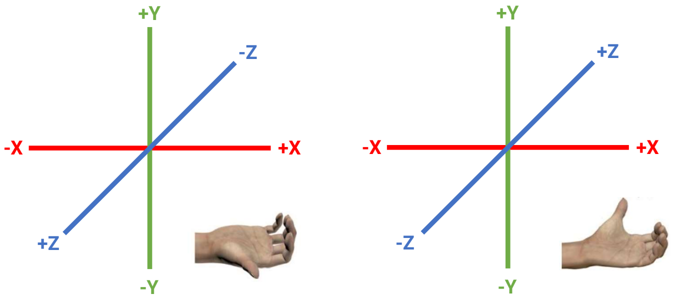
Geometry Representation

Introduction to Computer Graphics
Yu-Ting Wu

Define the 3D World

Description of the 3D World

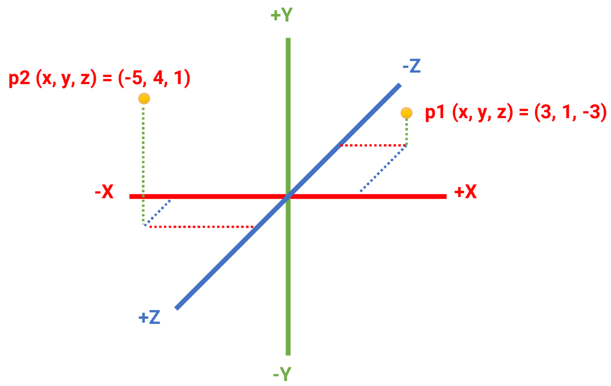
- 3D coordinate systems



OpenGL (Right-Hand-Side)

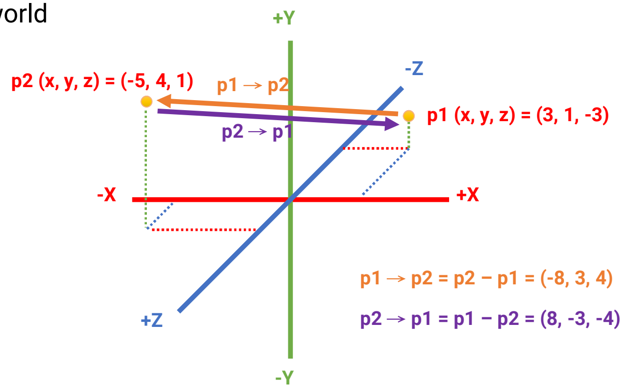
DirectX (Left-Hand-Side)

Points in 3D



Vector in 3D Space

- Use to represent direction (e.g., movement) in the 3D world



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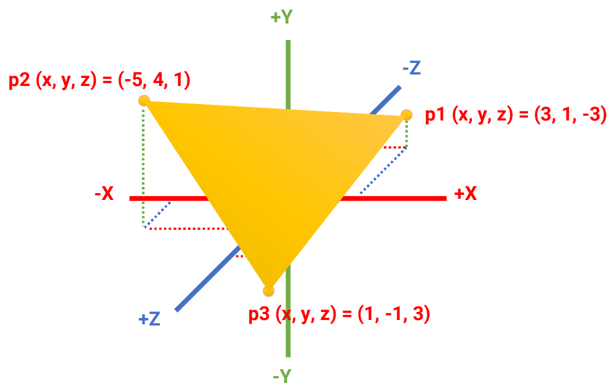
Triangles in 3D



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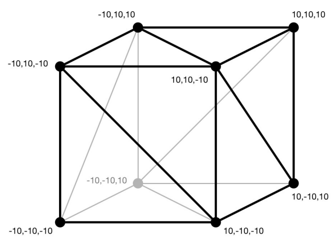
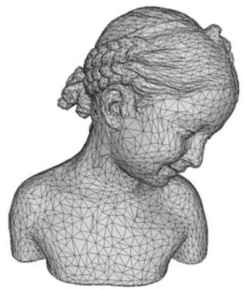
Triangle Mesh

- We can define the geometry of an object by specifying the coordinates of the vertices and their adjacencies



12 triangles



10K triangles

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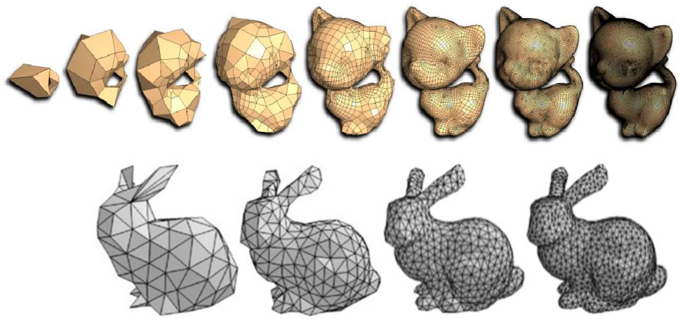
Triangle Mesh (cont.)

- Using more triangles can lead to higher-quality meshes
  - However, takes more time to render



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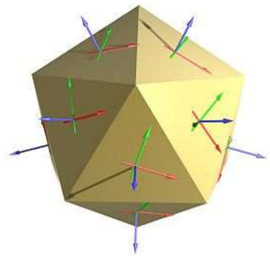
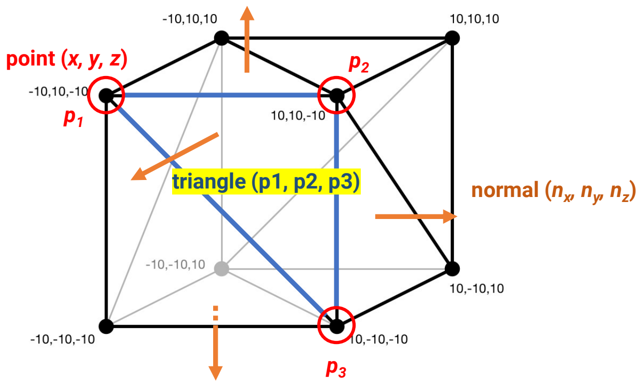
Scene Built with Triangle Mesh

Surface Normal

- A **surface normal** is a vector that is **perpendicular** to a surface at a particular position
- Represent the orientation of the face

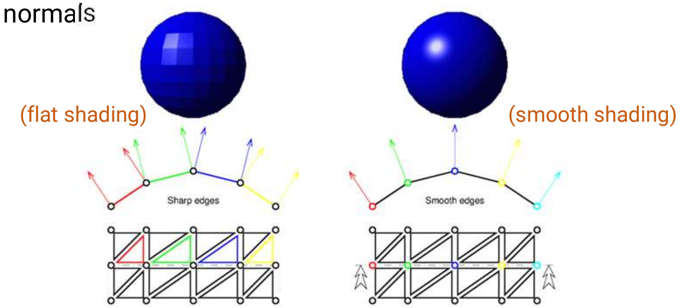


Point, Triangle, and Surface Normal



Vertex Normal

- Compute by **averaging** the surface normals of the faces that contain that vertex
- Can achieve much **smooth** shading than using triangle normals



3D Model Format

- A model is often stored in a file
- Common file format includes
  - **Wavefront (*.obj)**
  - Polygon file format (*.ply)
  - **Filmbox (*.fbx)**
  - MAX (*.max)
  - Digital Asset Exchange File (*.dae)
  - STereoLithography (*.stl)

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Example: Wavefront OBJ File Format

- cube.obj

cube.obj - 記事本

```
# Unit-volume cube with the same texture coordinates on each face.
#
# Created by Morgan McGuire and released into the Public Domain on
# July 16, 2011.
#
# http://graphics.cs.williams.edu/data
```
comments

```
mtllib default.mtl
```
specify material file

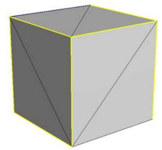
```
v -0.5 0.5 -0.5
v -0.5 0.5 0.5
v 0.5 0.5 0.5
v 0.5 0.5 -0.5
v -0.5 -0.5 -0.5
v -0.5 -0.5 0.5
v 0.5 -0.5 0.5
v 0.5 -0.5 -0.5
```
vertex position declaration

```
vt 0 1
vt 0 0
vt 1 0
vt 1 1
```
vertex texture coordinate declaration

```
vn 0 1 0
vn -1 0 0
vn 1 0 0
vn 0 0 -1
vn 0 0 1
vn 0 -1 0
```
vertex normal declaration

```
g cube
usemtl default

f -8/-4/-6 -7/-3/-6 -6/-2/-6
f -8/-4/-6 -6/-2/-6 -5/-1/-6
f -8/-4/-5 -4/-3/-5 -3/-2/-5
f -8/-4/-5 -3/-2/-5 -7/-1/-5
f -6/-4/-4 -2/-3/-4 -1/-2/-4
f -6/-4/-4 -1/-2/-4 -5/-1/-4
f -5/-4/-3 -1/-3/-3 -4/-2/-3
f -5/-4/-3 -4/-2/-3 -8/-1/-3
f -7/-4/-2 -3/-3/-2 -2/-2/-2
f -7/-4/-2 -2/-2/-2 -6/-1/-2
f -3/-4/-1 -4/-3/-1 -1/-2/-1
f -3/-4/-1 -1/-2/-1 -2/-1/-1
```
face data
(adjacency, submesh)



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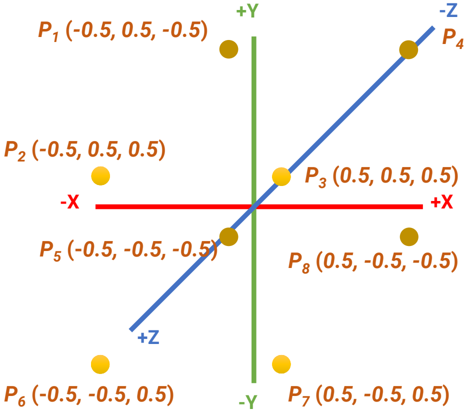
Example: Wavefront OBJ File Format (cont.)



P_1	v -0.5 0.5 -0.5
P_2	v -0.5 0.5 0.5
P_3	v 0.5 0.5 0.5
P_4	v 0.5 0.5 -0.5
P_5	v -0.5 -0.5 -0.5
P_6	v -0.5 -0.5 0.5
P_7	v 0.5 -0.5 0.5
P_8	v 0.5 -0.5 -0.5

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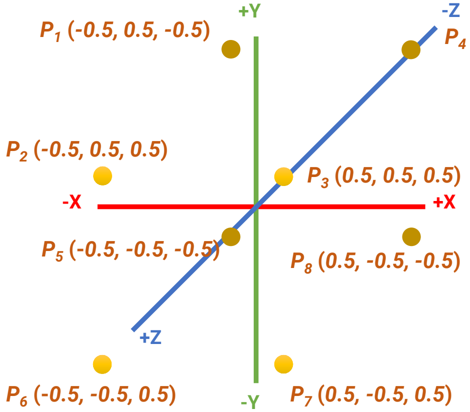
Example: Wavefront OBJ File Format (cont.)



F_1	f -8/-4/-6 -7/-3/-6 -6/-2/-6
F_2	f -8/-4/-6 -6/-2/-6 -5/-1/-6
F_3	f -8/-4/-5 -4/-3/-5 -3/-2/-5
F_4	f -8/-4/-5 -3/-2/-5 -7/-1/-5
F_5	f -6/-4/-4 -2/-3/-4 -1/-2/-4
F_6	f -6/-4/-4 -1/-2/-4 -5/-1/-4
F_7	f -5/-4/-3 -1/-3/-3 -4/-2/-3
F_8	f -5/-4/-3 -4/-2/-3 -8/-1/-3
	f -7/-4/-2 -3/-3/-2 -2/-2/-2
	f -7/-4/-2 -2/-2/-2 -6/-1/-2
	f -3/-4/-1 -4/-3/-1 -1/-2/-1
	f -3/-4/-1 -1/-2/-1 -2/-1/-1

vertex1 vertex2 vertex3
f *P/T/N* *P/T/N* *P/T/N*

P: index of vertex position
T: index of texture coordinate
N: index of vertex normal

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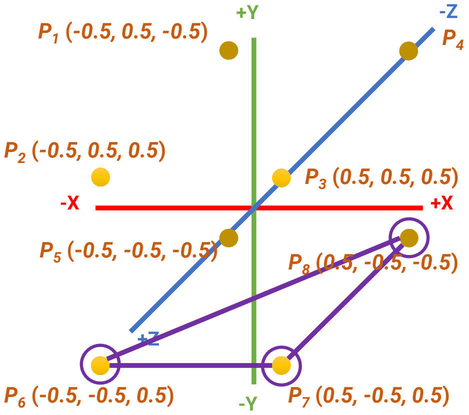
Example: Wavefront OBJ File Format (cont.)



F_1 … F_8

```
f -8/-4/-6 -7/-3/-6 -6/-2/-6
f -8/-4/-6 -6/-2/-6 -5/-1/-6
f -8/-4/-5 -4/-3/-5 -3/-2/-5
f -8/-4/-5 -3/-2/-5 -7/-1/-5
f -6/-4/-4 -2/-3/-4 -1/-2/-4
f -6/-4/-4 -1/-2/-4 -5/-1/-4
f -5/-4/-3 -1/-3/-3 -4/-2/-3
f -5/-4/-3 -4/-2/-3 -8/-1/-3
f -7/-4/-2 -3/-3/-2 -2/-2/-2
f -7/-4/-2 -2/-2/-2 -6/-1/-2
f -3/-4/-1 -4/-3/-1 -1/-2/-1
f -3/-4/-1 -1/-2/-1 -2/-1/-1
```

vertex1 vertex2 vertex3

f P/T/N P/T/N P/T/N

P: index of vertex position
T: index of texture coordinate
N: index of vertex normal

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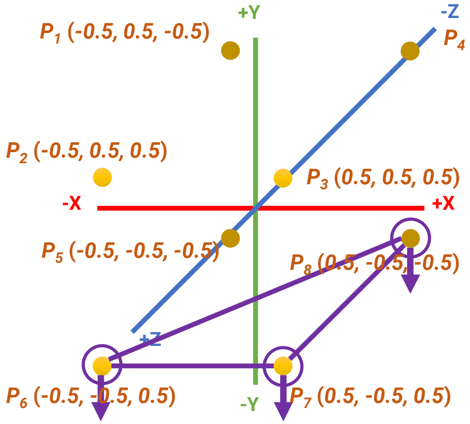
Example: Wavefront OBJ File Format (cont.)



F_1
```
f -8/-4/-6 -7/-3/-6 -6/-2/-6
```

N_1 … N_6
```
vn 0 1 0
vn -1 0 0
vn 1 0 0
vn 0 0 -1
vn 0 0 1
vn 0 -1 0
```

vertex1 vertex2 vertex3

f P/T/N P/T/N P/T/N

P: index of vertex position
T: index of texture coordinate
N: index of vertex normal

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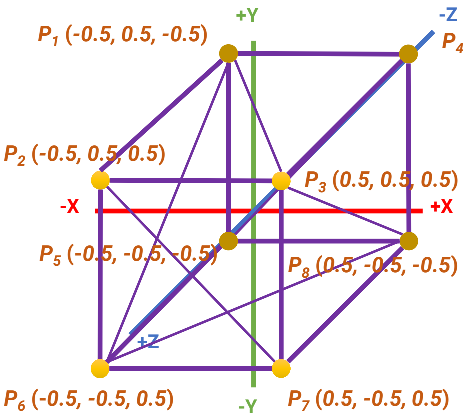
Example: Wavefront OBJ File Format (cont.)



F_1 … F_8

```
f -8/-4/-6 -7/-3/-6 -6/-2/-6
f -8/-4/-6 -6/-2/-6 -5/-1/-6
f -8/-4/-5 -4/-3/-5 -3/-2/-5
f -8/-4/-5 -3/-2/-5 -7/-1/-5
f -6/-4/-4 -2/-3/-4 -1/-2/-4
f -6/-4/-4 -1/-2/-4 -5/-1/-4
f -5/-4/-3 -1/-3/-3 -4/-2/-3
f -5/-4/-3 -4/-2/-3 -8/-1/-3
f -7/-4/-2 -3/-3/-2 -2/-2/-2
f -7/-4/-2 -2/-2/-2 -6/-1/-2
f -3/-4/-1 -4/-3/-1 -1/-2/-1
f -3/-4/-1 -1/-2/-1 -2/-1/-1
```

vertex1 vertex2 vertex3

f P/T/N P/T/N P/T/N

P: index of vertex position
T: index of texture coordinate
N: index of vertex normal

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Transformation

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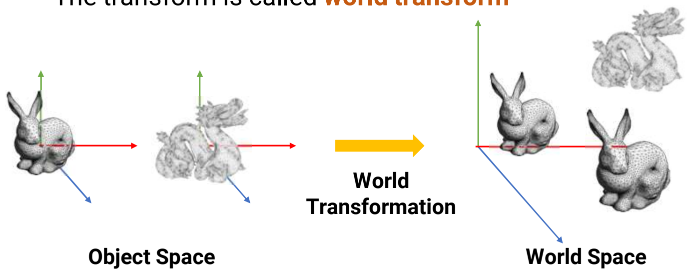
World Space and World Coordinate

- Objects are defined in **object space individually**
- When building a scene, each object is transformed to a **global** and **unique** space called **world space**
- The transform is called **world transform**



World Space and World Coordinate (cont.)

- Advantages for using “transformation”
  - **Reuse model:** design a model and use it in several scenes
  - **Memory saving:** store a 4x4 matrix instead of duplication of the entire models

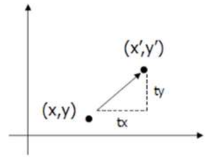
Common Transformations

- Translation
- Scaling
- Rotation

2D Translation

- Given a point ***p(x, y)*** and a translation offset ***T(t***$T(t_x, t_y)$, the new point ***p'(x', y')*** after translation is ***p' = p + T***

$$x' = x + t_x$$
$$y' = y + t_y$$



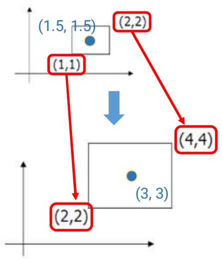
- **Can be represented as** Matrix-vector multiplication

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

2D Scaling

- Given a point ***p(x, y)*** and a scaling factor ***S(s_x, s_y)***, the new point ***p'(x', y')*** after scaling is ***p' = S p***

$$x' = x * s_x$$
$$y' = y * s_y$$

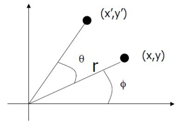


- Matrix-vector multiplication

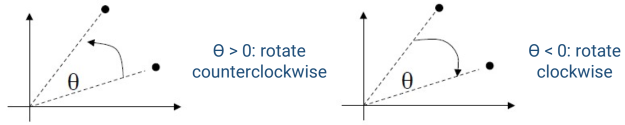
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

2D Rotation

- Given a point ***p(x, y)***, rotate it with respect to the **origin** by ***θ*** and get the new point ***p'(x', y')*** after rotation



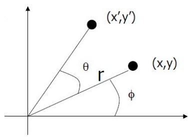
- First we define



2D Rotation (cont.)

- Given a point ***p(x, y)***, rotate it with respect to the **origin** by ***θ*** and get the new point ***p'(x', y')*** after rotation

$$x = r\cos(\phi) \qquad y = r\sin(\phi)$$
$$x' = r\cos(\phi + \theta) \qquad y' = r\sin(\phi + \theta)$$



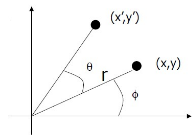
$$\begin{aligned} x' &= r\cos(\phi + \theta) \\ &= r\cos(\phi)\cos(\theta) - r\sin(\phi)\sin(\theta) \\ &= x\cos(\theta) - y\sin(\theta) \end{aligned}$$

$$\begin{aligned} y' &= r\sin(\phi + \theta) \\ &= x\sin(\phi)\cos(\theta) + r\cos(\phi)\sin(\theta) \\ &= y\cos(\theta) + x\sin(\theta) \end{aligned}$$

2D Rotation (cont.)

- Given a point ***p(x, y)***, rotate it with respect to the **origin** by ***θ*** and get the new point ***p'(x', y')*** after rotation

$$\begin{aligned} x' &= r\cos(\phi + \theta) \\ &= x\cos(\theta) - y\sin(\theta) \\ y' &= r\sin(\phi + \theta) \\ &= y\cos(\theta) + x\sin(\theta) \end{aligned}$$



- Matrix-vector multiplication

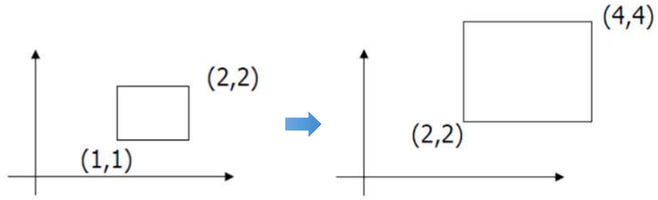
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

2D Translation, Scaling, and Rotation

- Translation $\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$
- Scaling $\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$
- Rotation $\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$
- Using a 3x3 matrix allows us to perform all transformations using matrix/vector multiplications
  - We can also **pre-multiply (concatenate)** all the matrices
- We call the *(x, y, 1)* representation the **homogeneous coordinate** for *(x, y)*

Revisit 2D Scaling

- The standard scaling matrix will only anchor at (0, 0)

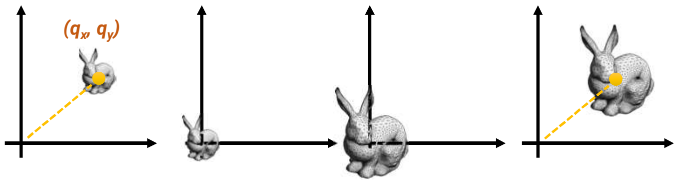


- What if we want the object to be scaled w.r.t its center?

Revisit 2D Scaling (cont.)

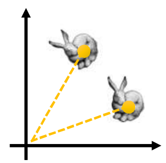
- Scaling about an arbitrary pivot point ***Q(q$_x$, q$_y$)***
  - Translate the objects so that Q will coincide with the origin: ***T(-q$_x$, -q$_y$)***
  - Scale the object: ***S(s$_x$, s$_y$)***
  - Translate the object back: ***T(q$_x$, q$_y$)***
- The final scaling matrix can be written as $\boxed{T(q)S(s)T(-q)}$

Concatenation of matrices



Revisit 2D Rotation

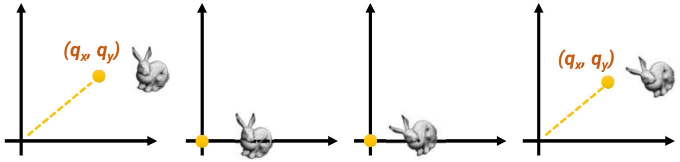
- The standard rotation matrix is used to rotate about the origin (0, 0)



- What if we want the object to be rotated w.r.t a specific pivot?

Revisit 2D Rotation (cont.)

- Rotate about an arbitrary pivot point ***Q(q$_x$, q$_y$)*** by ***θ***
  - Translate the objects so that Q will coincide with the origin: ***T(-q$_x$, -q$_y$)***
  - Rotate the object: ***R(θ)***
  - Translate the object back: ***T(q$_x$, q$_y$)***
- The final rotation matrix can be written as $\boxed{T(q)R(\theta)T(-q)}$



Translation (3D) and Scaling (3D)

- A 3D transformation is represented as a **4x4 matrix**, with **homogeneous coordinate**

translation
$$\begin{bmatrix}1 & 0 & t_x\\ 0 & 1 & t_y\\ 0 & 0 & 1\end{bmatrix} \Rightarrow \begin{bmatrix}1 & 0 & 0 & t_x\\ 0 & 1 & 0 & t_y\\ 0 & 0 & 1 & t_z\\ 0 & 0 & 0 & 1\end{bmatrix}$$

scaling
$$\begin{bmatrix}s_x & 0 & 0\\ 0 & s_y & 0\\ 0 & 0 & 1\end{bmatrix} \Rightarrow \begin{bmatrix}s_x & 0 & 0 & 0\\ 0 & s_y & 0 & 0\\ 0 & 0 & s_z & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

2D 3D

Rotation (3D)

rotation w.r.t x-axis
$$\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & \cos\theta & -\sin\theta & 0\\ 0 & \sin\theta & \cos\theta & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

2D
$$\begin{bmatrix}\cos(\theta) & -\sin(\theta) & 0\\ \sin(\theta) & \cos(\theta) & 0\\ 0 & 0 & 1\end{bmatrix}$$

rotation w.r.t y-axis
$$\begin{bmatrix}\cos\theta & 0 & \sin\theta & 0\\ 0 & 1 & 0 & 0\\ -\sin\theta & 0 & \cos\theta & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

rotation w.r.t z-axis
$$\begin{bmatrix}\cos\theta & -\sin\theta & 0 & 0\\ \sin\theta & \cos\theta & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

3D

3D Transformation

- Practice
  - Scale w.r.t a given pivot point
  - Rotate w.r.t a given pivot point

Spoiler

- There are other spaces
- We will introduce camera space, clip space, and NDC in the next slides

for building scene

Object Space
(Local Space)

World Space

for assisting rendering

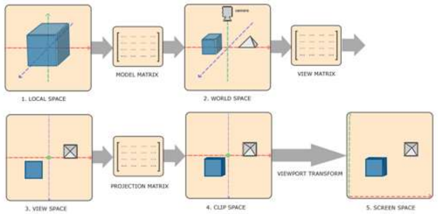
Camera Space
(View, Eye Space)

Clip Space

Normalized Device Coordinate (NDC)

Screen Space

for displaying



Introduction to Computer Graphics 2022

Any Questions?