EE20N: Structure and Interpretation of Systems and Signals **Fall 2013**

Lecture 10: October 1st

Lecturer: Kannan Ramchandran *Scribe: Rick Garcia*

10.1 Announcements

1. No class on Thursday, October 3rd!
2. Undergraduate Social Hour, Tuesday, 3:30-4:30 pm, Cory Courtyard
3. Midterm 1 Regrades: Need to be submitted by end of week.

10.2 Today

1. Impulse Response (needed for lab this week)
2. Convolution
3. Composition of LTI systems

10.3 Impulse Response

Definition 10.1. *The discrete impulse signal (Kronecker delta) can be defined as:*

$$\delta[n] = \begin{cases} 1 & \textit{if } n = 0 \\ 0 & \textit{if } n \neq 0 \end{cases}$$

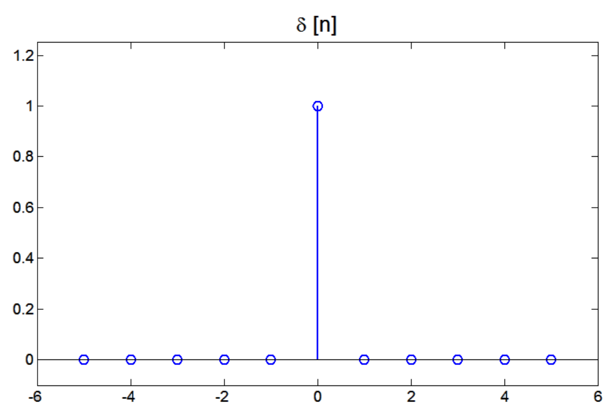


Figure 10.1: The Kronecker Delta, $\delta[n]$

Given an LTI system, the output when the input is an impulse signal is called the impulse response of the system. The impulse response tells everything you need to know about the system.

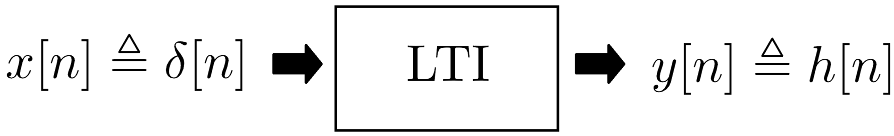


Figure 10.2: LTI systems are defined by their impulse response

Fact 10.1. *Any discrete time (DT) signal $x[n]$ can be represented as a linear combination of delayed and scaled copies of $\delta[n]$.*

Example 10.1. *This system consists of a summation of shifted delta functions:*

$$x[n] = \delta[n] + 2\delta[n-1] + \frac{1}{2}\delta[n-2]$$

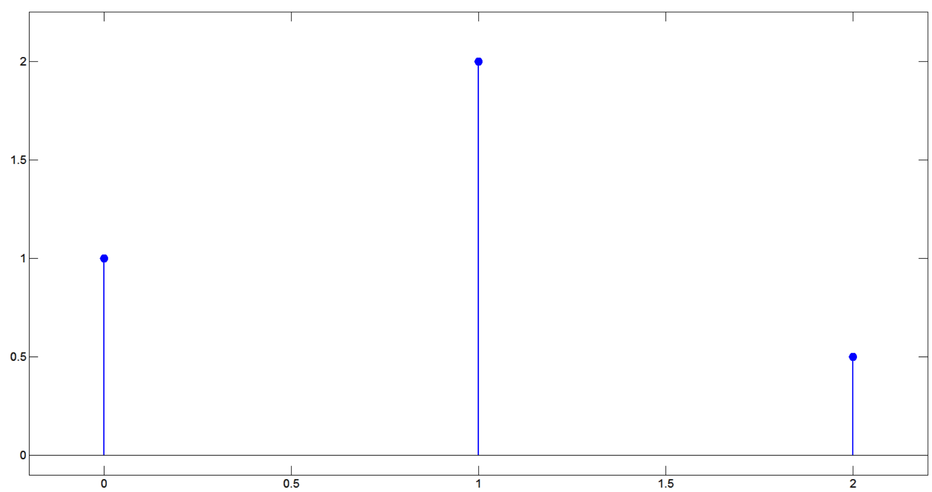


Figure 10.3: Example system

Any signal can be be represented as a sequence of scaled and shifted delta functions. Note that we actually mean any signal as opposed to "any signal" as in previous discussions.

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

Proposition 10.1. *If $x[n]$ is input to an LTI system with impulse response $h[n]$, then the output of that system is given by:*

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

The "sifting property" of the $\delta[n]$ shows that:

$$x[0]\delta[n] \Rightarrow \boxed{\text{LTI}} \Rightarrow x[0]h[n]$$

$$x[1]\delta[n-1] \Rightarrow \boxed{\text{LTI}} \Rightarrow x[1]h[n-1]$$

$$\vdots$$

$$\sum_{k=-\infty}^{\infty} x[k]\delta[n-k] \Rightarrow \boxed{\text{LTI}} \Rightarrow \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Example 10.2. *Consider the classic two-path wireless channel. We can represent the channel as a combination of delayed and scaled delta functions.*

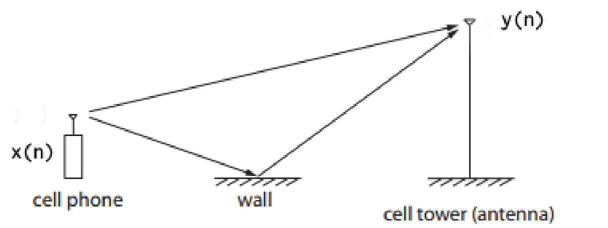


Figure 10.4: Two-path wireless channel

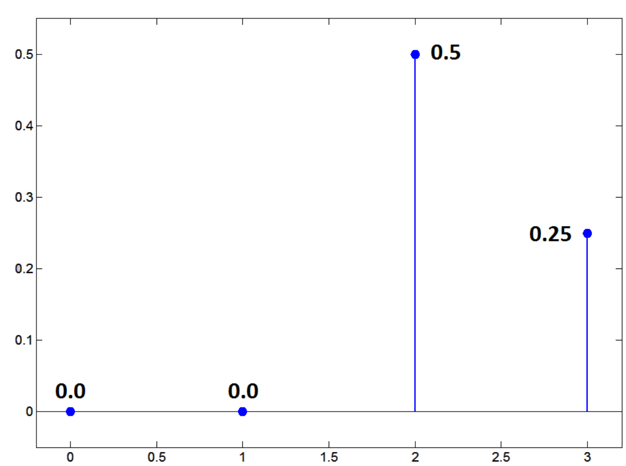


Figure 10.5: Plot of two-path wireless channel

Class example: find and plot the output when the input $x[n]$ is:

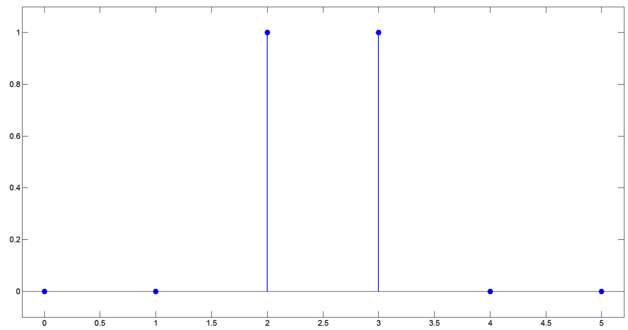


Figure 10.6: Input signal into two-path channel

The summed and shifted impulse responses can be used to construct the output signal, as shown in the following:

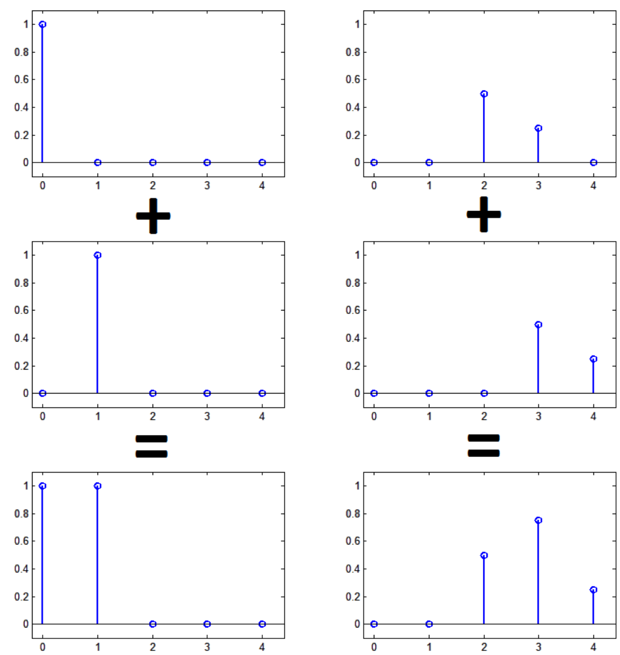


Figure 10.7: Illustration of summation of impulse responses

Example 10.3. *For this example, we look at a three sample $h[n]$ and a signal $x[n]$ of length 4.*

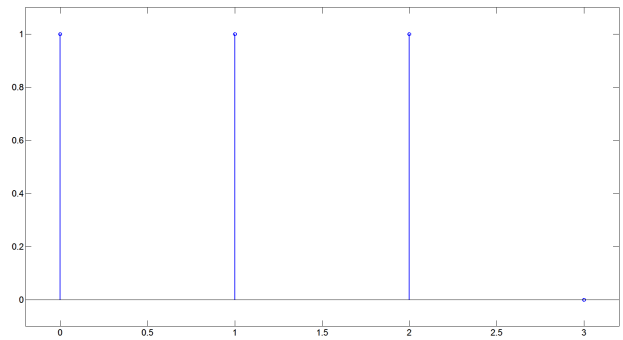


Figure 10.8: Signal $h[n]$

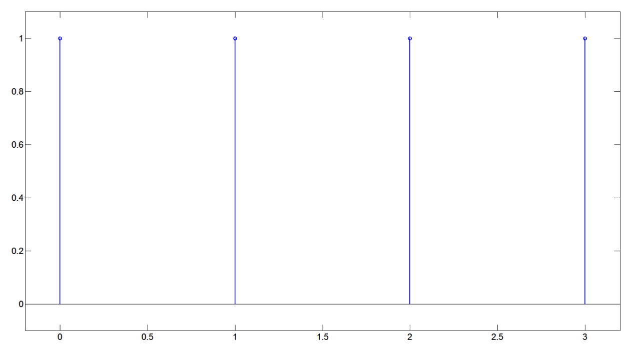
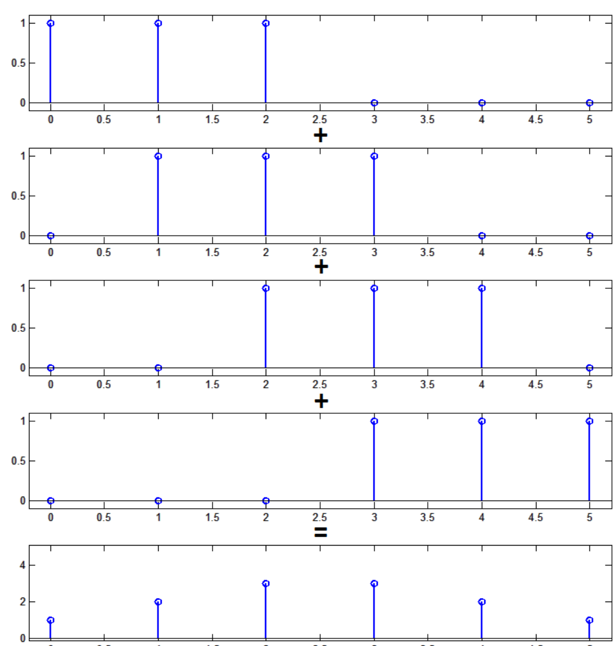


Figure 10.9: Signal $x[n]$

We solve for the output $y[n]$ of $x[n]$ passing through $h[n]$:



Definition 10.2. *The process of computing the output of an LTI system given an input $x[n]$ and the impulse response of the system $h[n]$ is known as convolution. It is defined as:*

$$y[n] \triangleq (x * h)[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

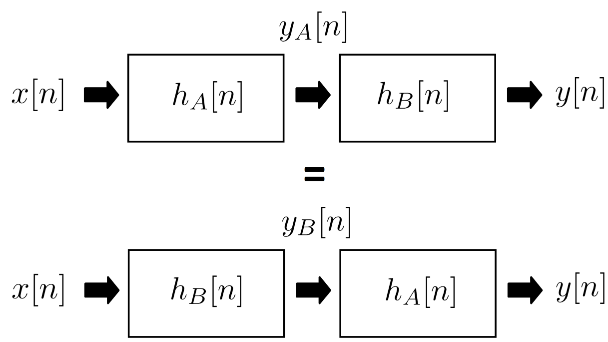
Question: What is $(h * x)[n]$?
Claim:$(x * h)[n] = (h * x)(n)$

$$(h * x)[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

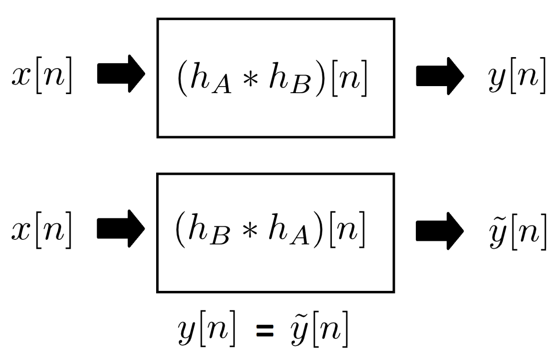
Let $l = n - k$

$$(h * x)(n) = \sum_{l=-\infty}^{\infty} h[n-l]x[l] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = (x * h)[n]$$

Therefore, convolution holds the property of commutivity. This is useful because it allows us to exchange the order of cascaded LTI systems without affecting the final output of the system.



You should convince yourself that the cascade of an LTI systems is also LTI.



Definition 10.3. *Causal system: A system is defined as causal in which for all n, $y[n]$ depends only on the input $x[m]$ for $m \leq n$.*

Theorem 10.1. *For an LTI system S, we have the following:*
If S is causal, then $h[n] = 0$, for all $n < 0$. The reverse is true as well.

Example 10.4. *Identify whether the following systems are causal or non-causal.*

1. $y[n] = \frac{1}{3}(x[n] + x[n-1] + x[n-2])$
   $h[n] = \frac{1}{3}(\delta[n] + \delta[n-1] + \delta[n-2])$
   Causal: The system output is only dependent on current and past values of the input $x[n]$.

2. $y[n] = \frac{1}{3}(x[n+1] + x[n] + x[n-1])$
   $h[n] = \frac{1}{3}(\delta[n+1] + \delta[n] + \delta[n-1])$
   Non-causal: In this system, the current output is dependent on a future value of the input, $\frac{1}{3}x[n+1]$. This makes the system non-causal.

10.4 Connecting Frequency and Impulse Response

Frequency Response View

$$x[n] = e^{iwn}$$

$$y[n] = H(w)e^{iwn}$$

Impulse Response View

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

By convolution property:

$$x[n-k] = e^{iw(n-k)}$$

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]e^{iw(n-k)} = e^{iwn} \sum_{k=-\infty}^{\infty} h[k]e^{-iwk}$$

$$H(w) = \sum_{k=-\infty}^{\infty} h[k]e^{-iwk}$$