MSBA Workshop: Calculus — A Few Things about Integrals

In each of the following situations, we want to compute the distance traveled by a person walking for 60 seconds. In each part:

- Sketch a plot of speed versus time.
- How is total distance traveled represented in this plot?
- Compute the total distance traveled over 60 seconds.

1. The person walks at a constant speed of 3 m/s for 60 seconds.

2. The person walks at a constant speed of 3 m/s for the first 20 seconds and 5 m/s for the next 40 seconds.

3. Their speed at t seconds is $0.1t$.

4. Their speed at t seconds is given by

$$f(t) = \begin{cases} 0.2t, & 0 < t < 20, \\ 4, & 20 < t < 40, \\ 12 - 0.2t, & 40 < t < 60 \end{cases}$$

5. Suppose their speed at t seconds is given by $400(t/60)^5(1 - t/60)^2$. How could you *approximate* the total distance traveled? How could you get a better approximation?

Integral

- An integral is commonly interpreted as an area under a curve

$$\int_a^b f(x)dx = \text{Area under curve } \ f \ \text{ over the interval } \ [a, b]$$

- You can approximate an integral $\int_a^b f(x)dx$> by chopping the interval $[a, b]$ into many small intervals, and approximating the area under the curve with the sum of the areas of many narrow rectangles with heights specified by f.
- Fundamental theorem of calculus: If F is a function whose derivative is f, that is $\frac{d}{dx}F(x) = f(x)$ then

$$\int_a^b f(x)dx = F(b) - F(a)$$

Integration in Python

```
from sympy import integrate, Symbol

# Define the variable
t = Symbol('t')

# Define the function to integrate
ft = 0.1 * t

# Calculate the integral of 0.1t from 0 to 60
integrate(ft, (t, 0, 60))
```

180.0

```
t = Symbol('t')

ft = 400 *(t / 60)**5 * (1 - t / 60)**2

integrate(ft, (t, 0, 60))
```

$\frac{1000}{7}$

Online integration tools

- WolframAlpha
- Symbolab
- Desmos