A Jacobian-free Newton-Krylov method applied to multi-phase flows

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Two common approaches are used when solving time dependent CFD problems, explicit and implicit schemes. Explicit schemes are known to be restricted to certain CFL conditions leading to the choice of small time steps. In contrast, implicit schemes are not as restricted to the CFL conditions and permit larger time steps to be chosen. Recent industrial thermal-hydraulic codes utilize the two-fluid model with a separation of the liquid and vapour phase, resulting in a system of four or six nonlinear equations (mass, momentum and energy for each phase) to describe the two-phase flow. The system of equations may numerically be solved in an implicit manner using Newtons method, but by doing so, the Jacobian matrix is required in order to update the solution at each time step. Due to the fact that the equation needed to be solved is a linear equation in the form of Ax = b, it can be solved quite effectively using powerful linear solvers. Therefore, calculating the Jacobian, numerically or analytically, is the most expensive task performed at each timestep. The requirement of the Jacobian also prevents the use of high accurate flux schemes. The current study is focused on the development of a Jacobian-free method to solve the system of equations seen in thermal-hydraulic codes. This is done by means of the Jacobian-Free Newton-Krylov (JFNK) method. In this approach, Newtons method is coupled with an iterative Krylov solver, generalized minimum residual (GMRES), which does not need explicit information of the Jacobian matrix to solve the linear equation. Instead, it uses an approximation of the Jacobian using the original system of equations. This technique saves computational time and allows the use of more accurate discretization schemes. Results are presented for a well-known numerical benchmark, the water faucet problem.