

6-input-6-output discrete-time model: $\mathbf{v}_m[k+1] = a_1 \mathbf{v}_m[k] + a_2 \mathbf{v}_m[k-1] + \mathbf{B}\{\mathbf{u}[k] + \mathbf{w}[k]\}$

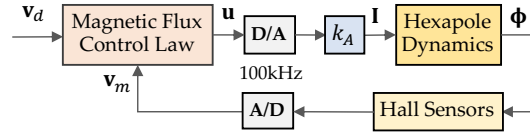
Tracking errors: $\delta \mathbf{v}[k] = \mathbf{v}_d[k-1] - \mathbf{v}_m[k]$ (one-step measurement delay)

Control objective: $\delta \mathbf{v}[k+1] = \lambda_c \delta \mathbf{v}[k]$, where $0 \leq \lambda_c < 1$

Control law: $\mathbf{u}[k] = \mathbf{B}^{-1}\{\mathbf{v}_{ff}[k] + \delta \mathbf{v}_{fb}[k] - \hat{\mathbf{w}}_T[k]\}$, where $\mathbf{w}_T[k] = \mathbf{B}\mathbf{w}[k]$ and $\hat{\mathbf{w}}_T[k] = \mathbf{B}\hat{\mathbf{w}}[k]$

$$\mathbf{v}_{ff}[k] = \mathbf{v}_d[k] - a_1 \mathbf{v}_d[k-1] - a_2 \mathbf{v}_d[k-2]$$

$$\delta \mathbf{v}_{fb}[k] = (a_1 - \lambda_c) \delta \hat{\mathbf{v}}[k] + a_2 \delta \hat{\mathbf{v}}[k-1]$$



Uncoupled error dynamics (control):

$$\mathbf{v}_m[k+1] = a_1 \mathbf{v}_m[k] + a_2 \mathbf{v}_m[k-1] + [\mathbf{v}_d[k] - a_1 \mathbf{v}_d[k-1] - a_2 \mathbf{v}_d[k-2]] + [(a_1 - \lambda_c) \delta \hat{\mathbf{v}}[k] + a_2 \delta \hat{\mathbf{v}}[k-1]] + [\mathbf{B}\mathbf{w}[k] - \hat{\mathbf{w}}_T]$$

$$\begin{aligned} \delta \mathbf{v}[k+1] &= a_1 \delta \mathbf{v}[k] + a_2 \delta \mathbf{v}[k-1] - a_1 \delta \hat{\mathbf{v}}[k] - a_2 \delta \hat{\mathbf{v}}[k-1] + \lambda_c \delta \hat{\mathbf{v}}[k] - [\mathbf{B}\mathbf{w}[k] - \hat{\mathbf{w}}_T] \\ &= \lambda_c \delta \hat{\mathbf{v}}[k] + a_1 \mathbf{e}_{\delta \mathbf{v}}[k] + a_2 \mathbf{e}_{\delta \mathbf{v}}[k-1] - \mathbf{e}_{\mathbf{w}_T}[k] \end{aligned}$$

Denote $\mathbf{s}_1[k] = \delta \mathbf{v}[k]$ and $\mathbf{s}_2[k] = \delta \mathbf{v}[k-1]$,

$$\begin{cases} \mathbf{s}_1[k+1] = \lambda_c \hat{\mathbf{s}}_1[k] + a_1 \mathbf{e}_{\mathbf{s}_1}[k] + a_2 \mathbf{e}_{\mathbf{s}_1}[k-1] - \mathbf{e}_{\mathbf{w}_T}[k] \\ \mathbf{s}_2[k+1] = \mathbf{s}_1[k] \end{cases}$$

Disturbance model: Employ the simplest disturbance model for \mathbf{w}_T , i.e.,

$$\mathbf{w}_T[k+1] = \mathbf{w}_T[k]$$

Estimator: An augmented state estimator, characterized by λ_e , is designed to achieve this objective.

$$\begin{cases} \hat{\mathbf{s}}_1[k+1] = \lambda_c \hat{\mathbf{s}}_1[k] + \ell_1 \{\delta \mathbf{v}[k] - \hat{\mathbf{s}}_1[k]\} \\ \hat{\mathbf{s}}_2[k+1] = \hat{\mathbf{s}}_1[k] + \ell_2 \{\delta \mathbf{v}[k] - \hat{\mathbf{s}}_1[k]\} \\ \hat{\mathbf{w}}_T[k+1] = \hat{\mathbf{w}}_T[k] + \ell_3 \{\delta \mathbf{v}[k] - \hat{\mathbf{s}}_1[k]\} \end{cases}$$

Error dynamics (estimation):

$$\begin{bmatrix} \mathbf{e}_{\mathbf{s}_1}[k+1] \\ \mathbf{e}_{\mathbf{s}_2}[k+1] \\ \mathbf{e}_{\mathbf{w}_T}[k+1] \end{bmatrix} = \begin{bmatrix} (a_1 - \ell_1) \mathbf{I} & a_2 \mathbf{I} & -\mathbf{I} \\ (1 - \ell_2) \mathbf{I} & \mathbf{0} & \mathbf{0} \\ -\ell_3 \mathbf{I} & \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{e}_{\mathbf{s}_1}[k] \\ \mathbf{e}_{\mathbf{s}_2}[k] \\ \mathbf{e}_{\mathbf{w}_T}[k] \end{bmatrix}$$

Assigning all eigenvalues to λ_e , the four feedback gains of the estimator are

$$\begin{cases} \ell_1 = 1 + a_1 - 3\lambda_e \\ \ell_2 = 1 + \lambda_e^2/a_2 \\ \ell_3 = -(1 - \lambda_e)^3 \end{cases}$$