Data Abstractions

Introduction to Computer

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Outline

- Arrays
- Lists
- Stacks
- Queues
- Trees

Data Structure Concepts

- Example: give an array, find the minimal element
  - How about doing this step 100 times
  - Sorting?
  - But if we need to insert a new element or update an old element?

- **Static** v.s. **dynamic** structures

Outline

- Arrays
- Lists
- Stacks
- Queues
- Trees

Homogeneous Arrays

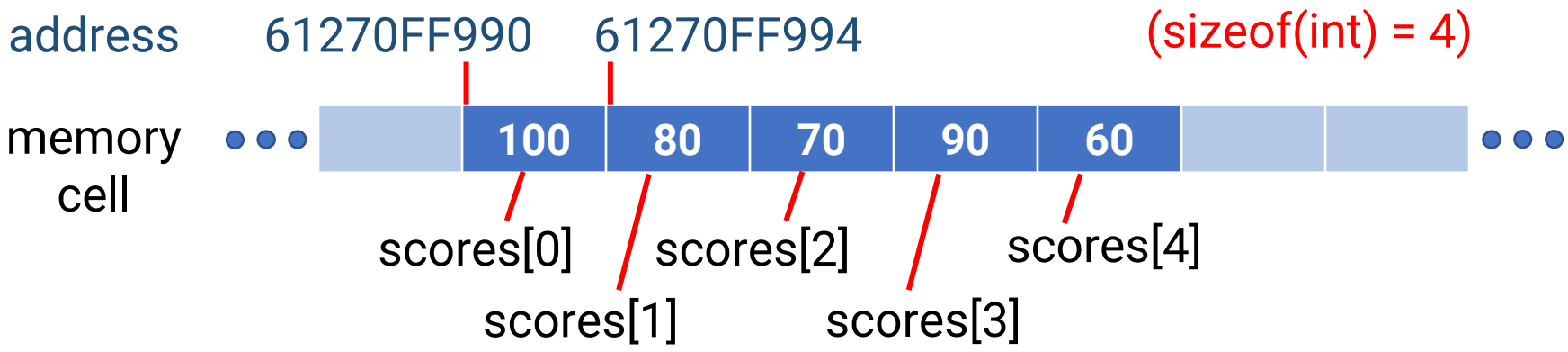
- Block of data whose entries are of the **same type (and size)**
- Indices are used to identify positions
- 1D array

```
int scores[5];
scores[0] = 100;
scores[1] = 80;
scores[2] = 70;
scores[3] = 90;
scores[4] = 60;
```

```
int* ptrScores = scores;
std::cout << ptrScores << std::endl;
std::cout << ptrScores + 1 << std::endl;
std::cout << ptrScores + 2 << std::endl;
std::cout << std::endl;
std::cout << ptrScores << std::endl;
std::cout << &(scores[0]) << std::endl;
std::cout << &(scores[1]) << std::endl;
std::cout << &(scores[2]) << std::endl;
```

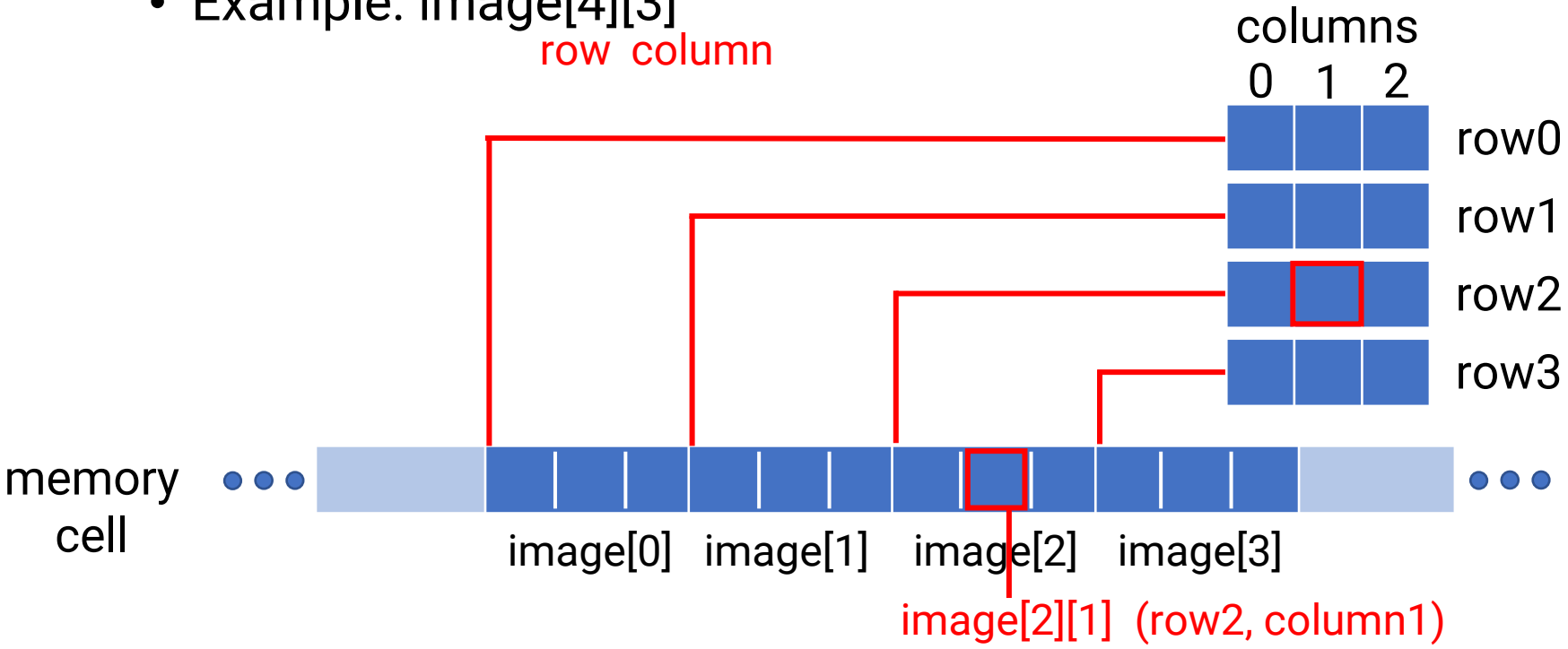
```
00000001000FFE60
00000001000FFE64
00000001000FFE68

00000001000FFE60
00000001000FFE60
00000001000FFE64
00000001000FFE68
```



Homogeneous Arrays (cont.)

- Block of data whose entries are of the **same type (and size)**
- 2D array: array of arrays
  - Consist of rows and columns
  - Example: image[4][3]



Homogeneous Arrays (cont.)

- **Array addresses**

x = 1000

x[0] = 1000	x[0][0] 1000	x[0][1] 1004	x[0][2] 1008	x[0][3] 1012
x[1] = 1016	x[1][0] 1016	x[1][1] 1020	x[1][2] 1024	x[1][3] 1028
x[2] = 1032	x[2][0] 1032	x[2][1] 1036	x[2][2] 1040	x[2][3] 1044

int x[3][4]

address polynomial **sizeof(int) = 4**

int x[1][2] = 1000 + 1 * 4 * 4 + 2 * 4 = 1016 + 2 * 4 = 1024

x[1]

Homogeneous Arrays (cont.)

- **Parameter passing**
  - Does it work?

```
void UpdateArray2D(int **ptrX)
{
    ptrX[2][3] = 5;
}

int main()
{
    int x[3][4];
    UpdateArray2D(x);
}
```

```
void UpdateArray2D(int ptrX[3][4])
{
    ptrX[2][3] = 5;
}

void UpdateArray2D(int ptrX[][4])
{
    ptrX[2][3] = 5;
}
```

```
E0167   類型 "int (*)[4]" 的引數與類型 "int **" 的參數不相容
C2664   'void UpdateArray2D(int **)': 無法將引數 1 從 'int [3][4]' 轉換為 'int **'
```

Why? no enough information for address polynomial
Need the number of elements per row

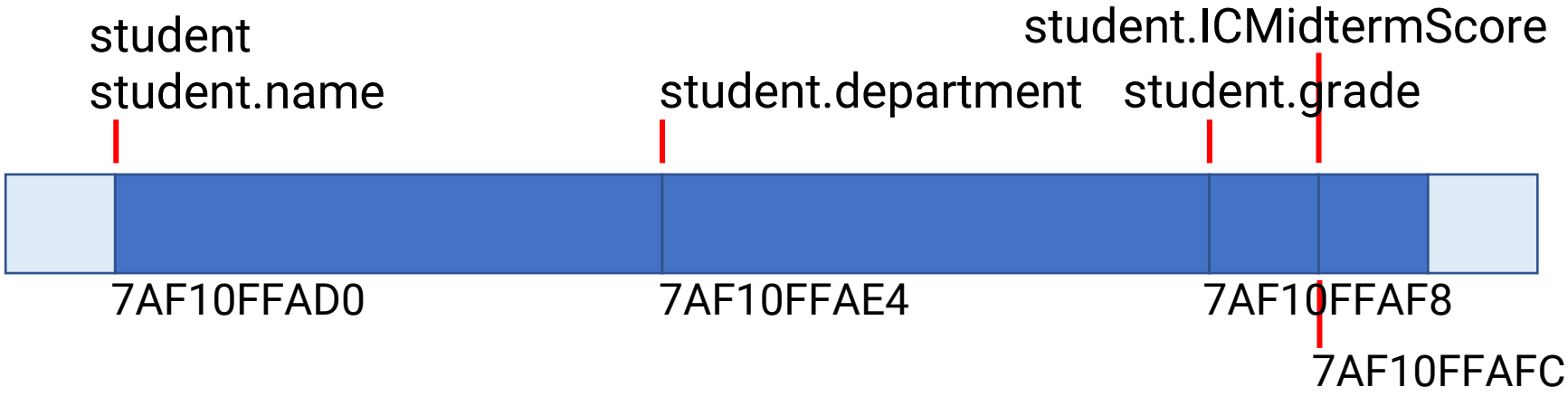
Heterogeneous Arrays

- Structure: a block of data items that might be of different types or sizes
- Each data item is called a field (accessed by name)

```
struct Student
{
    char name[20];
    char department[20];
    int grade;
    int ICMidtermScore;
};
```

```
Student student;
std::cout << &(student) << std::endl;
std::cout << &(student.name) << std::endl;
std::cout << &(student.department) << std::endl;
std::cout << &(student.grade) << std::endl;
std::cout << &(student.ICMidtermScore) << std::endl;
```

```
0000007AF10FFAD0
0000007AF10FFAD0
0000007AF10FFAE4
0000007AF10FFAF8
0000007AF10FFAFC
```



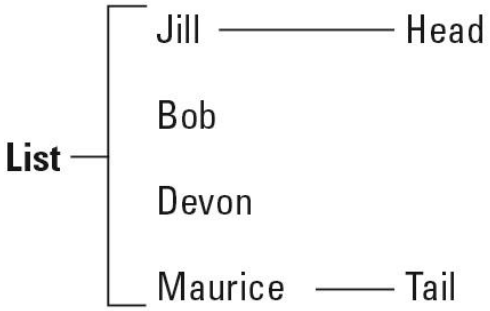
Outline

- Arrays
- Lists
- Stacks
- Queues
- Trees

Lists, Stacks, and Queues

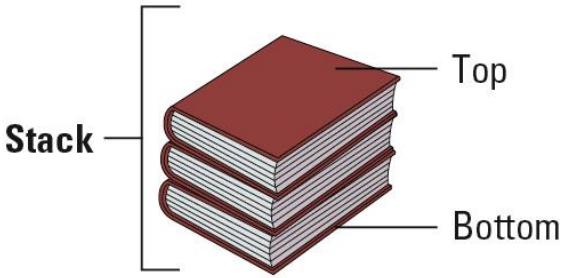
- List: a collection of data whose entries are arranged sequentially
- **Stacks** and **Queues** are specialized lists

List



a. A list of names

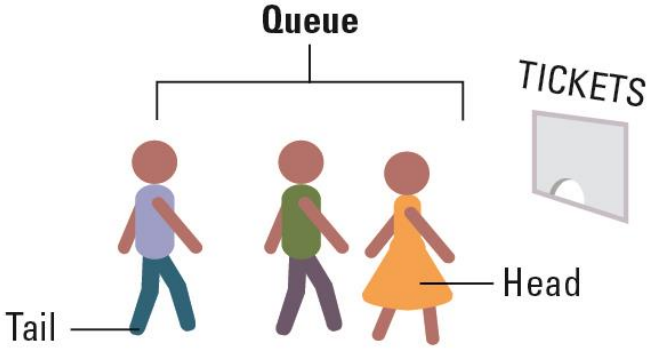
Stack



b. A stack of books

First-In-Last-Out

Queue



c. A queue of people

First-In-First-Out

Operations of a List

- **Empty()**
  - Return true if the list is empty
- **Size()**
  - Return the number of elements in the list
- **GetElement(index)**
  - Return the element with the given index
- **EraseElement(index)**
  - Remove the element at the index
- **Insert(index, data)**
  - Insert a new element with the given data at the given index

Storing Lists

- **Contiguous list (array)**
  - **Pros:** easy to implement, an excellent choice for static use
  - **Cons:** time-consuming for dynamic use, fragments may occur without careful implementation
  - Example: *L = (a, b, c, d, e)* using an array representation

a	b	c	d	e					

    - After deleting the third element, choices are
      - Leave the holes (time-consuming for later use)

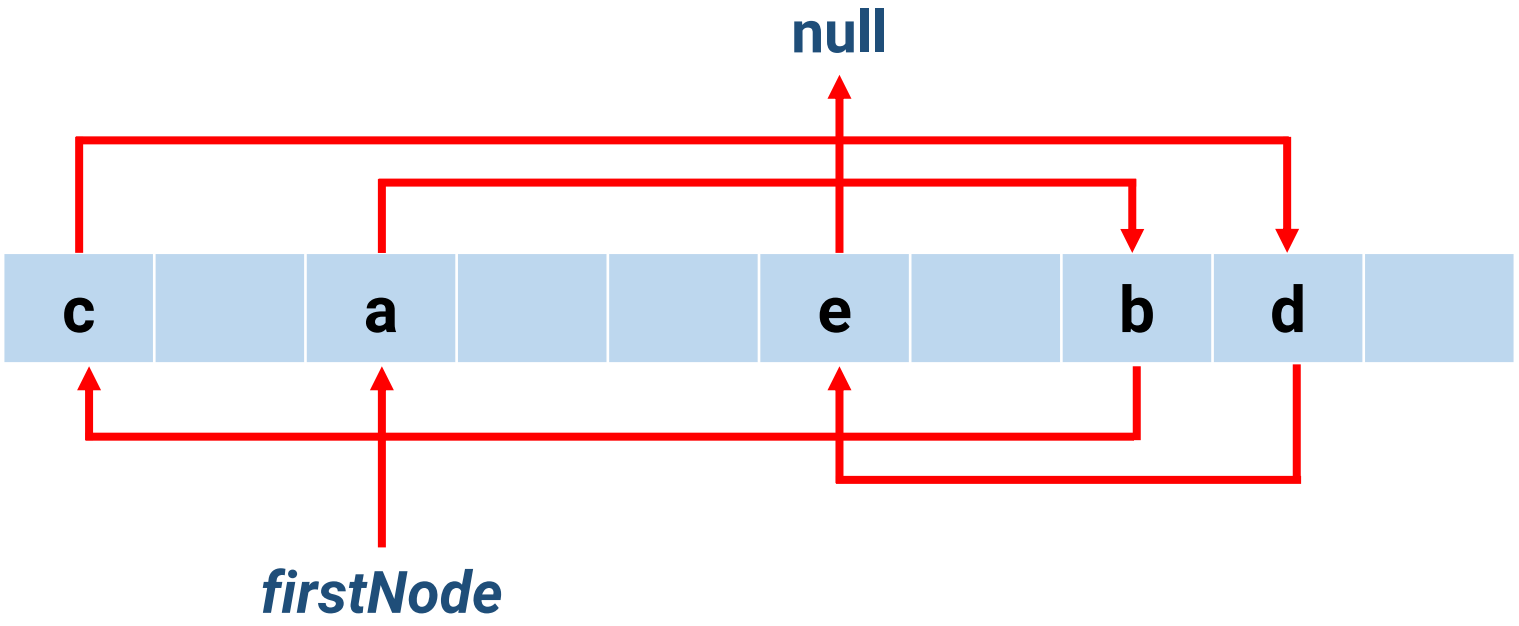
a	b		d	e					

      - Make it compact (easy for following OPs, but need to move)

a	b	d	e						

Storing Lists (cont.)

- **Linked list**
  - Head pointer: indicate the start
  - NULL pointer: indicate the end
  - Example: *L = (a, b, c, d, e)* using a linked representation



Storing Lists (cont.)

- **Linked list**
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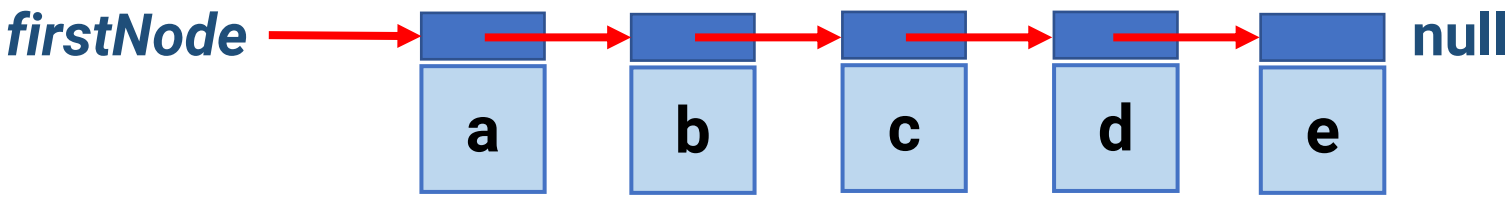
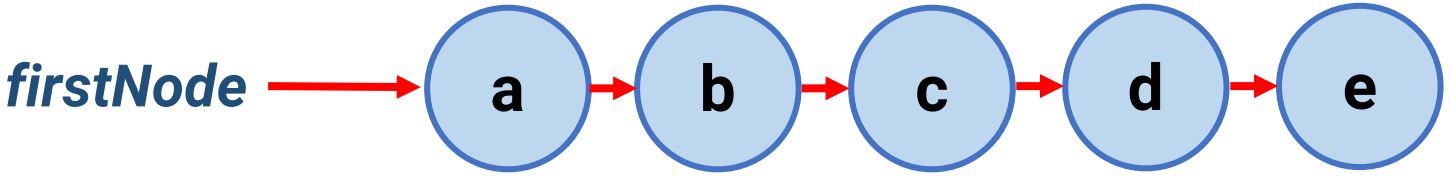
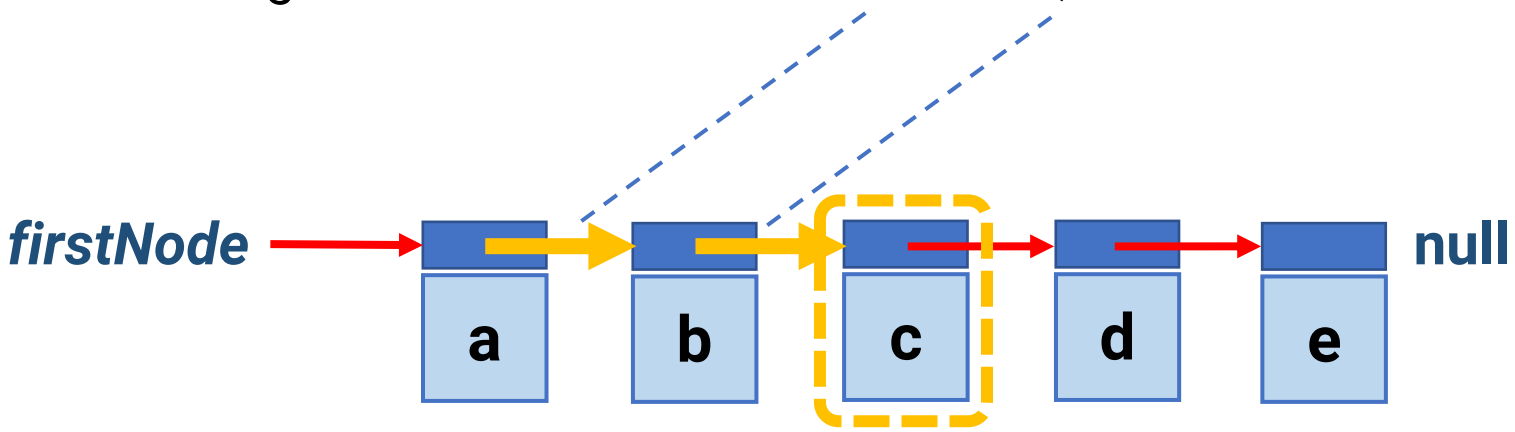




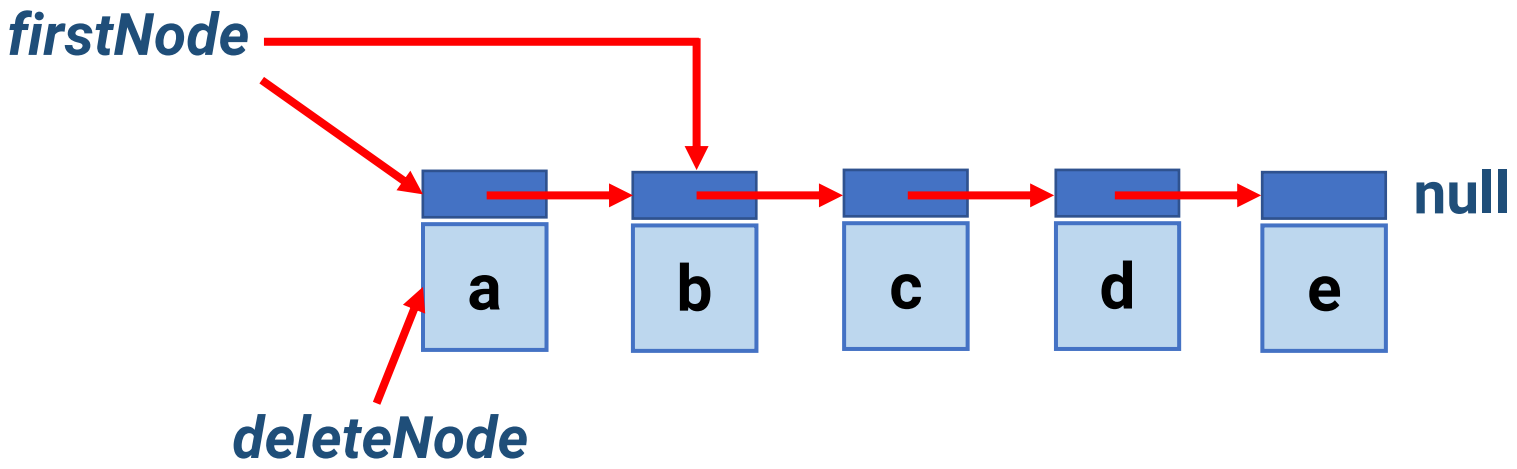
List: Get an Element with its Index

- **Procedure**
  - Start from the first node
  - List::GetElement(2) **assume the index starts with 0**
    - Target node = ***firstNode*→*next*→*next*;**



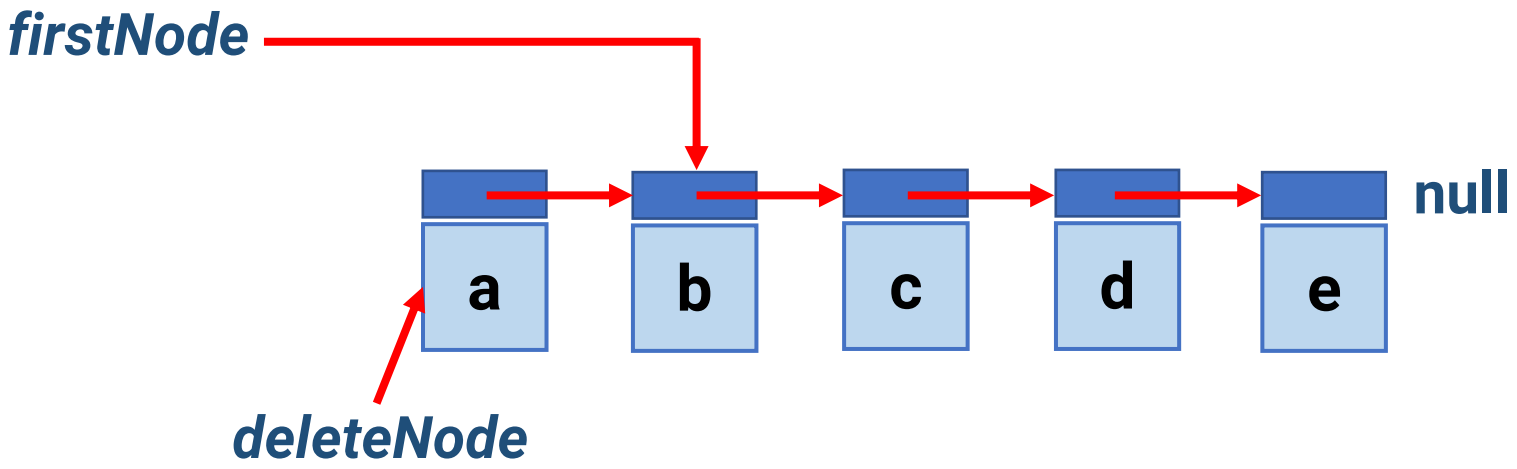
List: Erase an Element with its Index

- Case 1: erase the first element
- **Procedure**
  - Use a pointer to identify the first node (***deleteNode***)



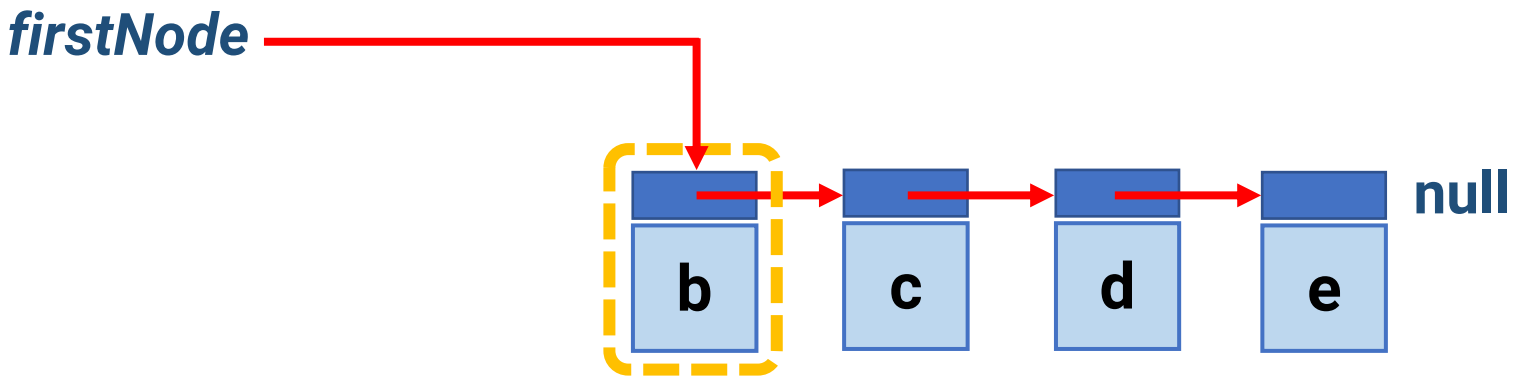
List: Erase an Element with its Index

- Case 1: erase the first element
- **Procedure**
  - Use a pointer to identify the first node (***deleteNode***)
  - Change ***firstNode*** pointer to the second node



List: Erase an Element with its Index

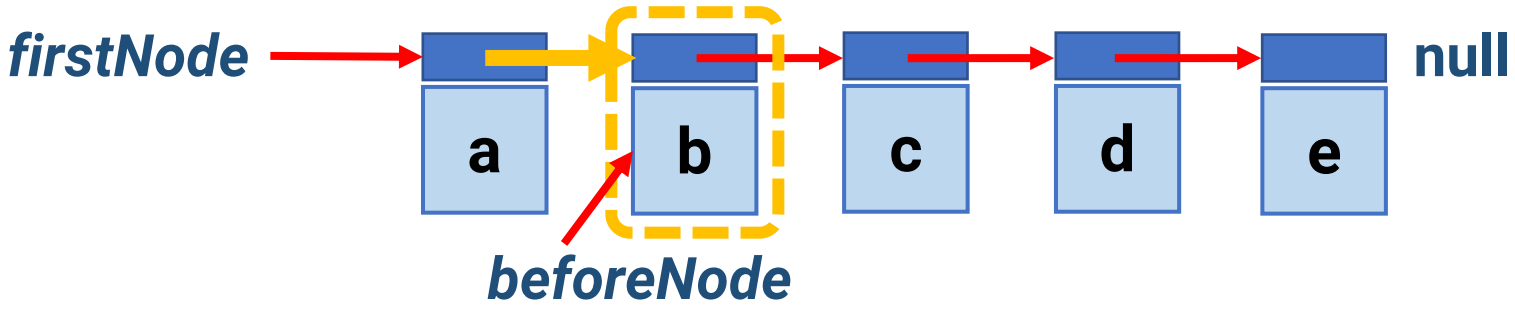
- Case 1: erase the first element
- **Procedure**
  - Use a pointer to identify the first node (***deleteNode***)
  - Change ***firstNode*** pointer to the second node
  - Delete the ***deleteNode***



List: Erase an Element with its Index (cont.)

- Case 2: an element that is not the first one
- **Procedure**
  - Get the node before the target index (***beforeNode***)

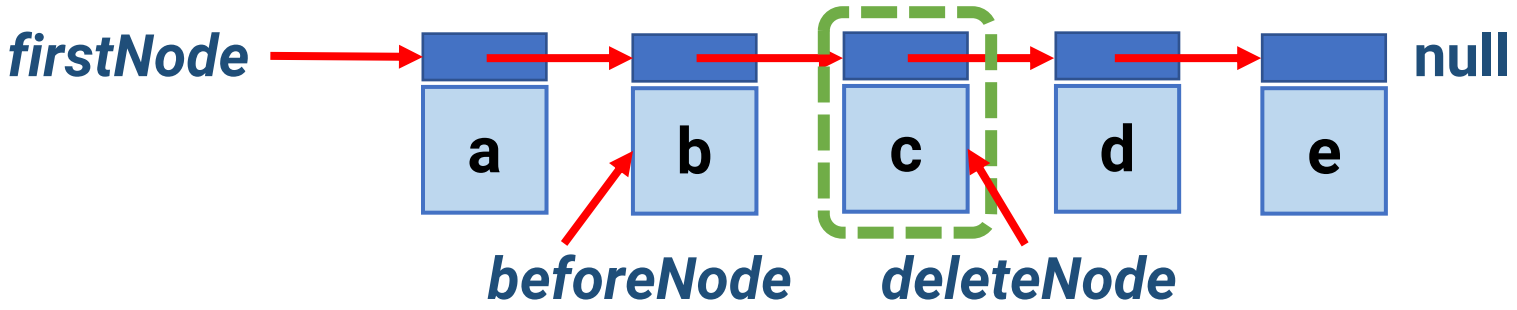
- Example: EraseElement(2)



List: Erase an Element with its Index (cont.)

- Case 2: an element that is not the first one
- **Procedure**
  - Get the node before the target index (***beforeNode***)
  - Identify the node to be deleted (***deleteNode***)

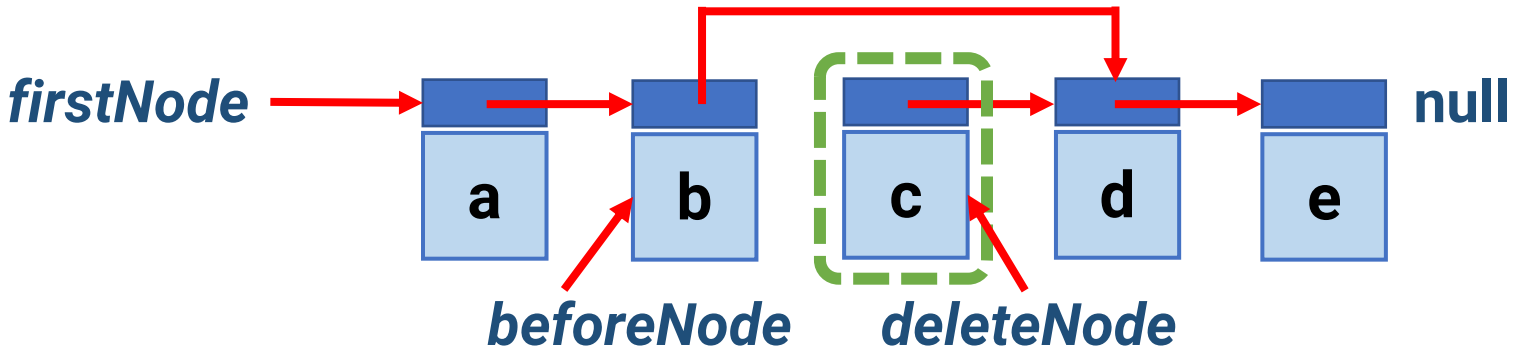
- Example: EraseElement(2)



List: Erase an Element with its Index (cont.)

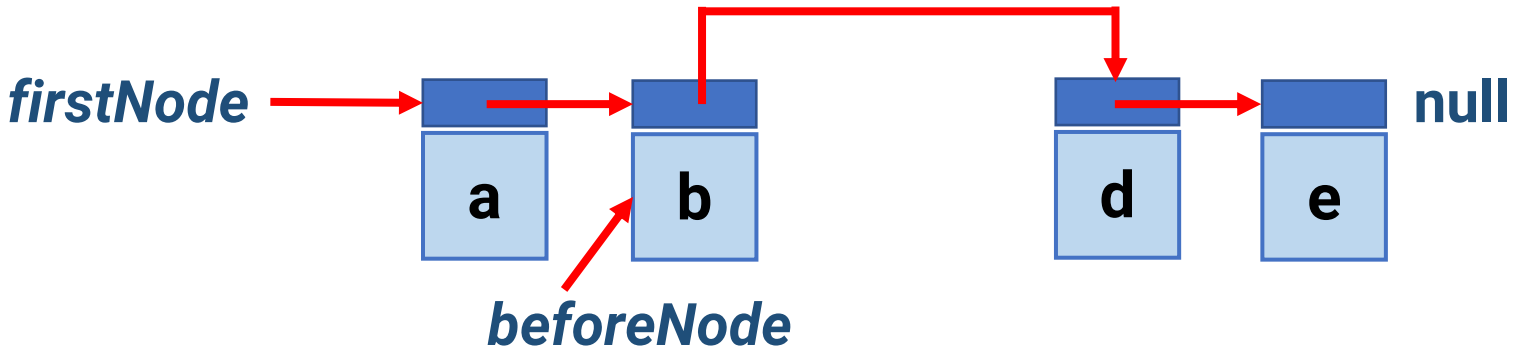
- Case 2: an element that is not the first one
- **Procedure**
  - Get the node before the target index (***beforeNode***)
  - Identify the node to be deleted (***deleteNode***)
  - Change pointer in ***beforeNode***

- Example: EraseElement(2)



List: Erase an Element with its Index (cont.)

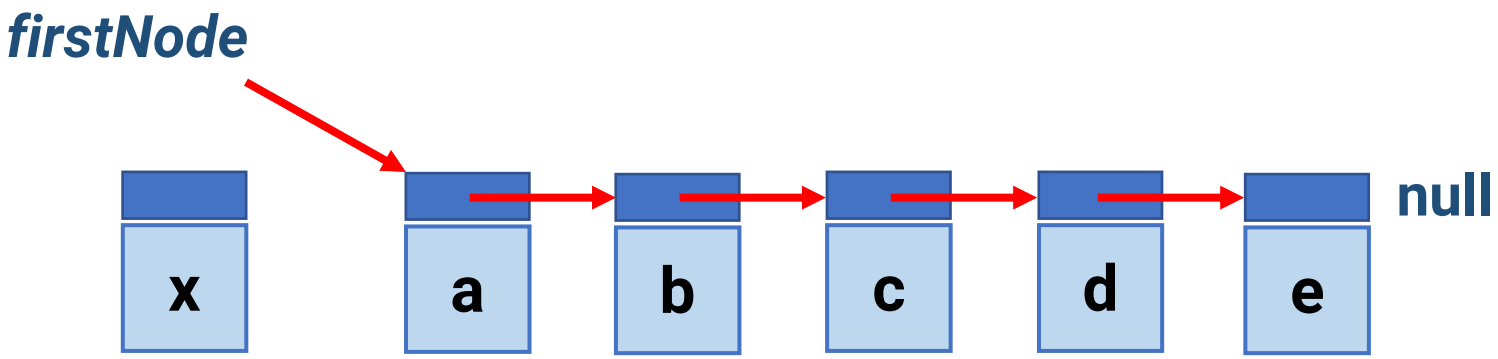
- Case 2: an element that is not the first one
- **Procedure**
  - Get the node before the target index (***beforeNode***)
  - Identify the node to be deleted (***deleteNode***)
  - Change pointer in ***beforeNode***
  - Delete the ***deleteNode***
- Example: EraseElement(2)



List: Insert a New Element

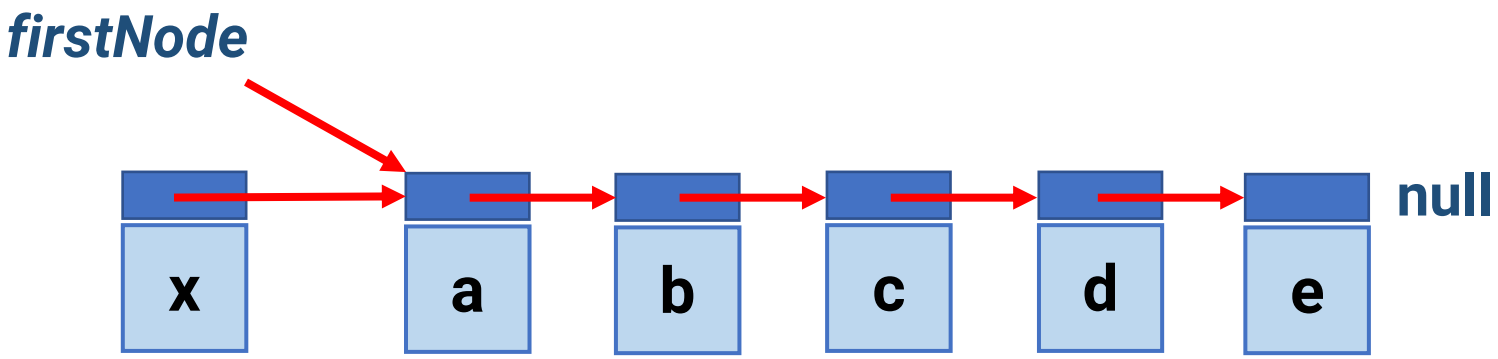
- Case 1: insert at front
- **Procedure**
  - Create a new node with the given data

- Example: Insert(0, x)



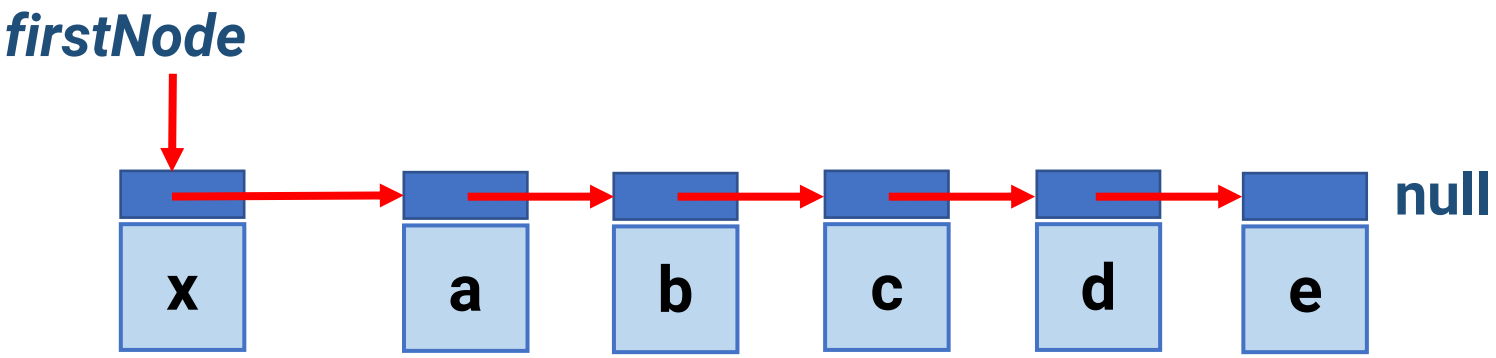
List: Insert a New Element (cont.)

- Case 1: insert at front
- **Procedure**
  - Create a new node with the given data
  - Set the pointer of the new node to the original first node
- Example: Insert(0, x)



List: Insert a New Element (cont.)

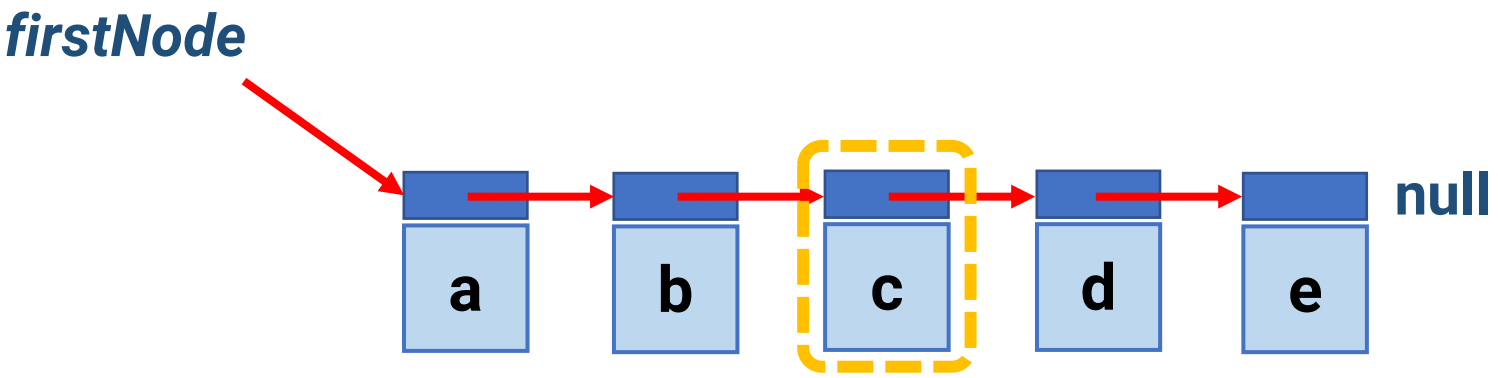
- Case 1: insert at front
- **Procedure**
  - Create a new node with the given data
  - Set the pointer of the new node to the original first node
  - Update the ***firstNode*** pointer
- Example: Insert(0, x)



List: Insert a New Element (cont.)

- Case 2: insert in the middle
- **Procedure**
  - Find the node before the target (***beforeNode***)

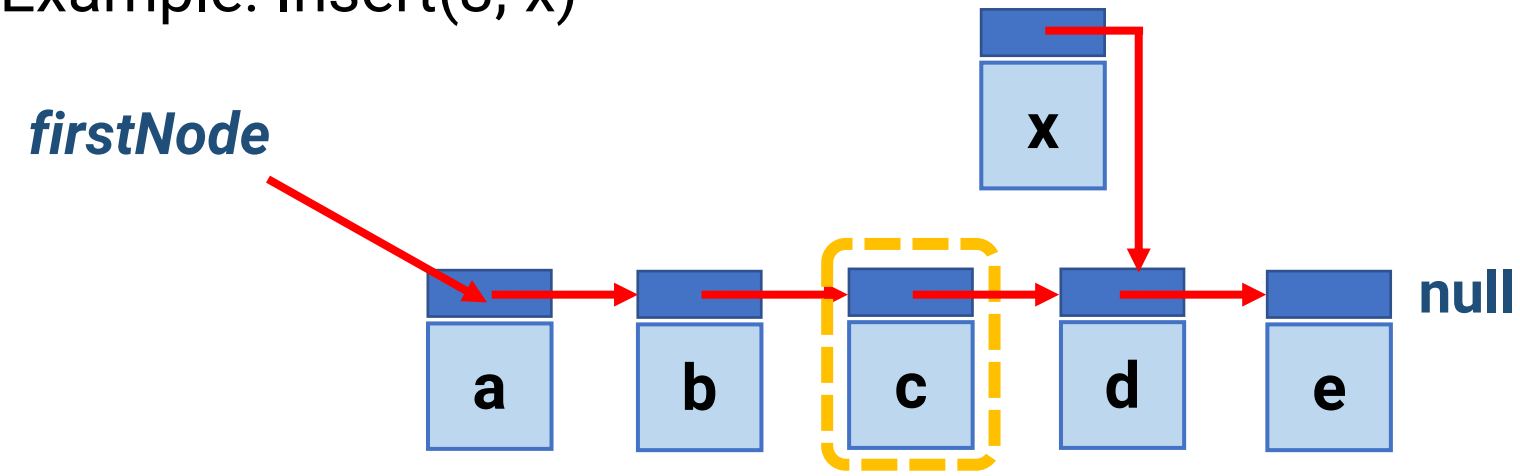
- Example: Insert(3, x)



List: Insert a New Element (cont.)

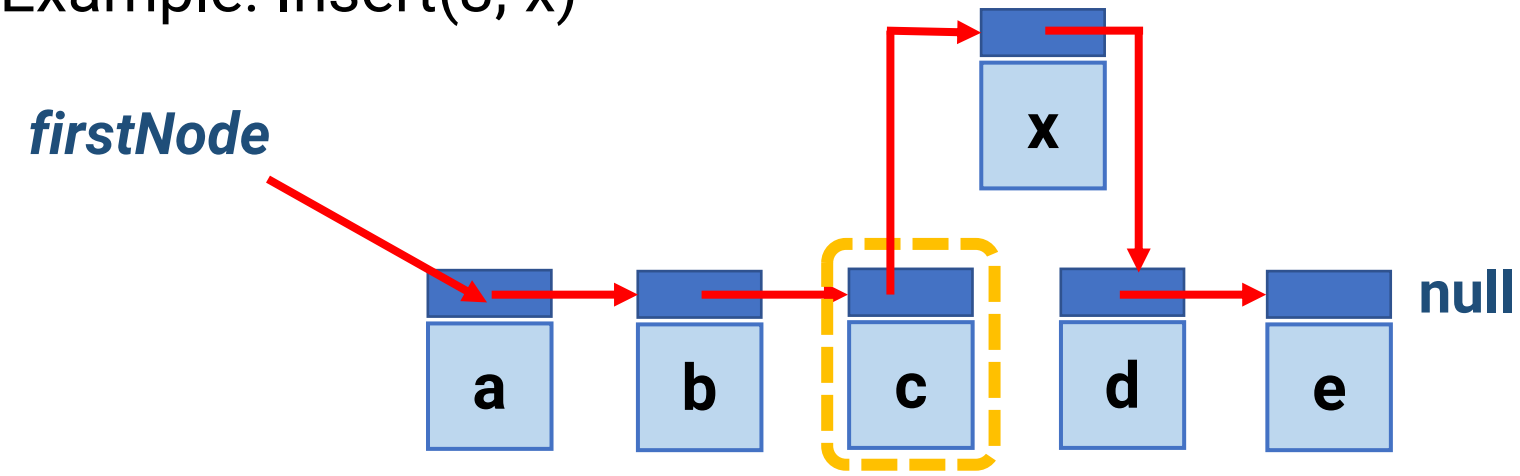
- Case 2: insert in the middle
- **Procedure**
  - Find t he node before the target (***beforeNode***)
  - Create a new node with the given data and set its pointer

- Example: Insert(3, x)



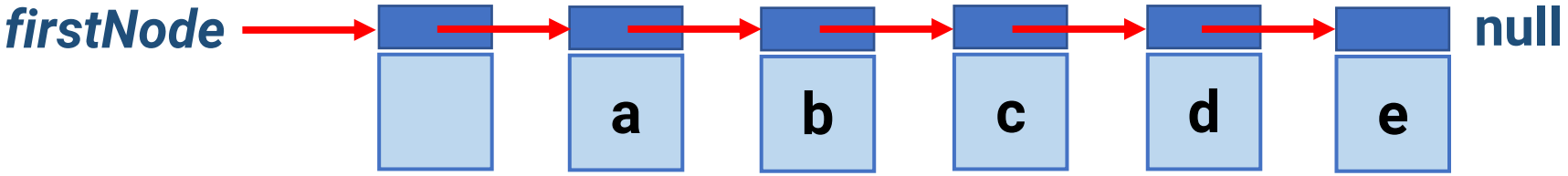
List: Insert a New Element (cont.)

- Case 2: insert in the middle
- **Procedure**
  - Find t he node before the target (***beforeNode***)
  - Create a new node with the given data and set its pointer
  - Change the pointer of ***beforeNode*** to the new node
- Example: Insert(3, x)

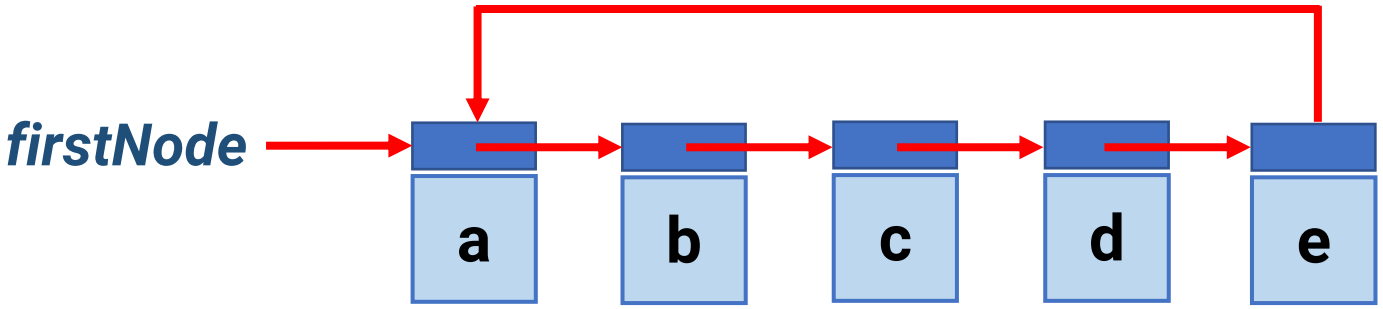


Variations

- List with a **dummy** header node

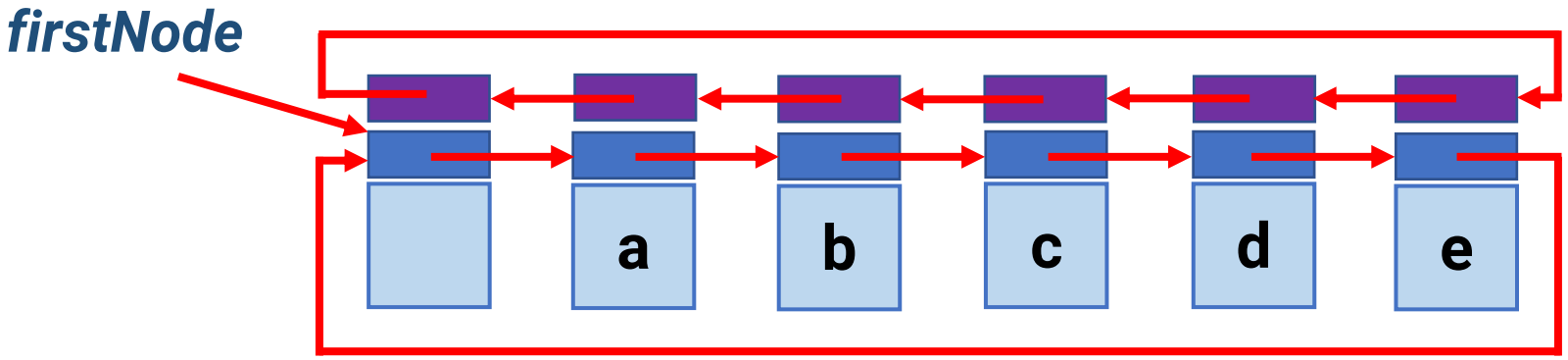


- Circular list



Variations (cont.)

- Doubly linked circular list with header
- Efficient for inserting at the end
- C++ Standard Template Library (STL) adopts this implementation (std::list)
  - https://en.cppreference.com/w/cpp/container/list



Outline

- Arrays
- Lists
- Stacks
- Queues
- Trees

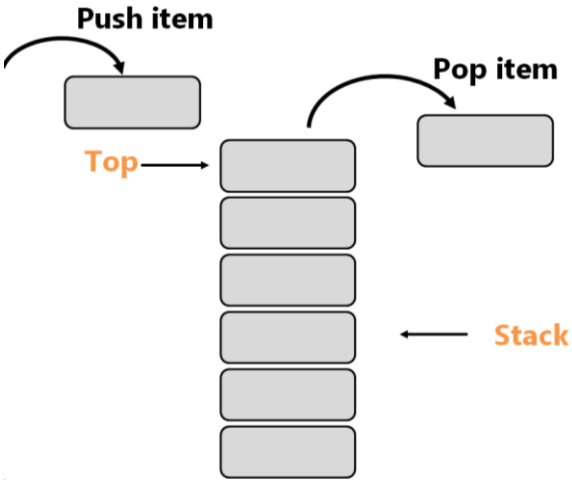
Stacks and Queues

- Special cases of linked list
  - **Stack**: record the stack point
  - **Queue**: record head and tail

- Both can be implemented either using contiguous memory (array) or linked list
  - Contiguous (array) implementation is more common

Stacks

- Stack: a list in which entries are removed and inserted only at the head
- **Last-in-first-out (LIFO)**
- Operations
  - Top: get the head of the list (stack)
  - **Pop**: to remove the entry at the top
  - **Push**: to insert an entry at the top



Storing Stacks

- Derive from array
  - Stack top is either left end or right end
  - When the top is left end

stack top

  - Push: Θ(n)
    - Need to move all elements right

  - Pop: Θ(n)
    - Need to move all elements left

Storing Stacks (cont.)

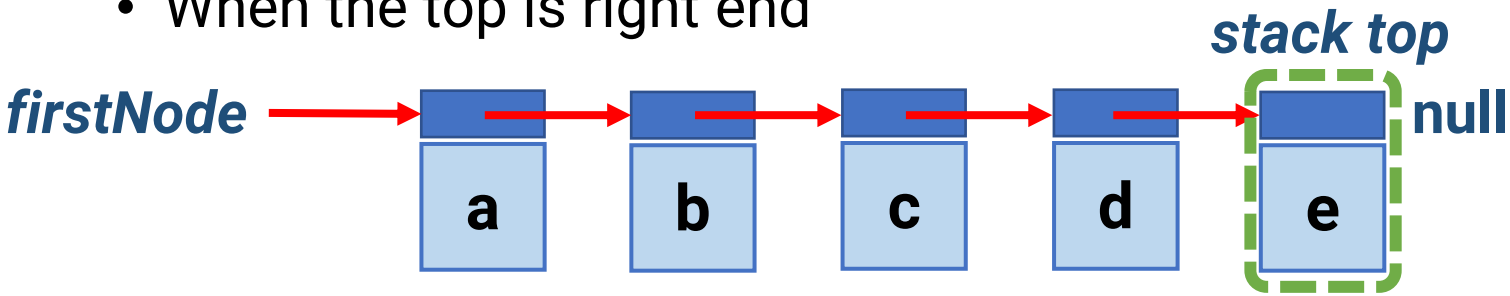
- Derive from array
  - Stack top is either left end or right end
  - When the top is right end

  - Push: Θ(1)
    - **No** need to move all elements right

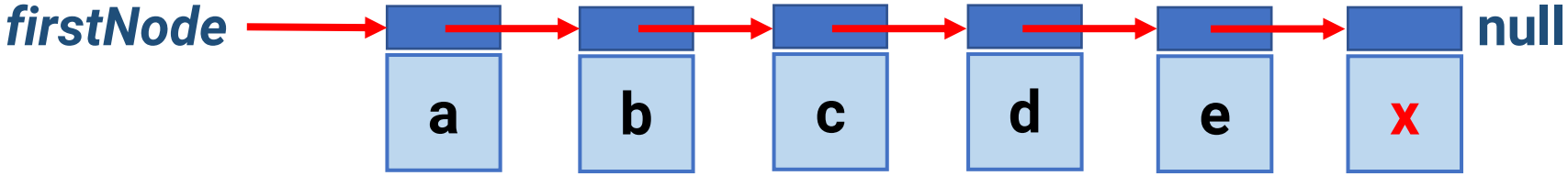
  - Pop: Θ(1)
    - **No** need to move all elements left

Storing Stacks (cont.)

- Derive from linked list
  - Stack top is either left end or right end
  - When the top is right end



  - Push: Θ(n)
    - Need to traverse all nodes



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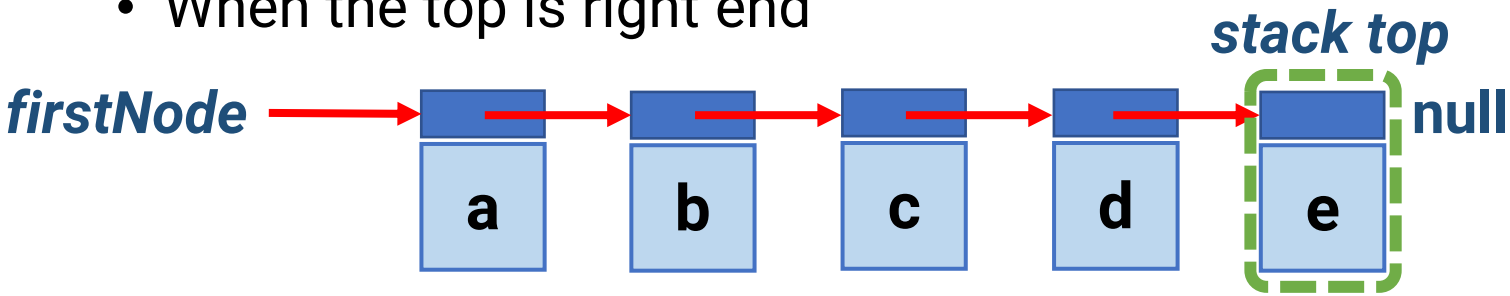
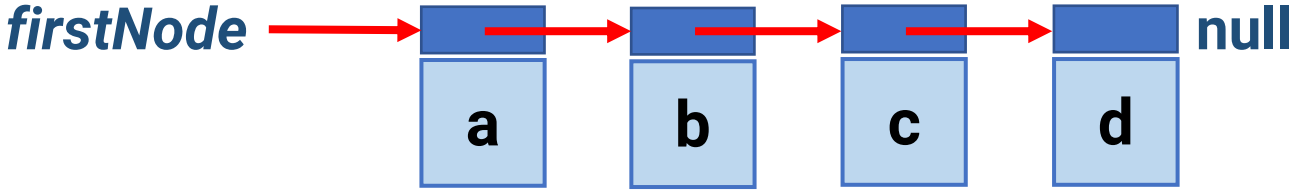
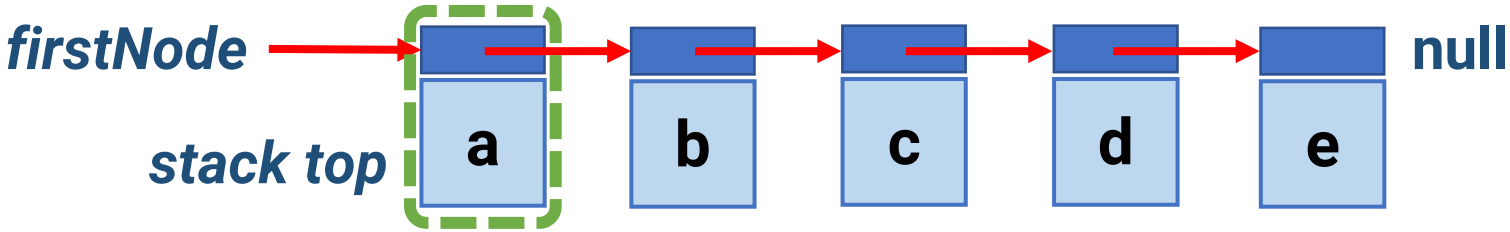


  - Pop: Θ(n)
    - Need to traverse all nodes



Storing Stacks (cont.)

- Derive from linked list
  - Stack top is either left end or right end
  - When the top is left end



  - Push: Θ(1)
    - Insert at front
  - Pop: Θ(1)
    - EraseElement at front

Parentheses Matching

(	(	(	a	+	b	)	*	c	+	d	-	e	)	/	(	f	+	g	)	-	(	h	+	j	)	)
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

- Output pairs ***(u, v)*** such that the left parenthesis at position ***u*** is matched with the right parenthesis at ***v***
  - (2, 6), (1, 13), (15, 19), (21, 25), (0, 26)

- Also report missing pair parentheses

(	a	+	b	)	)	*	(	(	c	+	d	)
0	1	2	3	4	5	6	7	8	9	10	11	12

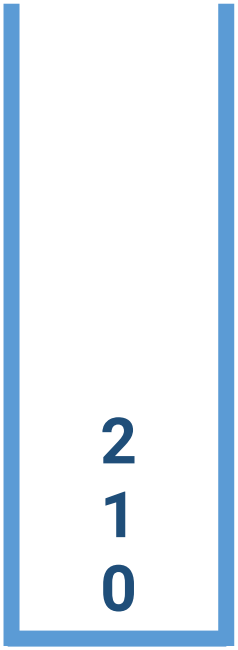
  - (0, 4), (missing, 5), (8, 12), (7, missing)

Parentheses Matching (cont.)

- Scan expression from left to right
- When a left parenthesis is encountered, **push** its position to the stack
- When a right parenthesis is encountered, **pop** matching position from the stack

Parentheses Matching (cont.)

(	(	(	a	+	b	)	*	c	+	d	-	e	)	/	(	f	+	g	)	-	(	h	+	j	)	)
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

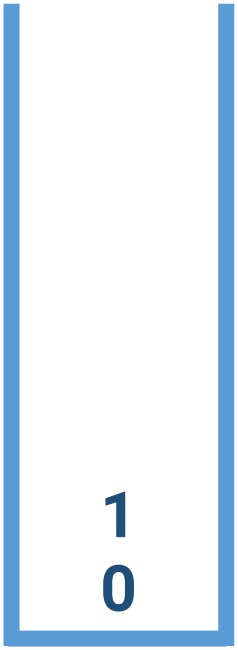


Actions

Push 0
Push 1
Push 2

Parentheses Matching (cont.)

(	(	(	a	+	b	)	*	c	+	d	-	e	)	/	(	f	+	g	)	-	(	h	+	j	)	)
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

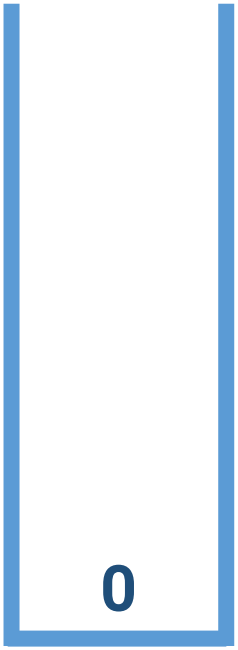


Actions

Push 0
Push 1
Push 2
Pop, output (2, 6)

Parentheses Matching (cont.)

(	(	(	a	+	b	)	*	c	+	d	-	e	)	/	(	f	+	g	)	-	(	h	+	j	)	)
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

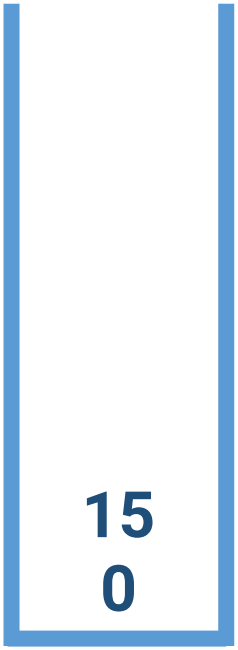


Actions

Push 0
Push 1
Push 2
Pop, output (2, 6)
Pop, output (1, 13)

Parentheses Matching (cont.)

(	(	(	a	+	b	)	*	c	+	d	-	e	)	/	(	f	+	g	)	-	(	h	+	j	)	)
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

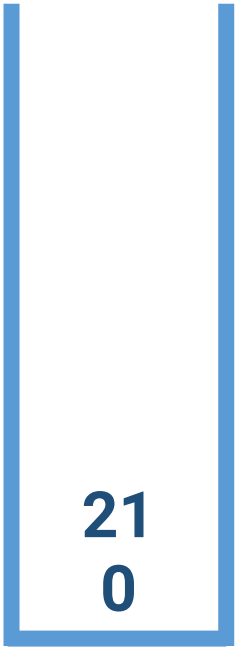


Actions

Push 0
Push 1
Push 2
Pop, output (2, 6)
Pop, output (1, 13)
Push 15

Parentheses Matching (cont.)

(	(	(	a	+	b	)	*	c	+	d	-	e	)	/	(	f	+	g	)	-	(	h	+	j	)	)
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

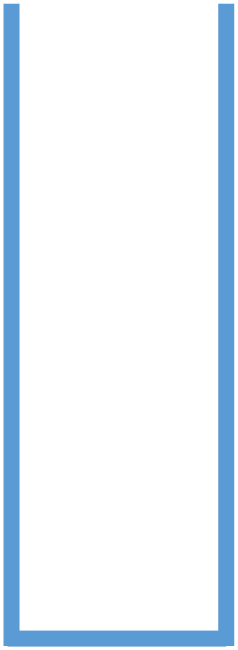


Actions

Push 0
Push 1
Push 2
Pop, output (2, 6)
Pop, output (1, 13)
Push 15
Pop, output (15, 19)
Push 21

Parentheses Matching (cont.)

(	(	(	a	+	b	)	*	c	+	d	-	e	)	/	(	f	+	g	)	-	(	h	+	j	)	)
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26



Actions

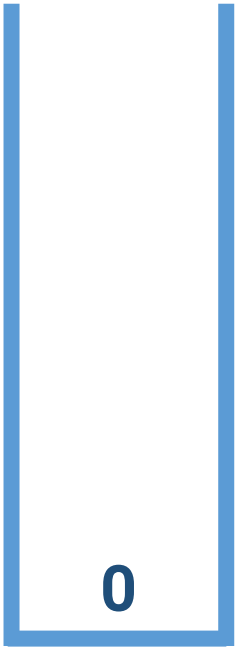
Push 0
Push 1
Push 2
Pop, output (2, 6)
Pop, output (1, 13)
Push 15
Pop, output (15, 19)
Push 21
Pop, output (21, 25)
Pop, output (0, 26)

Parentheses Matching (cont.)

(	a	+	b	)	)	*	(	(	c	+	d	)
0	1	2	3	4	5	6	7	8	9	10	11	12

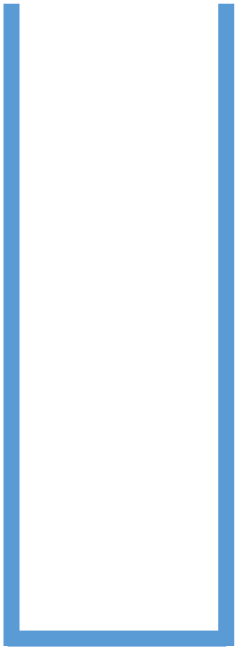
Actions

Push 0



Parentheses Matching (cont.)

(	a	+	b	)	)	*	(	(	c	+	d	)
0	1	2	3	4	5	6	7	8	9	10	11	12

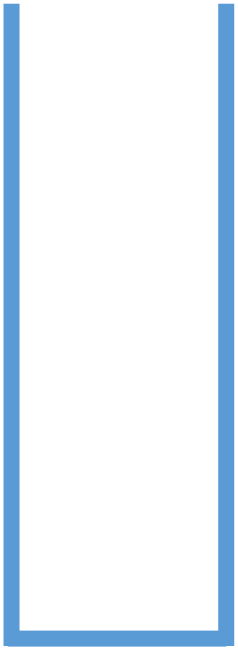


Actions

Push 0
Pop, output (0, 4)

Parentheses Matching (cont.)

(	a	+	b	)	)	*	(	(	c	+	d	)
0	1	2	3	4	5	6	7	8	9	10	11	12

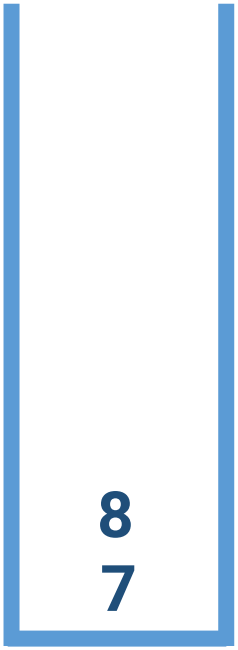


Actions

Push 0
Pop, output (0, 4)
Pop, error for stack is empty!

Parentheses Matching (cont.)

(	a	+	b	)	)	*	(	(	c	+	d	)
0	1	2	3	4	5	6	7	8	9	10	11	12

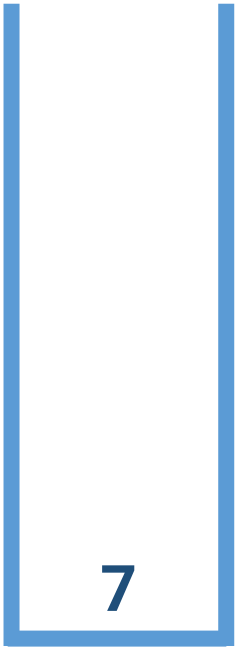


Actions

Push 0
Pop, output (0, 4)
Pop, error for stack is empty!
Push 7
Push 8

Parentheses Matching (cont.)

(	a	+	b	)	)	*	(	(	c	+	d	)
0	1	2	3	4	5	6	7	8	9	10	11	12



Actions

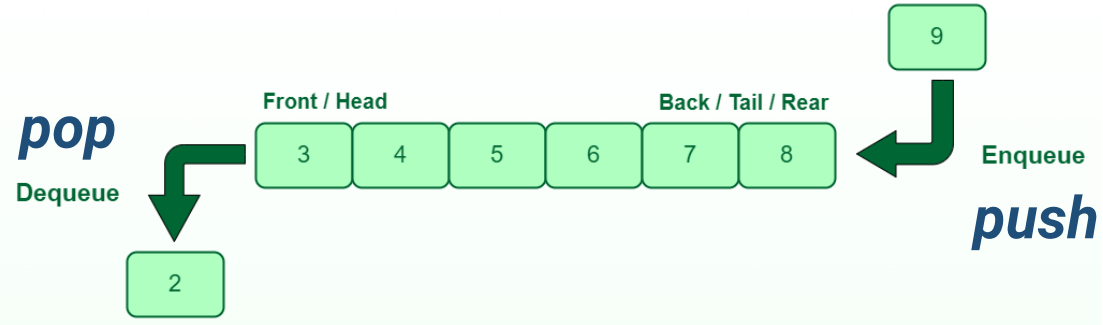
Push 0
Pop, output (0, 4)
Pop, error for stack is empty!
Push 7
Push 8
Pop, output (8, 12)
error for the left parenthesis at 7 is not matched any right parenthesis

Outline

- Arrays
- Lists
- Stacks
- Queues
- Trees

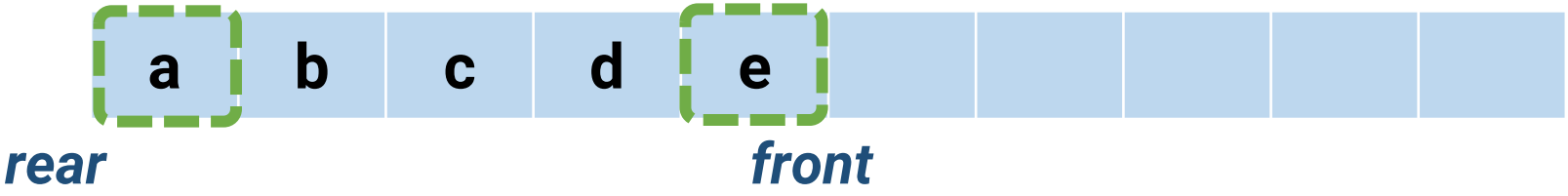
Queues

- Queue: a list in which entries are removed at the head and are inserted at the tail
- First-in-first-out (FIFO)
- Operations
  - Front: get the value of the front element
  - Back: get the value of the back element
  - Pop: remove the front element
  - Push: add an element at the back of the queue

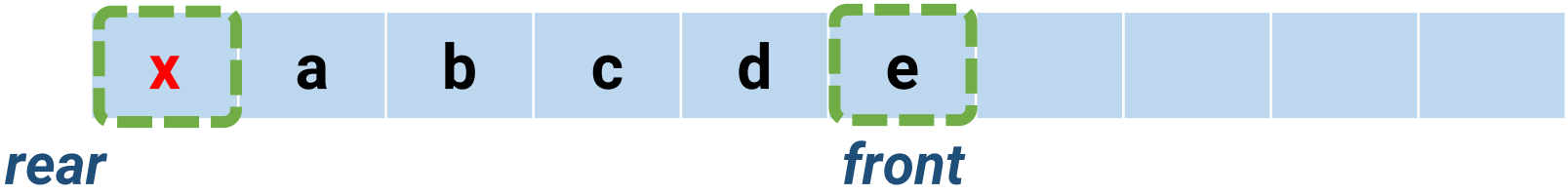


Storing Queues

- Derive from array
  - Two choices for the front and rear
  - When the front is right end and rear is left end

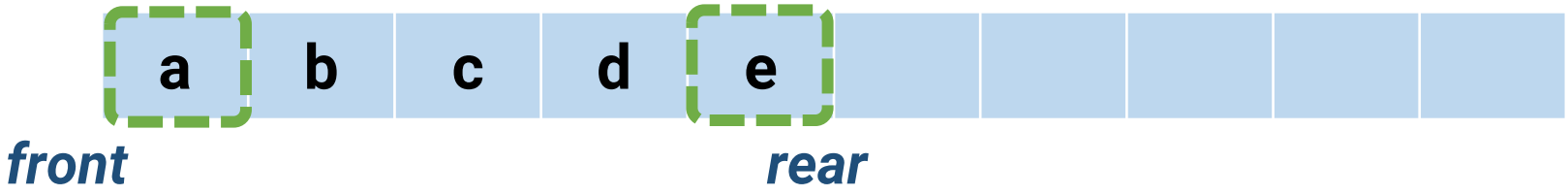


  - Front: Θ(1)
  - Back: Θ(1)
  - Pop: Θ(1)
  - Push: Θ(n)



Storing Queues (cont.)

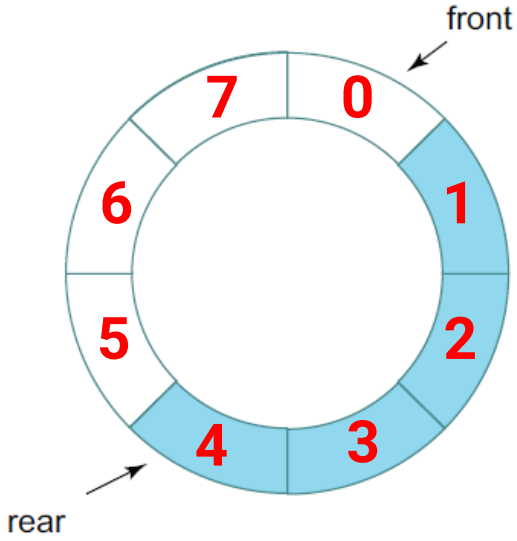
- Derive from linked list
  - Two choices for the front and rear
  - When the front is right end and rear is left end

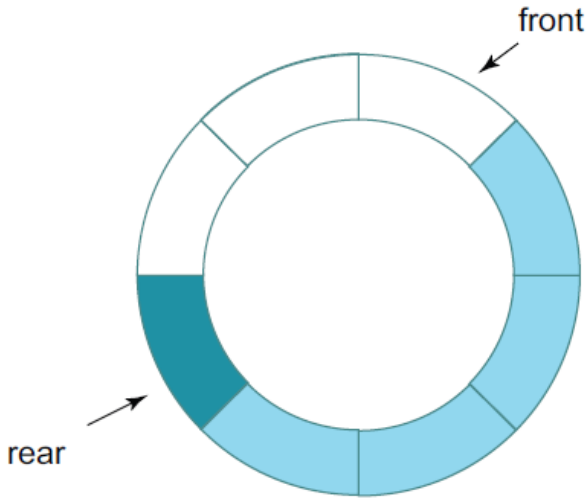


  - Front: Θ(1)
  - Back: Θ(1)
  - Pop: Θ(n)
  - Push: Θ(1)

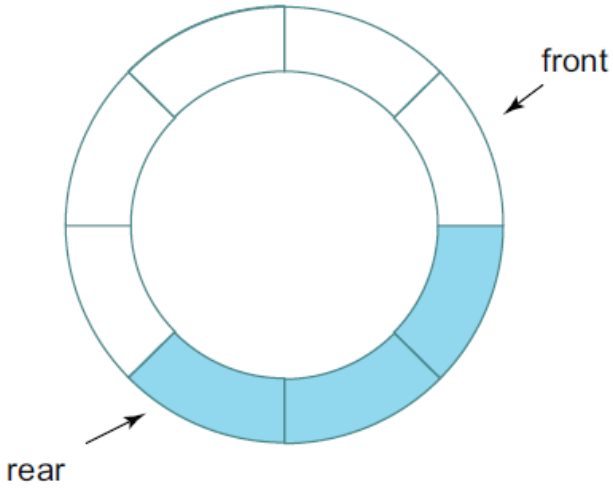
Storing Queues (cont.)

- Derive from linked list
  - We can do better (Θ(1) for both pushing and pop) by using a customized array representation
  - Circular + **mod** operation (w.r.t the array length)





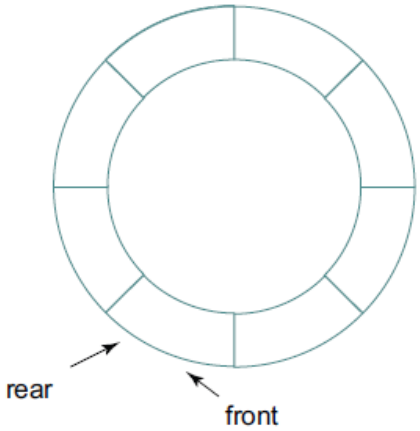
push
rear = (rear+1)%length

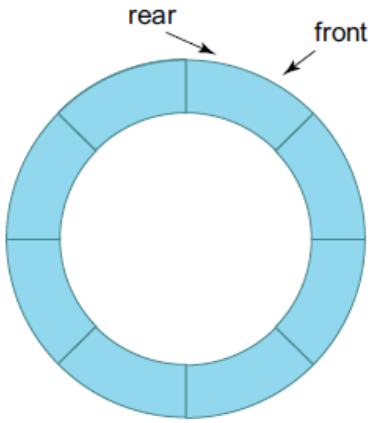


pop
front = (front+1)%length

Storing Queues (cont.)

- Derive from linked list
  - Handle **empty** and **full** queue (both **front == rear**)
  - Use a **size** variable
    - When pushing, ++size
    - When popping, --size
    - Queue is empty iff (size == 0)
    - Queue is full iff (size == length)





Outline

- Arrays
- Lists
- Stacks
- Queues
- Trees

Tree

- Lists are useful for serially ordered data
- Trees are useful for hierarchically ordered data

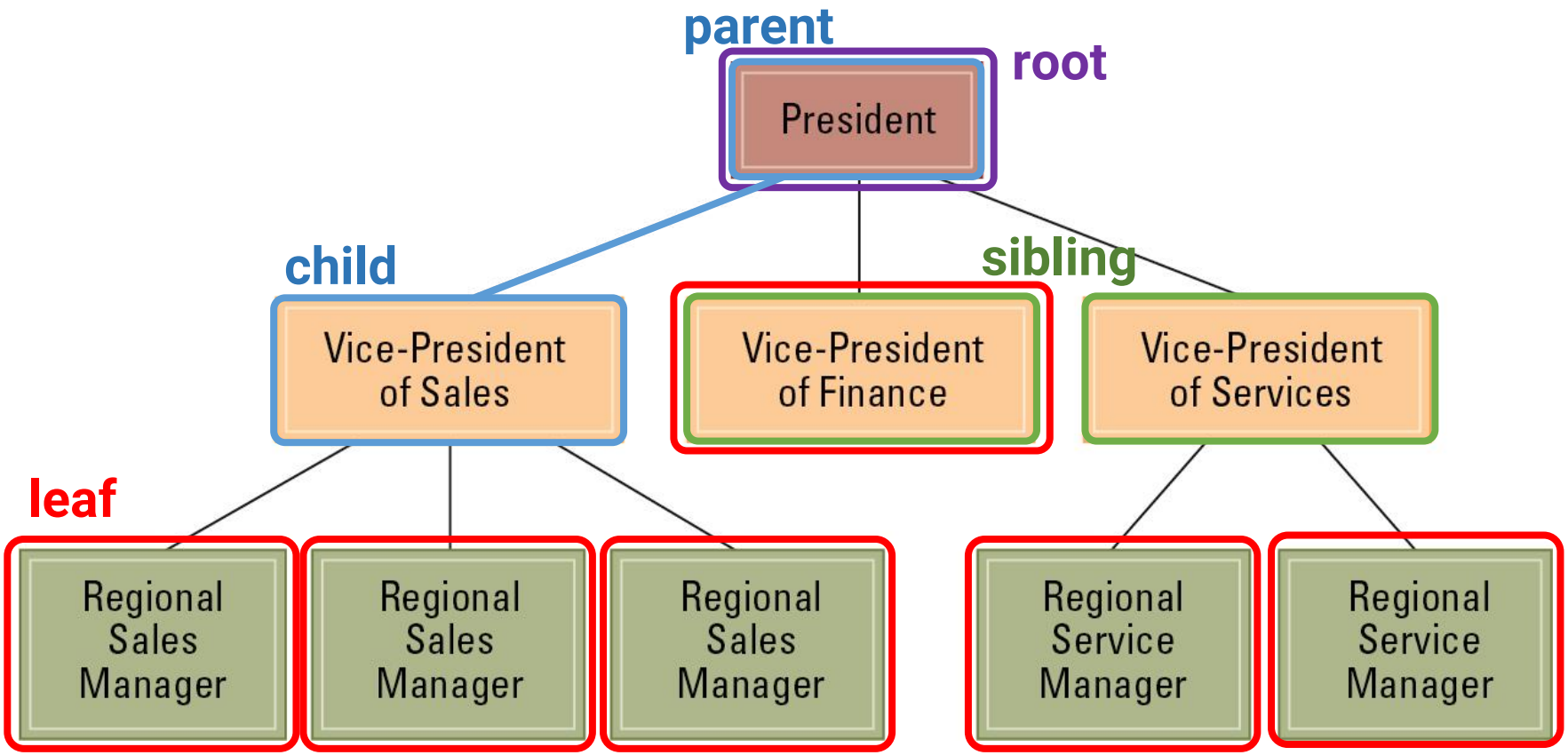
Terminology for a Tree

- **Node:** an entry in a tree

- **Parent:** the node immediately above a specified node
- **Child:** a node immediately below a specified node
- **Ancestor:** parent, parent of the parent, etc.
- **Descendent:** child, child of a child, etc.
- **Siblings:** nodes sharing a common parent

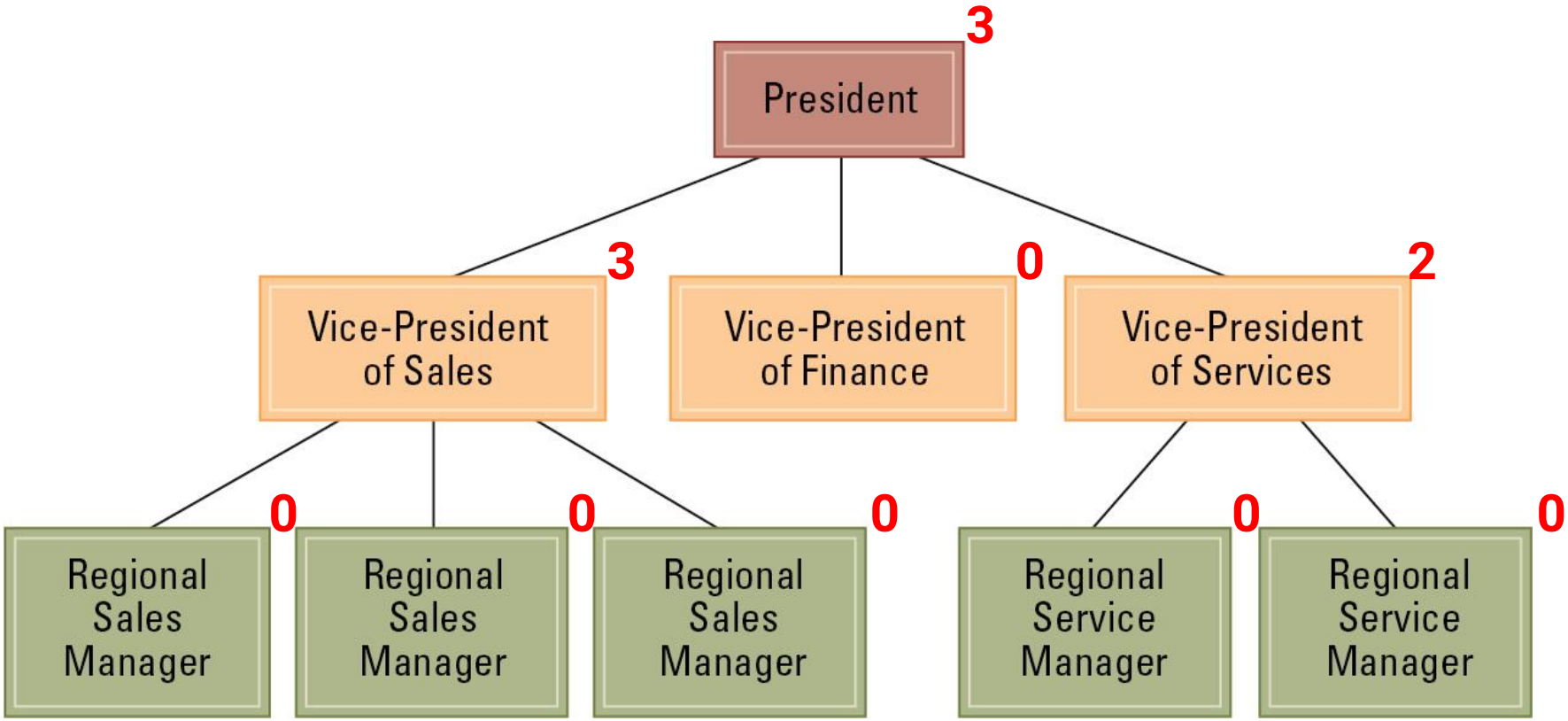
- **Root node:** the node at the top
- **Leaf node:** the node at the bottom (thus has no children)

Terminology for a Tree (cont.)



Terminology for a Tree (cont.)

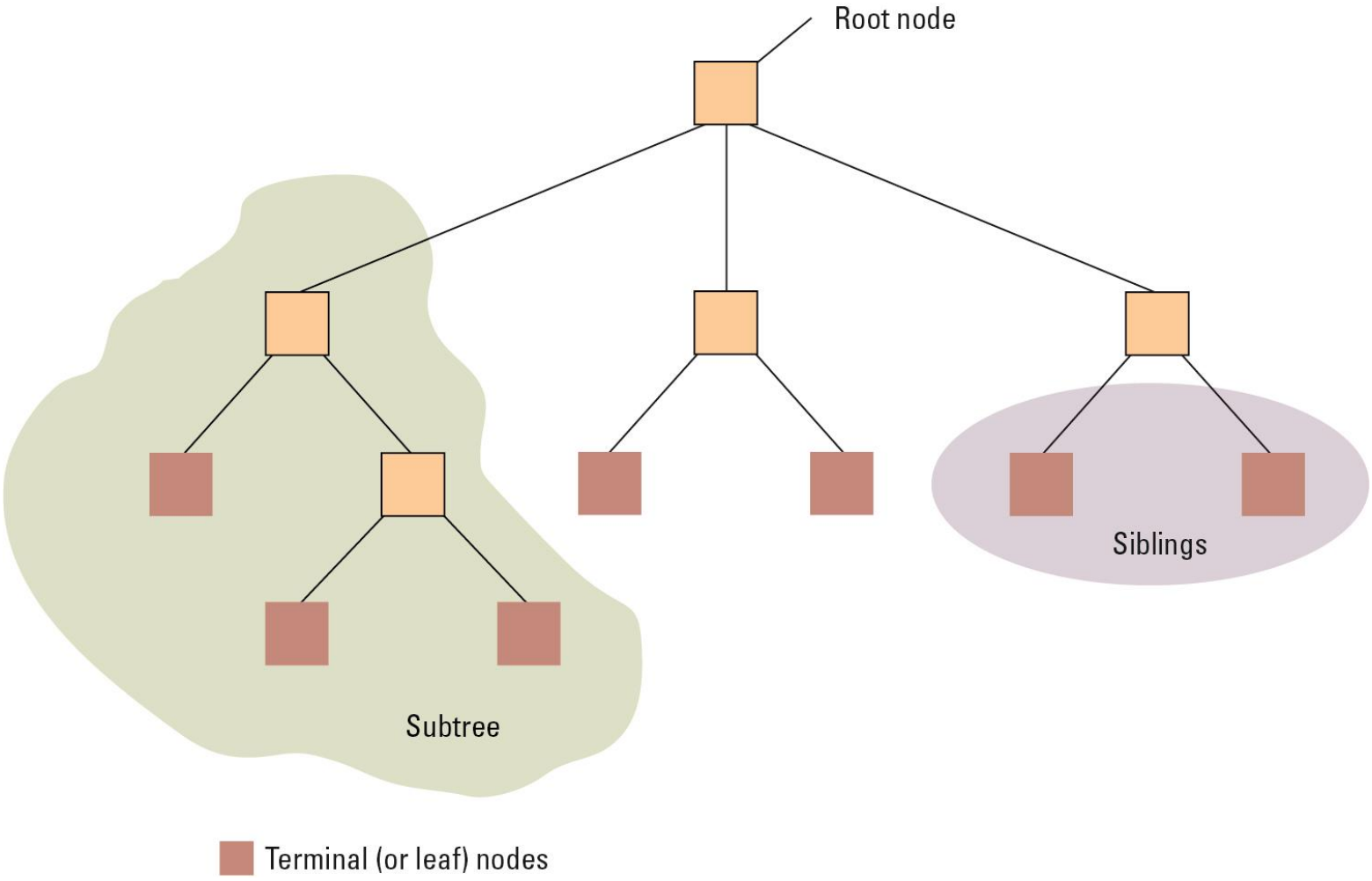
- **Degree:** number of children



Definition of Tree

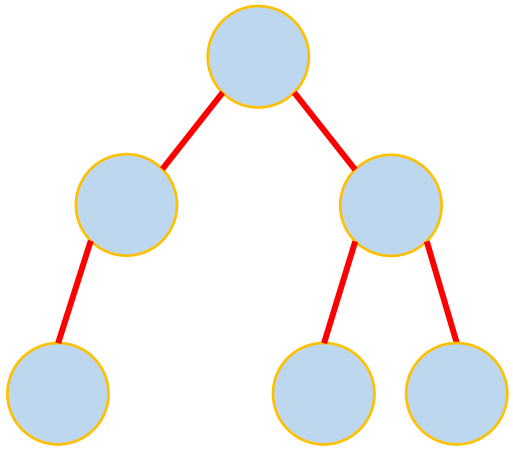
- **Recursive** definition
- A tree ***t*** is a finite non-empty set of elements
- One of these elements is called the **root**
- The remaining elements, if any, are partitioned into trees, which are called the subtrees of ***t***

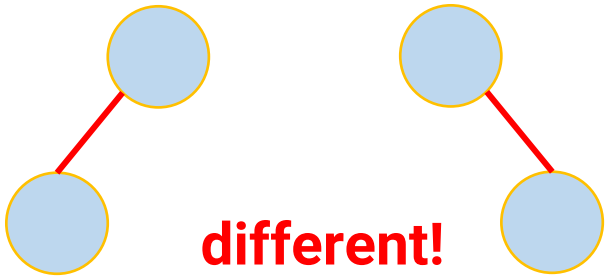
Definition of Tree (cont.)



Binary Trees

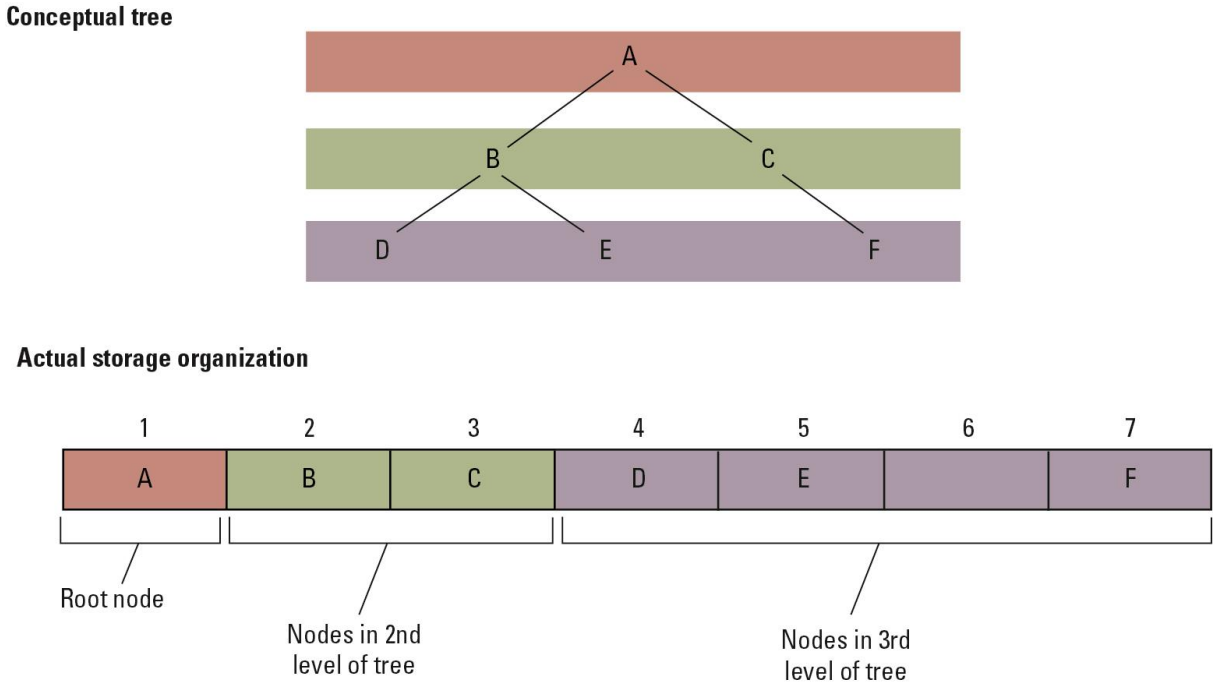
- Finite non-empty collection of elements
- A binary tree has a root element
- The remaining elements (if any) are partitioned into at most **two** binary trees
  - Called the **left** and **right** subtrees





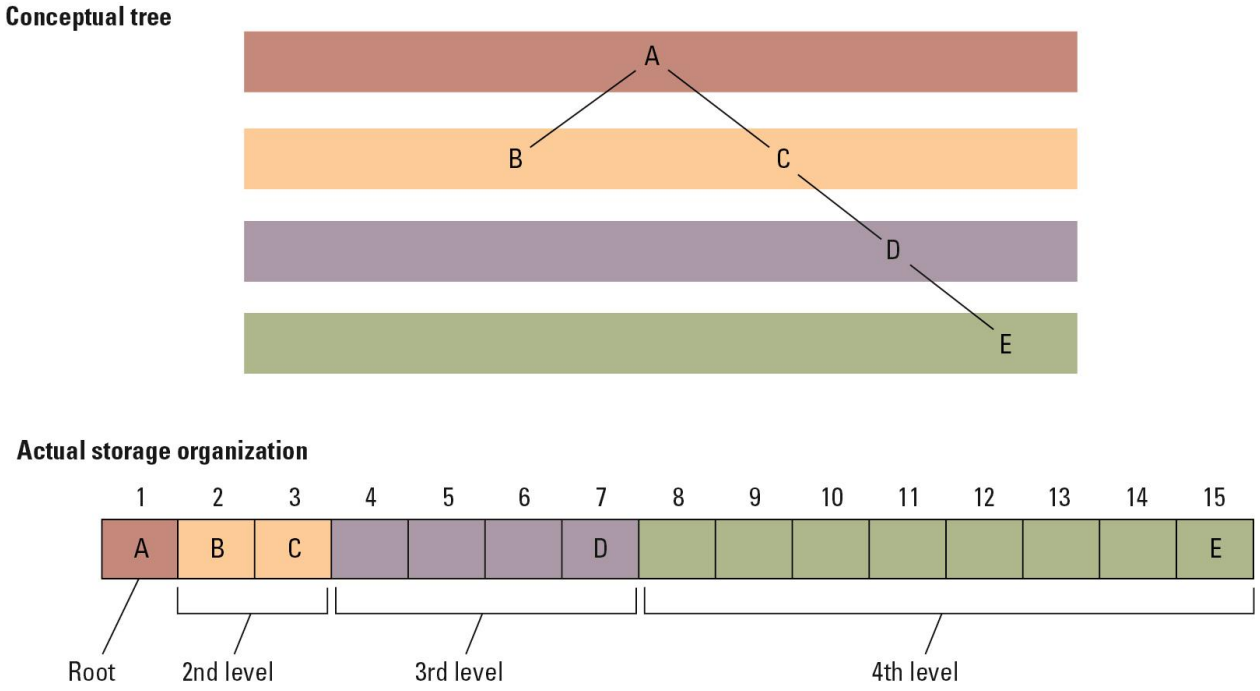
Storing Binary Trees

- **Contiguous array structure**
  - Root node: A[1]
  - Children of A[1]: A[2] and A[3]
  - Children of A[2] and A[3]: A[4], A[5], A[6], and A[7]



Storing Binary Trees

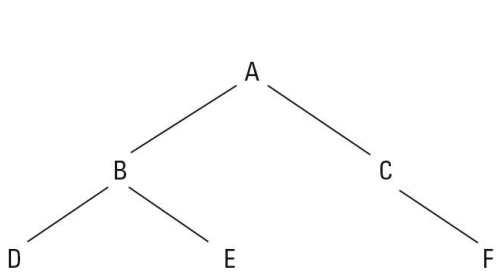
- **Contiguous array structure**
  - Root node: A[1]
  - Children of A[1]: A[2] and A[3]
  - Children of A[2] and A[3]: A[4], A[5], A[6], and A[7]

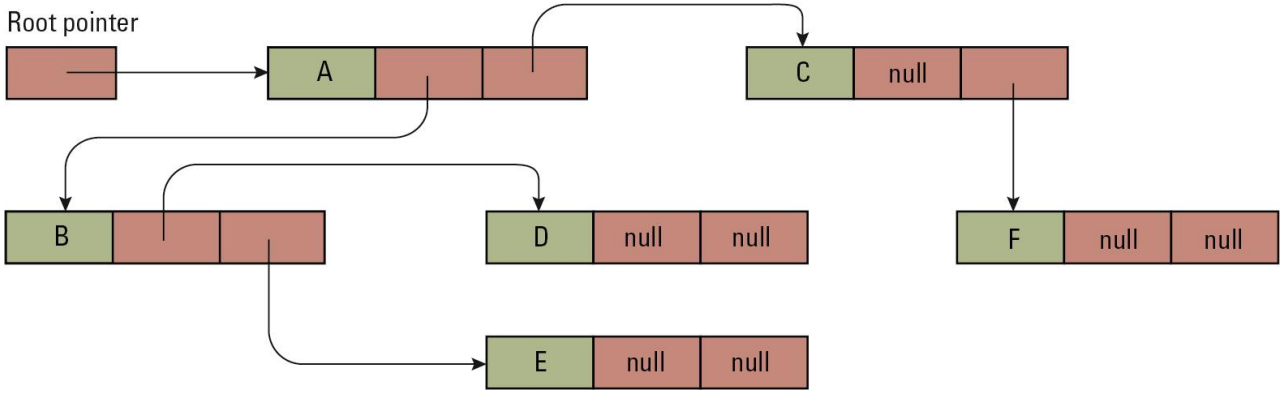


Storing Binary Trees

- **Linked structure**
  - Each node = data cells + two child pointers

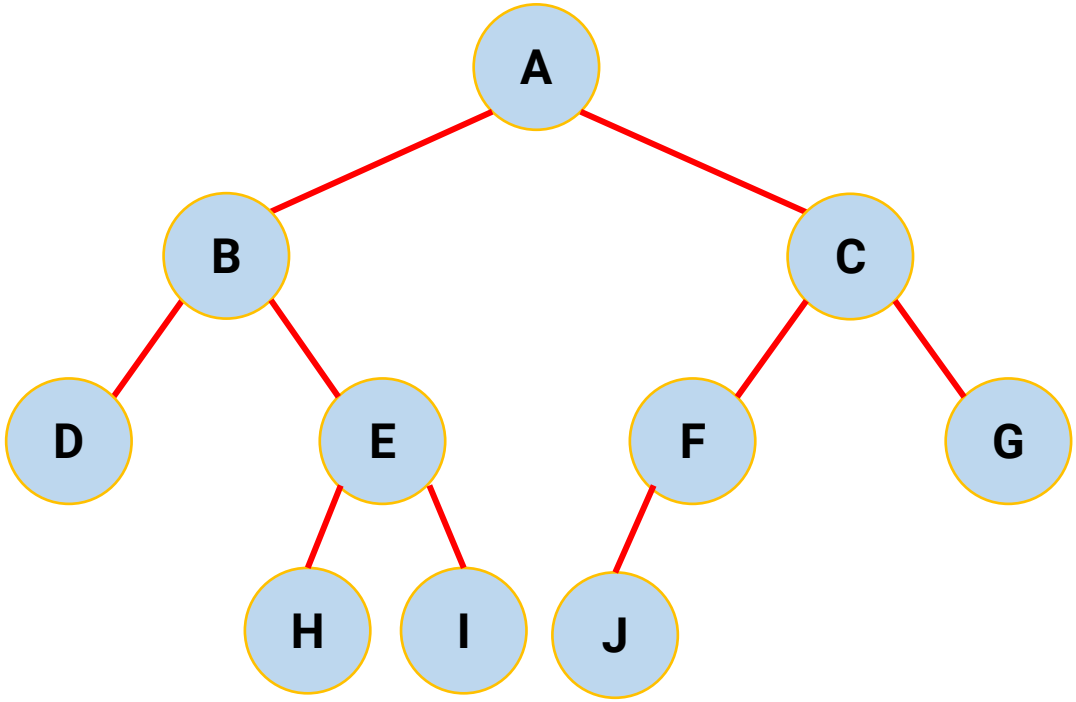
  - Accessed via a pointer to the root node





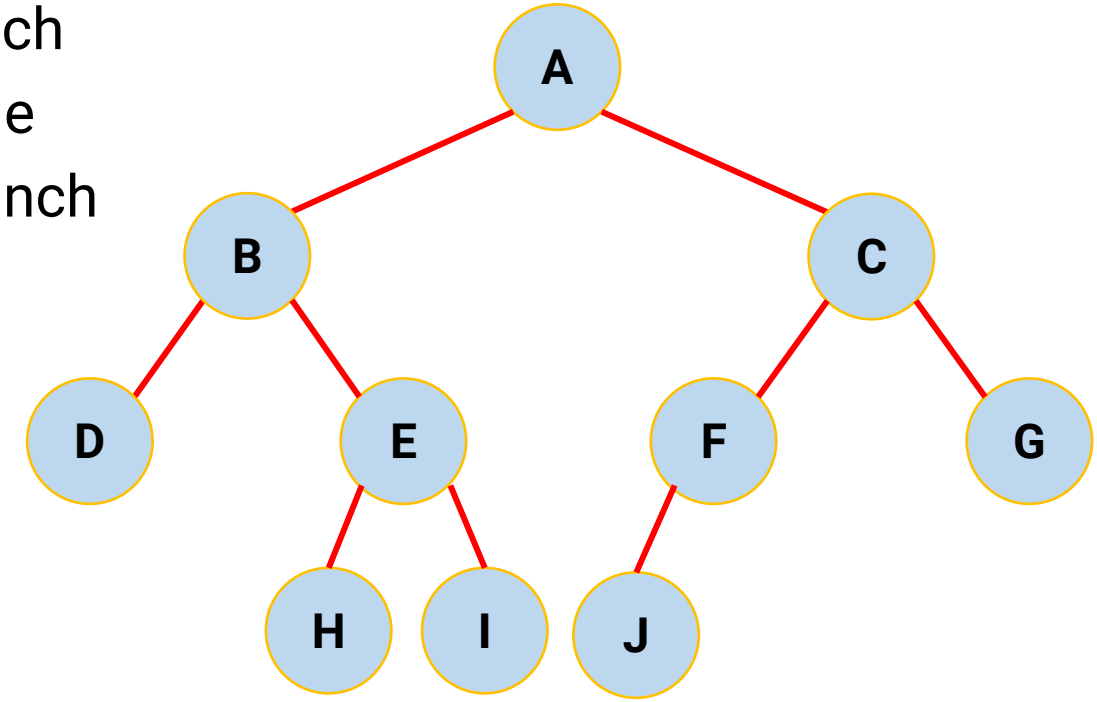
Traverse Binary Tree

- In-order
- Pre-order
- Post-order



Traverse Binary Tree

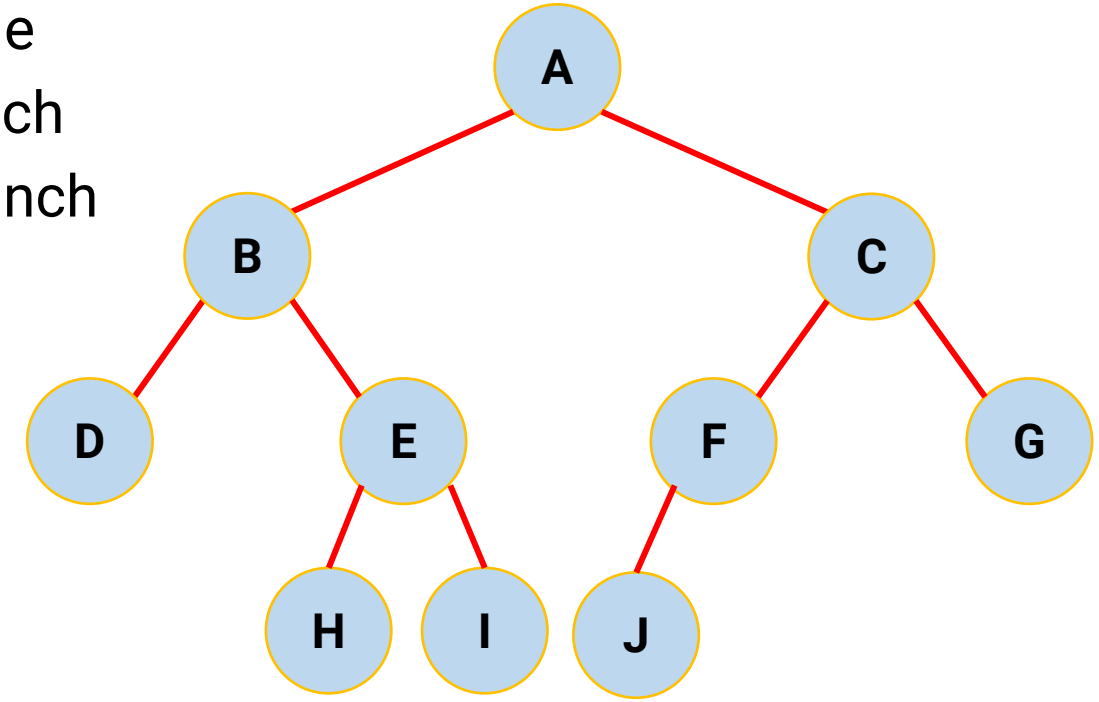
- In-order
  - Visit the left branch
  - Visit the root node
  - Visit the right branch



D ➔ B ➔ H ➔ E ➔ I ➔ A ➔ J ➔ F ➔ C ➔ G

Traverse Binary Tree (cont.)

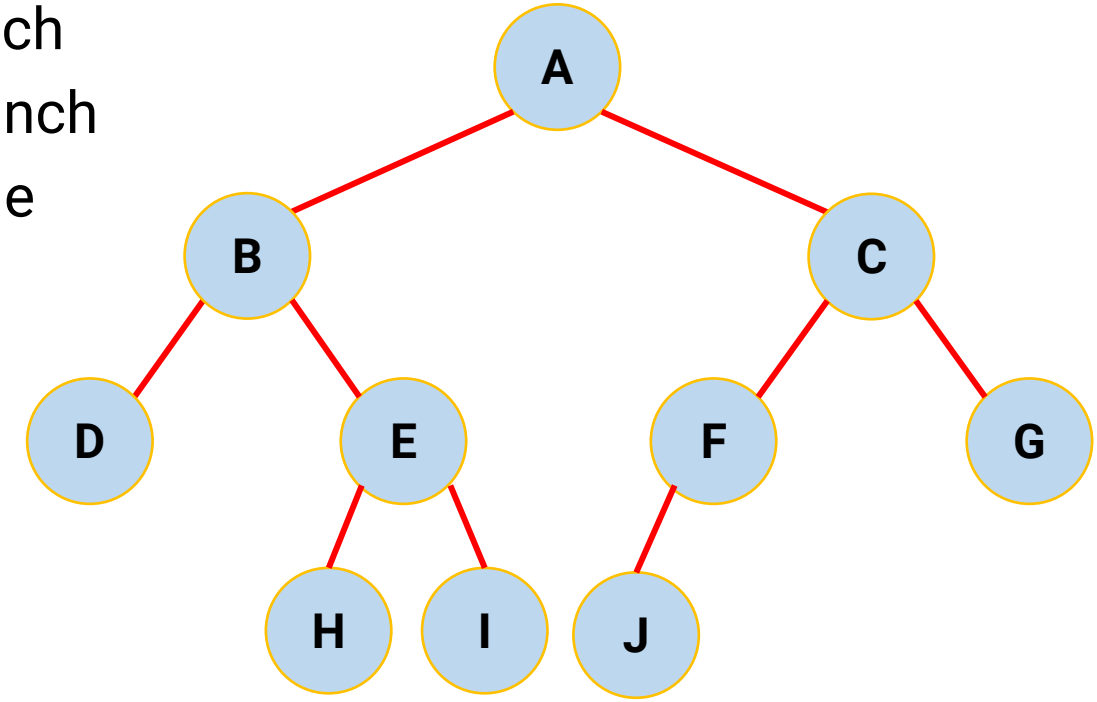
- Pre-order
  - Visit the root node
  - Visit the left branch
  - Visit the right branch



A → B → D → E → H → I → C → F → J → G

Traverse Binary Tree (cont.)

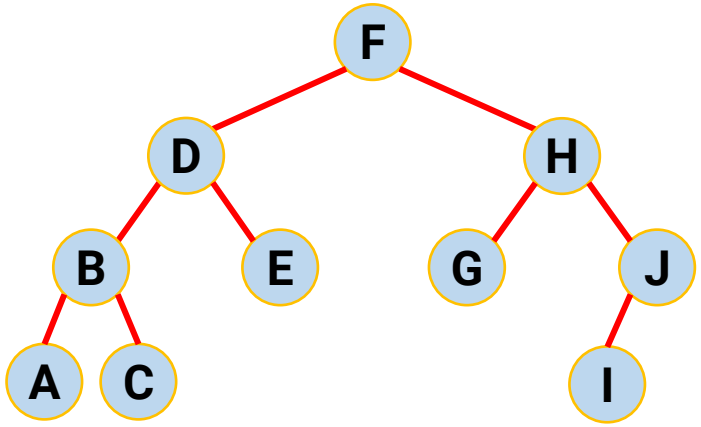
- Post-order
  - Visit the left branch
  - Visit the right branch
  - Visit the root node



D → H → I → E → B → J → F → G → C → A

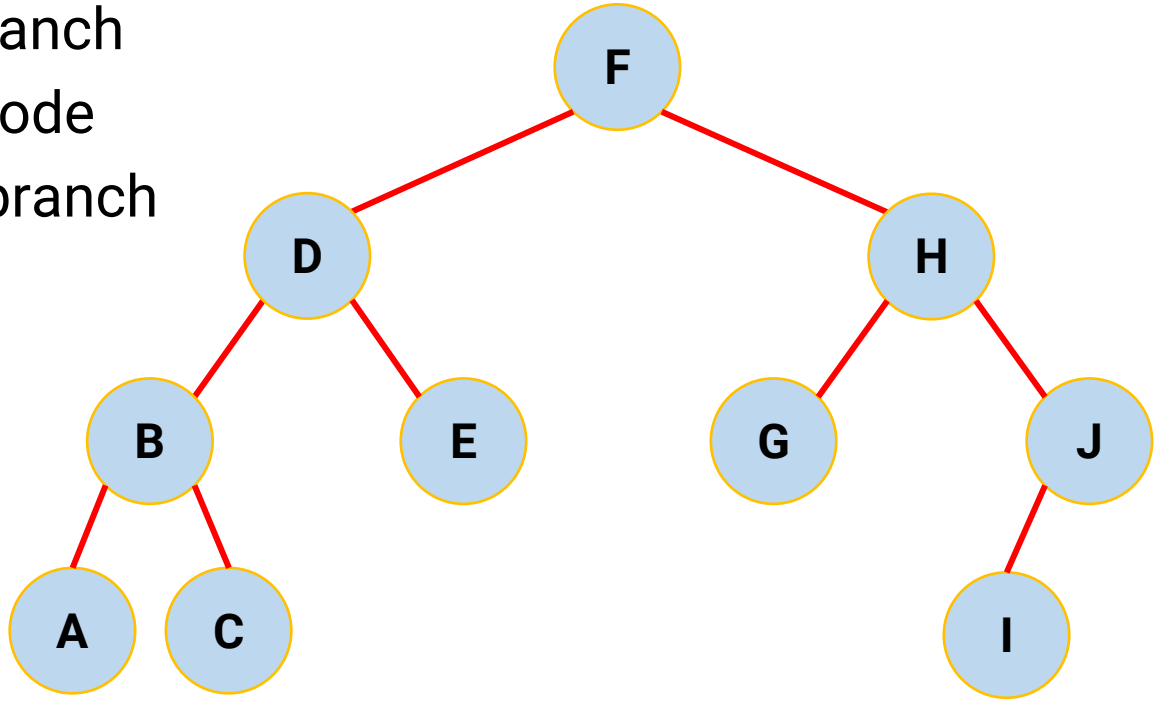
Binary Search Tree (BST)

- A binary tree
- Each node has a **(key, value)** pair
- For every node ***x***, all keys in the left subtree of ***x*** are smaller than that in ***x***
- For every node ***x***, all keys in the right subtree of ***x*** are greater than that in ***x***
- Operations
  - Traversal
  - Search
  - Insertion
  - Deletion



Traverse Binary Search Tree

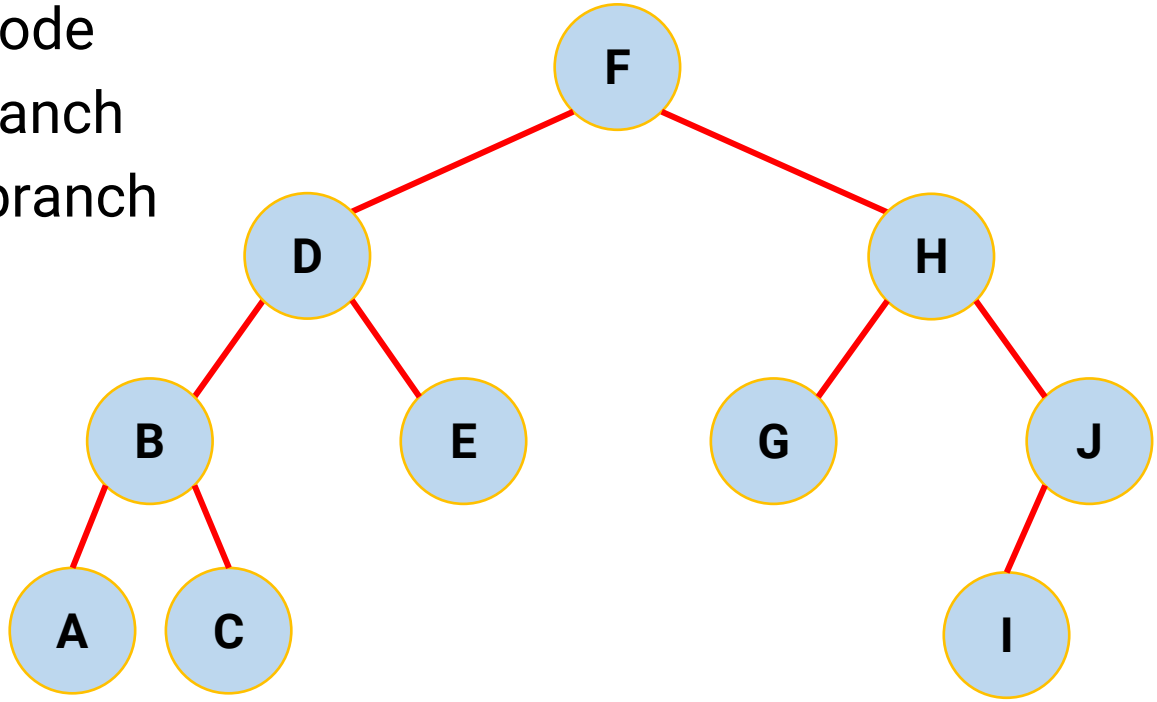
- In-order
  - Visit the left branch
  - Visit the root node
  - Visit the right branch



A ➔ B ➔ C ➔ D ➔ E ➔ F ➔ G ➔ H ➔ I ➔ J

Traverse Binary Search Tree (cont.)

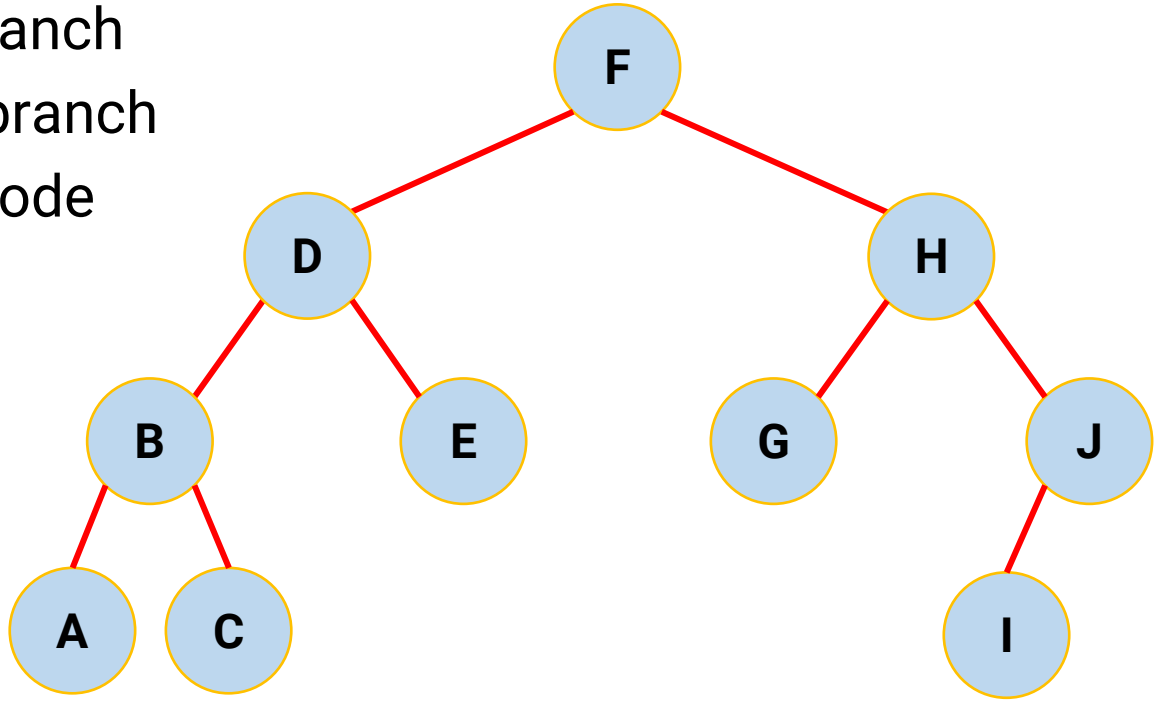
- Pre-order
  - Visit the root node
  - Visit the left branch
  - Visit the right branch



F ➔ D ➔ B ➔ A ➔ C ➔ E ➔ H ➔ G ➔ J ➔ I

Traverse Binary Search Tree (cont.)

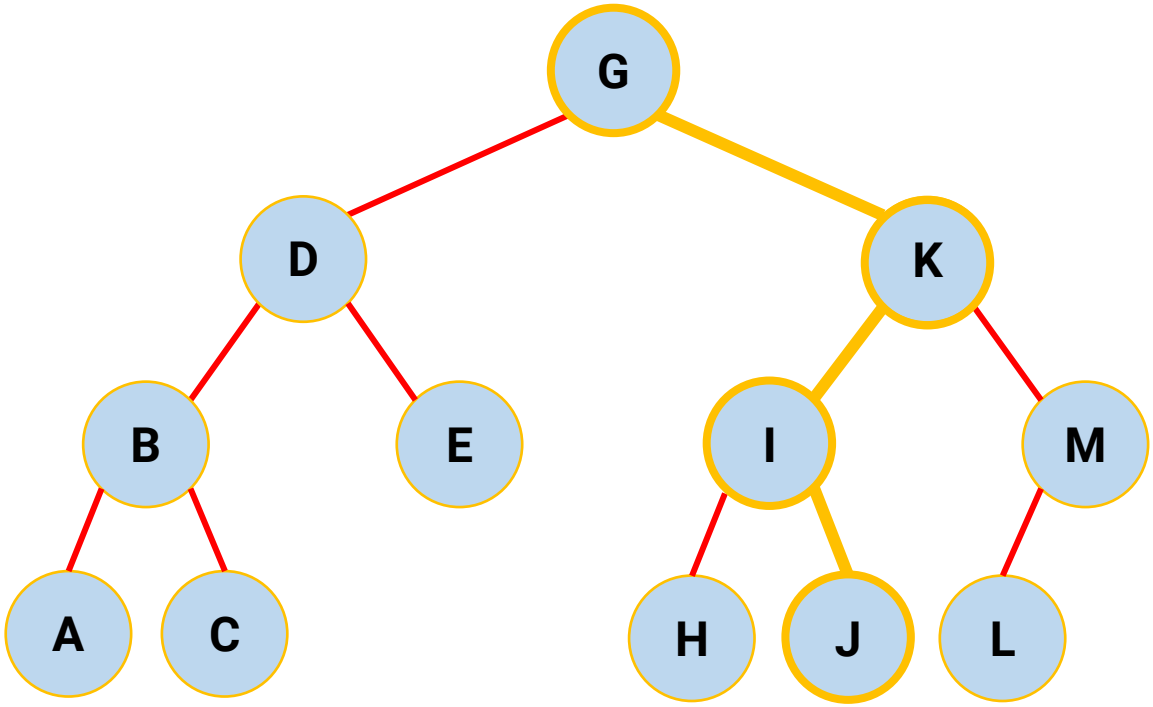
- Post-order
  - Visit the left branch
  - Visit the right branch
  - Visit the root node



A ➔ C ➔ B ➔ E ➔ D ➔ G ➔ I ➔ J ➔ H ➔ F

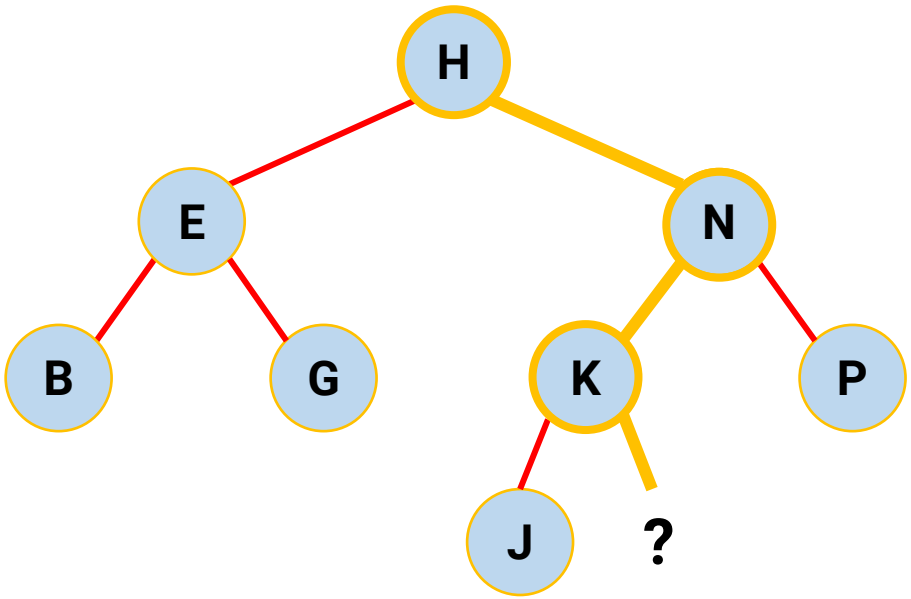
Search Binary Search Tree

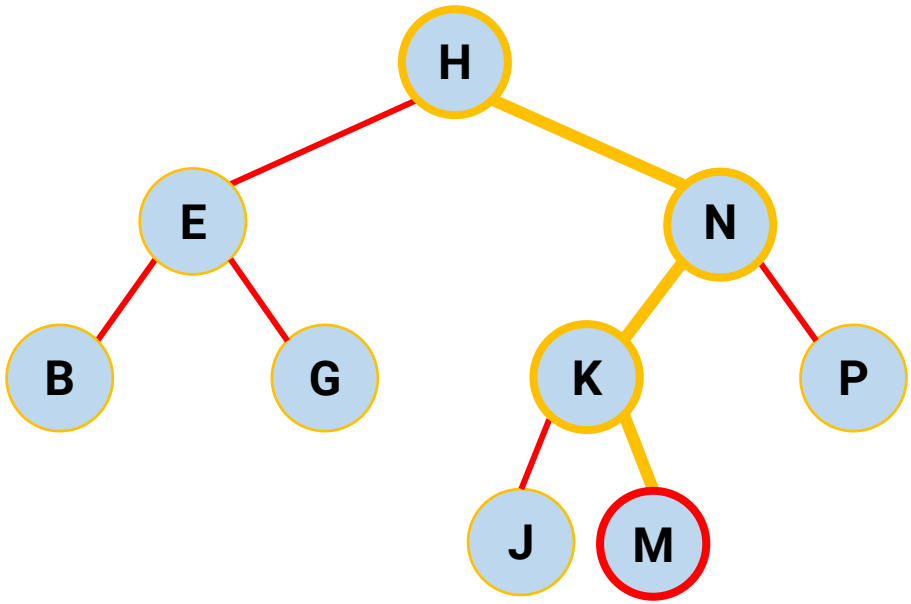
- Similar to binary search (but may not be half-half)
- Example: find J



Insertion in Binary Search Tree

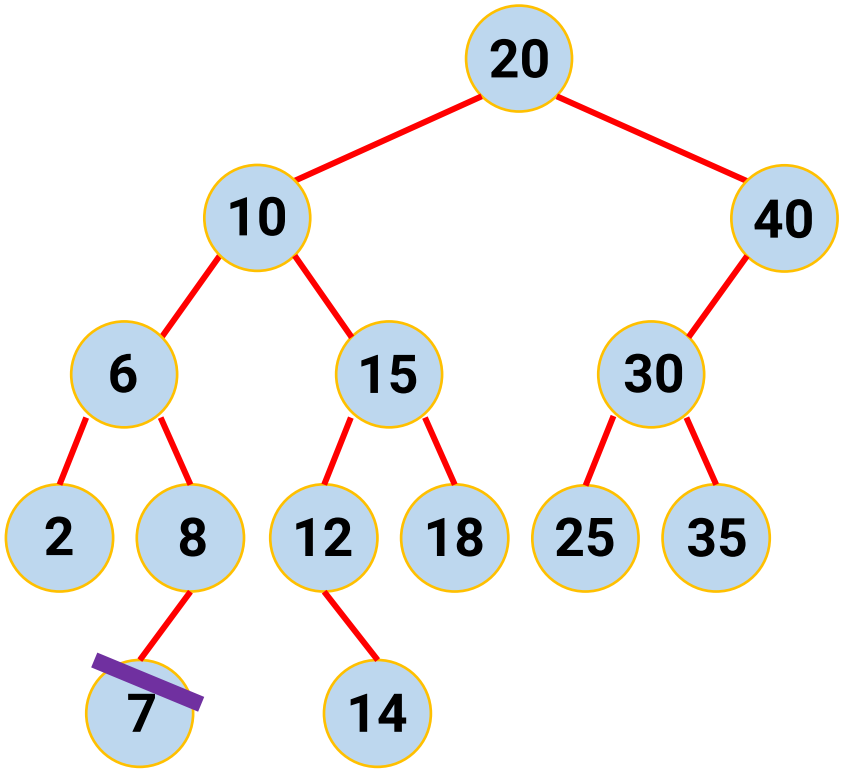
- First search (e.g., M) for the new entry until its absence is detected
- This is the position for the insertion

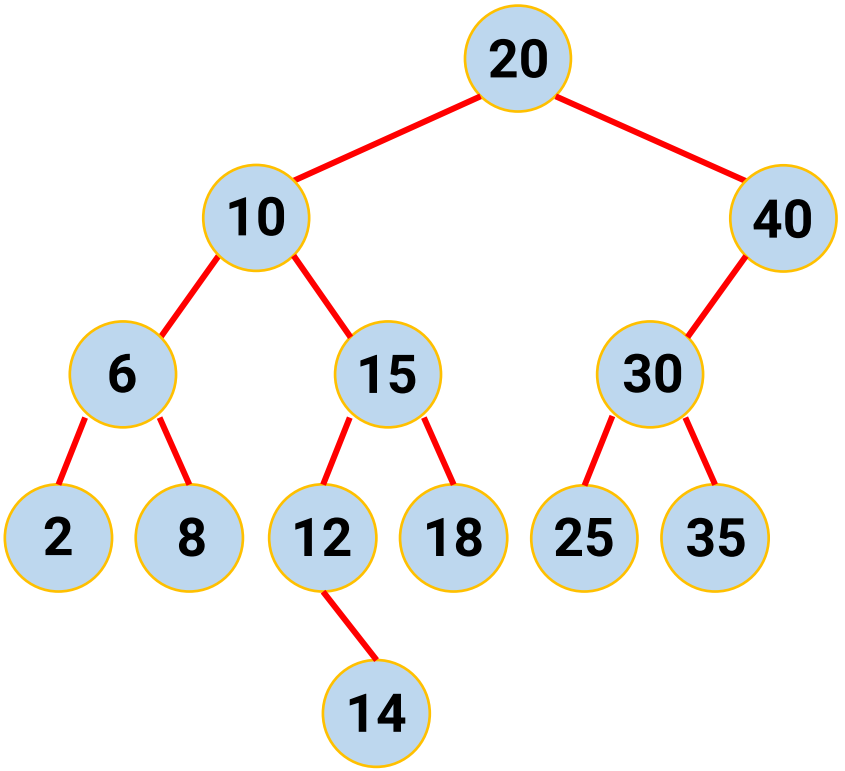




Deletion in Binary Search Tree

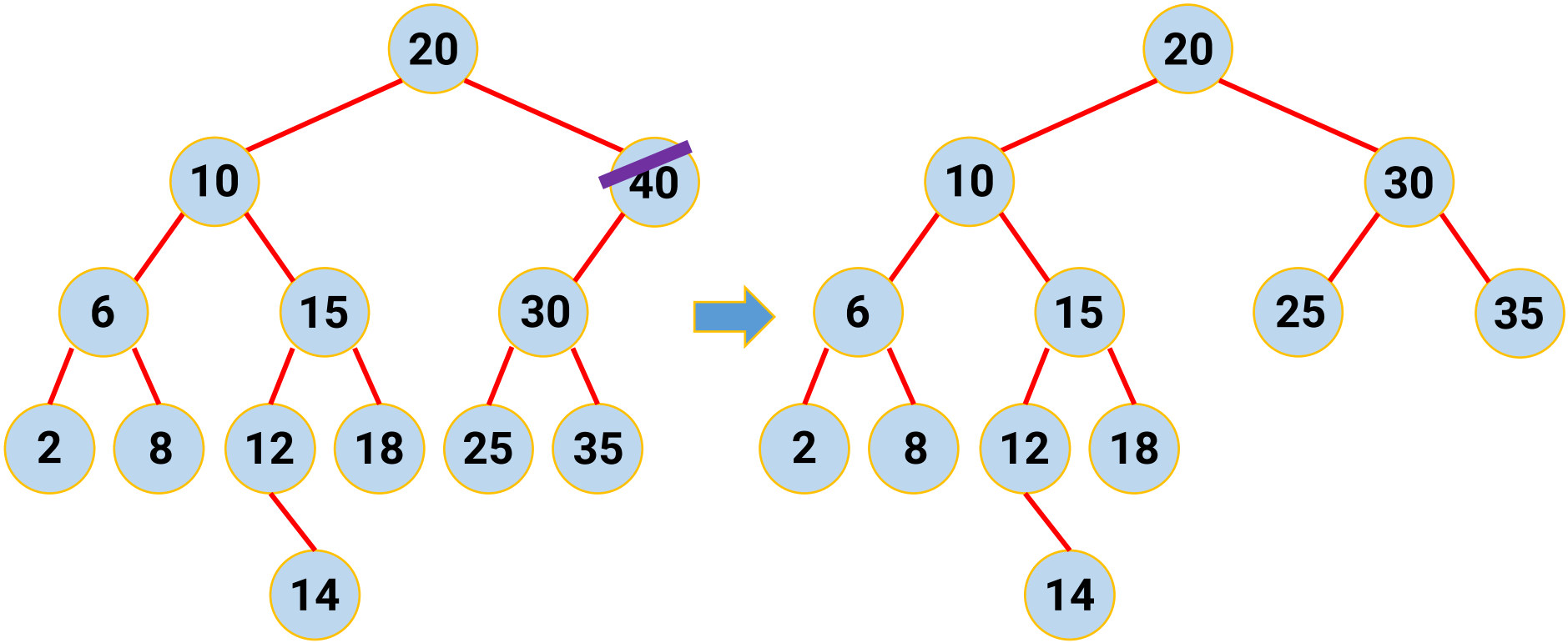
- Erase a leaf **element** whose key is 7





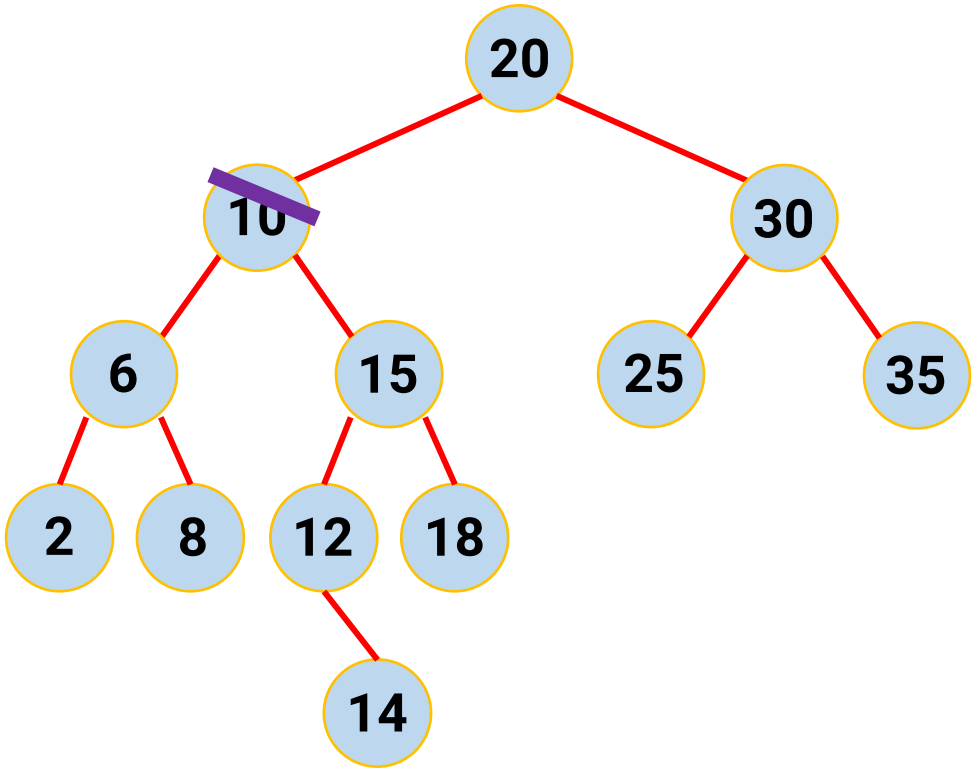
Deletion in Binary Search Tree (cont.)

- Erase a degree-1 node whose key is 40



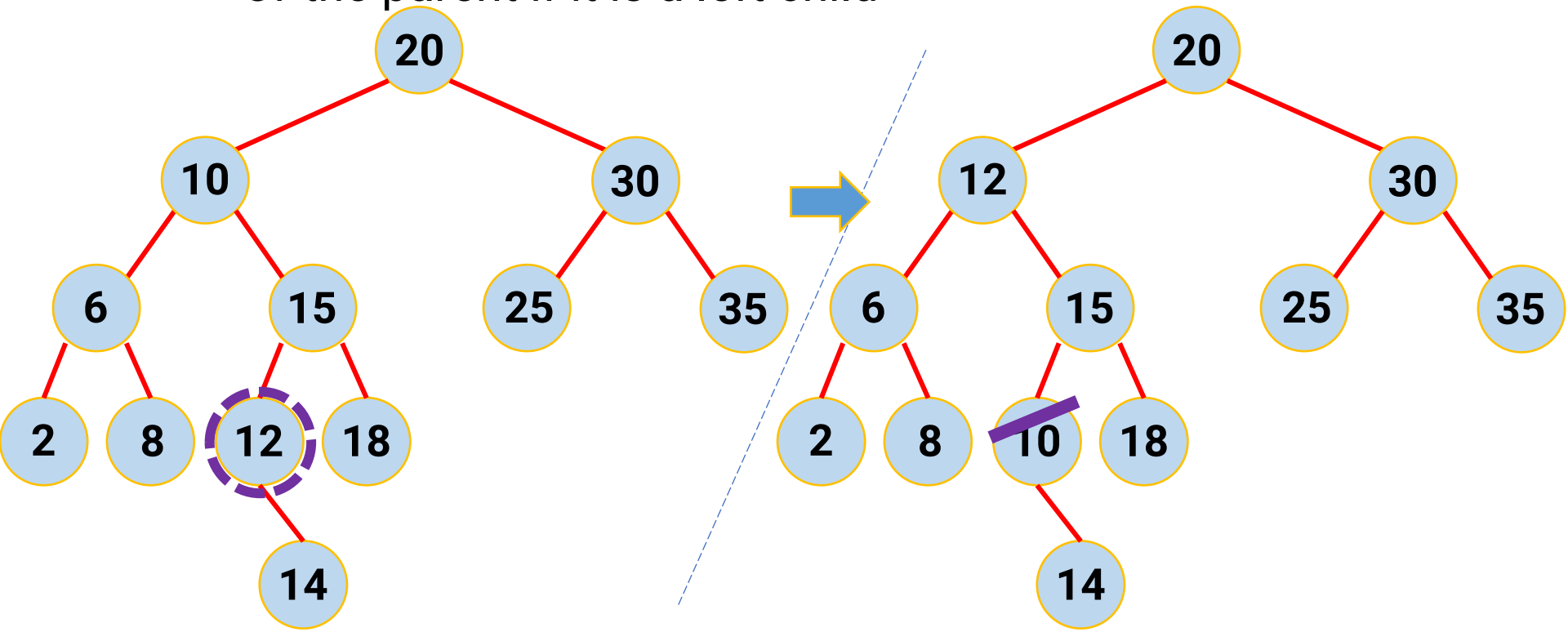
Deletion in Binary Search Tree (cont.)

- Erase a degree-2 node whose key is 10



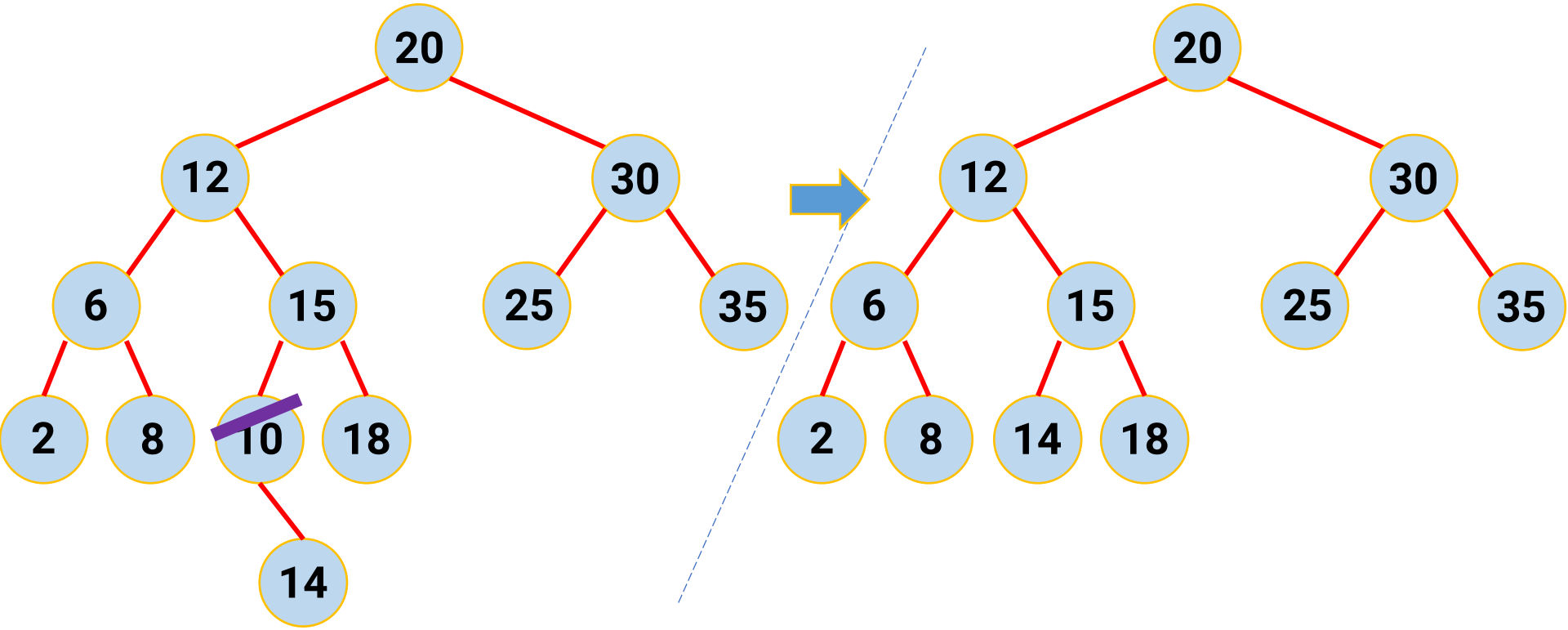
Deletion in Binary Search Tree (cont.)

- Swap its with its successor
  - The minimum node of the right subtree (keep going left)
  - Or the parent if it is a left child



Deletion in Binary Search Tree (cont.)

- Since Its successor has a degree of 1 or 0, we can simply cut and reconnect the rest of the tree



Any Questions?