

Solution:

Step	C10	C100	C1000	F	T	W	O	U	R
Initial setup Range	0, 1	0, 1	0, 1	1,2,3,4,5,6,7,8,9	1,2,3,4,5,6,7,8,9	0,1,2,3,4,5,6,7,8,9	0,1,2,3,4,5,6,7,8,9	0,1,2,3,4,5,6,7,8,9	0,1,2,3,4,5,6,7,8,9
C1000 1 Order: (1,0)	0,1	0,1	1	1	5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9
C100 1 Order: (1,0)	0,1	1	1	1	5,6,7,8,9	5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9
C10 1 Order: (1,0)	1	1	1	1	7,8,9	5,6,7,8,9	5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9
T 8 Order: (8,9,7)	1	1	1	1	8	5,6,9	7	0,2,3,5,6,9	4
W 6 Order: (6,5,9)	1	1	1	1	8	6	7	3	4

Explanation:

First I write down the constraints

$$\begin{cases}
 20 = R + 10 \cdot C_{10} \\
 2W + C_{10} = U + 10 \cdot C_{100} \\
 2T + C_{100} = O + 10 \cdot C_{1000} \\
 C_{1000} = F \\
 T, W, O, U, R \text{ satisfy Range Table \& They are not equal to each other} \\
 C_{10}, C_{100}, C_{1000} \in \{0, 1\}
 \end{cases}$$

Then at each row. Pick a variable & value.

Step	C10	C100	C1000	F	T	W	O	U	R
① Initial setup Range	0, 1	0, 1	0, 1	1,2,3,4,5,6,7,8,9	1,2,3,4,5,6,7,8,9	0,1,2,3,4,5,6,7,8,9	0,1,2,3,4,5,6,7,8,9	0,1,2,3,4,5,6,7,8,9	0,1,2,3,4,5,6,7,8,9
② C1000 1 Order: (1,0)	0,1	0,1	1	1	5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9
③ C100 1 Order: (1,0)	0,1	1	1	1	5,6,7,8,9	5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9
④ C10 1 Order: (1,0)	1	1	1	1	7,8,9	5,6,7,8,9	5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9
⑤ T 8 Order: (8,9,7)	1	1	1	1	8	5,6,9	7	0,2,3,5,6,9	4
⑥ W 6 Order: (6,5,9)	1	1	1	1	8	6	7	3	4

At row ①: $|C_{10}| = |C_{100}| = |C_{1000}| = 2$, I choose C_{1000} .

when $C_{1000} = 0$,

$$\begin{cases}
 20 = R + 10 \cdot C_{10} \\
 2W + C_{10} = U + 10 \cdot C_{100} \\
 2T + C_{100} = 0 + 10 \cdot C_{1000} \Rightarrow 2T = 0 - C_{100} \Rightarrow 2T \leq 9 \Rightarrow T \leq 4 \\
 C_{1000} = F \Rightarrow F = 0 \\
 T, W, U, R \text{ satisfy Range Table \& They are not equal to each other} \\
 C_{10}, C_{100} \in \{0, 1\}
 \end{cases}$$

The domain chart will be

Step	C10	C100	C1000	F	T	W	O	U	R
if C1000=0	0,1	0,1	0	Null	1,2,3,4	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9

See next page for $C_{1000} = 1$.

Step	C10	C100	C1000	F	T	W	O	U	R
Initial setup Range	0, 1	0, 1	0, 1	1,2,3,4,5,6,7,8,9	1,2,3,4,5,6,7,8,9	0,1,2,3,4,5,6,7,8,9	0,1,2,3,4,5,6,7,8,9	0,1,2,3,4,5,6,7,8,9	0,1,2,3,4,5,6,7,8,9
C1000 1 Order: (1,0)	0,1	0,1	1	1	5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9
C100 1 Order: (1,0)	0,1	1	1	1	5,6,7,8,9	5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9
C10 1 Order: (1,0)	1	1	1	1	7,8,9	5,6,7,8,9	5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9
T 8 Order: (8,9,7)	1	1	1	1	8	5,6,9	7	0,2,3,5,6,9	4
W 6 Order: (6,5,9)	1	1	1	1	8	6	7	3	4

when $C_{1000} = 1$,

$$\begin{cases}
 20 = R + 10 \cdot C_{10} \\
 2W + C_{10} = U + 10 \cdot C_{100} \\
 2T + C_{100} = O + 10 \cdot C_{1000} \Rightarrow \underline{2T = 10 + 0 - C_{100}} \Rightarrow \underline{2T \geq 9} \Rightarrow \underline{T \geq 5} \\
 \underline{C_{1000}} = F \Rightarrow F = 1 \\
 T, W, O, U, R \text{ satisfy Range Table \& They are not equal to each other} \\
 C_{10}, C_{100} \in \{0, 1\}
 \end{cases}$$

The domain chart will be

Step	C10	C100	C1000	F	T	W	O	U	R
if C1000=1	0,1	0,1	1	1	5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9

Easy to see $C_{1000} = 1$ is least-constraining

So the least-constrain order for C_{1000} is (1, 0), pick $C_{1000} = 1$

	Step	C10	C100	C1000	F	T	W	O	U	R
①	Initial setup Range	0, 1	0, 1	0, 1	1,2,3,4,5,6,7,8,9	1,2,3,4,5,6,7,8,9	0,1,2,3,4,5,6,7,8,9	0,1,2,3,4,5,6,7,8,9	0,1,2,3,4,5,6,7,8,9	0,1,2,3,4,5,6,7,8,9
②	C1000 1 Order: (1,0)	0,1	0,1	1	1	5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9
③	C100 1 Order: (1,0)	0,1	1	1	1	5,6,7,8,9	5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9
④	C10 1 Order: (1,0)	1	1	1	1	7,8,9	5,6,7,8,9	5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9
⑤	T 8 Order: (8,9,7)	1	1	1	1	8	5,6,9	7	0,2,3,5,6,9	4
⑥	W 6 Order: (6,5,9)	1	1	1	1	8	6	7	3	4

AK row ②

$$|C_{10}| = |C_{100}| = 2, \text{ I choose } C_{100}$$

when $C_{100} = 0$,

$$\left\{ \begin{array}{l} 20 = R + 10 \cdot C_{10} \\ 2W + C_{10} = U + 10 \cdot C_{100} \Rightarrow 2W = U - C_{10} \Rightarrow 2W \leq 9 \Rightarrow W \leq 4 \\ 2T + C_{100} = 0 + 10 \Rightarrow 2T \geq 10 \Rightarrow T \geq 5 \\ 1 = F \\ T, W, O, U, R \text{ satisfy Range Table \& They are not equal to each other} \\ C_{10} \in \{0, 1\} \end{array} \right.$$

The domain chart will be

Step	C10	C100	C1000	F	T	W	O	U	R
if C1000=1	0,1	0	1	1	5,6,7,8,9	0,2,3,4	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9

Step	C10	C100	C1000	F	T	W	O	U	R
Initial setup Range	0, 1	0, 1	0, 1	1,2,3,4,5,6,7,8,9	1,2,3,4,5,6,7,8,9	0,1,2,3,4,5,6,7,8,9	0,1,2,3,4,5,6,7,8,9	0,1,2,3,4,5,6,7,8,9	0,1,2,3,4,5,6,7,8,9
C1000 1 Order: (1,0)	0,1	0,1	1	1	5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9
C100 1 Order: (1,0)	0,1	1	1	1	5,6,7,8,9	5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9
C10 1 Order: (1,0)	1	1	1	1	7,8,9	5,6,7,8,9	5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9
T 8 Order: (8,9,7)	1	1	1	1	8	5,6,9	7	0,2,3,5,6,9	4
W 6 Order: (6,5,9)	1	1	1	1	8	6	7	3	4

When $G_{00} = 1$

$$\begin{cases}
 20 = R + 10 \cdot G_0 \\
 2W + C_{10} = U + 10 \cdot G_{00} \Rightarrow 2W = 10 + U - G_0 \Rightarrow 2W \geq 9 \Rightarrow W \geq 5 \\
 2T + C_{100} = O + 10 \Rightarrow 2T \geq 9 \Rightarrow T \geq 5 \\
 1 = F \\
 T, W, O, U, R \text{ satisfy Range Table \& They are not equal to each other} \\
 G_{10} \in \{0, 1\}
 \end{cases}$$

The domain chart will be

Step	C10	C100	C1000	F	T	W	O	U	R
if C1000=1	0,1	1	1	1	5,6,7,8,9	5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9	0,2,3,4,5,6,7,8,9

Easy to see when $G_{00} = 1$, $|W| = 5$

$G_{00} = 0$, $|W| = 4$

Thus pick $G_{00} = 1$.

Step	C10	C100	C1000	F	T	W	O	U	R
Initial setup Range	0, 1	0, 1	0, 1	1,2,3,4,5,6 ,7,8,9	1,2,3,4,5,6 ,7,8,9	0,1,2,3,4,5 ,6,7,8,9	0,1,2,3,4,5 ,6,7,8,9	0,1,2,3,4,5 ,6,7,8,9	0,1,2,3,4,5 ,6,7,8,9
C1000 1 Order: (1,0)	0,1	0,1	1	1	5,6,7,8,9	0,2,3,4,5,6 ,7,8,9	0,2,3,4,5,6 ,7,8,9	0,2,3,4,5,6 ,7,8,9	0,2,3,4,5,6 ,7,8,9
C100 1 Order: (1,0)	0,1	1	1	1	5,6,7,8,9	5,6,7,8,9	0,2,3,4,5,6 ,7,8,9	0,2,3,4,5,6 ,7,8,9	0,2,3,4,5,6 ,7,8,9
C10 1 Order: (1,0)	1	1	1	1	7,8,9	5,6,7,8,9	5,6,7,8,9	0,2,3,4,5,6 ,7,8,9	0,2,3,4,5,6 ,7,8,9
T 8 Order: (8,9,7)	1	1	1	1	8	5,6,9	7	0,2,3,5,6,9	4
W 6 Order: (6,5,9)	1	1	1	1	8	6	7	3	4

Do the same for rest rows. And we found a solution.

check

$$\begin{array}{r}
 867 \\
 + 867 \\
 \hline
 1734
 \end{array}$$

✓