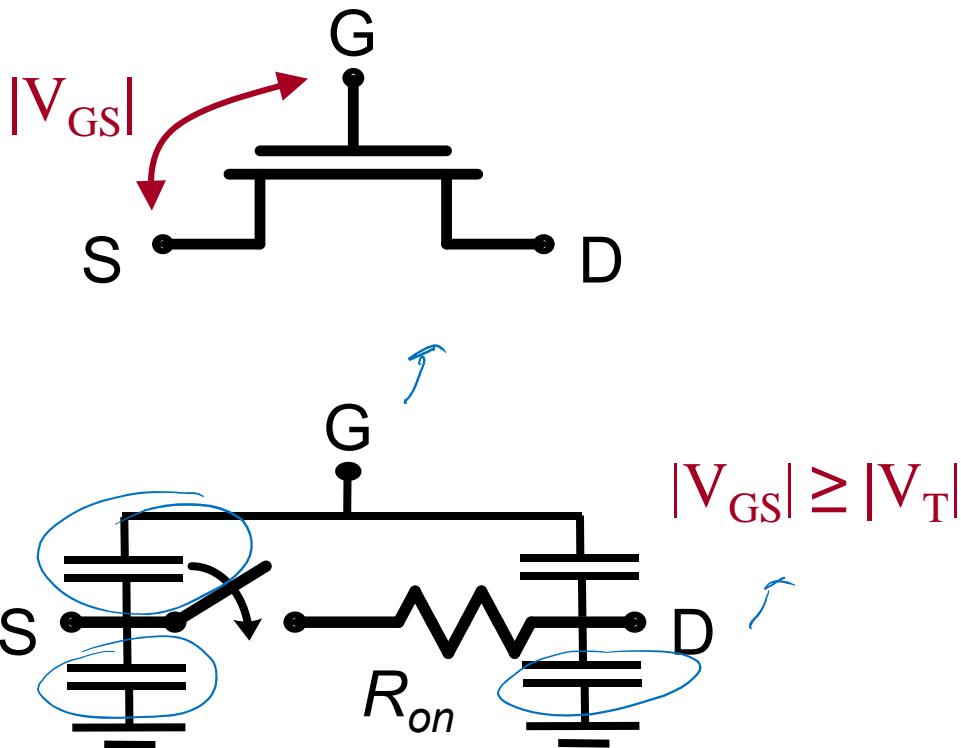
# *Summary: Complementary CMOS Properties*

- Full rail-to-rail swing
- Symmetrical VTC
- No static power dissipation
- Direct path current during switching

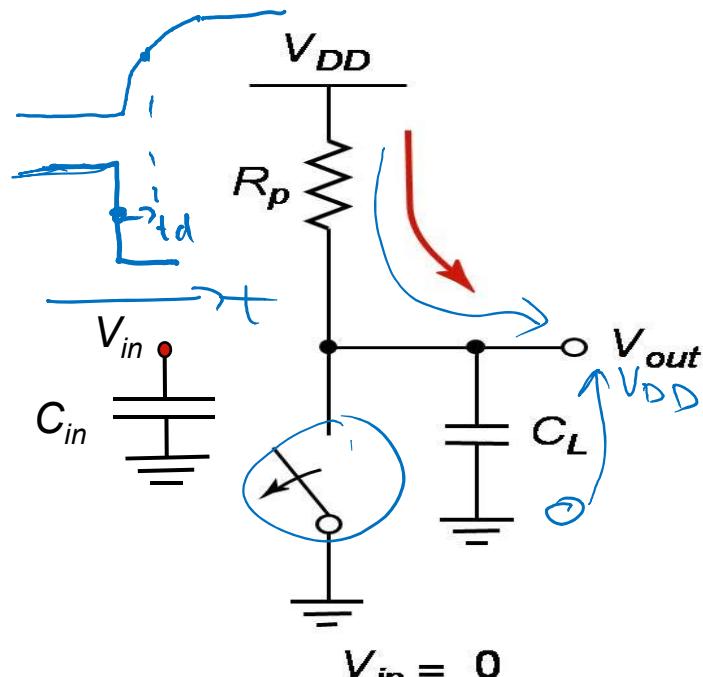


## Transient properties

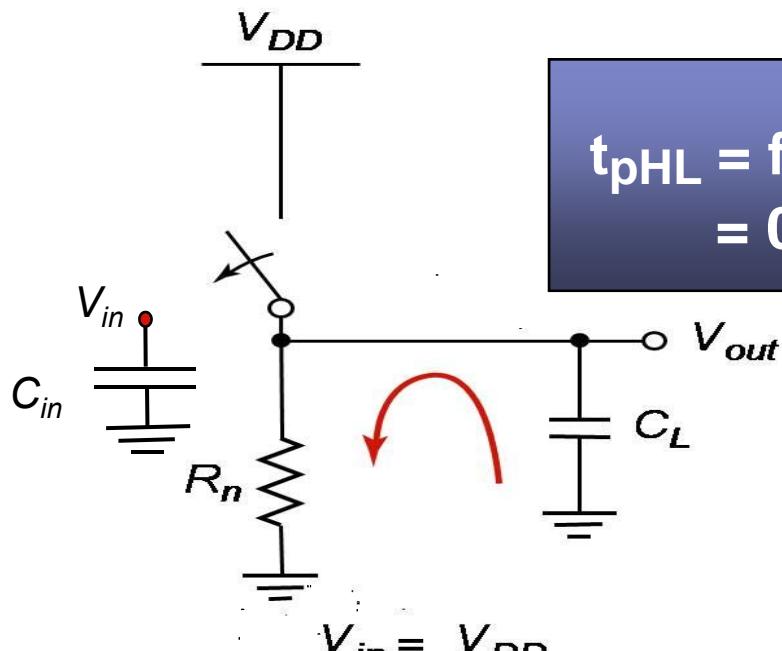
# The Switch Revisited



# The Switch Inverter: Transient Response



(a) Low-to-high

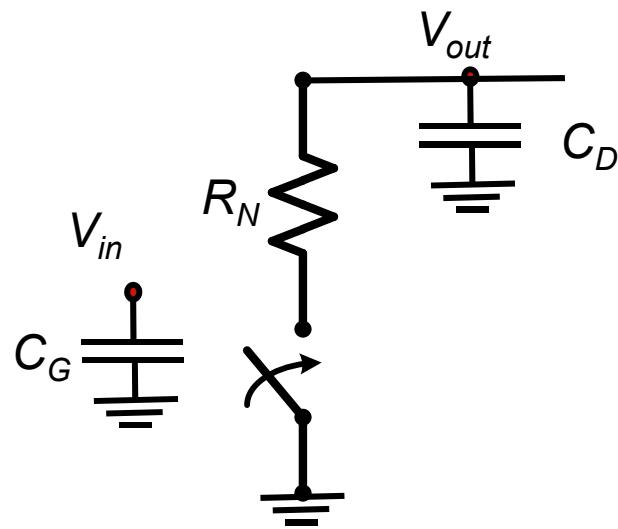


(b) High-to-low

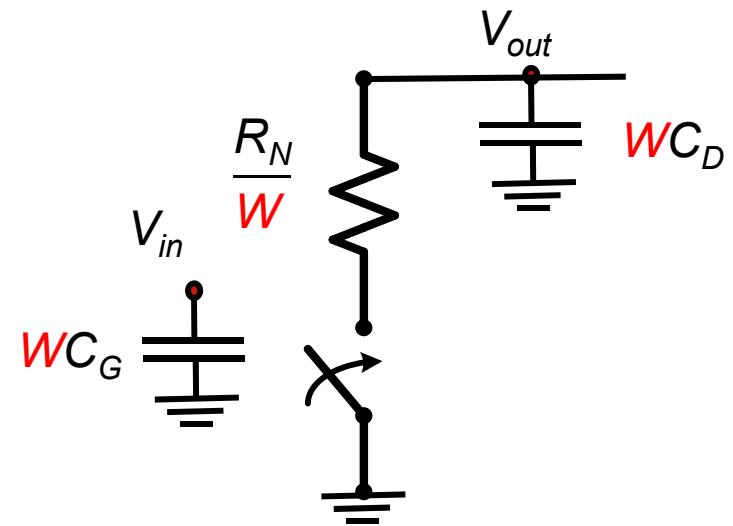
$$t_{pHL} = f(R_{on} C_L) \\ = 0.69 R_n C_L$$

# Switch Parasitic Model

The pull-down switch (NMOS)



Minimum-size switch

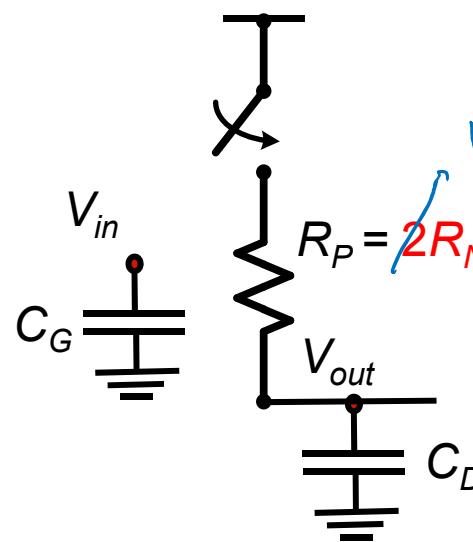


Sizing the transistor (factor  $W$ )

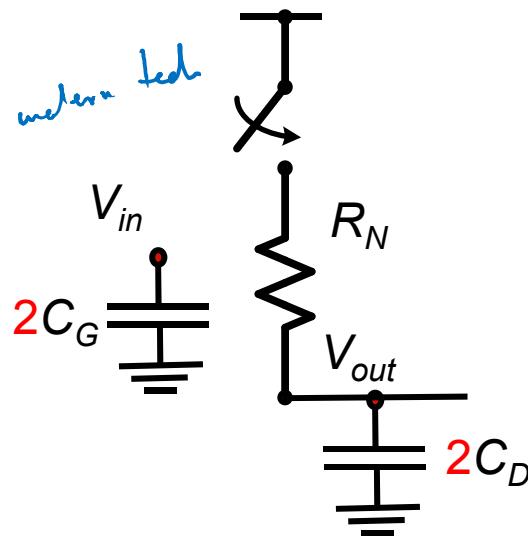
↑  
Changing the width

# Switch Parasitic Model

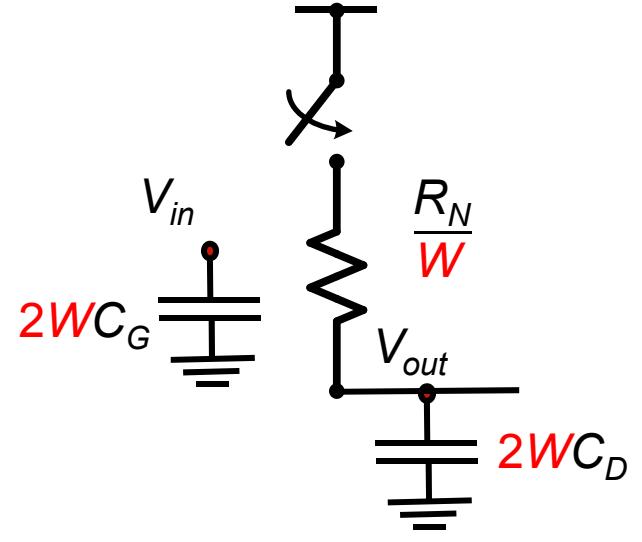
The pull-up switch (PMOS)



Minimum-size switch

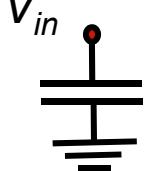


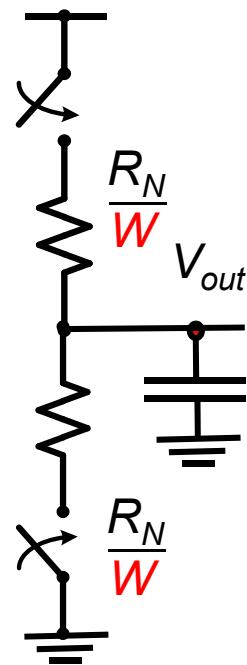
Sized for symmetry



General sizing

# Inverter Parasitic Model

$$C_{in} = 3WC_G$$




Drain and gate capacitance of transistor are **directly** related by process ( $\gamma \approx 1$ )

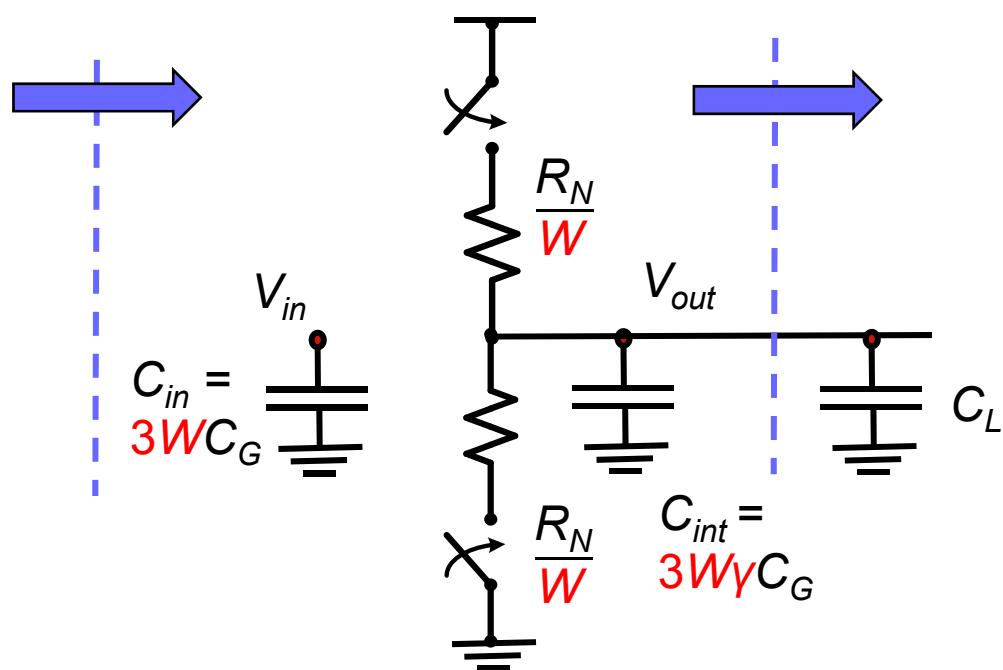
$$C_D = \gamma C_G$$

$$C_{int} = 3WC_D = 3W\gamma C_G$$

$$t_p = 0.69 \left( \frac{R_N}{W} \right) (3W\gamma C_G) = 0.69(3\gamma)R_N C_G$$

Intrinsic delay of inverter  
**independent of size**

# Inverter with Load Capacitance



$$\begin{aligned}
 t_p &= 0.69 \left( \frac{R_N}{W} \right) (C_{int} + C_L) \\
 &= 0.69 \left( \frac{R_N}{W} \right) (3W\gamma C_G + C_L) \\
 &= 0.69 (3C_G R_N) (\gamma + \frac{C_L}{C_{in}}) \\
 &= t_{inv} (\gamma + \frac{C_L}{C_{in}}) = t_0 (\gamma + f)
 \end{aligned}$$

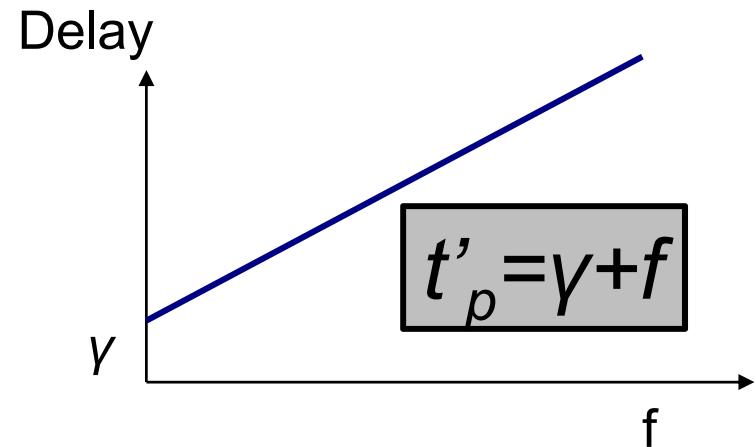
$f = \text{fanout} = \text{ratio between load and input capacitance of gate}$

# Inverter Delay Model

$$t_p = t_{inv}(\gamma + f)$$

$t_{inv}$  technology constant

- Can be dropped from expression
- Delay unit-less variable (expressed in unit delays)



Question: how does transistor sizing ( $W$ ) impact delay?