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EE20N

Homework 9

1.a) $[1,0,0](*_3)[1,2,3] = \sum_{m=0}^{2} x[m]\,y[n-m]_3$

$x = [1,0,0]$, $y = [1,2,3]$, $n = 0, 1, 2$

$= [x[0]y[0] + \cancel{x[1]y[2]} + \cancel{x[2]y[1]},\ x[0]y[1],\ x[0]y[2]]$

$= [1(1), 1(2), 1(3)]$

$= \boxed{[1, 2, 3]}$

b) $[1,1,1](*_3)[1,2,3] = [\underbrace{1(1)+1(3)+1(2)}_{n=0}, \underbrace{1(2)+1(1)+1(3)}_{n=1}, \underbrace{1(3)+1(2)+1(1)}_{n=2}]$

$= \boxed{[6, 6, 6]}$

c) $[1,2](*_1)[1,2] = [1+2](*_1)[1+2] = [3](*_1)[3] = 3\cdot 3 = \boxed{9}$

d) $[1,2,0,0](*_4)[1,2,0,0] = [1(1)+\cancel{2(0)}, 1(2)+2(1), 2(2), 0]$

1	2	0	0
1	0	0	2

1	2	0	0
2	1	0	0

1	2	0	0
0	2	1	0

1	2	0	0
0	0	2	1

$= \boxed{[1, 4, 4, 0]}$

Note: For part c), the N-point circular convolution of signals with length > N was note defined in lecture, and was also dismissed in office hours, so I will interpret this as requring the contraction of the convolting signals by summing the components. (Same as Matlab Result)

Circulant Matrix

2. $y[n] = \sum_{m=0}^{N-1} x[m]\, h[(n-m)_N]$

a) $N=3$

$y[0] = x[0]h[0] + x[1]h[2] + x[2]h[1]$

$y[1] = x[0]h[1] + x[1]h[0] + x[2]h[2]$

$y[2] = x[0]h[2] + x[1]h[1] + x[2]h[0]$

b) $\vec{y} = \begin{bmatrix} y[0] \\ y[1] \\ y[2] \end{bmatrix}$ $\quad \vec{x} = \begin{bmatrix} x[0] \\ x[1] \\ x[2] \end{bmatrix}$ $\quad \boxed{\vec{y} = M\vec{x}}$

$$\Rightarrow \underbrace{\begin{bmatrix} y[0] \\ y[1] \\ y[2] \end{bmatrix}}_{\vec{y}} = \underbrace{\begin{bmatrix} h[0] & h[2] & h[1] \\ h[1] & h[0] & h[2] \\ h[2] & h[1] & h[0] \end{bmatrix}}_{M} \cdot \underbrace{\begin{bmatrix} x[0] \\ x[1] \\ x[2] \end{bmatrix}}_{\vec{x}}$$

c) $[1,2,3,4,5,6,7,8,9] \circledast_9 [11,12,13,14,15,16,17,18,19]$

- since circular convolution is commutative, this is equal to $[11,12,13,14,15,16,17,18,19] \circledast_9 [1,2,3,4,5,6,7,8,9]$
- For the sake of brevity we will call
  $h[n] = [11,12,13,14,15,16,17,18,19]$
  $x[n] = [1,2,3,4,5,6,7,8,9]$

Notice from part b) that the bottom row of the circulant matrix is just the reverse of $h[n]$, and each row above is just a rotation to the left of the row underneath

(continued on next page)

circulant Matrix: length of $h = 9 \Rightarrow M$ is 9×9

$$M = \begin{bmatrix} 11 & 19 & 18 & 17 & 16 & 15 & 14 & 13 & 12 \\ 12 & 11 & 19 & 18 & 17 & 16 & 15 & 14 & 13 \\ 13 & 12 & 11 & 19 & 18 & 17 & 16 & 15 & 14 \\ 14 & 13 & 12 & 11 & 19 & 18 & 17 & 16 & 15 \\ 15 & 14 & 13 & 12 & 11 & 19 & 18 & 17 & 16 \\ 16 & 15 & 14 & 13 & 12 & 11 & 19 & 18 & 17 \\ 17 & 16 & 15 & 14 & 13 & 12 & 11 & 19 & 18 \\ 18 & 17 & 16 & 15 & 14 & 13 & 12 & 11 & 19 \\ 19 & 18 & 17 & 16 & 15 & 14 & 13 & 12 & 11 \end{bmatrix}$$

$8 = N-1 = 9-1$

so the circular convolution is: column vector

$(x \overset{9}{\circledast} h)[n] = y[n]$ let $\vec{y} = \langle y[0], \ldots, y[8] \rangle$ $\vec{x} = \langle x[0], \ldots, y$

such that

$$\vec{y} = M\vec{x} = \begin{bmatrix} 11 & \cdots & 12 \\ \vdots & \ddots & \vdots \\ 19 & \cdots & 11 \end{bmatrix} \begin{bmatrix} 1 \\ \vdots \\ 9 \end{bmatrix}$$

(Did this matrix multiplication with a calculator - matlab can be used too)

$$\boxed{y[n] = [651, 678, 696, 705, 705, 696, 678, 651, 615]}$$

Since the convolotion is commotative we can find $y[n]$ from $h[n]$ with the matrix M':

$$M'\vec{h} = \underbrace{\begin{bmatrix} 1 & 9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 \\ 2 & 1 & 9 & 8 & 7 & 6 & 5 & 4 & 3 \\ 3 & 2 & 1 & 9 & 8 & 7 & 6 & 5 & 4 \\ 4 & 3 & 2 & 1 & 9 & 8 & 7 & 6 & 5 \\ 5 & 4 & 3 & 2 & 1 & 9 & 8 & 7 & 6 \\ 6 & 5 & 4 & 3 & 2 & 1 & 9 & 8 & 7 \\ 7 & 6 & 5 & 4 & 3 & 2 & 1 & 9 & 8 \\ 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 9 \\ 9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 \end{bmatrix}}_{M'} \begin{bmatrix} 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \\ 19 \end{bmatrix}$$

$x \overset{9}{\circledast} h = h \overset{9}{\circledast} x = M^{-1}h =$

$$\boxed{[651, 678, 696, 705, 705, 696, 678, 651, 615]}$$

which confirms our answer and circulant matrices

3. $\vec{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $\vec{h} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

a) $x (*_3) h = [1, 2, 3] (*_3) [1, 2, 0]$

$$\begin{matrix} 1\ 2\ 3 & 1\ 2\ 3 & 1\ 2\ 3 \\ 1\ 0\ 2 & 2\ 1\ 0 & 0\ 1\ 2 \end{matrix}$$

$x (*_3) h = [1(1) + 3(2), (1)(2) + 2(1), 2(1) + 2(3)]$

$= \boxed{[7, 4, 7]}$

b) $\vec{x} * \vec{h} = [1(1), 1(2) + 1(2), 2(2) + 3(1), 2(3)]$

$= \boxed{[1, 4, 7, 6]}$

c) $\boxed{N = 4}$:

$\vec{x} (*_4) \vec{h} = [1, 2, 3, 0] (*_4) [1, 2, 0 0]$

$$\begin{matrix} 1\ 2\ 3\ 0 & 1\ 2\ 3\ 0 & 1\ 2\ 3\ 0 & 1\ 2\ 3\ 0 \\ 1\ 0\ 0\ 2 & 2\ 1\ 0\ 0 & 0\ 2\ 1\ 0 & 0\ 0\ 2\ 1 \end{matrix}$$

$\vec{x} (*_4) \vec{h} = [1(1), 1(2) + (2)(1), 2(2) + 3(1), 2(3)]$

$= [1, 4, 7, 6] = \vec{x} * \vec{h}$

when we take the 4-point circular convolution of $\vec{x}$ and $\vec{h}$ we get the same result as the linear convolution of $\vec{x}$ and $\vec{h}$

d) The linear convolution returns a matrix that is the length of the two vectors added together minus 1 since the last element of the resultant vector corresponds to when the input vectors only overlap by 1 element each Since we must zero pad our our input vectors to be the length of the resulting linear convolution vector, we need

$$\boxed{P = A + B - 1 \text{ such that } \vec{x} (*_P) \vec{h} = \vec{x} * \vec{h}}$$

4. $x \overset{N}{\circledast} h = IDFT(DFT(x) \cdot DFT(h))$

a) $\vec{F}$ = DFT matrix $\Rightarrow$ $DFT(x) = \vec{F}\vec{x}$

$x \circledast h = IDFT((\vec{F}\vec{x})\cdot(\vec{F}\vec{h}))$

$= \frac{1}{N} F^{*T}\left[(\vec{F}\vec{x})\cdot(\vec{F}\vec{h})\right] = F^{-1}\left[(F_x) \cdot (F x)\right]$

Notice that $F\vec{x}$ and $F\vec{h}$ are column vectors, so we need to use point-wise multiplication to create a column vector

b) $\overset{x}{[1,2,3]} \overset{3}{\circledast} \overset{h}{[4,5,6]}$

For the 3-point $(N=3)$ DFT, we have the matrix:

$$F = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & e^{-i\frac{2\pi}{3}} & e^{-i\frac{4\pi}{3}} \\ 1 & e^{-\frac{4\pi}{3}} & e^{-i\frac{2\pi}{3}} \end{bmatrix}$$

$$\omega^4 = e^{-i\frac{2\pi}{3}\cdot 4} = e^{-i\frac{8\pi}{3}} = e^{-i\frac{2\pi}{3}}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & -\frac{1}{2}+i\frac{\sqrt{3}}{2} & -\frac{1}{2}-i\frac{\sqrt{3}}{2} \\ 1 & -\frac{1}{2}-i\frac{\sqrt{3}}{2} & -\frac{1}{2}+i\frac{\sqrt{3}}{2} \end{bmatrix}$$

$$F\vec{x} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 6 \\ 1+2\omega+3\omega^2 \\ 1+2\omega^2+3\omega^4 \end{bmatrix}$$

$$F\vec{h} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{bmatrix}\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 15 \\ 4+5\omega+6\omega^2 \\ 4+5\omega^2+6\omega^4 \end{bmatrix}$$

$$x \overset{3}{\circledast} h = \frac{1}{N}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^{-1} & \omega^{-2} \\ 1 & \omega^{-2} & \omega^{-4} \end{bmatrix}\begin{bmatrix} 90 \\ (1+2\omega+3\omega^2)(4+5\omega+6\omega^2) \\ (1+2\omega^2+3\omega^4)(4+5\omega^2+6\omega^4) \end{bmatrix} = F^{-1}((F_x).*(Fh))$$

Doing this multiplication in Matlab:

$$\boxed{[1,2,3] \overset{3}{\circledast} [4,5,6] = [31, 31, 28]}$$

5. $(x \overset{N}{\circledast} h)[n] = \begin{bmatrix} \\ N \times N \\ \text{circulant} \end{bmatrix} \begin{bmatrix} x[0] \\ \vdots \\ x[N-1] \end{bmatrix}$ ← length N vector

a)

To do this multiplication, you need to do pointwise multiplication of $\vec{x} = (x[0], \dots, x[n-1])$ with each row of the circulant matrix. That means for each row, we need N operations, so with N rows

$$\boxed{N \times N = N^2 \text{ multiplication operations}}$$

b) N-point Cooley Tukey FFT (and IFFT) algorithm:

$$\frac{N}{2}\log_2 N \text{ multiplications}$$

N-point circular convolution

$$x \overset{N}{\circledast} h = IDFT(DFT(x) \cdot DFT(h))$$

is computed with one FFT for each IDFT and DFT, so in total, there are 3 FFT's which require

$$\boxed{\left(\frac{3N}{2}\log_2 N\right) + N \text{ mutiplications}}$$

where the extra N is for the point wise multiplication $F_x \cdot F_h$

c) 1 multiplication in 10^{-9} s $= 10^9$ multiplications / s

i) $N = 2$

circulant time: $(2^2)(10^{-9}) = \boxed{4 \times 10^{-9} \text{ s}}$

FFT time: $\left[\frac{3(2)}{2}\log_2 2 + 2\right] 10^{-9} = \boxed{5 \times 10^{-9} \text{ s}}$

ii) $N = 4$

circulant time: $(4^2)(10^{-9}) = \boxed{1.6 \times 10^{-8} \text{ s}}$

FFT time: $\left(\frac{3(4)}{2}\log_2 4 + 4\right) 10^{-9} = \boxed{1.6 \times 10^{-8} \text{ s}}$

iii) $N = 13250$

circulant time: $(13250)^2(10^{-9}) = \boxed{.176\ s}$

FFT time: $\left(\frac{3(13250)}{2}\log_2(13250) + 13250\right)10^{-9} = 2.8\times10^{-4}\ s$

iv) $N = 1000000$

circulant time: $(1\times10^6)^2(10^{-9}) = \boxed{1000\ s \approx 16.7\ min}$

FFT time: $\left(\frac{3(1\times10^6)}{2}\log_2(1\times10^6) + 1\times10^6\right)10^{-9} = \boxed{.0315}$

d) Clearly, the Cooley Tukey FFT algorithm for computing the circular convolution is better and more efficient than the circulant matrix method. In fact, the larger the signal length, the more efficient the FFT is compared to the circulant matrix, despite FFT being slower than the circulant for $N < 4$. (very small signals)

6. $x[n] = 256$ point signal
$h[n] = 32$ point filter response

a) $N = 256 + 32 - 1 = 287$

$287 - 256 = \boxed{31}$ zeros to pad $\boxed{x[n]}$

$287 - 32 = \boxed{255}$ zeros to pad $\boxed{h[n]}$

b) If we use a 256-point DFT, instead of the 287-point DFT for the zero padded signals (which would make cconv = linearconv), we can figure out how many samples are corrupted by thinking about the wrap around:

$$287 - 256 = 31$$

so the $\boxed{\text{first 31 samples}}$ are corrupted while the rest of the signal is good.

- In general, if we only zero pad the smaller signal to the length of the larger signal, the circular convolution will have

<u>length(smaller signal) - 1</u>

corrupted signals compared to the linear convolution. Additionally, the circularly convolted signal will be shorter than the full linear convolution by the same amount.

8: Midterm Corrections

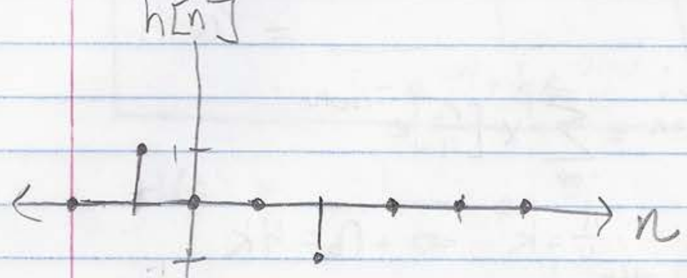
Q1b) Unit step response:

$$y_s[n] = u[n+1] - u[n-2]$$

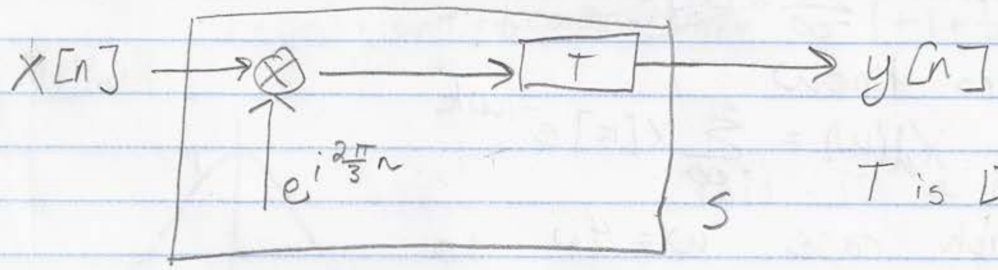
$$\delta[n] = u[n] - u[n-1]$$

by properties of LTI:

$$h[n] = S(\delta[n]) = S(u[n]) - S(u[n-1])$$
$$= y_s[n] - y_s[n-1] = u[n+1] - u[n-2] - u[n] + u[n-3]$$



Q2b



T is LTI system

$\boxed{\text{TRUE: } S \text{ is a linear System}}$

homogeneity:

$$x[n] \longrightarrow S \longrightarrow y[n] = T(x[n]e^{i\frac{2\pi}{3}n})$$
$$\hat{x}[n] = cx[n] \rightarrow S(\hat{x}) = S(cx[n]) = y[n] = T(\hat{x}e^{i\frac{2\pi}{3}n}) = T(cx[n]e^{i\frac{2\pi}{3}n})$$
$$= cT(x[n]e^{i\frac{2\pi}{3}n}) = cy[n]$$

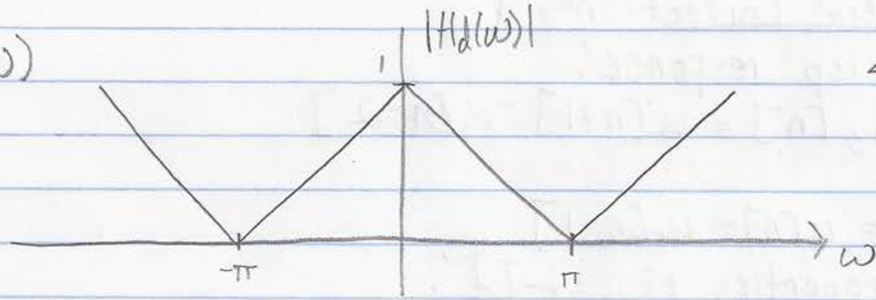
the system is <u>homogeneous</u>

Additivity:

$$x_1[n] \rightarrow S \longrightarrow y_1[n] = T(x_1[n]e^{i\frac{2\pi}{3}n})$$
$$x_2[n] \rightarrow S \longrightarrow y_2[n] = T(x_2[n]e^{i\frac{2\pi}{3}n}) \quad \text{by LTI of } T$$
$$x_1 + x_2 \rightarrow S \rightarrow y = T(x_1e^{i\frac{2\pi}{3}n} + x_2e^{i\frac{2\pi}{3}n}) = T(x_1e^{i\frac{2\pi}{3}n}) + T(x_2e^{i\frac{2\pi}{3}n})$$
$$= y_1 + y_2$$

the system is <u>additive</u>, so it is linear overall

Q3) $X_d(\omega)$



$\angle X_d(\omega) = 0 \ \forall \omega$

$$y[n] = \begin{cases} x\left[\frac{n}{4}\right] & \text{if } n \bmod 4 = 0 \\ 0 & \text{otherwise} \end{cases}$$

$Y_d(\omega) = H_d(\omega) X_d(\omega)$

by the analysis equation:

$$Y_d(\omega) = \sum_{-\infty}^{\infty} y[n] e^{-i\omega n} = \sum_{-\infty}^{\infty} x\left[\frac{n}{4}\right] e^{-i\omega n}$$

make the substitution $\frac{n}{4} = k \implies n = 4k$

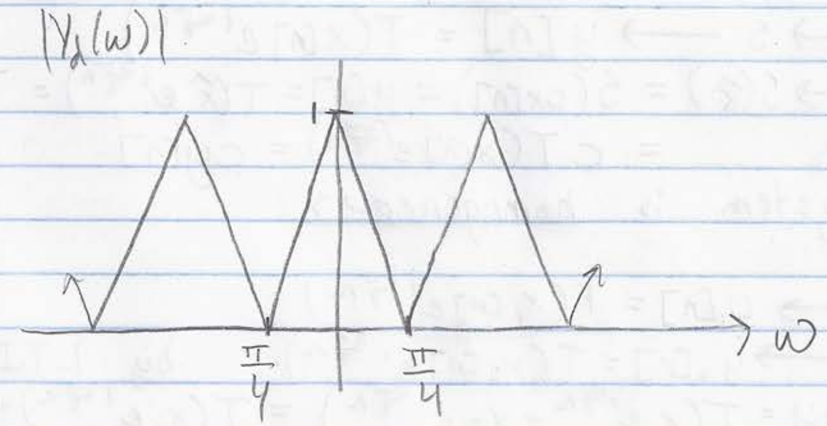
$$Y_d(\omega) = \sum_{-\infty}^{\infty} x[k] e^{-i\omega 4k}$$

we also know

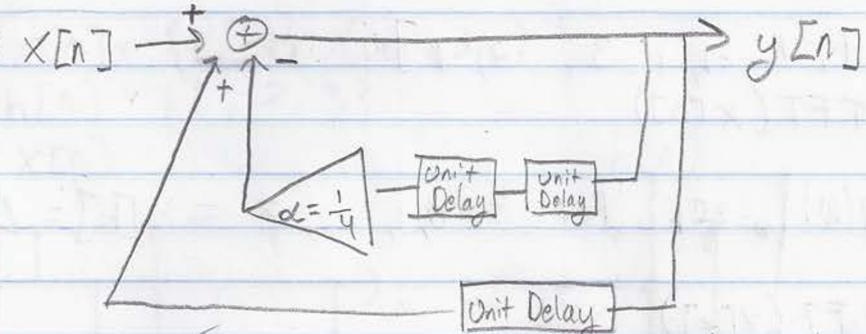
$$X_d(\omega) = \sum_{-\infty}^{\infty} x[k] e^{-i\omega k}$$

in which case $\omega = 4\omega$ so

$$\boxed{Y_d(\omega) = X_d(4\omega)}$$



Q4) a)

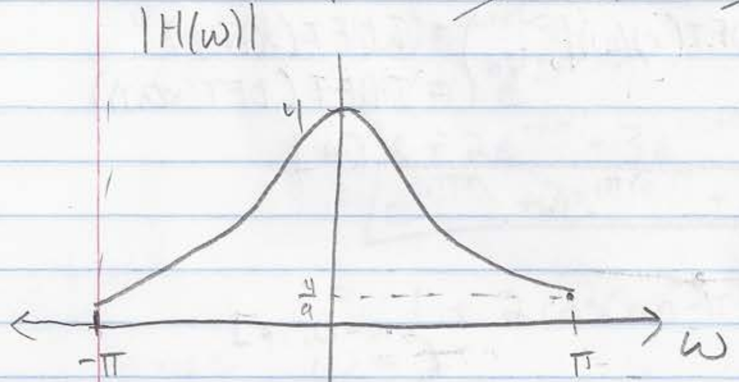


b) $H(\omega)\cancel{e^{i\omega n}} - H(\omega)\cancel{e^{i\omega n}}e^{-i\omega} + \frac{1}{4}H(\omega)\cancel{e^{i\omega n}}e^{-2i\omega} = \cancel{e^{i\omega n}}$

$$\boxed{H(\omega) = \frac{1}{1 - e^{-i\omega} + \frac{1}{4}e^{-2i\omega}}}$$

$$H(0) = \frac{1}{1 - 1 + \frac{1}{4}} = 4$$

$$H(\pi) = \frac{1}{1 - \cos\pi + \cancel{i\sin\pi} + \frac{1}{4}\cos 2\pi - \cancel{i\sin 2\pi}} = \frac{1}{1 + 1 + \frac{1}{4}} = \frac{4}{9}$$



This is a $\boxed{\text{Low Pass Filter}}$

6. $x[n] = [2, 1, 1, -1, 1, 3, -2, 1]$

$X_d(\omega) = DTFT(x[n])$

$Y[K] = X_d(\omega)\big|_{\omega = \frac{2\pi}{8}k}$ For $k = 0, 1, \ldots, 7$ $= X[k] = DFT(x[n])$

$y[n] = IDFT(Y[k])$

a) We know $Y[k]$ is the DFT of $y[n]$. We also know by definition that $Y[k]$ is just a sampled version of the $DTFT(y[n]) = Y_d(\omega)$. From this we can conclude that $X_d(\omega) = Y_d(\omega)$ since $Y[K]$ is just a sampled version of $X_d(\omega)$

$X_d(\omega) = DTFT(x[n])$

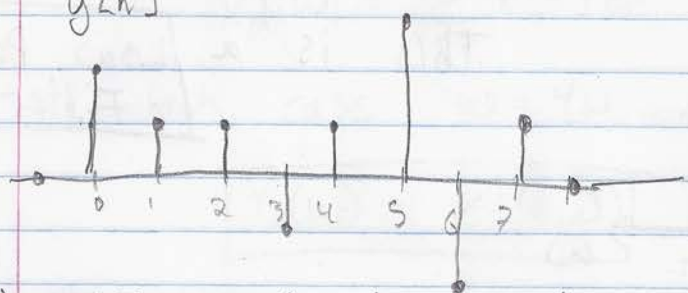
$Y_d(\omega) = DTFT(y[n])$

$X_d(\omega) = Y(\omega) = DTFT(x[n]) = DTFT(y[n])$

$\Rightarrow \boxed{y[n] = x[n]}$

or: $y[n] = IDFT(Y[k]) = IDFT\left(X_d(\omega)\big|_{\omega=\frac{2\pi}{8}k}\right) = IDFT(X[k])$
$= IDFT(DFT(x[n]))$
$= x[n]$

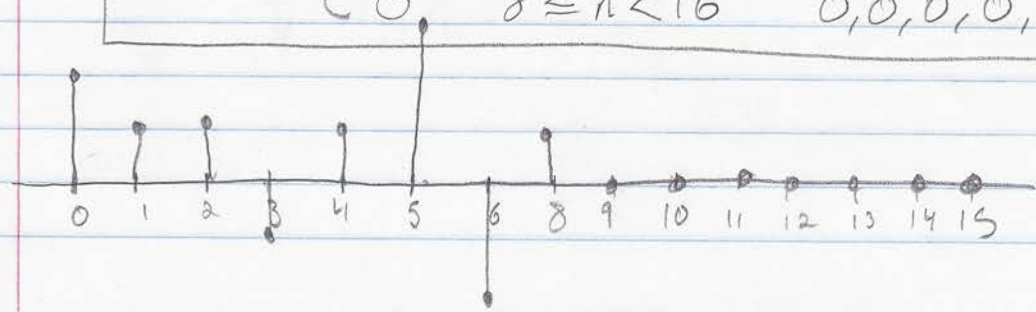
y[n]



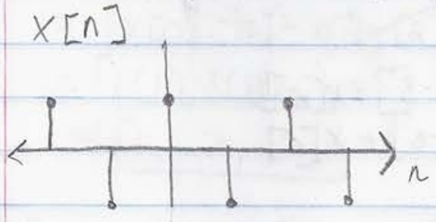
b) $Z[k] = DFT(z[n]) = X_d(\omega)\big|_{\omega = \frac{2\pi}{16}k} = X[k] = DFT(x[n])$ ($k = [1, 16]$)

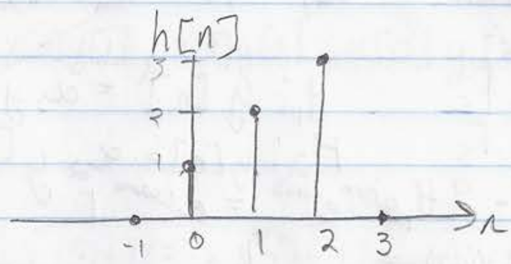
since $x[n]$ is 8 samples long, but we want to sample $X_d(\omega)$ 16 times, we need to $z[n]$ to be a zero padded $x[n]$ to increase our DFT resolution. We should add $16 - 8 = 8$ zeros to $x[n]$.

$$\boxed{z[n] = \begin{cases} x[n] & 0 \le n < 8 \\ 0 & 8 \le n < 16 \end{cases} = [2,1,1,-1,1,3,-2,1, 0,0,0,0,0,0,0,0]}$$



Q7) $x[n] = \{1, -1\}$ (periodic)
$h[n] = \{1, 2, 3\}$





$$x[n] = (-1)^n = (e^{i\pi})^n = e^{i\pi n}$$

so $\omega = \pi$

using the analysis equation:

$$H(\omega) = \sum_{-\infty}^{\infty} h[k]e^{-i\omega k} = \sum_{0}^{2} h[k]e^{-i\omega k}$$
$$= h[0]e^{-i\omega(0)} + h[1]e^{-i\omega(1)} + h[2]e^{-i\omega(2)}$$
$$= 1 + 2e^{-i\omega} + 3e^{-i\omega 2}$$

by LTI:

$$y[n] = H(\omega)e^{i\omega n}$$
$$= H(\pi)e^{i\pi n}$$
$$= (1 + 2e^{-i\pi} + 3e^{-i2\pi})e^{i\pi n}$$
$$= \boxed{e^{i\pi n} + 2e^{i\pi(n-1)} + 3e^{i\pi(n-2)}}$$

$$= -1 + 2\cos(\pi n - \pi) + 3\cos(\pi n - 2\pi)$$

since $\omega_0 = \pi$

$y[n]$ is periodic with $\boxed{P = \frac{2\pi}{\pi} = \frac{2\pi}{\omega_0} = 2}$

which is confirmed by the fact that $\cos(\pi n + \phi)$ is periodic in 2 samples. (ie $x[n]$)

8.b) $x(t) = \cos(8000\pi t) + e^{-20000i\pi t} + \sin(16000\pi t) + 16$

$f_s = 24\text{ kHz} \quad \alpha_1 = .9 \quad \alpha_2 = .9$

$f_d = \frac{f_c}{f_s} \qquad H_1: y[n] = \alpha_1 y[n-3] + x[n]$

$\qquad\qquad H_2: y[n] = \alpha_2 y[n-6] + x[n]$

$H_1: H_1 \cancel{e^{i\omega n}} - .9 H_1 \cancel{e^{i\omega n}} e^{-i\omega 3} = \cancel{e^{i\omega n}} 1$

$$H_1(\omega) = \frac{1}{1 - .9e^{-3i\omega}}$$

$H_2: H_2 \cancel{e^{i\omega n}} - .9 H_2 \cancel{e^{i\omega n}} e^{-i\omega 6} = \cancel{e^{i\omega n}} 1$

$$H_2(\omega) = \frac{1}{1 - .9e^{-i\omega 6}}$$

$$H_3 = \frac{1}{(1-.9e^{-i\omega 6})(1-.9e^{-i3\omega})} = \frac{1}{1 - .9e^{-3i\omega} - .9e^{-i\omega 6} + .81e^{-9i\omega})}$$

CT to complex exponential:

$x(t) = \frac{1}{2}e^{i(8000\pi t)} + \frac{1}{2}e^{-i(8000\pi t)} + e^{-20000i\pi t}$
$\quad + \frac{1}{2i}e^{i16000\pi t} - \frac{1}{2i}e^{-i16000\pi t} + 16e^0$

$x[n] = \frac{1}{2}e^{i2\pi\frac{4000}{24000}n} + \frac{1}{2}e^{-i2\pi\frac{4000}{24000}n} + e^{-i2\pi\frac{10000}{24000}n}$
$\quad + \frac{1}{2i}e^{i2\pi\frac{8000}{24000}n} - \frac{1}{2i}e^{-i2\pi\frac{8000}{24000}n} + 16$

since $x[n]$ is a complex exponential,

$y[n] = H_3 x[n] \qquad H_3(-\frac{\pi}{3}) = 5.263$

$$H_3(\tfrac{\pi}{3}) = \frac{1}{(1-.9e^{-i(\frac{\pi}{3})6})(1-.9e^{+i(\frac{\pi}{3})3})} = 5.263$$

$$H_3(\tfrac{-5}{6}\pi) = \frac{1}{(1-.9e^{+5\pi i})(1-.9e^{\frac{5\pi}{2}i})} = .29 + .26i \qquad |H_3(-\tfrac{5\pi}{6})| = .3912$$

$\qquad \angle H_3(-\frac{5\pi}{6}) = .7328$

$$H_3(\tfrac{2\pi}{3}) = \frac{1}{(1-.9e^{-4\pi i})(1-.9e^{-2\pi i})} = 100 \qquad H_3(-\tfrac{2\pi}{3}) = 100$$

$H_3(0) = 100$

$y[n] = (5.23)\frac{1}{2}(e^{i8000\pi t} + e^{-i8000\pi t}) + .391e^{-20000i\pi t + .7328}$
$\quad + 100(\frac{1}{2i})(e^{i16000\pi t} - e^{-i16000\pi t}) + 16(100)$

$$= \boxed{5.23\cos(8000\pi t) + .39e^{-i20000\pi t + .7328} + 100\sin(16000\pi t) + 1600}$$