

**THÈSE DE DOCTORAT
DE SORBONNE UNIVERSITÉ**

Spécialité : Physique

École doctorale n°564: Physique en Île-de-France

réalisée sous la direction de Alberto BRAMATI

au Laboratoire Kastler Brossel



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Sujet de la thèse :

Full optical control of quantum fluids of light in hot atomic vapors

soutenue le 2 octobre 2023

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Remerciements

Merci à tous

Part I

Theory of Microcavity Exciton Polaritons

Chapter 1

Microcavity Exciton Polaritons

Photons are massless particles, yet when confined within an optical cavity, they obtain an effective mass and exhibit a parabolic dispersion relation. However, two photons in the cavity do not interact with each other in the sense that they do not attract or repel each other as massive particles would do.

Whenever an electromagnetic field is shined on a material, the dipoles of the medium oscillate and change the refractive index seen by the field. At high intensity, the change in the refractive index can depend on the square of the electric field, which is then called the Kerr effect. Since a local variation of the refractive index deflects light, a high-intensity region in the material can modify the trajectory of an incoming beam. From this point of view, one can see how photon-photon interaction can arise in a nonlinear medium.

If the medium is chosen such that the resonance frequency of the dipoles matches the resonance of the optical microcavity, the light can be trapped in the sample and experience effective interaction through the medium. When the coupling of the light with the medium is strong enough to exceed the losses of the whole system, one can achieve the strong coupling regime. In this regime, the new eigenstates of the system are the so-called polaritons, which are a superposition of the photon and the dipole excitation of the medium. This hybrid state of matter inherits properties from both photons and matter excitations and forms the constitutive particles of the quantum fluid considered later.

In the present case, the strong coupling regime is achieved in a semiconductor microcavity by inserting two-dimensional quantum wells at the antinode of the electromagnetic field of a high-quality factor optical microcavity. In such devices, polaritons arise from the coupling between the cavity photons and the excitons of the quantum wells.

This chapter is dedicated to describing photons and excitons separately before showing how they couple in the sample to form polaritons. Then, from a microscopic description, one will derive the macroscopic equations of motion of the fluid and find that polaritons can be described in the usual Quantum Fluid framework with a driven dissipative Gross-Pitaevskii equation.

1.1 Microcavity Photons

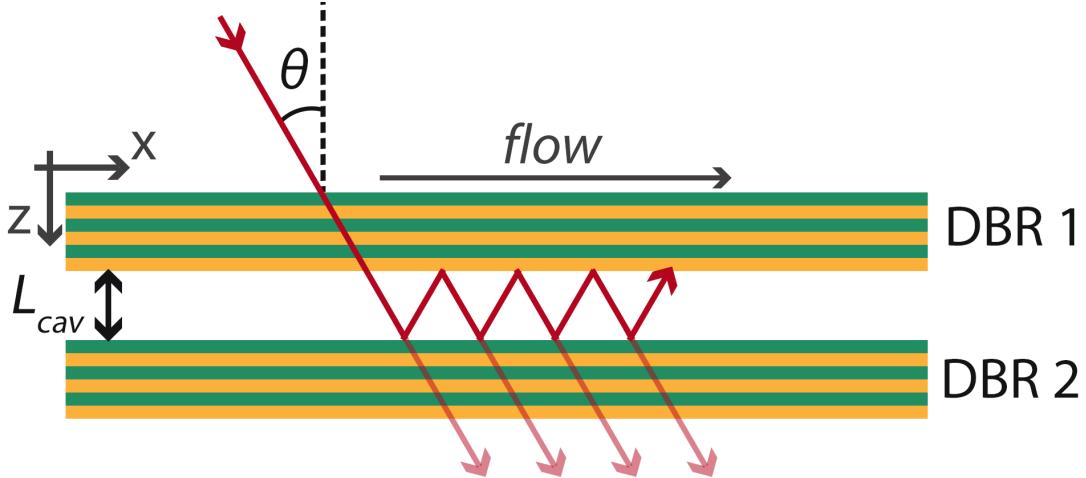


Fig. 1.1 Photon in a planar microcavity

Planar microcavity parameters: First, let us consider an electromagnetic field incident at an angle θ on a planar microcavity made with two mirrors with reflectivities R_1 and R_2 , separated by a distance L . The z-axis is normal to the mirrors as shown in Figure 1.1. The phase shift acquired during a single round trip in the cavity is $\Delta\phi(\theta) = 2nk_0L \cos(\theta)$, with n the refractive index of the medium and $k_0 = 2\pi/\lambda$ the wave vector of the field in vacuum. The interference between the multiple reflections sets the resonance condition of the cavity, and the transmission of the field can be written as:

$$T(\theta) = \frac{R_1 R_2}{1 + R_1 R_2 - \sqrt{R_1 R_2} \cos(\Delta\phi(\theta)/2)} \quad (1.1)$$

From this, one can define the decay rate of the field oscillation known as the quality factor:

$$Q = \frac{\omega_\gamma}{\Delta\omega_\gamma} \quad (1.2)$$

with ω_γ the resonance frequency of the cavity and $\Delta\omega_\gamma$ the linewidth of the resonance, which sets the lifetime of the photon in the cavity through $\tau_\gamma = 1/\Delta\omega_\gamma$ and depends on the mirrors' reflectivities. Another important parameter describing the cavity is its frequency resolution, which is encoded in the cavity finesse through:

$$\mathcal{F} = \pi \frac{\Delta\omega_\gamma}{\delta\omega_\gamma} = \pi \frac{\sqrt{R_1 R_2}}{1 - R_1 R_2} \quad (1.3)$$

where $\Delta\omega_\gamma$ is the Free Spectral Range (FSR) representing the frequency difference between two successive longitudinal modes of the cavity. To achieve strong coupling with the nonlinear medium, the photon needs to stay trapped in the cavity long enough to interact with the

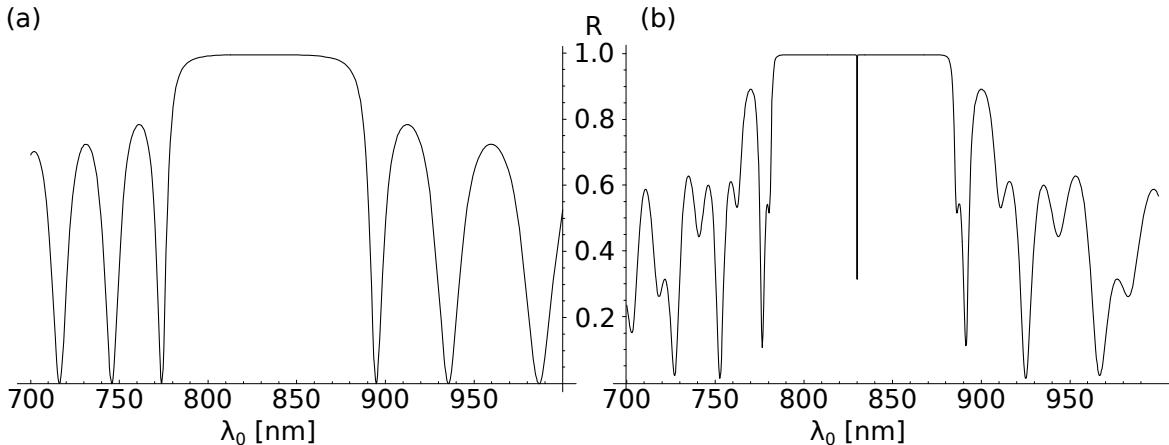


Fig. 1.2 **DBR and Fabry-Perot cavity reflectivity.** (a) Reflectivity R of a Bragg mirror of 20 pairs of $\text{Ga}_{0.9}\text{Al}_{0.1}\text{As}/\text{AlAs}$, illuminated at normal incidence for different wavelengths λ . The DBR has a reflectivity close to 1 over a large range of wavelengths called the stop-band, centered on $\lambda_0 = 836$ nm by tuning the thickness of each of the mirror layers. (b) Corresponding reflectivity of the optical cavity built from the facing of two DBR mirrors identical to that in (a) and separated by a distance $L_{\text{cav}} = 2\lambda_0/n_{\text{cav}}$, leading to the appearance of a very narrow resonance at λ_0 . Adapted from ??.

medium. In other words, the quality factor of the cavity must be very high, which means using mirrors with high reflectivities. This can be achieved with Distributed Bragg Reflectors (DBR) mirrors.

Distributed Bragg Reflectors. These reflectors consist of a series of N alternating layers composed of two materials with different refractive indices $n_1 < n_2$. This configuration causes partial reflection of the electromagnetic field at each of the N interfaces, resulting in a very high overall reflection coefficient, R , which is expressed as follows:

$$R = \left[\frac{(n_2/n_1)^{2N} - n_f/n_0}{(n_2/n_1)^{2N} + n_f/n_0} \right]^2, \quad (1.4)$$

with n_0 and n_f representing the refractive indices of the media before and after the mirrors. Figure 1.2 illustrates the evolution of R in relation to the wavelength of the electromagnetic field for one DBR mirror used in our system. This mirror is constructed with 20 pairs of $\text{Ga}_{0.9}\text{Al}_{0.1}\text{As}/\text{AlAs}$ layers, having refractive indices $n_2 = 3.48$ and $n_1 = 2.95$ respectively. The reflection coefficient R is 0.9985 across a broad wavelength range, known as the stop-band, centered around a wavelength of $\lambda_0 = 836$ nm. This central wavelength is achieved by setting the thickness of the layers to $d_{1,2} = \lambda_0/4n_{1,2}$. The microcavity used in the experiments is built by facing two of those DBR mirrors separated by a distance $L_{\text{cav}} = 3\lambda_0/2n_{\text{cav}}$, giving rise to a very narrow resonance at λ_0 with three field antinodes within the cavity. The parameters of the cavity are summarized in Equation 1.1.

Fabry-Perot cavity parameters					
R_1	R_2	n_1	n_2	n_{cav}	λ_0 (nm)
0.9992	0.9985	2.95	3.48	3.54	836

With the above-described DBR, the cavity has a finesse $\mathcal{F} = 2850$ from which we can infer the photon lifetime:

$$\delta\omega_\gamma = \frac{\Delta\omega_\gamma}{\mathcal{F}} = \frac{c}{n_{\text{cav}}L_{\text{eff}}} \frac{1-R}{\sqrt{R}}, \quad (1.5)$$

where $L_{\text{eff}} = L_{\text{cav}} + L_{\text{bragg}}$ is the effective length of the sample taking into account the penetration of the field in the mirrors.

$$L_{\text{Bragg}} = \frac{\lambda_0}{2} \frac{n_1 n_2}{n_{\text{cav}}(n_2 - n_1)} \quad (1.6)$$

which gives $\tau_\gamma = \frac{1}{\delta\omega_\gamma} = 5$ ps.

Transverse dynamics of the photon. The energy of the photon within the sample can be written as:

$$E_\gamma = \frac{\hbar c}{n_{\text{cav}}} \sqrt{k_x^2 + k_y^2 + k_z^2}. \quad (1.7)$$

The resonance condition of the cavity fixes the photon wavevector along the z component.

$$k_z = \frac{2\pi n_{\text{cav}}}{\lambda_0}. \quad (1.8)$$

In the paraxial approximation $k_x, k_y \ll k_z$, one can expand (1.7) in terms of $\frac{\|\mathbf{k}_\parallel\|}{\|\mathbf{k}_z\|}$ as:

$$E_\gamma = \frac{\hbar c}{n_{\text{cav}}} \sqrt{k_z^2 + k_\parallel^2} = E_0 \left(1 + (\hbar k_\parallel)^2 \frac{c^2}{2(n_{\text{cav}} E_0)^2} \right). \quad (1.9)$$

where $\mathbf{k}_\parallel = \mathbf{k}_x + \mathbf{k}_y$ is the wavevector in the transverse plane and $E_0 = hc/\lambda_0$ is the photon energy in free space. We can identify the effective mass of the photon by writing (1.9) with the usual form of the kinetic energy of a massive particle:

$$E_\gamma = E_0 + \frac{p_\parallel^2}{2m_\gamma} \quad (1.10)$$

where we identify $m_\gamma = \frac{\hbar n_{\text{cav}}^2}{\lambda_0 c}$ which is inversely proportional to the second derivative of the energy with respect to k_\parallel :

$$\frac{1}{m_\gamma} = \frac{1}{\hbar^2} \frac{\partial^2 E_\gamma}{\partial k_\parallel^2} \quad (1.11)$$

Momentum conservation: we mentioned the necessity to have a high quality factor to later achieve strong coupling. One could then ask why do we stick to planar designs, which are generally not the best solution to get a high Q factor since they are known to be unstable except for plane waves. The answer lies in the calculation just above. Indeed, the advantage of having a planar design is the translational invariance in the xy plane, which brings \mathbf{k}_{\parallel} conservation. As a consequence, if one shines a laser with a given \mathbf{k}_{\parallel} , light will behave in the cavity as a massive particle moving at velocity $v_{\gamma} = \hbar \mathbf{k}_{\parallel} / m_{\gamma}$. In the picture of creating a fluid whose flow is controlled, the planar design then appears as a wise choice.

A direct link between incidence angle and in plane momentum \mathbf{k}_{\parallel} : as shown in [Figure 1.1](#) the incidence angle $\theta = (\theta_x, \theta_y)$ of the incoming field is related to the in-plane momentum k_{\parallel} as :

$$k_x = k_0 \sin(\theta_x) \quad (1.12)$$

$$k_y = k_0 \sin(\theta_y) \quad (1.13)$$

Controlling the in plane momentum of photons and latter of polaritons then boils down to control the local incidence angle of the field, in other words, the transverse phase of the incoming beam. This is a crucial point for the experiments as it allows to control the flow of the fluid by changing the laser phase.

Conclusion: This section revealed how trapping light in a planar microcavity grants it effective mass and lifetime. The next section will explore the semiconducting media that can be inserted into these cavities, especially the bound electron-hole pairs called excitons that can be addressed by the trapped light.

1.2 Excitons in Semiconductors

1.2.1 Band theory in brief

Applying the Schrodinger equation to an atom reveals that the electrons energies can only take discrete values. However, N atoms sufficiently close to each other interact which lift the degeneracy and turns each energy state in a set of N separated levels (see [Figure 1.3](#)). Within a solid material, the density is so high (typically 10^{22} atoms per cm^3) that the spacing between energy levels tends to zero forming a continuous band of energy. The electrons fill the band from the lowest energy level up to the Fermi energy. The Fermi energy is the energy of the highest occupied state at zero temperature. The band structure of a material is then defined by the energy of the valence band, the conduction band and the band gap between them. The valence band is the highest energy band that is fully occupied at zero temperature, while the conduction band is the lowest energy band that is empty at zero temperature. The band gap is the energy difference between the conduction band and the valence band. Determining if a material is a metal, a semiconductor or an insulator is then a matter of comparing the band gap to the Fermi energy. For metals the conduction band and the valence band overlap, any electric potential difference puts then the electron into motion and create a current. For insulators and semiconducting materials the Fermi energy is in the band gap. For insulators the band gap is typically ~ 3 eV making the promotion of

an electron to the conduction band highly energy demanding. Finally, the semiconductors gap energy is accessible with photons in the visible domain making these materials perfect candidates for controlled light-matter interactions.

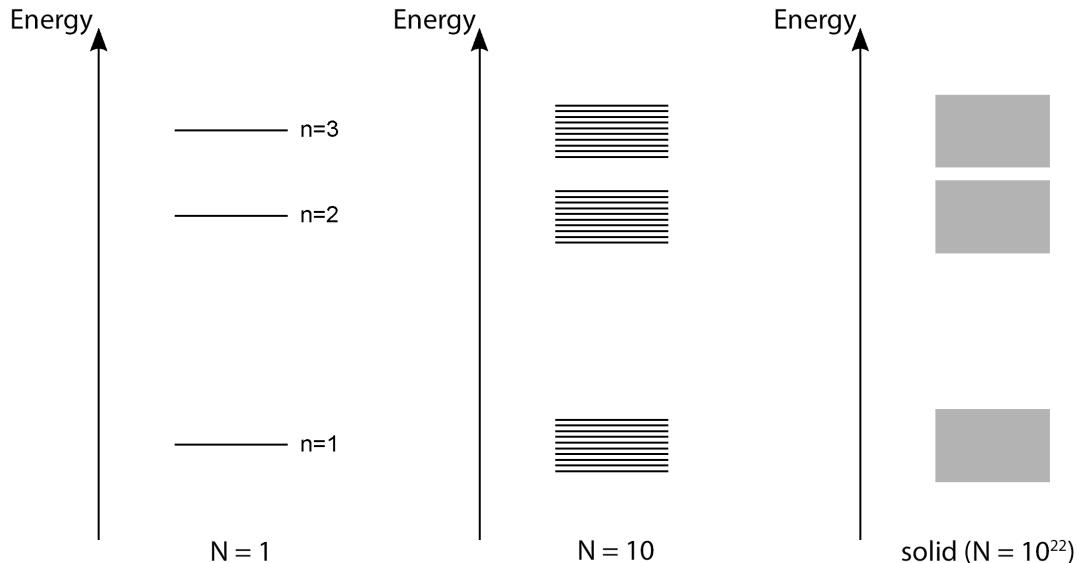


Fig. 1.3 **Energy levels of a set of N atoms.** As N increase the spacing between two successive levels tends to zero and eventually form a continuous band of energy.

1.2.2 Band structure of semiconductors

Finding the exact band structure of a material can be done by solving the Schrodinger equation for an electron in periodic potential. The Bloch theorem states that the wave function of such an electron can be written as a plane wave modulated by a periodic function. The periodic function is then expanded in a Fourier series and the Schrodinger equation is solved for each Fourier component. The exact band structure can be rather complicated but a simplified version can be obtained by considering the effective mass approximation [25]. In this picture, the dispersion relation of the electron in the material is typically represented in [Figure 1.4](#). In this case the minimum of the conduction band and the maximum of the valence band are located at the same point in the Brillouin zone. The material is then said to have a direct band gap.

1.2.3 Exciton phenomenological approach

As shown in [Figure 1.4](#) shining a photon whose energy exceed the gap energy can promote an electron from the valence band to the conduction band through the absorption of a photon. The disappearance of the electron in the valence band can be described equivalently as the creation of a virtual particle of opposite charge called hole [13]. If one scan shine a laser on a semiconductor and ramp up its frequency a narrow absorption peak is observed below the gap energy. The presence of this peak originate from the coulombic interaction between the

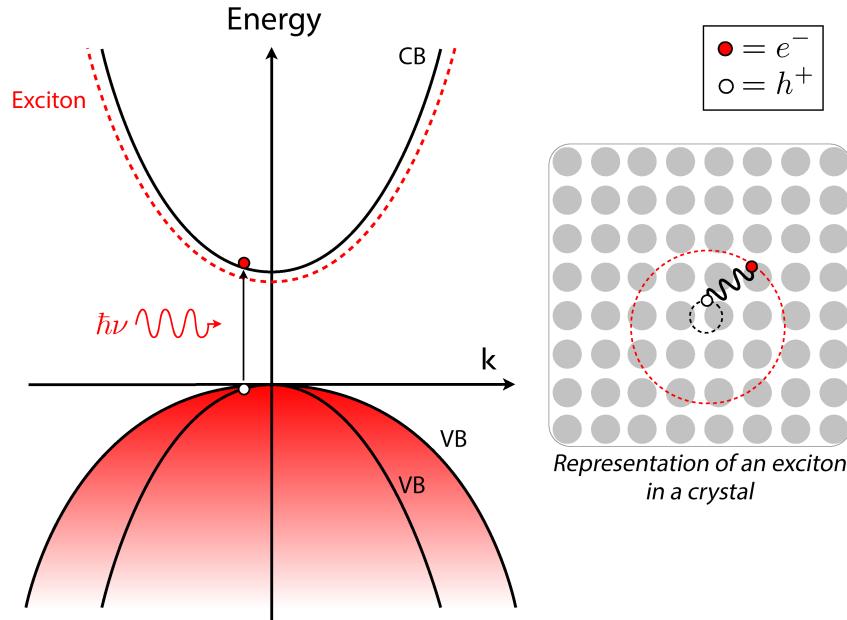


Fig. 1.4 **Band structure of a direct gap semiconductor.** The black solid line represent the conduction and valence band in a bulk semiconductor. The red dashed lines represent the dispersion relation of an exciton. The filling of the valence band by electrons is represented by the shaded red color.

created electron and the hole creating a bound state of the material. In terms of energy it's as if the gap energy is reduced by the binding energy of the exciton and the electron lies in a virtual band below the conduction band as represented by the red dashed lines in Figure 1.4. The exciton energy can then be written as :

$$E_X = E_g - E_b + \frac{\hbar^2 K^2}{2m_X} \quad (1.14)$$

where E_g is the gap energy, E_b the binding energy, $\hbar\vec{K}$ is the exciton momentum defined as the electron-hole pair center of mass momentum and m_X the exciton effective mass which is the sum of the electron and hole effective mass. Since E_b is the interaction energy between a electron and a hole it is, as we will see in the next section, very similar to the Rydberg energy series of the hydrogen atom, the hole playing the role of the proton. As a consequence, the same electronic structure exist for the exciton : 1s, 2s, 2p, etc state can be observed in the absorption spectrum of the material. However, the hole is quite different from the proton in the sense that it's actually a collective excitation of all the valence band electrons. Furthermore the exciton is a weakly bound state as the binding energy is typically a few meV. In GaAs or AlGaAs heterostructures which are the materials used in the present work E_b is around 4 meV due to their high dielectric constant $\epsilon_r \sim 12$. In these kind of samples, excitonic resonances can only be observed at cryogenic temperatures since the binding needs to exceeds the thermal energy $k_B T$ in order to survive the thermal fluctuations. For the

above mentionned structures we obtain the condition $T < 10$ K for excitons stability. Another important exciton characteristic is their huge Bohr Radius $a_X = \frac{\epsilon_r \hbar^2}{m_X e^2} = 11.6$ nm in GaAs which is much larger then the typical size of a unit crystalline cell $a_0 = 5.651$ Å. This means that the exciton wavefunction is delocalized over many unit cells. Such excitons, are called Wannier-Mott and contrast with Frenkel excitons which are localized on a single unit cell and are typically found in organic materials.

1.2.4 Electron-hole interactions and exciton formation

Excitons were said to be made of an electron and a hole bound by Coulombic interactions. However, all the interactions in the material are ultimately mediated by valence and conduction band electrons. The rising of a bound state in such a material is therefore not obvious. In other words moving from a conduction-valence band electrons to a electron-hole picture is not trivial and reveals interesting feature about the electron-hole interactions that will at the end enable exciton formation. In this section we will first describe all the possible interactions between the electrons of the semiconductor before showing how they can be recasted in terms of electron-hole interactions.

1.2.4.1 Electron scattering in semiconductors

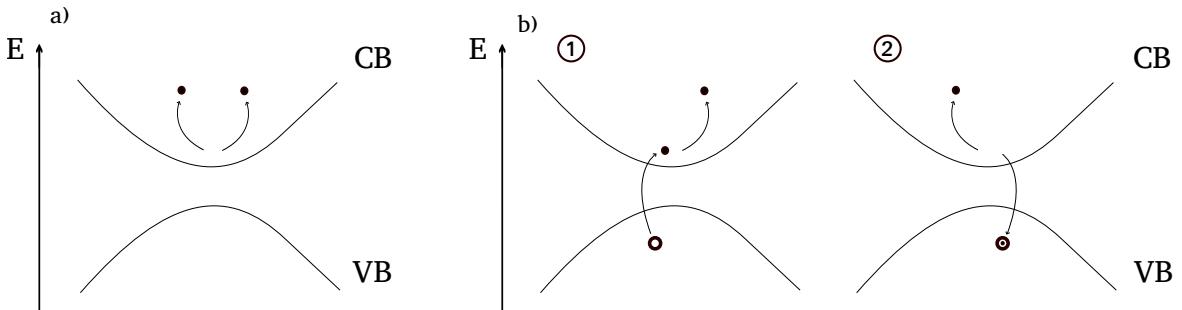


Fig. 1.5 **Drawing representing the scattering between two conduction band electrons.** a) Direct intraband scattering. Filled black dots represent conduction band electrons. b) Two steps scattering involving an electron-hole exchange or creation an annihilation of an exciton, referred as interband scattering. The empty black circle represent a valence band hole while the same circle with a small black dot inside stands for a valence band hole filled with an electron which is equivalent to a valence electron. a) and b) share the same final state which is a full valence band and two scattered conduction band electrons. Drawing inspired from [13].

Intraband scattering : to start with let's consider the direct scattering between two conduction band electrons as presented in Figure 1.5 a). This simple process is referred as intraband scattering since each electron stays in its own band. In the small momentum limit $\mathbf{q} \rightarrow 0$ it yields a potential of the form:

$$V_q \simeq \frac{4\pi e^2}{L^3 q^2} \quad (1.15)$$

in a sample of size L. This form of potential holds also for the scattering between two valence electrons or between a valence and a conduction band electron that stay in their respective bands.

Interband scattering : processes in which one or two electrons change band are also possible and are referred as interband scattering. In the case of a single interband jump the potential reads $(\frac{q}{\kappa})V_q = \frac{4\pi e^2}{L^3 q \kappa}$ where κ is a dimensionnal factor that depends on the material. It then behaves as $1/q$. When both electrons change band the potential is then $(\frac{q}{\kappa})^2 V_q$ proportionnal to q^0 and thus remaining finite when $q \rightarrow 0$. Remarkably, in both the single and two interband jump scenarios, the number of electrons in each band changes by one and two, respectively, making these transitions less likely to occur compared to number-conserving processes.

Indeed, in second order perturbation theory the transition amplitude between an initial state $|i\rangle$ and a final state $|f\rangle$ may be written as :

$$\Gamma_{i \rightarrow f}^{(2)} = \frac{2\pi}{\hbar} \left| \sum_m \frac{\langle f | V_q | m \rangle \langle m | V_q | i \rangle}{E_f - E_i} \right|^2 \delta(E_f - E_i) \quad (1.16)$$

Whenever an electron undergo a band jump the energy cost is of the order of the band gap $E_f - E_i \simeq E_g$. As a consequence the corresponding transition amplitude gets reduced by a factor $\frac{1}{E_g}$ and appears to be negligible. However a direct intraband scattering can happen in several interband jumps that will dress the direct Coulomb scattering as shown in [Figure 1.5 b\)](#) for the case of two jumps. When summing over all possible intermediate state as is [\(1.16\)](#) we end up with the same potential as the direct intraband scattering but reduced by dielectric constant factor ϵ_{sc} that depend on the material.

To clarify this idea let us now consider the scattering between a conduction and a valence band electron. The same scattering process can also occur in two steps. First, a conduction band electron is scattered in its band while a valence band electron is promoted to the conduction band. Since an electron changes band it brings a potential $V_q(q/\kappa)$. Secondly, the excited electron goes back to the valence band while a valence band electron is scattered in its band yielding again a potential $V_q(q/\kappa)$. This two steps interaction end up with the same initial and final states than direct intraband scattering and the number of electrons in each band is conserved. In terms of energy, the two scatterings behaving as $1/q$ potentials the overall potential behaves also as $1/q^2$ but with an additionnal $1/E_g$ factor coming from the intermediate interband jumps. More specifically the two steps contribution is : $(4\pi e^2/L^3 q^2) [(e^2/L^3 \kappa^2)/E_g]$.

It is also possible to make it a three steps scattering by adding an intermediate interband exchange after the first step : the previously excited conduction band electron returns to the valence band while a valence band electron is promoted to the conduction band. More explicitly the process is as follows :

- **Step 1 :** a valence band electron is promoted to the conduction band while a conduction band is scattered in its band. The potential reads $V_q(q/\kappa) \propto \frac{1}{q}$ and the energy cost is of the order of E_g .
- **Step 2 :** the previously excited conduction band electron returns to the valence band while another valence band electron is excited to the conduction band. The potential read $V_q(q/\kappa)^2 \propto q$ and the energy cost is of the order of $2E_g$.
- **Step 3 :** the excited electron goes back to the valence band while a valence band electron is scattered in its band. The potential read again $V_q(q/\kappa) \propto \frac{1}{q}$ and the energy cost is of the order of E_g .

The total energy budget is then : $(4\pi e^2/L^3 q^2) [(e^2/L^3 \kappa^2)/E_g]^2$. Remarkably, one might expect the overall potential to scale as $1/E_g^3$ since each step requires at least one electron to change band. However, the structure of perturbation theory ensures that the total energy denominator accounts for the number of coupled interband transitions, rather than treating each step independently. Step 2 effectively "couples" the transitions in such a way that two energy denominators are introduced, rather than three independent ones, which explains the $1/E_g^2$ dependence. This coupling reflects the fact that intermediate states are shared between adjacent steps in the perturbative sequence.

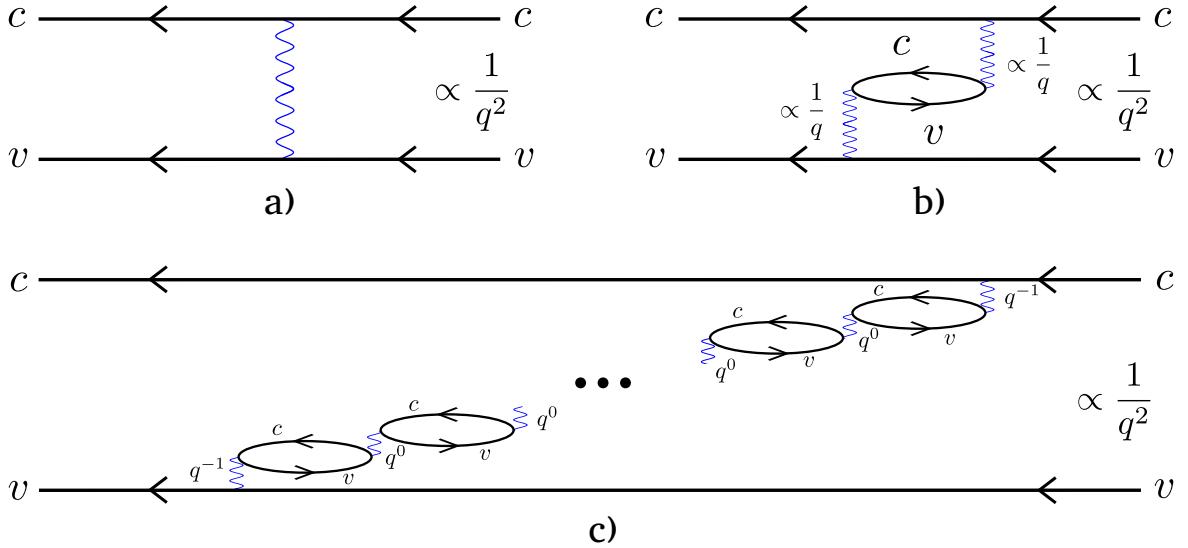


Fig. 1.6 **Feynman diagrams representing an intraband scattering between a conduction and a valence band electron.** a) Direct intraband scattering. The blue wavy lines represent the Coulomb interaction that make the electrons to change states. b) Two steps scattering involving an intermediate interband exchange that is represented by the bubble in the middle. c) Multiple steps scattering involving several interband exchanges represented by the numerous bubbles. Drawing inspired from [13].

Finally, the number of intermediate states can be arbitrarily increased by adding 'bubbles' of exchange interaction, as illustrated in the Feynman diagram in Figure 1.6 b). Each pair of

adjacent bubbles represents an interband interaction similar to the one described in Step 2 above. While it conserves the number of electrons in each band, it involves two interband jumps. The interaction between two bubbles is then $\propto q^0$ and is finite in the small momentum limit. By summing the contributions of processes with all possible numbers of bubbles, we obtain an effective Coulomb potential:

$$V_q = \frac{4\pi e^2}{L^3 q^2 \epsilon_{sc}} \quad (1.17)$$

where ϵ_{sc} is a dielectric constant stemming from the infinite sum over the number of bubbles. The discussion that we just made also holds for the intraband scattering between two conduction band electrons or two valence band electrons. Indeed, it graphically boils down to change the letter of the bands in the Feynman diagrams of [Figure 1.6](#) which doesn't change the overall energy budget of the process. Consequently, the potential (1.17) is valid for all intraband scattering processes. Based on the previous discussion we restrict the Coulomb potential that describe any scattering process to interactions that conserve the number of particle in each band, namely the intraband scattering potential between conduction and valence band electrons derived above. The general form of the potential is then in second quantization :

$$V_{coul} \simeq \frac{1}{2} \sum_{\mathbf{q} \neq 0} \frac{4\pi e^2}{L^3 q^2 \epsilon_{sc}} \sum_{\mathbf{k}_1, \mathbf{k}_2} \sum_{i_1, i_2} a_{i_1, \mathbf{k}_1 + \mathbf{q}}^\dagger a_{i_2, \mathbf{k}_2 - \mathbf{q}}^\dagger a_{i_2, \mathbf{k}_2} a_{i_1, \mathbf{k}_1} \quad (1.18)$$

where $i_1, i_2 \in (c, v)$. The creation operators $a_{c, \mathbf{k}}^\dagger$ and $a_{v, \mathbf{k}}^\dagger$ create respectively a conduction and a valence band electron with momentum \mathbf{k} . The $\mathbf{q} = 0$ contribution is not included and treated separately by the wise choice of an average electron-electron potential V_{e-e} that cancels it. More details can be found in the Appendix of [13]. It is worth noting that, in the conduction-valence electron framework, all intraband interactions are identical and repulsive. From this it is not obvious how a bound state like an exciton can arise in the material. However, as we will see in the next section, transitioning to the electron-hole description transforms one of these scattering interactions into an attractive interaction. This feature is central to the energy splitting between two types of excitons: bright excitons, which couple to light, and dark excitons, which do not.

1.2.4.2 From the conduction-valence electrons to electron-hole picture

So far we described the scattering process within the semiconductor using valence and conduction electrons. However, the exciton results from the binding between a conduction band electron and the absence of a valence band electron that we usually describe as a hole. While the conduction electron description stays the same, moving from one picture to the other require to define the hole creation operator that account for the removal of a valence band electron or equivalently the creation of a hole in the valence band.

$$a_{c, \mathbf{k}}^\dagger = a_{\mathbf{k}}, \quad (1.19a)$$

$$a_{v, \mathbf{k}}^\dagger = h_{-\mathbf{k}}^\dagger, \quad (1.19b)$$

where $h_{-\mathbf{k}}^\dagger$ creates a hole in the valence band with momentum $-\mathbf{k}$. The - sign is imposed by energy and momentum conservation laws. When we substitute the full valence band with a missing electron with momentum \mathbf{k}_v by a hole, the hole needs to have opposite charge $+|e|$, momentum $\mathbf{k}_h = -\mathbf{k}_v$ and energy $E_h(\mathbf{k}) = -E_v(\mathbf{k})$ to account for the "absence" of the valence electron. In the small momentum limit we have $E_v(\mathbf{k}) = E_{v0} + \frac{\hbar^2 k_v^2}{2m_v}$ with m_v the valence electron effective mass that is negative as visible in [Figure 1.4](#). The constraint on the energy $E_h = -E_v$ then imposes $\frac{\hbar^2 k_h^2}{2m_h} = -\frac{\hbar^2 k_v^2}{2m_v}$ making the effective mass of the hole positive. Its charge and mass being positive it can then undergo attractive interactions with a conduction band electron and eventually form an exciton. We already see how describing the valence band with hole instead of electrons can predict the formation of bound state. Let us now look how this feature arise from a microscopic point of view. The full crystal hamiltonian is :

$$\mathcal{H} = \mathcal{H}_0 + V_{coul} \quad (1.20)$$

with \mathcal{H}_0 the free electron hamiltonian and V_{coul} the Coulomb interaction between electrons. The free electron hamiltonian reads :

$$\mathcal{H}_0 = \sum_{\mathbf{k}} E_c(\mathbf{k}) a_{c,\mathbf{k}}^\dagger a_{c,\mathbf{k}} + \sum_{\mathbf{k}} E_v(\mathbf{k}) a_{v,\mathbf{k}}^\dagger a_{v,\mathbf{k}} \quad (1.21)$$

where $E_c(\mathbf{k})$ and $E_v(\mathbf{k})$ are the conduction and valence band energies respectively. The Coulomb interactions part can be decomposed in the following way :

$$V_{coul} = V_{cc} + V_{vv} + V_{cv} \quad (1.22)$$

Using the newly defined operators together with fermionic commutation relation $a_{i,\mathbf{k}}^\dagger a_{i,\mathbf{k}} = 1 - a_{i,\mathbf{k}} a_{i,\mathbf{k}}^\dagger$ the different scattering potentials are modified as we now show.

- The one body hamiltonian reads :

$$\mathcal{H}_0 = \sum_{\mathbf{k}} E_v(\mathbf{k}) + \sum_{\mathbf{k}} E_c(\mathbf{k}) a_{\mathbf{k}}^\dagger a_{\mathbf{k}} + \sum_{\mathbf{k}} -E_v(\mathbf{k}) h_{\mathbf{k}}^\dagger h_{\mathbf{k}}. \quad (1.23)$$

- The scattering between two conduction band electron is the same as before :

$$V_{cc} = \frac{1}{2} \sum_{\mathbf{q} \neq 0} V_{\mathbf{q}} \sum_{\mathbf{k}_1, \mathbf{k}_2} a_{\mathbf{k}_1 + \mathbf{q}}^\dagger a_{\mathbf{k}_2 - \mathbf{q}}^\dagger a_{\mathbf{k}_2} a_{\mathbf{k}_1} \equiv V_{ee} \quad (1.24)$$

- The scattering between two valence band electrons splits into three terms :

$$V_{vv} = -\frac{N_v}{2} \sum_{\mathbf{k} \neq 0} V_{\mathbf{k}} + \sum_{\mathbf{k}} \hat{h}_{\mathbf{k}}^\dagger \hat{h}_{\mathbf{k}} \sum_{\mathbf{k}' \neq \mathbf{k}} V_{\mathbf{k}' - \mathbf{k}} + V_{hh}. \quad (1.25)$$

The first term is constant and account for repulsive exchange interaction between all valence electrons. The second term comes from the scattering between a valence electron

with wavevector \mathbf{k} and all the other valence electrons. It adds a shift to the hole kinetic energy $E_h = -E_v$ that appears in the one body hamiltonian \mathcal{H}_0 . The last term is the hole-hole interaction that is repulsive and is the same as the valence electron-electron interaction with opposite wavevectors :

$$V_{hh} = \frac{1}{2} \sum_{\mathbf{q} \neq 0} V_{\mathbf{q}} \sum_{\mathbf{k}_1, \mathbf{k}_2} h_{\mathbf{k}_1 + \mathbf{q}}^\dagger h_{\mathbf{k}_2 - \mathbf{q}}^\dagger h_{\mathbf{k}_2} h_{\mathbf{k}_1} \quad (1.26)$$

- Finally, scattering between a conduction and a valence band electron is :

$$V_{cv} = \sum_{\mathbf{q} \neq 0} V_{\mathbf{q}} \sum_{\mathbf{k}_1, \mathbf{k}_2} a_{c, \mathbf{k}_1 + \mathbf{q}}^\dagger a_{v, \mathbf{k}_2 - \mathbf{q}}^\dagger a_{v, \mathbf{k}_2} a_{c, \mathbf{k}_1}. \quad (1.27)$$

Using that for non zero momentum transfer $[a_{v, \mathbf{k}_2 - \mathbf{q}}^\dagger, a_{v, \mathbf{k}_2}] = 0$ and introducing $\mathbf{k}'_2 = -(\mathbf{k}_2 - \mathbf{q})$ the potentials in terms of electron-hole reads :

$$V_{cv} = - \sum_{\mathbf{q} \neq 0} V_{\mathbf{q}} \sum_{\mathbf{k}_1, \mathbf{k}'_2} a_{\mathbf{k}_1 + \mathbf{q}}^\dagger h_{\mathbf{k}'_2 - \mathbf{q}}^\dagger h_{\mathbf{k}'_2} a_{\mathbf{k}_1} \equiv V_{eh}. \quad (1.28)$$

The latter require a special attention since it exhibits an attractive interaction and reveals the possibility of excitons formation which was not obvious in the conduction-valence electron picture.

Spin conservation in electron scattering

Scatterings processes mediated by the Coulomb interactions conserve the spin of the electron for both interband and intraband scatterings. As a consequence, the interactions between electrons and holes that we just exhibited are also spin preserving. The latter has important consequences to determine selection rules when looking at optical excitation of an electron-hole pair.

So far we demonstrated that replacing the full valence band by a hole looks quite convenient since it allow to forget about all the valence band electrons and exhibits the possibility of bound state formation. However, it must be done with caution since the remaining valence band electrons undergo the many intraband scattering processes described in the section and reduce the effective Coulombic intraband interaction by a factor $1/\epsilon_{sc}$. Remarkably, the intraband scatterings between valence and conduction band electrons V_{cv} are at the origin of the attractive electron-hole interactions while valence-valence interactions are responsible for the shift in the hole kinetic energy as expressed by [Equation 1.25](#). Let us now explain how the aforementioned electron-hole attractive interactions can lead to exciton formation.

1.2.4.3 The electron-hole hamiltonian

If we restrict the previous description to a one pair subspace namely to the interactions between a single electron and a single hole, electron-electron and hole-hole interactions can be safely dropped since they require two holes or electrons. The one pair hamiltonian then reads :

$$\mathcal{H}_{eh} = \mathcal{H}_e + \mathcal{H}_h + V_{eh}, \quad (1.29)$$

$$= \sum_{\mathbf{k}} E_c(\mathbf{k}) a_{\mathbf{k}}^\dagger a_{\mathbf{k}} + \sum_{\mathbf{k}} E_h(\mathbf{k}) h_{\mathbf{k}}^\dagger h_{\mathbf{k}} - \sum_{\mathbf{q} \neq 0} V_{\mathbf{q}} \sum_{\mathbf{k}_1, \mathbf{k}_2} a_{\mathbf{k}_1 + \mathbf{q}}^\dagger h_{\mathbf{k}_2 - \mathbf{q}}^\dagger h_{\mathbf{k}_2} a_{\mathbf{k}_1}. \quad (1.30)$$

Wannier exciton are the solutions $|x\rangle$ of the eigenproblem :

$$(\mathcal{H}_e + \mathcal{H}_h + V_{eh}) |i\rangle = E_i |i\rangle \quad (1.31)$$

with eigenvalue E_i . At this level the hamiltonian is very general and describe two coupled harmonic oscillators. Without going into details, this problem can be solved by following the same procedure as in classical mechanics. The core of the derivation consists in decoupling the two oscillators by introducing new coordinates that are the center of mass and relative coordinates. Since Coulombic interaction conserve momentum, the total momentum of the electron-hole pair $k_e + k_h$ stays constant and is equal to the exciton center of mass momentum \mathbf{Q} . Namely, we define :

$$\mathbf{Q} = \mathbf{k}_e + \mathbf{k}_h, \quad (1.32a)$$

$$\mathbf{q} = \gamma_h \mathbf{k}_e - \gamma_e \mathbf{k}_h, \quad (1.32b)$$

where $\gamma_e = 1 - \gamma_h = m_e / (m_e + m_h)$ is the reduced mass ratio and \mathbf{q} is the relative motion momentum. We then define the creation operator that creates an exciton $|x\rangle$ in state (\mathbf{Q}_i, ν_i) in terms of electron and hole operators :

$$b_{\mathbf{Q}_i, \nu_i}^\dagger = \sum_{\mathbf{p}} f_{\mathbf{p}}^{\nu_i} a_{\gamma_e \mathbf{Q}_i + \mathbf{p}}^\dagger h_{-\mathbf{p} + \gamma_h \mathbf{Q}_i}^\dagger \quad (1.33)$$

where $f_{\mathbf{p}}^{\nu_i} = \langle \mathbf{p} | \nu_i \rangle$ is the relative motion wave function. Let us show that the states $|\nu_i\rangle$ are the eigensolutions of a Hydrogen like problem ν_i being then related to the first quantum number of an electron orbitals. Injecting $|x\rangle = b_{\mathbf{Q}_i, \nu_i}^\dagger |0\rangle$, $|0\rangle$ being the vacuum state in [Equation 1.31](#) we obtain \mathcal{H}_{eh} in a diagonal form :

$$\mathcal{H}_{eh} b_{\mathbf{Q}_i, \nu_i}^\dagger |0\rangle = \sum_{\mathbf{p}} \left\{ \left(E_g + \frac{\mathbf{Q}_i^2}{2m_X} + \frac{\mathbf{p}^2}{2\mu_X} \right) f_{\mathbf{p}}^{\nu_i} - \sum_{\mathbf{q} \neq 0} V_{\mathbf{q}} f_{\mathbf{p}-\mathbf{q}}^{\nu_i} \right\} a_{\mathbf{p} + \gamma_e \mathbf{Q}_i}^\dagger h_{-\mathbf{p} + \gamma_h \mathbf{Q}_i}^\dagger |0\rangle \quad (1.34)$$

whose eigenstates are $b_{\mathbf{Q}_i, \nu_i}^\dagger |0\rangle$ with energy $E_{X_i} = E_g + \frac{\mathbf{Q}_i^2}{2M_X} + E_b^i$ if and only if $f_{\mathbf{p}}^{\nu_i}$ are solution of the Hydrogen-like problem :

$$\frac{\mathbf{p}^2}{2\mu_X} f_{\mathbf{p}}^{\nu_i} - \sum_{\mathbf{q} \neq 0} V_{\mathbf{q}} f_{\mathbf{p}-\mathbf{q}}^{\nu_i} = E_b^i f_{\mathbf{p}}^{\nu_i} \quad (1.35)$$

where $\mu_X^{-1} = m_e^{-1} + m_h^{-1}$ is the relative motion mass and $m_X = m_e + m_h$ the total mass. The exciton then appears as a bound state of the electron-hole pair with a binding energy

E_{b_i} whose center of mass is delocalized as a plane wave with momentum \mathbf{Q} . The binding energies E_b^i follows a Rydberg series $E_b^i = -\frac{R_X}{i^2}$ where R_X is the Rydberg constant of the exciton as defined in subsection 1.2.3 In the following section we will see how the excitons interact with light.

1.2.5 Exciton-photon interaction in bulk semiconductor:

The creation of an exciton can be mediated by the absorption of a photon by the material. On the other the annihilation of a exciton can lead to photon emission. As a start we describe this interaction with an usual electron-photon Hamiltonian in first quantization:

$$\mathcal{H}_{dip} = -e \frac{\mathbf{p}_e \cdot \mathbf{A}}{m_e}, \quad (1.36)$$

with \mathbf{p}_e the momentum operator of the electron and \mathbf{A} the potential vector of the electromagnetic field. Before going further let us have a look at what the symmetries of this Hamiltonian imply based on the Noether theorem.

- **Total momentum conservation :** translational invariance of the system implies that the total momentum of the system is conserved. The absorption of a photon with momentum $\hbar\vec{k}_c$ will create an exciton with the same momentum.
- **Total angular momentum conservation :** rotational invariance of the system implies that the total angular momentum of the system is conserved along the interaction.

The second point raises the question of the possible values that the exciton angular momentum can assume. To answer this question we need to get back to its microscopic constitutive particles : a conduction band electron and a hole in the valence band. In usual semiconductors a conduction band electron has an orbital angular momentum ($L = 0, L_z = 0$) while a valence band electron yields ($L = 1, L_z = (+1,0, -1)$) (see [25]). Since $J_z = J_z + S_z$ and the electron spin is $S_z = \pm 1/2$ we obtain for the valence band $J_z^v = (3/2, 1/2, -1/2, -3/2)$ and for the conduction band $J_z^c = \pm 1/2$. If we now move to the electron-hole picture the electron angular momentum remains unchanged $J_z^e = J_z^c = \pm 1/2$ while the hole angular momentum is the opposite of the valence band electron $J_z^h = -J_z^v = (3/2, 1/2, -1/2, -3/2)$. The exciton angular momentum is then $J_z = J_z^e + J_z^h = (-2, -1, 0, 1, 2)$. We can thus distinguish **bright** excitons with $J_z = (-1, 0, 1)$ that couple to light with polarization π, σ_+, σ_- and **dark** excitons with $J_z = (-2, 2)$ that do not couple to light. In the latter case the spin of the electron and the hole are aligned : $J_z = \pm 2$ excitons are made of $\pm 1/2$ electrons and $\pm 3/2$ holes. It means it corresponds to a situation in which the valence band electron flipped spin when it got promoted to the electron band. Yet, dark excitons can subsequently not undergo interband transitions that are spin preserving as mentionned in subsection 1.2.4.2. On the other hand, bright excitons result from interband transitions since both photon absorption and emission involve an electron changing band. In the framework of polariton which involve exciton creation through light one could think that since Coulomb inter and intraband scatterings are spin preserving, dark excitons are of no interest and won't play any role in the system. However, we will see that dark excitons can be created from bright excitons thanks to exchange carrier. that only bright excit

1.2.6 Exciton in 2D quantum well

Let us now get closer to the situation encountered in the sample used in the experiment. In such microcavities, excitons are confined in a planar quantum well which are made by stacking layers of semiconductor materials with different band gaps. The confinement in the plane is then ensured by the potential barrier between the layers. In our case a quantum well is made of InGaAs placed in between two layers of GaAs. InGaAs has a smaller band gap than the bulk GaAs because of the p-fraction of InAs doping as shown in [Figure 1.7 \(b\)](#). Both electrons and holes then remain confined in the region where the band gap is the smallest (see [Figure 1.7 \(a\)](#)). By repeating this "sandwiching" procedures it is possible to create many QW heterostructures as we shall see latter.

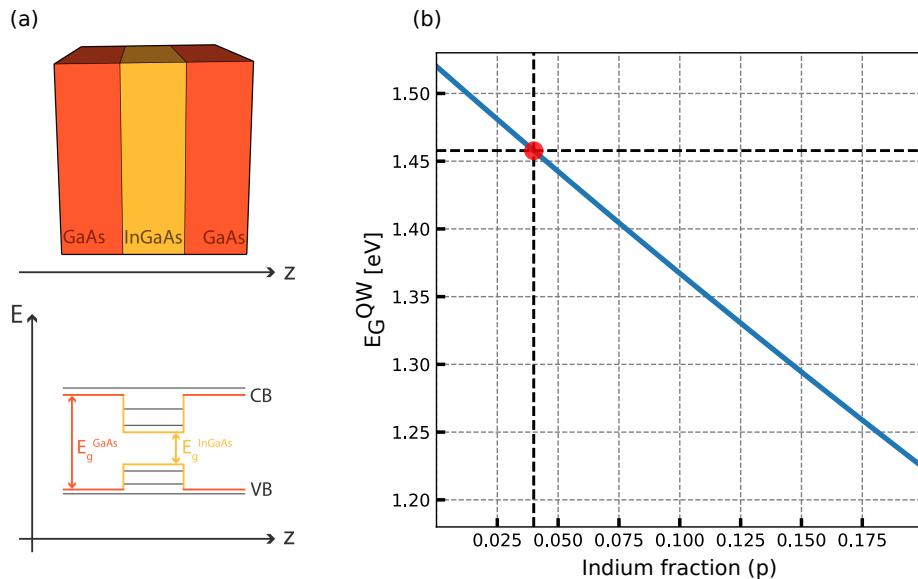


Fig. 1.7 **Schematic representation of a quantum well heterostructure.** a) Representation of a planar quantum well made of InGaAs placed in between two layers of GaAs. The confinement in the plane is ensured by the gap energy difference between the layers. b) Gap energy of the InGaAs quantum well as a function of the doping fraction of Indium. The red dot represent the doping in sample used in the experiment : $E_g^{InGaAs} = 1.46\text{eV} < E_g^{GaAs} = 1.52\text{eV}$.

Assuming the same geometry than the above figure the electron-hole hamiltonian taking into account the potential barrier is written in first quantization as :

$$\mathcal{H} = E_g^{QW} + \frac{\mathbf{p}_e^2}{2m_e} + \frac{\mathbf{p}_h^2}{2m_h} - \frac{e^2}{\epsilon_{sc}|r_e - r_h|} + V_e(z_e) + V_h(z_h) \quad (1.37)$$

where z is the growth axis of the quantum well and $V_e(z_e)$ and $V_h(z_h)$ are the potential barriers felt by the electron and the hole respectively, r_e and r_h are the electron and hole spatial coordinates. The system is invariant under translation in the xy -plane the excitons in plane wavevector k_{\parallel} can then take any value while the confinement in the z -direction quantize

the corresponding component of the wavevector k_z . This problem is treated by writting the exciton wavefunction $\Psi(x,y,z)$ as a two components product :

$$\Psi(x,y,z) = \psi(x,y,z)\phi(z). \quad (1.38)$$

The Schrodinger equation is then solved separately for $\psi(x,y)$ and $\phi(z)$. The confinement in z -direction is treated as a very basic quantum box problem. The electron and hole energies are then quantized as :

$$E_n^{e,h} = \frac{(\pi^2 n^2 \hbar^2)}{2m_{e,h} L_z^2} \quad (1.39)$$

where L_z is the width of the quantum well.

To determine the in-plane wavefunciton we follow the same procedure than for the exciton eigenproblem in the 3D case. The exciton is again a correlated pair whose center of mass is delocalized as a plane wave with momentum $\mathbf{Q} = \hbar\mathbf{k}_{||}$. The correspondig energy is then :

$$E_X^{xy} = E_g^{QW} + \frac{\hbar^2 \mathbf{k}_{||}^2}{2m_X} - \frac{R_X^{QW}}{n^2} \quad (1.40)$$

provided that we defined a new Rydberg constant R_X^{QW} taking into account the confinement in the z -direction. Remarkably, due to confinement the Bohr radius a_X^{QW} of the QW exciton is twice smaller than the bulk exciton Bohr radius. This is a direct consequence of the confinement that increases the electron-hole wavefunctions overlap and thus the binding energy of the exciton. The exciton total energy is then :

$$E_X(\mathbf{k}_{||}) = E_g^{QW} + E_n^e + E_n^h + \frac{\hbar^2 \mathbf{k}_{||}^2}{2m_X} - \frac{R_X^{QW}}{n^2} \quad (1.41)$$

The design of the sample used in the experimient is done such as the $n=1$ level is sufficiently far form the next level that we can restrict our description to the first 1s exciton state. In the following we will therefore drop the n index by setting it to one.

Bosonic nature of excitons. Since an exciton is made of two fermions its pseudo spin is an integer. However, when reaching regimes of high excitons densities the fermionic nature of their microscopic constitutives can no longer be neglected. The composite nature of excitons can then be safely dropped provided that the exciton density per unit area is such as $n_X(a_X^{QW})^2 \ll 1$. This condition state that the average distance between excitons is much larger than the exciton Bohr radius. In this regime the excitons can be treated as bosons, namely :

$$[b_{\mathbf{k}}, b_{\mathbf{k}}^\dagger] = 1 - O(n_X(a_X^{QW})^2) \simeq 1 \quad (1.42)$$

Optical selection rules in the planar quantum well. In bulk semiconductors, a single exciton mode couples exclusively to a single photon mode. In quasi-two-dimensional systems, due to

the breaking of translationnal invariance in the growth direction z the momentum conservation during a photon absorption or emission remains valid only for the in-plane component, the z -component being fixed by the quantum well width. Accordingly, an exciton with wavevector $\mathbf{k}_X = (\mathbf{k}_{\parallel}^X, k_z^X)$ will interact with a photon with the same in plane wavevector but with any value of k_z^{γ} . There is thus a density of states for radiative decay given by (see [53]):

$$\rho_{2D}(\mathbf{k}_{\parallel}, \hbar\omega) \equiv \sum_{k_z} \delta(\hbar\omega - \frac{\hbar c}{n_{QW}} \sqrt{\mathbf{k}_{\parallel}^2 + k_z^2}) \propto \frac{\hbar\omega}{\sqrt{(k_0^2 - \mathbf{k}_{\parallel}^2)}} \Theta(k_0 - |\mathbf{k}_{\parallel}|) \quad (1.43)$$

where where n_{QW} is the refractive index in the QW, $k_0 = n_{QW}\omega/c$ is the wavevector of the photon in the material and Θ is the Heaviside step function. Only states with $|\mathbf{k}_{\parallel}| < k_0$ can couple to light giving rise again to two class excitons.

- States with $|\mathbf{k}_{\parallel}^X| < k_0$ that couple to light. These states have a finite radiative lifetime $1/\gamma_X$ given by the Fermi Golden rule. They lie on the left of the light cone in the $\mathbf{k}_{\parallel} - \omega$ plane.
- States with $|\mathbf{k}_{\parallel}^X| > k_0$ that do not decay radiatively. They lie on the right of the photon line in the the $\mathbf{k}_{\parallel} - \omega$ plane. They are analog to surface modes as they have a vanishing electric field far from the QW. These states join the previously described excitons that could not couple to light because of their spin configuration to form a wider class of "dark" excitons.

This stands in contrast to bulk semiconductors, where a single exciton mode couples exclusively to a single photon mode. Nevertheless, if we also impose energy conservation between the exciton and the photon mode the maximum in-plane wavevector of radiative excitons gets modified and depend on the exciton energy. To see this let us consider an initial state made of an exciton of overall energy $E_X(\mathbf{k}_{\parallel}^X)$ and zero photon and a final state made of a single photon with energy $E_{\gamma}(\mathbf{k}_{\parallel}^{\gamma}) = \frac{\hbar c}{n_{QW}} \sqrt{(\mathbf{k}_{\parallel}^{\gamma})^2 + (k_z^{\gamma})^2}$ and no exciton. The energy conservation of this process reads :

$$E_X(\mathbf{k}_{\parallel}^X) = E_{\gamma}(\mathbf{k}_{\parallel}^{\gamma}) \quad (1.44a)$$

$$E_X^{QW} + \frac{(\hbar\mathbf{k}_{\parallel}^X)^2}{2m_X} = \frac{\hbar c}{n_{QW}} \sqrt{(\mathbf{k}_{\parallel}^{\gamma})^2 + (k_z^{\gamma})^2}, \quad (1.44b)$$

in which we set $E_X^{QW} = E_g^{QW} + E_X^{1s} = E_g^{QW} + E_1^e + E_1^h - R_X^{QW}$. Injecting the in-plane momentum conservation and imposing $k_z^{\gamma 2} \geq 0$ for the photon field to be propagative the above condition restricts the exciton that can couple to light through :

$$|\mathbf{k}_{\parallel}^X| \leq \frac{cm_X}{n_{QW}\hbar} \left(1 + \sqrt{1 - \frac{4n_{QW}E_X^{QW}}{m_Xc^2}} \right). \quad (1.45)$$

Since E_X^{QW} can be neglected with respect to the exciton mass energy m_Xc^2 the above condition taken at first order gives :

$$|\mathbf{k}_{\parallel}^X| \leq \frac{n_{QW} E_X^{QW}}{\hbar c}. \quad (1.46)$$

Excitons with higher wavevector are than forbidden to exchange energy with light. However, in our case we have $E_X^{QW} \simeq 1.485/\text{meV}$ and $n_{QW} \simeq 3.5$ leading to $|\mathbf{k}_{\parallel}^X| \leq 30/\mu\text{m}^{-1}$ which is way beyond the values addressed in the experiment. Now that the exciton formation is well established let us have a look at the mechanisms that can lead them to interact.

1.2.7 Exciton-exciton interaction

The scope of this section is to answer the question of what can lead two excitons in states (i,j) to end up in states (f,g) . Obviously, direct coulomb interactions between the excitons constitutive is an unambiguous valid answer. Nonetheless, an interaction giving the same result can also occur in the absence of any Coulomb interactions via pure carrier exchange. Namely, excitons i and j can exchange either their electron or hole. The latter is a direct consequences of the Pauli exclusion principle stating that two fermions cannot occupy the same quantum state.

1.2.7.1 Coulomb interaction

As explained in the previous section, holes and electrons undergo Coulomb interaction. As a consequence, the interaction between two excitons (e_1,h_1) and (e_2,h_2) consist of $V_{e_1e_2} + V_{h_1h_2} + V_{e_1h_2} + V_{h_1e_2}$. However, if the exciton were made of (e_1,h_2) and (e_2,h_1) the cross term in the interaction potential would be replaced by $V_{e_1h_1} + V_{e_2h_2}$. Because there are different possible ways to form excitons, the total interaction energy depends on the specific pairing choice. This makes it impossible to define a single, clean exciton-exciton interaction potential that works in all cases. Hence, treating them as pure bosons with a simple interaction potential fail because the effective exciton-exciton potential is not unique and depends on the microscopic way the excitons are formed. Nevertheless, it is clear that, through Coulomb interaction, excitons can change states and thus interact in the most general sense. This being said, we will as experimentalist, focus on the most important scattering process that is actually coming from exchange carrier among excitons and that is a unique feature of composite particle.

1.2.7.2 Fermionic exchange interaction

Let us consider two excitons $X_1 = |\mathbf{k}_e, \mathbf{k}_h\rangle$ and $X_2 = |\mathbf{k}'_e, \mathbf{k}'_h\rangle$. If their wavefunction start to overlap the two electrons for example might see their quantum state becoming almost equal which is forbidden by the Pauli exclusion principle. To regularize this situation the exciton can exchange their carrier to form $X'_1 = |\mathbf{k}'_e, \mathbf{k}_h\rangle$ and $X'_2 = |\mathbf{k}_e, \mathbf{k}'_h\rangle$. This is might look counterintuitive since exchanging two particles in the same quantum state doesn't seem to solve the problem. However, it's truly the exchange process itself put together with wavefunction symmetries that modify the quantum state configuration to avoid Pauli principle violation. Take two indistinguishable particles, in state ψ_a and ψ_b in a 1D system [10]. Normalization consideration aside, the wavefunction of the system can be written in two ways : $\psi_{ab} = \psi_a(x_1)\psi_b(x_2) \pm \psi_a(x_1)\psi_b(x_2)$. Depending on the plus or minus sign, the wavefunction

is symmetric or antisymmetric under position exchange. The two different configurations imply different physics. For instance the mean value of the particle interdistance is :

$$\langle(x_1 - x_2)^2\rangle = \langle x_1^2 \rangle + \langle x_2^2 \rangle \mp 2\langle x_1 x_2 \rangle, \quad (1.47)$$

provided the wavefunctions overlap $\langle x_1 x_2 \rangle \neq 0$, the expected value is decreased for bosons and increased for fermions. Although, it seems to act as a force repelling or attracting particles, it is rather a geometrical constraint arising from the Pauli principle and the quantum nature of the particle at stake. This is precisely what happened during carrier exchange and can, at the end, lead to scattering processes between excitons. Forgetting about this interaction and trying to fully bosonize excitons would then be a mistake [5]. As an example, some features experimentally measured like exciton stark effect, precisely needs to put exchange carrier on the table to be understood [23; 27; 34]. Furthermore, in a 2D quantum well system this scatterings are dominant at larger scale while direct Coulomb interactions are dominant at scales smaller than the Bohr radius of excitons. Therefore, by considering only radiative modes, i.e. dynamics occurring on scales much larger than the Bohr radius of excitons $ka_X \ll 1$, Coulomb interactions can be neglected. Despite all the differences with true bosons above mentioned and the hot debates still going on about polaritons interactions [12; 15] this enable us, as experimentalist, to escape from ambiguity and write the main exciton-exciton potential that will be relevant in the experiment reported in this work. Written in momentum space this exciton-exciton potential $V_{XX}(\mathbf{k})$ appear as four body contact interaction [42]. The corresponding hamiltonian follows as :

$$\mathcal{H}_{XX} = \frac{1}{2} \sum_{\mathbf{k}, \mathbf{k}', \vec{q}} V_{XX}(\vec{q}) \hat{b}_{\mathbf{k}+\vec{q}}^\dagger \hat{b}_{\mathbf{k}'-\vec{q}}^\dagger \hat{b}_{\mathbf{k}} \hat{b}_{\mathbf{k}'}. \quad (1.48)$$

The potential $V_{XX}(\vec{q})$ is a function of the exchanged momentum \vec{q} . However, it can be approximated by a constant in the limit of low wavevectors since, being four order of magnitude heavier than microcavities photons, their varies only slightly with \mathbf{k} in the range of the photon wavevector that excite them. Therefore V_{XX} can be assumed constant. The numerical calculation gives $A \times V_{XX} \simeq 3 \mu\text{eV}\cdot\mu\text{m}^2$ for our InGaAs system.

$$V_{XX} = \frac{e^2 a_X^{QW}}{\epsilon A}, \quad (1.49)$$

with A the quantization area equal to the quantum well surface. The numerical calculation gives $A \times V_{XX} \simeq 3 \mu\text{eV}\cdot\mu\text{m}^2$ in our GaAs-InGaAs system.

Dark excitons creation through carrier exchange.

As discussed in subsection 1.2.5 electron-hole pairs with a total spin of ± 1 are coupled to $\sigma \pm$ photons, meaning that photon absorption leads to the creation of only bright exciton states ± 1 . However, dark exciton states can also be present in the system, arising from fermion exchange processes. Namely, fermion exchange between two bright excitons $(+1, 1)$ made of $(\mp 1/2)$ electrons and $(\pm 3/2)$ holes leads to dark excitons $(+2, -2)$. Hence, even though photon absorption only create bright excitons, excitons not coupled to light are likely to be indirectly excited and participate to the overall interactions of the system. This being said,

we will in the following only tackle this problem from a phenomenological point of view and consider the exciton-exciton interaction as a unique potential V_{XX} whereas dark excitons will be accounted for through a reservoir.

1.3 Excitons-polaritons

So far we described separately microcavity photons and excitons both in bulk and 2D systems. Microcavity polaritons arise from the strong coupling between these two components. Remarkably, the half light half matter nature of polaritons allows them to inherit the best of both worlds namely the low photon effective mass and the strong excitons non linear interactions. In this section we will describe the formation of polaritons by describing how placing a 2D semiconducting quantum well in an optical microcavity to enhance the light-matter interactions. To start with let us have a look again at the Fermi Golden rule giving the spontaneous emission rate from a level $|i\rangle$ to a level $|f\rangle$ in a continuum of states per unit time in the presence of a weak coupling W :

$$\Gamma_{i \rightarrow f} = \frac{2\pi}{\hbar} \left| \langle f | W | i \rangle \right|^2 \rho(E_f) \delta(E_f - E_i) \quad (1.50)$$

where $\rho(E_f)$ is the density of states at energy E_f and $\delta(E_f - E_i)$ ensures energy conservation. Say, we want to "force" an exciton to recombine in a given photonic state $(\hbar\omega_\gamma, \mathbf{k}_\gamma)$. In a 2D quantum well, an exciton with in plane wavevector \mathbf{k}_\parallel^X is allowed to couple to a continuum of photon modes provided their wavevectors respect [Equation 1.46](#). Solving this problem boils down to changing the density of state ρ in order to suppress spontaneous emission in all unwanted modes. As usually done in cavity quantum electrodynamics, this problem is tackled by embedding the quantum well in a high finesse optical microcavity as shown in [Figure 1.8 \(a\)](#).

The role of the latter is to create a set of resonant modes that will act as a "filter" for the exciton recombination. The quantization axis of the optical microcavity is chosen to be the same as the quantum well and the light-matter interactions are optimized by placing the quantum well at the antinode of the electric field. The cavity length is chosen so that single photonic mode can be equal the excitonic energy. This matching can be seen by looking simultaneously at the exciton and photon dispersion relation as a function of the in plane wavevector and check that they cross each other at a finite wavevector. For instance in the case of [Figure 1.8 \(b\)](#) the cavity has been designed so that the exciton and photon energies are equal at $\mathbf{k}_\parallel = 0$. Several quantum well can then be stacked in the cavity to enhance the interactions provided that they are once again located at antinodes of the photonic field. In the experiment we use a $3\lambda/2$ made of two planar DBR in which are placed three $InGaAs/GaAs$ quantum wells. An extended description of the fabrication procedure to build such a sample can be found in [31]. However, the main features mentioned so far can be summarized in a few points :

- The cavity is made by embedding semiconducting quantum wells in between two distributed bragg reflectors forming a high finesse optical microcavity.
- The length of the cavity and of the wells are chosen so a single longitudinal mode of the electric field is resonant only with the 1s state of the exciton.

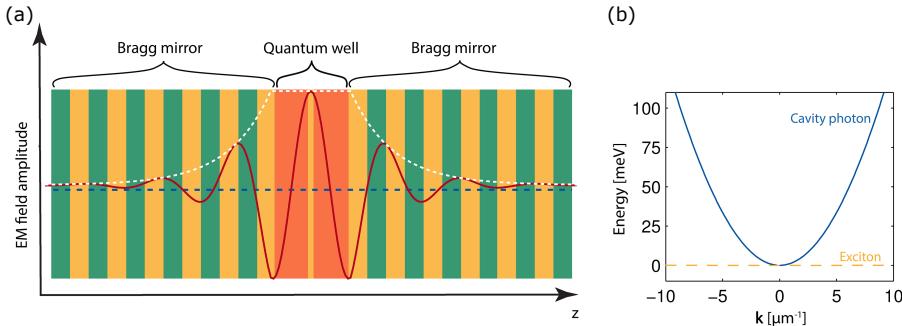


Fig. 1.8 **Schematic representation of a quantum well embedded in an optical microcavity.** The quantum wells are placed at the antinode of the electric field of the cavity. The cavity is designed to have a single mode that matches the exciton energy. (b) Dispersions relation of the exciton (yellow) and photon (blue) modes in a typical microcavity. Since the exciton effective mass m_X is 10^4 times larger than the photon mass the exciton dispersion looks flat with respect to the photon. The cavity has been designed so the $\mathbf{k}_{\parallel} = 0$ energies are matched.

- The exciton are confined in the xy -plane in each quantum well and the exciton density can be tuned by changing the number of quantum wells.
- Momentum conservation still holds in the plane of the cavity. Namely, an incoming photon with wavevector \mathbf{k}_{\parallel} will create an exciton with the same in-plane wavevector.

It appears then that the relevant motion of the interacting particles at stake lies in the xy -plane since the wavevectors in the z -direction are fixed by the cavity parameters. Consequently, for sake of clarity, we will only refer to the in-plane wavevector in the following by dropping the \parallel label and just write \mathbf{k} .

Now that some efforts were put in enhancing exciton-photon interactions let us see the different coupling regimes that we can be reached in such samples.

Coupling regimes. At first approximation the problem can be thought as two bath of particles exchanging quanta at a rate Ω_R . Depending on the efficiency of this exchange compared to the damping rate of each bath γ_X and γ_γ one has to revisit the way to describe the system. Indeed, when $\Omega_R \ll \gamma_X, \gamma_\gamma$ the system is said to be in the weak coupling regime. It can be understood in terms of lifetimes : both excitons and photons lifetimes are too short compared to the characteristic time of the exchange $1/\Omega_R$. The energy leaks out of the system before it can change from one state to the other. The exciton-photon interaction is in this case irreversible. If $\Omega_R \gg \gamma_X, \gamma_\gamma$ the system is said to be in the strong coupling regime. In a given photonic (or excitonic) lifetime $1/\gamma_{cav}$, the answer to the question "where is the energy ?" is no longer clear. Indeed the energy have had enough time to coherently go back and forth between exciton and photon states plenty of times. Translating this in quantum mechanics language simply means that the system is in a superposition of exciton and photon states. Polariton then rise in this regime as a superposition of microcavity photons and excitons exchanging energy at the so called Rabi frequency Ω_R .

Linear Hamiltonian. Let us write formally the coupling between excitons and photons described above. First we tackle the weak optical excitation scheme in which non linear interactions among excitons can be neglected. The corresponding linear Hamiltonian is given by :

$$\mathcal{H}_{lin} = \sum_{\mathbf{k}} \left[E_X(\mathbf{k}) b_{\mathbf{k}}^\dagger b_{\mathbf{k}} + E_\gamma(\mathbf{k}) a_{\mathbf{k}}^\dagger a_{\mathbf{k}} + \frac{\hbar\Omega_R}{2} a_{\mathbf{k}}^\dagger b_{\mathbf{k}} + a_{\mathbf{k}} b_{\mathbf{k}}^\dagger \right] \quad (1.51)$$

where $b_{\mathbf{k}}^\dagger$ and $a_{\mathbf{k}}^\dagger$ are the creation operators of the exciton and photon modes respectively. The first two terms account for the energies of the excitons and the photons while the terms $b_{\mathbf{k}}^\dagger a_{\mathbf{k}}$ and $a_{\mathbf{k}}^\dagger b_{\mathbf{k}}$ account for the linear exciton-photon coupling yielding an interaction energy $\hbar\Omega_R$. Obviously this hamiltonian is not diagonal in the excitons-photons operators basis. Excitons-polaritons then arise as the two eigenstates of \mathcal{H}_{lin} : the upper and the lower polaritons whose creation and annihilation operators are $(u_{\mathbf{k}}^\dagger, u_{\mathbf{k}})$ and $(p_{\mathbf{k}}^\dagger, p_{\mathbf{k}})$ respectively. These operators can be found by an unitary transformation on the exciton-photon basis that was extensively used by Hopfield [20] :

$$\begin{bmatrix} p_{\mathbf{k}} \\ u_{\mathbf{k}} \end{bmatrix} = \begin{bmatrix} C_{\mathbf{k}} & X_{\mathbf{k}} \\ X_{\mathbf{k}} & -C_{\mathbf{k}} \end{bmatrix} \begin{bmatrix} \hat{a}_k \\ \hat{b}_k \end{bmatrix}, \quad (1.52)$$

where $C_{\mathbf{k}}$ and $X_{\mathbf{k}}$ are the Hopfield coefficients for photons and excitons respectively. The unitarity of the transformation is reflected by the relation $C_{\mathbf{k}}^2 + X_{\mathbf{k}}^2 = 1$. As a consequence, they can be interpreted as the fractions of excitons and photons in both modes. More precisely they depend on the excitons/photons energies and the Rabi frequency as :

$$C_{\mathbf{k}}^2 = \frac{\sqrt{\Delta E_{X-\gamma}^2 + \hbar^2\Omega_R^2} - \Delta E_{X-\gamma}}{2\sqrt{\Delta E_{X-\gamma}^2 + \hbar^2\Omega_R^2}}, \quad (1.53a)$$

$$X_{\mathbf{k}}^2 = \frac{\sqrt{\Delta E_{X-\gamma}^2 + \hbar^2\Omega_R^2} + \Delta E_{X-\gamma}}{2\sqrt{\Delta E_{X-\gamma}^2 + \hbar^2\Omega_R^2}}. \quad (1.53b)$$

where $\Delta E_{X-\gamma}(\mathbf{k}) = E_\gamma(\mathbf{k}) - E_X(\mathbf{k})$ is the detuning between the exciton and the photon energies at \mathbf{k} . In this basis the linear hamiltonian can be written in its diagonal form.

$$\mathcal{H}_{lin} = \sum_{\mathbf{k}} \left[E_{LP}(\mathbf{k}) u_{\mathbf{k}}^\dagger u_{\mathbf{k}} + E_{UP}(\mathbf{k}) p_{\mathbf{k}}^\dagger p_{\mathbf{k}} \right] \quad (1.54)$$

where the eigenvalues $E_{LP}(\mathbf{k})$ and $E_{UP}(\mathbf{k})$ are the lower and upper polariton dispersion relation. Their analytical expressions are given by :

$$E_{LP}(\mathbf{k}) = \frac{E_X(\mathbf{k}) + E_\gamma(\mathbf{k})}{2} - \frac{\sqrt{\Delta E_{X-\gamma}^2 + \hbar^2 \Omega_R^2}}{2}, \quad (1.55)$$

$$E_{UP}(\mathbf{k}) = \frac{E_X(\mathbf{k}) + E_\gamma(\mathbf{k})}{2} + \frac{\sqrt{\Delta E_{X-\gamma}^2 + \hbar^2 \Omega_R^2}}{2}. \quad (1.56)$$

Tuning the exciton-photon fraction. As it can be seen the weight of photon and exciton in a given polaritonic state can be then tuned by changing the exciton-photon detuning $\Delta E_{X-\gamma}$. Particurlarly, when $\Delta E_{X-\gamma} = 0$ we have $C_{\mathbf{k}}^2 = X_{\mathbf{k}}^2 = \frac{1}{2}$ meaning that the exciton and the photon are equally mixed in the polariton states. In practice, there are two ways to change the exciton-photon detuning. The first tuning knob, which is the most obvious, is the wavevector \mathbf{k} . Since the exciton dispersion is flat with respect to the photon it boils down to move on the photon dispersion curve. Still, exciting a polariton state with a given set of Hopfield coefficients will fix the wavevector and, in some case might not be even possible. For instance if the photonic dispersion is above the exciton energy at all wavevector as in [Figure 1.9 b₁](#), it is impossible to reach the purely hybrid configuration $\Delta E_{X-\gamma} = 0$. Creating an arbitrary polaritonic state thus require a second knob to tune $\Delta E_{X-\gamma}$. It is generally done by introducing a small wedge between the two DBR that will change the photon resonance condition as shown in [Figure 1.9 a\)](#) : a change in the cavity length L will shift vertically the photon disperions. Luckily, this comes for free with the usual Molecular Beam Epitaxy method used to grow the sample. The principle is to heat the target material to its sublimation temperature to produce a molecular beam that is then deposited on a substrate to form thin atomic layers that stack on top of each other. In practice, the substrate is located on a spinning disk to avoid the molecule to clusterise. If the spinning disk is stopped, molecules will accumulate in the center of the beam and form a "hill". By taking advantages of this features it is possible to change progressively the thickness of the layer and thus the cavity resonance condition. A given sample has in fact a wide range of exciton-photon detuning that can be adressed in the lab by changing the working point on the cavity.

In general, when the exciton-photon detuning is too positive the UP branch recovers the excitons curve while the LP branch recovers the photons curve. Conversely, when the exciton-photon detuning is negative the UP branch recovers the photons curve while the LP branch recovers the excitons curve. This is not very suprising since a great detuning means that the exciton and the photon are far from each other making the coupling less efficient. At the zero detuning point $\Delta E_{X-\gamma} = 0$ the coupling is maximum and the splitting between the lower and upper polariton branch is equal to the Rabi energy frequency $\hbar \Omega_R$. Besides, the anticrossing between the two branches is a direct consequence of the strong coupling regime and can be find even in classical systems like coupled pendulums.

The suitable range of detuning for which the polariton concept is still relevant and that we can adress in the experiment is shown in [Figure 1.10](#)

Relaxation. The strong coupling condition $\Omega_R \gg \gamma_X, \gamma_{cav}$ can also be seen by directly taking into account the finite lifetime of excitons and photons, introducing imaginary energies $E_\gamma - i\gamma_{cav}$ and $E_X - i\gamma_X$ as in [11]. Doing so, the polariton dispersions relation become :

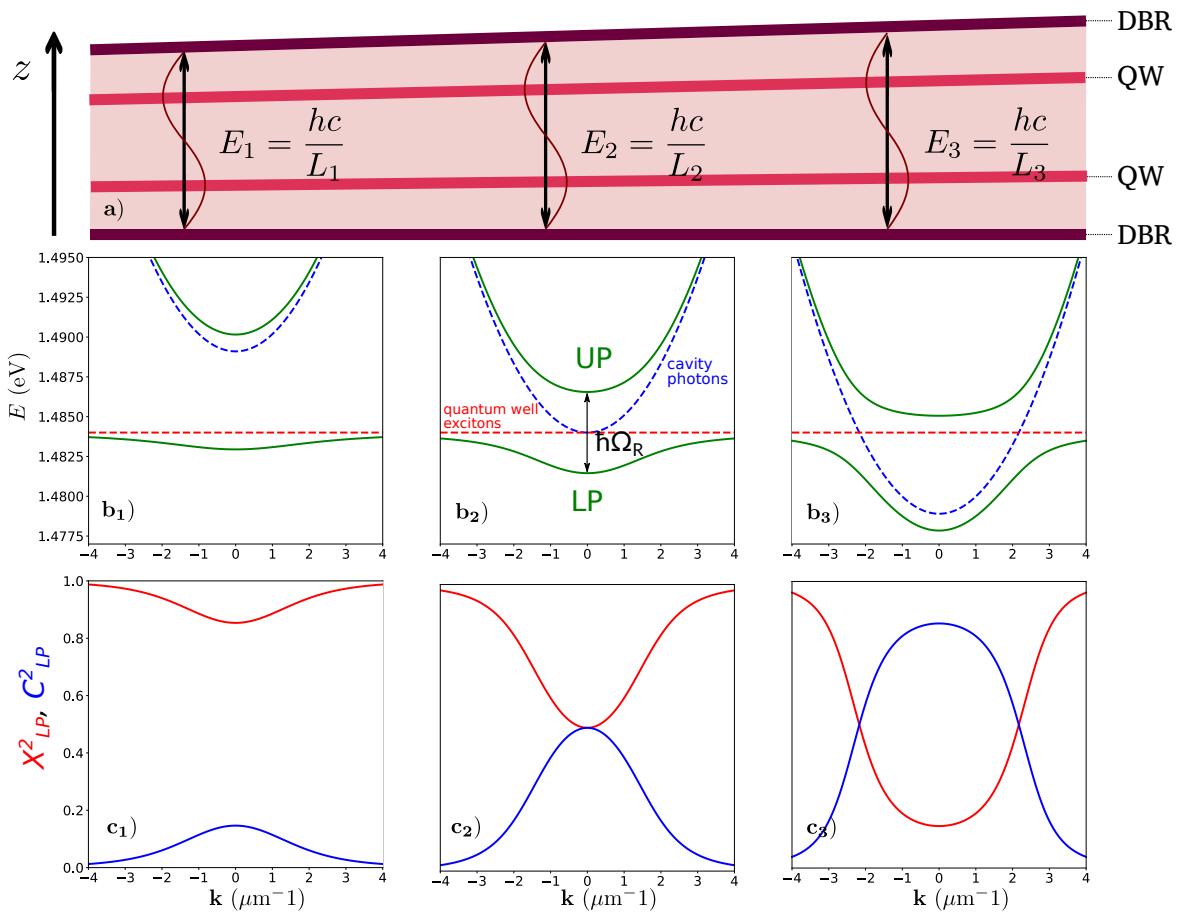


Fig. 1.9 **Scheme of an optical microcavity with a wedge, polaritons dispersion relations and Hopfield coefficients.** a) Side view of a double quantum well semiconducting microcavity. From left to right the photon resonant energy decreases as the length of the cavity increases. The exciton energy remains constant since it is fixed by the width of the quantum well. We take as example three point in the sample $i = 1,2,3$ with increasing cavity length. At each point the wavy lines represent the optical standing wave stemming from the resonance conditions $\lambda_i = L_i$. The corresponding energies E_i illustrate the three different regimes namely : $\Delta E_{X-\gamma}^{(1)}(\mathbf{k} = 0) > 0$, $\Delta E_{X-\gamma}^{(2)}(\mathbf{k} = 0) = 0$, $\Delta E_{X-\gamma}^{(3)}(\mathbf{k} = 0) < 0$. Each E_i corresponds to the subplots b_i) and c_i). b₁), b₂), b₃) Dispersion relations of the lower (LP) and upper polaritons (UP) in a typical microcavity for each value of $\Delta E_{X-\gamma}^{(i)}$. The exciton energy is the yellow dashed line while the photon energy is the blue dashed line. Depending on the location on the sample $i = 1,2,3$ the photon dispersion curve gets shifted vertically yielding different values of the Hopfield coefficient. c₁), c₂), c₃) Corresponding Hopfield coefficients as a function of the in-plane wavevector for the three different regimes. Adapted from [8].

$$\begin{aligned} E_{UP,LP}(\mathbf{k}) &= \frac{E_\gamma(\mathbf{k}) + E_X(\mathbf{k})}{2} - i\hbar \frac{\gamma_{cav} + \gamma_X}{2} \\ &\pm \frac{1}{2} \sqrt{[\hbar\Omega_R]^2 + [\Delta E_{X-\gamma} - i\hbar(\gamma_{cav} - \gamma_X)]^2}. \end{aligned} \quad (1.57)$$

At zero detuning we obtain :

$$E_{UP,LP}(\mathbf{k}) = E_X(\mathbf{k}) - i\hbar \frac{\gamma_{cav} + \gamma_X}{2} \pm \frac{1}{2} \sqrt{(\hbar\Omega_R)^2 + (\hbar\gamma_{cav} - \hbar\gamma_X)^2}. \quad (1.58)$$

The existence of two distinct energies for the two polaritons modes thus depends on the relative values of Ω_R and $|\gamma_{cav} - \gamma_X|$. If the coupling constant Ω_R is smaller than the difference between the relaxation constants, the two eigenenergies share the same real part, and the degeneracy between the exciton and the cavity mode remains unbroken. Being a supersposition of excitons and photons the polaritons also have a relaxation rate that can then be inferred from the imaginary part of their energy :

$$\gamma_{UP}(\mathbf{k}) = X_{\mathbf{k}}^2 \gamma_{cav} + C_{\mathbf{k}}^2 \gamma_X, \quad (1.59a)$$

$$\gamma_{LP}(\mathbf{k}) = X_{\mathbf{k}}^2 \gamma_X + C_{\mathbf{k}}^2 \gamma_{cav}. \quad (1.59b)$$

Once again, the hopfield coefficient have a strong impact on the polariton relaxation rate. In the range of detuning available in our sample we measure relaxation rates of the order of 70 μeV which correspond to a lifetime $\tau \sim 10 \text{ ps}$. This value is beyond the time response of a wide class of instruments and require generally both pulsed lasers and ultrafast detection devices to be resolved as in [50]. The present work is not dedicated to time resolved experiments as we rather use a continuous wave laser to constantly compensate for the losses and reach a steady state from which we extract observable quantities. This system is then highly out of equilibrium. Although this feature seems limiting at first sight it can actually turn into an asset whenever one wants to study dynamical instabilities. Indeed, in conservative systems, instabilities are difficult to study precisely because they tend to make the system unstable. In the case of polaritons, the losses can reduce their effect while keeping their signature visible in the steady state of the system [7].

Effective mass. By analogy with what was done for the photons in section 1.1 it is possible to affiliate an effective mass to the polaritons by taking the second derivative at the bottom of the polariton dispersion relation, namely :

$$\frac{1}{m_{UP}} = \frac{X_0^2}{m_X} + \frac{C_0^2}{m_\gamma}, \quad (1.60a)$$

$$\frac{1}{m_{LP}} = \frac{X_0^2}{m_\gamma} + \frac{C_0^2}{m_X}. \quad (1.60b)$$

Reminding that $C_0^2 + X_0^2 = 1$ and that $\frac{m_\gamma}{m_X} \ll 1$. It can be cast in the following form :

$$m_{UP} \approx \frac{m_\gamma}{X_0^2} \quad (1.61a)$$

$$m_{LP} \approx \frac{m_X}{C_0^2} \quad (1.61b)$$

It can be seen that polaritons inherit the low effective mass of the photons making their transport easier than for excitons. This is particularly interesting for the realisation of polaritonic circuits in the framework of quantum information processing [30]. The excitonic part of the polariton also bring its own features to the table. In particular, the strong exciton-exciton non linear interactions that will turn light into a fluid of interacting particles. This is the subject of the next section.

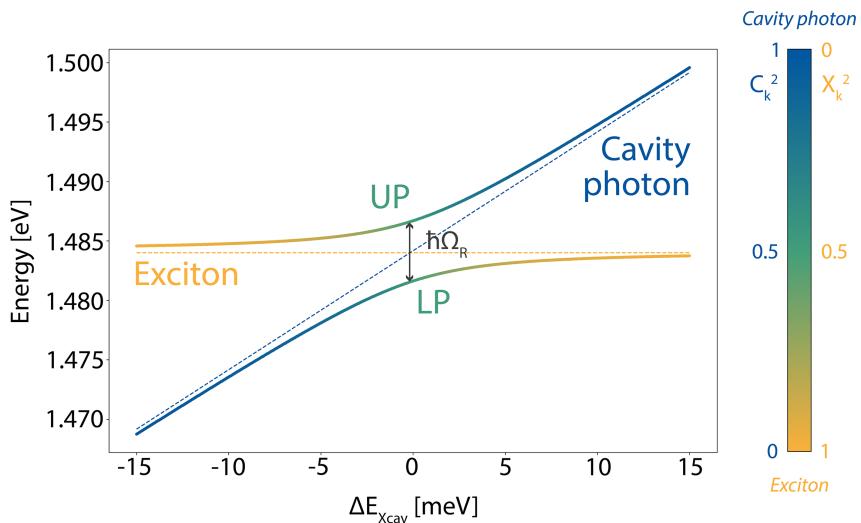


Fig. 1.10 **Anticrossing in the strong coupling regime.** Energies of the upper (UP) and lower (LP) polariton branches at $\mathbf{k} = 0\mu\text{m}^{-1}$, with respect to the exciton-photon detuning at zero wavevector $\Delta E_{X-\gamma}(\mathbf{k} = 0)$ and the color-coded squared modulus of the Hopfield coefficient X^2 (exciton) and C^2 (photon). The yellow and blue dashed lines are respectively the bare exciton and photon energies. At $\Delta E_{X-\gamma} = 0$ meV, the coupling is the better yielding an energy splitting equal to the Rabi energy $\hbar\Omega_R$. Adapted from [31]

1.3.1 Polariton interactions

It is now clear that shinning a strong laser with the right frequency on a semiconducting microcavity will create plenty of polaritons. Since they have an excitonic part, they may undergo many body effects that were not taken into account in the linear hamiltonian model derived earlier. More precisely, polaritons experience all the exciton-exciton interactions described in subsection 1.2.7 as well as additional features coming from their coupling with light. Remarkably, in high density regime the Pauli principle has to apply again on the electrons and the holes leading to carrier exchanges. In the absence of light these fermionic

exchanges are actually the dominant scatterings processes while the direct Coulomb scattering is negligible as was explained in subsection 1.2.7. We remind the expression of the resulting exchange hamiltonian :

$$\mathcal{H}_{XX} = \frac{1}{2} \sum_{\mathbf{k}, \mathbf{k}', \mathbf{q}} V_{XX} [p_{\mathbf{k}+\mathbf{q}}^\dagger p_{\mathbf{k}'-\mathbf{q}}^\dagger p_{\mathbf{k}} p_{\mathbf{k}'}] \quad (1.62)$$

When the excitons are strongly coupled to cavity photon these carrier exchanges give rise to an additional saturation potential describing what was called a "photon-assisted exchange scattering" in the proposal of [12].:

$$\mathcal{H}_{sat} = \frac{1}{2} \sum_{\mathbf{k}, \mathbf{k}', \mathbf{q}} V_{sat} [a_{\mathbf{k}+\mathbf{q}}^\dagger b_{\mathbf{k}'-\mathbf{q}}^\dagger b_{\mathbf{k}} b_{\mathbf{k}'} + h.c.] , \quad (1.63)$$

where V_{sat} is the saturation potential that can be computed by applying the Usui transformation on the quantum wells [49].:

$$V_{sat} = \frac{\hbar \Omega_R}{A n_{sat}}, \quad (1.64)$$

in which n_{sat} is the saturation density computed from the 2D exciton bohr radius as $n_{sat} = 1/a_X^{QW}$ and A the macroscopic quantization area. This scattering can be understood from a microscopic point of view. Consider two excitons, i_1 and i_2 , in a first step they exchange their carrier without any Coulomb process giving two other excitons i'_1 and i'_2 . In a second step, one of the exciton, let us say i'_1 find itself in the same state than a neighbouring exciton. Because of the Pauli principle it can not stay in this state. Luckily it is coupled to cavity photon which enable it to get out of this irregular situation by emitting a photon. To end up with a description of interactions in the polariton basis we invert the unitary transformation in Equation 1.52 :

$$\hat{b}_k = X_k p_{\mathbf{k}} - C_k u_{\mathbf{k}}, \quad (1.65a)$$

$$\hat{a}_k = C_k p_{\mathbf{k}} + X_k u_{\mathbf{k}}, \quad (1.65b)$$

and inject these expressions into \mathcal{H}_{XX} and \mathcal{H}_{sat} . We end up with a single polariton-polariton potential that is a combination of hopfield coefficients, V_{sat} and V_{XX} . For the lower polaritons, which are the one we excite in the experiment and to which we will restrict in the following we obtain an effective polariton-polariton potential V_{pp} as :

$$V_{pp} = |X_{\mathbf{k}}|^4 V_{XX} + 2|X_{\mathbf{k}}|^2 X_{\mathbf{k}} C_{\mathbf{k}} V_{sat}. - \quad (1.66)$$

Once again it depends on the exciton-photon fraction through the Hopfield coefficients, and, in the purely hybrid situation $\Delta E_{X-\gamma}=0$, yields $A \times V_{pp} = 1 \text{ } \mu\text{eV } \mu\text{m}^{-2}$ in our sample.

Non linear LP hamiltonian. Finally, taking into account the polariton-polariton interactions abovementioned the linear hamiltonian for Lower Polaritons can be completed as follows :

$$\mathcal{H}_{LP} = \sum_{\mathbf{k}} \hbar \omega_{LP}(\mathbf{k}) p_{\mathbf{k}}^\dagger p_{\mathbf{k}} + \frac{1}{2} \sum_{\mathbf{k}, \mathbf{k}', \mathbf{q}} V_{pp} [p_{\mathbf{k}+\mathbf{q}}^\dagger p_{\mathbf{k}'-\mathbf{q}}^\dagger p_{\mathbf{k}} p_{\mathbf{k}'}]. \quad (1.67)$$

Energy renormalization. As it can already be seen in the LP non linear hamiltonian, the bare polartion energies are no longer eigenvalues of the system in the presence of interactions. To give a simple picture of this feature let us restrict the description to two polariton modes $p_{\mathbf{k}_1}$ and $p_{\mathbf{k}_2}$. If the system is prepapred in such a way as to populate one of the two modes, \mathbf{k}_2 , this macroscopic reservoir of polaritons acts as an external potential for the polaritons in the poorly populated mode \mathbf{k}_1 . Consequently, the energy of the polaritons in mode \mathbf{k}_1 increases by an amount corresponding to the potential created by the polaritons in mode \mathbf{k}_2 . This is a phenomenon of energy renormalization.

To be more quantitative, we can write the Hamiltonian of the system while restricting it to these two modes. In this case, \mathcal{H}_{int} is the sum of $p_{\mathbf{k}_1}^\dagger p_{\mathbf{k}_2}^\dagger p_{\mathbf{k}_1} p_{\mathbf{k}_2}$ and $p_{\mathbf{k}_2}^\dagger p_{\mathbf{k}_1}^\dagger p_{\mathbf{k}_2} p_{\mathbf{k}_1}$. Since $p_{\mathbf{k}_2}$ and $p_{\mathbf{k}_1}$ are eigenstates of the system, they commute, meaning that the two previous term account for the same scattering. The time evolution of $p_{\mathbf{k}_1}$ can then be described by the Heisenberg equation :

$$i\hbar \frac{d}{dt} p_{\mathbf{k}_1} = [p_{\mathbf{k}_1}, \mathcal{H}_{LP}] = [p_{\mathbf{k}_1}, \mathcal{H}_{lin}] + [p_{\mathbf{k}_1}, \mathcal{H}_{int}]. \quad (1.68)$$

the interacting term being :

$$[p_{\mathbf{k}_1}, \mathcal{H}_{int}] = \frac{V_{pp}}{2} p_{\mathbf{k}_2}^\dagger p_{\mathbf{k}_2} p_{\mathbf{k}_1} = \frac{\hbar g_{LP}}{A} \hat{N}_2 p_{\mathbf{k}_1}. \quad (1.69)$$

\hat{N}_2 is the number operator in mode \mathbf{k}_2 and $\hbar g_{LP} = V_{pp} A / 2$ is the polariton-polariton interaction constant. Since, the mode \mathbf{k}_2 is macroscopically populated, we can replace \hat{N}_2 by its expectation value $\langle \hat{N}_2 \rangle$, which exhibit the polariton density in mode \mathbf{k}_2 , $n_2 = \langle \hat{N}_2 \rangle / A$. As a result, the master equation for the \mathbf{k}_1 gets renormalized by interactions with the \mathbf{k}_2 mode as :

$$i\hbar \frac{d}{dt} p_{\mathbf{k}_1} = [\hbar \omega_{LP}(\mathbf{k}_1) + \hbar g_{LP} n_2] p_{\mathbf{k}_1}. \quad (1.70)$$

where $\omega_{LP}(\mathbf{k}_1) = E_{LP}(\mathbf{k}_1)$ is the bare energy of the polariton with wavevector \mathbf{k}_1 . Although this is a simplified model with only two modes, the equation demonstrates how interactions between polaritons can lead to the renormalization of their energies. More generally, this phenomenon takes place between all modes in the system, including self-interactions of individual modes. The latter closely resembles the optical Kerr effect, where a field modifies the refractive index of the medium through which it propagates.

1.4 Polariton dynamics

To summarize, microcavity exciton-polaritons are hybrid light-matter quasiparticles that combine characteristics of both components. Their photonic part endows them with an

exceptionally low effective mass and allows us to manipulate them using light. Conversely, their excitonic part provides them with the exotic properties associated with electrons in semiconductors. Although the resulting interactions are varied and stem from different origins, they can, to a first approximation, be described by a single four-body contact potential, V_{pp} .

In this framework, polaritons can be viewed as weakly interacting bosons, distinguished by their inherent losses, making their behavior an out-of-equilibrium problem. In the following, we will explore their dynamics when one or a few modes are macroscopically populated, revealing remarkable phenomena typically associated with true many-body bosonic systems, such as superfluidity and Bose-Einstein condensation (BEC). However, whenever necessary, we will return to their composite fermionic nature to account for unique features that may not be captured by the mean field approximation.

1.4.1 Mean field approximation

In order to describe the dynamics of the polariton fluid of light, it is convenient to use the mean field approximation as it is usually done for quantum fluids [38]. This assumption is valid when the number of particles is large $N \sim N + 1$ and when the major part of them occupy the same quantum state. Obviously, the latter can happen only in bosonic systems since fermions obey the Pauli exclusion principle. In atomic systems, the macroscopic occupation occurs in the ground state of the system, and, provided that the temperature is low enough, happens spontaneously due to bosonic stimulation. In the case of polaritons, the situation is dramatically different since the system is out of equilibrium and the particles have to be continuously pumped to compensate for the losses. The mean field of the system is then rather a steady state than a ground state at equilibrium. This being said, it is still possible to define a macroscopic occupation of a given mode in the system. Whenever this happens, the theoretical procedure boils down to replacing the field operator $\hat{\psi}(\mathbf{r},t) = \sum_k \varphi_k \hat{p}_k$ that annihilates a particle at (\mathbf{r},t) , by its expectation value $\langle \hat{\psi}(\mathbf{r},t) \rangle$. This idea is supported by the fact that, in such situations, adding or removing a particle from the system does not change its state. The system is then described by a single classical wavefunction whose square modulus gives the density of particles in the system $|\psi(\mathbf{r},t)|^2 = n(\mathbf{r},t)$ and with a well defined phase accounting for long range coherence. However, it doesn't mean that the system has became fully classical. Indeed, its quantum nature is hidden in the fluctuations around the mean field and often manifest itself through collective behavior. Accounting for these fluctuations is done by adding a small perturbation to the mean field :

$$\hat{\psi}(\mathbf{r},t) = \langle \psi(\mathbf{r},t) \rangle + \delta\psi(\mathbf{r},t). \quad (1.71)$$

For now let us focus on the description of the mean field dynamics and justify more quantitatively the relevance of describing a polariton fluid with a single wavefunction.

One wavefunction to rule them all. A pioneering result in the early stages of quantum mechanics is the so called wave-particle duality proposed by Louis de Broglie [14] in 1924. It states that particles can exhibit both wave and particle properties. More precisely, to any particle with momentum \mathbf{p} can be associated a wavelength $\lambda = h/|\mathbf{p}|$. This wavelength is the characteristic of the wave associated with the particle and is called the de Broglie wavelength. As a result if two particle are separated by a distance smaller than their de

Broglie wavelength their corresponding wavefunction will sum and possibly give rise to wave like effect as interferences. This idea was later confirmed by the famous double slit experiment in which electrons were sent through a double slit and displayed an interference pattern.

If one apply this idea to a great number of particle in the same quantum states whose inter-particle distance is smaller than λ it is no longer possible to distinguish them and they can be described by a single wavefunction. At thermal equilibrium the momentum of a particle follows a Maxwell Boltzmann distribution and the de Broglie wavelength is of the order of the thermal de Broglie wavelength :

$$\lambda_T = \frac{h}{\sqrt{2\pi m k_B T}}, \quad (1.72)$$

where T is the temperature m the particle mass and k_B the Boltzmann constant. From this simple formula, one can see why reaching long range coherence in a system often require trapping and cooling procedures. In the case of polaritons, a typical polariton-polariton inter-distance is about $0.1 \mu\text{m}$ while the small polaritons effective mass and the cryogenic temperature ($\sim 4K$) yields a thermal de Broglie wavelength of the order of $1 \mu\text{m}$ which validates the mean field approximation. Now that we have a single order parameter to describe the system, let us see how it evolves in time.

1.4.2 Driven dissipative Gross-Pitaevskii Equation.

In the context of Bose gas this problem was first tackled by Gross [16] and Pitaevskii [37] to describe the structure of quantized vortices in liquid Helium. The resulting equation is known as the *Gross-Pitaevskii equation* or non-linear Schrödinger equation :

$$i\hbar \frac{\partial}{\partial t} \psi(\mathbf{r}, t) = \left(-\frac{\hbar^2}{2m} \nabla_{\mathbf{r}}^2 + V_{ext}(\mathbf{r}) + \hbar g |\psi(\mathbf{r}, t)|^2 \right) \psi(\mathbf{r}, t), \quad (1.73)$$

where g is the interaction constant, $\nabla_{\mathbf{r}}$ the kinetic energy and $V_{ext}(\mathbf{r})$ is the external potential experienced by the particles. To extend this equation to the out of equilibrium polariton case, a first formulation in terms of exciton and photon reveals to be enlightening to identify the various corrections that must be considered. Losses are incorporated through the relaxation rates γ_X and γ_{cav} while the continuous injection of photons in the system is accounted for by a pumping term $F_p(\mathbf{r}, t)$ in the photon field master equation. Considering these terms we end up with a set of coupled equation for the exciton and photon fields $\psi_X(\mathbf{r}, t)$, $\psi_{\gamma}(\mathbf{r}, t)$:

$$i\hbar \frac{d}{dt} \begin{bmatrix} \psi_{\gamma}(\mathbf{r}, t) \\ \psi_X(\mathbf{r}, t) \end{bmatrix} = \begin{bmatrix} \hbar F_p(\mathbf{r}, t) \\ 0 \end{bmatrix} + \left(\mathcal{H}_{lin}(\mathbf{r}) + \begin{bmatrix} V_{\gamma}(\mathbf{r}) - i\hbar \frac{\gamma_{cav}}{2} & 0 \\ 0 & V_X(\mathbf{r}) - i\hbar \frac{\gamma_X}{2} + \hbar g_{XX} n_X(\mathbf{r}, t) \end{bmatrix} \right) \begin{bmatrix} \psi_{\gamma}(\mathbf{r}, t) \\ \psi_X(\mathbf{r}, t) \end{bmatrix}, \quad (1.74)$$

where $V_{\gamma}(\mathbf{r})$ and $V_X(\mathbf{r})$ are the mean external potentials felt by photons and excitons respectively, $g_{XX} = A V_{XX} / 2\hbar$ is the exciton interaction strength and $n_X = |\psi_X|^2$ is the exciton

density. Additionally, $\mathcal{H}_{lin}(\mathbf{r})$ corresponds to the linear Hamiltonian from [Equation 1.54](#), expressed in real space by substituting \mathbf{k} with $i\nabla_{\mathbf{r}}$.

$$\mathcal{H}_{lin}(\mathbf{r}) = \begin{bmatrix} \hbar\omega_X & \hbar\Omega_R/2 \\ \hbar\Omega_R/2 & \hbar\omega_{\gamma}(-i\nabla_{\mathbf{r}}) \end{bmatrix}. \quad (1.75)$$

as explained earlier, the exciton dispersion relation can be safely considered constant with respect to the photon one, $\hbar\omega_X(\mathbf{k}) = \hbar\omega_X(0)$. We can then transition to the polariton basis, obtaining two decoupled equations for the lower and upper polariton fields, $\psi_{LP}(\mathbf{r},t)$ and $\psi_{UP}(\mathbf{r},t)$. Focusing on the lower polariton branch, this leads to the so-called *driven-dissipative Gross-Pitaevskii equation*:

$$i\hbar\frac{\partial}{\partial t}\psi_{LP}(\mathbf{r},t) = \left[\hbar\omega_{LP}^0 - \frac{\hbar^2}{2m_{LP}}\nabla_{\mathbf{r}}^2 + V_{LP}(\mathbf{r}) + \hbar g n(\mathbf{r},t) - i\hbar\frac{\gamma_{LP}}{2} \right] \psi_{LP}(\mathbf{r},t) + \hbar\eta_{LP}F_p(\mathbf{r},t) \quad (1.76)$$

$$:= \mathcal{H}_{LP}\psi_{LP}(\mathbf{r},t) + \hbar\eta_{LP}F_p(\mathbf{r},t), \quad (1.77)$$

where V_{LP} is the external potential experienced by the polaritons which depend on those felt by photons and excitons through the Hopfield coefficients $V_{LP}(\mathbf{r}) = C_{\mathbf{k}}^2 V_{\gamma} + X_{\mathbf{k}}^2 V_X$, \mathbf{k} being here the pump wavevector. The term $\hbar\omega_{LP}^0$ is the polaritons energy at zero wavevector while the $\hbar^2\nabla_{\mathbf{r}}^2/2m_{LP}$ is again the kinetic energy. Finally, the prefactor η_{LP} account for the coupling efficiency of the pump photons whithin the system. Since the presence of a fluid in the sample change the optical resonance through non linear interaction this term is generally not constant. However, in first approximation it is linked the reflectivity of the sample front mirror and can be considered as a constant. A full derivation of this equation as well as a complete discussion on its validity can be found in [\[3\]](#).

Dark reservoir. Along our description of excitons we mentionned the presence of dark excitons that are not directly coupled to light because of spin selection rules. As a consequence, one could think that they are not involved in the polariton dynamics. It is actually not the case. Indeed, eventhough they do not interact with photons they can still interact with bright excitons through the many complex electron-hole interactions described in [subsection 1.2.7](#). Forgetting them in the description of the polariton dynamics would then be a mistake [\[32; 47\]](#). To derive a single master equation starting from the many microscopic interactions between photons, dark and bright excitons would require a full spin resolved description and is beyond the scope of this manuscript. Nevertheless, we will account for their presence phenomenologically by considering them as a reservoir of density n_r coupled to polaritons through a relaxation term γ_r . In terms of equations it reads as :

$$i\hbar\frac{\partial}{\partial t}\psi_{LP}(\mathbf{r},t) = \left[\hbar\omega_{LP}^0 - \frac{\hbar^2}{2m_{LP}}\nabla_{\mathbf{r}}^2 + V_{LP}(\mathbf{r}) + \hbar g n(\mathbf{r},t) + \hbar g_r n_r(\mathbf{r},t) - i\hbar\frac{\gamma_{LP} + \gamma_{in}}{2} \right] \psi_{LP}(\mathbf{r},t) + \hbar\eta_{LP}F_p(\mathbf{r},t), \quad (1.78)$$

$$\frac{\partial}{\partial t} n_r = -\gamma_r n_r + \gamma_{in} n. \quad (1.79)$$

As it will be shown in the following the presence of a dark reservoir is of great importance when it comes to describing the quantum fluctuations of the polariton fluid.

1.4.3 Excitation scheme

So far, the derivation of the master equation was initiated by stating that a polaritonic state could be macroscopically populated through optical pumping. However, the pump term was not yet defined in the sense that we did not specify the frequency of the pump photons with respect to the polariton dispersion relation. The latter is crucial to determine what are the mechanism leading to long range coherence.

1.4.3.1 Off resonance excitation

In the case off resonant case the pump laser is highly blue detuned with respect to the polariton energy, close to a reflectivity minimum above the stop band of the DBRs (see [Figure 1.2](#)). The rise of a macroscopic population then occurs in the bottom of the LP branch through polaritons relaxation. More precisely, the injected photons create highly excited electron-hole pairs that relax through the emission of phonons. Lowering their energy they can emit photons and, at some point, eventually strongly couple to them to form polaritons. At this stage the polaritons are incoherent since the coherence of the pump laser got lost in the many relaxation processes. Then, the incoherent polariton undergo scatterings and relax toward the minimum energy states available. If the pump intensity is increased, the population in the minimum of the LP branch will "accelerate" the relaxation of the other polaritons through bosonic stimulations. If the stimulation is efficient enough with respect to the losses, the bottom of the LP branch gets macroscopically populated and obtain a long range order which can be seen on [Figure 1.11](#). Just like in atomic systems, the phase of the condensate wave function is chosen randomly through $U(1)$ symmetry breaking and is not inherited from the pump laser. Such a phase transition in polariton planar microcavity was first observed in 2006 by Kasprzak et al. [24].

Some major differences with atomic BECs are worth mentioning. First, the temporal phase of an atomic BEC is defined by the chemical potential and is related to the number of atoms in the system while the polaritonic system can never reach equilibrium. Its temporal phase result from a complex interplay between the pump laser and the losses of the system : the phase transition is driven by the pump intensity rather than the temperature. transverse

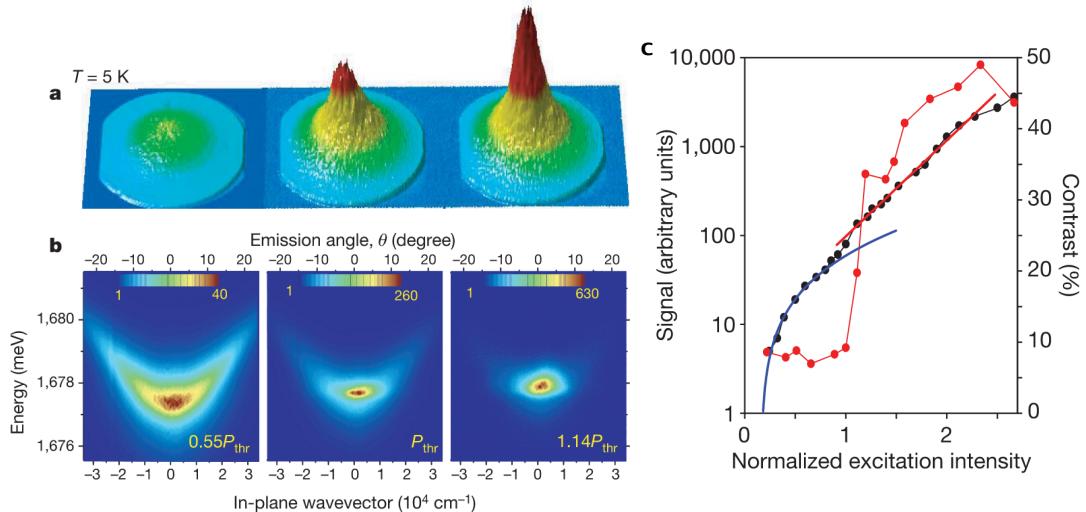


Fig. 1.11 Bose-Einstein condensation of polaritons. **a)** Far field emission of the fluid for increasing pump intensity. Above threshold, a strong emission in the zero wavevector mode is observed. **b)** Polariton dispersion for increasing pump intensities. Above threshold the bottom of the LP branch is macroscopically populated. **c)** Spatial correlation measurements using a Michelson interferometer. Solid red circles indicate correlations between two spots separated by 6mm (2.5 times the thermal de Broglie wavelength) within the condensate as a function of the excitation power. The correlation exhibits a threshold-like behaviour. The variation of the ground-state emission intensity, normalized to the excitation power, is shown for comparison (solid black circles). The solid blue line is a quadratic fit of the data demonstrating the occurrence of particle-particle interaction below threshold. Above threshold, the solid red line is an exponential fit demonstrating the strong stimulation of the relaxation by the high occupancy factor of the ground state. Adapted from [24].

1.4.3.2 Resonant excitation.

In the resonant case, the pump laser is tuned near the polariton energy. The population of the polaritons is then directly driven by the pump laser. In particular due to momentum and energy conservation, the spatial and temporal coherence of the pump laser are transferred to the polaritons fluid. In that case the phase of the fluid is fixed by the pump and therefore is not randomly chosen. In this excitation scheme the fluid should then not be called a condensate. However, it can still exhibit superfluidity and other quantum fluid properties because it possesses the required properties, namely : a macroscopic occupation of a single quantum state, long range coherence and weak interactions. In this work we will only use the resonant scheme to take advantage of the tunability of the pump to create fluid with arbitrary densities and velocity profiles. Before detailing the great versatility of this system, let us first discuss a consequence of the resonant excitation scheme which will be of great interest in the following.

Optical bistability. Consider a Fabry-Perot cavity filled with a non-linear Kerr medium,

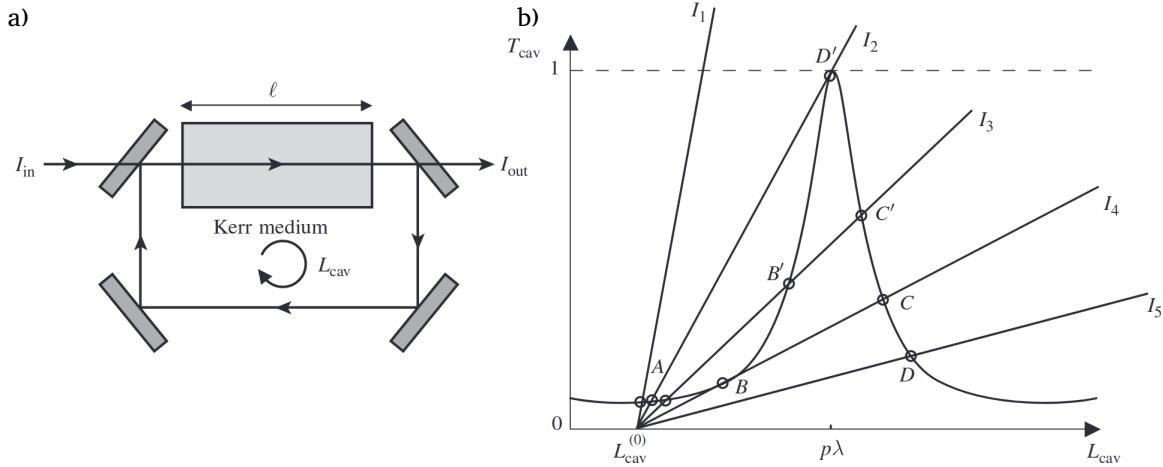


Fig. 1.12 **Optical bistability.** a) Schematic of a ring cavity filled with a Kerr medium. The cavity has a length $L_{cav}^{(0)}$ and the Kerr medium a length ℓ . b) Transmission of the cavity as a function of the optical length of the cavity L_{cav} . The straight lines represent the values T_{cav} as a function of L_{cav} for different values of I_{in} . The operating points lie at the intersections of the cavity resonance curve and the straight lines. Depending on I_{in} one or three operating point can be found. Adapted from [17].

through which a weak resonant laser beam is shone. As the laser intensity is low, the refractive index within the cavity is the same as the bare medium. It means that the optical length of the cavity is an integer multiple of half the laser wavelength. If the laser intensity is increased, the non-linear Kerr effect turns on and the refractive index of the medium increases. The laser is then no longer resonant with the cavity which reduce the intensity of the intracavity field and counteract on the refractive index. This interplay between non-linearities and resonance conditions is at the heart of what is called optical bistability. Since this phenomenon is not proper to polaritonic system, we will, to understand it more in details, follow the very general derivation done in [17].

To avoid first complications due to cross Kerr effect we consider a ring cavity of geometrical length L filled with Kerr medium of length ℓ as shown in Figure 1.12 a). The optical length of such a cavity is :

$$L_{cav} = L + (n_0 - 1)\ell + n_2 I_{cav} \ell, \quad (1.80)$$

where n_0 is the refractive index of the medium, n_2 the non linear refractive index and I_{cav} the intracavity intensity. As explained for a planar cavity in section 1.1, the cavity has transmission peaks whenever the optical path of a round trip in the cavity (a photon going back and forth) is an integer multiple of the laser wavelength λ . In the case of a ring cavity this condition reads :

$$T_{cav} = \frac{1}{1 + \frac{4F^2}{\pi^2} \sin^2 \left(\frac{kL_{cav}}{2} \right)}, \quad (1.81)$$

where F is the cavity finesse and $k = 2\pi/\lambda$ the wavevector of the laser. The transmission of the cavity as a function of L_{cav} is plotted in [Figure 1.12 b](#)). On the other hand the transmission of the cavity is defined as :

$$T_{cav} = \frac{I_{out}}{I_{in}} = T \frac{I_{cav}}{I_{in}} = \frac{T}{n_2 l} \frac{L_{cav} - L_{cav}^{(0)}}{I_{in}}, \quad (1.82)$$

where T is the transmission of the outport mirror and $L_{cav}^{(0)}$ the optical length of the cavity at low intensity $I_{cav} \approx 0$. An operating point is then defined as the intersection of the transmission curve [Equation 1.81](#) and the straight line of [Equation 1.82](#). Graphically the slope of the linear relation between T_{cav} and L_{cav} depend on the value of I_{in} . As a consequence several regime are possible depending on the value of the input intensity as shown in [Figure 1.12 b](#)). For low intensity as I_1 , the operating point is unique and the system is in a stable regime but yields a low transmission. For higher input intensity like I_3 , the system exhibits three operating points. However, it can be shown that the intersection points with a negative derivative between B and D are unstable. At even higher intensity the system has again a single operating point and a poor transmission. When the input intensity lies between I_2 and I_4 the system is said to be in a bistable regime.

In the case of microcavity-polariton the same behavior can be observed as the exciton-exciton non-linear interactions are of the same nature as the Kerr effect. Analytically, polariton bistability can be found by looking at the steady state of the driven-dissipative Gross-Pitaevskii equation in the presence of a driving pump term nearly resonant with the LP branch. In first approximation, it can be written as a plane wave $F_p(\mathbf{r}, t) = F_p e^{i\mathbf{k}_p \cdot \mathbf{r} - i\omega_p t}$. As it is usually done for equations including a resonant forcing term, we look for solution of the same form namely, plane wave with the same phase, $\psi(\mathbf{r}, t) = \sqrt{n_0} e^{i\mathbf{k}_p \cdot \mathbf{r} - i\omega_p t}$. Inserting this ansatz into [Equation 1.4.2](#) in the absence of external potential $V_{LP} = 0$ one obtain the steady state equation :

$$\left[\omega_p - \omega_{LP} - \frac{\hbar k_p^2}{2m_{LP}} - gn_0 - g_r n_r + i \frac{\gamma_{LP}}{2} \right] \sqrt{n_0} = \eta_{LP} F_p^0, \quad (1.83)$$

while the reservoir rate equation gives :

$$n_r = \frac{\gamma_{in}}{\gamma_r} n_0. \quad (1.84)$$

To eliminate the reservoir density we follow the procedure done in [47] and introduce an effective interaction strength $g_e = g + g_r \frac{\gamma_{in}}{\gamma_r}$ so that $g_r n_r + gn = g_e n$. We can then rewrite [Equation 2.17](#) as :

$$\left[\omega_p - \omega_{LP} - \frac{\hbar k_p^2}{2m_{LP}} - g_e n_0 + i \frac{\gamma_{LP}}{2} \right] \sqrt{n_0} = \eta_{LP} F_p^0. \quad (1.85)$$

It is the equivalent of the equation of state of a conservative system with the difference that, in this case, it results of a complex equilibrium between interactions, pumping and losses. Notice that the contribution of the dark reservoir just appear as a correction on the interaction strength and thus doesn't change much the phenomenology discussed earlier. Nonetheless, it has quantitative consequences on the hysteresis cycle that can be used to probe polariton-dark excitons interactions with optical means, notably through two photon excitation to overcome spin forbidden transitions [44]

To end up with an equation linking the pump intensity to the polariton density one can multiply this equation by its complex conjugate which gives :

$$n \left[\frac{\gamma_{LP}^2}{4} + (\delta(k_p) - g_e n)^2 \right] = \eta_{LP}^2 I, \quad (1.86)$$

where $\delta(k_p)$ is the effective detuning between the pump laser and the LP branch at the pump wavevector in a parabolic approximation, $\delta(k_p) = \omega_p - \omega_{LP} - \hbar k_p^2 / 2m_{LP}$ and I the pump intensity. Being a polynomial of degree 3 in the polariton density, this equation have up to three solutions. Three distinct real solutions can be found only if $\delta > \sqrt{3/2}\gamma_{LP}$, as explained in the previous paragraph, one of this solution is known to be unstable. However, in some peculiar cases, namely when the fluid dimension is ramped down from 2D to 1D this solution can be explored and is responsible for the emergence of first order phase transition as investigated in [28]. This being said, we don't take into account this solution in the present work since the dimension of the fluid will always be 2D. The evolution of n as a function of I in this regime is plotted in [Figure 1.13](#). The system is in a bistable regime when the curve exhibits two stable solutions for a given intensity : one at low density and one at high density. It's worth noticing that the actual state in which the system is depend on its past history. More precisely, if the incident intensity is initially low, since the laser is blue detuned with the polariton energy, the photon injection whithin the sample is poor and a few polariton are created. However, their presence in the cavity tends to blueshift their own resonance energy as explained in [subsection 1.3.1](#). As the intensity is ramped up, the system will follow the low density branch until the energy renormalisation is sufficient to reach the point B. At this point the system is compelled to jump to the high density branch and will suddenly move from point B to point C. If the pump intensity is then decreased, the situation is rather different since many polaritons are already present in the sample and support the laser injection and thus polariton creation. The system will follow the high density branch untill it reaches the point D' where interactions can no longer compensate for the losses making it fall again on the low density branch. A particular attention must be given to this so called turning point D'. Indeed, as it can be seen on [Figure 1.12](#) it's the only point at which the laser is exactly resonant with the cavity filled with non linearities. This point can only be reached by travelling on the whole hysteresis loop. The corresponding density can determined by solving $\frac{dI}{dn} = 0$ which gives the discriminants :

$$\Delta = g_e^2 \left(\delta^2 - \frac{3\gamma_{LP}^2}{2} \right). \quad (1.87)$$

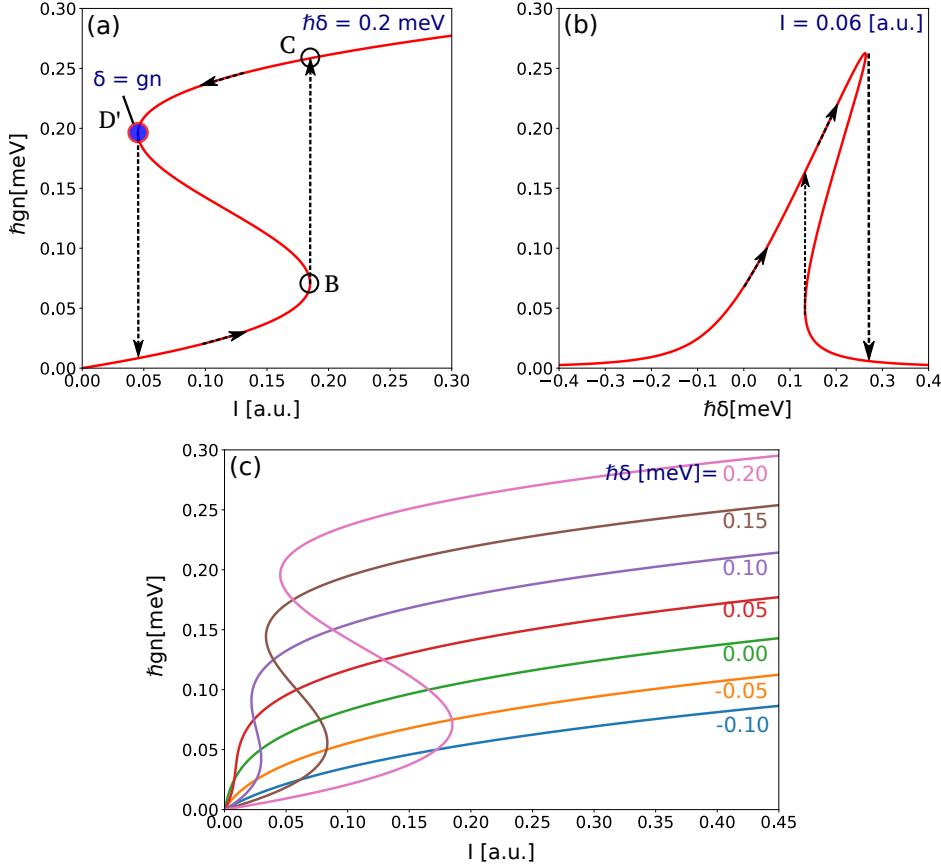


Fig. 1.13 **Optical bistability in a polariton fluid.** a) The polariton density as a function of the pump intensity when $\delta > \frac{\sqrt{3}\gamma_{LP}}{2}$. The blue point represent the so called turning point at which the detuning is exactly equal to the interaction energy. b) Polariton density as a function of δ for a fixed pump intensity. c) Polariton density as a function of the pump intensity for different values of δ , bistability is observed when $\delta > \frac{\sqrt{3}\gamma_{LP}}{2}$.

The bistable regime require two distinct solution for $\frac{dI}{dn} = 0$ which require $\Delta > 0$ and provide the aforementioned condition $\delta > \sqrt{3/2}\gamma_{LP}$. From this one can find that the density corresponding to the turning point fulfill $\delta(k_p) = g_{eff}n = gn_0 + grn_r$.

Conversely, when the detuning is too small with respect to the system losses the system is monostable and said to be in the optical limiter regime. In the bistable regime, the system exhibits

All optical control. In the context of analog gravity, the ability to easily control and monitor the fluid of light is a great asset. Indeed, the study of Hawking radiation in a quantum fluid require the creation of an acoustic horizon as well as the possibility to measure what is emitted by the latter. In particular, it is crucial to achieve two goals.

- 1. Create and monitor a stable fluid with an arbitrary flow profile and density since the mean field and especially its velocity profile are the analog of the curved spacetime we want to study.

- 2. Probe the fluid collective excitations. These small perturbations on top of the mean field are the fluid counterpart of the quantum fields fluctuations considered in the Hawking radiation theory.

With planar microcavity polaritons this is done fully optically since the fluid is created by the pump laser and is monitored by collecting the photons leaving the sample. The first point is allowed by the resonant excitation scheme. The velocity of a polariton being $\mathbf{v}_{LP} = \frac{\hbar\mathbf{k}_{LP}}{m_{LP}}$ momentum conservation enable to create a fluid with a given velocity by just changing the pump incidence angle (see [section 1.1](#)). This correspondance also holds for the light outgoing the sample : the recombination of a polariton with wavevector \mathbf{k} produce a photon with the same wavevector. Although the relation between the pump intensity and the polariton density is not trivial as we shall see in the next section, it is also a practical control knob that can be used to tune the fluid speed of sound. The second point can again be tackled with optics means since the struture of collective excitations is ultimately hidden in the fluctuations of the photonic field outgoing the sample. As a consequence, the large variety of tools and methods developped in the field of optics can be used to probe the fluid. Microcavity polaritons, are liying at the crossroad of optics and quantum fluids, definitely justifying the designation "quantum fluid of light".

Chapter 2

Hawking radiation in a polariton quantum fluid

Now that polaritons have been introduced, we can start to discuss the theoretical framework that will be used to describe Hawking radiation in a polariton quantum fluid. In this chapter, we will first establish the original hydrodynamical analogy between the propagation of acoustic waves in a moving fluid and scalar fields in curved spacetime. This analogy is at the heart of what motivated the study of Hawking radiation in analog systems, leading to a first variety of experiment in classical systems such as water tanks. These great experiments, [43; 52] demonstrated positive to negative frequency conversion of shallow surface waves at the vicinity of a sonic horizon. However the temperature of classical fluids is too high to quantize their collective excitation field which prevent the study of quantum effects. More precisely, Hawking radiation is expected to create entanglement between the emitted modes which can not be tested with water waves. Quantum fluids appear then as promising candidate since the fluctuations around the ground or steady state require a quantum treatment to be understood. In fact we will see that tackling a fluid with trans-critical flow with the usual Bogoliubov theory of condensed matter also predict particle creation from vacuum. Studying the effect from this point of view will reveal a strong robustness of the effect beyond the original hydrodynamical approach and widen the range of regimes where Hawking radiation can be observed.

2.1 The hydrodynamical analogy

In the previous chapter, we established that the dynamics of microcavity polaritons can be described by a driven dissipative Gross-Pitaevskii equation. By writing the wavefunction in term of phase and density the Gross-Pitaevskii equation can be cast into a continuity equation and an Euler equation for a non-barotropic fluid in the pump rotating frame. Starting from the GPE a system pumped by a laser of the general form $F_p e^{i\phi(x,t)}$ lead the fluid to a

stationnary state whose wavefunction $\psi(x,t) = \sqrt{n_0}e^{i\phi(x,t)}$ phase and modulus respect :

$$\partial_t \phi + \frac{m_{LP} v_0^2}{2\hbar} + \frac{\hbar}{2m_{LP}} \frac{\partial_x^2 \sqrt{n_0}}{\sqrt{n_0}} + V_{LP} + gn + g_r n_r + \frac{\Re[F_p e^{-i\theta}]}{m_{LP}} = 0 \quad (2.1)$$

$$\partial_t n_0 + \partial_x(n_0 v) = \gamma n_0 - 2\Im[F_p e^{-i\theta}] \sqrt{n_0} \quad (2.2)$$

If we neglect the pump and dissipation terms which is basically the situation of a conservative fluid, we recover the original equation describing sound waves in a convergent flow. This is precisely the equation that was used in the initial analogy made by W. Unruh [48]. However, the out of equilibrium nature of the microcavity polaritons brings different phenomenology. In particular, perturbations propagating in the fluid are not necessary sound waves as we will see latter. It means that the dispersion of these perturbation is not linear and can remarkably exhibit a gap which can be associated to massive excitations. To avoid making approximation and encapsulate as much as possible the complexity of the system it is convenient to make the same calculation in the pump rotating frame. It equivalently means that all frequencies will be taken with respect to the pump. In practice this is done by writting the pump $F_p(x) = F_p e^{i\theta_p(x)}$ and look for steady state wavefunction of the same form $\psi(x,t) = \sqrt{n_0}e^{i\theta_p(x)}$. Plugging this into the GPE we obtain that the phase and density of the fluid must fullfill the following equations :

$$\left[-\frac{\hbar}{2m_{LP}} \nabla^2 - i\frac{\hbar}{2} (\boldsymbol{\nabla} \cdot \mathbf{v}_0) - i\hbar(\mathbf{v}_0 \cdot \boldsymbol{\nabla}) - \delta(v_0) + g_r n_r + gn_0 - i\frac{\hbar\gamma}{2} \right] \sqrt{n_0} + |F_p| = 0, \quad (2.3)$$

with $\hbar\delta(v_0) = \hbar\omega_p - \hbar\omega_0 - m_{LP}v_0^2/2$ (note that this is just $\delta(k_p)$ in a homogeneous configuration where $\mathbf{v}_0 = \hbar\mathbf{k}_p/m_{LP}$). To study the low energy collective perturbations of the fluid state we linearize the system around the steady state by writing the wavefunction $\psi(x,t) = (\sqrt{n_0} + \delta\psi(x,t))e^{i\theta_p(x)}$. Injecting this expression in [Equation 1.79](#) and using [Equation 2.3](#) we obtain at linear order in $\delta\psi$:

$$i\hbar(\partial_t + \mathbf{v}_0 \cdot \boldsymbol{\nabla})\delta\psi = \left[-\frac{\hbar}{2m^*} \nabla^2 + \rho - i\sigma \right] \phi + gn_0\delta\psi^*, \quad (2.4)$$

where

$$\rho := 2gn_0 - \delta(v_0) + g_r n_r \quad \text{and} \quad \sigma := \hbar/2\boldsymbol{\nabla} \cdot \mathbf{v}_0.$$

Note that here we have assumed that there is no external potential without loss of generality since it can be included in the definition of ρ .

Ici il faut finir le calcul et notamment cette histoire de masse et de Klein gordon de mes couilles

2.2 At the heart of particle creation : Bogoliubov transformation

2.2.1 The ambiguity of vacuum definition

The quantum vacuum state is defined as the quantum state of a system with the lowest possible energy. Eventhough this definition is commonly used and looks quite intuitive, it in

fact implies some deep properties. First, a minimum energy state exists. In a finite dimension problem, this is obvious since the spectrum has also a finite dimension but whenever the dimension of the Hilbert space is infinite, the existence of a minimum energy state is a priori not guaranteed. Secondly, it seems to depend on the system which suggests that the vacuum is not a universal concept. This is particularly true in the context of quantum field theory in curved spacetime where the vacuum state is observer dependent. For instance, in the Unruh effect, an accelerating observer detect a thermal radiation while an inertial observer sees 'standard' vacuum. This ambiguity is as at the root of particle creation phenomena like the Hawking effect and once it is understood, make these effects appear less surprising.

2.2.1.1 The harmonic oscillator

To start with, let us consider the usual harmonic oscillator describing a massive particle in a one dimensionnal harmonic potential and see how the defintion vacuum arise in this simple case [9]. The Hamiltonian of the particle of mass m is :

$$H = \frac{P^2}{2m} + \frac{1}{2}m\omega^2 X^2 \quad (2.5)$$

Where P and X are the momentum and position operators satisfying the canonical commutation relation $[X, P] = i\hbar$. It is convenient to define the dimensionless operators :

$$\hat{P} = \frac{1}{\sqrt{\hbar m \omega}} P, \quad (2.6a)$$

$$\hat{X} = \sqrt{\frac{m\omega}{\hbar}} X, \quad (2.6b)$$

which respect the commutation relation $[\hat{X}, \hat{P}] = i$. The Hamiltonian is then written as :

$$H = \hbar\omega \hat{H} \quad (2.7)$$

with :

$$\hat{H} = \frac{1}{2} (\hat{P}^2 + \hat{X}^2). \quad (2.8)$$

The corresponding eigenproblem is usually tackled by introducing the creation and annihilation operators :

$$\hat{a} = \frac{1}{\sqrt{2}} (\hat{X} + i\hat{P}), \quad (2.9a)$$

$$\hat{a}^\dagger = \frac{1}{\sqrt{2}} (\hat{X} - i\hat{P}), \quad (2.9b)$$

which satisfy the commutation relation $[\hat{a}, \hat{a}^\dagger] = 1$. Taking the definition of \hat{a} and \hat{a}^\dagger we look a the quantity $\hat{a}^\dagger \hat{a}$:

$$\hat{a}^\dagger \hat{a} = \frac{1}{2} (\hat{X} - i\hat{P}) (\hat{X} + i\hat{P}) \quad (2.10)$$

$$= \frac{1}{2} (\hat{X}^2 + \hat{P}^2 + i(\hat{X}\hat{P} - \hat{P}\hat{X})) \quad (2.11)$$

$$= \frac{1}{2} (\hat{X}^2 + \hat{P}^2 - 1). \quad (2.12)$$

Comparing with [Equation 2.8](#) we see that :

$$\hat{H} = \hat{a}^\dagger \hat{a} + \frac{1}{2}. \quad (2.13)$$

We then naturally introduce the operator $\hat{N} = \hat{a}^\dagger \hat{a}$. Eventhough this operator is known to be the number operator, we will on purpose treat it without prior knowledge and see what can be learned from the derivation. From [Equation 2.13](#) we see that the an eigenstate $|\phi_\nu^i\rangle$ of \hat{N} with eigenvalue ν is also an eigenstate of \hat{H} with eigenvalue $\nu + \frac{1}{2}$. Solving the problem then boils down to find the spectrum of \hat{N} .

Spectrum determination. The operator \hat{N} is hermitian and its eigenstates form a complete basis of the Hilbert space. Let us show two usefull lemmas resulting directly from the definition of \hat{N} :

Lemma 1. *The eigenvalues of \hat{N} are positive or zero.*

Proof. Let us consider any eigenstate $|\phi_\nu\rangle$ of \hat{N} . The square of norm of the state $\hat{a}|\phi_\nu\rangle$ is

$$\|\hat{a}|\phi_\nu\rangle\|^2 = \langle\phi_\nu| \hat{a}^\dagger \hat{a} |\phi_\nu\rangle \geq 0 \quad (2.14)$$

Using the definition of \hat{N} we have :

$$\langle\phi_\nu| \hat{a}^\dagger \hat{a} |\phi_\nu\rangle = \nu \langle\phi_\nu| \phi_\nu\rangle \geq 0. \quad (2.15)$$

Since $\langle\phi_\nu| \phi_\nu\rangle > 0$ we have $\nu \geq 0$. □

Lemma 2. *Let $|\phi_\nu\rangle$ be an eigenstate of \hat{N} with eigenvalue ν .*

- $\nu = 0$ if and only if $\hat{a}|\phi_\nu\rangle = 0$.
- If $\nu > 0$ then $\hat{a}|\phi_\nu\rangle$ is an eigenstate of \hat{N} with eigenvalue $\nu - 1$.

Proof. If $\nu = 0$, [Equation 2.15](#) implies that $\langle\phi_\nu| \phi_\nu\rangle = 0$ which imposes that $|\phi_\nu\rangle = 0$. Reciprocally, if $\hat{a}|\phi_\nu\rangle = 0$ a multiplication by \hat{a}^\dagger gives $\hat{N}|\phi_\nu^i\rangle = 0$. Since $|\phi_\nu\rangle$ is not zero we conclude that $|\phi_\nu\rangle$ is an eigenvector of \hat{N} with eigenvalue $\nu = 0$.

Consider now that $\nu > 0$, we now know that $\hat{a}|\phi_\nu\rangle$ is not zero. By using the identity $[\hat{N}, \hat{a}] = -\hat{a}$ we obtain :

$$\hat{N}\hat{a}|\phi_\nu\rangle = \hat{a}\hat{N}|\phi_\nu\rangle - \hat{a}|\phi_\nu\rangle = (\nu - 1)\hat{a}|\phi_\nu\rangle, \quad (2.16)$$

which shows that $\hat{a}|\phi_\nu\rangle$ is an eigenstate of \hat{N} with eigenvalue $\nu - 1$. □

Let us summarize what we know so far :

- The operator \hat{N} has a non negative spectrum.
- If ν is an eigenvalue of \hat{N} then $\nu - 1$ is also an eigenvalue of \hat{N} .

This two properties are in fact sufficient to show that the spectrum of \hat{N} are the positive or zero integers.

Assume, by contradiction, that there is a positive non integer eigenvalue ν associated with the eigenvector $|\phi_\nu\rangle$. Then, by the second lemma, $\nu - 1$ is also a non integer eigenvalue of \hat{N} generated by the action of \hat{a} on $|\phi_\nu\rangle$. By iterating this process we obtain a sequence of eigenvalues $\nu, \nu - 1, \nu - 2, \dots$ which is impossible since the spectrum is bounded from below. More precisely, it exists a positive integer p such that $\nu - p < 0$ which contradicts the fact that the spectrum is non negative. We conclude that the spectrum of \hat{N} is the positive or zero integers.

2.3 Elementary excitations of the fluid : Bogoliubov theory

Unlike in classical systems, where Hawking radiation can only be triggered by external disturbances, its analog in quantum fluids may instead arise from vacuum fluctuations. In this context, perturbations of the quantum fluid are emitted at the horizon and propagate in opposite directions, mirroring the behavior expected near a gravitational event horizon. However, it is important to recognize that at this stage, the analogy with black hole physics—initially derived from hydrodynamical equations—should not be taken as a strict equivalence but rather as an inspiration for exploring particle creation in a quantum fluid.

Indeed, treating the collective excitations of a transonic Bose-Einstein condensate within the framework of Bogoliubov theory already predicts the spontaneous emission of modes at the horizon from vacuum fluctuations. Fundamentally, both Stephen Hawking's original calculation and its quantum fluid analog rely on a Bogoliubov transformation between two sets of operators [19], which ultimately leads to the mixing of creation and annihilation operators.

This section will establish the structure of the collective excitation spectrum and demonstrate how specific fluid configurations can give rise to mode emission from vacuum. Furthermore, we will show that the Hawking effect remains robust even when deviating from the original hydrodynamical analogy, and it does not necessarily require the excitations to be sound waves. In fact, the intrinsically out-of-equilibrium nature of the polariton system reveals a phenomenology far richer than that of conservative systems, extending the possibility of observing Hawking radiation to massive excitations.

2.3.1 Linearization of the Gross-Pitaevskii equation

Let us consider a monochromatic pump described by a plane wave $F_p(\mathbf{r},t) = F_p e^{i(\mathbf{k}_p \cdot \mathbf{r} - \omega_p t)}$. As explained in the previous chapter, this will drive the system to a steady state of the form $\psi_0(\mathbf{r},t) = \psi_0 e^{i(\mathbf{k}_p \cdot \mathbf{r} - \omega_p t)}$ and respecting :

$$\left[\omega_p - \omega_{LP} - \frac{\hbar k_p^2}{2m_{LP}} - g |\psi_{LP}^0|^2 + i \frac{\gamma_{LP}}{2} \right] \psi_{LP}^0 = \eta_{LP} F_p^0. \quad (2.17)$$

We now look for the elementary excitations following the Bogoliubov prescription which consists in linearizing the GPE around ψ_0 . We write the wavefunction as :

$$\psi(\mathbf{r},t) = [\psi_0(\mathbf{r}) + \delta\psi(\mathbf{r},t)] e^{i(\mathbf{k}_p \mathbf{r} - \omega_p t)}. \quad (2.18)$$

The second term in this writing can be interpreted as the polaritons that are not in the steady state and therefore have a global phase different from the pump. This imply that they can have different energy and momentum than the fluid and undergo scatterings or spontaneous effects since their dynamics is not fixed by the pump. It is worth noticing that this term do not originate from the out of equilibrium nature of the system and would be not zero even at zero temperature due to interactions. In the case of Bose Einstein condensates, this linearization process actually introduces a state that is not physical in the sense that it suggest the breaking of the $U(1)$ symmetry of the system or equivalently that the condensates picked a phase [4]. The true physical state is recovered through a statistical mixtures of all these "symetry broken states" which then rather appear as intermediate state convenient for calculation. In the case of a polariton fluid pumped quasi resonantly the situation is different since the mean field phase is fixed by the pump.

Injecting [Equation 2.18](#) in the GPE, using [Equation 2.17](#) and keeping only the linear terms in $\delta\psi$ we obtain :

$$\begin{aligned} i\hbar\partial_t\delta\psi = & \left(\hbar\omega_{LP}^0 - \hbar\omega_p - \frac{\hbar^2}{2m_{LP}} [\nabla^2 + 2i\mathbf{k}_p \nabla - \mathbf{k}_p^2] + \hbar g_r n_r + 2\hbar g |\psi_0|^2 + \frac{i\hbar\gamma}{2} \right) \delta\psi \\ & + \hbar g \psi_0^2 \delta\psi^*. \end{aligned} \quad (2.19)$$

Yet, this equation is not stricly speaking linear in $\delta\psi$ since there is a term involving its complex conjugated $\delta\psi^*$. A way to solve this problem is to decompose $\delta\psi$ in its real and imaginary part and to obtain two independant equations. An equivalent procedure, more common in the litterature, is to find an equation on the complex conjugated and consider $\delta\psi^*$ as an independant variable. This is the approach we will follow here. The equation on $\delta\psi^*$ is obtained by simple complex conjugation of [Equation 2.19](#) :

$$\begin{aligned} -i\hbar\partial_t\delta\psi^* = & \left(\hbar\omega_{LP}^0 - \hbar\omega_p - \frac{\hbar^2}{2m_{LP}} [\nabla^2 - 2i\mathbf{k}_p \nabla - \mathbf{k}_p^2] + \hbar g_r n_r + 2\hbar g |\psi_0|^2 - \frac{i\hbar\gamma}{2} \right) \delta\psi^* \\ & + \hbar g \psi_0^{*2} \delta\psi. \end{aligned} \quad (2.20)$$

For the sake of clarity we define the operator :

$$\mathcal{H}_{bog} = \hbar\omega_{LP}^0 - \hbar\omega_p - \frac{\hbar^2}{2m_{LP}} [\nabla^2 + 2i\mathbf{k}_p \nabla - \mathbf{k}_p^2] + \hbar g_r n_r + 2\hbar g |\psi_0|^2. \quad (2.21)$$

We can now write the fully linearized problem under its matrix form :

$$i\hbar\partial_t \begin{pmatrix} \delta\psi \\ \delta\psi^* \end{pmatrix} = (\mathcal{L}_B + \frac{i\hbar\gamma}{2}\mathbf{I}_2) \begin{pmatrix} \delta\psi \\ \delta\psi^* \end{pmatrix} \quad (2.22)$$

where \mathcal{L}_B is the Bogoliubov matrix defined as :

$$\mathcal{L}_B = \begin{pmatrix} \mathcal{H}_{bog} & \hbar g\psi_0^2 \\ -\hbar g\psi_0^{*2} & -\mathcal{H}_{bog}^* \end{pmatrix} \quad (2.23)$$

Since \mathcal{L}_B is time independant the usual procedure to solve [Equation 2.22](#) is to diagonalize the bogoliubov matrix and write any generic solution in the basis of eigenmodes defined by :

$$\mathcal{L}_B \begin{pmatrix} u_i \\ v_i \end{pmatrix} = (\hbar\omega_i + \frac{i\hbar}{2}) \begin{pmatrix} u_i \\ v_i \end{pmatrix}. \quad (2.24)$$

As the entire the spectrum is fully determined by the eigenvalues of \mathcal{L}_B we will drop the losses term in the following for the sake of simplicity and take them into account at the end of the calculation by multiplying the solutions by $\exp(-\gamma t/2)$:

Bogoliubov matrix symmetries. As in general for quadratic hamiltonian of bosonic systems [4] the Bogoliubov matrix is not hermitian. Therefore its eigenvalues are not necessarily real and \mathcal{L}_B is not even ensured to be diagonalizable. However, it can easily be shown that the following symmetry is respected :

$$\mathcal{L}_B^\dagger = \eta^{-1} \mathcal{L}_B \eta \quad \text{with} \quad \eta = \eta^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (2.25)$$

It means that the Bogoliubov operator is "hermitian" for the inner product :

$$\langle \vec{X}_1, \vec{X}_2 \rangle = (\vec{X}_1^*)^T \eta \vec{X}_2, \quad (2.26)$$

It means that for any vectors \vec{X}_1 and \vec{X}_2 we have $\langle \vec{X}_1, \mathcal{L}_B \vec{X}_2 \rangle = \langle \mathcal{L}_B \vec{X}_1, \vec{X}_2 \rangle$. Consequently the inner product between two modes $|\phi\rangle = (u_\phi, v_\phi)^T$ and $|\psi\rangle = (u_\psi, v_\psi)^T$:

$$\langle \psi | \phi \rangle_B := \int d\mathbf{r} [\psi^\dagger(\mathbf{r}) \eta \psi(\mathbf{r})] = \int d\mathbf{r} [u_\psi^*(\mathbf{r}) u_\phi(\mathbf{r}) - v_\psi^*(\mathbf{r}) v_\phi(\mathbf{r})], \quad (2.27)$$

is conserved in time. This product induce a modified norm for the modes :

$$\|\psi\|_B := \langle \psi | \psi \rangle_B. \quad (2.28)$$

It's worth noticing that since the inner product defined in [Equation 2.27](#) is neither positive nor definite, the induced norm of a given mode $|\psi\rangle$ can be positive, negative or zero. Let

us now see what the symmetry [Equation 2.25](#) implies on the spectrum of \mathcal{L}_B . Consider an eigenvector $|\psi\rangle = (u_k, v_k)^T$ of \mathcal{L}_B with eigenvalue $\hbar\omega_k$:

$$\mathcal{L}_B \begin{pmatrix} u_k \\ v_k \end{pmatrix} = \hbar\omega_k \begin{pmatrix} u_k \\ v_k \end{pmatrix}. \quad (2.29)$$

Direct substitution of [Equation 2.25](#) in the eigenvalue equation gives directly :

$$\mathcal{L}_B^\dagger \begin{pmatrix} u_k \\ -v_k \end{pmatrix} = \hbar\omega_k \begin{pmatrix} u_k \\ -v_k \end{pmatrix}. \quad (2.30)$$

From this, we obtain that $\hbar\omega_k^*$ is also an eigenvalue of \mathcal{L}_B with since we have the relation :

$$\det(\mathcal{L}_B - \hbar\omega_k^* \mathbf{I}_d) = [\det(\mathcal{L}_B^\dagger - \hbar\omega_k \mathbf{I}_d)]^* = 0. \quad (2.31)$$

The Bogoliubov matrix as well as its adjoint operator also respect the symmetry :

$$\mathcal{L}_B^* = -\sigma^{-1} \mathcal{L}_B \sigma \quad \text{with} \quad \sigma = \sigma^{-1} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad (2.32)$$

The latter implies that $-\hbar\omega_k^*$ is also an eigenvalue of \mathcal{L}_B :

$$\mathcal{L}_B \begin{pmatrix} v_k^* \\ u_k^* \end{pmatrix} = -\hbar\omega_k^* \begin{pmatrix} v_k^* \\ u_k^* \end{pmatrix}. \quad (2.33)$$

which can be again obtained by direct substitution. Putting together all this properties we find that $\hbar\omega_k$, $-\hbar\omega_k$, $\hbar\omega_k^*$ and $-\hbar\omega_k^*$ are simultaneously eigenvalues of \mathcal{L}_B . A special attention must be given to the fact that if $|\psi\rangle = (u_k, v_k)^T$ is an eigenvector of \mathcal{L}_B with eigenvalue $\hbar\omega_k$ the eigenvector $|\phi\rangle = (v_k^*, u_k^*)^T$ linked to $-\hbar\omega_k^*$ has opposite norm.

$$\langle \psi | \psi \rangle_B = \langle u_k | u_k \rangle - \langle v_k | v_k \rangle = -\langle \phi | \phi \rangle_B. \quad (2.34)$$

Orthogonality condition. An orthogonality condition can be derived by calculating the quantity $\langle \psi_i | \eta \mathcal{L}_B | \psi_j \rangle - \langle \psi_j | \eta \mathcal{L}_B | \psi_i \rangle^*$. On one hand we find this term to be zero because the symmetry of [Equation 2.25](#) equivalently means that $\eta \mathcal{L}_B$ is hermitian. On the other hand we find :

$$\langle \psi_i | \eta \mathcal{L}_B | \psi_j \rangle - \langle \psi_i | \eta \mathcal{L}_B | \psi_j \rangle^* = \hbar\omega_i \langle \psi_i | \eta | \psi_j \rangle - \hbar\omega_j^* \langle \psi_i | \eta | \psi_j \rangle, \quad (2.35)$$

$$= (\hbar\omega_i - \hbar\omega_j^*) \langle \psi_i | \eta | \psi_j \rangle, \quad (2.36)$$

which gives the orthogonality condition :

$$(\hbar\omega_i - \hbar\omega_j^*) \langle \psi_i | \eta | \psi_j \rangle = 0 \quad (2.37)$$

showing that the modified scalar product $\langle \cdot | \cdot \rangle_B$ between two eigenvectors with different eigenvalues is zero and that eigenmodes with complex eigenvalues have a vanishing norm. The modes with non zero norm are then associated to non zero real eigenvalues.

At the end, if we normalize the eigenvectors to unity in the usual sense, the spectrum of \mathcal{L}_B can be split in three parts :

- The S_+ family of modes $|\delta\psi_k^+\rangle = (u_k, v_k)^T$ with positive norm : $\langle u_k|u_k\rangle - \langle v_k|v_k\rangle = +1$ and real eigenvalues $\hbar\omega_k > 0$
- The S_- family of modes $|\delta\psi_k^-\rangle = (v_k^*, u_k^*)^T$ with negative norm : $\langle v_k^*|v_k^*\rangle - \langle u_k^*|u_k^*\rangle = -1$ and real eigenvalues $-\hbar\omega_k > 0$
- The S_0 family of modes with zero norm : $\langle u_k|u_k\rangle - \langle v_k|v_k\rangle = 0$ and zero or complex eigenvalues.

Note that the eigenvectors of the S_- subspace are expressed in terms of the u_k and v_k component of the S_+ vectors to show reflect the dual structure of the solution space. The modes with zero norm are usually related to abnormal modes. One of them is the vector whose component are ψ_0 and $-\psi_0$ which satisfy the Bogoliubov equations with a zero eigenvalue because ψ_0 obeys the Gross Pitaevskii equation. It corresponds to global change of the phase condensate and is very similar to the appearance of the Goldstone mode when the $U(1)$ phase symmetry of a system is broken. Other modes of S_0 are related to dynamical instabilities due to non zero imaginary part of their eigenvalues.

Dynamical stability. The free evolution of a mode is given by $\exp(-i\omega_k t)$ so it remains bounded in time provided that the imaginary part of $\hbar\omega_k$ is negative or zero. This is known as the dynamical stability condition. In fact, this condition must be refined in the polaritonic case since as one can see from [Equation 2.22](#), the eigenvalues already contain a non zero imaginary part due to the inherent losses in the system. Since eigenvalues come in pairs of complex conjugated the stability condition apply both on $\Im(\hbar\omega_k)$ and $\Im(\hbar\omega_k^*) = -\Im(\hbar\omega_k)$ which gives :

$$|\Im(\hbar\omega_k)| \leq \frac{\hbar\gamma}{2} \quad \text{for all } k. \quad (2.38)$$

In practice, the losses can damp dynamical instabilities that would destroy a conservative system while still exhibiting precursors of the instabilities which make polariton system very useful for the study of such phenomena [7]. This being said, the present experiments always take place in regime where these modes are not present and the system is stable.

At the end, if we restrict our description to modes with non zero norm, a general solution of the Bogoliubov equation can be written as :

$$|\psi\rangle = \begin{pmatrix} \delta\psi \\ \delta\psi^* \end{pmatrix} = \sum_{\mathbf{k} \in S_+} b_{\mathbf{k}} \begin{pmatrix} u_k \\ v_k \end{pmatrix} e^{-i\omega_k t} + b_{\mathbf{k}}^* \begin{pmatrix} v_k^* \\ u_k^* \end{pmatrix} e^{i\omega_k t} \quad (2.39)$$

where

$$b_{\mathbf{k}} = \int d\mathbf{r} u_k^*(\mathbf{r}) \delta\psi(\mathbf{r}) - v_k^*(\mathbf{r}) \delta\psi(\mathbf{r}). \quad (2.40)$$

The redundancy of the first line being the complex conjugated of the second is the price we pay now for our initial assumption that considered $\delta\psi$ and $\delta\psi^*$ as independent variables.

The advantage of the procedure followed here is the natural emergence of the inner product of [Equation 2.27](#) from the symmetries of \mathcal{L}_B . It allowed for the definition of a norm and the establishment of orthogonality among the eigenvectors, thereby revealing the structure of the solution space as the direct sum of two dual subspaces. This decomposition results in norms with opposite signs and paired complex-conjugated eigenvalues. Another possible approach would have been to directly look for solutions as linear combination of plane wave with opposite frequencies as it is done in [\[36\]](#). This method however does not provide a natural way to define a norm and as we shall see later, it's the sign of the norm and not the sign of the eigenvalues that is relevant for mode classification. The last advantage of the approach followed here is that the derivation is closer to the full quantum derivation and will ease the latter quantization of the classical fields obtained here. Taking into account the dissipative part of [Equation 2.22](#) and dropping the redundancy a generic expression for the fluctuations of the order parameter is :

$$\delta\psi(\mathbf{r},t) = \sum_{\mathbf{k} \in S_+} \left[A_{\mathbf{k}} u_k(\mathbf{r}) e^{-i\omega_k t} + A_{\mathbf{k}}^* v_k^*(\mathbf{r}) e^{i\omega_k t} \right] e^{-\gamma t/2}. \quad (2.41)$$

Particle-hole symmetry. Under this form one might be tempted to interpret the fluctuations order parameter has a superposition of independent plane waves with opposite directions and frequencies. This would be a mistake. Indeed, the coefficient u_k and v_k^* are coupled through the Bogoliubov matrix and respect the particle hole symmetry [Equation 2.32](#). The first term can be interpreted as the addition of a particle with energy $\hbar\omega_k$ and momentum $\hbar\mathbf{k}$ while the second term can be interpreted as the addition of a hole with energy $-\hbar\omega_k^*$ and momentum $-\hbar\mathbf{k}$. The coupling between the two terms reflect that a fluctuation of the system is particle-hole hybrid excitation, while the norm $\langle \cdot | \cdot \rangle_B$ of the mode reflects the relative weight of the particle and hole components.

- If $\int d\mathbf{r} \|u_k\|^2 - \|v_k\|^2 > 0$ the mode is mostly made of u_k and can be interpreted as a particle excitation.
- If $\int d\mathbf{r} \|u_k\|^2 - \|v_k\|^2 < 0$ the mode is mostly made of v_k^* and can be interpreted as a hole excitation.

In conservative systems this symmetry is often related to the fluctuation of the number of particles in the condensed state. When the system is out of equilibrium this interpretation seems less straightforward since the number of particles in the mean field is not strictly speaking fixed in time. However, these modes still account for small deviation from the pump mode (\mathbf{k}_p, ω_p) both in frequency and wavevector. To clarify that, imagine that an experimentalist had the possibility to track simultaneously the incoming laser and its copy that went through the microcavity. Since the linewidth of the laser is much smaller than the microcavity resonance the latter doesn't act as a filter. Hence, the experimentalist would first notice that the two beams share the random events from which the shot noise of the laser originates. Then, he would notice that the polariton field exhibit additional fluctuations, namely the random emission of photons with a frequency and wavevector that are not the same as the pump. These photons originate from the bogoliubov modes of the fluid due to interactions. The photonic nature of this system then appear again as a great asset because the fluctuations of the system are translated in the noise spectrum of the outcoming laser.

2.4 Homogeneous fluid

So far, the description of the collective excitations spectrum remained quite general and we didn't use the homogeneity of system. In this case ψ_0 is invariant under translation meaning the eigenvectors of \mathcal{L}_B are just plane waves. The S_+ vectors then may be written as :

$$u_k(\mathbf{r}) = U_k \cdot \exp(i\mathbf{k}\mathbf{r}) \quad (2.42a)$$

$$v_k(\mathbf{r}) = V_k \cdot \exp(i\mathbf{k}\mathbf{r}). \quad (2.42b)$$

Before going further in the calculation, let us comment the ansatz of the total field including the fluctuations in terms of reference frame. To obtain the bogoliubov matrix we set our description in the rotating frame of the pump by factorizing the total field : $\psi(r,t) = (\psi_0 + \delta\psi)e^{i(\mathbf{k}_p\mathbf{r} - \omega_p t)}$. Consequently, all the dynamics on $\delta\psi$ we might extract from the equation derived earlier happen in the moving frame where the fluid is at rest. However, in practice all the accessible observables are measured in the lab frame and are consequently dressed by the motion of the fluid. To directly predict the excitation spectrum as we expect to measure it in the lab we explicitly write the wavefunction as a function of the wavevector measured in the lab frame \mathbf{k} as : $\psi(r,t) = \psi_0 e^{i(\mathbf{k}_p\mathbf{r} - \omega_p t)} + u_k e^{i\mathbf{k}\mathbf{r}}$ and look for their dynamics in the pump rotating frame by writing again $\psi(r,t) = (\psi_0 + u_k e^{i(\mathbf{k}-\mathbf{k}_p)\mathbf{r}})e^{i(\mathbf{k}_p\mathbf{r} - \omega_p t)}$. Under this form the wavevector $\delta k = \mathbf{k} - \mathbf{k}_p$ appear as the wavevector of the fluctuations in the moving frame. To obtain the corresponding Bogoliubov matrix we first express the action of \mathcal{H}_{bog} on the fluctuations plane wave : $u_k e^{i((\mathbf{k}-\mathbf{k}_p)\mathbf{r})}$:

$$\mathcal{H}_{bog} u_k(\mathbf{r}) = \left[\hbar\omega_{LP}^0 - \hbar\omega_p - \frac{\hbar^2}{2m_{LP}} [-(\mathbf{k} - \mathbf{k}_p)^2 - 2\mathbf{k}_p(\mathbf{k} - \mathbf{k}_p) - \mathbf{k}_p^2] + \hbar g_r n_r + 2\hbar g n_0 \right] U_k e^{i(\mathbf{k}-\mathbf{k}_p)\mathbf{r}} \quad (2.43)$$

$$= \left[-\hbar\delta(k_p) + \frac{\hbar^2(\mathbf{k} - \mathbf{k}_p)^2}{2m_{LP}} + \hbar g_r n_r + 2\hbar g n_0 \right] U_k e^{i(\mathbf{k}-\mathbf{k}_p)\mathbf{r}} + \frac{\hbar^2 \mathbf{k}_p(\mathbf{k} - \mathbf{k}_p)}{m_{LP}} U_k e^{i(\mathbf{k}-\mathbf{k}_p)\mathbf{r}}. \quad (2.44)$$

where $\delta(k_p) = \omega_p - \omega_{LP}^0 - \frac{\hbar k_p^2}{2m_{LP}}$ is the detuning between the pump and the LP polariton dispersion at \mathbf{k}_p provided we stay in the low wavevector parabolic approximation. The second term proportionnal to $(\mathbf{k} - \mathbf{k}_p)^2$ is the kinetic energy shows that due to the fluid motion, the Bogoliubov spectrum observed in the lab frame is as expected centered on the fluid wavevector. The last term describe the temporal frequency shift due to the fluid motion $\Delta\omega = \hbar^2 \mathbf{k}_p(\mathbf{k} - \mathbf{k}_p)/m_{LP} = \hbar \mathbf{v}_0(\mathbf{k} - \mathbf{k}_p)$ where $\mathbf{v}_0 = \hbar \mathbf{k}_p/m_{LP}$ is the fluid velocity. This term is precisely the term that emerge naturally in the Doppler effect demonstration. Doing the same procedure for the complex conjugated the Bogoliubov matrix can be written as :

$$\begin{aligned}\mathcal{L}_B &= \begin{pmatrix} -\hbar\delta(k_p) + \frac{\hbar^2(\mathbf{k} - \mathbf{k}_p)^2}{2m_{LP}} + \hbar g_r n_r + 2\hbar g n_0 & \hbar g n_0 e^{2ik_p} \\ -\hbar g n_0^{-2ik_p} & \hbar\delta(k_p) - \frac{\hbar^2(\mathbf{k} - \mathbf{k}_p)^2}{2m_{LP}} - \hbar g_r n_r - 2\hbar g n_0 \end{pmatrix} \\ &\quad + \begin{pmatrix} \hbar \mathbf{v}_0(\mathbf{k} - \mathbf{k}_p) & 0 \\ 0 & \hbar \mathbf{v}_0(\mathbf{k} - \mathbf{k}_p) \end{pmatrix} \\ &= \mathcal{L}'_B + \hbar \mathbf{v}_0(\mathbf{k} - \mathbf{k}_p) \mathbf{I}_2\end{aligned}\tag{2.45}$$

where the doppler effect appears naturally in the second diagonal matrix and shifts the spectrum of \mathcal{L}'_B in the fluid rest frame. Looking at the roots of the $\det(\mathcal{L}'_B - X\mathbf{I}_2) = 0$ we find the eigenvalues in the fluid frame through :

$$(\hbar\omega'_k)^2 = \left(\frac{\hbar^2(k - k_p)^2}{2m_{LP}} - \hbar\delta(k_p) + \hbar g_r n_r + 2\hbar g n_0 \right)^2 - (\hbar g n_0)^2,\tag{2.46}$$

From this we obtain the two branches of the so called bogoliubov dispersion relation in the free falling frame :

$$\begin{aligned}\omega'_B^\pm(k) &= \pm \sqrt{\left(\frac{\hbar(k - k_p)^2}{2m_{LP}} - \delta(k_p) + g_r n_r + 2g n_0 \right)^2 - (gn_0)^2} \\ &= \pm \sqrt{\left(\frac{\hbar\delta k^2}{2m_{LP}} - \delta(k_p) + g_r n_r + 2g n_0 \right)^2 - (gn_0)^2}.\end{aligned}\tag{2.47}$$

while in the laboratory frame it yields :

$$\begin{aligned}\omega_B^\pm(k) &= \mathbf{v}_0 \cdot (\mathbf{k} - \mathbf{k}_p) + \omega'_B^\pm(k) \\ &= \mathbf{v}_0 \cdot (\mathbf{k} - \mathbf{k}_p) \pm \sqrt{\left(\frac{\hbar(k - k_p)^2}{2m_{LP}} - \delta(k_p) + g_r n_r + 2g n_0 \right)^2 - (gn_0)^2}.\end{aligned}\tag{2.48}$$

This expression is exactly what we would have obtained by directly applying a Galilean transformation to the spectrum in the free falling frame. With this form the dual structure of the solution space exhibited earlier appear less clear. In fact, the motion of the fluid breaks the symmetry and the bogoliubov matrix no longer respects [Equation 2.32](#). However, while changing the reference frame doppler shifts the eigenvalues, it leaves the eigenvectors untouched. More precisely, if $(u_{k'}, v_{k'})^T$ is an eigenvector with eigenvalues $\hbar\omega'_k$ in the fluid frame it is also an eigenvector in the lab frame for the shifted frequency. As a consequence, the norm of the eigenvectors is the same and our description of the solution space remains relevant.

2.4.0.1 Bogoliubov mode energy

The gravitational Hawking effect involve the emission of particles with opposite energy and momentum at the horizon. In our analog system, we demonstrated the existence of modes with opposite frequency and norm. However, the concept of "opposite" is in our case only relative to the fluid energy and momentum. Consequently, what we called negative frequencies so far are actually positive frequencies that can be measured in the lab. The concept of negative energy can nevertheless be understood as Landau did to describe the emission of phonons in a superfluid [26]. Consider a fluid in motion with a velocity \mathbf{v}_0 and a Bogoliubov spectrum $\omega'_B(k)$ in the fluid frame. In the moving frame, the total energy of the system before any excitations are created is just the internal energy of the superfluid while in the lab frame it is the sum of the internal energy and the kinetic energy of the fluid. Landau's criterion results from analyzing when excitations can be spontaneously created without increasing the total energy. The fundamental condition for this to happen is:

$$v_0 > v_c = \min_k \frac{\omega'_B(k)}{k} \quad (2.49)$$

When $v_0 > v_c$ certain excitations become energetically favorable, meaning that their spontaneous creation extracts energy from the bulk flow. From an external observer it seems that these modes carry energy away from the bulk motion or in other words, that the fluid is loosing energy.

Let us clarify this more quantitatively for the modes of interest here. To compute the energy of a given mode we introduce the energy fonctionnal of the system [4]:

$$E = \int d\mathbf{r} \left(\frac{\hbar^2}{2m_{LP}} |\nabla \psi|^2 + \frac{g_e}{2} |\psi|^4 \right) \quad (2.50)$$

where we set the external potential to zero. **TODO : check how it is modified by losses**
Inserting the expression of the total field including the fluctuations it is possible to expand the energy in terms of the fluctuations $E = E^{(0)} + E^{(1)} + E^{(2)}$. The first order vanishes because ψ_0 minimizes the energy fonctionnal. The second order term is given by :

$$E^{(2)} = \int d\mathbf{r} \left(\frac{\hbar^2}{2m_{LP}} |\nabla \delta\psi|^2 + [2g_e |\psi_0|^2 - \delta(k_p)] |\delta\psi|^2 + \frac{g_e}{2} (\psi_0^*)^2 \delta\psi^2 + \frac{g_e}{2} \psi_0^2 (\delta\psi^*) \right) \quad (2.51)$$

which can be cast in the vectorial form

$$E^{(2)} = \frac{1}{2} \langle \delta\psi | \eta \mathcal{L}_B | \delta\psi \rangle \quad (2.52)$$

where

$$\begin{aligned} \eta \mathcal{L}_B = & \begin{pmatrix} -\hbar\delta(k_p) + \frac{\hbar^2(\mathbf{k} - \mathbf{k}_p)^2}{2m_{LP}} + \hbar g_r n_r + 2\hbar g n_0 & \hbar g n_0 e^{2ik_p} \\ \hbar g n_0^{-2ik_p} & -\hbar\delta(k_p) + \frac{\hbar^2(\mathbf{k} - \mathbf{k}_p)^2}{2m_{LP}} + \hbar g_r n_r + 2\hbar g n_0 \end{pmatrix} \\ & + \begin{pmatrix} \hbar \mathbf{v}_0(\mathbf{k} - \mathbf{k}_p) & 0 \\ 0 & -\hbar \mathbf{v}_0(\mathbf{k} - \mathbf{k}_p) \end{pmatrix}. \end{aligned} \quad (2.53)$$

Expanding the fluctuations in the eigenvectors basis of \mathcal{L}_B we obtain :

$$\begin{aligned} E^{(2)} &= \frac{1}{2} \sum_{k,l} \langle \delta\psi_k | \eta \mathcal{L}_B | \delta\psi_l \rangle = \frac{1}{2} \sum_k \langle \delta\psi_k | \eta | \delta\psi_k \rangle \hbar\omega_k \\ &= \frac{1}{2} \sum_k \langle \delta\psi_k | \eta | \delta\psi_k \rangle (\hbar\omega'_k + \hbar\mathbf{v}_0(\mathbf{k} - \mathbf{k}_p)). \end{aligned} \quad (2.54)$$

Let us analyze the case of a static fluid or equivalently what we obtain in the co moving frame. Setting $\mathbf{k}_p = 0$ the energy contribution is :

$$\frac{1}{2} \langle \delta\psi_k | \eta \mathcal{L}'_B | \delta\psi_k \rangle = \frac{1}{2} \hbar\omega'_k \langle \delta\psi_k | \delta\psi_k \rangle_B. \quad (2.55)$$

When the system operates on the high density branch of the bistability loop one can show that the operator $\eta \mathcal{L}'_B$ is positive definite. This means that the left hand side of [Equation 2.55](#) is real and stricly positive which imposes that $\langle \delta\psi_k | \delta\psi_k \rangle_B$ is not zero. We can thus divide the whole expression by the norm and we deduce that the $\hbar\omega'_k$ is also real. Finally, the positivity of $\eta \mathcal{L}'_B$ imply that the norm and the eigenvalue of a mode share the same sign. Hence, in the frame where the fluid is at rest the energy of a mode is always positive even if the corresponding frequency is negative.

In the lab frame, the eigenvalue must be replaced by its counterpart in the lab frame $\hbar\omega_k = \hbar\omega'_k + \hbar\mathbf{v}_0(\mathbf{k} - \mathbf{k}_p)$ while the norm remains unchanged. If the fluid velocity is large enough one can see that the doppler shift can give rise to negative energies which remind the above discussion on the Landau criterion. Indeed, above a certain velocity, a mode emitted in the fluid with positive energy can appear as a negative contribution for an external observer sat in the lab frame.

2.4.1 Bogoliubov spectrum characterization

2.4.1.1 Static fluid

Let us start our description with the case of a static fluid. The dispersion is the same in both reference frame which can be seen by setting $\mathbf{k}_p = 0$ in [Equation 2.48](#) :

$$\begin{aligned} \omega_B^\pm(k) &= \pm \sqrt{\left(\frac{\hbar k^2}{2m_{LP}} - \delta(0) + g_r n_r + 2gn_0 \right)^2 - (gn_0)^2 - \frac{i\gamma}{2}} \\ &= \pm \sqrt{\left(\frac{\hbar k^2}{2m_{LP}} - \delta(0) + g_r n_r + 3gn_0 \right) \left(\frac{\hbar k^2}{2m_{LP}} - \delta(0) + g_r n_r + gn_0 \right) - \frac{i\gamma}{2}}. \end{aligned} \quad (2.56)$$

Where we put the losses term back in the expression. This expression is in general drastically different from conservative systems where the Bogoliubov spectrum is linear. This feature is a direct consequence of interactions and thermodynamical equilibrium which imposes that the energetic cost of adding a particles to the system μ is equal to the interaction energy due to the presence of the other bosons gn_0 . Whenever the system is perturbed, it is thus more energetically favorable to dissipate energy through a collective excitation – phonons – rather than to add a particle to the condensate.

The polaritons dynamics is instead dictated by the interplay between losses, pumping and interactions which can give rise to optical bistability. As a consequence, the collective excitations are not necessarily sound waves and can be massive or even energetically unstable. Indeed, one can see from the second line of [Equation 2.56](#) that one of the term in the square root might be negative depending on the values of the parameters. In the following, we will discuss the modifications of elementary excitations depending on the fluid excitation regime.

Interplay with the bistability. As demonstrated in the previous chapter, the system exhibits two distinct density regimes depending on the value of $\delta(k_p)$:

1. *Optical bistability*, when $\delta(k_p) > \sqrt{3}\gamma_{LP}/2$, which allows for two different interaction regimes depending on the pump intensity: a high-density regime characterized by strong interactions and a low-density regime with weak interactions.
2. *Optical limiter*, when $\delta(k_p) < \sqrt{3}\gamma_{LP}/2$. This case will not be considered here, as the various types of elementary excitations it entails are also present in the bistable regime.

To explore the different regimes of collective excitation along the bistability loop, we fix the detuning to be $\delta(0) = 0.2\text{meV}$ and vary the pump intensity. The corresponding values that gn_0 can take are plotted in [Figure ?? a\)](#). At each operating point the collective excitation spectrum is calculated using [Equation 2.56](#) and the results are shown in the subpanel b) of [Figure ??](#). Note that for the discussion we arbitrarily set the reservoir contribution to $g_r n_r = \alpha gn_0$ with $\alpha = 1.84$ based on previous measurement in the team [8]. The value of α is expected to depend on sample parameters and more specifically on the exciton-photon detunning $\omega_X - \omega_\gamma$. In practice, the value we chose doesn't change much the discussion since the only relevant effect the reservoir has on the spectrum is to increase the total blueshift due to interaction. However, it must be taken into account whenever one wants to extract quantitative value from a measured spectrum as we shall see in the next chapter.

Linear spectrum. Let us first examine the most distinctive regime, which establishes a direct connection with conservative systems and was, in fact, the primary motivation for exploring this platform in the context of analog gravity experiments [7]. When the fluid operates at the turning point of the bistability (D) we demonstrated in the previous chapter that the detuning compensates the total interaction blueshift $\delta(0) = g_r n_r + gn_0$. Plugging this constraint in [Equation 2.56](#) we obtain :

$$\omega_B(k) = \pm \sqrt{\frac{\hbar k^2}{2m_{LP}} \left(\frac{\hbar k^2}{2m_{LP}} + gn_0 \right)}. \quad (2.57)$$

We introduce the healing length $\xi = \sqrt{\hbar^2/mgn_0}$ which is the characteristic length scale of the system below which the collective description of the fluid breaks down and microscopic effects must be taken into account. The spectrum exhibits two trends depending whether k is smaller or larger than $1/\xi$ as shown on the (D) plot of [Figure ?? b\)](#). At low wavevector $k \ll 1/\xi$ the spectrum is linear, for the normal branch we have :

$$\Re(\omega_B^+(k)) \sim c_s |k| \quad (2.58)$$

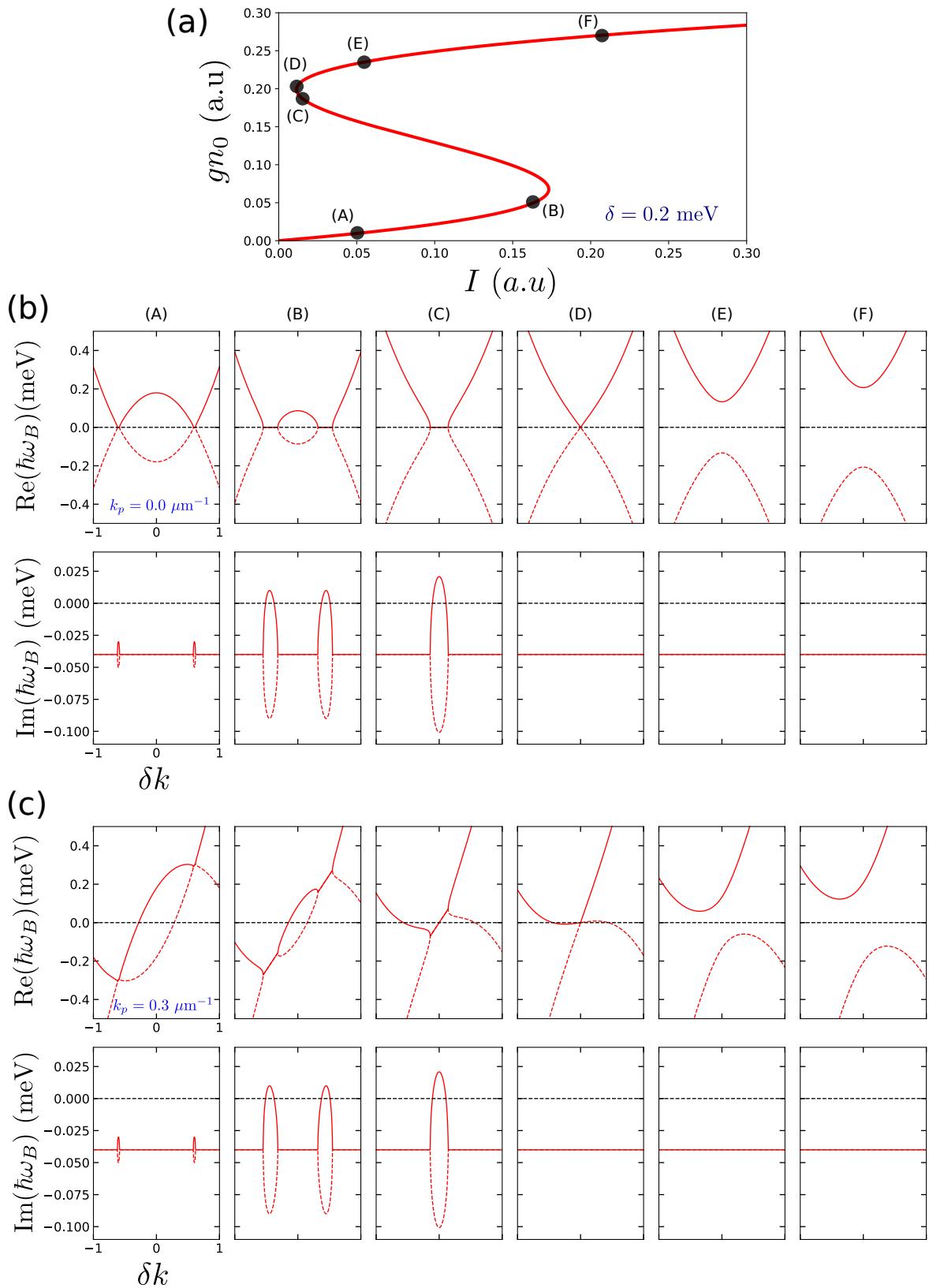


Fig. 2.0 **Analytical Bogoliubov dispersion relations.** (a) Bistability curve of the fluid density n_0 as a function of the pump intensity I , calculated from [Equation 2.17](#), for a pump detuning $\hbar\delta = 0.2$ meV. (b) Positive (solid red lines) and negative (dashed red lines) solutions of the Bogoliubov dispersion relation in the fluid reference frame, computed from [Equation 2.48](#), at the pump wavevector $\mathbf{k}_p = 0$ (zero flow velocity), at the pump detuning $\delta(0) = 0.2$ meV and for pump intensities indicated by the points A, ..., F in (a). Upper line: real part of the energy of elementary excitations. The black dotted line highlights the pump energy. Lower line: imaginary part of the energy of the elementary excitations. As long as it remains negative (below the black dashed line), the Bogoliubov solutions are dynamically stable. Note that near and all along the negative slope branch of the bistability, the modes are unstable, giving rise at the point B and C respectively to modulation instabilities (coupling of the positive solution with the negative solution at $\mathbf{k} \neq 0$) and Kerr instabilities (coupling of the positive and the negative solutions with the pump mode at $\mathbf{k} = 0$). At the other points A, D, E, F, the system losses fix the imaginary part at $-\gamma_{LP}/2$, stabilizing the fluid. (c) Same considerations as for (b) but at a pump wavevector $k_p = 0.5 \mu\text{m}^{-1}$. The fluid speed of flow is no longer zero, leading to an asymmetrization of the Bogoliubov solutions. In both (a) and (b) cases, the linearization of the solutions at low- k , relating the generation of phonon-type elementary excitations, appears only at the turning point D. Adapted from [8]

where $c_s = \sqrt{\hbar g n_0 / m_{LP}}$ is the speed of sound in the fluid. For the ghost branch, represented by the red dashed line c_s is replaced by $-c_s$. At this operating point the driving laser is forcing the system exactly at its proper energy and doesn't fix the phase of the excitations. In this regime, the long range interactions are dominated by collective deformation of the fluid –phonons– that propagate with the same group velocity $\partial\omega/\partial k = c_s$ regardless their wavevector. The idea of having –in the low wavevector limit – a fixed velocity at all frequencies remind the light cone with the difference that the sound velocity is not the same in all the reference frames. The linearity is a typical feature of quantum fluids and give rise to spectacular collective behavior such as superfluidity [1] as well as topological protection of quantized vortices and dark solitons [31]. The bogoliuvov spectrum was reported in several experiments [47; 51] with different experimental methods while the linearity have been measured with an unprecedented resolution in the group [7]. The latter experiment was performed with a spectroscopy method recently developped [6] that we will describe and use in the next chapter.

In the high wavevector limit, $k \gg 1/\xi$, we recover the standard parabolic dispersion blueshifted by the interaction.

$$\Re(\omega_b^+(k)) \sim \frac{\hbar k^2}{2m_{LP}} + gn. \quad (2.59)$$

It's worth noticing that at even higher wavevector the parabolic approximation is not true anymore and the full LP dispersion must be plugged in the bogoliubov matrix. In this case one would observe that the bogoliubov spectrum is flattened by the exciton line at high wavevector [15].

Massive excitations regime. If the system is brought further along the high density branch ie if $gn_0 + grn_r > \delta(0)$ a gap opens in the spectrum and linearity is lost as it can be seen in plot (E) and (F). A limited development of the spectrum around $k = 0$ gives the parabolic approximation:

$$\Re(\omega_b^+(k)) \sim \omega_B^+(0) + \frac{\hbar k^2}{2m_{LP}} \frac{(2gn_0 + grn_r - \delta(0))}{\sqrt{(2gn_0 + grn_r - \delta(0))^2 - (gn_0)^2}}. \quad (2.60)$$

This enable to provide the Bogoliubov modes with an effective mass :

$$m_{\text{det}} = m_{LP} \frac{\sqrt{(2gn_0 + grn_r - \delta(0))^2 - (gn_0)^2}}{(2gn_0 + grn_r - \delta(0))}. \quad (2.61)$$

Note that in the limit $gn_0 + grn_r \rightarrow \delta(0)$, the system recovers the massless scenario described earlier. The emergence of a gap in the spectrum can be understood as follows: when the system is driven too strongly relative to the turning point, the phase of the Bogoliubov excitations tends to become locked to the phase of the pump. This induces a form of "rigidity" in the fluid's phase, implying that introducing a phase fluctuation into the system requires a minimum energy.

In both of the previous regimes, the terms in the square root of [Equation 2.56](#) remain positive at all wavevectors meaning the imaginary part of full dispersion never deviate from $\gamma/2$ as shown in the subpanels of [Figure ?? b\)](#) ensuring the stability of the system.

Low density regime. Conversely, when the system is operating on the lower branch of the hysteresis loop, the amount of interaction is not sufficient to compensate for the detuning. Equivalently, it means that for some value of the wavevector the term in the square root of [Equation 2.56](#) becomes negative giving rise to additional contribution to the imaginary part of the spectrum. Note that it doesn't necessarily mean that the system is unstable. Indeed, an instability requires that the overall imaginary part of ω_B^\pm is positive. At very low density like in (A), the inherent losses of the system are sufficient to damp the instability and the imaginary part remains negative at all k . If the pump strength is ramped up near the lower branch turning point (B) the imaginary part of the spectrum might become positive at some wavevector and make the system unstable. Note that the point at which this happens depend on the experimental parameters. Finally, the point (C), located between the two branches, has an imaginary part that is highly positive at low wavevectors. This is the reason why this branch of excitation is often referred as unstable and can not be observed.

The discussion above also holds for the case of a moving fluid provided we set our description in the comoving frame and replace k by $\delta k = k - k_p$ in [Equation 2.56](#).

2.4.1.2 Moving polaritons at finite k_p

In this section we consider again a fluid in motion at $\mathbf{k}_p \neq 0$ that we describe in the lab frame. In terms of regime of collective excitations nature, the phenomenology is similar to the one described above and the discussion can be carried out in the same way. The only difference is that the dispersion relation is now centered around the fluid wavevector \mathbf{k}_p and "tilted" by the doppler effect. Consequently, the bogoliubov spectrum graphically loses an axis of symmetry which is a graphical manifestation that \mathcal{L}_B no longer respects the symmetry of [Equation 2.25](#). Again we set $\delta(k_p) = 0.2\text{meV}$ and we plot the dispersion relation for the same operating points than in the static case, the results are presented in the subpanels of [Figure ?? c\)](#). Note that the spectra are displayed as function of the wavevector in the fluid frame since [Equation 2.48](#) mathematically depends only on δk enabling to compare the spectra in the two reference frame.

In the linear case $gn_0 + grn_r = \delta(k_p)$, the low wavevector slope is modified by the fluid velocity :

$$\begin{aligned} \Re(\omega_b^+(\delta k)) &\sim (c_s + \mathbf{v}_0)|\delta k| \quad \text{for } \delta k > 0 \\ \Re(\omega_b^+(\delta k)) &\sim (c_s - \mathbf{v}_0)|\delta k| \quad \text{for } \delta k < 0 \end{aligned} \tag{2.62}$$

From this, one can see that when the fluid is supersonic $\mathbf{v}_0 > c_s$ as in plot (D) some modes of the normal branch –the positive norm modes– are pulled down to the negative frequency domain. Symmetrically, some ghost branch modes –the negative norm modes– are pulled up to the positive frequency domain. In both case, the excitations carry negative energy as explained in [subsubsection 2.4.0.1](#), in this regime superfluidity is lost and any defect in the fluid flow can create excitations [1]. If the pump intensity is increased further like in (E)

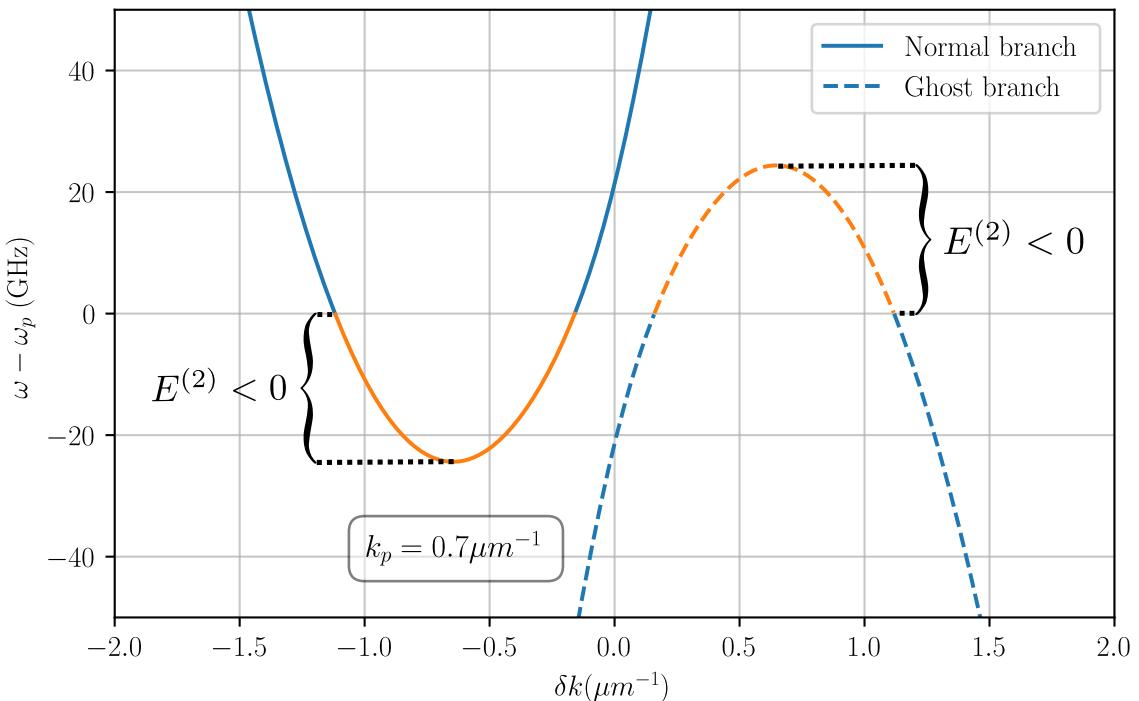


Fig. 2.1 Bogoliubov spectrum for a fluid in motion at $k_p = 0.7 \mu m^{-1}$ and $\delta(k_p) = 0.2$ meV. The dashed line represent the ghost branch ie the negative norm branch while the solid line represent the normal branch ie the positive norm branch. The blue color represents the positive energy modes while the orange represent the negative energy modes resulting from the product of a norm and a frequency with opposite signs.

and (F), a gap opens again in the spectrum, the excitations are massive while the negative energy modes may disappear due to the gap opening. However, it doesn't mean that the possibility to create excitations with negative energy is lost as soon as the spectrum is no longer linear. Indeed, increasing further the pump velocity can lead to situations where the spectrum is massive but still has negative energy modes. A typical example of this case is presented in [Figure 2.1](#), where the pump wavevector was set to $k_p = 0.7\mu\text{m}^{-1}$ and the other parameters stayed the same. A gap is present but the doppler effect is strong enough to excite negative energy modes on both branches which are represented by the orange lines. Unlike the linear case, it is not possible to define a sound velocity for the low wavevector excitations. Consequently, the critical velocity to exceed for the appearance of negative energy modes doesn't match the velocity $\sqrt{\hbar g n_0 / m_{LP}}$ that we still refer to as the sound velocity by analogy with linear case. Moreover, since the velocity affects the dispersion itself through the detuning $\delta(k_p)$, the usual procedure of applying the Landau criterion—where the energy of excitations is modified solely through a Doppler shift $\omega_B(k) \rightarrow \omega'_B(k) + v_0 k$ —is no longer valid. In this case, the spectrum itself is inherently altered by the fluid motion, meaning that the flow velocity does not simply act as a shift parameter but fundamentally changes the nature of the excitations. As a result, the criterion must be modified to account for this direct dependence. In practice, the knowledge of an analytical expression for the critical velocity is not necessary and knowing that there exist one is sufficient as we will see in the experimental section.

It is now clear that depending on its velocity and parameters the fluid can exhibit a rich variety of collective excitations. Let us summarize what we learned so far.

Summary

- The polariton fluid exhibits a rich variety of collective excitations depending on the pump intensity and detuning through the bistability cycle. The different modes are characterized by their norm and frequency.
- There exist a peculiar operating point at which the spectrum is linear allowing for the definition of a sound velocity $c_s = \sqrt{\hbar g n_0 / m_{LP}}$.
- The spectrum can also be massive if the system is pumped further along the high density branch and the effective mass depend on the detuning and the pump intensity.
- In the low density regime, the spectrum can become unstable and exhibit a positive imaginary part.
- Putting the fluid in motion can lead to the appearance of negative energy modes. The critical velocity for this to happen is a complex function of the detuning and the interactions. It doesn't match the speed of sound c_s as soon as the system doesn't operate at the turning point of the hysteresis loop.

In particular, if we restrict ourselves to high density regime where, inherent losses apart, the bogoliubov dispersion remains real, the fluid can be either subcritical—all the modes carry positive energy—or supercritical—some modes carry negative energy. The long way

we have made to prove that quite simple feature paves the way to already understand how Hawking radiation can be observed in a polariton fluid. Indeed, creating an interface between a subcritical and a supercritical region will mix positive and negative energy modes just as a gravitational event horizon would. Obviously, showing that it can lead to paired correlated emission from vacuum will require to quantize the modes as we will see in the next section.

2.5 Hawking radiation in a transcritical fluid

2.5.1 Inhomogeneous pumping

We consider a driving pump defined as follow :

$$F_p(r,t) = \begin{cases} F_u e^{i(k_u x - \omega_p t)} & \text{if } x < x_h \\ F_d e^{i(k_d x - \omega_p t)} & \text{if } x > x_h \end{cases} \quad (2.63)$$

where x_h is an arbitrary position that will define the interface between the two regions. The wavevectors are chosen so that $0 < k_u < k_d$ and k_d is sufficiently high to create a subcritical flow. The fluid is flowing toward the $x > 0$ direction and the label "u" and "d" refer respectively to "upstream" and "downstream" region of the fluid with respect to the interface. We want to study Hawking emission of propagating modes so we set the laser frequency in order to ensure bistability in both region namely $\delta(k_u), \delta(k_d) > \sqrt{3\gamma_{LP}/2}$, and the pump amplitudes F_u and F_d so the fluids operate on the high density branch of their respective hysteresis cycle. Note that the pump is two dimensionnal but only depend on x . As we will justify it in the experimental section, the sample is in some extent also translationally invariant in the y direction. Henceforth, we restrict our description to an effective 1D model. Solving the GPE far from the interface gives two homogeneous regions with different asymptotic velocities $v_u = \hbar k_u / m_{LP}$ and $v_d = \hbar k_d / m_{LP}$, and different densities n_u and n_d . We also account for fluctuations around each of these solutions the same way we did in the previous section which gives the asymptotic wavefunctions :

$$\psi(x,t) = \begin{cases} (\sqrt{n_u} + \delta\psi_u) e^{i(k_u x - \omega_p t)} & \text{if } x \ll x_h \\ (\sqrt{n_d} + \delta\psi_d) e^{i(k_d x - \omega_p t)} & \text{if } x \gg x_h \end{cases} \quad (2.64)$$

Note that continuity condition at x_h makes the definition of the wavefunction near the interface unclear for now. Typical density and velocity profiles are shown in ???. For the sake of clarity, the speed of sound is plotted allowing direct comparison with the fluid velocity. We remind nonetheless that the previous section made clear that the relevant velocity to cross generally differ from c_s .

We wish to solve the scattering problem for the Bogoliubov modes at the interface. In each homogeneous region, fluctuations are plane wave oscillating at ω of the form :

$$\begin{pmatrix} u_k(x) \\ v_k(x) \end{pmatrix} = \begin{pmatrix} U_k^u \\ V_k^u \end{pmatrix} e^{i(k+k_{u,d})x}. \quad (2.65)$$

The wavevector k and frequency ω satisfy the Bogoliubov dispersion relation $\omega = \omega_B(k)$ that we remind here :

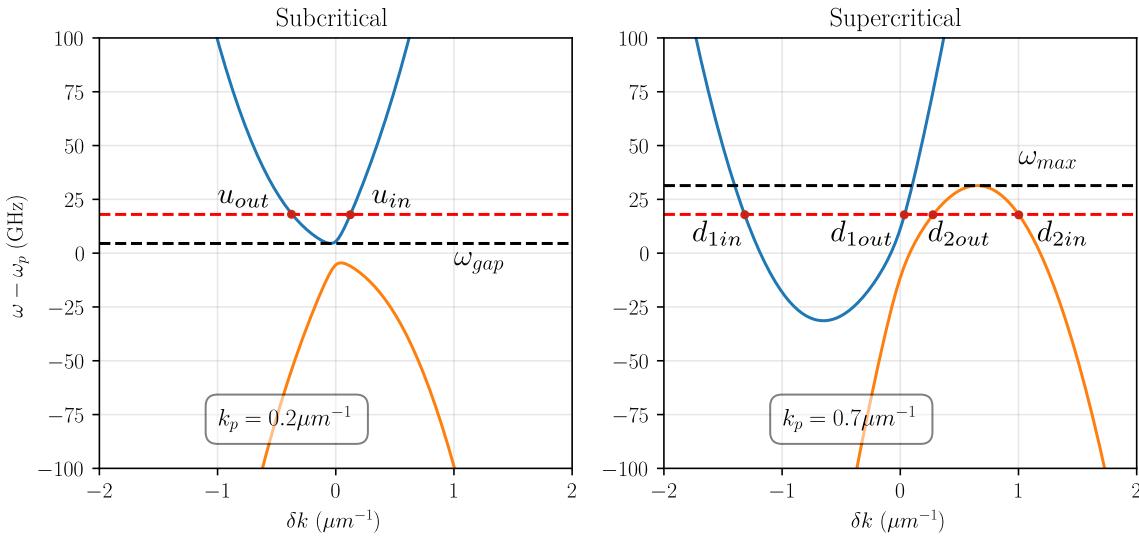


Fig. 2.2 **Bogoliubov spectrum in the upstream and downstream regions.** The Bogoliubov spectrum in the asymptotic upstream region (a) and downstream region (b) of the fluid, calculated from Equation 2.48 with $\delta(0) = 30$ (GHz), $k_u = 0.2 \mu m^{-1}$ and $k_d = 0.7 \mu m^{-1}$.

$$\omega_B(k) = v_{u,d}k \pm \sqrt{\left(\frac{\hbar k^2}{2m_{LP}} - \delta(k_{u,d}) + g_r n_r + 2gn_0\right)^2 - (gn_0)^2 - \frac{i\gamma}{2}} \quad (2.66)$$

where $v_{u,d} = \hbar k_u/m_{LP}$ are the fluid velocities in the upstream and downstream regions. The corresponding dispersions are plotted in Figure 2.2. We see that, in the subcritical region ($x < x_h$), positive- and negative-norm modes are still found exclusively at positive and negative frequency, respectively. However, in the supercritical region ($x > x_h$), some positive-norm modes are dragged to negative frequencies (and reciprocally for negative-norm modes) by the Doppler effect.

Since there is redundancy of the information with respect to the zero frequency line let us fix $\omega > 0$ and look for the wavevectors k satisfying $\omega = \omega_B(k)$ in each region.

Subcritical region. In the upstream region $x < x_h$, for any $\omega > 0$ there always exist two real solution corresponding to positive norm modes as visible on the left panel of Figure 2.2. Depending on the sign of their group velocity $v_g = \partial\omega/\partial k$, we label the solution "out" if it propagates away from the interface and "in" if it goes toward the interface. Taking the square of Equation 2.66 the problem can be cast under a real coefficient polynomial equation of degree 4 in k . As a consequence, there exist two additional complex conjugate solutions. Depending on the sign of their imaginary part, they are either exponentially growing or decaying.

Supercritical region. In the downstream region $x > x_h$, the situation is different and require a disjunct analysis. If $\omega > \omega_{max}$ the situation is the same than in the upstream

region. There exist two propagating solutions d_{1in} and d_{1out} lying on the normal branch with opposite group velocity as well as two non propagating modes. When $\omega < \omega_{max}$, in addition to the positive norm solution, the doppler shift pull two negative norm solutions d_{2in} and d_{2out} herited from $\omega = \omega_B^-(k)$

Scattering solutions-in global modes A general function to describe the fluctuations at ω is obtained by the linear combination of all the possible propagating solutions :

$$\begin{pmatrix} u_k(x) \\ v_k(x) \end{pmatrix}_{u,d} = \left(\sum_{i \in \text{in}} \beta_i^{\text{in}} e^{ik_i^{\text{in}} x} \begin{pmatrix} U_{k_i^{\text{in}}} \\ V_{k_i^{\text{in}}} \end{pmatrix} + \sum_{i \in \text{out}} \beta_i^{\text{out}} e^{ik_i^{\text{out}} x} \begin{pmatrix} U_{k_i^{\text{out}}} \\ V_{k_i^{\text{out}}} \end{pmatrix} \right)_{u,d} \quad (2.67)$$

From this expression we can form the so called scattering solutions. Such a solution reflect how a given mode impinging on the interface is scattered into the other channels. For instance, for the case of an incoming mode from the upstream region, the scattering solution is a piece-wise function given by :

$$\bullet \quad \begin{pmatrix} u_k(x) \\ v_k(x) \end{pmatrix}_u = \beta_u^{\text{in}} e^{ik_u^{\text{in}} x} \begin{pmatrix} U_{k_u^{\text{in}}} \\ V_{k_u^{\text{in}}} \end{pmatrix} + \beta_u^{\text{out}} e^{ik_u^{\text{out}} x} \begin{pmatrix} U_{k_u^{\text{out}}} \\ V_{k_u^{\text{out}}} \end{pmatrix} \text{ for } x \leq x_h \quad (2.68)$$

$$\bullet \quad \begin{pmatrix} u_k(x) \\ v_k(x) \end{pmatrix}_d = \beta_{d2}^{\text{out}} e^{ik_{d2}^{\text{out}} x} \begin{pmatrix} U_{k_{d2}^{\text{out}}} \\ V_{k_{d2}^{\text{out}}} \end{pmatrix} + \beta_{d1}^{\text{out}} e^{ik_{d1}^{\text{out}} x} \begin{pmatrix} U_{k_{d1}^{\text{out}}} \\ V_{k_{d1}^{\text{out}}} \end{pmatrix} \text{ for } x \geq x_h. \quad (2.69)$$

A given solution and its derivative has to be a continuous in space which imposes matching condition at the interface. This condition will fix the coefficients β_i^{in} and β_i^{out} that can be interpted as reflection and transmission coefficents of the incoming wave at the inteface. These modes are definded on the whole space and solve the equations of motion everywhere. From the three *in* modes u_{in} d_{1in} and d_{2in} in the $\omega_{gap} < \omega < \omega_{max}$ regime it is possible to construct three scattering solutions. At a given frequency ω one can verify that these solutions are orthonormal with respect to the Bogoliubov inner product :

$$\langle \delta\psi_i^{\text{in}}(\omega) | \delta\psi_j^{\text{in}}(\omega') \rangle_B = \int dx u_{i,\omega}^*(x) u_{j,\omega'}(x) - v_{i,\omega}^*(x) v_{j,\omega'}(x) = \text{sign}(\langle \psi_i^{\text{in}} | \psi_i^{\text{in}} \rangle_B) \delta_{ij} \delta(\omega - \omega'). \quad (2.70)$$

where $i,j \in \{u_{in}, d_{1in}, d_{2in}\}$ and sign is the sign function. As defined, the *in* global modes (GM) form a basis of the Bogoliubov problem valid at any x in the system.

Outgoing global modes. It also possible to define the global solution of an outgoing wave. Take the outgoing mode d_{1out} in the downstream region, in the scattering picture it results from the the transmission of an ingoing upstream mode u_{in} and the reflection of the two ingoing downstream modes d_{2in} and d_{1in} . Doing this procedure in the $\omega_{gap} < \omega < \omega_{max}$ range, the three *out* modes define three outgoing GM. Just as the *in* solutions, the *out* ones form an orthonormal basis of the solution space.

Since we found two basis describing the same set it is possible to express the *out* modes in terms of the *in* modes. Each three *out* modes are linear combination of the three *in* modes which define the so called 3×3 scattering matrix $S(\omega)$:

$$\begin{pmatrix} \beta_u^{\text{out}}(\omega) \\ \beta_{d1}^{\text{out}}(\omega) \\ \beta_{d2}^{\text{out}}(\omega) \end{pmatrix} = S(\omega) \begin{pmatrix} \beta_u^{\text{in}}(\omega) \\ \beta_{d1}^{\text{in}}(\omega) \\ \beta_{d2}^{\text{in}}(\omega) \end{pmatrix}. \quad (2.71)$$

Note that in $\omega > \omega_{\max}$ case the $d_{2\text{out}}$ mode is not present and the scattering matrix should be 2×2 . However, to keep the notation the same everywhere, we also take into account the evanescent modes and allow coefficient some coefficient to be zero in the scattering matrix.

Since the orthonormal structure of the Bogoliubov modes was built with the modified norm $\langle \cdot | \cdot \rangle_B$ it imply that S is unitary for the corresponding extented metric ν :

$$S^\dagger(\omega) \eta S(\omega) = \eta. \quad (2.72)$$

where $\nu = \text{diag}(1,1, -1)$. The unitary condition ensure energy conservation even in the presence of negative energy modes. Indeed, the -1 sign in the third diagonal term account for norm conversion and originate from the scattering of positive norm modes into negative norm modes. The square modulus of a given coefficient $|S_{ij}|^2$ gives the probability of reflexion or transmission of the mode i into the mode j .

To make this point clear, let us tackle the example of the scattering of an incoming mode u_{in} with energy $\hbar\omega$ in the upstream region. There is a probability $|S_{uu}|^2$ that the mode get reflected in u_{out} and a probability $|S_{d1u}|^2$ that it gets transmitted into d_{1out} both of this mode have the same energy. The remaining term $|S_{u2}|^2$ is the probability that the mode get transmitted into a negative norm mode d_{2out} which has the opposite energy $-\hbar\omega$. The unitary condition [Equation 2.72](#) ensures that the total energy is conserved $1 = |S_{uu}|^2 + |S_{d1u}|^2 - |S_{d2u}|^2$.

It is interesting to stress that the asymptotic classification of modes as *in* or *out* mirrors the procedure used in the study of quantum fields in black hole spacetimes, where a causal, time-oriented interpretation is adopted: in modes represent incoming configurations in the remote past, while out modes correspond to outgoing configurations in the distant future. This structure forms the basis of the Bogoliubov transformation formalism, which relates the two mode bases and captures the mixing between positive and negative frequency components that underlies processes such as particle production.

2.5.2 Modes quantization

So far, we described the fluctuations of the order parameter as classical fields. This approach show that the bogoliubov modes can exist in the system as solutions to the equations of motion. The scattering matrix a priori tells us how an incoming wavepacket is scattered at the interface. Yet, our classical description require an external perturbation to inject non zero amplitude in a given mode. Hence it doesn't predict what would happen with highly non classical state as an input, like the vacuum state. To achive this, we need to quantize the modes.

Quadratic Hamiltonian. The first step is to write the field operator :

$$\hat{\psi}(x,t) = \psi_0(x,t) \hat{a}_{\psi_0} + \hat{\delta}\psi(x) \quad (2.73)$$

where \hat{a}_{ψ_0} is the operator that annihilate a polariton in the mean field state and $\delta\hat{\psi}(x)$ is the fluctuation operator that annihilate a fluctuation. Pluggin this ansatz into the GPE in its second quantization form 1.67 and going to first order in the fluctuations, one finds that the hamiltonian has a quadratic form in the fluctuation operator. As a consequence, the Heisenberg equations of motion for the Bogoliubov operator are linear in the fluctuations [4]. This property is of great importance. Indeed, it allows to use a very simple quantization procedure. The fully quantum decomposition of the fluctuation operator is obtained from its classical counterpart 2.39 by promoting the coefficients b_k to bosonic operators. A positive norm modes coefficient is promoted to an annihilation operator \hat{b}_k while a negative norm mode has to be replaced by a creation operator. Under this prescription the classical fluctuations field becomes :

$$\delta\hat{\psi}(x,t) = \sum_{k \in S_+} \left[u_k(x)\hat{b}_k e^{-i\omega_k t} + v_k^*(x)\hat{b}_k^\dagger e^{i\omega_k t} \right] \quad (2.74)$$

with the commutation relation

$$\|\phi_i\|_B \|\phi_j\|_B [\hat{b}_i, \hat{b}_j^\dagger] = \|\phi_i\|_B \delta_{ij} \quad (2.75)$$

which is the usual bosonic commutation relation for positive norm modes while for negative norm modes the annihilation operator and the creation operator are exchanged. To encode this property in a convenient way in the notation we rewrite the fluctuation field in a way that also count modes with negative norm at the cost of a redundancy.

$$\delta\hat{\psi}(x,t) = \sum_{k \in S_+} \left[u_k(x)\hat{b}_k e^{-i\omega_k t} + v_k^*(x)\hat{b}_k^\dagger e^{i\omega_k t} \right] + \sum_{k \in S_-} \left[u_k(x)\hat{b}_k^\dagger e^{-i\omega_k t} + v_k^*(x)\hat{b}_k e^{i\omega_k t} \right] \quad (2.76)$$

This writing has the advantage of making explicit that for negative norm modes the frequency ω_k is associated to annihilation operators reflecting the particle-hole symmetry : the creation of a particle a yields the same contribution as the annihilation of a hole.

This procedure remains valid regardless the basis initially chosen and can especially be applied to the global modes exhibited earlier. The fully quantized fluctuations field for fluctuations can then be expressed in terms of *in* global operators as :

$$\delta\hat{\psi}(x,t) = \int d\omega \sum_{j \in \{u_{in}, d_{1in}\}} [\hat{a}_j(\omega)u_{j,\omega}(x) + \hat{a}_j^\dagger(\omega)v_{j,\omega}^*(x)] + \int d\omega [\hat{a}_{d2}^\dagger(\omega)u_{d2,\omega}(x) + \hat{a}_{d2}(\omega)v_{d2,\omega}^*(x)] \quad (2.77)$$

where the operator $\hat{a}_j(\omega)$ is the operator that annihilates a global *in* mode in channel j at frequency ω . An equivalent expression can be written for the *out* global modes noted \hat{b}_i . The two sets of operators are again related by the scattering matrix :

$$\begin{pmatrix} \hat{b}_u(\omega) \\ \hat{b}_{d1}(\omega) \\ \hat{b}_{d2}^\dagger(\omega) \end{pmatrix} = S(\omega) \begin{pmatrix} \hat{a}_u(\omega) \\ \hat{a}_{d1}(\omega) \\ \hat{a}_{d2}^\dagger(\omega) \end{pmatrix} \quad (2.78)$$

Few comments are in order at this stage. First, since we deal with quadratic Hamiltonian, the matrix is exactly the same obtained in the classical case. From an experimental point of view this property is a crucial asset. Indeed, it means one can reconstruct the full matrix and hence all its quantum properties by just solving the scattering problem for classical modes which are much easier to engineer than quantum state. Any quantum correlation between the modes are in fact encoded in the matrix coefficient under phase and amplitude value of the reflexion-transmission coefficients. Of course, solving the scattering problem would only be a way to infer correlations between the modes but not to measure them. Still, it gives information about what you can expect from a given analog horizon.

Secondly, while the *in* and *out* operators describe the same physical system they generate two different Fock states and thus two distinct vacuum states $|0_{\text{in}}\rangle$ and $|0_{\text{out}}\rangle$, as explained in the harmonic oscillator example. This raises the question of whether a preferential basis can be found to provide a unequivocal description of the system. The answer is no. Indeed, unlike the Harmonic oscillator the system doesn't exhibit symmetries that justify one choice over the other. This being said, the *in* and *out* basis remain wise choices to describe the system since they are natural basis to infer what can be measured in the lab.

Finally, the transformation induced by the scattering matrix between the *in* and *out* operators is a Bogoliubov transformation where the normalisation condition is hidden in the unitarity condition 2.72. We already discussed how such a transformation is the heart of any particle creation process and in particular of the gravitational Hawking radiation. From an academic point of view, it is simultaneosuly satisfying and puzzling that this simple transformation can help understanding the behavior of drastically different objects : a gravitational black hole and a transcritical quantum fluid. However one must remain careful with the analogy and never mistakenly deduce that the polariton fluid is "like a black hole". Indeed, a quantum fluid and a black hole are obviously too dissimilar to make that statement true and any quantitative measurement made on analog system may never be transposed to gravitational objects. But what these two systems do have in common is the presence of a horizon that mixes positive and negative energy modes. The polariton platform should then rather be seen as a system to study quantum field theory predictions –initially made for gravitational event horizons– through the universality of the underlying mathematical structure, a bogoliubov transformation.

2.5.3 Spontaneous emission

Now that we have a fully quantized description of the system, expectation values of physical observables can be straightforwardly computed by plugging the desired input quantum state and look how it is modified by the scattering matrix. Let us start with the most spectacular and that Stephen Hawking demonstrated in its original work [19] : spontaneous emission of correlated pairs from vacuum.

We are interested in the expectation value of the outgoing mode that is radiated by the interface away from the supercritical region \hat{b}_u . Its decomposition in terms of the *in* modes is :

$$\hat{b}_u = S_{uu}\hat{a}_u + S_{ud_1}\hat{a}_{d_1} + S_{ud_2}\hat{a}_{d_2}^\dagger. \quad (2.79)$$

The corresponding number operator $\hat{N}_u^{out} = \hat{b}_u^\dagger \hat{b}_u$ then reads :

$$\begin{aligned}\hat{N}_u^{out} = & |S_{uu}|^2 \hat{a}_u^\dagger \hat{a}_u + |S_{ud_1}|^2 \hat{a}_{d_1}^\dagger \hat{a}_{d_1} + |S_{ud_2}|^2 \hat{a}_{d_2}^\dagger \hat{a}_{d_2} \\ & + S_{uu}^* S_{ud_1} \hat{a}_u^\dagger \hat{a}_{d_1} + S_{uu}^* S_{ud_2} \hat{a}_u^\dagger \hat{a}_{d_2} + S_{ud_1}^* S_{uu} \hat{a}_u^\dagger \hat{a}_{d_1} \\ & + S_{ud_1}^* S_{ud_2} \hat{a}_{d_1}^\dagger \hat{a}_{d_2} + S_{ud_2}^* S_{uu} \hat{a}_{d_2} \hat{a}_u + S_{ud_2}^* S_{ud_1} \hat{a}_{d_2} \hat{a}_{d_1}.\end{aligned}\quad (2.80)$$

Using the bosonic commutation relation for the negative norm term $|S_{ud_2}|^2 \hat{a}_{d_2} \hat{a}_{d_2}^\dagger = |S_{ud_2}|^2 (1 + \hat{a}_{d_2}^\dagger \hat{a}_{d_2})$, the expectation value of the number operator in the *in* vacuum state $|0_{in}\rangle$ is :

$$\langle 0_{in} | \hat{N}_u^{out} | 0_{in} \rangle = |S_{ud_2}|^2 \quad (2.81)$$

In the $\omega \notin [\omega_{gap}, \omega_{max}]$ frequency range, the d_2 mode is not present and there is no emission. Conversely, when $\omega_{gap} < \omega < \omega_{max}$ the negative energy mode d_2 is present and yields a non zero emission from vacuum. This is the so-called Hawking effect. In fact, this radiation is correlated with all the other outgoing modes and one could even show tripartite entanglement among them [21]. Nonetheless, this is beyond the scope of this work and we will not go into the details of this calculation. Usually in the literature, correlation at the horizon between the out mode are inferred by computing the two point density correlation function [35; 41] :

$$g^{(2)}(x, x') = \frac{\langle \hat{\delta\psi}^\dagger(x) \hat{\delta\psi}^\dagger(x') \hat{\delta\psi}(x) \hat{\delta\psi}(x') \rangle}{\langle \hat{\delta\psi}^\dagger(x) \hat{\delta\psi}(x) \rangle \langle \hat{\delta\psi}^\dagger(x') \hat{\delta\psi}(x') \rangle} \quad (2.82)$$

Plenty of numerical study in the group have been performed [22], and demonstrated that the interface of transcritical flow of polariton can indeed exhibits Hawking radiation signature in its density correlation map.

An experimental measurement of this correlation map has already been performed in Bose Einstein Condensate of Rb atoms [46]. In this experiment the same analogue black hole is created 7400 times, each shot provide a realization of the density fluctuation map. The two point density correlation function is then computed by taking the average of $\hat{\delta\psi}^\dagger(x) \hat{\delta\psi}^\dagger(x') \hat{\delta\psi}(x) \hat{\delta\psi}(x')$. In the case of a polariton fluid the density fluctuations map is harder to obtain due to the out of equilibrium nature of the system. Indeed, due to the finite lifetime of the polaritons $1/\gamma \sim 10$ ps, a single image of the fluid, even with the fastest camera, is already the results of many realization of the fluctuations which hence average to zero. In the conservative case, a given shot of the experiment freezes the system at given time when the trap is released. Therefore, looking for spontaneous emission in the polariton fluid is not a trivial task. As a first approach one can look for a stimulated version of the experiment which, in the light of the previous section, can be theoretically grasped by injecting a coherent state in the scattering problem. **pas sur de ce que je raconte ici**

2.5.4 Stimulated emission

Consider a coherent state input state $|\alpha_{in}\rangle$ in the upstream region eigenstate of the annihilation operator \hat{a}_u with eigenvalue $\alpha_u \in \mathbb{C}$. By just using that $\hat{a}_u |\alpha_{in}\rangle = \alpha_u |\alpha_{in}\rangle$ the expectation value in this state is straightforwardly computed as :

$$\langle \alpha_{in} | \hat{N}_u^{out} | \alpha_{in} \rangle = |S_{uu}|^2 |\alpha_u|^2 + |S_{ud_2}|^2 |\alpha_u|^2 \quad (2.83)$$

The presence of a coherent input state give rise to an additionnal term $|S_{uu}|^2|\alpha_u|^2$ which is the "classical" reflexion of the input mode on the interface. To obtain a full scattering picture we also compute what is transferred in the other outgoing modes. The calculation are similar and we find :

$$\langle \alpha_{in} | \hat{N}_{d_1}^{out} | \alpha_{in} \rangle = |S_{d_1u}|^2|\alpha_u|^2 + |S_{d_1d_2}|^2 \quad (2.84)$$

for the d_1 outgoing mode and

$$\langle \alpha_{in} | \hat{N}_{d_2}^{out} | \alpha_{in} \rangle = |S_{d_2u}|^2|\alpha_u|^2 + |S_{d_2d_1}|^2 \quad (2.85)$$

for the d_2 outgoing mode. This negative energy mode can be interpreted as the Hawking partner of the positive energy mode u_{out} studied in the first place. If we assume that $|\alpha_u|$ is large enough, the first term in each of the above equation dominates the second one which gives :

$$\frac{\langle \alpha_{in} | \hat{N}_i^{out} | \alpha_{in} \rangle}{|\alpha_u|^2} \approx |S_{iu}|^2 \text{ for } i \in \{u, d_1, d_2\}. \quad (2.86)$$

This quantity is accessible in the lab since it boils down to measuring the classical reflexion and transmission coefficients of the interface when a coherent state is sent toward the interface from the upstream region. Eventhough this feature is not triggered by quantum vacuum fluctuations it is still a manifestation of the Hawking effect. To understand this one must compare the scenarios whether ω is in $[\omega_{gap}, \omega_{max}]$ or not. In the latter case, energy conservation is simply $|S_{uu}|^2|\alpha_u|^2 + |S_{d_1u}|^2|\alpha_u|^2 = |\alpha_u|^2$ and reflects that each of the output channels carry a fraction of the incoming energy. While when $\omega \in [\omega_{gap}, \omega_{max}]$ the same condition reads $|S_{uu}|^2|\alpha_u|^2 + |S_{d_1u}|^2|\alpha_u|^2 - |S_{d_2u}|^2|\alpha_u|^2 = |\alpha_u|^2$ due to the negative energy mode. This directly implies $|S_{uu}|^2|\alpha_u|^2 + |S_{d_1u}|^2|\alpha_u|^2 \geq |\alpha_u|^2$ which means that the two positive energy channels carry more energy than what was injected, the excess being compensated by the negative energy mode. This amplification means that the incoming coherent state extracted energy from the transcritical region by exciting a negative energy mode. This is the same argument that led to the prediction of rotationnal superradiance in black hole physics [19] as well as in classical electromagnetic systems [55].

2.6 Conclusion

In this chapter, we have explored the theoretical framework underlying the observation of Hawking radiation in a polariton quantum fluid. Starting from the Gross-Pitaevskii equation, we derived the Bogoliubov spectrum of collective excitations, highlighting the rich variety of regimes depending on the fluid's density, velocity, and interaction strength. We demonstrated how the interplay between positive- and negative-energy modes leads to the possibility of particle creation at a transcritical interface, analogous to the Hawking effect in black hole physics.

The quantization of Bogoliubov modes revealed the fundamental role of the scattering matrix in describing the mixing of positive- and negative-norm modes at the horizon. This

mixing not only enables spontaneous emission from vacuum but also provides a framework to study stimulated emission, where coherent input states amplify the Hawking-like radiation. Importantly, we showed that the amplification of outgoing modes is driven by the presence of negative-energy modes.

These results establish a theoretical foundation for the experimental investigation of analog Hawking radiation in polariton systems. By leveraging the properties of polariton fluids, such as their out-of-equilibrium nature and tunable parameters, this platform offers a promising avenue to probe fundamental aspects of quantum field theory in curved spacetime. In the next chapter, we will focus on the experimental implementation of these concepts and the challenges associated with observing Hawking radiation in practice.

Chapter 3

Optical generation and spectroscopy of arbitrary acoustic horizons

The study of particle creation in the presence of highly curved spacetime has been a subject of interest for many years. The most famous example is the Hawking radiation [19], which predicts the creation of particles from the vacuum in the vicinity of a black hole event horizon enabling the black hole to evaporate. The obvious difficulty to test this prediction experimentally is double. First, the blackness of such an object makes it hard to spot with a telescope meaning one have to rely rather on the peculiar behavior of visible object moving in the gravitational field of such a supermassive object. The optical observation of a black hole took almost one century since the first prediction of gravitational collapse by Subrahmanyan Chandrasekhar in 1920. It required the synchronization of nine telescope across the world to obtain an optical system whose optical aperture is the size the diameter of earth. the black body temperature of

As mentionned in the previous chapter, the creation of a sonic horizon in a polariton fluid can lead to spontaneous emission of Bogoliubov modes provided the downstream region exhibits a collective excitation spectrum with negative energy modes. Furthermore, it was shown that the strength of the emitted signal depends strongly on the curvature of the horizon, or in other words, its steepness. To ensure that such effect is in principle observable in the laboratory, one needs to fully characterize the mean field of the fluid and locally probe its excitation spectrum. This chapter is dedicated to the description of the full optical generation of arbitrary transonic fluids as well as the characterization of the excitation spectrum on both sides of the horizon. This measurement revealed for the first time negative energy modes in a supersonic quantum fluid, validating the possibility to observe particle creation in a polariton fluid.

The first part of the chapter will focus on the generation of mean fields with arbitrary velocity profiles through the shaping of the pump laser phase. In the second part, I will present the pump probe spectroscopy method used to locally measure the collective excitation spectrum, as well as the results obtained for several transonic fluids with different parameters. The results obtained are reported in Ref ??

3.1 Optical generation of an arbitrary fluid velocity field

Set the coordinates x and y to describe the microcavity plane. As explained in Chapter 2, translational invariance in the xy -plane ensure in-plane momentum conservation along photon absorption, while the wavevectors along the z direction are fixed by the cavity and quantum well lengths. Furthermore, in this experiment, the laser beam is set to be quasi-resonant with the lower polariton branch. As a consequence, the transverse phase of the laser is directly imprinted on the polariton field. Indeed, in the low wavevector limit, the lower polariton branch can be safely approximated by a parabola, namely :

$$\omega_{LP}(\mathbf{k}) = \omega_{LP}^0 + \frac{\hbar k^2}{2m_{LP}}. \quad (3.1)$$

The group velocity of a polariton is then $\mathbf{v} = \frac{\partial \omega_{LP}}{\partial k} = \frac{\hbar \mathbf{k}}{m_{LP}} = \frac{\hbar \mathbf{k}_p}{m_{LP}}$ where k_p is the in-plane wavevector of the pump laser. In the case of a plane wave we can write $k_p = \vec{\nabla}\theta(\mathbf{r})$ where $\theta(r)$ is the spatial phase. This can safely be generalized to more complex spatial phase profiles, which provides a direct link between the driving laser phase and the velocity of the fluid :

$$\mathbf{v} = \frac{\hbar \vec{\nabla}\theta(\mathbf{r})}{m_{LP}}. \quad (3.2)$$

3.1.1 Waterfall configuration

A first realisation of a 1D acoustic black hole in a polaritonic system was done in [35] with the so-called waterfall configuration. The setup was composed of a microcavity etched to be truly one dimensional (a microwire) and a defect was added in the middle of the wire to act as an external attractive potential and force the polariton flow to accelerate. The pump was only shone in the region upstream of the defect. The downstream region of the flow was then made of polaritons that propagate ballistically and whose speed is fixed by the interaction energy in the upstream region. Even though this configuration provides a natural fluid acceleration, the horizon geometry depends greatly on the defect shape and cannot be tuned easily. Furthermore, the collective excitation spectrum and the presence of negative energy modes were not investigated, which prevented to conclude on the nature of the signal observed in the experiment. In this chapter, we present a different approach to generate such flows with an effective 1D fluid fully optically created, as well as a full characterization of the Bogoliubov spectrum.

3.1.2 Target velocity profile

Before analyzing whether the fluid exceeds a critical velocity, the primary objective is to generate a flow exhibiting two homogeneous regions separated by a sharp transition, each characterized by a well-defined velocity. For clarity, the region preceding the transition will be referred to as the upstream region, with velocity v_{up} while the region following the transition will be designated as the downstream region, with velocity v_d . To model this configuration, a

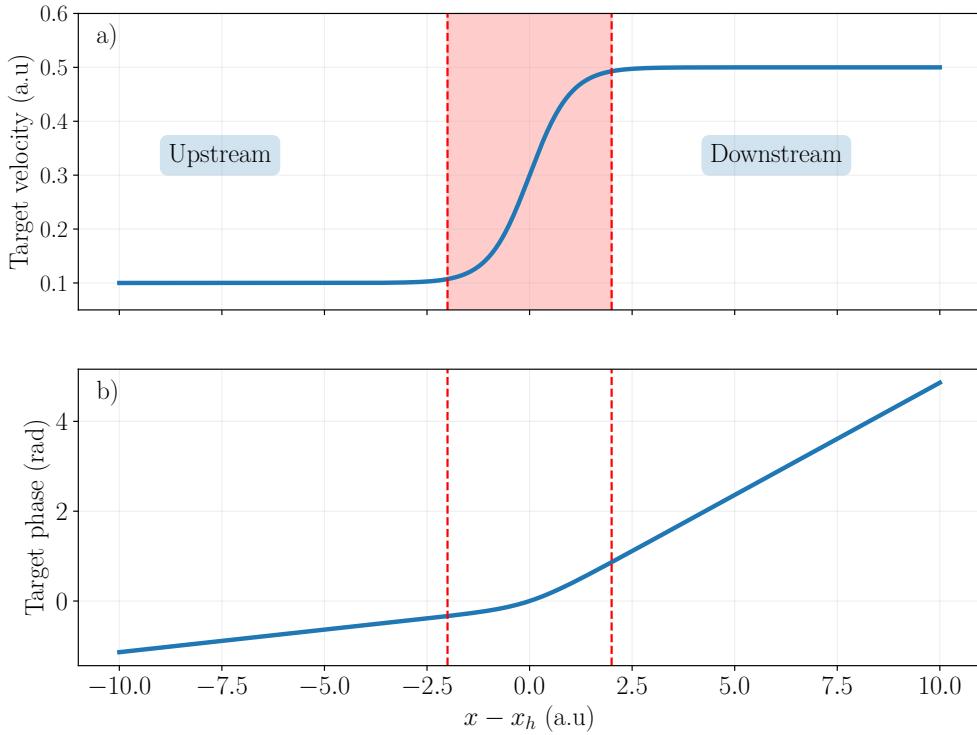
target velocity profile is arbitrarily defined as follows:⁶

$$v(x) = \frac{v_d - v_{up}}{2} \tanh\left(\frac{x - x_h}{w_h}\right) + \frac{v_{up} + v_d}{2} \quad (3.3)$$

where x_h and w_h are the position and width of the transition respectively. This profile is represented in [Figure 3.1 a\)](#). One can then verify that :

$$\lim_{x-x_h \ll -w_h} v(x) = v_{up}, \quad (3.4a)$$

$$\lim_{x-x_h \gg +w_h} v(x) = v_d. \quad (3.4b)$$



[Fig. 3.1 a\)](#) Target velocity profile for input parameters $v_{up} = 0.1$, $v_d = 0.5$, $x_h = 0$ and $w_h = 1$ in arbitrary units. The red shaded area represent the transition region.
[b\)](#) Corresponding phase profile to be imprinted on the pump laser.

Such velocity profile provides great flexibility in the choice of the upstream and downstream velocities as well as the steepness and the position of the transition. From this, one can determine the phase that must be imprinted on the pump laser to generate such a flow profile by simple integration of [Equation 3.3](#), which gives :

$$\phi(x) = \frac{m_{LP}}{\hbar} \int v(x) dx = \frac{m_{LP}}{\hbar} \left(\frac{v_d - v_{up}}{2} w_h \ln(\cosh\left(\frac{x - x_h}{w_h}\right)) + \frac{v_{up} + v_d}{2} x \right). \quad (3.5)$$

The latter is plotted in [Figure 3.1 b\)](#). The derivative of this curve at the position x corresponds to the local wavevector of the pump laser, or equivalently, this curve provides a direct side view of the wavefront of the driving laser. Indeed, the wavefront is defined as the surface of constant phase of the beam. Notably, considering that the driving laser also possesses a wavevector along the z direction, determined by the resonance conditions, the isophase surfaces of the total laser phase θ_{laser} are given by:

$$\theta_{laser}(r) = k_z z + \phi(x) = cst \quad (3.6)$$

$$= \frac{2\pi n_{cav}}{\lambda_0} z + \theta(x) = cst, \quad (3.7)$$

which is inverted as $z \propto \phi(x)$. Making a fluid with the desired velocity field then boils down to be able to imprint the phase [Equation 3.5](#) on the laser that resonantly excites the fluid. As we will see in the next section, this can be done with a Spatial Light Modulator (SLM).

A simplified scheme showing how the printing of the target wavefront in the sample plane creates the desired flow. To verify that the fluid has the expected phase we use the off-axis interferometry method whose principle is explained in [subsection A.1.2](#). As an example, the measured phase corresponding to the mean field displayed in [Figure 3.2 b\)](#) is shown in [Figure 3.3](#).

Controlling the local intensity. Being able to modify on demand the intensity of the pump laser along with the phase is also crucial since the polariton fluid density, and thus the regime of collective excitations, depends on the input intensity as explained in [??](#). Notably, if the system operates at the turning point of the upper bistability loop, it exhibits a linear collective excitation spectrum, and the speed of sound is directly linked to the detuning of the laser with respect to the lower polariton branch as $gn + g_r n_r = \delta(k_p)$. More generally, the possibility to move onto the higher branch of the bistability enables control over the gap opening in the Bogoliubov spectrum, transitioning from a linear, massless spectrum to a parabolic, massive one. Once again, this can be achieved using the SLM by locally adjusting the height of the phase grating under the target phase profile, as explained in [subsection A.1.4](#). This adjustment reduces the number of photons directed into the first diffracted order. This method is widely used to create top-hat intensity profiles and is of significant interest in metrology and cold-atom experiments to minimize systematic effects [\[33\]](#).

To summarize, it is possible to generate and monitor a fluid with arbitrary density and velocity flow by shaping the pump laser both in phase and intensity. We hence have a full optical way to create effective space-time on which we can study the propagation of the collective excitations of the fluid near the transition region.

3.1.3 Effective 1D fluid

So far, the description of the fluid both in velocity and density was one dimensional despite the fluid is actually a 2D system. This assumption relies on the fact that the fluid wavefunction is invariant under translation in the y direction in the region of interest represented by the

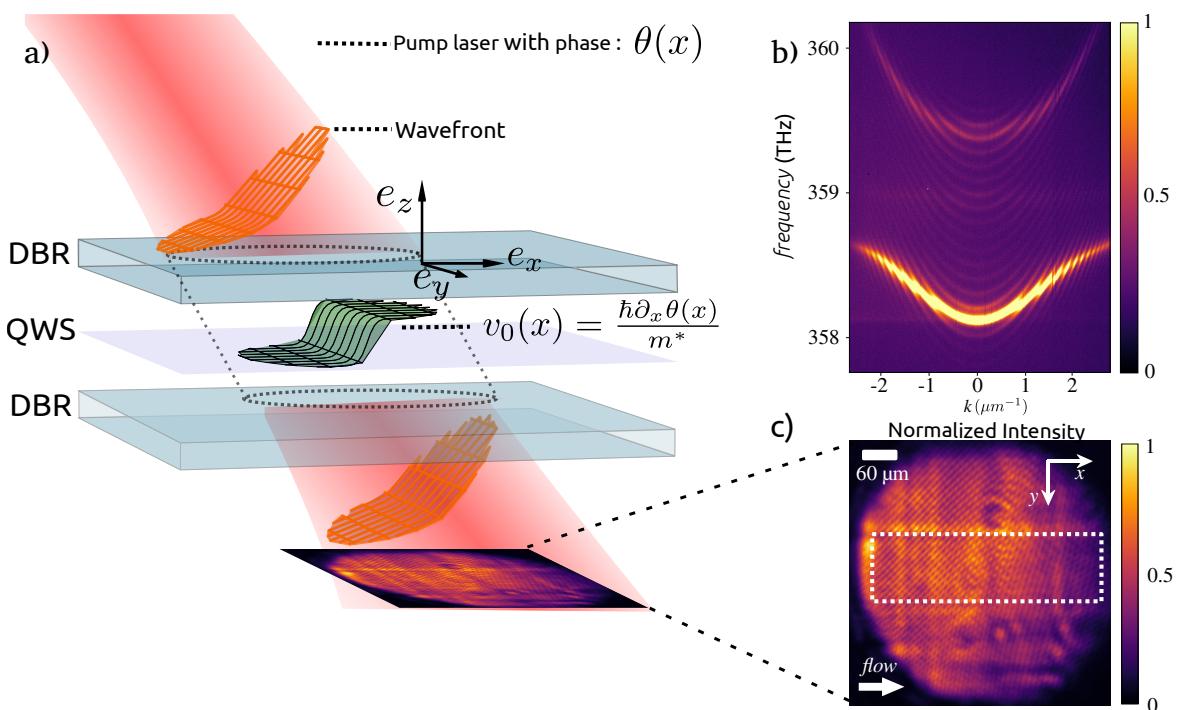


Fig. 3.2 Simplified scheme of the generation of a transonic fluid. a) The pump laser is shaped with the target phase profile and sent on the microcavity. The corresponding fluid velocity field is represented by the black surface lying on the Quantum Wells (QWs). c) The outgoing photons are collected and the sample plane is imaged on a CCD camera to obtain the fluid density map. b) Measured Lower and Upper polariton dispersion at the working point C6 – D6 at which the experiment was run. By fitting this dispersions, the effective LP mass can be extracted $m_{LP} = 7.0 \times 10^{-35}$ kg

white dashed rectangle of **Figure 3.3**. This area is about $130\mu\text{m}$ long and $30 - 40\mu\text{m}$ large. Cuts of both the phase and density in the direction orthogonal to the propagation are shown in **Figure 3.3 b) and c)**. Periodic modulations are present because of back reflection on the protection screen of the camera sensor and are present even without passing through the sample. These modulations can be removed using Fourier filtering which allows to compute the variation of the phase with respect to 2π , $\sigma_y^{\text{ph}}/2\pi = 0.3\%$ as well as the relative intensity variation $\sigma_y^I/\langle I \rangle = 3\%$. As a consequence, we can safely assume that the fluid is effectively 1D in the region of interest. This in contrast with the work [35] previously mentioned. In the present configuration, the shape of the horizon is fixed by the phase of the exciting laser and can be tuned at will which is crucial to study the effect of the horizon steepness on the emitted signals. Moreover, the presence of the pump laser in the downstream region brings a double advantage. First, the velocity of the fluid can be changed and does not depend on the interaction energy of the upstream region, allowing to study many fluid configurations. Secondly, this experiment does not suffer from the exponential density decay as in the ballistic propagation [45]. As a consequence, it is possible to probe the collective excitations spectrum in any region of the fluid using the high resolution pump probe spectroscopy method that we will describe in the next section. This method is based on the measurement of the response of the system to a weak perturbation. It is hence somehow proportional to the interaction energy of the fluid gn and better suited to pumped fluid [7].

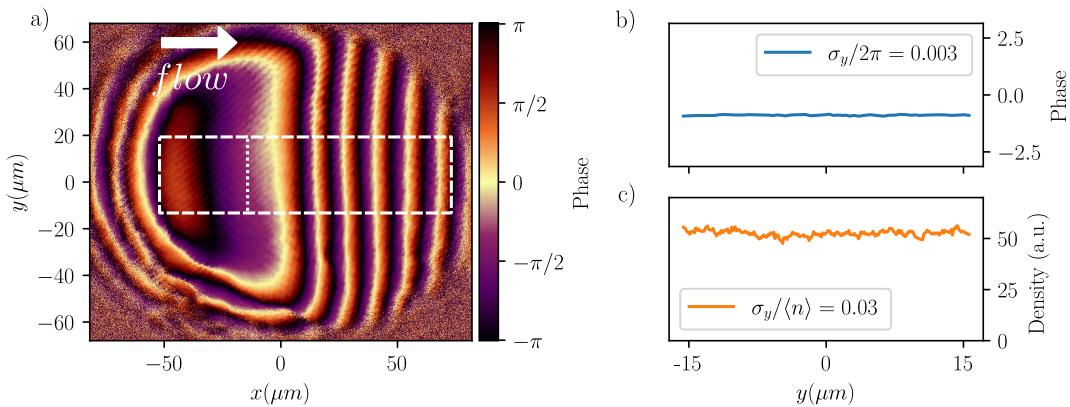


Fig. 3.3 a) Measured phase of the mean field shown in **Figure 3.2**. The curvature present in the upstream region is due to the non linear self focusing. The region on interest in which we assume translational invariance is represented by the white dashed rectangle. b) Cut of the phase in the y direction represented by the dotted white line in the rectangle. The variation to the phase with respect to 2π are equal to 0.3%. c) Cut of the intensity of the mean field in the y direction at the same location than b). The corresponding relative intensity variation is equal to 0.3%.

3.2 Experimental spectroscopy of the collective excitation spectrum

The analog of the Hawking radiation in a polariton fluid is the spontaneous emission of Bogoliubov modes from the horizon. When the system is operating at the turning point of the bistability the Bogoliubov spectrum is gapless and linear which enables to define a speed of sound $c_s = \sqrt{\hbar g n / m_{LP}}$ and speak without ambiguity of sonic excitation in the low wavevector limit. In this regime, the condition to observe negative energy modes coincides with the fluid being supersonic as explained in the previous chapter. However, creating a stable fluid at the turning point is quite challenging and the measurement of the spectrum linearity requires experimental techniques that are hard to implement on a moving fluid [8]. Nevertheless, linearity is not mandatory to observe Hawking radiation since it only requires the mixing between positive and negative energy modes which can be achieved without operating at the turning point. Besides, a gap opening in the Bogoliubov physics brings new physics on the table since it widens the study of quasiparticle creation to the case of massive particles. In this section, we discard the necessity to have sonic excitations and measure the presence of negative energy modes in a wide range of fluid configurations exploring different asymptotic velocities and horizon steepness.

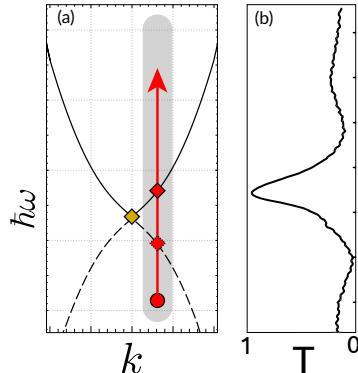


Fig. 3.4 Simplified scheme of the pump probe spectroscopy method. a) At a given in plane wavevector represented by the red arrow, the probe frequency is scanned around the mean field frequency and is transmitted when it resonates with the Bogoliubov branch. b) Typical transmission spectrum of the probe at the wavevector of a).

3.2.1 High resolution pump-probe spectroscopy method

The measurement of the collective excitations spectrum consists in scanning the mean field resonance by looking at the transmission of a weak probe laser at different incidence angles. First, polariton fluid is created with a driving laser as described in the previous section. A probe laser is then sent on the sample with a well defined incidence angle corresponding to a precise in plane wavevector, as explained in section 1.1. The frequency of the probe is then scanned around the mean field frequency. When the probe frequency matches the frequency of a collective excitation mode, the probe creates a perturbation on top of the fluid with the same energy and wavevector and it is detected as photons that ultimately escape from the

cavity. Frequency scans are then repeated for different incidence angles providing each time a transmission spectrum of the probe as shown in [Figure 3.4](#). To make sure that we are in the weak perturbation regime the intensity of the probe is set two orders of magnitude below the pump. To collect only the signal emitted in the perturbation mode, a tunable pinhole is placed in the Fourier plane of the collection path and track the probe in-plane wavevector position to filter out the unwanted photons. Finally, the probe intensity is modulated at f_{mod} in order to isolate its transmission from the very strong signal of the pump. The remaining light is then sent on a photodiode connected to a Spectrum analyzer that demodulates the signal at f_{mod} to obtain the transmission spectrum. At the end, all the scans are put together to reconstruct the full Bogoliubov dispersion as typically shown in [Figure 3.6](#).

3.3 Experimental setup

The setup used in this experiment is shown in [??](#). The sample is a microcavity consisting of three InGaAs quantum wells sandwiched between two highly reflecting planar GaAs-AlGaAs Bragg mirrors. To optimize the light matter coupling, the quantum wells, separated by GaAs barriers, are located at the three antinodes of the cavity, which has a finesse on the order of 3000. A complete map of this sample can be found at the end of this manuscript in [??](#) along with exciton-photon detuning measurements. This map enables to perform an experiment over several days at the same working point and overcome the problem due to daily cooling and warming procedures imposed by the open cycle cryostat. Furthermore, despite rigorous and complete characterization of the sample, the quality of certain measurements can be substantially enhanced by looking phenomenologically the right working point fitted to the experiment.

The experiment described in this chapter was run on the working point $C5 - D6$. The set up is divided in three main paths, the pump path, the probe path and the collection path.

- **The pump path** represented by the blue box is used to create the steady states of the experiment. The polariton fluid is generated by a circularly polarized CW Ti:sapphire laser with a sub-MHz linewidth. This laser can be precisely frequency tuned around the LP resonance energy centered around 836 nm in our sample. An acousto-optic modulator (AOM) together with a Proportional-integral-derivative feedback on the AOM driving RF are used to stabilize the laser intensity and make sure the experiment is constantly run at a single fluid density. The stabilized Gaussian beam is reflected on the spatial light modulator (SLM) which imprints the target phase $\theta_p(x) = \int v_0(x)dx$ that determines the fluid velocity $v_0(x)$ at each point. The SLM plane is imaged on the input plane of the cavity with two focal-length matched telescopes ($2f - 2f$ configuration).
- **The probe path** represented by the red box is used to measure the collective excitation spectrum. The probe is another tunable CW Ti:Sapphire laser with the same polarization than the pump beam. The probe is sent on the sample with a well defined incidence angle controlled by another SLM on which a tunable step blazed grating is imprinted. The frequency of the laser is scanned over 220 GHz around the mean field frequency and monitored by a high resolution wavemeter. An AOM and a Proportional-integral-derivative feedback are also used to both stabilize the intensity along the scan and apply a $f_{mod} = 5$ MHz intensity modulation.

- **The collection path** located after the sample is used to collect the output signals coming from the fluid. A microscope objective sends the outgoing field on a 50:50 BS. One part is directed to an optical system to make the image both in real and momentum space of the field. For both lasers, a pick-up is made after their respective AOM to have a phase reference and perform off axis interferometry measurement. Another part is sent to the apparatus described in [subsection 3.2.1](#) to measure the transmission spectrum of the probe. The tunable pinhole is made with a DMD and is placed in the Fourier plan of a lens, selecting only photons arriving at the position of the pinhole and sending them to the collection photodiode. Given the relative size of the probe mode and the size of a pixel of the DMD, the resolution of the pinhole can go down to $\delta k = 0.0005 \mu\text{m}^{-1}$. The collection photodiode is then connected to a spectrum analyzer set in zero span at f_{mod} to demodulate the signal and obtain the transmission spectrum of the probe. On both path a set of $\lambda/4$, $\lambda/2$ and PBS allows to control and filter the polarization of the signal.

Scan resonances analysis. The maximum value and the linewidth of the probe transmission and reflection peaks are directly related to the real and imaginary parts of the energy $\hbar\omega_b$ of the Bogoliubov dispersion relation. If we consider plane wave like excitations

$$\psi_{LP}(t) \propto \exp(-i\omega_b t) = \exp(-i\Re(\omega_b)t) \cdot \exp(i\Im(\omega_b)t), \quad (3.8)$$

one easily verifies by taking its temporal Fourier transform that its spectral density has the following Lorentz-distribution law

$$I(\omega) = |\psi_{LP}(\omega)|^2 \propto \frac{1}{(\omega - \Re(\omega_b))^2 + \left(\frac{\Im(\omega_b)}{2}\right)^2}. \quad (3.9)$$

At each probe wavevector k_{pr} the transmission spectrum $I_{k_{pr}}(\omega)$ is fitted with a Lorentzian function to extract the real and imaginary parts of the Bogoliubov dispersion $\omega_b(k_{pr})$

3.4 Experimental results

3.4.1 Homogeneous fluid

We first study homogeneous fluids with non zero velocities to see the effect of the Doppler shift on the collective excitations spectrum. In this section, the pump is a gaussian beam which creates fluid having a spatial extension of approximately $150 \mu\text{m}$. The input angle of the pump is controlled in the same way as the probe by printing a tunable step blazed grating on the SLM. In each measurement, the effective detuning $\delta(k_p) = \omega_p - \omega_{LP}^0 - \hbar k_p^2 / 2m_{LP}$ is high enough to be in the bistable regime and the input pump intensity is chosen so the system operates on the higher branch of the hysteresis curve quite far from the turning point. The probe is set to be two order of magnitude weaker than the pump and it has the same spatial extension and polarization.

Direct normal branch measurement. We create a set of homogeneous fluids with increasing in plane wavevectors k_p while keeping the pump frequency constant. For each fluid, we use

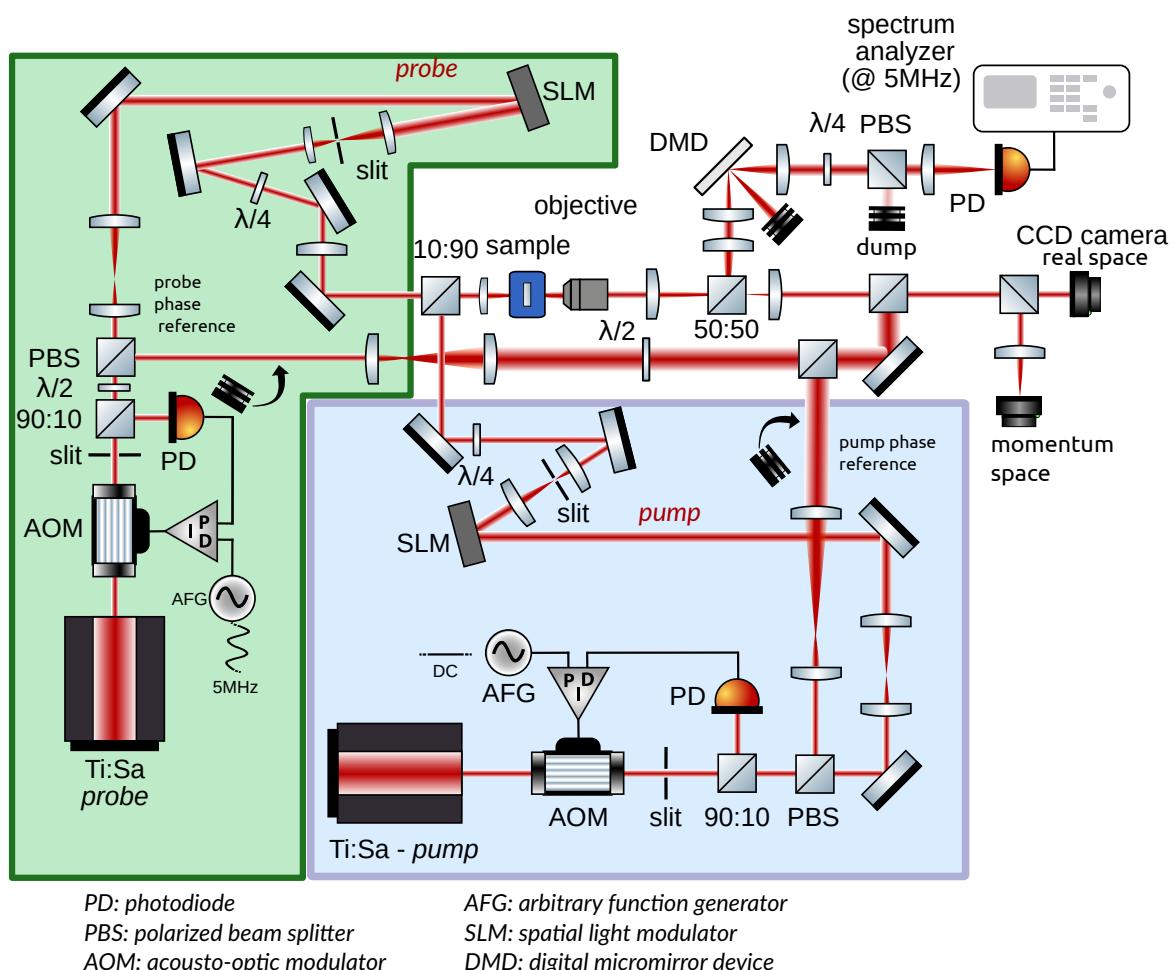


Fig. 3.5 Experimental setup

the pump probe spectroscopy method to measure the probe transmission spectrum as shown on [Figure 3.6](#). The DMD is programmed to display pinholes that track the wavevector of the probe laser along the scans. In this case the system measures the direct transmission of the probe. The first measurement a) is made with the pump laser turned off which means in the absence of interactions within the sample. The probe transmission then scan the bare cavity resonances and recover the parabolic shape at low wavevector of LP branch. From this measurement we extract the detuning between the pump laser and the LP branch at zero wavevector $\delta(0) = 33$ GHz. The blueshift due to interaction lifts the dispersion a) to higher energies whereas the Doppler shift modifies the resonances according to $\omega' = \omega + \mathbf{v}_p \delta \mathbf{k}$ where \mathbf{v}_p is the fluid velocity $\mathbf{v}_p = \hbar \mathbf{k}_p / m_{LP}$. As the fluid velocity increases, the branch is bended and moved toward the pump wavevector. For $k_p \geq 0.3$ we already see some resonances lying under the pump energy. As explained in the previous chapter, the energy sign of a collective mode is given by :

$$\text{sign}(E) = \text{sign}(\omega_B) \times \text{sign}(Q_\phi), \quad (3.10)$$

where Q_ϕ is the norm of the mode defined in ?? and $\text{sign}(\omega_B)$ is taken with respect to the pump energy. Since the normal branch has a positive norm, the resonances located below the pump frequency are negative energy modes. As it can be seen, the ghost branch is not visible through direct excitation. This feature has a double origin : first the ghost branch is not optically resonant with the cavity which makes photon injection difficult at its energy. Secondly, the Bogoliubov coefficient v_k of the negative norm branch is very small compared to u_k . In the presence of a coherent state like the probe, the populations then scale as $v_k^2 |\alpha|^2 \ll u_k^2 |\alpha|^2$ where α is the intracavity amplitude of the probe.

Ghost branch indirect measurement. To overcome the difficulty to directly excite the ghost branch, we take advantage of the strong nonlinearities of the system to induce population through four wave mixing process. Indeed, because of the bosonic nature of Bogoliubov excitations, the occupation of the normal branch in the probe wavevector mode should stimulate the parametric conversion of two pump polaritons into a Bogoliubov pair with opposite wavevectors and energies with respect to the pump, namely :

$$(k_p, k_p) \rightarrow (k_p + \Delta k, k_p - \Delta k), \quad (3.11a)$$

$$(\omega_p, \omega_p) \rightarrow (\omega_p + \Delta\omega, \omega_p - \Delta\omega). \quad (3.11b)$$

The emission in the ghost branch is then boosted by a factor $(u_k v_k)^2 |\alpha|^2$ [15]. In practice, this measurement can be done by changing the positions of the tunable pinhole so it tracks wavevectors opposite to the probe with respect to the pump. More precisely, when a scan at $k_{pr} = k_p + \Delta k$ is made, the pinhole is set at $k_{pr} = k_p - \Delta k$ to collect the signal emitted in the conjugated mode.

Here we create again a homogeneous fluid with wavevector $k_p = 0.6 \mu\text{m}^{-1}$. We then perform direct measurement of the normal branch and indirect measurement of the ghost branch. The results are shown in [Figure 3.7 a\) and b\)](#) respectively. The modulations on the right side of the direct measurement come from interferences in the substrate of the sample

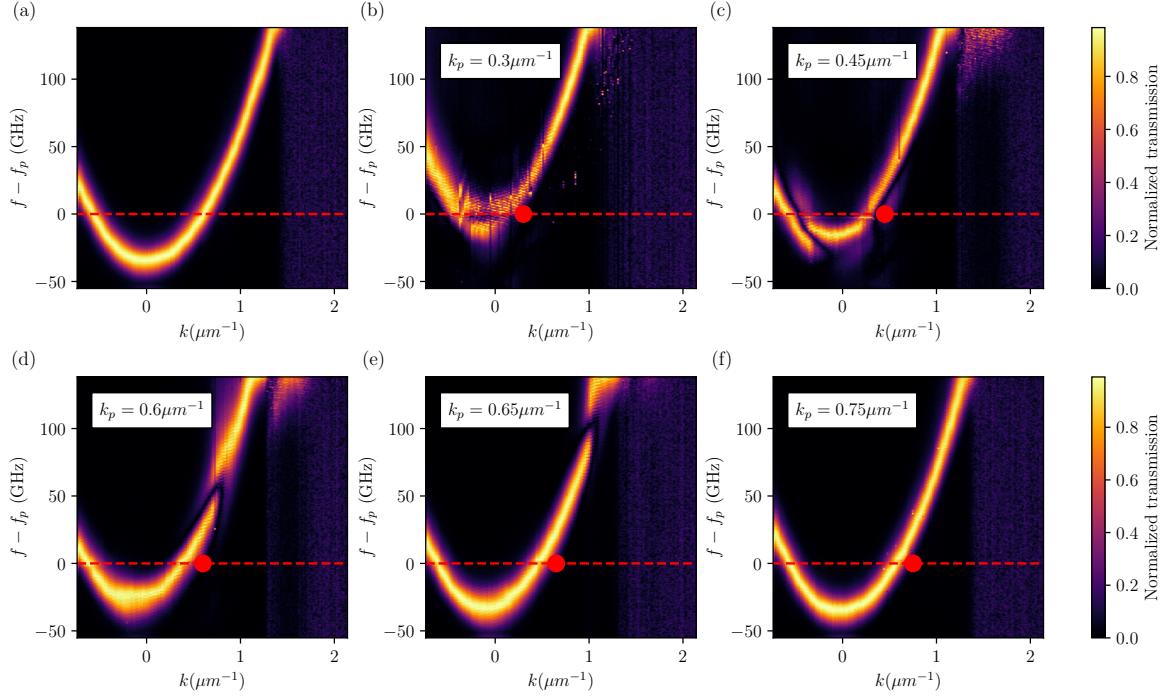


Fig. 3.6 a) Bare cavity dispersion measurement with the pump probe spectroscopy method in the absence of fluid. The parabolic shape of the Lower Polariton branch is observed. The red dashed line represent the mean field frequency that was kept constant for all the measurements. From this we can measure the detuning between the pump laser and the LP branch at $k = 0$, $\delta(0) = 33$ GHz. b),c),d),e) and f) measured Bogoliubov normal branches for homogeneous fluid with velocities 0.75 , 0.98 , 1.23 , 1.29 and $1.43 \mu\text{m ps}^{-1}$ respectively, corresponding to the pump wavevector values in the inset of each figure.

that act as a low Q-factor Fabry Perot [8]. Each scan is normalized to its own maximum for the sake of clarity but the strength of the indirect signals is approximately 1 – 10% of the direct one. The noise present on several scan of b) corresponds to parasitic signals coming from other scatterings process that may send photons at the position of the pinhole and that, in the case of direct measurement, are negligible with respect to the resonance. The reason why the ghost branch does not appear as the symmetric of the normal branch is because the axis k and $f - f_p$ represents the wavevector and frequency of the injected probe and not the emitted signal. However, the pinhole positions first ensure us that the signal is emitted in the opposite wavevector mode. To verify that it is also symmetric in frequency, we send the photons collected through the pinhole into a spectrometer to measure their energy. This two tests allow to confirm that the signals taken through indirect measurement in b) indeed come from the ghost branch. As a consequence, it is possible to symmetrize the indirect data around the pump wavevector and energy to plot the full collective excitation spectrum shown in Figure 3.7 c). First, one can check that the two branches are indeed symmetric with respect to the pump represented by the red dot.

Once again this measurement is an additional proof that for fluid velocities high enough,

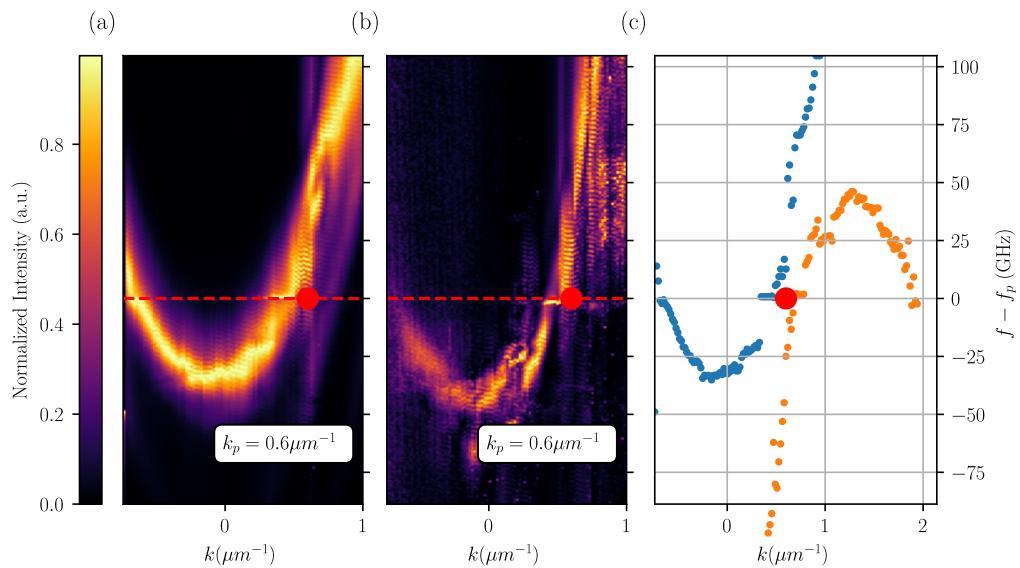


Fig. 3.7 **a)** Direct measurement of the normal branch of the Bogoliubov spectrum for a homogeneous fluid with in plane wavevector $k_p = 0.6 \mu m^{-1}$. **b)** Indirect measurement of the ghost branch of the Bogoliubov spectrum for the same fluid. **c)** Extracted resonance energy for each probe wavevector. The blue point are extracted from the direct measurement and correspond directly to the resonances of **a**). The orange points are extracted from the indirect measurement. Each couple (k_{pr}, ω) extracted from b) is symmetrized around the pump position in phase space (k_p, ω_p) to account for FWM process.

negative energy modes can be excited on both positive (normal) and negative (ghost) norm branches. It's worth noticing that no effort were put in this experiment to reach the turning point of the bistability loop which did not prevent the observation of negative energy modes. This relaxes the experimental constraints to observe Hawking radiation in a polariton fluid and widens the initial analogy that was made for sonic excitation in a supersonic fluid. More precisely it can be reformulated as follows : a fluid exhibiting a sharp transition between a region with positive energy modes, and another one with negative energy modes available, should experience emission of correlated Bogoliubov modes at the transition. These modes are not necessarily phonons and the velocity to exceed to observe them is not necessarily the speed of sound, but rather a critical velocity that depends more generally on the fluid parameters. Although this statement seems unclear with respect to the sonic version, it is actually more general and broaden the range of experimental configurations at which particle creation can be observed. In the following we will rather speak of trans critical flow and critical velocity instead of "sonic".

3.4.2 Smooth transition geometry

We first study a smooth transition between the sub- and supercritical flow regions. We implement the target velocity profile [Equation 3.3](#) with a transition width $w_H = 20 \mu\text{m}$. We study three profiles, with three values for the up- and downstream flow velocities v_u and v_d . All the measurements are made with $\delta(0) = 56 \text{ GHz} \rightarrow \hbar\delta(0) = 0.2 \text{ meV}$. At this detuning the system is in the bistable regime in both the upstream and downstream regions with a spatial extension of $120 \mu\text{m}$. For each configuration, the phase of the created fluid is measured by off-axis interferometry as described in [subsection A.1.2](#). By unwrapping the phase profile, as the one typically shown in [Figure 3.8 a\)](#), and taking the gradient in the x direction, we obtain the fluid velocity profile $v(x)$. The measured velocity profiles are represented by the solid lines in [Figure 3.8 b\)](#) showing three different asymptotic downstream velocities v_d with a single upstream velocity v_u . To selectively measure the upstream or downstream region the diameter of the probe beam is reduced to half the spatial extension of the fluid and the probe focused on the region of interest. The corresponding excitation spectra are shown in [Figure 3.8 c\)-f\)](#). The upstream region is shown in [Figure 3.8 c\)](#) and the downstream regions in [Figure 3.8 d\)-f\)](#).

3.4.2.1 Speed of sound measurement

Even though the system does not operate in a regime of sonic collective excitation we call $c_s = \sqrt{\hbar g n / m_{LP}}$ the speed of sound. The value of gn is extracted from the simultaneous fit of the normal and ghost branch of each spectrum with the Bogoliubov dispersion relation :

$$\omega_b^\pm(\delta k) = -\frac{i\gamma}{2} + v_0(x)\delta k(x) \pm \sqrt{\left(\frac{\hbar\delta k(x)^2}{2m^*} - \delta(k_p) + 2gn(x) + g_r n_r\right)^2 - (gn(x))^2}, \quad (3.12)$$

with $\delta k(x) = k - k_p(x)$. The only unkown parameters in this equation are gn and $g_r n_r$. However by taking the steady states solution of the Gross Pitaevskii equation coupled to a

reservoir [Equation 1.4.2](#) we obtain $n_r \propto n$. Based on the work [7; 8] made on the same sample and the assumption that the reservoir contribution depends on the exciton-photon detuning. We use the value $g_r n_r \approx 1.8gn$ of work [8] and end with a single free parameter gn to fit the spectra. The fit are represented by the colored dashed line in [Figure 3.8](#). The output of the fit gives in fact a value of $gn(r_{pr})$ where r_{pr} is the position of the probe. However, this value can serve as calibration to reconstruct the full $gn(r)$ profile. Indeed we can safely assume that the output measured intensity is proportional to the fluid density as $I_{out}(r) = an(r)$ and that a is a constant that depend on the exciton photon detuning through the hopfield coefficients and complex cavity parameters. Nonetheless, given the spatial extension of the fluid with respect to the wedge of the sample $0.04 \mu\text{eV } \mu\text{m}^{-1}$, these paramateres can be considered as constant in the region of interest. We can then calibrate the speed of sound measurement through :

$$c_s(r) = \sqrt{\frac{I_{out}(r)}{I_{out}(r_{pr})}} c_s(r_{pr}). \quad (3.13)$$

The $c_s(x)$ map obtained for the different configurations are represented by the dashed lines in [Figure 3.8 b\)](#). The speed of sound is found to be quite constant which yields a smooth transition between the upstream and downstream regions.

3.4.2.2 Critical velocity

In conservative fluids, sub- and supercritical flows are discriminated by the Mach number $M := v_0/c_s = 1$. Instead, in our driven-dissipative fluid, the minimum velocity to have a supercritical dispersion is larger than c_s . As a result, the condition $M = 1$, which defines the acoustic horizon in the hydrodynamic limit does not necessarily coincide with the condition to excite negative energy waves in the system. In other words the position at which the velocities exceed c_s in [Figure 3.8 b\)](#) does not define the horizon of the fluid. Indeed, when the system does not operate at the turning point of the bistability we have $gn > \delta(k_p) - g_r n_r$, which opens a gap in the dispersion. We rewrite the dispersion in the laboratory frame in a form that allows to easily identify the effect of the gap on the spectrum,

$$\begin{aligned} \omega_B^\pm(\delta k) &= v_0(x)\delta k(x) - i\frac{\gamma}{2} \pm \left[\left(\frac{\hbar\delta k(x)^2}{2m_{LP}} \right)^2 + \frac{\hbar\delta k(x)^2}{m_{LP}} (2gn_0 - \delta(k_p) + g_r n_r) \right. \\ &\quad \left. + (gn_0 - \delta(k_p) + g_r n_r)(3gn_0 - \delta(k_p) + g_r n_r) \right]^{-1/2} \\ &= v_0(x)\delta k(x) - i\frac{\gamma}{2} \pm \sqrt{\left(\frac{\hbar\delta k(x)^2}{2m_{LP}} \right)^2 + c_B^2 \delta k(x)^2 + m_{det}^2 c_B^4} \end{aligned} \quad (3.14)$$

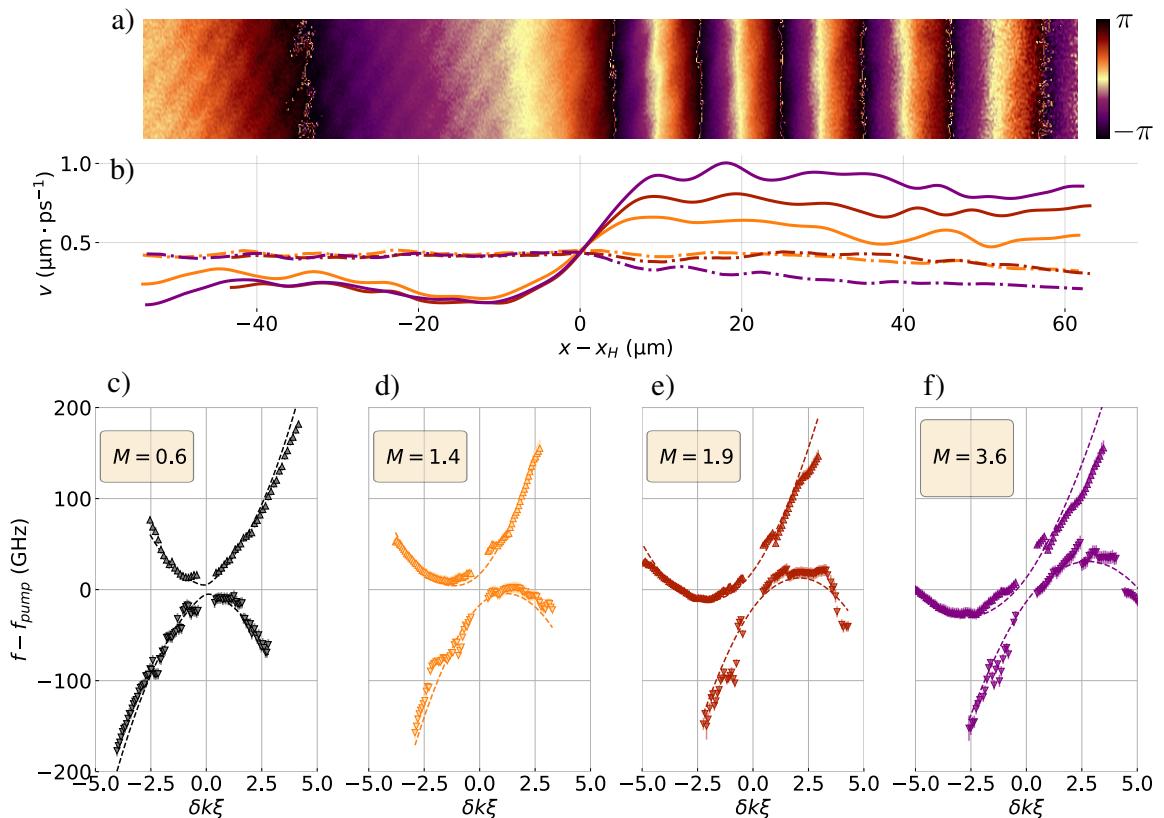


Fig. 3.8 **Smooth horizon.** a) Typical measured phase of the fluid $\theta(x)$ inside the region of interest as defined in Figure 3.3. b) Fluid velocity profiles. Solid lines, $v_0(x)$; dot-dashed lines, $c_s(x)$. Orange, $M_u = 0.6$ and $M_d = 1.4$; red, $M_u = 0.4$ and $M_d = 1.9$; purple $M_u = 0.5$ and $M_d = 3.6$. **Excitation spectra** Eq. (3.12): c) typical upstream region; d)-f) downstream region. Up-triangles, positive-norm branch ω_B^+ ; down-triangles, negative-norm branch ω_B^- ; dashed lines, fit with free parameter gn . Error bars mostly come from the uncertainty in the determination of the center of the lorentzian function used to fit each scan.

where we defined :

$$c_B = \sqrt{\frac{\hbar(2gn_0 - \delta(k_p) + g_r n_r)}{m_{LP}}}, \quad (3.15a)$$

$$m_{\text{det}} = m_{LP} \frac{\sqrt{(gn_0 - \delta(k_p) + g_r n_r)(3gn_0 - \delta(k_p) + g_r n_r)}}{(2gn_0 - \delta(k_p) + g_r n_r)}. \quad (3.15b)$$

These two quantities have a well defined physical meaning as follows. On the one hand, $m_{\text{det}} c_B^2$ is the energy of $k = 0$ modes of ϕ , introducing a mass gap which vanishes when the system operates at the turning point of the bistability. On the other hand, ?? is a hyperbolic PDE whose characteristic curves in a fluid at rest are given by $|\mathbf{x}| = c_B t$. These characteristic curves limit the speed at which the excitations ϕ can propagate information, defining the sound cones in full analogy to light cones. However, due to the mass gap, the speed of propagation of excitations of ϕ is $v_g = d\omega/dk \leq c_B$. The latter can be seen as the limiting speed of propagation of $k \rightarrow \infty$ perturbations. This is also in full analogy to propagation of modes of a massive relativistic scalar field, which travel at subluminal speeds, reaching only the speed of light in the limit of infinite momentum. From [Equation 3.14](#) we see that the field mass m_{det} increases $|\text{Re}(\omega_B^\pm)|$ at all k , meaning that the critical velocity to have a Doppler effect large enough to excite negative energy waves is larger than c_s . The value of the critical velocity $v_c = \hbar k_c/m_{LP}$ at which negative energy waves become available at positive frequency can be computed as follows.

For $k_p > k_c$, there will be negative norm modes available at positive frequencies, and there will be two values $\delta k_{1,2}$ at which each dispersion branch intersects the horizontal axes. These $\delta k_{1,2}$ depend on the value of k_p, gn and $g_r n_r$. The critical wavenumber k_c , and correspondingly the critical velocity $v_c = \hbar k_c/m_{LP}$, is obtained when the two intersection points merge, so that $\delta k_1 = \delta k_2 = \delta k_0$. In practice, we compute numerically the normal branch $\omega_B^\pm(\delta k)$ spectrum for different values of k_p and gn while keeping the zero momentum detuning constant and equal to $\delta(0) = 56$ GHz as in the experiment. On each computed spectrum, we verify if it exhibits negative frequency values. We obtain the map shown in [Figure 3.9](#), indicating if a fluid with given velocity and interaction energy is super-critical. The points c), d), e) and f) correspond to the measured spectrum of [Figure 3.8](#). As it can be seen, the upstream region and downstream region with the smaller velocity do not lie in the super-critical regime whereas e) and f) do. This is in agreement with the fact that the critical velocity is larger than the speed of sound since all the regime yielding super-critical flows are located on the right side of the black dashed line corresponding to the acoustic case $c_s = \hbar k_p/m_{LP} = \sqrt{\hbar gn/m_{LP}}$. Moreover, the map predicts that configuration d) is subcritical despite a Mach number $M_d = 1.34$ greater than one.

Cutoff spatial frequency. The absence of experimental data points at wavevectors $\delta k < 0.10\mu\text{m}^{-1}$ is a consequence of the finite size of the probe beam $\delta_x = 60\mu\text{m}$ which fixes the spatial extension of the probe in momentum space scaling as $2\pi/\delta_x = 0.10\mu\text{m}^{-1}$. At very small wavevector, the probe and its conjugated counterpart start to overlap and photons coming from both branches are collected by the pinhole. This prevent the effective discrimination between the two modes in the ouput signal of the photodiode. In practice the cutoff frequency is computed by a simple optical Rayleigh criterion stating that two points are resolved if the distance between them is larger than the FWHM of the point spread function. In our case this directly gives $\delta k_{\text{cut}} = 2\pi/\delta_x = 0.10\mu\text{m}^{-1}$.

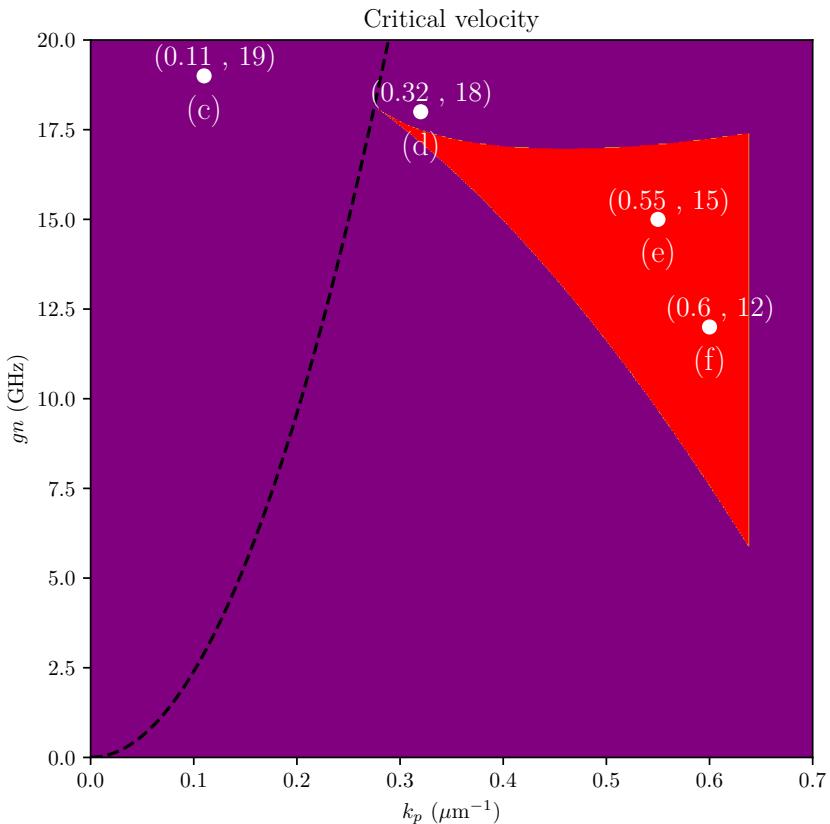


Fig. 3.9 Critical velocity map as a function of k_p and gn while the reservoir contribution was fixed as $g_r n_r \approx 1.8 = gn$. For each couple (k_p, gn) the Bogoliubov spectrum [Equation 3.12](#) is computed : red points correspond to super critical flows and purple point to sub-critical flows. The points c), d), e) and f) correspond to the measured spectrum of [Figure 3.8](#). The black dashed line represent the acoustic case where the critical velocity coincides with the speed of sound $c_s = \hbar k_s / m_{LP} = \sqrt{\hbar g n / m_{LP}}$

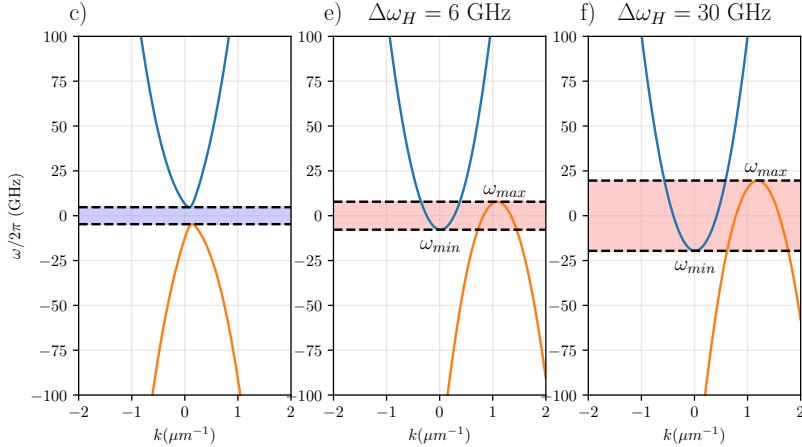


Fig. 3.10 c), e), f) analytical Bogoliubov dispersion corresponding to the fit of Figure 3.8 c), e) and f). The red range in e) and f) represents the frequency range at which negative energy modes are available that can mix with the positive energy modes outside the gap represented in blue whose value $\Delta\omega_H$ are displayed above each plot.

3.4.2.3 Paired emission frequency range

The paired emission of correlated mode at the horizon requires the mixing of positive and negative energy modes. Consequently the finite velocity in the downstream region sets a maximum frequency at which the effect can occur [?]. This frequency is given by the maximum energy of the negative energy modes that can be excited in the downstream region and is ultimately set by the asymptotic downstream velocity v_d and the interaction energy gn . Increasing this range result in a larger number of correlated pairs emitted and is expected to enhanced the overall correlation signal [?]. In Figure 3.10 we plot the analytical Bogoliubov dispersion corresponding to the measurement of Figure 3.8 c), e) and f). Paired emission is then possible in the red range of Figure 3.10 where negative energy modes can mix with the positive energy mode of the upstream spectrum, located outside the gap represented in blue.

The overall frequency range at which Hawking emission is possible is given by :

$$\Delta\omega_H := 2\omega_{max} - \Delta\omega_{gap} \quad (3.16)$$

where ω_{max} is the maximum energy of the negative energy modes that can be excited in the downstream region and $\Delta\omega_{gap}$ the width of the gap in the upstream spectrum. We measure $\Delta\omega_H=6$ GHz for the e) configuration and 30 GHz for the fastest flow. Hence by changing the asymptotic downstream speed our system enables to tune the number of states available for paired emission. Let us now show that we can control another crucial parameter of the Hawking radiation, the steepness of the horizon.

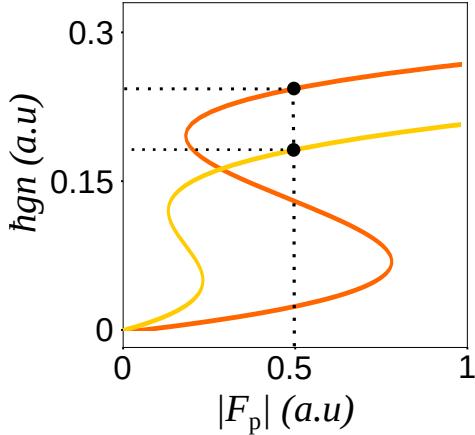


Fig. 3.11 Bistability cycle for two different values of the effective detuning $\delta(k_p^{(1)})$ (yellow) $\leq \delta(k_p^{(2)})$ (orange) when $k_p^{(2)} \leq k_p^{(1)}$. The vertical dashed line represents the input intensity of the pump laser whereas the black dots represent the operating points of the system for each detuning. The region with the higher wavevector yields a lower gn value.

3.4.3 Steep horizon geometry

The sharpness of the transition is a crucial parameter to study the strength of correlation of the emitted modes. It can be evaluated with the following equation :

$$\kappa := \frac{1}{2c_s(x)} \frac{d}{dx} [v_0^2(x) - c_s^2(x)]|_{x_H}, \quad (3.17)$$

the hydrodynamic equivalent to surface gravity [2]. However the discussion in the previous section pointed out that the position x_H at which the fluid velocity exceed the speed of sound does not define the horizon of the fluid, the derivative in the definition of κ should then be taken at the position where the critical velocity is reached and involves the critical velocity instead of c_s . In practice this brings only a negligible correction to the value of κ .

Increasing the surface gravity. The steepness of the horizon can be tuned in two ways. First, by reducing the width of the transition w_H in the target velocity profile. Second, by taking advantage of the dependence of the bistability cycle on the effective detuning $\delta(k_p)$. As shown in Figure 1.12 b) the value of gn with a fixed input intensity increases linearly with the effective detuning until the turning point is reached. More precisely, when the pump wavevector is increased, $\delta(k_p) = \omega_p - \omega_{LP}^{(0)} - \hbar k_p^2 / 2m_{LP}$ decreases and so does gn as shown in Figure 3.11. In some sense the microcavity naturally reduce the density of the fluid when the pump wavevector is increased, moving the laser away from resonance. In practice, we tune wisely the energy of the pump laser and the asymptotic upstream and downstream target wavevevectors. The different values of $\delta(k_{up})$ and $\delta(k_d)$ can lead to region with different gn values as shown on Figure 3.11. Here we create a trans critical flow with a sharper transition in order to increase the value

Here we exploit the phenomenology and implement the target profile Equation 3.3 with a large difference between v_d and v_u , a transition width $w_H = 20 \mu\text{m}$ and a detuning

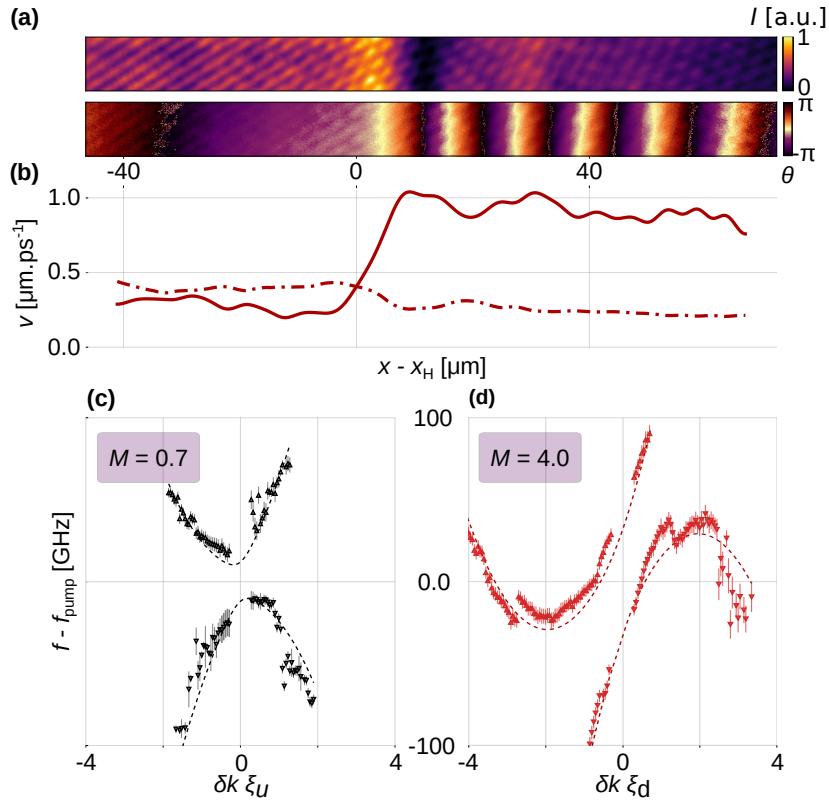


Fig. 3.12 **Steep horizon.** (a) Measured fluid density (top) and phase (bottom). (b) Measured fluid velocity profile. Solid line, $v_0(x)$; dashed line, $c_s(x)$. **Excitation spectra** Eq. (3.12): (c) upstream region. (d) downstream region; dashed lines, fit with free parameter g_n . Error bars represent the uncertainty in the determination of the center of the lorentzian when each scan is fitted by a lorentzian function to find the resonance energy. The M values in the inset of each spectrum is the computed Mach number in the corresponding region.

$\delta(0) = 71 \text{ GHz} \rightarrow \hbar\delta(0) = 0.3 \text{ meV}$ Figure 3.12 (b) shows $v_0(x)$ (solid lines) as well as $c_s(x)$ (dot-dashed lines). The two supercritical fluids created in the smooth configuration have an horizon steepness κ of 0.07 ps^{-1} (red) and 0.08 ps^{-1} (purple) whereas in the steep geometry implemented here we compute $\kappa = 0.11 \text{ ps}^{-1}$. The excitation spectra are shown in Figure 3.12 c) and d). The frequency range at which paired emission is possible is roughly the same than in the smooth case f) $\Delta\omega_H = 28 \text{ GHz}$ whereas the horizon steepness gets increased by 25%. This independent tuning of κ and the paired emission spectrum is not possible in conservative quantum fluids such as atomic BECs, for example. The fine control over the waterfall horizon geometry we demonstrate here is interesting to test recent tunneling models for the Hawking effect [40].

3.4.4 Quasi-normal mode configuration

The target velocity profile Equation 3.3 can be modified to create complex horizon geometry by imprinting a velocity peak in the transition region. More precisely, we define the *Quasi-*

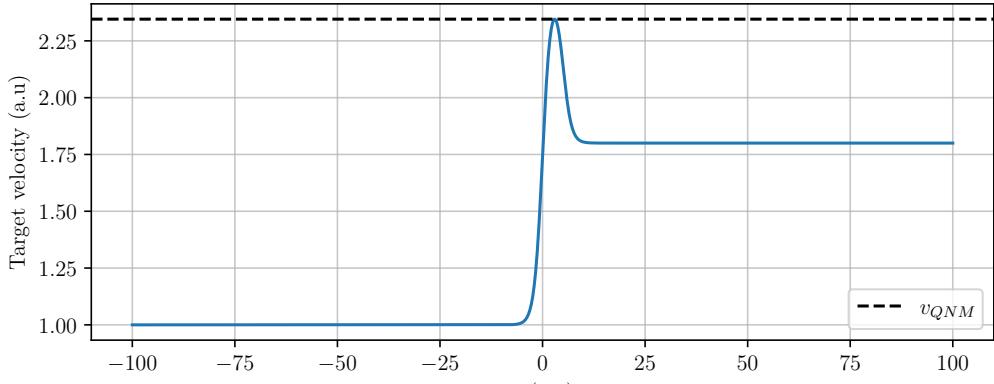
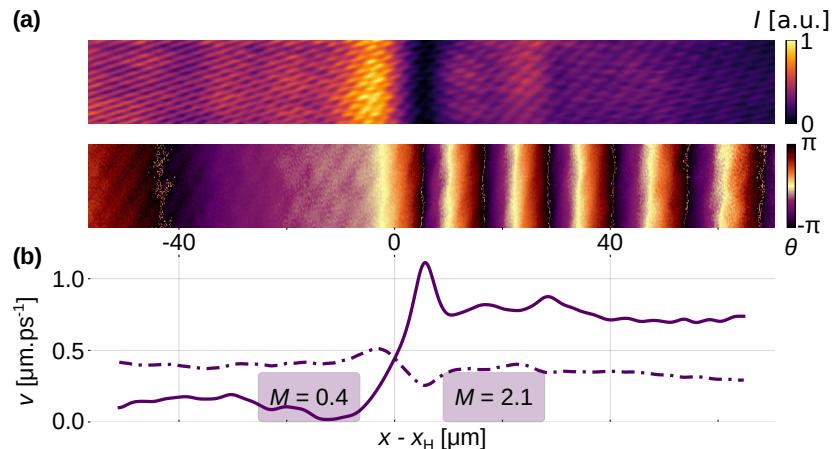


Fig. 3.13 Typical QNM target velocity profile.

Fig. 3.14 Quasi-normal mode horizon. (a) Measured fluid density (top) and phase (bottom). (b) Measured fluid velocity profile. Solid line, $v_0(x)$; dashed line, $c_s(x)$.

Normal Mode (QNM) target velocity profile as :

$$v(x) = \frac{v_{QNM} - v_{up}}{2} \tanh\left(\frac{x - x_1}{w_1}\right) + \frac{v_d - v_{QNM}}{2} \tanh\left(\frac{x - x_2}{w_2}\right) + \frac{v_{up} + v_d}{2} \quad (3.18)$$

where v_{QNM} is the velocity of the peak, v_{up} and v_d are the upstream and downstream velocities, x_1 and x_2 are the position of the first and second transition and w_1 and w_2 are their width. The typical corresponding shape is shown in Figure 3.13. With the same asymptotic parameters as in the smooth horizon configuration of Figure 3.8 e) we create a fluid with the QNM target velocity profile.

First, the asymptotic velocities are as expected, not modified by the presence of the peak, while the surface gravity is greatly enhanced as $\kappa = 0.19 \text{ ps}^{-1}$, almost twice the value obtained in the steep configuration. But the most peculiar feature of the QNM configuration is the density dip that comes with the velocity peak. For the reason explained in the previous section, the velocity peak imprinted here is associated with a density dip. As the dip is

surrounded by two regions of higher density, it forms an effective resonator for Bogoliubov excitation. This leads to the appearance of a metastable excitation mode of ϕ called a quasi-normal mode: here specifically, a negative-energy standing wave can establish itself inside the resonator (at $\omega_{\text{QNM}} > \max(\omega_B^-|_{x>x_H})$) and tunnel-couple with positive-energy propagating waves on either side [22]. This quasi-normal mode is spontaneously excited by quantum vacuum fluctuations of ϕ . As a result, the Hawking spectrum is predicted to peak at the frequency of the quasi-normal mode, bearing signatures of the near-horizon geometry. In some sense, the QNM configuration is a quantum analogue of the black hole ringdown phase, where the black hole emits gravitational waves with a characteristic frequency and damping time [?] that depend on its parameters.

3.5 Conclusion

We demonstrated full optical generation and characterization of mean field with arbitrary velocity profiles in a polariton fluid which act as an effectively curved spacetime for Bogoliubov excitations. In each configuration, we were able to measure the collective excitations spectrum of the fluid and state or not if Hawking radiation could be observed. We showed that the critical velocity to observe negative energy modes is not necessarily the speed of sound but rather a critical velocity that depends on the fluid parameters. This widens the range of experimental configurations at which particle creation can be observed. The great versatility of our experimental methods also showed that the steepness of the horizon can be controlled independently of the paired emission spectrum. Finally, we demonstrated the possibility to create complex horizon geometry that could yield peculiar behaviors like Quasi Normal Modes. The great advantage of the system presented here is the large range of field theory scenarii that can be tested in a single setup. Even though trans-critical polariton fluids are far from encapsulating what truly happens at a black hole horizon, its a great platform to study effects predicted by field theory in curved spacetime, regardless they were originally thought for black holes or not.

3.5.1 Outlook

Chapter 4

Experimental observation of stimulated Hawking radiation in a polariton quantum fluid

In the previous chapter, we demonstrated the experimental realization of a transcritical flow in a polariton quantum fluid through precise optical pump shaping. This allowed us to engineer the fluid's density and velocity profiles to form a transcritical region, a key ingredient for the emergence of a sonic horizon. Additionally, we presented experimental evidence of negative-energy modes in this region, confirming the necessary conditions for the observation of the Hawking effect.

Building on these achievements, this chapter focuses on the experimental observation of stimulated Hawking radiation in a polariton fluid. Stimulated emission provides a controlled way to probe the Hawking effect by injecting a coherent state into the upstream region and measuring its scattering into outgoing modes. This approach allows us to directly study the amplification of positive-energy modes and the role of negative-energy modes in the transcritical region. The experimental realization of stimulated Hawking radiation involves several key steps. First, we describe the setup used to inject coherent states into the polariton fluid and the techniques employed to measure the outgoing modes. Next, we detail the partial reconstruction of the scattering matrix, which encodes the mixing of positive- and negative-energy modes and provides direct evidence of energy amplification. Finally, we present the results of these measurements, discuss the agreement with theoretical predictions and the robustness of the stimulated Hawking effect. The results presented in this chapter not only validate the theoretical predictions of stimulated emission but also establish a pathway toward the observation of spontaneous Hawking radiation. By leveraging the unique properties of polariton fluids, this work demonstrates the potential of these systems as a platform for exploring fundamental aspects of quantum field theory in curved spacetime and beyond.

4.1 Interferometric measurement of the scattering matrix

We aim at solving the scattering problem of a coherent state injected in the upstream region impinging on the horizon. Doing so, it is possible to partially reconstruct the scattering

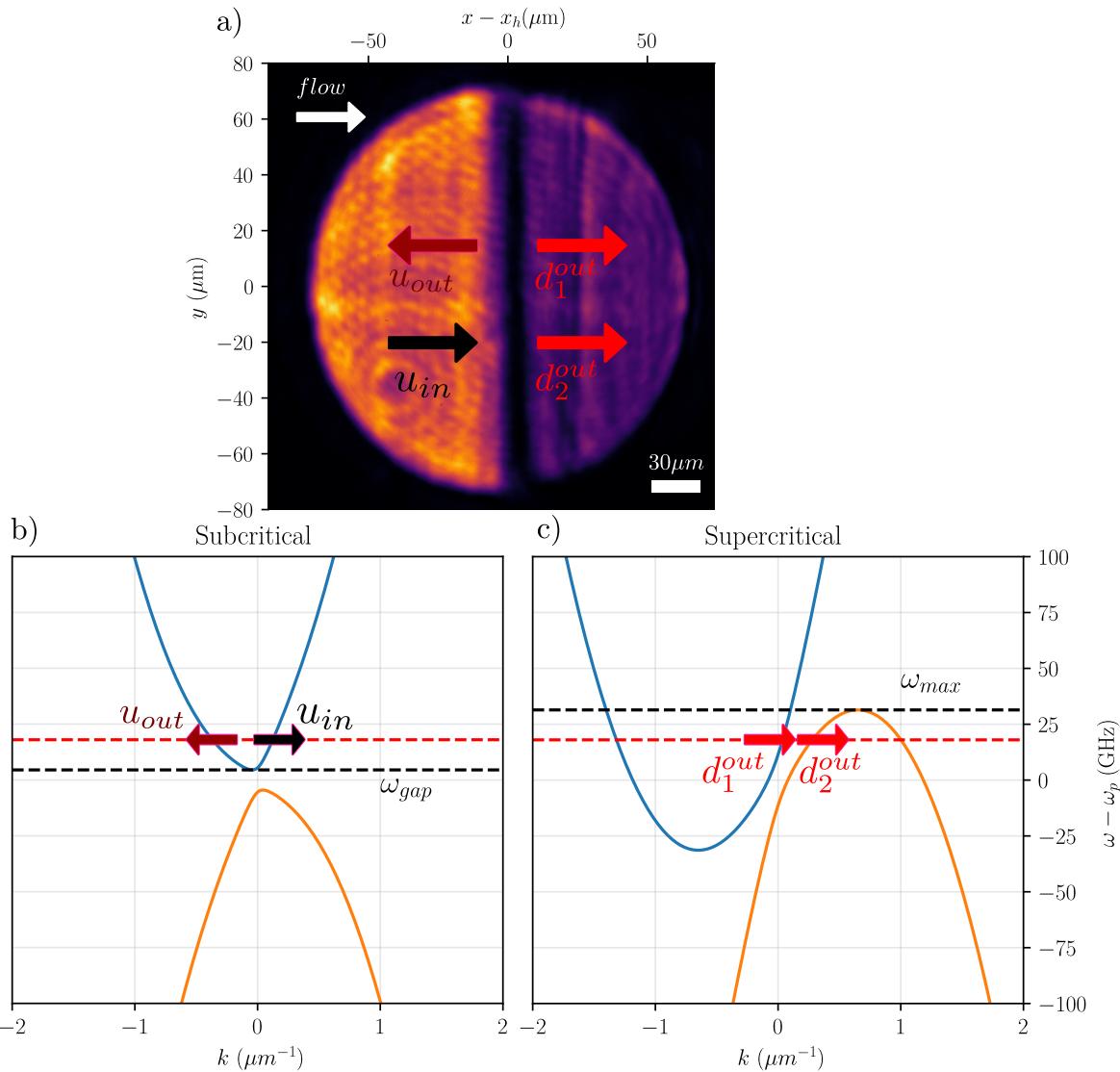


Fig. 4.1 **a)** Real space image of a transcritical flow of polaritons with detuning $\delta(0) = 29\text{GHz}$ and wavevectors $k_u = 0.2\mu\text{m}^{-1}$ and $k_d = 0.6\mu\text{m}^{-1}$. The black arrow represents the impinging mode u_{in} while the blue arrows represent the outgoing modes u_{out} , d_1^{out} and d_2^{out} . The direction of the arrow reflects the sign of the group velocity of each mode. **b)** Typical Bogoliubov dispersion relation in the upstream subcritical region. The arrows correspond to the modes displayed on the left side of a). **c)** Typical Bogoliubov dispersion relation in the downstream supercritical region. The arrows correspond to the modes displayed on the right side of a)

matrix. This approach is complementary of what has already been done in other works in which the stimulating field was coming from the transcritical region. ????????

4.1.1 Challenges of the measurement

Let us take a transcritical flow of polaritons with typical density profile shown in [Figure 4.1 a\)](#). The bright region on the left correspond to the upstream subcritical region while the other side of the interface is the transcritical region. Typical spectra corresponding to each region are shown in [b\)](#) and [c\)](#). The previous chapter made clear that it is possible to locally excite the Bogoliubov dispersion by shinning a weak probe laser at the right couple (ω, k) . Consider then a coherent state $|\alpha_{in}\rangle$ in the u_{in} mode of [b\)](#) that is, an upstream mode impinging on the interface. If its frequency ω_{in} is within $[\omega_{gap}, \omega_{max}]$ we expect to measure one reflected mode u_{out} and two transmitted modes d_1^{out} and d_2^{out} . The latter is the negative energy mode responsible for Hawking radiation. Each of this mode has a different wavevector k_i .

Experimentally, this situation corresponds to four distinct optical signals exiting the sample at different angles, or equivalently, to four spatially separated spots in the Fourier plane of the cavity. To illustrate the inherent challenges associated with such measurements, we present in [Figure 4.2 b\)](#) the typical expected locations of the scattered signals in the Fourier plane corresponding to the fluid configuration of [Figure 4.1 a\)](#). The two bright spots correspond to the regions of the fluid characterized by the wavevectors k_u and k_d . Two principal experimental challenges can be identified.

First, the expected position of the mode d_1^{out} nearly coincides with the pump signal in the downstream region. Since the probe intensity must remain at least two orders of magnitude lower than that of the pump to preserve the perturbative regime, the resulting signal-to-noise ratio is extremely low. Consequently, a direct measurement of the d_1^{out} mode becomes unfeasible.

Second, in addition to the desired signals at frequency ω_{in} , all modes are subject to four-wave mixing processes due to nonlinear interactions with the pumped polaritons. As a result, depending on the fluid parameters, the conjugated modes generated by these interactions may spatially overlap with the reflected or transmitted modes. For example, as illustrated in [Figure 4.2](#), the conjugate of the injected mode u_{in} appears at the same location in momentum space as the reflected mode u_{out} . Consequently, placing a pinhole at this position does not permit one to distinguish between the two contributions.

The central challenge of this measurement is thus to detect weak signals on top of a strong background while ensuring that these signals originate from genuine transmission and reflection at the interface –rather than from spurious scattering events or nonlinear mixing. Therefore, the critical experimental objective is to optimize the signal-to-noise ratio, where “noise” refers to all undesired signals. This is achieved by acting on two main experimental parameters:

- **Enhancement of the signal:** As discussed in the previous chapter, this can be accomplished by increasing the steepness of the horizon.
- **Suppression of the pump background:** The photonic nature of the system allows us to exploit two key properties of light –polarization and coherence. Polarization filtering can be applied in the detection path, while coherence enables the use of interferometric techniques, as will be detailed in the following section.

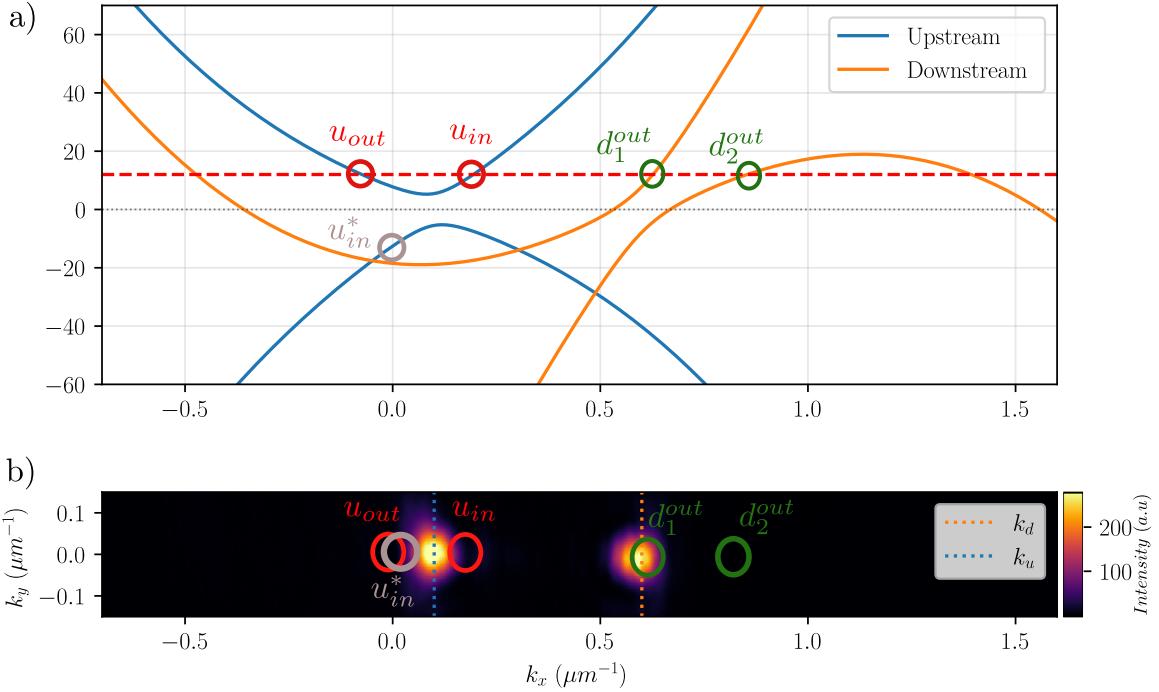


Fig. 4.2 **a)** Analytical Bogoliubov dispersion of both regions calculated with the parameters of the fluid of [Figure 4.1 a\)](#). To reflects what happen in practice, the two dispersion are plotted in the same graph as a function of the wavevector in the laboratory frame k . The upstream dispersion is centered at k_u while the downstream one is centered at k_d . The red dashed line corresponds to the energy of the injected mode. The wavevectors of the reflected-transmitted is obtained at the intersections of this line with the Bogoliubov branches and reported in the momentum space **b)**. The red (resp. green) circle represent the expected location of the upstream (resp. downstream) modes. Finally, the grey circle represent the conjugated of the injected signal resulting from four wave mixing. **b)** Momentum space of [Figure 4.1 a\)](#) : the two bright spots correspond to the two region of the fluid with wavevectors k_u and k_d . The colored circle are reported from **a)**.

Electronic filtering. One possible strategy to suppress the pump background is inspired by the method used to measure the excitation spectrum in the previous chapter. By modulating the probe beam intensity at a frequency ω_{mod} accessible in the electronic domain, the signal can later be demodulated, isolating the component arising from the probe-induced excitation. However, in the current context, what previously served as an advantage becomes a limitation. Indeed, the four-wave mixing signals also carry the modulation, thus failing to resolve the overlap problem discussed above. To overcome this, we turn to the optical domain, specifically to interference-based detection schemes.

4.1.2 Interferometric reconstruction of the Fluctuation Field

The measurement is based on the same interferometric technique previously employed to reconstruct the mean field. The key distinction here is that the method is now applied to the probe field instead. Although this may appear to be a minor adjustment, it represents a significant conceptual shift that enables a comprehensive characterization of the fluctuations. Before detailing its experimental implementation in the context of the scattering problem, we first examine the insights that such a measurement can provide.

We again consider the generic situation presented above namely, a coherent state injected in the upstream region. The full optical field transmitted through the sample can be composed of :

- The mean field ψ_0 oscillating at the pump frequency ω_p .
- The fluctuation field oscillating at the probe frequency ω_{in} resulting from the scattering of u_{in} on the interface.
- The conjugated field oscillating at the opposite of the probe frequency $-\omega_{in}$ resulting from the four wave mixing of each $+\omega_{in}$ mode with the pump.

When the probe field is superimposed with a phase reference beam at the probe frequency—obtained as a pick-off from the probe laser—the only components that generate interference fringes are those oscillating at $+\omega_{in}$. Each scattered mode is characterized by a distinct wavevector k_i , leading to a unique spatial frequency in the resulting interferogram. As an illustrative example, [Figure 4.3 a\)](#) displays two regions of such an interferogram. The first lies within the probe injection area, where the fringe spacing is inversely proportional to $|\Delta k|$, with $\Delta k = k_{u_{in}} - k_{\text{ref}}$ denoting the wavevector mismatch between the injected mode and the reference beam. The second region is located downstream, beyond the probe injection area. The mere presence of interference fringes in this region already confirms that the injected mode has propagated through the interface, while the fringe spacing provides direct information about the wavevector of the transmitted mode.

This interpretation is supported by the spatial Fourier transforms of the two regions, shown in [d\)](#). The low- Δk peak corresponds to the continuous background signal—primarily the pump field and the conjugated modes at $-\omega_{in}$ —which do not interfere with the reference beam. The central peaks represent the spatial frequencies of the propagating components in each region. Notably, the downstream region exhibits a higher wavevector, consistent with the scattering picture.

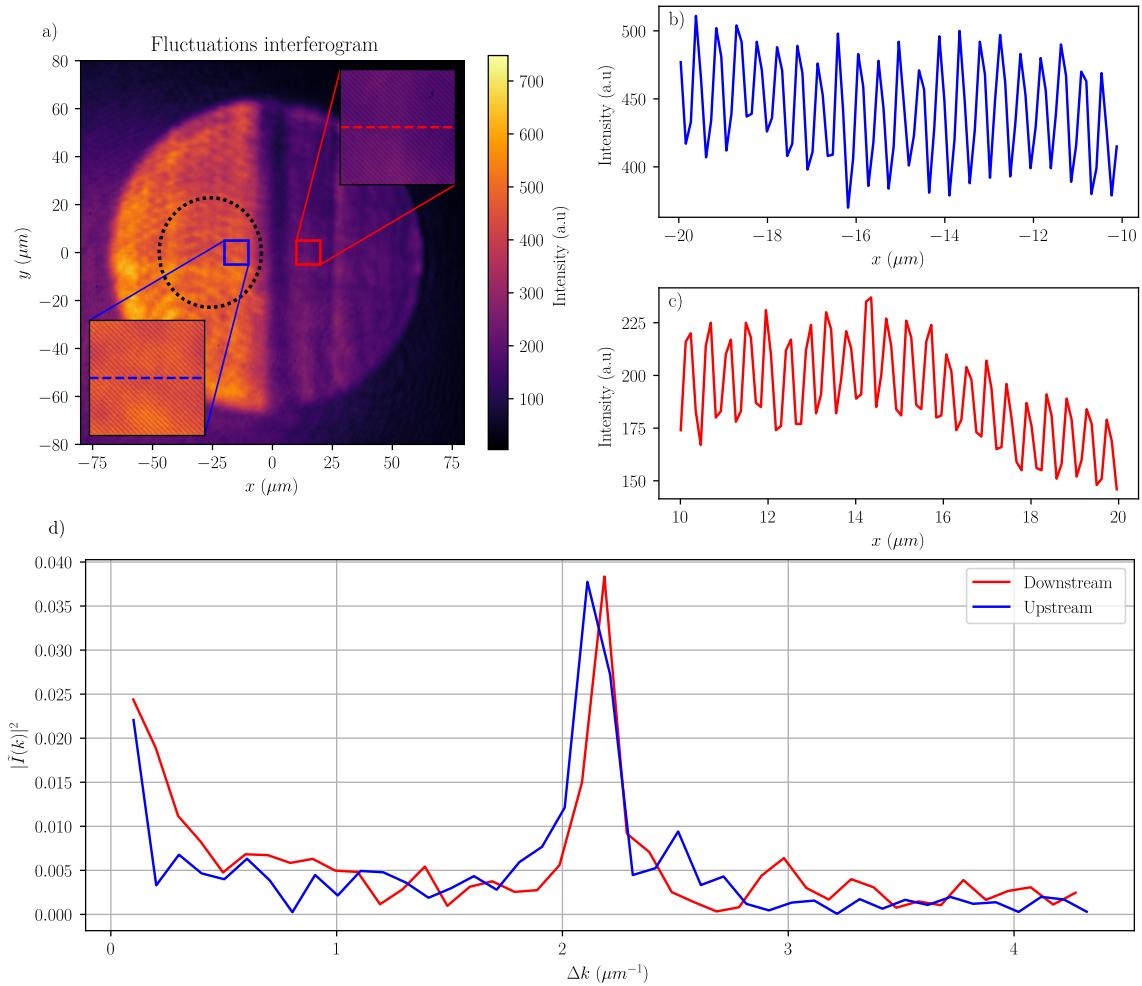


Fig. 4.3 **a)** Interferogram resulting from the supersposition of Figure 4.1 **a)** and a reference beam from the probe laser. The black dashed circle represent the injected probe location. The red inset (resp. blue) is a zoom on the upstream (resp. downstream) region near the interface. **b)** Cut of the interferogram along the red dashed line of the red inset. **c)** Cut of the interferogram along the blue dashed line of the blue inset. **d)** Fourier transforms of the cuts **b)** and **c)**. The blue curve corresponds to the upstream region with spatial frequency $\Delta k_u \approx 2.11 \mu\text{m}^{-1}$. The red curve corresponds to the downstream region with spatial frequency $\Delta k_d = 2.19 \mu\text{m}^{-1}$.

This preliminary analysis already highlights the power of the technique. By applying numerical masks at different spatial locations, one can selectively extract the Fourier components of the fluctuations. Put more generally, a single interferogram enables full reconstruction of the fluctuation field at the frequency of the injected perturbation. As a result, this method proves particularly well-suited for addressing the scattering problem, as it inherently overcomes all the challenges previously outlined :

- Conjugated modes and background contributions from the pump are inherently filtered out, as they do not interfere with the reference beam and therefore do not generate fringes.
- Each scattered mode gives rise to a distinct spatial frequency in the interferogram, enabling their individual identification and selective extraction.
- The interference with the coherent reference beam enhances the signal-to-noise ratio, thereby facilitating the detection of weak scattered components.

The interferometric technique outlined above provides a powerful framework for reconstructing the fluctuation field and isolating the scattered modes. By leveraging the distinct spatial frequencies of the scattered components and filtering out unwanted contributions, this method enables a precise characterization of the scattering process. In the following section, we detail the experimental implementation of this approach, focusing on the setup and procedures required to reconstruct the scattering matrix and extract the key parameters governing the stimulated Hawking effect.

4.2 Experimental implementation

The experiment is conducted in the same microcavity as in the previous chapter, operating at the same working point (C5-D6) and using an identical optical setup. However, whereas the previous measurements focused on characterizing the fluid's excitation spectrum, the present objective is to detect the scattered signals generated at the horizon interface. To this end, we first investigate configurations in which the horizon is as sharp as possible, even at the cost of significantly reducing the downstream density.

4.2.1 Ballistic configuration

This is accomplished by illuminating only the upstream region with the pump laser, set at a wavevector $k_p = 0.02 \mu\text{m}^{-1}$. As a result, the downstream region consists of polaritons propagating ballistically beyond the pumped area. The pump is detuned by $\delta(0) = 26 \text{ GHz}$ in order to reach the high-density regime of optical bistability. Additionally, the Gaussian intensity profile of the pump beam is truncated into a half-disk shape with a diameter of $150 \mu\text{m}$, thereby generating a sharp intensity gradient at the edge of the pumped region. The resulting mean-field profile is presented in [Figure 4.4](#).

As shown on the corresponding intensity and velocity profiles [see [Figure 4.5 a\)](#)], the downstream region exhibits an exponentially decaying density accompanied by an increasing flow velocity. This behavior naturally arises from the conservation of current, as described by the continuity equation [2.1](#), in the absence of external pumping. More precisely, it can be

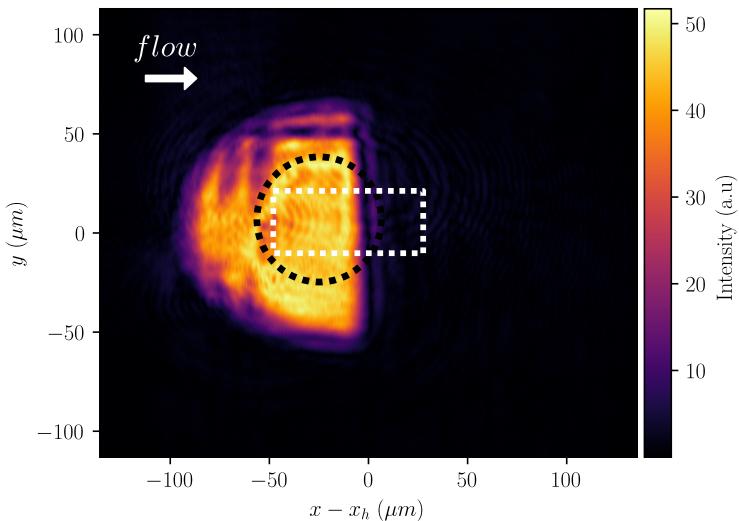


Fig. 4.4 **a)** Real space image of a transcritical flow of polaritons with detuning $\delta(0) = 48$ GHz. The pump is shined only in the upstream region with wavevector $k_p = 0.02 \mu\text{m}^{-1}$. The downstream region is made of polaritons propagating ballistically. The black dashed circle represent the location of the injected probe while the white dashed rectangle represent the region of interest (ROI) for the density and velocity profiles.

shown that away from the pump spot in slowly varying region where the quantum pressure can be neglected, the Gross Pitaevskii equation takes the form of generalized Bernoulli equation for driven dissipative fluid. The wavefunction then takes the form [54]:

$$\psi_0(x \gg x_h) = \sqrt{n_0} e^{-\frac{x}{l_d}} e^{ik_{fluid}x}, \quad (4.1)$$

where $1/l_d = \gamma_{LP}/2v_g$ is the spatial decaying rate resulting from the product between the fluid group velocity $v_g = \hbar k_{fluid}/m_{LP}$ and the polariton lifetime $1/\gamma_{LP}$. The local wavevector of the polariton fluid k_{fluid} is determined by the equation:

$$\hbar\omega_p = \hbar\omega_{LP} + \frac{\hbar^2 k_{fluid}^2}{2m} + gn_0(x) + g_r n_r(x). \quad (4.2)$$

Taking the low density limit $gn_0, g_r n_r \rightarrow 0$, the asymptotic value of k_{fluid} is fixed by the detuning $\delta(0)$ in the pumped region as $k_d := k_{fluid}(x \gg x_h) = \sqrt{2m\delta(0)/\hbar}$. This evolution can also be intuitively understood through an optical analogy: photons propagating from the pumped region encounter a zone of lower polariton density, which corresponds to a higher effective refractive index. According to Snell-Descartes laws of refraction, the wavevectors of the photons are bent toward the normal of the interface, effectively resulting in an increase in the group velocity. The corresponding velocity profile measured by off axis interferometry is depicted in [Figure 4.5 b\)](#). The horizon then forms spontaneously due to the simultaneous increasement of the flow velocity and the decreasement of the speed of sound.

TODO : add the speed of sound measurement; In this configuration the surface gravity is $\kappa \approx 18 \text{ ps}^{-1}$ which is two order of magnitude higher than what was obtained in the steepest horizon of the previous chapter. The price we pay for this enhancement is the appearance of oscillations in the fluid density and velocity profiles. Indeed, since the pump is no longer present to fix the phase of the system and inject density, the fluid is more sensitive to fabrication defects in the microcavity each of them acting as a scattering center.

4.2.2 Measurement of the Bogoliubov spectrum

In order to determine whether a detected signal originates from genuine scattering at the interface or from a spurious scattering event, we begin by characterizing the Bogoliubov excitation spectrum of the fluid in each region independently. This is done by employing a technique mixing the high resolution pump probe spectroscopy detailed in the previous chapter and the interferometric technique described in the previous section.

First, the probe beam is centered in the upstream region close to the interface as represented by the black dashed circle in [Figure 4.4 a\)](#). A slighth spatial overlap with the downstream region across the interface is voluntarily introduced to have acces to the spectrum of both regions on a single frequency scan as we will see later. To remain in the perturbative regime, the probe intensity is kept two orders of magnitude lower than the pump intensity. Then, the whole field is superimposed with a collimated phase reference beam originating from the probe laser. As a rule of thumb, the diameter of the reference is increased to be three times larger than the diameter of the probe beam in order to have the flattest possible

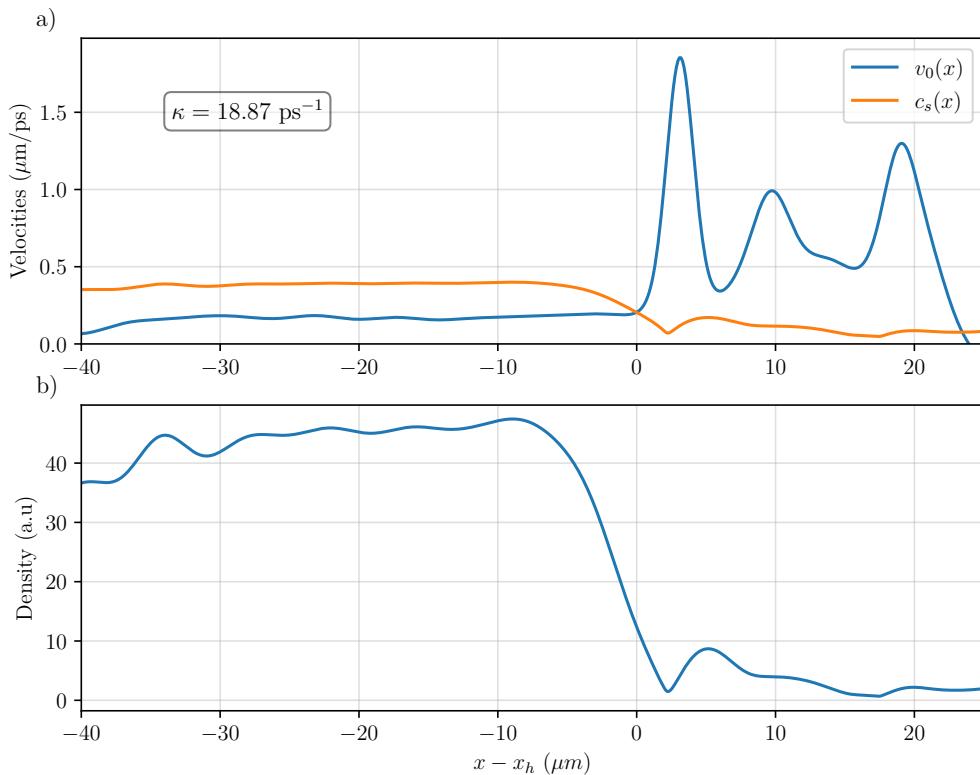


Fig. 4.5 **a)** **Density profile of the polariton fluid taken in the region of interest.** The profile is obtained by taking an average along the y -axis on the whole ROI represented by the white dashed rectangle of Figure 4.4. **b)** **Velocity profiles of the fluid in the ROI.** The blue curve corresponds to the velocity of the fluid obtained by off axis interferometry. The orange curve corresponds to $c_s = \sqrt{gn_0/m_{LP}}$ calibrated from the value of gn_0 fitted from the upstream Bogoliubov spectrum.

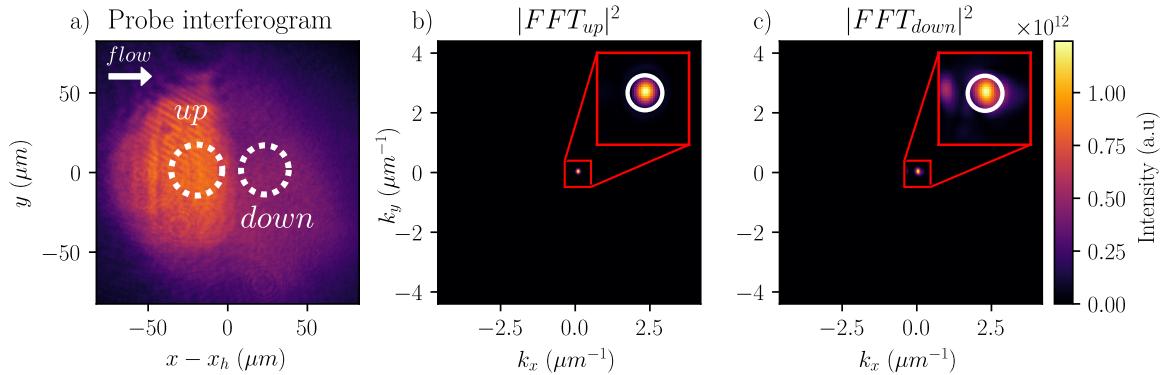


Fig. 4.6 **a)** **Probe interferogram.** The probe is located in the upstream region as in [Figure 4.4](#). The white dashed circle represent the numerical mask applied to isolate each region. **b)** Shifted fourier transform of the masked interferogram of the upstream region. The red inset is a zoom on a $0.2 \mu m^{-1}$ region around the probe spot where the solid white circle correspond to the numerical pinhole applied to isolate the probe signal. **c)** Shifted fourier transform of the masked interferogram of the downstream region. The red inset is a zoom on a $0.2 \mu m^{-1}$ region around the probe spot where the solid white circle correspond to the numerical pinhole applied to isolate the probe signal.

phase front. The angle difference between the two beams is set to obtain the best spatial frequency resolution as explained in [subsection A.1.2](#).

Secondly, the wavevector of the probe beam is tunned from $-1.0 \mu m^{-1}$ to $1.0 \mu m^{-1}$ by 60 steps of $\Delta k = 0.046 \mu m^{-1}$. At each step, the probe frequency is scanned 130 GHz around the pump frequency. The main difference with the previous experiment arises in the detection scheme. Indeed, instead of modulating the probe intensity to latter isolate the signal by electronic filtering, we use the interferometric technique aforementioned. A single frequency scan consist in a series of 80 interferograms recorded on a CCD camera. A full dataset is then made of 80×60 interferograms where each image has 2056×2464 pixels encoded on 16 bits for the best dynamical range. Obviously, compared to the 600 hundreds points per energy scan available with the previous technique, the present procedure suffer from a lack of frequency resolution. This being said, frequency resolution was never a limitation and this drawback is largely compensated by the advantages of the interferometric technique. In particular, it allows to measure almost all the relevant observables in a single data set as we shall see now.

Spectrum reconstruction. For each interferogram, a Gaussian numerical mask of width $40 \mu m$ is (see [4.4](#)) applied either in the upstream or downstream region to spatially isolate the area of interest. A two-dimensional Fourier transform is then performed on the masked interferograms, yielding the fluctuation spectrum at the probe frequency within the selected region. Initially, our objective is to extract the Bogoliubov dispersion relation in each region, which—as discussed in the previous chapter—can be inferred from the transmission spectrum of the probe across the sample. To this end, an additional numerical pinhole, centered on the probe wavevector and with a radius matched to the probe spot, is applied in the Fourier

domain of the interferogram, as illustrated in [Figure 4.6](#). The integral over the masked fourier planes provides the transmission of the probe in the corresponding region.

In summary, for each probe configuration defined by the wavevector-frequency pair (k_{in}, ω_{in}) , this procedure yields the transmitted probe intensity $I_{u,d}(k_{in}, \omega_{in})$ in both upstream and downstream regions. At this stage, the reason for positioning the probe beam to slightly overlap with the downstream region becomes evident : it allows for the direct excitation of Bogoliubov modes with minimal influence from the interface. This is essential for benchmarking the dispersion relation of modes transmitted from upstream, and for confirming that they coincide with those excited locally in the downstream region. By repeating this procedure for all probe configurations, we obtain the Bogoliubov dispersion in each region, as shown in [??](#). As anticipated, only the normal branch is accessible via direct excitation. This is not a limitation but rather a deliberate feature of the method, which was specifically chosen for its ability to isolate the positive frequency signals. Besides, knowledge of the normal branch alone is sufficient to reconstruct the full Bogoliubov spectrum, as its structure is inherently symmetric. Moreover, the existence of the ghost branch, particularly its positive-frequency domain which is essential for the Hawking effect in the supercritical regime, was established in the previous chapter. Consequently, we will not try to measure it in the present work.

The upstream spectrum shown in [Figure 4.7 a\)](#) exhibits a gap and a slight doppler shift consistent with the pump wavevector in the pumped upstream region $k_p = 0.023 \mu\text{m}^{-1}$. The data points are fitted with the Bogoliubov dispersion relation [2.48](#) where gn_0 is the only fitting parameter and the dark reservoir is handled the same way as in the previous chapter. We obtain $gn_0 = 9.6 \pm 0.1 \text{ GHz}$ and find a good agreement of the model with the experimental points. The downstream spectrum shown in [b\)](#) possess negative frequency modes which is typical of a supercritical flow. As explained earlier, the absence of pumping imposes the condition [4.2](#) that can be recast in the form $\delta(k_{fluid}) = gn_0 + g_r n_r$. It is the counterpart of the turning point condition in the pumped region implying that the Bogoliubov spectrum is expected to be linear far from the interface :

$$\omega_B(k) = v_d(k - k_d) \pm \sqrt{\frac{\hbar(k - k_d)^2}{2m_{LP}} \left(\frac{\hbar(k - k_d)^2}{2m_{LP}} + gn_0 \right)}, \quad (4.3)$$

where $v_d = \hbar k_d / m_{LP}$ is the asymptotic velocity of the fluid measured by off axis interferometry. The fit gives $gn_0 = 1.20(1) \text{ GHz}$. As visible, the agreement is not as good as in the upstream region. This is due to the fact that in the unpumped region, the fluid is not homogeneous anymore close to the interface as shown in [Figure 4.5 b\)](#). Homogeneity is recovered far from the interface where the density is low enough. But whenever this condition is satisfied, there is no fluid left to probe. Consequently, the signal measured mainly comes from the transient region where looking for the bogoliubov modes as plane waves is already an approximation. To obtain a better agreement, a complete diagonalization of the Bogoliubov matrix around the steady state would be necessary in this region. However, this is not the goal of this work and the supercritical feature we are interested in remain robust against this approximation. This two measurements prove together the presence of an analogue horizon in between the two regions. Now that the spectrum of collective excitation is known on both side of the interface. Let us now turn to the measurement of the scattering matrix.

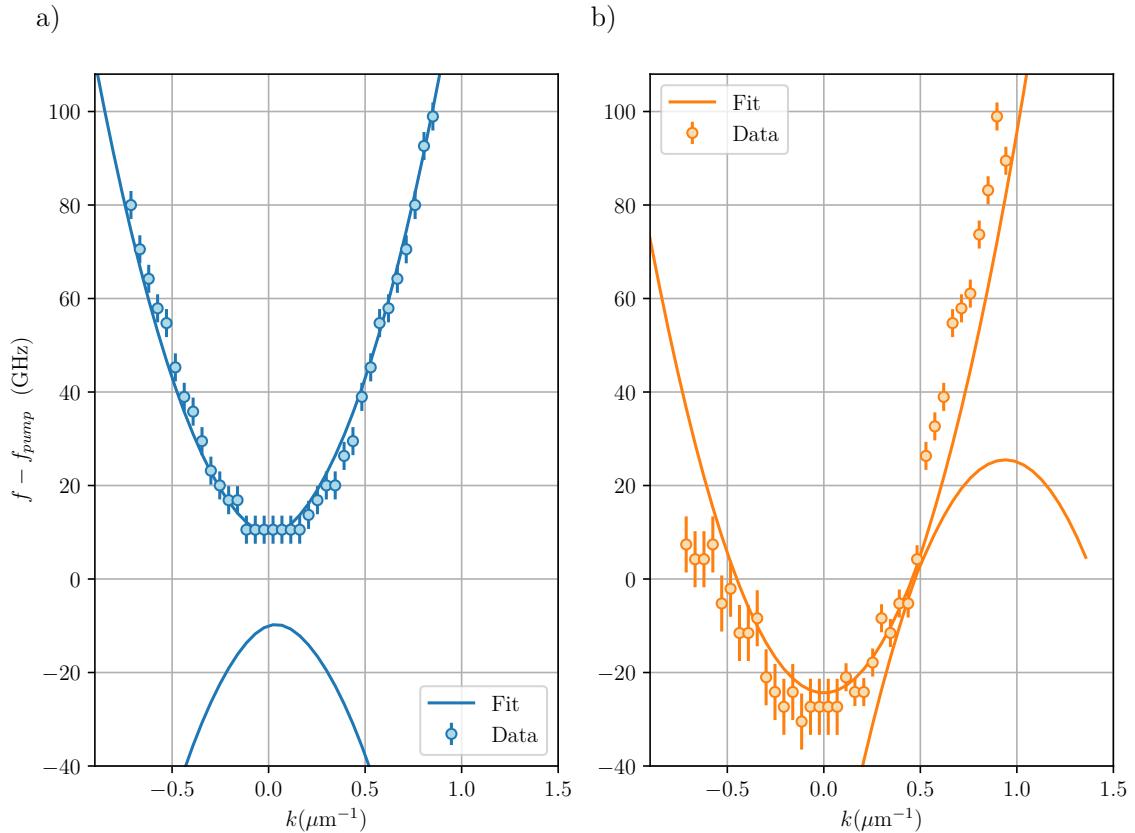


Fig. 4.7 **a)-b)** **Upstream-Downstream Bogoliubov spectrum.** The scattered points correspond to the maxima of the transmission spectrum of the probe in the upstream-downstream region. The blue-orange solid line is the analytical bogoliubov dispersion plotted with the experimental parameters and the non linear interactions gn_0 obtained from the fit of the data with the normal bogoliubov branch. The upstream region fit gives $gn_0 = 9.6 \pm 0.1$ GHz while in the downstream we obtain $gn_0 = 1.2 \pm 0.01$ GHz. The vertical errorbars are mainly originating from the frequency resolution of the measurement while the horizontal uncertainties are due to the probe extension in momentum space $\sigma_k = 0.04 \mu\text{m}^{-1}$.

4.2.3 Measurement of the scattering matrix

4.2.3.1 Detection of the scattered modes

We analyze the emergence of transmitted or reflected modes resulting from an incoming perturbation propagating toward the horizon. We focus on the scattering of the $u_{in}(k)$ modes. From the maxima of the upstream Bogoliubov spectrum, we select the set of configurations (k, ω) where the probe is exciting upstream modes with positive group velocity, i.e., $\partial\omega/\partial k > 0$. For each configuration, we compute the Fourier transform of the probe field in both upstream and downstream regions, following the same separation procedure employed for the Bogoliubov spectrum reconstruction. In contrast to the earlier analysis, we now apply a numerical anti-pinhole to suppress the intense contribution of the u_{in} mode at the probe position. A peak-detection algorithm is then used to identify the positions of the scattered modes in the Fourier plane and to extract their corresponding wavevectors. Each identified wavevector is compared with the Bogoliubov spectrum to confirm its physical relevance and to rule out spurious scattering events. This process is systematically repeated for all configurations associated with u_{in} . The resulting scattering data are summarized in **Figure 4.8 a)**. As observed, the modes detected both in the upstream and downstream regions are in good agreement with their respective Bogoliubov relations.

This measurement demonstrates that the interface scatters the incoming mode as expected from the Bogoliubov theory. However, $d2_{out}$ is not detected in the downstream region. This is due to its negative norm stating the major part of the mode amplitude is on the conjugated branch. Indeed, in the bogoliubov basis the excited $d2_{out}$ mode is written as :

$$\begin{pmatrix} u_{k_{d2}^{out}} \\ v_{k_{d2}^{out}} \end{pmatrix} e^{-i\omega_{prt}t} = \begin{pmatrix} U_{k_{d2}^{out}} \\ V_{k_{d2}^{out}} \end{pmatrix} e^{ik_{d2}^{out}x} e^{-i\omega_{prt}t}, \quad (4.4)$$

where ω_{pr} is the probe frequency. According to **2.41** the corresponding fluctuation of the order parameter ie the physical quantity experimentally accessible, is :

$$\delta\psi_{d2^{out}}(x,t) = U_{k_{d2}^{out}} e^{ik_{d2}^{out}x} e^{-i\omega_{prt}t} + V_{k_{d2}^{out}}^* e^{-ik_{d2}^{out}x} e^{i\omega_{prt}t}. \quad (4.5)$$

Due to its negative norm the bogoliubov amplitudes are such as $\int |U_{k_{d2}^{out}}|^2 - |V_{k_{d2}^{out}}|^2 = -1$ meaning that $|V_{k_{d2}^{out}}|^2$ is significantly greater than $|U_{k_{d2}^{out}}|^2$. Consequently, its larger amplitude is at opposite frequency and wavevector, namely, in the $d2_{out}^*$ mode on the normal branch. This explains why the interferometric technique, which can detect only modes at the probe frequency fails to detect the weak amplitude of the $d2_{out}$ mode. To measure it, we use an imaging spectrometer Princeton Instruments SpectraPro SP-2750 mounted with a 1200 lines/mm grating together with a high sensitive CCD camera Teledyne PIXIS 1024. By imaging the Fourier plane of the entire field on the entrance slit of the spectrometer it is possible to directly obtain the spectrum of the fluid including both the pump and the probe contributions. As the pump is generally two orders of magnitude brighter than the probe, it quickly saturates the camera sensor preventing to detect weak probe signals. To overcome this issue, the pump is filtered out with a rasor blade placed in the real space of the field before the spectrometer. The blade is positioned in order to block the bright upstream region while letting the weak downstream region pass through. This allows to set a long exposure time without being blinded by the pump. A typical spectrum taken with a 4 seconds exposure time is shown in ?? a). By superimposing the image with the downstream Bogoliubov dispersion

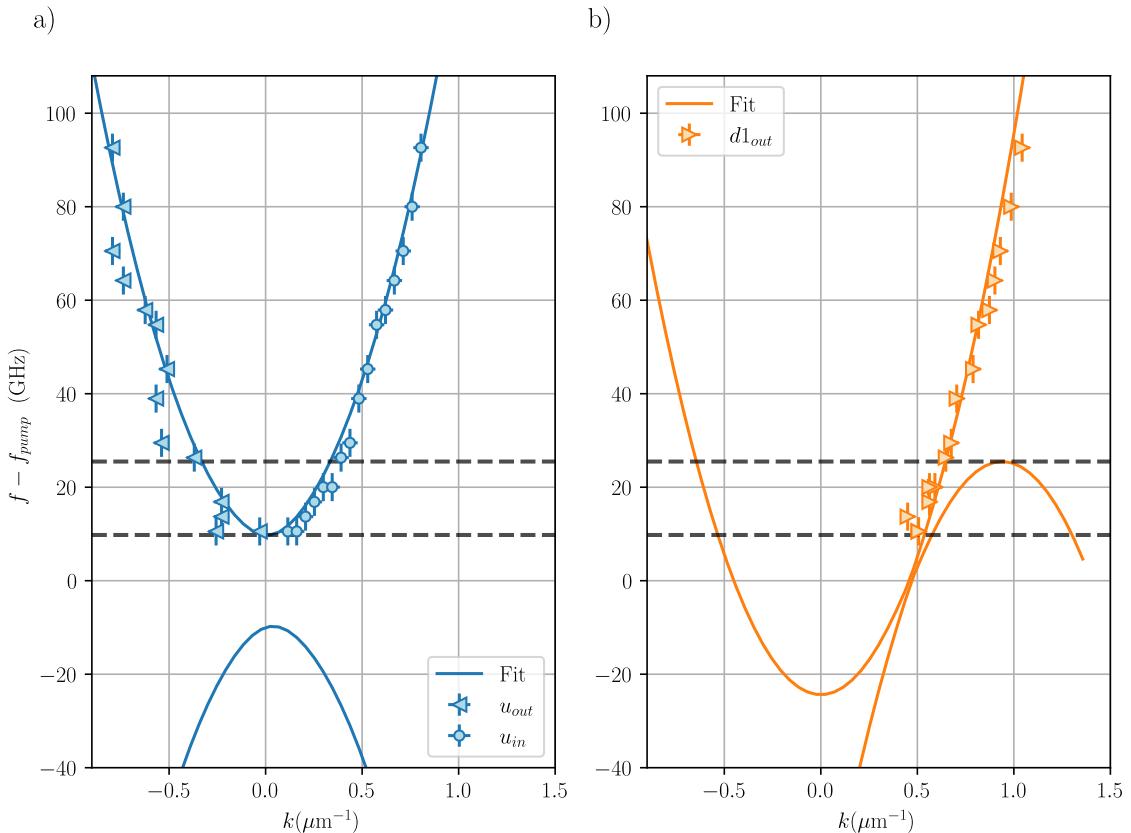


Fig. 4.8 **a)** **Measurement of the reflected modes u_{out}** The u_{in} modes excited by the probe are represented by the blue dots. The errorbars are the same than in Figure 4.7. For each u_{in} mode the wavevector of the maximum detected in the upstream region is represented by the blue left oriented triangle. The errorbars are the same than for the u_{in} modes with an additionnal contribution on the steeming from our momentum space resolution $\Delta = 0.027 \mu\text{m}^{-1}$ during the peak detection algorithm. The blue solid line is the fit of the upstream Bogoliubov dispersion obtained in Figure 4.7 a) The black dashed lines defines the frequency range at which negative-positive energy mixing is possible. **b)** **Measurement of the transmitted modes.** The wavevector of the maximum detected in the downstream region is represented by the orange right oriented triangle. The errorbars are the same than in a). The orange solid line is the fit of the downstream Bogoliubov dispersion obtained in Figure 4.7 b).

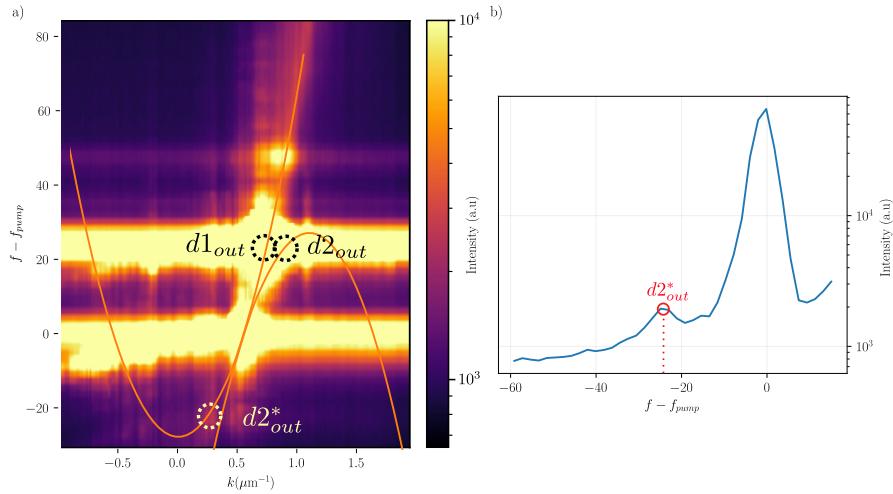


Fig. 4.9 **a) Spectrum of the downstream region of the fluid.** The spectrum is taken by integrating the whole downstream region of the fluid during 4 seconds with an imaging spectrometer. The orange solid line represent the fit of the Bogoliubov dispersion relation obtained in [Figure 4.7 b\)](#). The dashed black line is the expected location of the $d2_{out}$ mode. The red dashed circle is the $d2^*_{out}$ mode, conjugated of $d2_{out}$. **b) Vertical cut of the $d2^*_{out}$ mode.** The intense peak on the right corresponds to the remaining pump contribution. The red dashed vertical line locates the $d2^*_{out}$ mode.

obtained earlier, the different modes are identified and their positions are represented by colored dashed circles. Remarkably, an intensity peak is observed at the $d2^*_{out}$ mode, as visible on the vertical cut b). It is worth noticing that this peak disappears when the probe is exciting an input mode outside of the negative-positive energy mixing region. Finally, an additionnal peak is observed at twice the probe frequency. We interpret it as the result of a four wave mixing process where two $d1_{out}$ polaritons scatters into a pump mode polariton and a higher energy mode :

$$\begin{aligned} (\omega_{pr}, \omega_{pr}) &\rightarrow (0, 2\omega_{pr}) \\ (k_{d1}^{out}, k_{d1}^{out}) &\rightarrow (k_{d1}^{out} - \Delta k, k_{d1}^{out} + \Delta k), \end{aligned} \quad (4.6)$$

where $\Delta k = k_{d1}^{out} - k_d$ is the wavevector mismatch between the pump in the downstream region and the $d1_{out}$ mode. This process is a bosonic stimulation triggered by the high occupation of the pump state and is particularly efficient because of the linearity of the Bogoliubov spectrum provinding a great phase matching

4.2.4 A signature of amplification

As explained in [chapter 2](#), if the incoming mode is modeled as a coherent state $|\alpha_{in}\rangle$ with amplitude α_{in} , it is possible to compute the average number of quanta in each output mode.

4.2.4.1 Input and output modes discrimination

Consider the perturbation field $\delta\psi$ when the probe is exciting a given input mode in the upstream region. In a first approach we restrict our description to what is experimentally accessible with our interferometric method, namely, the field at the probe energy only. In a very general way, the wavefunction can be decomposed as follows:

$$\begin{aligned}\delta\psi(x,t) &= \delta\psi_{in}(x,t) + \delta\psi_{res}(x,t) \\ &= \delta\psi_{in}(x,t) + \delta\psi_{scat}(x,t) + \delta\psi_{err}(x,t).\end{aligned}\quad (4.7)$$

where $\delta\psi_{in}$ is the ideal incoming mode and $\delta\psi_{res}$ is the residual field. The latter is made of all the other modes that are not in the *in* mode. The term ψ_{scat} represents all the scatterings that may occur in the fluid. In particular, it contains the outgoing modes u_{out} , d_1^{out} and d_2^{out} that are the result of the scattering process. Furthermore, the residual field also accounts for deviations of the injected probe beam in the cavity from the perfect gaussian beam defined by $\delta\psi(x,t)$. This contribution $\delta\psi_{err}(x,t)$ is purely optical and can be thought as a bias when ψ_{in} is extracted from the true probe beam as we shall see now.

Input mode. The safest way to define the input while minimizing the bias is to perform the Fourier transform of the perturbation field and use modal decomposition. As in the previous section we first detect the amplitude peak of the injected mode that is the major contribution in the Fourier space. Cuts along both axis of the detected peak are shown in **Figure 4.10 a) and b)**. A few comments can already be done. First, one can notice that the peak is centered at $\mathbf{k} = (k_x^{in}, k_y^{in}) = (0.23, 0.0)$ reflecting the positive group velocity of the injected mode. On the other hand, the peak is not perfectly symmetric and is slightly deformed in the k_x direction. It's worth stressing that it is hard to determine whether this elongated shape already contains information about the scattering process or if it's a broadening resulting from the fluid motion only. To overcome this issue, we use the cut of the peak in the k_y direction using that the fluid is still translationnaly invariant in this direction as explained in [chapter 2](#). The y cut is then fitted with a 1D gaussian amplitude of the form $f(k_y) = A \exp(-(k_y - k_y^0)^2 / (2\sigma^2))$ and the result is shown in a). As visible, the fit matches well the data. The values of the amplitude A and the width σ are then used to define a 2D gaussian function with the same amplitude and width, centered at \mathbf{k} :

$$A_{fit}(k_x, k_y) = A \exp\left(-\frac{(k_x - k_x^0)^2}{2\sigma^2} - \frac{(k_y - k_y^0)^2}{2\sigma^2}\right). \quad (4.8)$$

To end the definition of the input mode complex field, we need to provide it with a phase. To do so, we look at the phase of the $\delta\tilde{\psi}$ in the vicinity of the peak. Close to the maximum amplitude, the phase is well defined and is approximated by a planar surface by fitting the signal with a function of the form $\theta_{fit}(k_x, k_y) = ak_x + bk_y + c$ where a , b and c are the fitting parameters. In fact encode the position of the in mode in real space and could also be determined by computing the barycenter of the probe intensity in real space. This fitting procedure is equivalent to neglect any curvature in the mode wavefront or in other words to assume that the mode is a plane wave. Of course, fluctuations around this ideal phase are expected and may contain information about the scattering process. To take this into

account we perform a last step consisting in projecting the overall field $\delta\tilde{\psi}$ on the ideal input mode we just defined ie $\delta\tilde{\psi}_{\text{fit}}(\mathbf{k}) = \mathbf{A}_{\text{fit}}(\mathbf{k})\exp(i\theta_{\text{fit}}(\mathbf{k}))$. The resulting field is then given by :

$$\delta\tilde{\psi}_{in} = \langle \delta\tilde{\psi}_{\text{fit}} | \delta\tilde{\psi} \rangle \delta\tilde{\psi}_{\text{fit}}. \quad (4.9)$$

Residual field. From the above expression, we are able to define the residual field as the difference between the injected mode and the overall field $\delta\tilde{\psi}_{res} = \delta\tilde{\psi} - \delta\tilde{\psi}_{in}$. Note that the way $\delta\tilde{\psi}_{in}$ is built through a scalar product ensures that the residual field is orthogonal to the *in* mode provided $\delta\tilde{\psi}_{fit}$ is normalized :

$$\begin{aligned} \langle \delta\tilde{\psi}_{in} | \delta\tilde{\psi}_{res} \rangle &= \langle \delta\tilde{\psi}_{\text{fit}} | \delta\tilde{\psi} \rangle (\langle \delta\tilde{\psi}_{\text{fit}} | \delta\tilde{\psi} \rangle - \langle \delta\tilde{\psi}_{\text{fit}} | \delta\tilde{\psi}_{in} \rangle) \\ &= \langle \delta\tilde{\psi}_{\text{fit}} | \delta\tilde{\psi} \rangle (\langle \delta\tilde{\psi}_{\text{fit}} | \delta\tilde{\psi} \rangle - \langle \delta\tilde{\psi}_{\text{fit}} | \delta\tilde{\psi}_{\text{fit}} \rangle \langle \delta\tilde{\psi}_{\text{fit}} | \delta\tilde{\psi} \rangle) \\ &= 0. \end{aligned} \quad (4.10)$$

In this manner, the basis chosen for our description is as close as possible to the Bogoliubov plane wave basis explicitated in [chapter 2](#). This is a strong assumption that will be discussed in the next section. Now that the different fields are defined,

So far, the field description was done on the full space instead of a local analysis in the interface vicinity. Their respective intensities are displayed in [Figure 4.10 c\)](#) and [d\)](#).

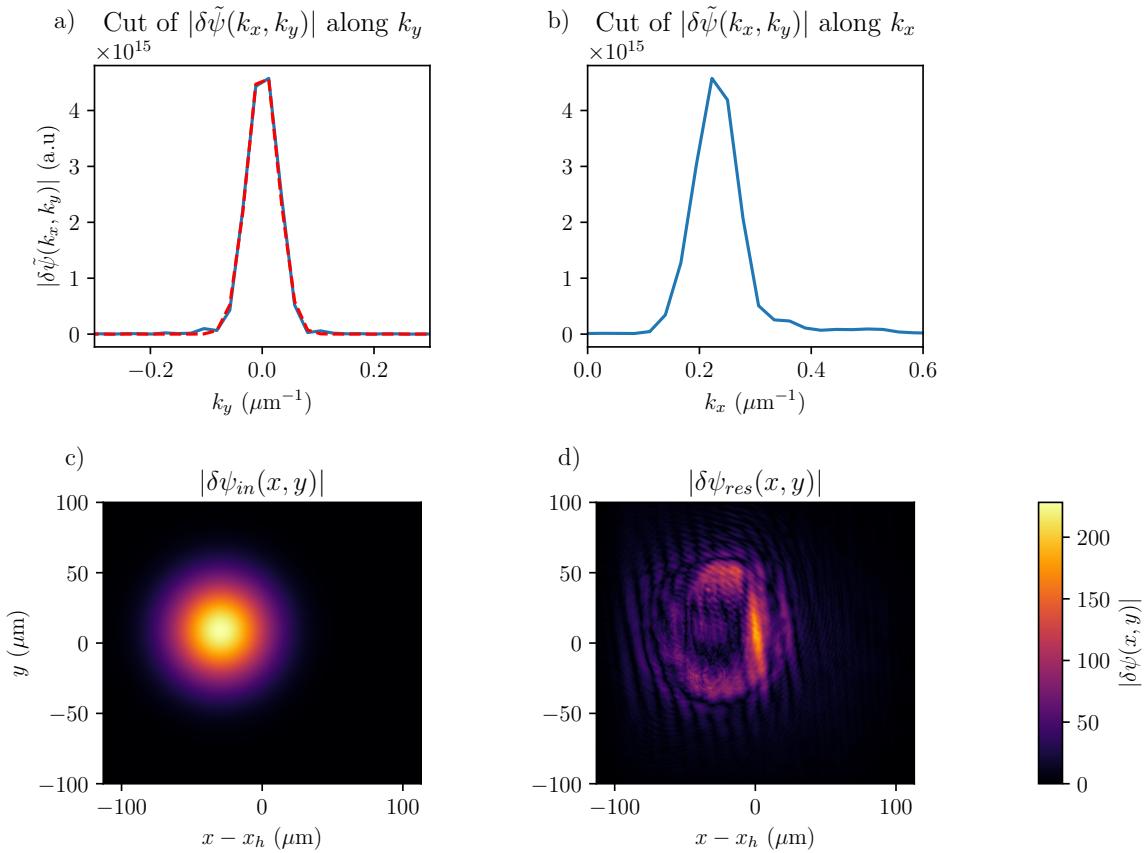


Fig. 4.10 **a)** **Cut of the injected mode in the k_y direction.** The blue curve is the data and the red dashed curve is the gaussian fit. The model has a R-squared value of 0.98. The amplitude A and width σ are extracted from the fit. **b)** **Cut of the injected mode in the k_x direction.** **c)** **Reconstructed input mode in real space.** Obtained by applying the inverse fourier transform to $\delta\psi_{in}(k_x, k_y)$. **d)** **Reconstructed residual field in real space.** Obtained by applying the inverse fourier transform to $\delta\psi_{res}(k_x, k_y)$.

Chapter 5

Bogoliubov modes correlations in polariton quantum fluid

The study of collective excitations in quantum fluids is fundamental to understand nonequilibrium dynamics and many-body interactions. Furthermore, the analog Hawking radiation on which this work is focused, is expected to create non classical correlations between bogoliubov modes, the event horizon acting as a two mode squeezer. The simplest manifestation of paired correlated emission can be observed in the density fluctuations second order correlation function [18; 22; 35; 46]. This observable exhibits correlation pattern in the density fluctuations of the fluid on both side of the horizon. Yet, this has been done only numerically in polaritonic system since the experimental equivalent would require to resolve the polariton lifetime $\sim 10\text{ps}$. However the strong photonic component of this system suggest that correlations between collective excitations mode must have an optical signature that could be addressed with all the tools developped in quantum optics. In this work, we report the first experimental measurement of correlations between collective excitation modes—Bogoliubov modes—in a static and homogeneous quantum fluid of microcavity exciton-polaritons. By using a balanced detection set-up, we measure intensity correlation between the normal and ghost branches, probing the fluctuation dynamics of polariton fluids and extracting the spectral correlations of Bogoliubov excitations. We observe a clear enhancement of the intensity correlations when the polariton fluid operates near the turning point of the bistability due to the emergence of correlated phonon-like excitations. These correlations, seeded by quantum and thermal fluctuations, provide insights into the role of the nonlinear and phononic interactions in the collective excitations of a polariton quantum fluid.

Appendices

Appendix A

Mean field shaping and monitoring

A.1 Phase printing

Controlling precisely the phase of a laser beam is a common but challenging task in optics. The most basic way to do it is by applying spatial filtering in the fourier plane of a lens. The phase of the beam after a second collimating lens is then given by the convolution product of the beam phase and the mask inverse fourier transform. This method then show its limits when one wants to generate complex phase profile since its depend on the mask form. A way to overcome this problem is the use of Digital Micromirror Device (DMD) which is an array of micro-mirrors that can be individually controlled to reflect the light or not. By putting this array in the fourier plan of a lens it is possible to create spatial filtering with arbitrary shapes. However, this method suffers from high losses and diffraction of the light on the individual mirrors that tend to add unwanted noise. This being said, DMD are very powerfull devices and allow to do a great amount of things at low cost. A wide range of possible methods are referenced in the great work [39].

A.1.1 Spatial light modulator.

In this work, we use a Spatial Light Modulator (SLM) which is a liquid crystal display that can be used to modulate the phase of a laser beam. The principle is to apply a voltage on each pixel of the SLM to change the orientation of the liquid crystal molecules. The phase shift is then given by the difference of the optical path of light going through the different pixels as shown in [Figure A.1](#).

By shinning a flat phase collimated beam on the SLM which displays the target phase profile the beam gets reflected carrying the desired wavefront. However the efficiency of the SLM is not perfect and whenever light is shone on it, some photons might not see it and not be phase modulated. To overcome this difficulty, we first write a blazed phase grating on the SLM screen on top of which the wanted profile is set. All the photons that did interact with the liquid crystals are then mostly diffracted on the first order of the grating. Doing so, an efficiency of 60-70% can be reached. This contrast with the usual 80% usually claimed on manufacturer datasheets that actually correspond to the efficiency in all orders. A typical phase profile encoded on the SLM is shown in [Figure A.2](#). A gray value of zero correspond to no shift while 255 corresponds to 2π . The gray map corresponding to the target phase

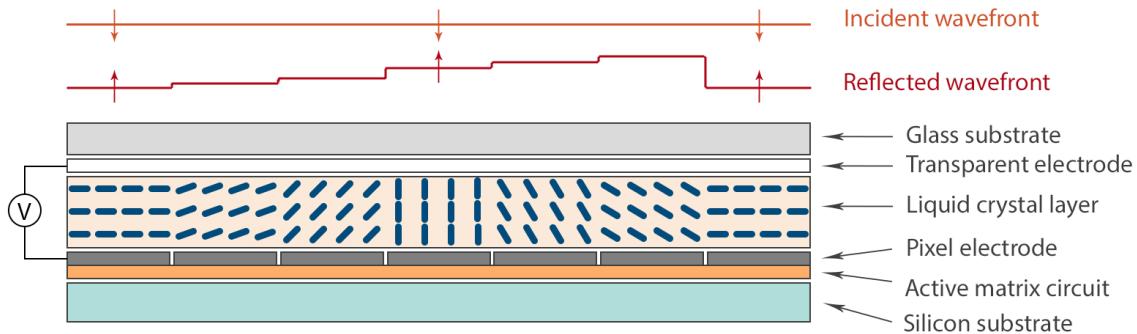


Fig. A.1 Principle of a Spatial Light Modulator.

profile has been unwrapped for the sake of clarity and is shown in [Figure A.2 c](#)). The final gray map displayed on the SLM screen is represented in [Figure A.2 e](#)).

To imprint precisely the resulting wavefront in the cavity plane two $2f - 2f$ optical telescopes are used in series to image the SLM screen with an overall demagnification of 1/130 while avoiding short focal length that would introduce optical aberrations. An adjustable slit is placed in the fourier plane of the first telescope to filter out the unwanted diffracted orders.

A.1.2 Phase measurement

A.1.3 Off axis interferometry

The principle is the following : the beam whose phase is to be measured is recombined on a CCD camera with a flat phase reference beam. An angle is voluntary introduced between the two beam to create an interference pattern on the camera plane. The phase difference between the two beams is then encoded in the size of the fringes and can be recovered by performing a fourier transform on the interferogram and demodulate the signal around the satellite peak created by the angle between the two beams [29].

A.1.4 Controlling the local intensity

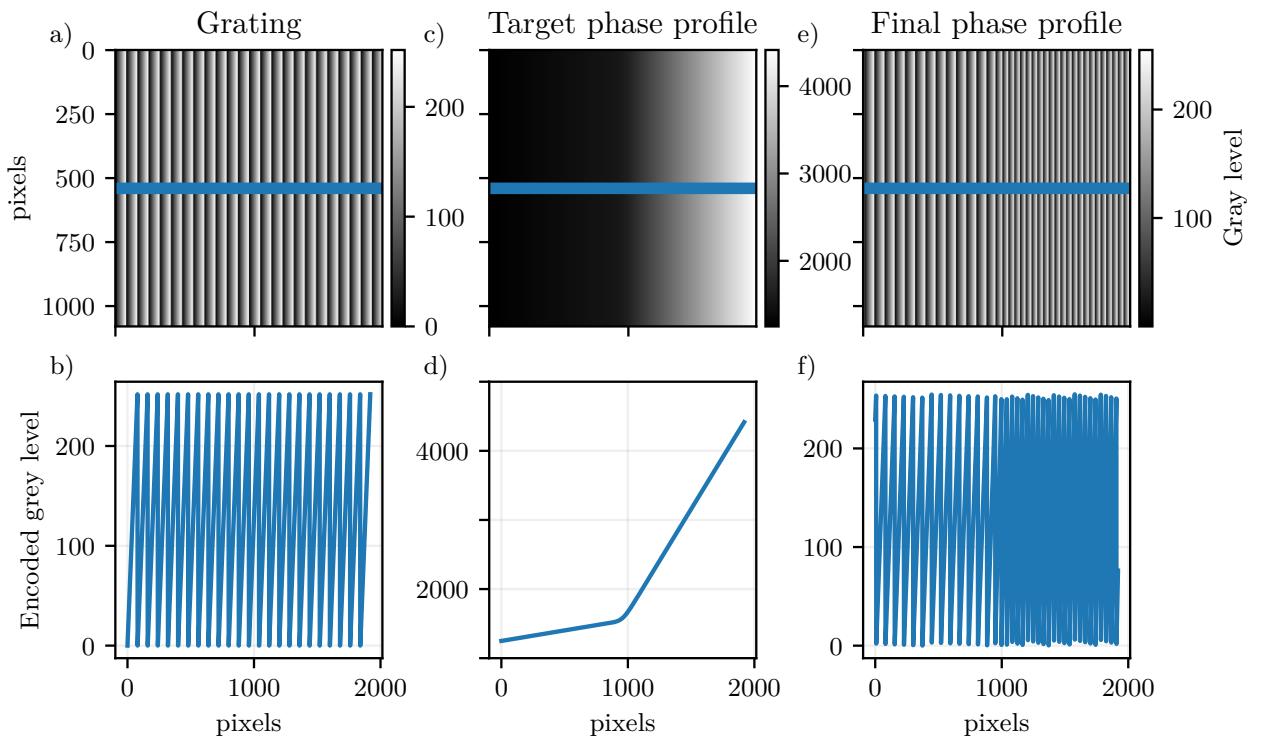


Fig. A.2 *Typical profile encoded on the SLM to generate the target velocity profile. a) Gray map of a phase blazed grating encoded on the SLM screen and b) corresponding cut in the x direction. c) Unwrapped grey map of the target phase profile and d) corresponding cut in the x direction. e) Final gray map encoded on the SLM screen and f) corresponding cut in the x direction. This map is the sum of the blazed grating and the target phase profile modulus 255.*

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Sujet : Contrôle tout optique de fluides quantiques de lumière en vapeur atomique chaude

Résumé : To do

Mots clés : Optique quantique, fluides quantiques de lumière, superfluidité, Falque, contrôle tout optique, Gravité Analogue, Rayonnement de Hawking, Création de particules, Theorie des champs, Espace-temps courbe, Trous noirs

Subject : Full optical control of quantum fluids of light in hot atomic vapors

Abstract: To do

Keywords : Quantum optics, quantum fluids of light, quantum fluids, superfluidity, Falque full optical control, Analogue Gravity, Hawking Radiation , Particles Creation, Quantum field theory, Curved spacetime, Black Holes
