1. Optimal Inverse Model

Source: `MTCtrl_FdBack_Pos2Current_HsVolt.sv`

1.1. Stage 1: Coordinate Transform

1.1.1. Input Preprocessing

$\boldsymbol{M}_{\mathrm{Om}}$, $\boldsymbol{F}_{\mathrm{Om}}$: position and force in measurement coordinates

$\boldsymbol{R}$: 3×3 transformation matrix (measurement ⟶ actuation)

$$\boldsymbol{M}_{\mathrm{Oa}} = \boldsymbol{R} \cdot \boldsymbol{M}_{\mathrm{Om}}$$

$$\boldsymbol{F}_{\mathrm{Oa}} = \boldsymbol{R} \cdot \boldsymbol{F}_{\mathrm{Om}}$$

$$F_x = F_{\mathrm{d},x} \times \frac{10}{\mathrm{FGain}_x}, \quad F_y = F_{\mathrm{d},y} \times \frac{10}{\mathrm{FGain}_y}, \quad F_z = F_{\mathrm{d},z} \times \frac{10}{\mathrm{FGain}_z}$$

1.1.2. Spherical Coordinates

F_x, F_y, F_z: desired force (in actuation coordinates, `oFDsir_Oa_AftScale`)

$$\theta = \arctan\left(\frac{F_y}{F_x}\right)$$

$$\varphi = \arctan\left(\frac{F_z}{\sqrt{F_x^2 + F_y^2}}\right)$$

$$|F| = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

1.2. Stage 2: Octant Detection

M_x, M_y, M_z: measured position (in actuation coordinates)

M_z	M_y	M_x	OCT
+	+	+	1
+	+	−	2
+	−	−	3
+	−	+	4
−	+	+	5
−	+	−	6
−	−	−	7
−	−	+	8

1.3. Stage 3: Address Calculation

1.3.1. Angle Correction by Octant

θ' correction:

OCT	θ'
1, 5	$\pi + \theta$
4, 8	$\pi - \theta$
2, 6 ($\theta \geq 0$)	$2\pi - \theta$
2, 6 ($\theta < 0$)	$\lvert\theta\rvert + \pi$

3, 7 ($\theta \geq 0$)	$\theta + \pi$
3, 7 ($\theta < 0$)	$2\pi + \theta$

φ' correction:
- OCT 1-4: $\varphi' = \frac{\pi}{2} + \varphi$
- OCT 5-8: $\varphi' = \frac{\pi}{2} - \varphi$

1.3.2. Grid Index Calculation

Δ_θ, Δ_φ: angle step size

$$\text{intThe} = \left\lfloor \frac{\theta'}{\Delta_\theta} - 0.5 \right\rfloor \quad // \ \theta \text{ grid index}$$

$$\text{intPhi} = \left\lfloor \frac{\varphi'}{\Delta_\varphi} - 0.5 \right\rfloor \quad // \ \varphi \text{ grid index}$$

$$\text{addr}_1 = \text{intThe} \times 1860 + \text{intPhi} \times 60$$

$$\text{addr}_2 = \text{addr}_1 + 1860$$

$$\text{fracThe} = \frac{\theta'}{\Delta_\theta} - \text{intThe} \quad // \ \theta \text{ interpolation ratio}$$

$$\text{fracPhi} = \frac{\varphi'}{\Delta_\varphi} - \text{intPhi} \quad // \ \varphi \text{ interpolation ratio}$$

1.4. Stage 4: Polynomial Expansion

M_x, M_y, M_z: measured position (in actuation coordinates, `oMoa_um`)

R_{norm}: normalization radius

$C[k]$: coefficients from SDRAM

$$P_0 = 1, \quad P_1 = \frac{|M_x|}{R_{\text{norm}}}, \quad P_2 = \frac{|M_y|}{R_{\text{norm}}}, \quad P_3 = \frac{|M_z|}{R_{\text{norm}}}$$

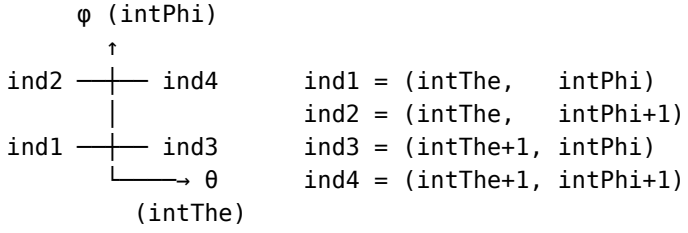
$$P_4 = P_1 \cdot P_2, \quad P_5 = P_1 \cdot P_3, \quad P_6 = P_2 \cdot P_3$$

$$P_7 = P_1^2, \quad P_8 = P_2^2, \quad P_9 = P_3^2$$

$$I_n = \sum_{k=0}^{9} C_n[k] \cdot P_k \quad // \ \text{n} = 1\text{~}6$$

1.5. Stage 5: Bilinear Interpolation

4 lookup points from SDRAM:



I_{ind1}, I_{ind2}, I_{ind3}, I_{ind4}: current values from Stage 4

$$I_{\text{mid13}} = I_{\text{ind1}} + \text{fracThe} \cdot (I_{\text{ind3}} - I_{\text{ind1}})$$

$$I_{\text{mid24}} = I_{\text{ind2}} + \text{fracThe} \cdot (I_{\text{ind4}} - I_{\text{ind2}})$$

$$I_{\text{interp}} = I_{\text{mid13}} + \text{fracPhi} \cdot (I_{\text{mid24}} - I_{\text{mid13}})$$

$$I_{\text{final}} = I_{\text{interp}} \cdot \sqrt{|F|}$$

1.6. Stage 6: Octant Current Remap

OCT	[0]	[1]	[2]	[3]	[4]	[5]
1	I_1	I_2	I_3	I_4	I_5	I_6
2	I_2	I_1	I_3	I_4	I_5	I_6
3	I_2	I_1	I_4	I_3	I_5	I_6
4	I_1	I_2	I_4	I_3	I_5	I_6
5	I_1	I_2	I_3	I_4	I_6	I_5
6	I_2	I_1	I_3	I_4	I_6	I_5
7	I_2	I_1	I_4	I_3	I_6	I_5
8	I_1	I_2	I_4	I_3	I_6	I_5

1.7. Stage 7: KI Matrix Transform

$$I_{\text{out}} = \boldsymbol{K}_{6\times6} \cdot I_{\text{theo}}$$

Current Mode (`iHsCtrl_bol = 0`): output is `oI_out_n`, proceeds to Stage 8

- `iInvMdl_Meth = 0`: no transform
- `iInvMdl_Meth = 1`: KImeasInv
- `iInvMdl_Meth = 2`: KImdfyInv

Hall Sensor Mode (`iHsCtrl_bol = 1`): output is `oHsVd_VECT6`, skips Stage 8

- KIhallInv

1.7.1. KImeasInv

$\boldsymbol{K}_{\text{ImeasInv}} \times \boldsymbol{K}_{\text{Itheo}}$:

$$\begin{pmatrix} 1.344 & -0.396 & -0.316 & -0.164 & -0.133 & -0.334 \\ -0.283 & 0.802 & -0.030 & -0.231 & -0.246 & -0.011 \\ -0.293 & -0.140 & 1.267 & -0.380 & -0.169 & -0.286 \\ -0.037 & -0.225 & -0.279 & 0.822 & -0.228 & -0.052 \\ 0.006 & -0.245 & -0.064 & -0.231 & 0.825 & -0.291 \\ -0.325 & -0.129 & -0.295 & -0.166 & -0.396 & 1.311 \end{pmatrix}$$

1.7.2. KImdfyInv

$\boldsymbol{K}_{\text{ImdfyInv}} \times \boldsymbol{K}_{\text{Itheo}}$:

$$\begin{pmatrix} 1.090 & -0.328 & -0.293 & -0.096 & -0.065 & -0.308 \\ -0.285 & 0.833 & -0.074 & -0.200 & -0.216 & -0.058 \\ -0.272 & -0.074 & 1.027 & -0.313 & -0.103 & -0.266 \\ -0.081 & -0.196 & -0.282 & 0.851 & -0.199 & -0.093 \\ -0.045 & -0.214 & -0.103 & -0.201 & 0.855 & -0.293 \\ -0.301 & -0.061 & -0.275 & -0.098 & -0.328 & 1.063 \end{pmatrix}$$

1.7.3. KIhallInv

$\hat{\boldsymbol{D}}_H^{-1}$:

$$\mathrm{diag}(2.252, 2.749, 2.246, 2.294, 2.832, 2.295)$$

1.8. Stage 8: Current to DAC

QVgain: current to voltage gain

$\mathrm{DAScale}_{\mathrm{perV}}$: voltage to DAC scale

$\mathrm{DAScale}_{\mathrm{0V}}$: DAC zero offset

$$V_{\mathrm{out}} = I_{\mathrm{out}} \cdot \mathrm{QVgain}$$

$$V_{\mathrm{scale}} = V_{\mathrm{out}} \cdot \mathrm{DAScale}_{\mathrm{perV}}$$

$$\mathrm{DAC}_{\mathrm{16b}} = \mathrm{FP2INT}(V_{\mathrm{scale}}) + \mathrm{DAScale}_{\mathrm{0V}}$$