Sequence Labeling with Hidden Markov Models

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Intro to NLP, Fall 2019

Sequence Labeling/Tagging Problem

- Input: a sequence of **T** words (e.g. a sentence)
- Output: a sequence of **T** labels/tags, one for each word
- In contrast, text classification:
    - Input: a sequence of **T** words
    - Output: **1** label

A naive implementation

- Treat each **T** word as independent
- Apply a classifier to each input word independently
- But this ignores sequence structure
  - e.g. maybe some labels are more likely to follow others

Part-of-Speech Tagging

- Input: The grand jury commented on a number of topics
- Ouput: DT JJ NN VBD IN DT NN IN NNS

Named Entity Recognition

- Task: Find text spans that refer to a proper name and label its type, e.g.
  - [George Washington PERSON] was the first president.
  - [Washington ORGANIZATION] won the World Series
- Input: George Washington was the first president
- Output: B-PER I-PER O O O O

Running Example: Ice Cream & Global Warming (courtesy of Jason Eisner)

- It’s 2799. You’re a climatologist studying the history of global warming. You have Jason Eisner’s diary from 2007, which lists how many ice creams he ate every day.
  - Assume there are two kinds of days: Cold (C) and Hot (H)
- Task: given a sequence of diary observations O, figure out the correct hidden sequence Q of weather states.
  - e.g. Jason ate 3 icecreams on Day 1, 1 icecream on Day 2, and 3 icecreams on Day 3. What’s the weather on those three days?

This lecture

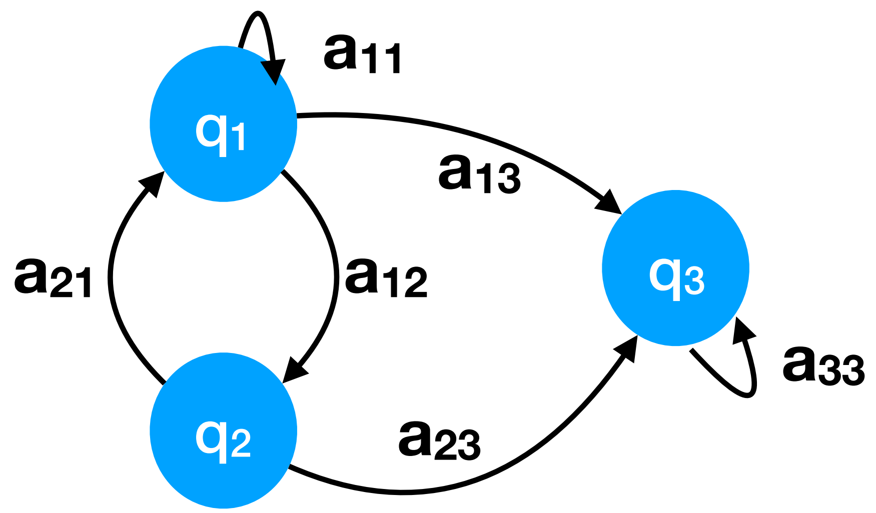
- We’ll discuss in-depth **Hidden Markov Models (HMM)**
  - Later in the course, we will also cover Conditional Random Fields (CRF), etc.
- HMM is a very useful pedagogical tool to illustrate:
  - How to **probabilistically model** the sequence labeling problem
  - How to efficiently compute with **Dynamic Programming**
  - How a model’s parameters can be learned by either **supervised** and **unsupervised** learning (for the latter, we’ll focus on **Expectation-Maximization**)

Outline

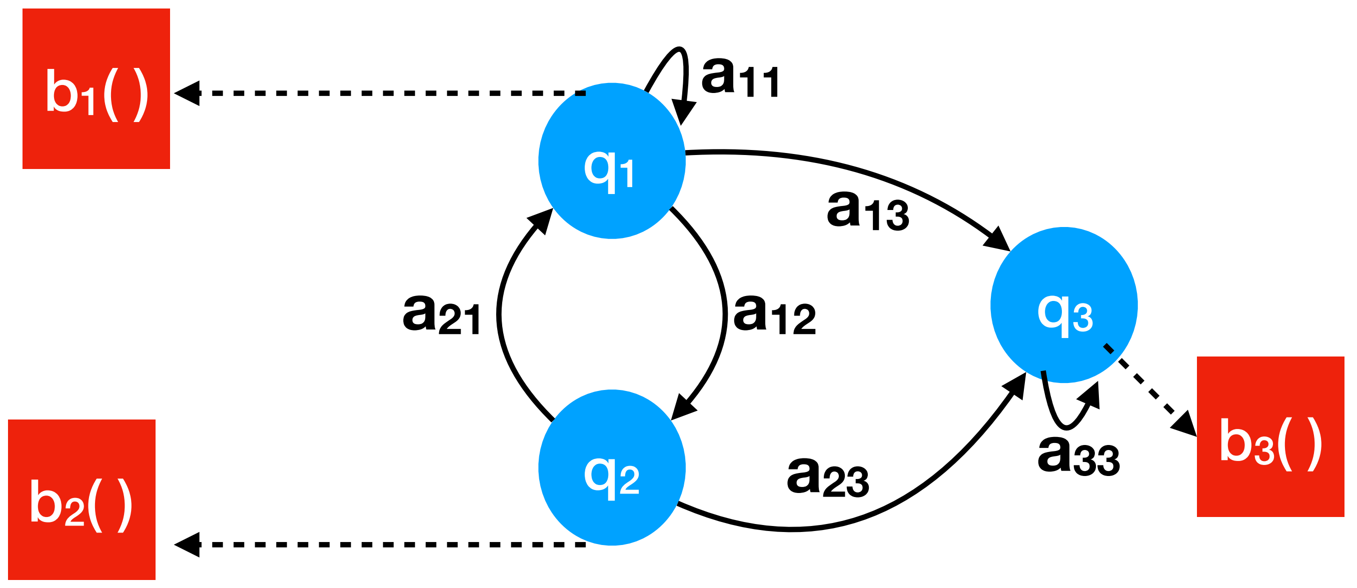
- Motivation/Examples
- HMM: Basic Definition & Three Problems
- Problem 1: Likelihood
- Problem 2: Decoding
- Problem 3: Learning
    - Supervised
    - Unsupervised: EM

Markov Chains

- A Markov Chain is specified by:
  - $Q = q_1 q_2 \ldots q_N$: a set of N states
  - $A = a_{11} a_{12} \ldots a_{ij} \ldots a_{NN}$: a transition probability matrix
  - $\pi = \pi_1 \pi_2 \ldots \pi_N$: initial probability distribution
- Markov Assumption: $P(q_i | q_1 \ldots q_{i-1}) = P(q_i | q_{i-1})$



- A **Hidden Markov Model (HMM)** is specified by:
  - $Q = q_1 q_2 \ldots q_N$: a set of N states
  - $A = a_{11} a_{12} \ldots a_{ij} \ldots a_{NN}$: a transition probability matrix
  - $\pi = \pi_1 \pi_2 \ldots \pi_N$: initial probability distribution
  - $B = b_i(o_t)$: emission probabilities
  - $O = o_1 o_2 \ldots o_T$: a sequence of T observations
- Markov Assumption: $P(q_i|q_1 \ldots q_{i-1}) = P(q_i|q_{i-1})$
- Output Independence: $P(o_i|q_1 \ldots q_T, o_1 \ldots o_T) = P(o_i|q_i)$



Three Problems for HMM

	Problem 1: Likelihood	Problem 2: Decoding	Problem 3: Learning
Given	HMM parameters λ = (A,B) Observation O	HMM parameters λ = (A,B) Observation O	Supervised: O and Q Unsupervised: O
Goal	Likelihood $P(O\|\lambda)$	Most likely hidden sequence Q	HMM parameters λ = (A,B) that maximize likelihood
Method	Forward Algorithm	Viterbi Algorithm	Supervised: Count Unsupervised: Forward-Backward Algorithm

Running Example: Ice Cream

- HMM has two states: Cold (C) and Hot (H)
- We have a sequence of integer observations, e.g.
    - **3 1 3** (3 icecream on day 1, 1 icecream on day 2,…)
- Likelihood: What’s the probability of 3 1 3 occurring under our HMM model?
- Decoding: What’s the most likely weather sequence for 3 1 3? Hot Cold Hot?
- Learning: How to estimate the HMM parameters?
    - Supervised: given 3 1 3 and H C H
    - Unsupervised: given 3 1 3 only

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Likelihood: $P(O|\lambda = (A,B))$

- Joint probability is easy:

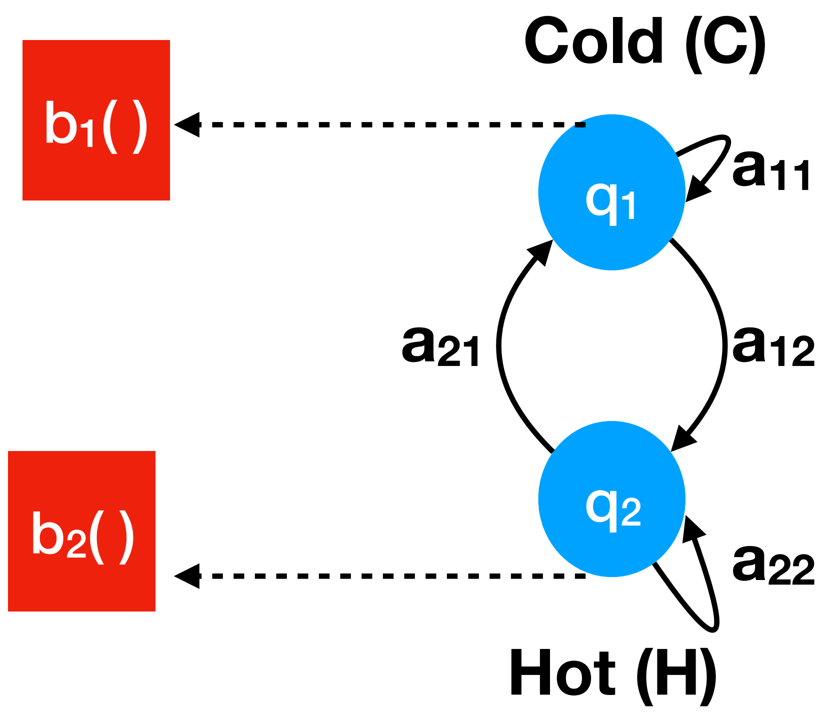
$$P(O,Q) = P(O|Q)P(Q) = \prod_{t=1}^{T} P(o_t|q_t) \times \prod_{t=1}^{T} P(q_t|q_{t-1})$$

Note: λ = (A,B) is implicit; dropped for notational simplicity

- For example: Q = H H C, O = 3 1 3

Then $P(O,Q) = P(3|H)P(1|H)P(3|C) \times P(H|start)P(H|H)P(C|H)$

$= b_2(3)\ b_2(1)\ b_1(3) \times \pi_2\ a_{22}\ a_{21}$



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- Since we don't know Q, sum over it

$$P(O) = \sum_{allQ} P(O,Q)$$

e.g. P(O=313) = P(O=313,Q=HHH)+P(O=313,Q=CCC)+…

For N states and T-length sequence, there are N^T hidden sequences!

Efficient enumeration by Dynamic Programming: Forward Algorithm

- Prepare a trellis data structure
- Each cell represents probability of being in state j after seeing the first t observations:

$$\alpha_t(j) = P(o_1, o_2, \ldots, o_t, q_t = j|\lambda)$$

- DP subproblem recursion:

$$\alpha_t(j) = \sum_{i=1}^{N} \alpha_{t-1}(i) a_{ij} b_j(o_t)$$

$$\alpha_t(j) = P(o_1, o_2, \ldots, o_t, q_t = j|\lambda)$$

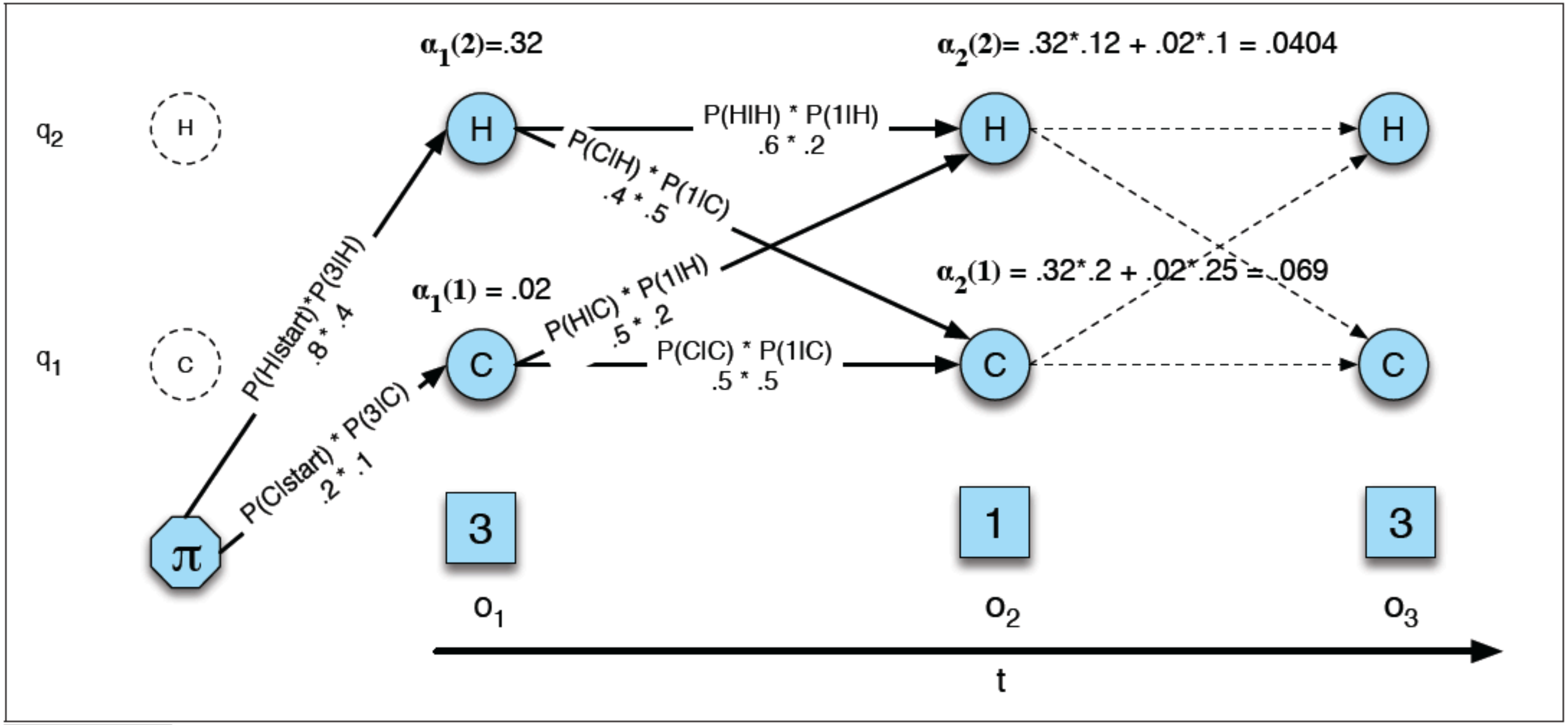

Figure A.5 The forward trellis for computing the total observation likelihood for the ice-cream events *3 1 3*. Hidden states are in circles, observations in squares. The figure shows the computation of $\alpha_t(j)$ for two states at two time steps. The computation in each cell follows Eq. A.12: $\alpha_t(j) = \sum_{i=1}^{N} \alpha_{t-1}(i)a_{ij}b_j(o_t)$. The resulting probability expressed in each cell is Eq. A.11: $\alpha_t(j) = P(o_1, o_2 \ldots o_t, q_t = j|\lambda)$.

**Jurafsky & Martin (2019) Speech & Language Processing, 3rd ed.
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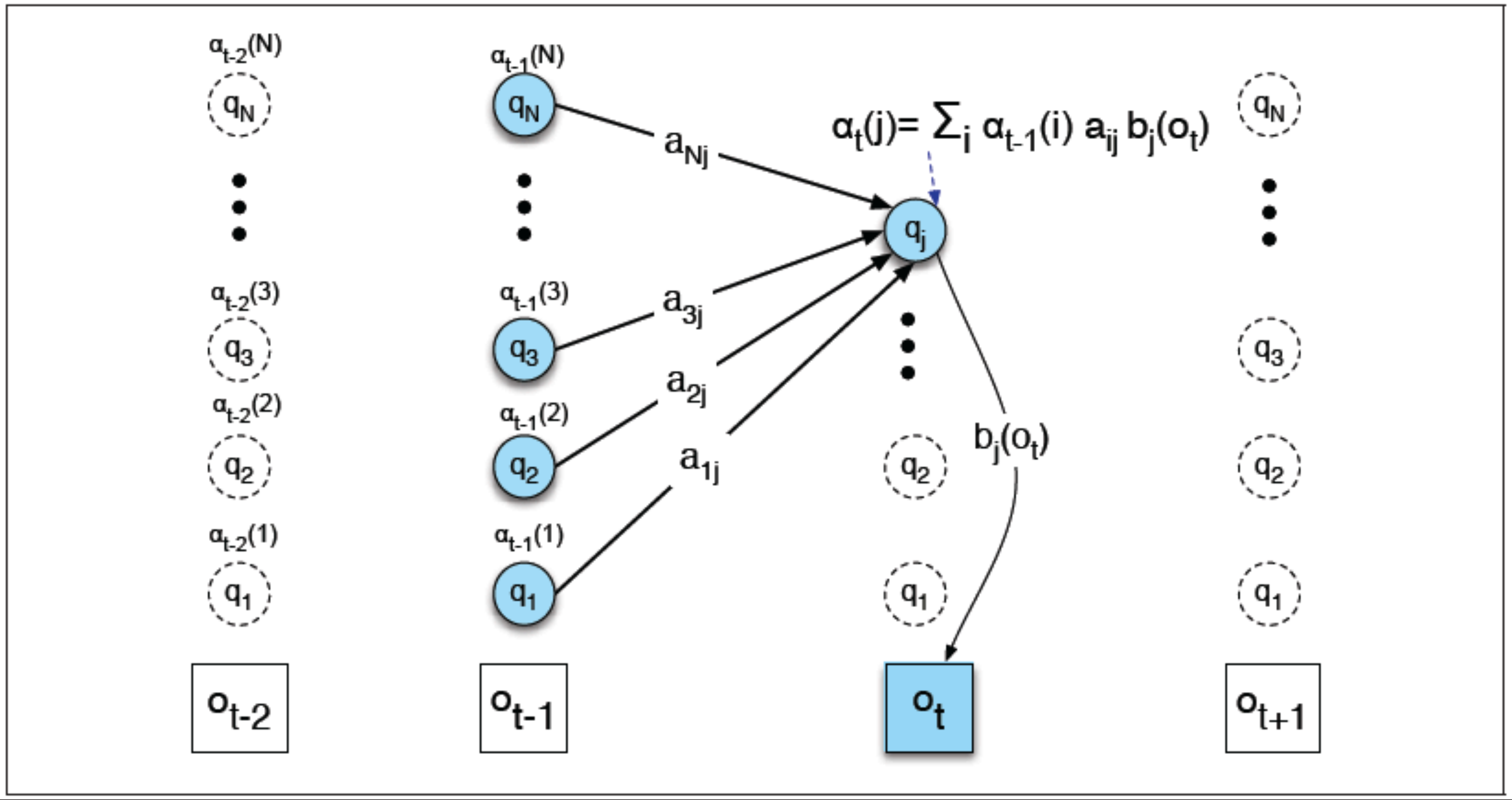

```
function FORWARD(observations of len T, state-graph of len N) returns forward-prob

  create a probability matrix forward[N,T]
  for each state s from 1 to N do                                ; initialization step
      $forward[s,1] \leftarrow \pi_s * b_s(o_1)$
  for each time step t from 2 to T do                            ; recursion step
    for each state s from 1 to N do
              $forward[s,t] \leftarrow \sum_{s'=1}^{N} forward[s',t-1] * a_{s',s} * b_s(o_t)$
  $forwardprob \leftarrow \sum_{s=1}^{N} forward[s,T]$          ; termination step
 return forwardprob
```

Figure A.7 The forward algorithm, where $forward[s,t]$ represents $\alpha_t(s)$.

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Decoding (Viterbi Algorithm): very similar to Likelihood Computation (Forward Algo.)

- Likelihood: Each cell in trellis represents probability of being in state j after seeing the first t observations:

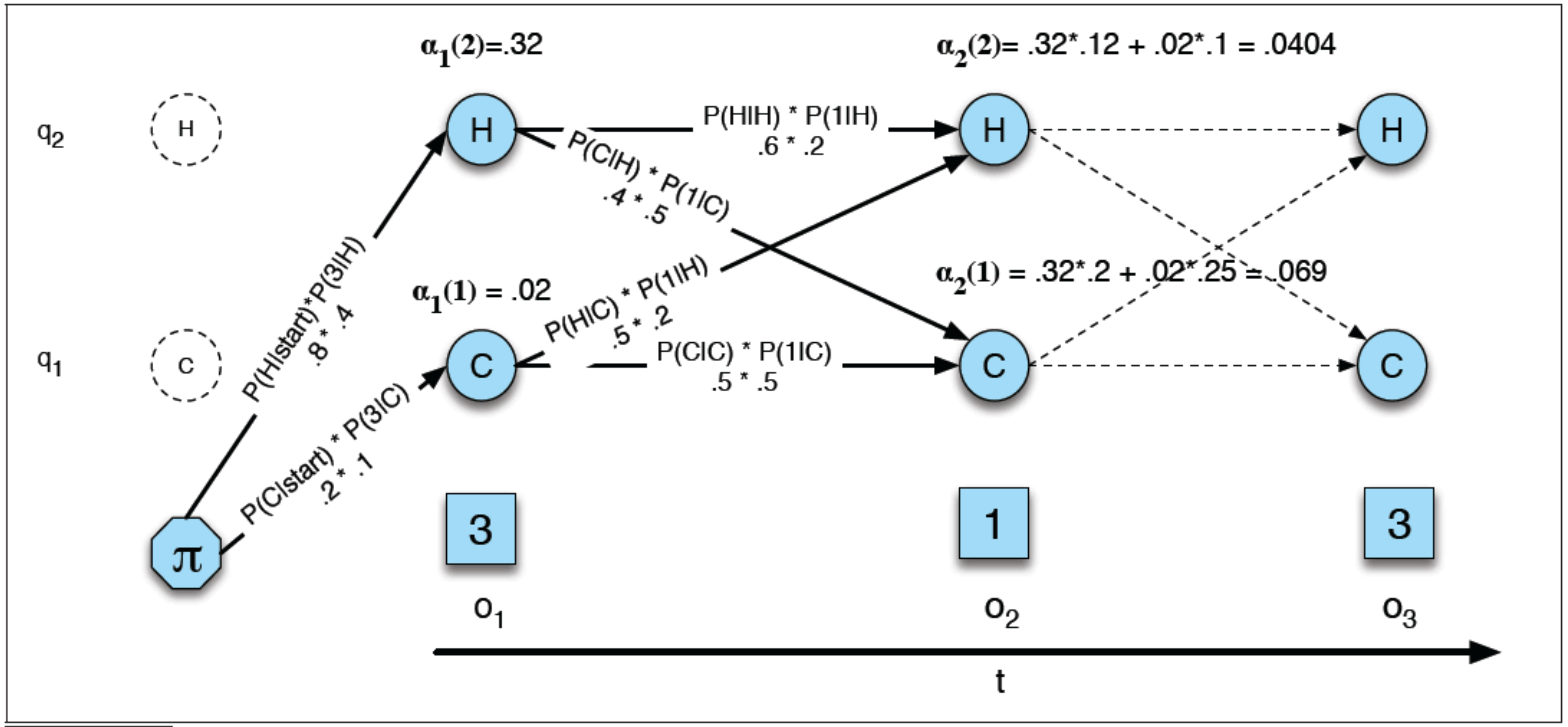
$$\alpha_t(j) = P(o_1, o_2, \ldots, o_t, q_t = j|\lambda)$$

- Decoding: Each cell in trellis represents probability of being in state j after seeing the first t observations *and* passing through the most probable state sequence $q_1..q_{t-1}$

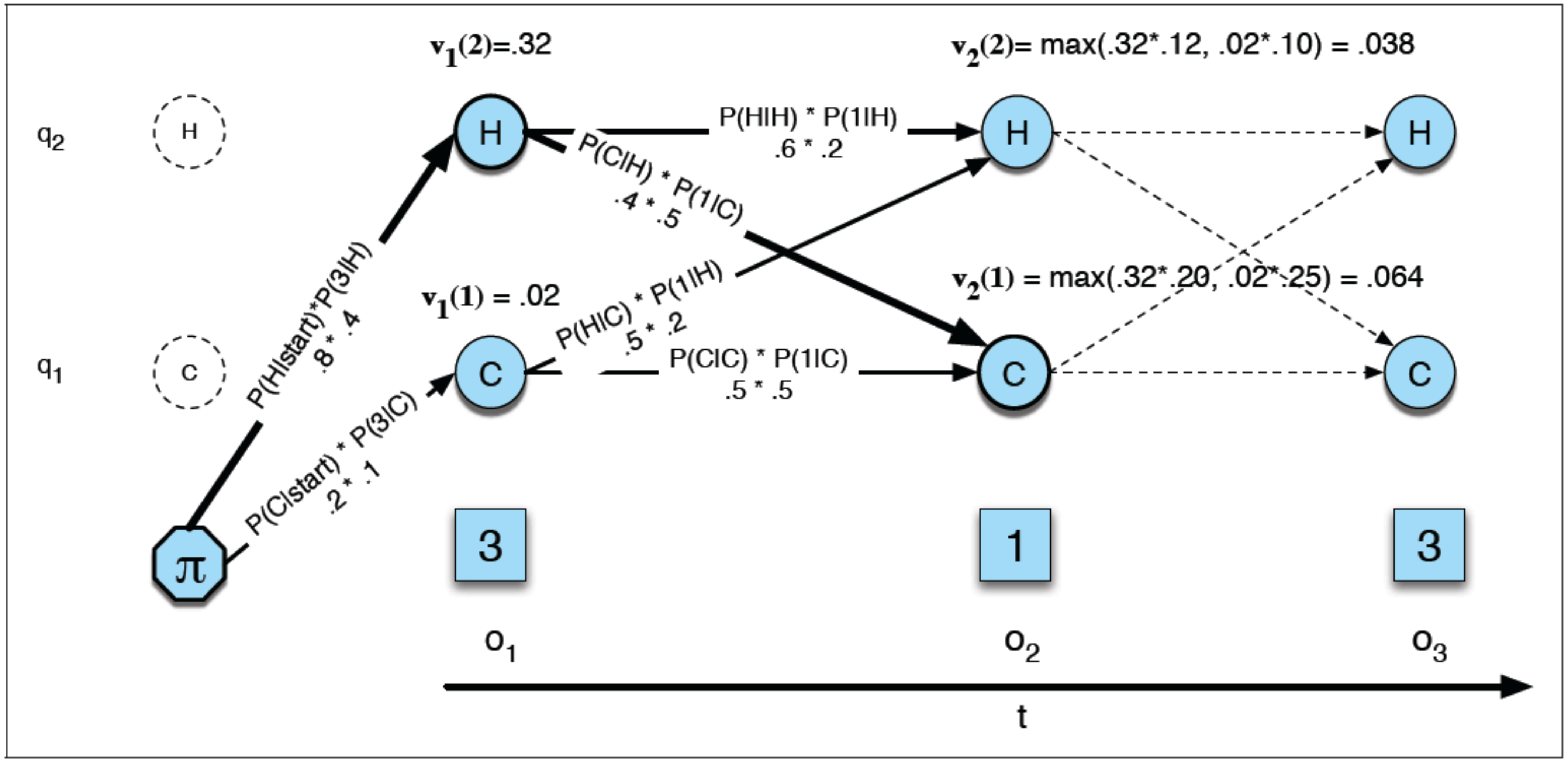
$$v_t(j) = \max_{q_1,\ldots,q_{t-1}} P(o_1, o_2, \ldots, o_t, q_1, \ldots, q_{t-1}, q_t = j, |\lambda)$$

- DP subproblem recursion: $v_t(j) = \max_{i=1,\ldots,N} v_{t-1}(i) a_{ij} b_j(o_t)$

Forward Algo:
Computes likelihood of observation



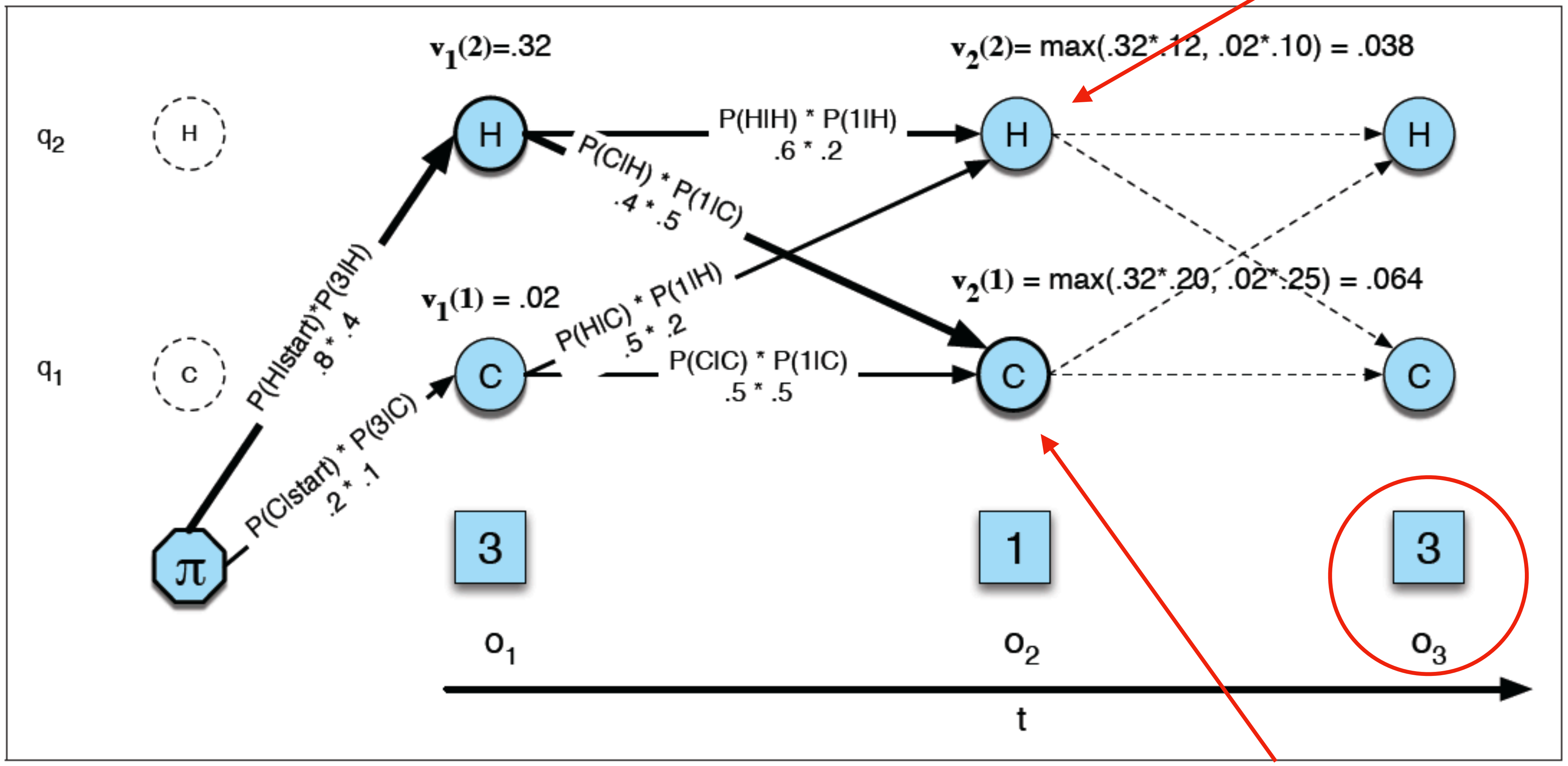
Viterbi Algo:
Decode most probable states given observation



Note: we're just changing sum to max. In later lectures we'll discuss general structures & semirings

Note: even though the HMM equation with $P(q_t|q_{t-1})$ seems unidirectional, it actually models the whole sequence and both future/past observations matter

Suppose o_3=3 picks H at t=2



If o_3=1 it's possible there may be a different best path at t=2

```
**function** VITERBI(*observations* of len $T$,*state-graph* of len $N$) **returns** *best-path*, *path-prob*

create a path probability matrix *viterbi[N,T]*
**for** each state $s$ **from** 1 **to** $N$ **do**                    ; initialization step
        $viterbi[s,1] \leftarrow \pi_s * b_s(o_1)$
        $backpointer[s,1] \leftarrow 0$
**for** each time step $t$ **from** 2 **to** $T$ **do**                 ; recursion step
    **for** each state $s$ **from** 1 **to** $N$ **do**
        $viterbi[s,t] \leftarrow \max_{s'=1}^{N} viterbi[s',t-1] * a_{s',s} * b_s(o_t)$
        $backpointer[s,t] \leftarrow \operatorname{argmax}_{s'=1}^{N} viterbi[s',t-1] * a_{s',s} * b_s(o_t)$
$bestpathprob \leftarrow \max_{s=1}^{N} viterbi[s,T]$                   ; termination step
$bestpathpointer \leftarrow \operatorname{argmax}_{s=1}^{N} viterbi[s,T]$              ; termination step
$bestpath \leftarrow$ the path starting at state *bestpathpointer*, that follows backpointer[] to states back in time
**return** *bestpath*, *bestpathprob*
```

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Supervised Learning

- Suppose we're given a dataset with both O and Q.

3	3	2	1	1	2	1	2	3
hot	hot	cold	cold	cold	cold	cold	hot	hot

- It's easy to estimate HMM paramaters by counting:

e.g.
P(hot) = Cnt(hot)/9
P(3|hot)=Cnt(3,hot)/Cnt(hot)

Emission Probability (B):

$$P(1|hot) = 0/4 = 0 \qquad p(1|cold) = 3/5 = .6$$
$$P(2|hot) = 1/4 = .25 \qquad p(2|cold = 2/5 = .4$$
$$P(3|hot) = 3/4 = .75 \qquad p(3|cold) = 0$$

Initial Probability:

$$\pi_h = 1/3$$
$$\pi_c = 2/3$$

Transition Probability (A):

$$p(hot|hot) = 2/3 \qquad p(cold|hot) = 1/3$$
$$p(cold|cold) = 2/3 \qquad p(hot|cold) = 1/3$$

Unsupervised Learning

- We're only given observations O.

  3 3 2 1 1 2 1 2 3

- How to estimate HMM parameters? Don't know hidden states associated with observation, can't get counts.

- The Baum-Welch / Forward-Backward Algorithm does this by iteratively estimating the counts, and using this to re-derive initial/transition/emission probabilities.

- This is an instance of Expecation-Maximization (EM) Algo.

- Forward Probability

$$\alpha_t(j) = P(o_1, o_2, \ldots, o_t, q_t = j|\lambda)$$

- Backward Probability

$$\beta_t(i) = P(o_{t+1}, o_{t+2}, \ldots, o_T|q_t = i, \lambda)$$

- Recursion for Backward Probability

$$\beta_t(i) = \sum_{j=1}^{N} a_{ij} b_j(o_{t+1}) \beta_{t+1}(j) \qquad P(O|\lambda) = \sum_{j=1}^{N} \pi_j b_j(o_1) \beta_1(j)$$

$$\hat{a}_{ij} = \frac{\text{expected number of transitions from state } i \text{ to state } j}{\text{expected number of transitions from state } i}$$

$$P(O|\lambda) = \sum_{j=1}^{N} \alpha_t(j)\beta_t(j)$$

$$\xi_t(i,j) = P(q_t = i, q_{t+1} = j|O,\lambda) = \frac{\alpha_t(i)\, a_{ij} b_j(o_{t+1})\beta_{t+1}(j)}{\sum_{j=1}^{N} \alpha_t(j)\beta_t(j)}$$

$$\text{not-quite-}\xi_t(i,j) = P(q_t = i, q_{t+1} = j, O|\lambda) = \alpha_t(i)\, a_{ij} b_j(o_{t+1})\beta_{t+1}(j)$$

$$\hat{a}_{ij} = \frac{\sum_{t=1}^{T-1} \xi_t(i,j)}{\sum_{t=1}^{T-1} \sum_{k=1}^{N} \xi_t(i,k)}$$

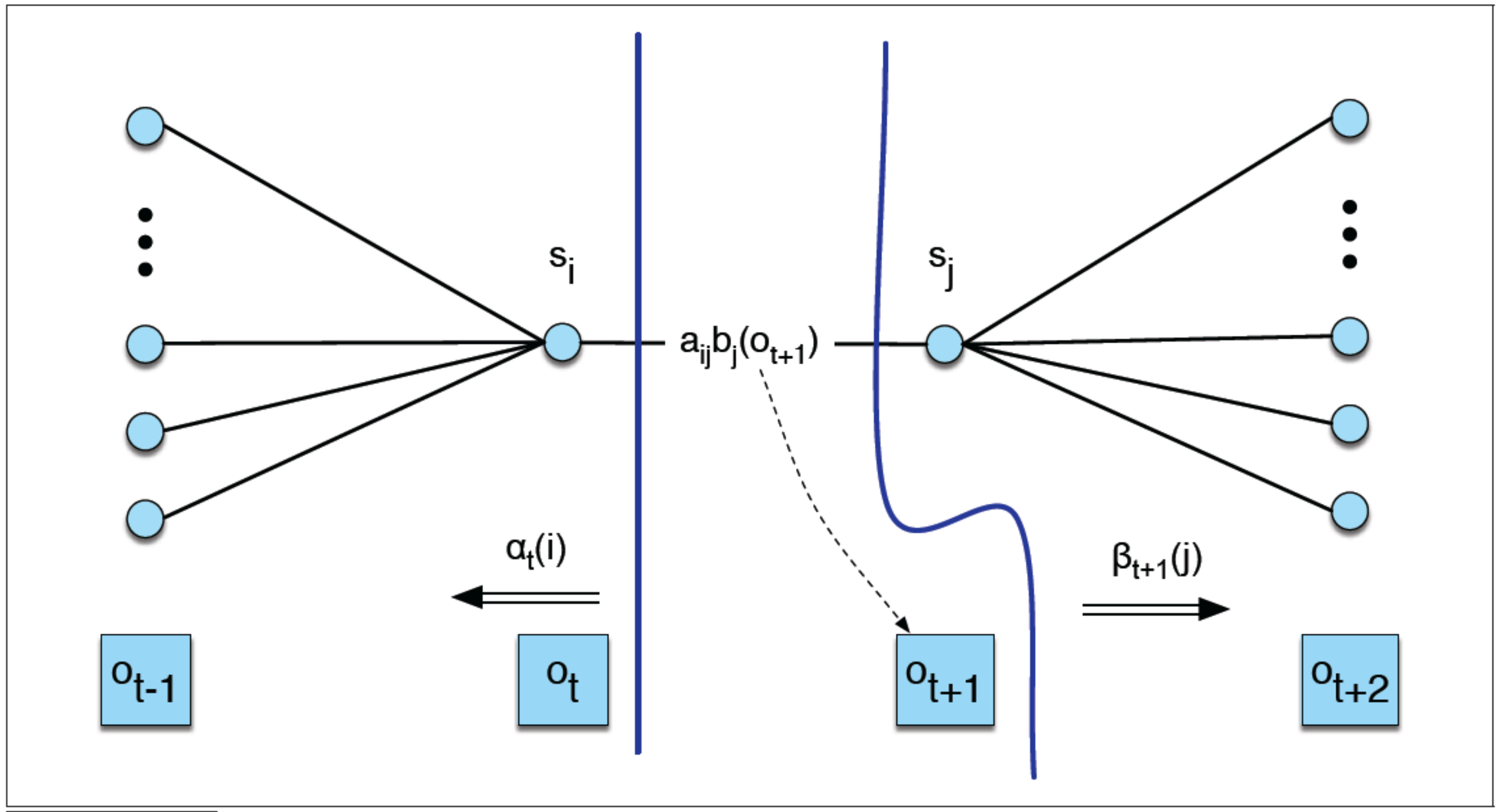

Figure A.12 Computation of the joint probability of being in state i at time t and state j at time $t+1$. The figure shows the various probabilities that need to be combined to produce $P(q_t = i, q_{t+1} = j, O|\lambda)$: the α and β probabilities, the transition probability a_{ij} and the observation probability $b_j(o_{t+1})$. After Rabiner (1989) which is ©1989 IEEE.

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$$\hat{b}_j(v_k) = \frac{\text{expected number of times in state } j \text{ and observing symbol } v_k}{\text{expected number of times in state } j}$$

$$\gamma_t(j) = P(q_t = j|O,\lambda) = \frac{P(q_t = j, O|\lambda)}{P(O|\lambda)} = \frac{\alpha_t(j)\beta_t(j)}{P(O|\lambda)}$$

$$\hat{b}_j(v_k) = \frac{\sum_{t=1\ s.t.O_t=v_k}^{T} \gamma_t(j)}{\sum_{t=1}^{T} \gamma_t(j)}$$

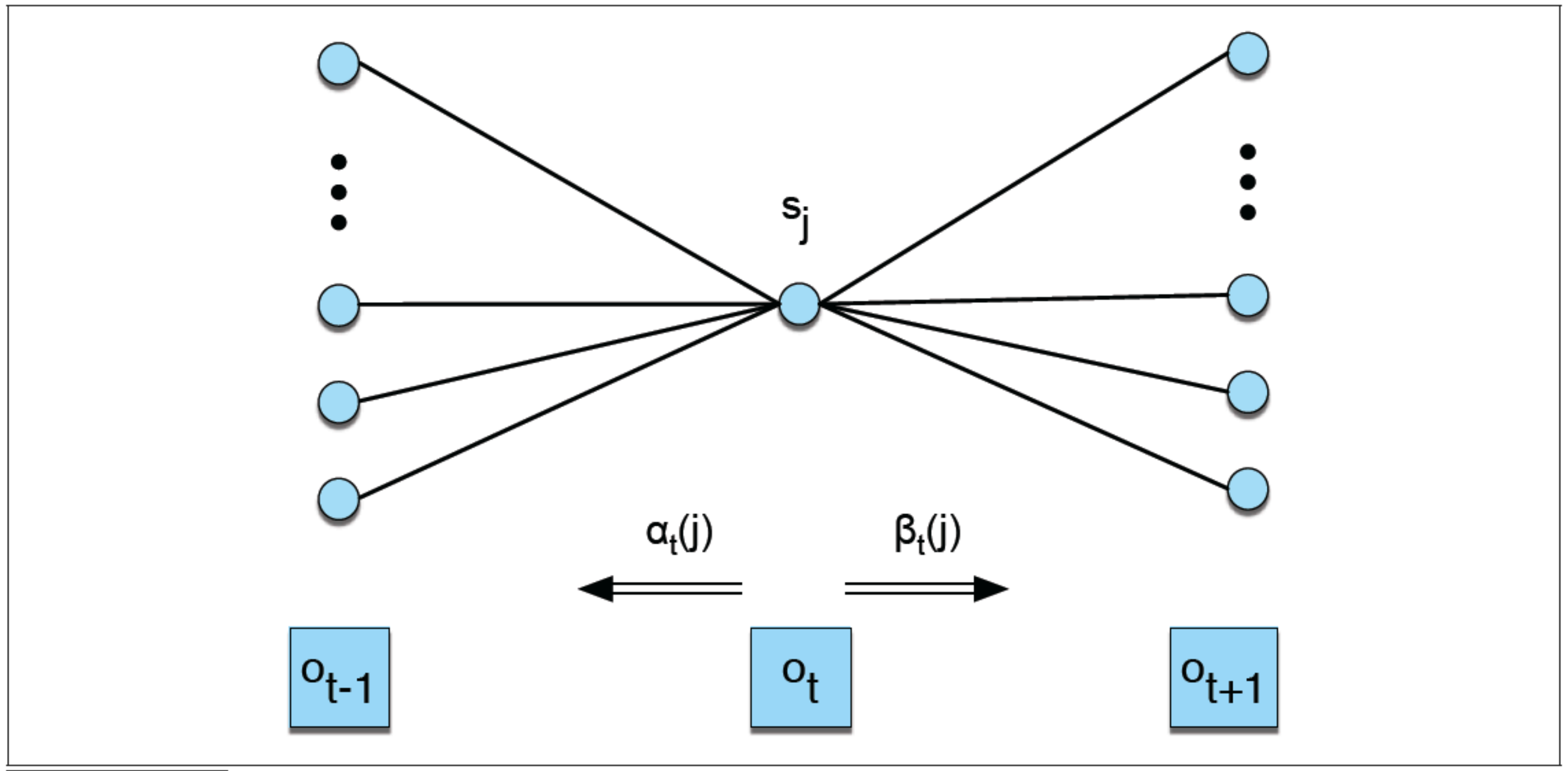

Figure A.13 The computation of $\gamma_t(j)$, the probability of being in state j at time t. Note that γ is really a degenerate case of ξ and hence this figure is like a version of Fig. A.12 with state i collapsed with state j. After Rabiner (1989) which is ©1989 IEEE.

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```
**function** FORWARD-BACKWARD(*observations* of len *T*, *output vocabulary V*, *hidden state set Q*) **returns** *HMM=(A,B)*

  **initialize** *A* and *B*
  **iterate** until convergence
    **E-step**
      $\gamma_t(j) = \frac{\alpha_t(j)\beta_t(j)}{\alpha_T(q_F)} \quad \forall\, t \text{ and } j$
      $\xi_t(i,j) = \frac{\alpha_t(i)\, a_{ij} b_j(o_{t+1})\beta_{t+1}(j)}{\alpha_T(q_F)} \quad \forall\, t,\ i, \text{ and } j$
    **M-step**
      $\hat{a}_{ij} = \frac{\sum_{t=1}^{T-1} \xi_t(i,j)}{\sum_{t=1}^{T-1} \sum_{k=1}^{N} \xi_t(i,k)}$
      $\hat{b}_j(v_k) = \frac{\sum_{t=1 s.t.\, O_t=v_k}^{T} \gamma_t(j)}{\sum_{t=1}^{T} \gamma_t(j)}$
  **return** *A*, *B*
```

Notes:
- Each iteration increases data likelihood
- In practice, initialization may be important

Figure A.14 The forward-backward algorithm.

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