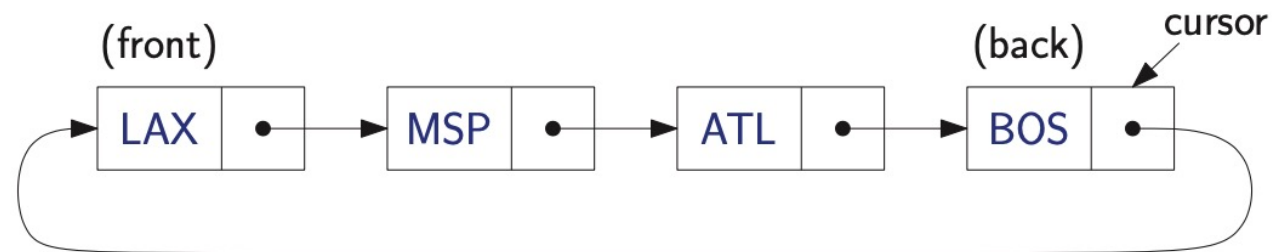


CIRCULARLY LINKED LIST: IMPLEMENTATION

- The method “add” puts *e* in a node and adds it to the beginning of the list
- i.e., if "N" denotes the node that "cursor" points to, then the method “add” creates a new node whose element equals "e", and inserts it right after N.

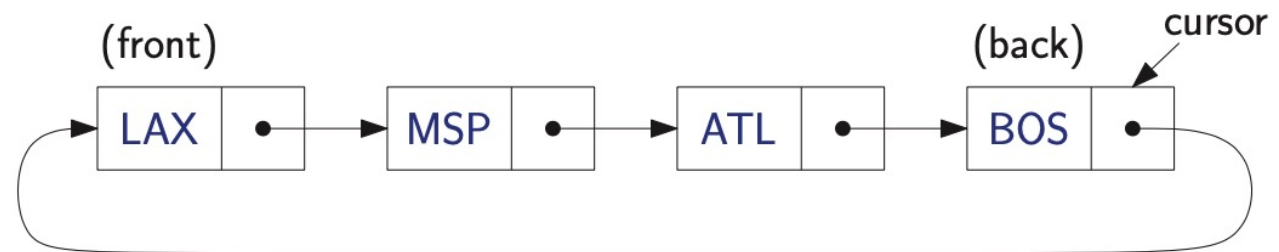
```
void CircleList::add(const Elem& e) {  
    CNode* v = new CNode; // create a new node  
    v->elem = e;  
    if (cursor == NULL){ // handling the special case when the list is empty  
        v->next = v; // v points to itself  
        cursor = v;  
    }  
    else { // if the list is not empty  
        v->next = cursor->next;  
        cursor->next = v;  
    }  
}
```



CIRCULARLY LINKED LIST: IMPLEMENTATION

- The method "remove" deletes the node at the beginning of the list.
- i.e., if "N" denotes the node that "cursor" points to, then the method "remove" deletes the node right after N (unless N was the only node in the list, in which case N itself is deleted).

```
void CircleList::remove() {  
    CNode* old = cursor->next; // the node being removed  
    if (old == cursor) { // if "true", it implies that the list has only one node  
        cursor = NULL; // indicating that the list is now empty  
    }  
    else { // if the list does not contain just a single node  
        cursor->next = old->next;  
    }  
    delete old;  
}
```



CIRCULARLY LINKED LIST: **PLAYLIST EXAMPLE**

To help illustrate the use of our CircleList implementation, consider how building a simple interface of a **playlist** in a music player. Here, the track at the curser is marked with a star (*)

```
int main() {  
    CircleList playList;           // [ ] (the list is empty)  
    playList.add("Faint");          // [Faint*]  
    playList.add("Numb");           // [Numb, Faint*]  
    playList.add("In the End");     // [In the End, Numb, Faint*]  
    playList.advance();             // [Numb, Faint, In the End*]  
    playList.advance();             // [Faint, In the End, Numb*]  
    playList.remove();              // [In the End, Numb*]  
    playList.add("Castle Of Glass"); // [Castle Of Glass, In the End, Numb*]  
    return EXIT_SUCCESS;  
}
```



RECURSION



RECURSION

- Another way to achieve repetition, other than loops, is through **recursion**
- Recursion is **when a function repeatedly calls itself**
 - until it reaches a **base case**



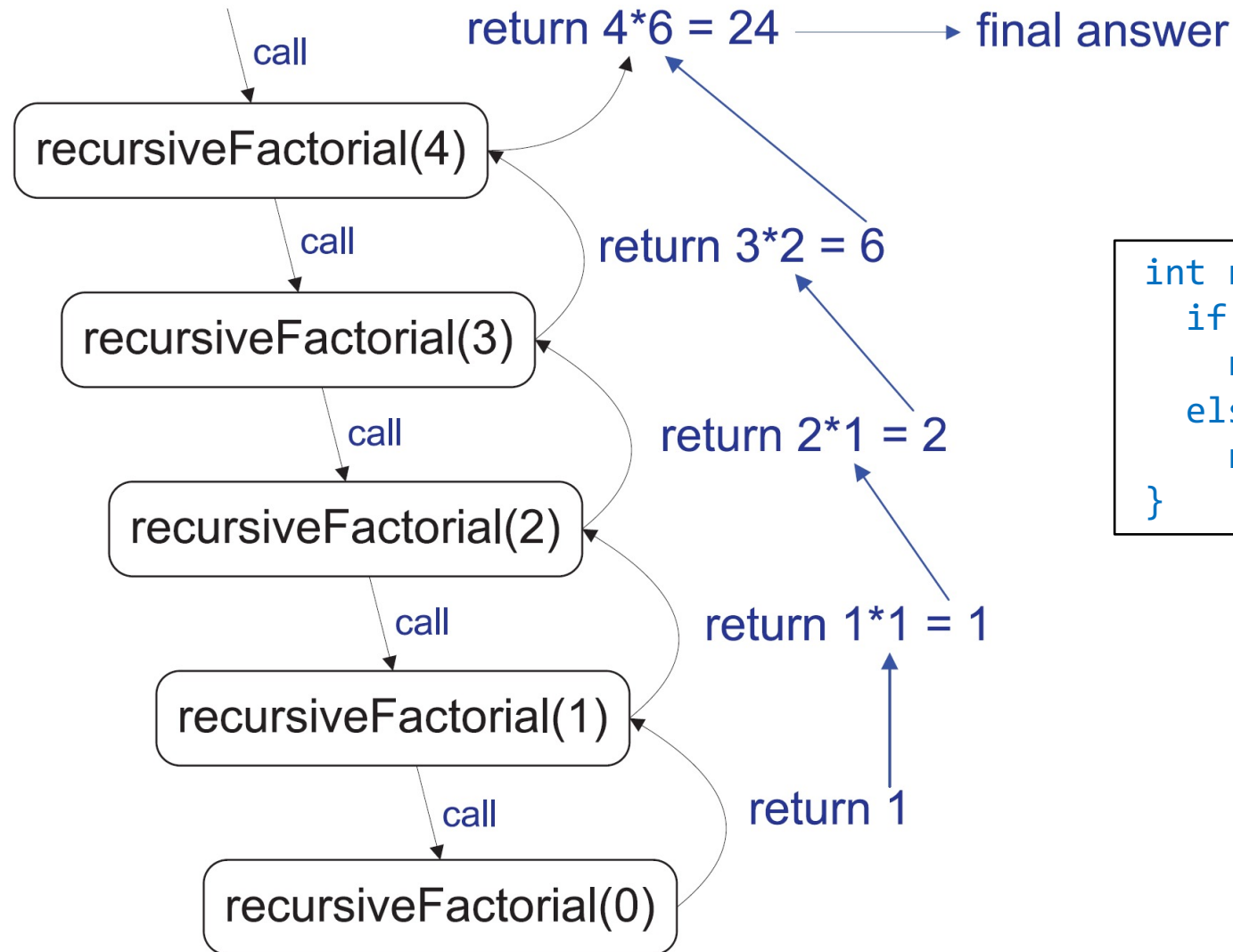
EXAMPLE: FACTORIAL USING RECURSION

- The factorial, $n!$, is defined as:
 - If $n > 0$ then $n! = 1 \times 2 \times 3 \times 4 \times 5 \times \dots \times n \equiv (n-1)! * n$
 - If $n = 0$ then $0! = 1$

How would you implement this recursively in C++?

```
int recursiveFactorial(int n) {  
    if (n == 0) // checks if we reach the base case  
        return 1;  
    else // if not, make a recursive call  
        return n * recursiveFactorial(n-1);  
}
```


EXAMPLE: FACTORIAL – RECURSIVE TRACE



```
int recursiveFactorial(int n){  
    if (n == 0)  
        return 1;  
    else  
        return n*recursiveFactorial(n-1);  
}
```

EXAMPLE: LINEAR SUM USING RECURSION

Algorithm LinearSum(A, n):

Input: Array A and integer $n \geq 1$, such that A has at least n elements

Output: The sum of the first n integers in A

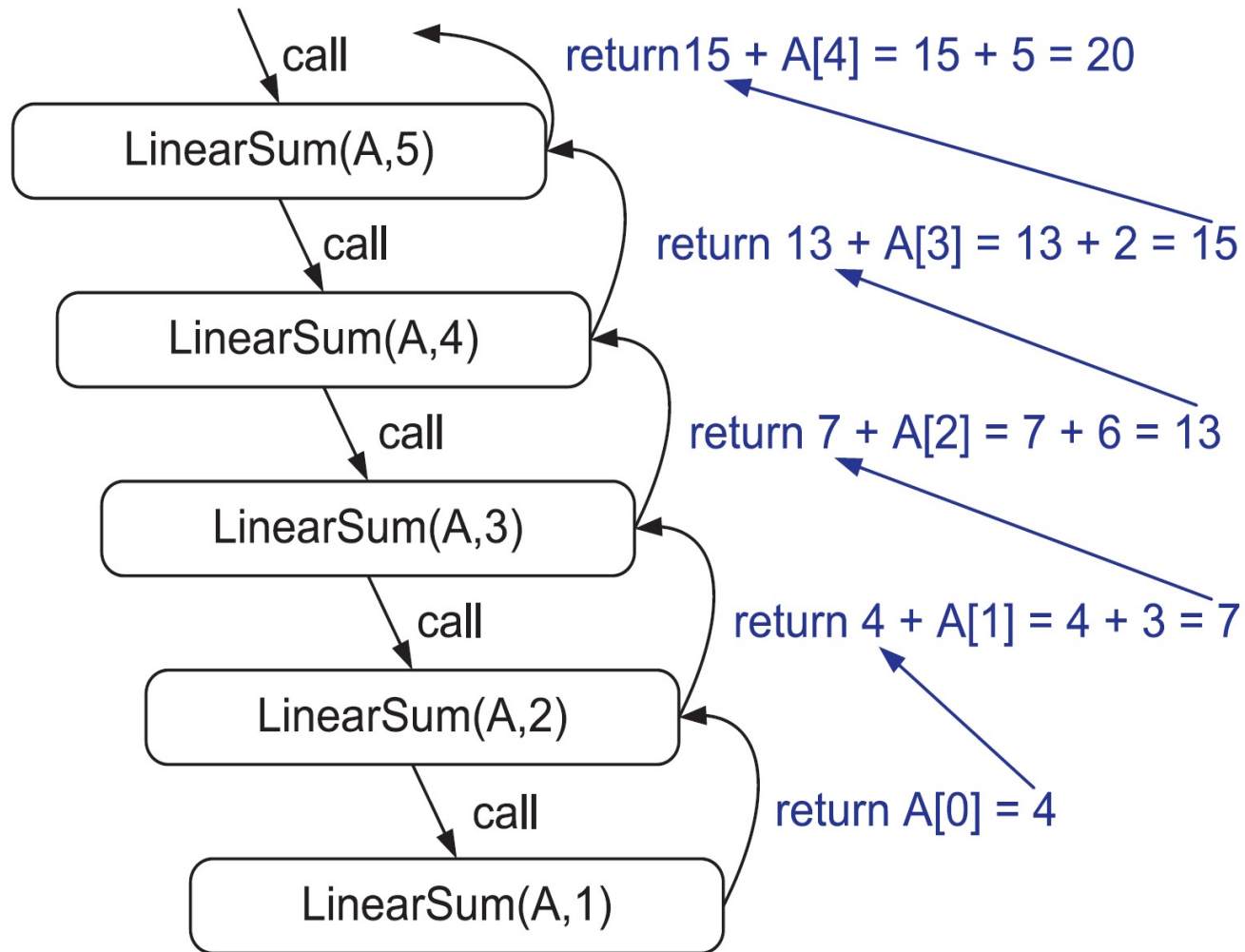
How would you implement this recursively in C++?

```
int LinearSum(int *A,int n){  
    if( n == 1 )  
        return A[0];  
    else  
        return LinearSum(A,n-1) + A[n-1];  
}
```

e.g. $A =$

0	1	2	3	4
4	3	6	2	5

EXAMPLE: LINEAR SUM – RECURSIVE TRACE



```
int LinearSum(int *A,int n){  
    if( n == 1 )  
        return A[0];  
    else  
        return LinearSum(A,n-1) + A[n-1];  
}
```

In this example:
 $A = \{4,3,6,2,5\}$ and $n=5$

EXAMPLE: REVERSING AN ARRAY

Algorithm ReverseArray(A, i, j):

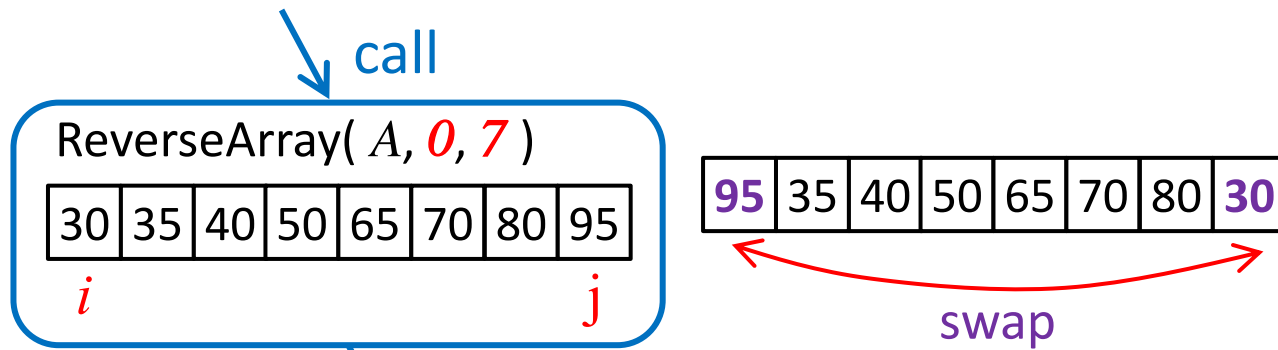
Input: Array A and non-negative integer indices i and j

Output: The reversal of the elements in A starting at index i and ending at j

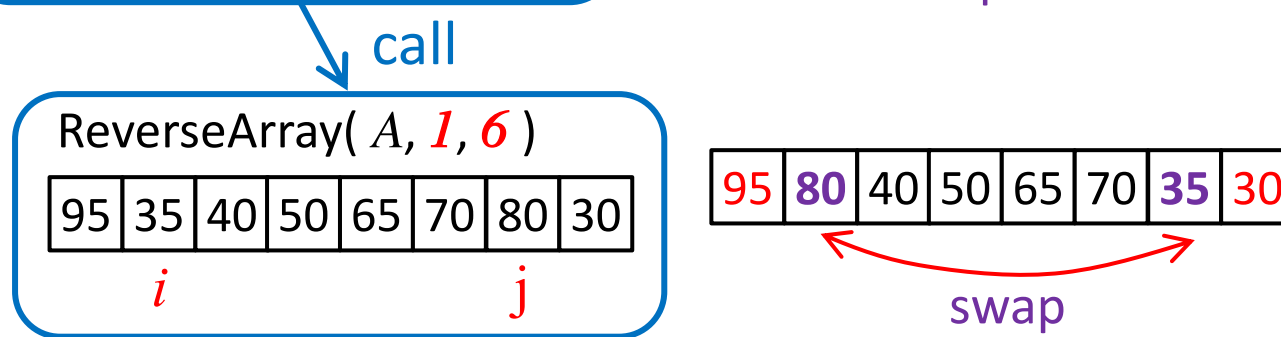
How would you implement this recursively?

i							j
30	35	40	50	65	70	80	95

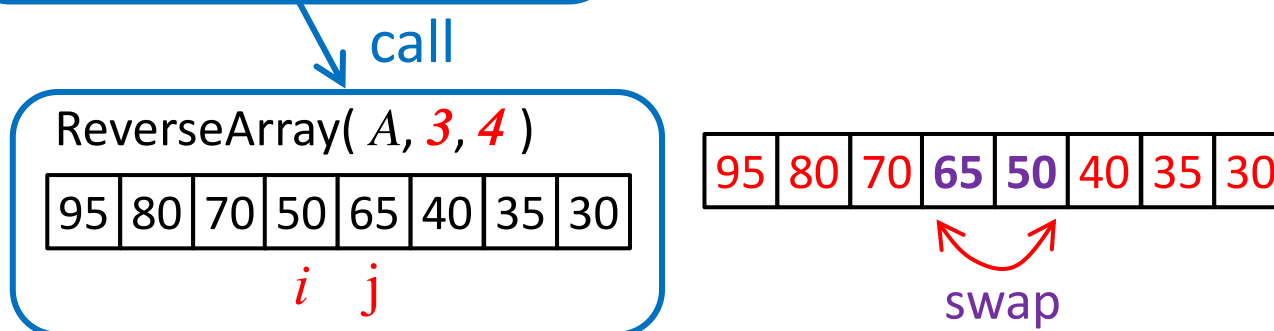
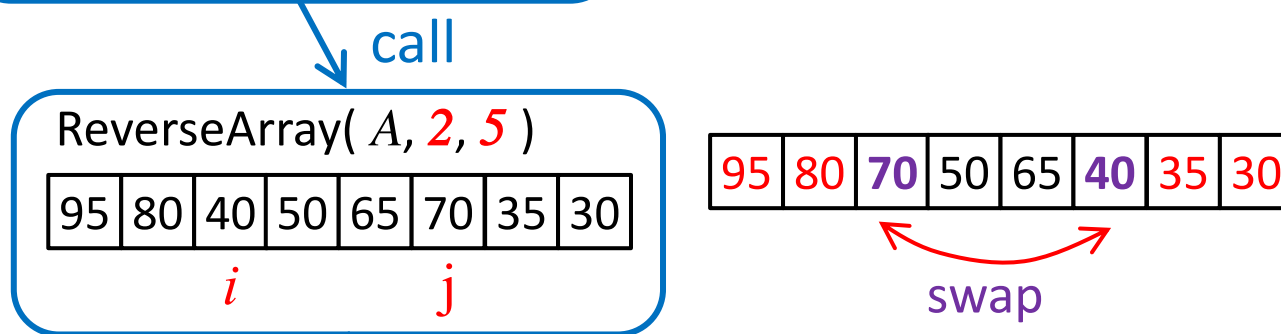
```
void ReverseArray(int *A, int i, int j){  
    if(i < j){  
        swap(A, i, j); //you define a swap function before  
        ReverseArray(A, i+1, j-1);  
    }  
}
```



```
void ReverseArray(int *A, int i, int j)
{
    if(i < j){
        swap(A, i, j);
        ReverseArray(A, i+1, j-1);
    }
}
```



```
void swap(int *A, int i, int j){
    int temp;
    temp = A[i];
    A[i] = A[j];
    A[j] = temp;
}
```



BINARY RECURSION

- When an algorithm makes **two** recursive calls, we say it uses **binary recursion**
- For example, these two calls can be used to solve **two similar halves of a problem**
- Let us revisit the problem of *summing the n elements of an integer array...*

EXAMPLE: BINARY RECURSION – SUM

Algorithm BinarySum(A, i, n):

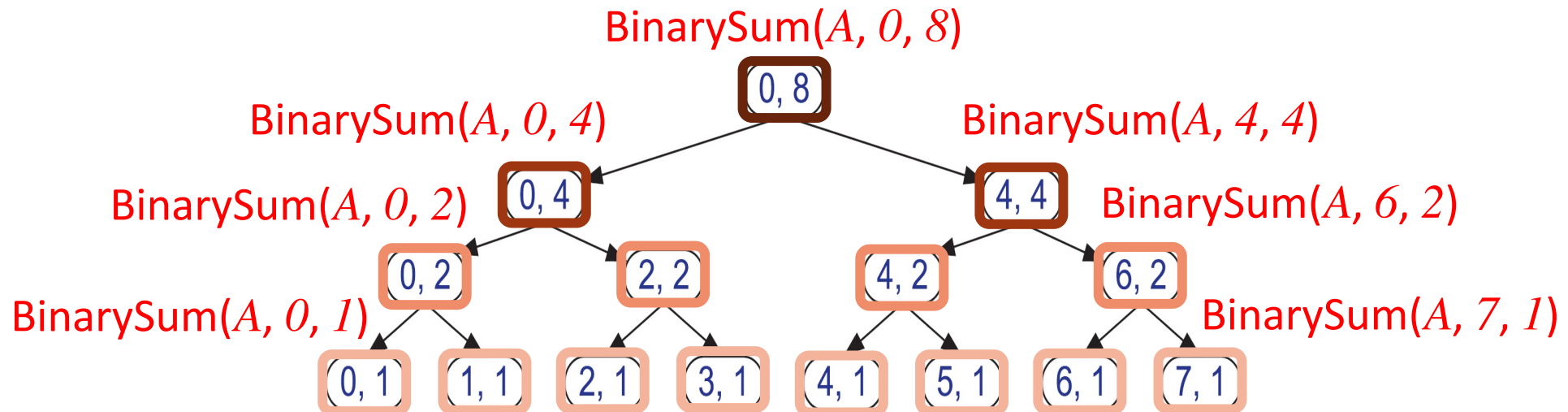
Input: An array A and integers i and n

Output: The sum of the n integers in A starting at index i

if $n = 1$ **then**

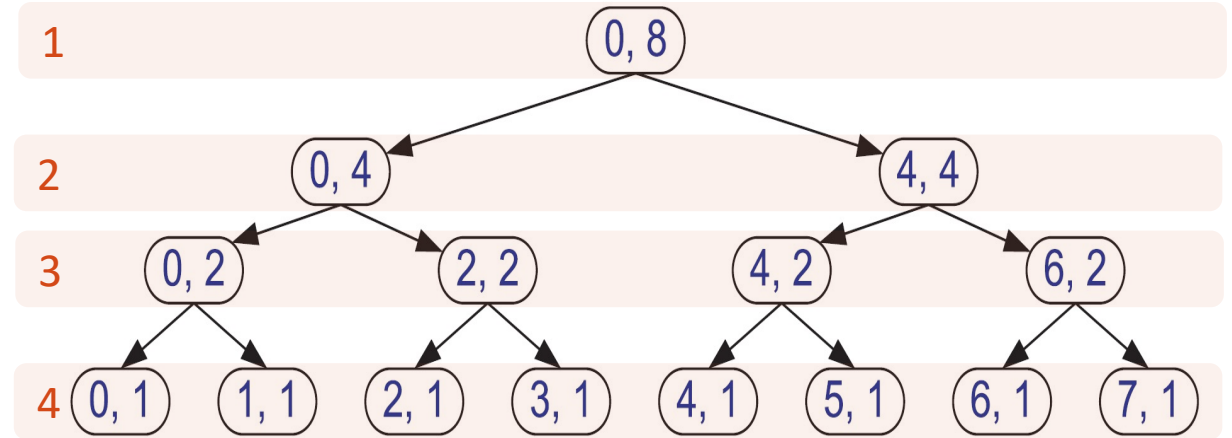
return $A[i]$

return BinarySum($A, i, \lfloor n/2 \rfloor$) + BinarySum($A, i + \lfloor n/2 \rfloor, \lfloor n/2 \rfloor$)



EXAMPLE: BINARY RECURSION – SUM

In this example, the maximum number of iterations we need to sum up 8 elements is $1 + \log_2 n$, which is at most 4.



This is a big improvement over the number of iterations needed by “LinearSum”, where you need n iterations for n elements.

