6 Distributions of Random Variables (A Brief Introduction)

- The **joint (probability) distribution** of a collection of random variables identifies the possible values that the random variables can take and their relative likelihoods.
- We will see many ways of describing a distribution, depending on how many random variables are involved and their types (discrete or continuous).
- In the context of multiple random variables, the distribution of any one of the random variables is called a **marginal distribution**.

Example 6.1. Roll a fair four-sided die twice. Let X be the sum of the two dice, and let Y be the larger of the two rolls (or the common value if both rolls are the same).

1. Construct a table and plot displaying the marginal distribution of Y.

2. Describe the distribution of Y in terms of long run relative frequency.

3. Describe the distribution of Y in terms of relative degree of likelihood.

4. Construct a table and plot displaying the joint distribution of X and Y.

5. Construct a table and plot displaying the marginal distribution of X.

- The expected value $\mathrm{E}(X)$ of a discrete random variable X is defined by the probability-weighted average according to the underlying probability measure.
- The expected value of a random variable can be interpreted as the long-run average value of the random variable

Exercise 6.1. Consider the matching problem with $n = 4$: objects labeled 1, 2, 3, 4, are placed at random in spots labeled 1, 2, 3, 4, with spot 1 the correct spot for object 1, etc. Let the random variable X count the number of objects that are put back in the correct spot. Let P denote the probability measure corresponding to the assumption that the objects are equally likely to be placed in any spot, so that the 24 possible placements are equally.

1. Find the distribution of X by creating an appropriate table and plot.

2. Compute $\mathrm{E}(X)$.

3. Is the value from part 2 the most likely value of X? Explain.

4. Is the value from part 2 the value that we would “expect” to see for X in a single repetition of the phenomenon? Explain.

5. Explain in what sense the value from part 2 is “expected”.