Clock 0-5:

```
S_k1[i] = HsVd2_VECT6[i] - iHsVm_VECT6[i]
a1_m_Vd_d1[i] = iAnew[0] × HsVd1_VECT6[i]
a2_m_Vd_d2[i] = iAnew[1] × HsVd2_VECT6[i]
lamc_M_S1_k1_esti[i] = S1_k1_esti[i] × ilambda
```

Clock 9-14:

```
Output_diff[i] = S_k1[i] - S1_k1_esti[i]
```

Clock 15-23:

```
Vd_s_a1Vd1[i] = iHsVd_VECT6[i] - a1_m_Vd_d1[i]
Vd_s_a1Vd1_s_a2Vd2[i] = Vd_s_a1Vd1[i] - a2_m_Vd_d2[i]

mVolt_6x1_3 = Output_diff
mTransf_MAT6x6_3 = i_A1matrix_s_lambda

mVolt_6x1_2 = Output_diff
mTransf_MAT6x6_2 = i_L2_esti
```

Clock 18-23:

```
S1_esti_correct[i] = L1 × Output_diff[i]  // L1 = iAnew[2]
```

Clock 24-32:

```
S2_esti_correct[i] = L2 × Output_diff[i]  // L2 = iAnew[3]
```

Clock 25-30:

```
S1_esti[i] = lamc_M_S1_k1_esti[i] + S1_esti_correct[i]
```

Clock 34-39:

```
S2_esti[i] = S1_k1_esti[i] + S2_esti_correct[i]
a1lamc_m_S1_esti[i] = a1_s_lamc × S1_esti[i]  // a1_s_lamc =
iAnew[4]
```

Clock 41-46:

```
Vd_s_a1Vd_s_a2Vd2_p_a1lamc_m_S1_esti[i] =
    Vd_s_a1Vd1_s_a2Vd2[i] + a1lamc_m_S1_esti[i]
a2_m_S2_esti[i] = iAnew[1] × S2_esti[i]
```

Clock 49-54:

```
u_before_Binv[i] = Vd_s_a1Vd_s_a2Vd2_p_a1lamc_m_S1_esti[i] +
a2_m_S2_esti[i]
```

Clock 56-67:

```
one_p_Betam_m_westi_d1[i] = one_p_Betam × westi_d1[i]

Betam_m_w2esti_d1[i] = iBeta_m × w2esti_d1[i]

w1_before_correct[i] = westi_d1[i] + w2esti_d1[i]
```

Clock 64:

```
mVolt_6x1_4 = u_before_Binv
mTransf_MAT6x6_4 = i_inv_B       // B⁻¹
```

Clock 92-93

```
w_correct = Rst_6x6_6x1_3
w2_correct = Rst_6x6_6x1_2
```

Clock 101-118:

```
westi[i] = w1_before_correct[i] + w_correct[i]

w2esti[i] = w2esti_d1[i] + w2_correct[i]
```

Clock 133:

```
u_before_w = Rst_6x6_6x1_4
```

Clock 134-147:

```
  u_total_new[i] = u_before_w[i] - westi[i]
```

```
westi_d1[i] ≤= westi[i]
w2esti_d1[i] ≤= w2esti[i]
S1_k1_esti[i] ≤= S1_esti[i]
S2_k1_esti[i] ≤= S2_esti[i]
HsVd2_VECT6[i] ≤= HsVd1_VECT6[i]
HsVd1_VECT6[i] ≤= iHsVd_VECT6[i]
```

FPGA 控制器核心數學運算表達式

1. 基本控制變數定義

狀態變數**:**

- S_k**: 追蹤誤差 (tracking error)**
- S_1^{esti}**:** 狀態估測器輸出
- S_2^{esti}**:** 延遲狀態估測器
- w^{esti}**:** 擾動估測
- w_2^{esti}**:** 二階擾動估測

2. 主要控制律運算

Step 1: 追蹤誤差計算

$$S_k = V_d - V_m$$

Step 2: 前饋控制

$$V_{\text{ff}} = V_d - a_1 \times V_{d-1} - a_2 \times V_{d-2}$$

Step 3: 狀態估測誤差

$$e = S_k - S_1^{\text{esti}_{k-1}}$$

Step 4: 估測器更新

$$S_1^{\text{esti}} = \lambda_c \times S_1^{\text{esti}_{k-1}} + L_1 \times e$$

$$S_2^{\text{esti}} = S_1^{\text{esti}_{k-1}} + L_2 \times e$$

Step 5: 回饋控制

$$u_{\text{fb}} = (a_1 - \lambda_c) \times S_1^{\text{esti}} + a_2 \times S_2^{\text{esti}}$$

Step 6: 控制輸入**(**未解耦**)**

$$u_{\text{before}} = V_{\text{ff}} + u_{\text{fb}}$$

Step 7: 矩陣解耦

$$u_{\text{decoupled}} = B^{-1} \times u_{\text{before}}$$

Step 8: 擾動補償

$$u_{\text{final}} = u_{\text{decoupled}} - w^{\text{esti}}$$

3. 擾動估測器運算

擾動估測更新**:**

$$w_1^{\text{before}} = w^{\text{esti}_{k-1}} + w_2^{\text{esti}_{k-1}}$$

$$w^{\text{correct}} = (\text{A1matrix\_s\_lambda}) \times e$$

$$w_2^{\text{correct}} = (\text{L2\_esti}) \times e$$

$$w^{\text{esti}} = w_1^{\text{before}} + w^{\text{correct}}$$

$$w_2^{\text{esti}} = w_2^{\text{esti}_{k-1}} + w_2^{\text{correct}}$$

增強版本 **(with β_m):**

$$w_1^{\text{before}} = (1 + \beta_m) \times w^{\text{esti}_{k-1}} + \beta_m \times w_2^{\text{esti}_{k-1}}$$

4. 關鍵系統參數

固定參數**:**

- $a_1 = 1.595052025060797$ **(**來自系統極點 p_1**)**
- $a_2 = -0.599079946700523$ **(**來自系統極點 p_2**)**
- $\lambda_c = 0.9391$ **(**控制頻寬 **1000 Hz)**
- $\lambda_w = 0.7304$ **(**估測器頻寬 **5000 Hz)**
- β_m**:** 模型參數 **(**可調**)**

5. 估測器增益計算

Augmented Version (當前活躍版本**):**

$$L_1 = -\left(-\beta + (1+\beta)^2 - 4(1+\beta)\lambda_w + 6\lambda_w^2 - \frac{\lambda_w^4}{\beta}\right)$$

$$L_2 = -\left(1+\beta-4\lambda_w-(1+\beta)\frac{\lambda_w^4}{\beta^2}+4\frac{\lambda_w^3}{\beta}\right)$$

$$L_3 = \text{tune\_L3}$$

(來自 ThreeD_sine_theta)

iAnew 參數映射**:**

- **`iAnew[0]` = a_1 = 1.595052025060797**
- **`iAnew[1]` = a_2 = −0.599079946700523**
- **`iAnew[2]`** = L_1 **(**單一增益值**)**
- **`iAnew[3]`** = L_2 **(**單一增益值**)**
- **`iAnew[4]`** = $a_1 - \lambda_c$

6. 矩陣運算

解耦矩陣 **B^(-1):**

$$B^{-1} = \begin{pmatrix} 1254.4 & 365.9 & 362.7 & 509.3 & 510.7 & 478.1 \\ 354.7 & 1162.4 & 387.9 & 669.9 & 692.5 & 448.6 \\ 427.5 & 453.9 & 1258.9 & 439.3 & 488.0 & 520.8 \\ 442.5 & 654.6 & 353.8 & 1460.7 & 800.5 & 448.8 \\ 414.2 & 572.5 & 337.4 & 687.5 & 1274.4 & 374.7 \\ 412.2 & 400.9 & 368.5 & 474.4 & 407.4 & 1371.6 \end{pmatrix} \times L_3$$

擾動估測矩陣**:**

$$\text{A1matrix\_s\_lambda} = B^{-1} \times L_1$$

$$\text{L2\_esti} = B^{-1} \times L_2$$

7. 完整控制輸入計算流程 **(Control Effort Computation)**

Step 1: 追蹤誤差計算

$$\delta \boldsymbol{v}[k] = \boldsymbol{v}_d[k] - \boldsymbol{v}_m[k]$$

Step 2: 前饋控制項 **(Feedforward)**

$$\boldsymbol{v}_{\text{ff}}[k] = \boldsymbol{v}_d[k+1] - a_1\boldsymbol{v}_d[k] - a_2\boldsymbol{v}_d[k-1]$$

Step 3: 估測追蹤誤差 **(**使用增強狀態估測器**)**

$$\widehat{\delta \boldsymbol{v}}[k] = \hat{\boldsymbol{s}}_1[k]$$

Step 4: 回饋控制項 **(Feedback)**

$$\delta \boldsymbol{v}_{\mathrm{fb}}[k] = (a_1 - \lambda_c)\widehat{\delta \boldsymbol{v}}[k] + a_2\widehat{\delta \boldsymbol{v}}[k-1]$$

Step 5: 總控制輸入 **(**未解耦**)**

$$\boldsymbol{v}_{\mathrm{total}}[k] = \boldsymbol{v}_{\mathrm{ff}}[k] + \delta \boldsymbol{v}_{\mathrm{fb}}[k]$$

Step 6: 矩陣解耦與擾動補償

$$\boldsymbol{u}[k] = \boldsymbol{B}^{-1}\boldsymbol{v}_{\mathrm{total}}[k] - \hat{\boldsymbol{w}}[k]$$

其中 $\boldsymbol{B}^{-1}$ 為預先計算的解耦矩陣，$\hat{\boldsymbol{w}}[k]$ 來自擾動估測器

Step 7: 最終控制輸出 **(Control Effort)**

$$\boldsymbol{u}_{\mathrm{final}}[k] = \boldsymbol{B}^{-1}\Big[\boldsymbol{v}_d[k+1] - a_1\boldsymbol{v}_d[k] - a_2\boldsymbol{v}_d[k-1] + (a_1 - \lambda_c)\widehat{\delta \boldsymbol{v}}[k] + a_2\widehat{\delta \boldsymbol{v}}[k-1]\Big] - \hat{\boldsymbol{w}}[k]$$

此控制輸入將產生磁通：

$$\boldsymbol{\varphi}[k] = \boldsymbol{B}\boldsymbol{u}_{\mathrm{final}}[k]$$

進而產生磁力：

$$\boldsymbol{f}_m = g_\varphi \boldsymbol{\varphi}^T \boldsymbol{L}(\boldsymbol{p})\boldsymbol{\varphi}$$

iAnew

```
Anew[0][0] = A1matrix[0][0] = 1.595052025060797
Anew[0][1] = A2matrix[0][0]=-0.599079946700523;
Anew[0][2] = 1 + beta_model + A1matrix[0][0] - 4 * lambda_w;
Anew[0][3] = lambda_w*lambda_w*lambda_w*lambda_w / (A2matrix[0]
[0] * beta_model) + 1;
Anew[0][4] = A1matrix[0][0] - lambda_inner;
```

```
// Augmented version (currently active in PT3DView.cpp line
3330-3331)
tune_L1 = -(-beta_model + (1 + beta_model)*(1 + beta_model)
          - 4 * (1 + beta_model)*lambda_w + 6 *
lambda_w*lambda_w
          - lambda_w*lambda_w*lambda_w*lambda_w / beta_model);

tune_L2 = -( 1 + beta_model - 4 * lambda_w
          - (1 +
beta_model)*lambda_w*lambda_w*lambda_w*lambda_w
            / (beta_model*beta_model)
          + 4 * lambda_w*lambda_w*lambda_w / beta_model);

tune_L3 = pdlg6->ThreeD_sine_theta;
```

> **Augmented Version (活躍版本):**
>
> $$\mathrm{tune}_{\mathrm{L1}} = -\left(-\beta + (1+\beta)^2 - 4(1+\beta)\lambda + 6\lambda^2 - \frac{\lambda^4}{\beta}\right)$$
>
> $$\mathrm{tune}_{\mathrm{L2}} = -\left(1 + \beta - 4\lambda - (1+\beta)\frac{\lambda^4}{\beta^2} + 4\frac{\lambda^3}{\beta}\right)$$
>
> $$\mathrm{tune}_{\mathrm{L3}} = \mathrm{ThreeD\_sine\_theta}$$

> 其中：$\beta = \text{beta\_model}, \lambda = \text{lambda\_w}$

其他版本 (已註解):

```
// Simple version (PT3DView.cpp line 3321-3322, 3324-3325)
tune_L1 = 2 * lambda_w - 1 - beta_model;
tune_L2 = lambda_w*lambda_w / beta_model - 1;

// Polynomial version (PT3DView.cpp line 3332-3333)
tune_L1 = (lambda_w - 1)*(lambda_w - 1)*(lambda_w - 1)*(lambda_w
+ 3);
tune_L2 = -(lambda_w - 1)*(lambda_w - 1)*(lambda_w -
1)*(lambda_w - 1);

// M-delay version (PT3DView.cpp line 3326-3327)
tune_L1 = ((2*lambda_w -
(1+beta_model))*((1+beta_model)*(A1matrix[0][0]-lambda_inner) +
beta_model)
          - (lambda_w*lambda_w - beta_model)*(A1matrix[0][0]-
lambda_inner))
          / ((1+beta_model)*(A1matrix[0][0]-lambda_inner) +
beta_model
          + (A1matrix[0][0]-lambda_inner)*(A1matrix[0][0]-
lambda_inner));

tune_L2 = ((2*lambda_w - (1+beta_model))*(A1matrix[0][0]-
lambda_inner)
          + (lambda_w*lambda_w - beta_model))
          / ((1+beta_model)*(A1matrix[0][0]-lambda_inner) +
beta_model
          + (A1matrix[0][0]-lambda_inner)*(A1matrix[0][0]-
lambda_inner));
```

inv_B

```
inv_B = [
  [1254.4,  365.9,  362.7,  509.3,  510.7,  478.1],
  [ 354.7, 1162.4,  387.9,  669.9,  692.5,  448.6],
  [ 427.5,  453.9, 1258.9,  439.3,  488.0,  520.8],
  [ 442.5,  654.6,  353.8, 1460.7,  800.5,  448.8],
  [ 414.2,  572.5,  337.4,  687.5, 1274.4,  374.7],
  [ 412.2,  400.9,  368.5,  474.4,  407.4, 1371.6]
 * tune_L3;
```

A1matrix_s_lambda

```
A1matrix_s_lambda = [
  [1254.4,  365.9,  362.7,  509.3,  510.7,  478.1],
  [ 354.7, 1162.4,  387.9,  669.9,  692.5,  448.6],
  [ 427.5,  453.9, 1258.9,  439.3,  488.0,  520.8],
  [ 442.5,  654.6,  353.8, 1460.7,  800.5,  448.8],
  [ 414.2,  572.5,  337.4,  687.5, 1274.4,  374.7],
  [ 412.2,  400.9,  368.5,  474.4,  407.4, 1371.6]
] * tune_L1 * tune_L3;
```

L2_esti

```
L2_esti = [
  [1254.4,  365.9,  362.7,  509.3,  510.7,  478.1],
  [ 354.7, 1162.4,  387.9,  669.9,  692.5,  448.6],
  [ 427.5,  453.9, 1258.9,  439.3,  488.0,  520.8],
  [ 442.5,  654.6,  353.8, 1460.7,  800.5,  448.8],
  [ 414.2,  572.5,  337.4,  687.5, 1274.4,  374.7],
  [ 412.2,  400.9,  368.5,  474.4,  407.4, 1371.6]
] * tune_L2 * tune_L3;
```

論文理論算法數學表達式總結

1. 六輸入六輸出磁通控制系統 **(Six-Input-Six-Output Magnetic Flux Control)**

系統離散模型 **(**方程式 **6):**

$$\boldsymbol{v}_m[k+1] = a_1\boldsymbol{v}_m[k] + a_2\boldsymbol{v}_m[k-1] + \boldsymbol{B}\{\boldsymbol{u}[k] + \boldsymbol{w}[k]\}$$

其中：

- $\boldsymbol{v}_m$**: 6×1 Hall sensor** 電壓向量
- $\boldsymbol{u}$**: 6×1** 控制輸入向量
- $\boldsymbol{w}$**: 6×1** 擾動向量
- $a_1 = 0.9898$**,** $a_2 = 0.6053$ **(**系統極點**)**
- $\boldsymbol{B}$**: 6×6** 輸入矩陣

2. 控制律設計 **(**方程式 **7)**

控制目標**:** $\delta\boldsymbol{v}[k+1] = \lambda_c\delta\boldsymbol{v}[k]$**,** 其中 $0 \leq \lambda_c < 1$

控制律**:**

$$\boldsymbol{u}[k] = \boldsymbol{B}^{-1}\{\boldsymbol{v}_{\text{ff}}[k] + \delta\boldsymbol{v}_{\text{fb}}[k]\} - \hat{\boldsymbol{w}}[k]$$

前饋控制**:**

$$\boldsymbol{v}_{\text{ff}}[k] = \boldsymbol{v}_d[k+1] - a_1\boldsymbol{v}_d[k] - a_2\boldsymbol{v}_d[k-1]$$

回饋控制**:**

$$\delta\boldsymbol{v}_{\text{fb}}[k] = (a_1 - \lambda_c)\widehat{\delta\boldsymbol{v}}[k] + a_2\widehat{\delta\boldsymbol{v}}[k-1]$$

3. 追蹤誤差動態 **(**方程式 **8)**

$$\delta\boldsymbol{v}[k+1] = \lambda_c\delta\boldsymbol{v}[k] - \boldsymbol{B}\boldsymbol{e}_w[k] + (a_1 - \lambda_c)\boldsymbol{e}_{\text{delta v}}[k] + a_2\boldsymbol{e}_{\text{delta v}}[k-1]$$

其中：

- $\boldsymbol{e}_w = \boldsymbol{w} - \hat{\boldsymbol{w}}$**:** 擾動估測誤差
- $\boldsymbol{e}_{\text{delta v}} = \delta\boldsymbol{v} - \widehat{\delta\boldsymbol{v}}$**:** 狀態估測誤差

4. 增強狀態估測器 (方程式 9)

$$\hat{\boldsymbol{s}}_1[k+1] = \lambda_c \hat{\boldsymbol{s}}_1[k] + \boldsymbol{L}_1\{\delta \boldsymbol{v}[k] - \hat{\boldsymbol{s}}_1[k]\}$$

$$\hat{\boldsymbol{s}}_2[k+1] = \hat{\boldsymbol{s}}_1[k] + \boldsymbol{L}_2\{\delta \boldsymbol{v}[k] - \hat{\boldsymbol{s}}_1[k]\}$$

$$\hat{\boldsymbol{w}}[k+1] = \hat{\boldsymbol{w}}[k] + \delta\hat{\boldsymbol{w}}[k] + \boldsymbol{L}_3\{\delta \boldsymbol{v}[k] - \hat{\boldsymbol{s}}_1[k]\}$$

$$\delta\hat{\boldsymbol{w}}[k+1] = \delta\hat{\boldsymbol{w}}[k] + \boldsymbol{L}_4\{\delta \boldsymbol{v}[k] - \hat{\boldsymbol{s}}_1[k]\}$$

5. 估測器回饋矩陣 (方程式 11)

$$\boldsymbol{L}_1 = (2 + a_1 - 4\lambda_e)\boldsymbol{I}$$

$$\boldsymbol{L}_2 = \left(1 + \frac{\lambda_e^4}{a_2}\right)\boldsymbol{I}$$

$$\boldsymbol{L}_3 = -(1-\lambda_e)^3(3+\lambda_e)\boldsymbol{B}^{-1}$$

$$\boldsymbol{L}_4 = -(1-\lambda_e)^4\boldsymbol{B}^{-1}$$

6. 最優磁通分配 (Optimal Flux Allocation)

Hall sensor 磁力模型 **(**方程式 **3):**

$$\boldsymbol{f}_{m(\boldsymbol{p},\boldsymbol{v}_H)} = g_H \boldsymbol{v}_H^T \hat{\boldsymbol{D}}_H^T \boldsymbol{L}\left(\frac{\boldsymbol{p}}{\ell}\right)\hat{\boldsymbol{D}}_H \boldsymbol{v}_H$$

最優電壓分配 **(**方程式 **5):**

$$\boldsymbol{v}_{\text{opt}}(\boldsymbol{f}_d, \boldsymbol{p}) = \sqrt{\frac{f_d}{g_H}}\hat{\boldsymbol{D}}_H^{-1}\hat{\boldsymbol{\varphi}}_{\text{opt}}(\varphi, \theta, \boldsymbol{p})$$

7. 關鍵系統參數

控制器參數**:**

- $\lambda_c = 0.9391$ → 控制頻寬 **1000 Hz**
- $\lambda_e = 0.7304$ → 估測器頻寬 **5000 Hz**
- 採樣頻率**: 100 kHz (**採樣週期 **10 μs)**

系統極點 **(z-domain):**

- $p_1 = 0.9898 \rightarrow$ 主導極點
- $p_2 = 0.6053 \rightarrow$ 次要極點
- 開迴路頻寬**: 160 Hz**
- 閉迴路頻寬**: 1000 Hz**

$$\boldsymbol{u}[k] = {\boldsymbol{B}_{\mathbf{tune}}}^{-1}\{\boldsymbol{v}_{\mathrm{ff}}[k] + \delta\boldsymbol{v}_{\mathrm{fb}}[k]\} - \hat{\boldsymbol{w}}[k]$$

$$\boldsymbol{v}_{\mathrm{ff}}[k-1] = \boldsymbol{v}_d[k] - a_1\boldsymbol{v}_d[k-1] - a_2\boldsymbol{v}_d[k-2]$$

$$\delta\boldsymbol{v}_{\mathrm{fb}}[k] = (a_1 - \lambda_c)\widehat{\delta\boldsymbol{v}}[k] + a_2\widehat{\delta\boldsymbol{v}}[k-1]$$

$$\hat{\boldsymbol{s}}_1[k] = \lambda_c\hat{\boldsymbol{s}}_1[k-1] + \boldsymbol{L}_1\{\delta\boldsymbol{v}[k-1] - \hat{\boldsymbol{s}}_1[k-1]\}$$

$$\hat{\boldsymbol{s}}_2[k] = \hat{\boldsymbol{s}}_1[k-1] + \boldsymbol{L}_2\{\delta\boldsymbol{v}[k-1] - \hat{\boldsymbol{s}}_1[k-1]\}$$

$$\hat{\boldsymbol{w}}[k] = \hat{\boldsymbol{w}}[k-1] + \delta\hat{\boldsymbol{w}}[k-1] + \boldsymbol{L}_3\{\delta\boldsymbol{v}[k-1] - \hat{\boldsymbol{s}}_1[k-1]\}$$

$$\delta\hat{\boldsymbol{w}}[k] = \delta\hat{\boldsymbol{w}}[k-1] + \boldsymbol{L}_4\{\delta\boldsymbol{v}[k-1] - \hat{\boldsymbol{s}}_1[k-1]\}$$

$$L_1 = 1 + \beta + a_1 + 4\lambda_e$$

$$L_2 = 1 + \frac{\lambda_e^4}{a_2\beta}$$

$$L_3 = -\left(-\beta + (1+\beta)^2 - 4(1+\beta)\lambda_e + 6\lambda_e^2 - \frac{\lambda_e^4}{\beta}\right)\boldsymbol{B}_{\mathbf{tune}}^{\mathbf{-1}}$$

$$L_4 = -\left(1 + \beta - 4\lambda_e - (1+\beta)\frac{\lambda_e^4}{\beta^2} + 4\frac{\lambda_e^3}{\beta}\right)\boldsymbol{B}_{\mathbf{tune}}^{\mathbf{-1}}$$

$$*\,\beta = \lambda_e^2$$