EE20N: Structure and Interpretation of Systems and Signals **Fall 2013**

Lecture 02: September 3

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2.1 Announcements

Administrative

- HW Party Office Hours are Tuesday, 3-5 in Wozniak Lounge. (Starting week of 9/9).
- Labs start on 9/9

Homework

- Homework 01: Due next Thursday, 9/12.
- Reading: Lee and Varaiya, Chapter 1, 7. Many Cheerful Facts.pdf, Sections 2.1-2.5, 3.1-3.2.

Getting Feedback

This class is very large, so it is difficult to gauge students' understanding of the topics covered. Make sure to give us feedback through email, office hours, etc., "to help us help you" so that we can adjust the pace and content of lectures, homeworks, discussions, and so on.

2.2 Systems

A system transforms one signal to another (see Fig. 2.1). A system may be designed/engineered by humans (eg. radio) or may already be present in nature (eg. human ear).

Figure 2.1: System

Example 2.1. *Fig. 2.2 is an example of a number of systems involved in communicating a video from one smartphone to the base station. The Image Stabilization , Compression/Decompression and Modulation/Demodulation systems are engineered while the Wireless Channel is given by nature.*

In this course, we shall study various systems that are designed to achieve certain goals such as signal processing, communication and control (eg. Google's self-driving cars).

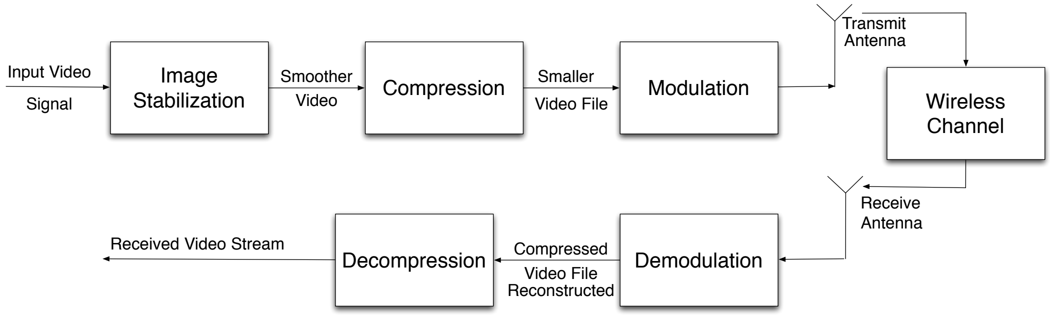

Figure 2.2: Communication System

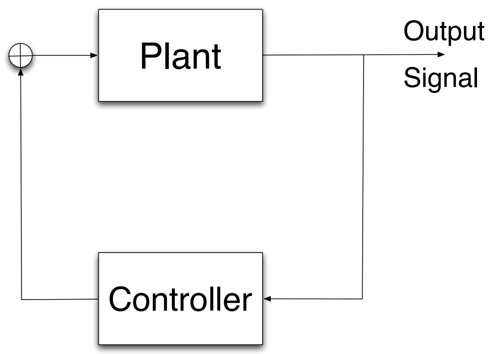

Figure 2.3: Control System

Example 2.2. *Fig. 2.3 shows a Controller system that acquires feedback from the output signal to produce a control signal that drives the Plant. In the self-driving car example, the output signal may be the distance of the car from the nearest car, and the control signal may be steering commands.*

Example 2.3. *A vision is the future "Internet of Things" with billions of wireless sensors that sense the environment, communicate with each other and to the wired network to perform various control tasks.*

The purpose of this course is to study a common set of mathematical tools for understanding signals and designing systems.

2.3 Linear Systems

In this course, our objective lies in analysis and design, so we will look at systems that can are useful models of a wide range of engineering problems. The most important class is the class of linear systems.

A linear system is represented by the block diagram in Figure 2.4 and also satisfies the following properties:

1. Homogeneity
   If $x(t)$, $y(t)$ is a valid input-output pair of a particular system, then for any scalar $c \in \mathbb{R}$, $cx(t)$, $cy(t)$ is also a valid input-output pair of that system (Figure 2.5).

2. Additivity
   If $x_1(t)$, $y_1(t)$ and $x_2(t)$, $y_2(t)$ are valid input-output pairs (Figure 2.6), respectively, of a

Figure 2.4: A linear system with input $x(t)$ and output $y(t)$.

Figure 2.5: Homogeneity of a system, where $c \in \mathbb{R}$.

particular system, then $x_1(t) + x_2(t)$, $y_1(t) + y_2(t)$ is also a valid input-output pair of that system (Figure 2.7).

Figure 2.6: A system with input-output pairs $x_1(t)$, $y_1(t)$, and $x_2(t)$, $y_2(t)$.

Figure 2.7: Additivity of a system.

These two properties form the Superposition Principle of real-valued signals.

Example 2.4. *Speech. Suppose Kannan and Giulia are talking simultaneously; you hear the superposition of the two voices.*

Example 2.5. *Wireless Channel. Shown below.*

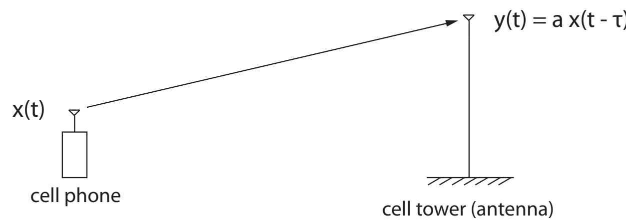

Figure 2.8: A simple wireless channel, viewed as a system.

In Figure 2.8, a modulated signal $x(t)$ is transmitted radially outward from a cell-phone and received at a cell-phone tower as the signal $y(t)$. Note the general form of $y(t)$:

$$y(t) = ax(t - \tau) \tag{2.1}$$

The term a is an attenuation term and is generally small, as signals that travel distances tend to decrease in strength over time. For simplicity, a is a constant (that is, $a \in \mathbb{R}$); in other words, both the cell phone and the cell tower are stationary.

The term $\tau \in \mathbb{R}$ denotes a time-shift of length τ seconds. This time-shift τ is preceded by a negative sign, by convention, because the signal is being *delayed* in time. This delay comes from the time it takes for the signal to traverse the distance between the transmitter and receiver (Figure 2.9). Mathematically,

$$\text{Signal delay} = \frac{\text{Distance traveled}}{\text{Speed of light}} \tag{2.2}$$

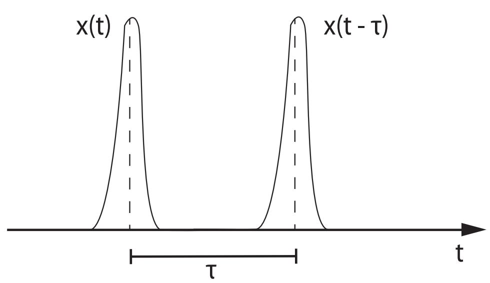

Figure 2.9: A signal $x(t)$ and a delayed version of the signal, $x(t - \tau)$.

We can prove the linearity the wireless system described above by checking that both properties of linearity hold:

- Homogeneity: $\hat{x}(t) = cx(t)$ as input gives scaled output $\hat{y}(t) = a\hat{x}(t-\tau) = c(ax(t-\tau)) = cy(t)$
- Additivity: If $x_1(t), y_1(t) = ax_1(t - \tau)$ and $x_2(t), y_2(t) = ax_2(t - \tau)$ are input-output pairs of the system, then the input signal $\hat{x}(t) = x_1(t) + x_2(t)$ has output signal $\hat{y}(t) = (ax_1(t - \tau)) + (ax_2(t - \tau)) = y_1(t) + y_2(t)$.

Remark 2.5. *A student asked during lecture whether there are systems that are homogeneous but not additive, and vice versa. This is a great question, and a rather subtle one! For a system that* *is homogeneous but not additive, consider the following:*

$$y(t) = \begin{cases} \frac{x^2(t)}{x(t-1)}, & \text{if } x(t-1) \neq 0 \\ 0, & \text{if } x(t-1) = 0 \end{cases}$$

Can you show that this system is homogeneous but not additive?

On the other hand, most systems that are additive are also homogeneous. This is easy to show when the scaling factors c is rational. However, for a system that is additive but not homogeneous, consider the system $y(x(t)) = x^*(t)$, i.e. the output is the complex conjugate of the input. This system is additive. Then consider scaling factor $c = i$, where $i = \sqrt{-1}$. Then $y(ix(t)) = [ix(t)]^* = -ix^*(t)$, while $i \cdot y(x(t)) = ix^*(t)$, so the two are not equal and the system is not homogeneous.

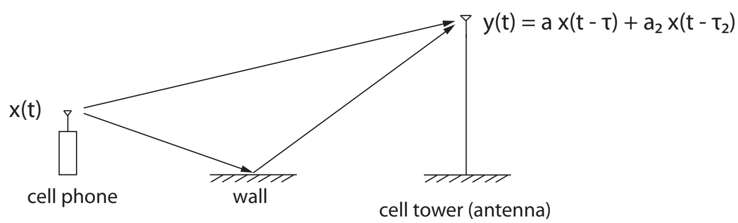

Figure 2.10: A more complex wireless channel.

Example 2.6. *A more complicated wireless channel, shown in Figure 2.10. Check for yourself that this system is also linear.*

Linear systems are useful models also because they are often good approximations of non-linear systems around a nominal operating point. (Think Taylor series expansion, where the linear term is the first-order approximation of a non-linear function.)

2.4 Signals as functions

We now move our discussion back to signals. Last lecture, we mentioned the mathematical definition of a signal as a function of time and/or space. This time, we will elaborate further on this mathematical definition.

Definition 2.1. *A function maps each value in a set (a domain) to a value in another set (a range).*

Note that a function does not assign one value in a domain to multiple values in a range. "That's not a function; that's confusion" (D. Tse, 1/24).

Example 2.7. *Continuous-time or -space and continuous-valued signal (Domain: $\mathbb{R}$, Range: $\mathbb{R}$), shown in Figure XX.*

Example 2.8. *Continuous-time signal with non-negative domain and continuous-valued (Domain: $[0, \infty) = \mathbb{R}^+$, Range: $\mathbb{R}$), shown in Figure XX.*

Example 2.9. *Finite-duration continuous-time signal (Domain: $[0, T]$, Range: $\mathbb{R}$), shown in Figure XX.*

Example 2.10. *Discrete-time or -space continuous-valued signal (Domain: Integers* $= \{\dots, -2, -2, 0, -1, -2, \dots\}$ $\mathbb{Z}$*, Range* $= \mathbb{R}$*), shown in Figure XX.*

Example 2.11. *Discrete-time continuous-valued signal with non-negative domain (Domain: Natural numbers* $= \mathbb{N} = \{0, 1, 2, 3, \dots, \}$*, Range:* $\mathbb{R}$*), shown in Figure XX.*

Note that a system is also a function whose domain and range are signals.

Why do we deal with both continuous-time and discrete-time signals? As engineers, both representations are equally important. For example, speech is a continuous-time signal, but engineers typically perform speech analysis on a computer, which discretizes the speech to store in memory. Later in this course, we will learn how to sample a signal; that is, to transform a signal from continuous-time to discrete-time while preserving signal information.