6-input-6-output discrete-time model: $\mathbf{v}_m[k+1] = a_1\mathbf{v}_m[k] + a_2\mathbf{v}_m[k-1] + \mathbf{B}\{\mathbf{u}[k] + \mathbf{w}[k]\}$

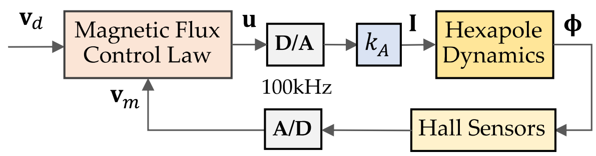
Tracking errors: $\delta\mathbf{v}[k] = \mathbf{v}_d[k-1] - \mathbf{v}_m[k]$ (one-step measurement delay)

Control objective: $\delta\mathbf{v}[k+1] = \lambda_c\delta\mathbf{v}[k]$, where $0 \le \lambda_c < 1$

Control law: $\mathbf{u}[k] = \mathbf{B}^{-1}\{\mathbf{v}_{ff}[k] + \delta\mathbf{v}_{fb}[k] - \hat{\mathbf{w}}_T[k]\}$, where $\mathbf{w}_T[k] = \mathbf{B}\mathbf{w}[k]$ and $\hat{\mathbf{w}}_T[k] = \mathbf{B}\hat{\mathbf{w}}[k]$

$\mathbf{v}_{ff}[k] = \mathbf{v}_d[k] - a_1\mathbf{v}_d[k-1] - a_2\mathbf{v}_d[k-2]$

$\delta\mathbf{v}_{fb}[k] = (a_1 - \lambda_c)\delta\hat{\mathbf{v}}[k] + a_2\delta\hat{\mathbf{v}}[k-1]$



Uncoupled error dynamics (control):

$$\mathbf{v}_m[k+1] = a_1\mathbf{v}_m[k] + a_2\mathbf{v}_m[k-1] + \left[\mathbf{v}_d[k] - a_1\mathbf{v}_d[k-1] - a_2\mathbf{v}_d[k-2]\right] + \left[(a_1 - \lambda_c)\delta\hat{\mathbf{v}}[k] + a_2\delta\hat{\mathbf{v}}[k-1]\right] + [\mathbf{B}\mathbf{w}[k] - \hat{\mathbf{w}}_T]$$

$$\begin{aligned}\delta\mathbf{v}[k+1] &= a_1\delta\mathbf{v}[k] + a_2\delta\mathbf{v}[k-1] - a_1\delta\hat{\mathbf{v}}[k] - a_2\delta\hat{\mathbf{v}}[k-1] + \lambda_c\delta\hat{\mathbf{v}}[k] - [\mathbf{B}\mathbf{w}[k] - \hat{\mathbf{w}}_T] \\ &= \lambda_c\delta\hat{\mathbf{v}}[k] + a_1\mathbf{e}_{\delta\mathbf{v}}[k] + a_2\mathbf{e}_{\delta\mathbf{v}}[k-1] - \mathbf{e}_{\mathbf{w}_T}[k]\end{aligned}$$

Denote $\mathbf{s}_1[k] = \delta\mathbf{v}[k]$ and $\mathbf{s}_2[k] = \delta\mathbf{v}[k-1]$,

$$\begin{cases}\mathbf{s}_1[k+1] = \lambda_c\hat{\mathbf{s}}_1[k] + a_1\mathbf{e}_{\mathbf{s}_1}[k] + a_2\mathbf{e}_{\mathbf{s}_1}[k-1] - \mathbf{e}_{\mathbf{w}_T}[k] \\ \mathbf{s}_2[k+1] = \mathbf{s}_1[k]\end{cases}$$

Disturbance model: Employ a second-order disturbance model for $\mathbf{w}_T$, i.e., $\mathbf{w}_1[k] = \mathbf{w}_T[k]$,

$$\begin{cases}\mathbf{w}_1[k+1] = (1+\beta)\mathbf{w}_1[k] - \beta\mathbf{w}_2[k] \\ \mathbf{w}_2[k+1] = \mathbf{w}_1[k]\end{cases}$$

Estimator: An augmented state estimator, characterized by λ_e, is designed to achieve this objective.

$$\begin{cases}\hat{\mathbf{s}}_1[k+1] = \lambda_c\hat{\mathbf{s}}_1[k] + \ell_1\{\delta\mathbf{v}[k] - \hat{\mathbf{s}}_1[k]\} \\ \hat{\mathbf{s}}_2[k+1] = \hat{\mathbf{s}}_1[k] + \ell_2\{\delta\mathbf{v}[k] - \hat{\mathbf{s}}_1[k]\} \\ \hat{\mathbf{w}}_1[k+1] = (1+\beta)\hat{\mathbf{w}}_1[k] - \beta\hat{\mathbf{w}}_2[k] + \ell_3\{\delta\mathbf{v}[k] - \hat{\mathbf{s}}_1[k]\} \\ \hat{\mathbf{w}}_2[k+1] = \hat{\mathbf{w}}_1[k] + \ell_4\{\delta\mathbf{v}[k] - \hat{\mathbf{s}}_1[k]\}\end{cases}$$

Error dynamics (estimation):

$$\begin{bmatrix}\mathbf{e}_{\mathbf{s}_1}[k+1] \\ \mathbf{e}_{\mathbf{s}_2}[k+1] \\ \mathbf{e}_{\mathbf{w}1}[k+1] \\ \mathbf{e}_{\mathbf{w}2}[k+1]\end{bmatrix} = \begin{bmatrix}(a_1 - \ell_1)\mathbf{I} & a_2\mathbf{I} & -\mathbf{I} & \mathbf{0} \\ (1 - \ell_2)\mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -\ell_3\mathbf{I} & \mathbf{0} & (1+\beta)\mathbf{I} & -\beta\mathbf{I} \\ -\ell_4\mathbf{I} & \mathbf{0} & \mathbf{I} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{e}_{\mathbf{s}_1}[k] \\ \mathbf{e}_{\mathbf{s}_2}[k] \\ \mathbf{e}_{\mathbf{w}1}[k] \\ \mathbf{e}_{\mathbf{w}2}[k]\end{bmatrix}$$

Assigning all eigenvalues to λ_e, the four feedback gains of the estimator are

$$\begin{cases}\ell_1 = (1 + \beta + a_1 - 4\lambda_e) \\ \ell_2 = [1 + \lambda_e^4/(a_2\beta)] \\ \ell_3 = -\left\{-\beta + (1+\beta)^2 - 4(1+\beta)\lambda_e + 6\lambda_e^2 - \dfrac{\lambda_e^4}{\beta}\right\} \\ \ell_4 = -\left\{(1+\beta) - 4\lambda_e - (1+\beta)\dfrac{\lambda_e^4}{\beta^2} + 4\dfrac{\lambda_e^3}{\beta}\right\}\end{cases}$$

6-input-6-output discrete-time model: $\mathbf{v}_m[k+1] = a_1\mathbf{v}_m[k] + a_2\mathbf{v}_m[k-1] + \mathbf{B}\{\mathbf{u}[k] + \mathbf{w}[k]\}$

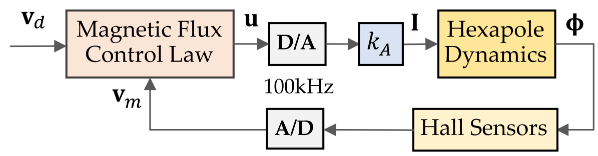
Tracking errors: $\delta\mathbf{v}[k] = \mathbf{v}_d[k-1] - \mathbf{v}_m[k]$ (one-step measurement delay)

Control objective: $\delta\mathbf{v}[k+1] = \lambda_c\delta\mathbf{v}[k]$, where $0 \le \lambda_c < 1$

Control law: $\mathbf{u}[k] = \mathbf{B}^{-1}\{\mathbf{v}_{ff}[k] + \delta\mathbf{v}_{fb}[k] - \widehat{\mathbf{w}}_T[k]\}$, where $\mathbf{w}_T[k] = \mathbf{B}\mathbf{w}[k]$ and $\widehat{\mathbf{w}}_T[k] = \mathbf{B}\widehat{\mathbf{w}}[k]$

$\mathbf{v}_{ff}[k] = \mathbf{v}_d[k] - a_1\mathbf{v}_d[k-1] - a_2\mathbf{v}_d[k-2]$

$\delta\mathbf{v}_{fb}[k] = (a_1 - \lambda_c)\delta\hat{\mathbf{v}}[k] + a_2\delta\hat{\mathbf{v}}[k-1]$



Uncoupled error dynamics (control):

$$\mathbf{v}_m[k+1] = a_1\mathbf{v}_m[k] + a_2\mathbf{v}_m[k-1] + \left[\mathbf{v}_d[k] - a_1\mathbf{v}_d[k-1] - a_2\mathbf{v}_d[k-2]\right] + \left[(a_1-\lambda_c)\delta\hat{\mathbf{v}}[k] + a_2\delta\hat{\mathbf{v}}[k-1]\right] + [\mathbf{B}\mathbf{w}[k] - \widehat{\mathbf{w}}_T]$$

$$\begin{aligned}\delta\mathbf{v}[k+1] &= a_1\delta\mathbf{v}[k] + a_2\delta\mathbf{v}[k-1] - a_1\delta\hat{\mathbf{v}}[k] - a_2\delta\hat{\mathbf{v}}[k-1] + \lambda_c\delta\hat{\mathbf{v}}[k] - [\mathbf{B}\mathbf{w}[k] - \widehat{\mathbf{w}}_T] \\ &= \lambda_c\delta\hat{\mathbf{v}}[k] + a_1\mathbf{e}_{\delta\mathbf{v}}[k] + a_2\mathbf{e}_{\delta\mathbf{v}}[k-1] - \mathbf{e}_{\mathbf{w}_T}[k]\end{aligned}$$

Denote $\mathbf{s}_1[k] = \delta\mathbf{v}[k]$ and $\mathbf{s}_2[k] = \delta\mathbf{v}[k-1]$,

$$\begin{cases}\mathbf{s}_1[k+1] = \lambda_c\hat{\mathbf{s}}_1[k] + a_1\mathbf{e}_{\mathbf{s}_1}[k] + a_2\mathbf{e}_{\mathbf{s}_1}[k-1] - \mathbf{e}_{\mathbf{w}_T}[k] \\ \mathbf{s}_2[k+1] = \mathbf{s}_1[k]\end{cases}$$

Disturbance model: Employ a second-order disturbance model for $\mathbf{w}_T$, i.e.,

$$\begin{cases}\mathbf{w}_T[k+1] = \mathbf{w}_T[k] + \delta\mathbf{w}_T[k] \\ \delta\mathbf{w}_T[k+1] = \delta\mathbf{w}_T[k]\end{cases}$$

Estimator: An augmented state estimator, characterized by λ_e, is designed to achieve this objective.

$$\begin{cases}\hat{\mathbf{s}}_1[k+1] = \lambda_c\hat{\mathbf{s}}_1[k] + \ell_1\{\delta\mathbf{v}[k] - \hat{\mathbf{s}}_1[k]\} \\ \hat{\mathbf{s}}_2[k+1] = \hat{\mathbf{s}}_1[k] + \ell_2\{\delta\mathbf{v}[k] - \hat{\mathbf{s}}_1[k]\} \\ \widehat{\mathbf{w}}_T[k+1] = \widehat{\mathbf{w}}_T[k] + \delta\widehat{\mathbf{w}}_T[k] + \ell_3\{\delta\mathbf{v}[k] - \hat{\mathbf{s}}_1[k]\} \\ \delta\widehat{\mathbf{w}}_T[k+1] = \delta\widehat{\mathbf{w}}_T[k] + \ell_4\{\delta\mathbf{v}[k] - \hat{\mathbf{s}}_1[k]\}\end{cases}$$

Error dynamics (estimation):

$$\begin{bmatrix}\mathbf{e}_{\mathbf{s}_1}[k+1] \\ \mathbf{e}_{\mathbf{s}_2}[k+1] \\ \mathbf{e}_{\mathbf{w}_T}[k+1] \\ \mathbf{e}_{\delta\mathbf{w}_T}[k+1]\end{bmatrix} = \begin{bmatrix}(a_1-\ell_1)\mathbf{I} & a_2\mathbf{I} & -\mathbf{I} & \mathbf{0} \\ (1-\ell_2)\mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -\ell_3\mathbf{I} & \mathbf{0} & \mathbf{I} & \mathbf{I} \\ -\ell_4\mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{e}_{\mathbf{s}_1}[k] \\ \mathbf{e}_{\mathbf{s}_2}[k] \\ \mathbf{e}_{\mathbf{w}_T}[k] \\ \mathbf{e}_{\delta\mathbf{w}_T}[k]\end{bmatrix}$$

Assigning all eigenvalues to λ_e, the four feedback gains of the estimator are

$$\begin{cases}\ell_1 = (2 + a_1 - 4\lambda_e) \\ \ell_2 = [1 + \lambda_e^4/a_2] \\ \ell_3 = -(1-\lambda_e)^3(3+\lambda_e) \\ \ell_4 = -(1-\lambda_e)^4\end{cases}$$

6-input-6-output discrete-time model: $\mathbf{v}_m[k+1] = a_1\mathbf{v}_m[k] + a_2\mathbf{v}_m[k-1] + \mathbf{B}\{\mathbf{u}[k] + \mathbf{w}[k]\}$

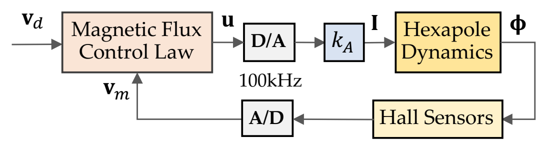
Tracking errors: $\delta\mathbf{v}[k] = \mathbf{v}_d[k-1] - \mathbf{v}_m[k]$ (one-step measurement delay)

Control objective: $\delta\mathbf{v}[k+1] = \lambda_c\delta\mathbf{v}[k]$, where $0 \le \lambda_c < 1$

Control law: $\mathbf{u}[k] = \mathbf{B}^{-1}\{\mathbf{v}_{ff}[k] + \delta\mathbf{v}_{fb}[k] - \widehat{\mathbf{w}}_T[k]\}$, where $\mathbf{w}_T[k] = \mathbf{B}\mathbf{w}[k]$ and $\widehat{\mathbf{w}}_T[k] = \mathbf{B}\widehat{\mathbf{w}}[k]$

$\mathbf{v}_{ff}[k] = \mathbf{v}_d[k] - a_1\mathbf{v}_d[k-1] - a_2\mathbf{v}_d[k-2]$

$\delta\mathbf{v}_{fb}[k] = (a_1 - \lambda_c)\delta\hat{\mathbf{v}}[k] + a_2\delta\hat{\mathbf{v}}[k-1]$



Uncoupled error dynamics (control):

$$\mathbf{v}_m[k+1] = a_1\mathbf{v}_m[k] + a_2\mathbf{v}_m[k-1] + \left[\mathbf{v}_d[k] - a_1\mathbf{v}_d[k-1] - a_2\mathbf{v}_d[k-2]\right] + \left[(a_1-\lambda_c)\delta\hat{\mathbf{v}}[k] + a_2\delta\hat{\mathbf{v}}[k-1]\right] + [\mathbf{B}\mathbf{w}[k] - \widehat{\mathbf{w}}_T]$$

$$\begin{aligned}\delta\mathbf{v}[k+1] &= a_1\delta\mathbf{v}[k] + a_2\delta\mathbf{v}[k-1] - a_1\delta\hat{\mathbf{v}}[k] - a_2\delta\hat{\mathbf{v}}[k-1] + \lambda_c\delta\hat{\mathbf{v}}[k] - [\mathbf{B}\mathbf{w}[k] - \widehat{\mathbf{w}}_T] \\ &= \lambda_c\delta\hat{\mathbf{v}}[k] + a_1\mathbf{e}_{\delta\mathbf{v}}[k] + a_2\mathbf{e}_{\delta\mathbf{v}}[k-1] - \mathbf{e}_{\mathbf{w}_T}[k]\end{aligned}$$

Denote $\mathbf{s}_1[k] = \delta\mathbf{v}[k]$ and $\mathbf{s}_2[k] = \delta\mathbf{v}[k-1]$,

$$\begin{cases}\mathbf{s}_1[k+1] = \lambda_c\hat{\mathbf{s}}_1[k] + a_1\mathbf{e}_{\mathbf{s}_1}[k] + a_2\mathbf{e}_{\mathbf{s}_1}[k-1] - \mathbf{e}_{\mathbf{w}_T}[k] \\ \mathbf{s}_2[k+1] = \mathbf{s}_1[k]\end{cases}$$

Disturbance model: Employ the simplest disturbance model for $\mathbf{w}_T$, i.e.,

$\mathbf{w}_T[k+1] = \mathbf{w}_T[k]$

Estimator: An augmented state estimator, characterized by λ_e, is designed to achieve this objective.

$$\begin{cases}\hat{\mathbf{s}}_1[k+1] = \lambda_c\hat{\mathbf{s}}_1[k] + \ell_1\{\delta\mathbf{v}[k] - \hat{\mathbf{s}}_1[k]\} \\ \hat{\mathbf{s}}_2[k+1] = \hat{\mathbf{s}}_1[k] + \ell_2\{\delta\mathbf{v}[k] - \hat{\mathbf{s}}_1[k]\} \\ \widehat{\mathbf{w}}_T[k+1] = \widehat{\mathbf{w}}_T[k] + \ell_3\{\delta\mathbf{v}[k] - \hat{\mathbf{s}}_1[k]\}\end{cases}$$

Error dynamics (estimation):

$$\begin{bmatrix}\mathbf{e}_{\mathbf{s}_1}[k+1] \\ \mathbf{e}_{\mathbf{s}_2}[k+1] \\ \mathbf{e}_{\mathbf{w}_T}[k+1]\end{bmatrix} = \begin{bmatrix}(a_1-\ell_1)\mathbf{I} & a_2\mathbf{I} & -\mathbf{I} \\ (1-\ell_2)\mathbf{I} & \mathbf{0} & \mathbf{0} \\ -\ell_3\mathbf{I} & \mathbf{0} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{e}_{\mathbf{s}_1}[k] \\ \mathbf{e}_{\mathbf{s}_2}[k] \\ \mathbf{e}_{\mathbf{w}_T}[k]\end{bmatrix}$$

Assigning all eigenvalues to λ_e, the four feedback gains of the estimator are

$$\begin{cases}\ell_1 = 1 + a_1 - 3\lambda_e \\ \ell_2 = 1 + \lambda_e^3/a_2 \\ \ell_3 = -(1-\lambda_e)^3\end{cases}$$