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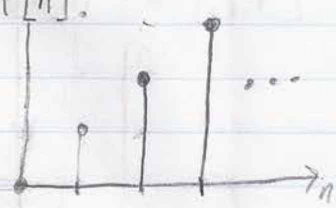
EE20N

## Homework 5

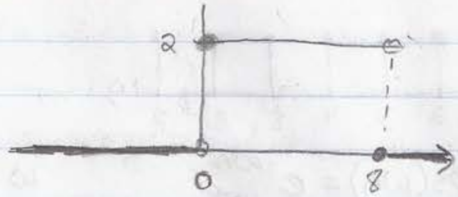
[8.18] 1. $u[n]$ = unit step function $= \begin{cases} 1 & 0 \le n \\ 0 & \text{otherwise} \end{cases}$

$h[n]$:



$h[n] = n\,u[n]$

$p[n] = \begin{cases} 2 & 0 \le n < 8 \\ 0 & \text{otherwise} \end{cases}$




$w = H(p)$

$p[n]$ can be expressed as a linear combination of $u(n)$:

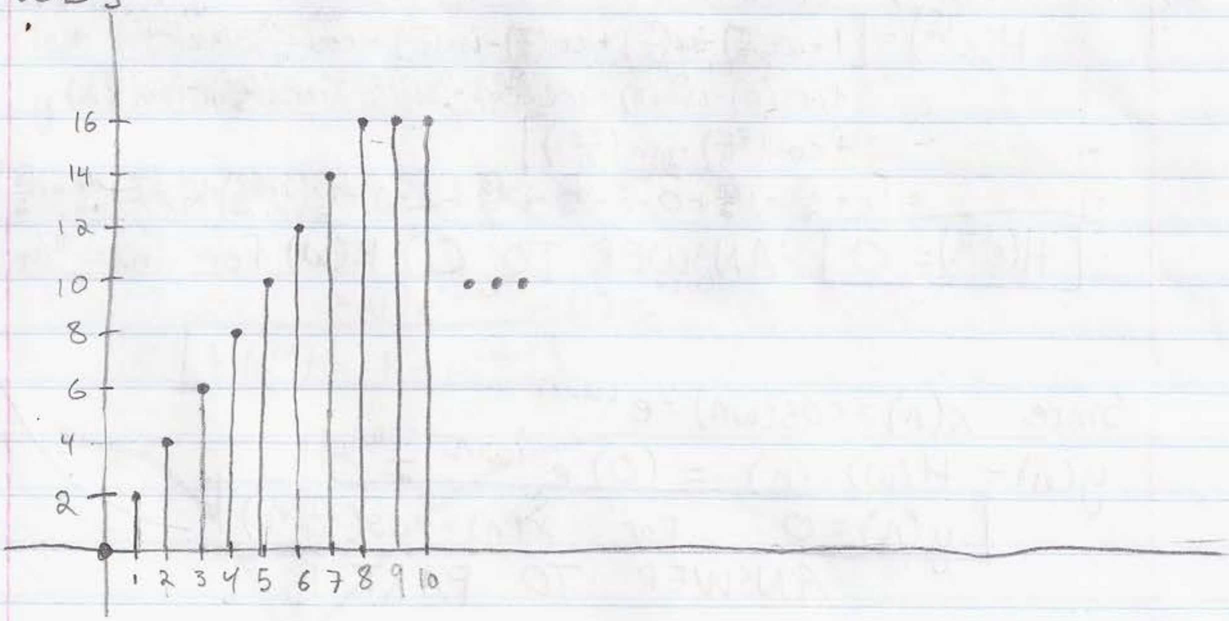
$p[n] = 2(u[n] - u[n-8])$

so $y(n) = H(p) = 2(h[n] - h[n-8]) = 2(n u(n) - (n-8)u(n-8)) = w(n)$

$w(0) = 2(0(1) + 8u(-8)) = 0$ $\quad w(2) = 4$ $\quad w(9) = 2(9-1) = 16$

$w(1) = 2(1 + 7(0)) = 2$ $\quad w(8) = 2(8) = 16$ $\quad w(10) = 2(10-2) = 16$

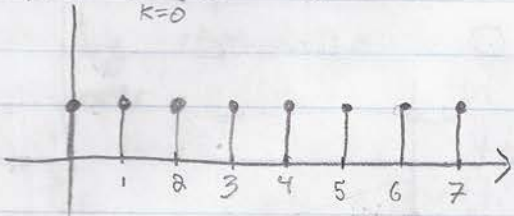
$w[n]$




$$\boxed{w[n] = \begin{cases} 2n & 0 \le n < 8 \\ 16 & n \ge 8 \\ 0 & n < 0 \end{cases}}$$

[9.1] 2. $h[n] = \sum_{k=0}^{7} \delta(n-k)$

a)




zero elsewhere ($n \notin [0,7]$)

b) $x(n) = \cos(\omega n) = e^{i\omega n}$ $\qquad \omega = \frac{\pi}{4}$ radians/sample

$$y(n) = (x * h)(n) = \sum_{-\infty}^{\infty} h(k)\, x(n-k)$$

$$= \sum_{0}^{7} h(k)\, e^{i\omega n} e^{-i\omega k} = e^{i\omega n} \sum_{0}^{7} h(k) e^{-i\omega k}$$

$$= e^{i\omega n} \sum_{0}^{7} e^{-i\omega k} = e^{i\omega n} H(e^{i\omega})$$

c) $$H(e^{i\omega}) = \left(1 + e^{-i\omega} + e^{-i\omega 2} + e^{-i\omega 3} + e^{-i\omega 4} + e^{-i\omega 5} + e^{-i\omega 6} + e^{-i\omega 7}\right)$$

$$H(e^{i\frac{\pi}{4}}) = \Big[1 + \cos(\tfrac{\pi}{4}) - i\sin(\tfrac{\pi}{4}) + \cos(\tfrac{\pi}{2}) - i\sin(\tfrac{\pi}{2}) + \cos(\tfrac{3\pi}{4}) - i\sin(\tfrac{3\pi}{4})$$
$$+ \cos(\pi) - i\sin(\pi) + \cos(\tfrac{5\pi}{4}) - i\sin(\tfrac{5\pi}{4}) + \cos(\tfrac{3\pi}{2}) - i\sin(\tfrac{3\pi}{2})$$
$$+ \cos(\tfrac{7\pi}{4}) - i\sin(\tfrac{7\pi}{4})\Big]$$
$$= \Big[\cancel{1} + \cancel{\tfrac{\sqrt{2}}{2}} - \cancel{i\tfrac{\sqrt{2}}{2}} + 0 - \cancel{i} - \cancel{\tfrac{\sqrt{2}}{2}} - \cancel{i\tfrac{\sqrt{2}}{2}} - \cancel{1} - 0 - \cancel{\tfrac{\sqrt{2}}{2}} + \cancel{i\tfrac{\sqrt{2}}{2}} + 0 + \cancel{i} + \cancel{\tfrac{\sqrt{2}}{2}} + \cancel{i\tfrac{\sqrt{2}}{2}}\Big]$$

$\boxed{H(e^{i\frac{\pi}{4}}) = 0}$ ANSWER TO C) $H(\omega)$ For $\omega = \pi/4$

Since $x(n) = \cos(\omega n) = e^{i\omega n}$

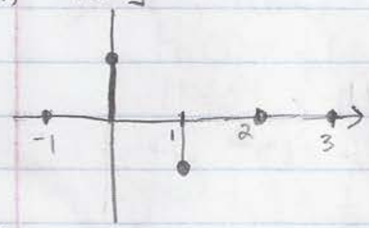
$y(n) = H(\omega)x(n) = (0)\, e^{i\omega n} = 0$

$\boxed{y(n) = 0 \quad \text{For} \quad x(n) = \cos(\tfrac{\pi}{4} n)}$

ANSWER TO PART B

[9.10] 3. $h[n] = \delta[n] - \delta[n-1]$

a) $h[n]$




b) $x[n] = 1$

$$y[n] = (x * h)(n) = \sum_{-\infty}^{\infty} x(k)h(n-k) = \sum_{-\infty}^{\infty} h(k)x(n-k)$$

$$= \sum_{0}^{1} h(k)x(n-k) = h(0)x(n) + h(1)x(n-1) = (1)x(n) - (1)x(n-1)$$

$$= 1 - 1 = 0 \quad \Rightarrow \boxed{y[n] = 0}$$

c) let $x(n) = e^{i\omega n}$

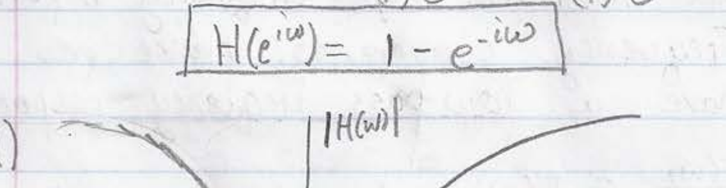
$$y(n) = (h * x)(n) = \sum_{-\infty}^{\infty} h(k)\, e^{i\omega n} e^{-i\omega k}$$

$$\Rightarrow H(e^{i\omega}) = \sum_{0}^{1} h(k)\, e^{-i\omega k}$$

$$= (1)e^{-i\omega 0} - (1)e^{-i\omega(1)}$$

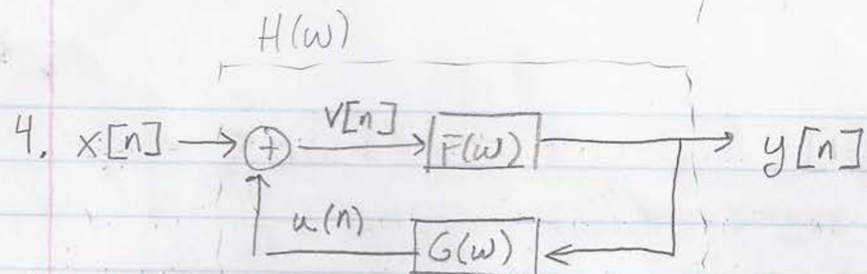
$$\boxed{H(e^{i\omega}) = 1 - e^{-i\omega}}$$

d)




This is a highpass filter – it gains high frequencies and attenuates low frequencies.
we can consider the input $x(n) = 1$ as a periodic signal with $\omega = 0$. We see that the frequency response $H(\omega) = H(\omega = 0) = 0$, so the output is completely attenuated $\Rightarrow y[n] = 0 \;\forall n$.

4.




a) $V[n] = x[n] + u[n] \qquad y[n] = V[n]\, F(w)$

$u[n] = G(w)\, y[n]$

$\Rightarrow V[n] = x[n] + G(w)\, y[n]$

$$\frac{y[n]}{F(w)} = x[n] + G(w)\, y[n]$$

$$y[n]\left(\frac{1}{F(w)} - G(w)\right) = x[n]$$

$$\boxed{H(w) = \frac{y[n]}{x[n]} = \frac{1}{\frac{1}{F(w)} - G(w)} = \frac{F(w)}{1 - F(w)G(w)}}$$

b) When we say we want $y[n]$ to be as smooth (or constant) as possible, we mean to say that we want $y$ to have mostly very low frequency components and almost none or no high frequency components. Thus we want our filter to accept a general input with multiple frequency components (from fourier decomposition etc) but the filter should attenuate high frequency inputs and pass/gain low frequency components. Hence we need H(w) to have a low pass frequency response (Filter)

c) $G(w) = 0$

From A: $H(w) = \frac{F(w)}{1 - F(w)(0)} = F(w)$ when there is no feedback

* since $u[n] = G(w)\, y[n] = 0\, y[n] = 0$:

$v[n] = x[n] + 0 = x[n]$

$\Rightarrow y[n] = .5(v[n] + v[n-1]) = .5(x[n] + x[n-1])$

Let $x[n] = e^{iwn}$: $\Rightarrow y[n] = H(w)e^{iwn} = H(w)x[n]$

$\Rightarrow H(w)\cancel{e^{iwn}} = .5(\cancel{e^{iwn}} + \cancel{e^{iwn}}e^{-iw})$

$\boxed{H(w) = \frac{1}{2}(1 + e^{-iw})} = F(w)$ [even with Feedback]

d) $x[n] = 12 + \cos(.9\pi n)$

$y[n] = H(e^{i\omega})\, x[n] = H(0)\,12 + |H(.9\pi)| \cos(.9\pi n + \angle H(.9\pi))$

$H(0) = \frac{1}{2}(1 + e^{-i0}) = 1$

$|H(0)| = 1 \qquad \angle H(0) = 0$

$H(.9\pi) = \frac{1}{2}(1 + e^{-.9\pi i}) = \frac{1}{2}\left(1 + \cos(.9\pi) - i\sin(.9\pi)\right)$

$|H(.9\pi)| = \frac{1}{2}\left(\sqrt{\cos^2(.9\pi) + 2\cos(.9\pi) + 1 + \sin^2(.9\pi)}\right)$

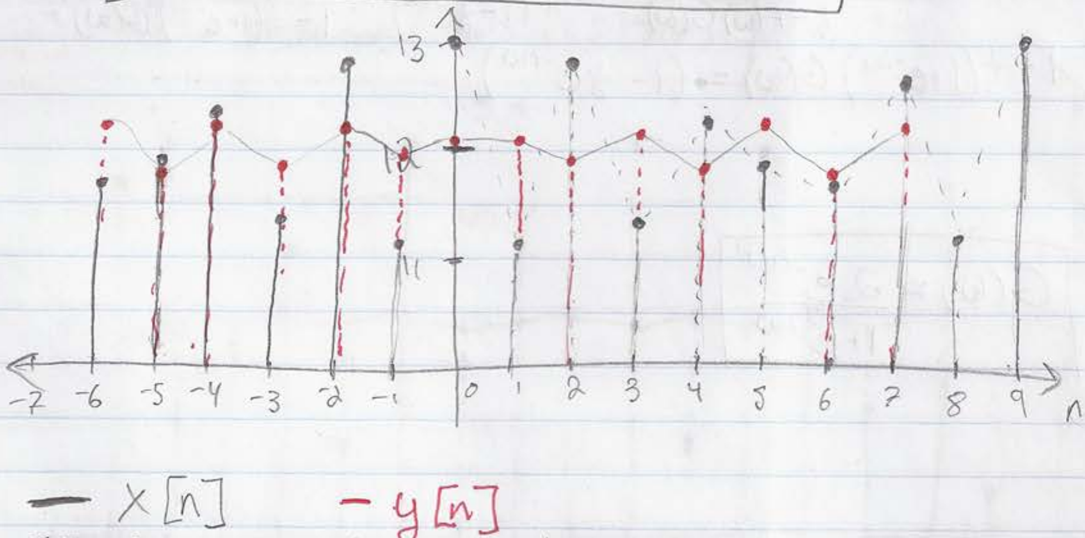
$= \frac{1}{2}\sqrt{2 + 2\cos(.9\pi)} \approx .1564$

$\angle H(.9\pi) = \tan^{-1}(0) + \tan^{-1}\left(\frac{-\sin(.9\pi)}{1 + \cos(.9\pi)}\right)$

$\approx 0 - 1.4137 = -1.4137$

$y[n] = (1)12 + .1564 \cos(.9\pi n - 1.4137)$

$= \boxed{12 + .1564\cos(.9\pi n - 1.4137)}$




y[n] is closer to the desired signal, which is an attenuated, almost constant signal

5. $y_F[n] = \frac{1}{4}(x[n] + x[n-1]) + \frac{1}{2} y_F[n-1]$

First, consider without Feedback to Find $F(\omega)$:

$$y_F[n] = \frac{1}{4}(x[n] + x[n-1])$$

$$H(\omega) \cancel{e^{i\omega n}} = \frac{1}{4}(\cancel{e^{i\omega n}} + \cancel{e^{i\omega n}} e^{-i\omega})$$

$$H(\omega) = F(\omega) = \frac{1}{4}(1 + e^{-i\omega})$$

Now, with feedback:

$$H(\omega) \cancel{e^{i\omega n}} = \frac{1}{4}(\cancel{e^{i\omega n}} + \cancel{e^{i\omega n}} e^{-i\omega}) + \frac{1}{2} H(\omega) \cancel{e^{i\omega n}} e^{-i\omega}$$

$$H(\omega)\left(1 - \frac{1}{2} e^{-i\omega}\right) = \frac{1}{4}(1 + e^{-i\omega})$$

$$\boxed{H(\omega) = \frac{1}{4} \frac{(1 + e^{-i\omega})}{(1 - \frac{1}{2} e^{-i\omega})}}$$

since $H(\omega) = \dfrac{F(\omega)}{1 - F(\omega) G(\omega)} = \dfrac{\cancel{\frac{1}{4}(1 + e^{-i\omega})}}{\cancel{4}(1 - \frac{1}{2} e^{-i\omega})} = \dfrac{\cancel{\frac{1}{4}(1 + e^{-i\omega})}}{1 - \frac{1}{4}(1 + e^{-i\omega}) G(\omega)}$

$$\cancel{1} - \frac{1}{4}(1 + e^{-i\omega}) G(\omega) = (\cancel{1} - \frac{1}{2} e^{-i\omega})$$

$$\boxed{G(\omega) = \frac{2e^{-i\omega}}{1 + e^{-i\omega}}}$$

b) $x[n] = 12 + \cos(.9\pi n)$

$y[n] = H(0)12 + H(.9\pi)\cos(.9\pi n)$

$= |H(0)|12 + |H(.9\pi)|\cos(.9\pi n + \angle H(.9\pi))$

$|H(0)| = \frac{1}{4}\frac{(1+e^0)}{(1-\frac{1}{2}e^0)} = \frac{1\,(2)}{4(\frac{1}{2})} = 1 \qquad \angle H(0) = 0$

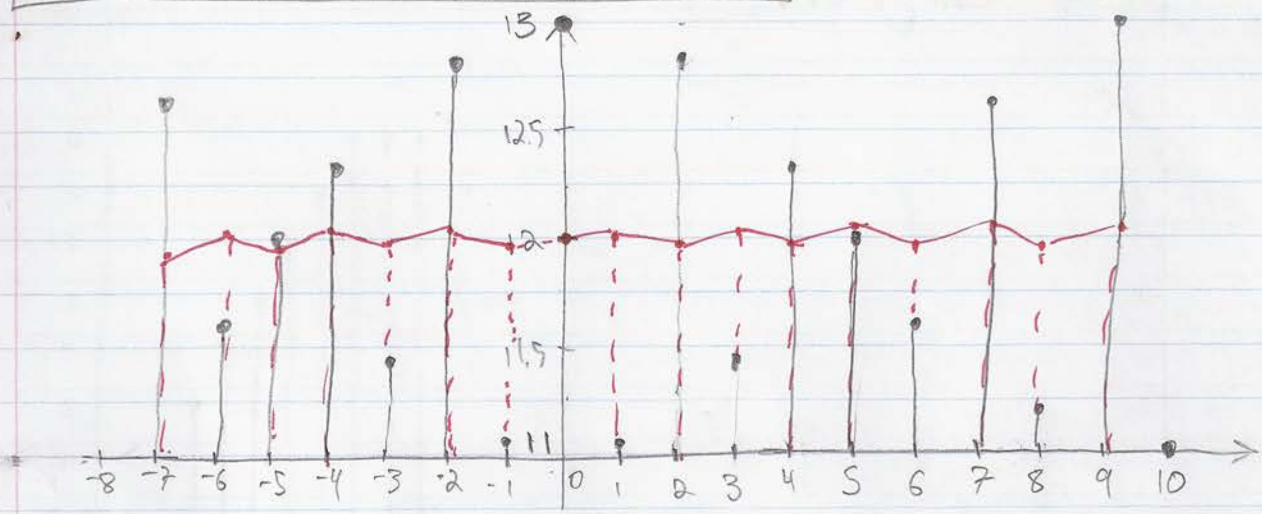
$|H(.9\pi)| = \left|\frac{1}{4}\frac{(1+e^{-.9\pi i})}{(1-\frac{1}{2}e^{-.9\pi i})}\right| = \frac{1}{4}\left(\sqrt{(1+\cos(.9\pi))^2 + \sin^2(.9\pi)}\right) / \left(\sqrt{(1-\frac{1}{2}\cos(.9\pi))^2 + \frac{1}{2}\sin^2(.9\pi)}\right)$

$= \frac{1}{4}\frac{(.31287)}{(1.49)} = .05244$

$\angle H(.9\pi) = \tan^{-1}(0) + \tan^{-1}\left(\frac{-\sin(.9\pi)}{1+\cos(.9\pi)}\right) - \tan^{-1}\left(\frac{+\frac{1}{2}\sin(.9\pi)}{-\frac{1}{2}\cos(.9\pi)+1}\right)$

$= 0 - 1.4137 - .104 = -1.5177$

$\boxed{y_F[n] = 12 + .0524\cos(.9\pi n - 1.5177)}$




We should use the signal $y_F[n]$ for our computer because after $x[n]$ has been filtered with feedback, $y_F[n]$ is nearly constant (more constant than $y[n]$). The max/min of $y_F[n] = 12 \pm .0524$, whereas $y[n] = 12 \pm .1564$. Clearly, feedback makes our low pass filter attenuate better.

c) As in lab 3, feedback improved the performance of our low pass filter. Without feedback, we can think of our system as "averaging" our input function, since it takes an input $x(n)$, sums it with $x(n-1)$, and then scales it by some factor less than 1 (in this case, 1/4). Our feedback essentially allows our system to repeat this averaging process, since the output becomes part of the input as well. Thus by repeating this "averaging" process over and over again with feedback, we get closer and closer to the actual average of the cosine function, which we know is zero. Of cource, our DC term 12 volts has no frequency ($\omega = 0$), so the output is very nearly the constant voltage of 12, with tiny amounts of oscillation. Hence, our feedback filter works as the best low pass filter to convert the AC power supply into DC voltage for the computer.