Analysis of "A Linear Programming Methodology for the Optimization of Electric Power-Generation Schemes"

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Introduction:

The paper we investigated is interested specifically in finding appropriate electric power-generation schemes. With today's increasing industrial nature, the demand for steady electrical supplies have increased drastically. To ensure a continuous, dedicated electric supply, it's important to have an in-house generation scheme. However, these costs can be expensive and without exploring all the variables, it's possible to spend too much on a generator scheme. In an effort to prevent this from happening, this paper considers the following:

1. Optimum number of power generation units
2. The reliability of each unit
3. The cost invested to build and maintain the power generation units
4. Fuel cost to operate the power generation units

This paper presents a mathematical model consisting of linear and integer programming in order to minimize the cost of an independent power generation array. This array is expected to consistently be able to fulfill an average load demand.

Motivation:

The paper presents a potential solution to a problem which is dealt with in many major industries, such as manufacturing, oil and gas, etc. The loss of power for even a day could cost hundreds of thousands or even millions of dollars for an industrial company and put their future at risk, so it's essential that a power generation grid is chosen carefully. It's important to find a grid that can survive unexpected failures to prevent this huge detriment to a company. Choosing the right generator array can be difficult given the number of different considerations, so creating a mathematical model can help to minimize struggle and human error. Additionally, if a model is created, modifying it for future use is relatively simple and can continue to work to save the company time and money.

Problem Formulation:

SETS:

The sets of interest in this problem formulation are as follows:

TYPE: the different generator types available for each test case, associated with Nc in the problem

NUMBER: the total number of generation units for each type installed, associated with Nu in this problem

DECISION VARIABLES:

The decision variables are associated with two main factors: the **number** of generators installed in an array and the **type** of generators installed in an array:

X_i: binary decision variable associated with the installation of the generator array with capacity index *i*, $\{\frac{1\ if\ installed}{0\ if\ not\ installed}\}$

X_{ij}: binary decision variable associated with the installation of unit number *j* with capacity index *I*, $\{\frac{1\ if\ installed}{0\ if\ not\ installed}\}$

Two additional variables, the power generated by each unit and the average unserved power, should be included as well:

P_{ij}: real power generated by each unit [MW]

USP_i: average unserved power [MW]

The parameters are discussed throughout the problem formulation, wherever relevant, and will be labeled.

OBJECTIVE FUNCTION:

Minimize cost=(Investment Costs+Fuel Costs+Operation and Maintenance Costs+Unavailability Costs)

The four categories of the objective function are written in the paper as:

Investment Costs:

As written in the paper, the investment costs look like this:

$$\sum_{j \in NUMBER} Cap_i X_{ij} - Dmax X_i = Reserve_i\ [\forall\ \mathrm{i}\ \in\ \mathrm{TYPE}]$$

Decision variables:

X_i: binary decision variable associated with the installation of the generator array with capacity index *i*, $\{\frac{1\ if\ installed}{0\ if\ not\ installed}\}$

X_{ij}: binary decision variable associated with the installation of unit number *j* with capacity index *I*, $\{\frac{1\ if\ installed}{0\ if\ not\ installed}\}$

Parameters:

Cap_i: available capacity of the generators [MW]

Dmax: maximum system load [MW]

Brief explanation: This is calculating reserve power, the amount of power in the system currently produced by operating generators minus the maximum load on the system at any given time, **but does not belong as an investment cost!**

Instead, the investment costs should look like this:

$$\sum_{i \in TYPE} \sum_{j \in NUMBER} C_{ij} X_{ij}$$

Decision variables:

X_{ij}: binary decision variable associated with the installation of unit number *j* with capacity index *I*, $\{\frac{1\ if\ installed}{0\ if\ not\ installed}\}$

Parameters:

C_{ij}: investment cost coefficient [$/year]

Brief explanation: The cost of purchasing the generators for the array chosen, essentially multiplies the investment cost coefficient for an individual generator times the total number of generators in the system. This actually provides the investment cost coefficients as provided in the data tables and makes logical sense as a cost.

Fuel Costs:

$$\sum_{i \in TYPE} \sum_{j \in NUMBER} Cc_i P_{ij}$$

Decision variables:

P_{ij}: real power generated by each unit [MW]

Parameters:

Cc_i: fuel cost coefficient [$/MW/year]

Brief explanation: The cost of fuel needed for the generator to produce the amount of power desired, calculated by multiplying the fuel cost coefficient by the power produced by each generating unit. Note, the original parameter had units of dollars per kW per year, but was modified here for units to match. The units are discussed later as well in our model formulation.

Operation and Maintenance Costs:

$$\sum_{i \in TYPE} \sum_{j \in NUMBER} Cf_i X_{ij} + Cv_i P_{ij}$$

Decision variables:

X_{ij}: binary decision variable associated with the installation of unit number *j* with capacity index *I*, $\{\frac{1\ if\ installed}{0\ if\ not\ installed}\}$

P_{ij}: real power generated by each unit [MW]

Parameters:

Cf_i: operation and maintenance fixed cost coefficient [$/year]

Cv_i: operation and maintenance variable cost coefficient [$/MW/year]

Brief explanation: These are the fixed and variable costs of operation and maintenance. Fixed costs are based on the number of units, variable costs is based on the total power generated by the units. Note, the original variable cost parameter had units of dollars per kW per year, but was modified here for units to match. The units are discussed later as well in our model formulation.

Unavailability Costs:

$$\sum_{i \in TYPE} Cun_i USP_i$$

USP_i: average unserved power [MW] defined by the following equation:

$$USP_i = (Cap_i - Dmax)X_i - \sum_{j \in NUMBER} Cap_i X_{ij} \; [\forall \text{ i } \in \text{ TYPE}]$$

Decision variables:

X_i: binary decision variable associated with the installation of the generator array with capacity index *i*, $\{\frac{1 \; if \; installed}{0 \; if \; not \; installed}\}$

X_{ij}: binary decision variable associated with the installation of unit number *j* with capacity index *I*, $\{\frac{1 \; if \; installed}{0 \; if \; not \; installed}\}$

Parameters:

Cap_i: available capacity of the generators [MW]

Dmax: maximum system load [MW]

Cun_i: unavailability cost coefficient [$/MW/year]

Brief explanation: These costs are associated with the unavailability of units, assumed proportional to the average unserved power. The average unserved power is found by calculating the remaining power in each generator array less the capacity of each generator unit. Essentially, this component details what would happen if a generator were to fail and the energy loss that would take place in that case.

CONSTRAINTS

Power Balance at Node Zero:

$$\sum_{i \in TYPE} P_{i0} = Dav$$

Decision variables:

P_{i0}: power at the ‘zero node’, which comprises all the power in the system of generators [MW]

Parameters:

Dav: average system load [MW]

Brief explanation: The total power of all the different generation arrays must equal the average system load.

Concrete: For test case 1,

$P_{GT35,0}$+ $P_{GT10,0}$+$P_{PG5371,0}$+$P_{PG6551,0}$=14.1 MW

Power Balance in Fictitious Nodes:

$$\sum_{j \in NUMBER} P_{ij} = P_{i0}, [\forall \text{ i } \in \text{ TYPE}]$$

Decision variables:

P_{ij}: real power generated by each unit [MW]

P_{i0}: power at the ‘zero node’, which comprises all the power in the system of generators [MW]

Brief explanation: The power generated by each unit in the array should be equal to the total power for the array

Concrete: Nu=5

$P_{GT35,1}$+ $P_{GT35,2}$+ $P_{GT35,3}$+ $P_{GT35,4}$+ $P_{GT35,5}$=$P_{GT35,0}$

Maximum Power for Each Group of Generators:

$$P_{i0} \leq nCap_i X_i \text{ [}\forall \text{ i} \in \text{TYPE, } \forall \text{ j} \in \text{NUMBER]}$$

Decision variables:

X_i: binary decision variable associated with the installation of the generator array with capacity index *i*, $\{\frac{1\ if\ installed}{0\ if\ not\ installed}\}$

P_{i0}: power at the ‘zero node’, which comprises all the power in the system of generators [MW]

Parameters:

n: Maximum number of units that can be selected

Cap_i: available capacity of the generators [MW]

Brief explanation: the total power of one chosen generator grouping cannot exceed the total capacity of the grouping

Concrete: $P_{GT35,0} \leq n*13.2\ MW*X_{GT35}$

Minimum Capacity of the Generator Array

$$\sum_{i \in TYPE} \sum_{j \in NUMBER} Cap_i X_{ij} \geq Dmax$$

Decision variables:

X_{ij}: binary decision variable associated with the installation of unit number *j* with capacity index *I*, $\{\frac{1\ if\ installed}{0\ if\ not\ installed}\}$

Parameters:

Cap_i: available capacity of the generators [MW]

Dmax: maximum system load [MW]

Brief explanation: the total capacity of one generator array must be able to exceed maximum demand

Concrete: Nu=4 for every generator array, for test case 1,

$19.2\ MW*X_{GT10,1} + 19.2\ MW*X_{GT10,2} + 19.2\ MW*X_{GT10,3} + 19.2\ MW*X_{GT10,4} + 13.2\ MW*X_{GT35,1} + 13.2\ MW*X_{GT35,2} + 13.2\ MW*X_{GT35,3} + 13.2\ MW*X_{GT35,4} + 20.5\ MW*X_{PG5371,1} + 20.5\ MW*X_{PG5371,2} + 20.5\ MW*X_{PG5371,3} + 20.5\ MW*X_{PG5371,4} + 30.5\ MW*X_{PG6551,1} + 30.5\ MW*X_{PG6551,2} + 30.5\ MW*X_{PG6551,3} + 30.5\ MW*X_{PG6551,4} \geq 22.5\ MW$

Limits on the Power Generated by Each Machine:

$$P_{ij} \leq Cap_i X_{ij} \ [\forall\ i \in \text{TYPE},\ \forall\ j \in \text{NUMBER}]$$

Decision variables:

X_{ij}: binary decision variable associated with the installation of unit number *j* with capacity index *I*, $\{\frac{1\ if\ installed}{0\ if\ not\ installed}\}$

P_{ij}: real power generated by each unit [MW]

Parameters:

Cap_i: available capacity of the generators [MW]

Brief explanation: The real power generated by each unit must be less than or equal to the capacity of the unit as the power cannot exceed what the generator is capable of producing

Concrete:

$P_{GT10,1} \leq 19.2$ MW*$X_{GT10,1}$

"Radial" Constraint at Node Zero:

$$\sum_{i \in TYPE} X_i = 1$$

Decision variables:

X_i: binary decision variable associated with the installation of the generator array with capacity index *i*, $\{\frac{1\ if\ installed}{0\ if\ not\ installed}\}$

Brief explanation: The model is constrained to select only one type of generator for its array.

Concrete: For test case 1,

$X_{GT35}+X_{GT10}+X_{PG5371}+X_{PG6551}=1$

Strengths and Weaknesses:

One of the strengths of this model was the ability to include unavailability costs in their calculations. These costs can be very detrimental if not properly addressed and, without them, the model doesn't have redundancies built into the system to prevent a potentially catastrophic outage caused by having only a single generator unit. The decision variables reflect this as, in the original model, all decision variables are binary and help to reflect these states. It also restricts the final generator array to one type of generator, which is easier for the company to utilize because it cuts down on variations in materials, operating differences, offers a greater chance of a bulk discount, and helps synchronize with the grid.

The major weaknesses lie more with the paper than the model itself, but there are places where these two concepts coincide:

1. The paper itself does not properly reflect the elements of the model. The objective function is split into four parts, one of which is investment costs. This should be related to the investment costs of the generators chosen, but instead reflects power reserve, explained later in unavailability costs.
2. The recorded variables are not well explained; Dmax, which should be the maximum demand on the generator system, is indexed with 'i' without a need.
3. The data tables and the results are not consistent with the information presented. For instance, the fuel costs are given in units of $/kW/year while the majority of the paper uses MW, but if these units were converted, the calculation for fuel cost would no longer match the one given in results. An assumption is given for variable operating costs but the values used do not match those given in the assumption and no interpretation is given.
4. Two test cases are presented, but the second test case has improperly labeled data tables, increasing the confusion.

5. While a new generator array scheme is chosen in test case 2, the results of the cost breakdown for the two test cases are the same, which does not make sense considering the resultant cost reported is different and the data given does not make sense with the values reported.

6. The unavailability costs are arbitrarily set to zero, which might make sense if this model were an attempt to demonstrate an actual cost perspective to a customer but does not make sense for minimizing costs, as a different generator scheme would be predicted if this portion is neglected and if the costs of unavailability are arbitrarily made very high, the generator scheme in the first test case also changes. It makes sense to report or try to utilize this cost instead.

It's hard to know exactly where the model weaknesses lie in the face of information reporting that is inconsistent or incorrect. The main weakness in the model is the confusion caused by misinformation. An additional weakness is the confusion in the indices. Instead of using an integer decision variable to find the number of units in each generator array, the units are treated as separate elements. To attempt to improve the model, one thing we did was modify the decision variable Xij, previously a binary variable associated with 'installation of unit number j with capacity index i' to instead be an integer decision variable that calculates the number of units present. Doing this cut down on indices and confusion in the paper as a whole, very useful when the paper itself is hard to digest. The model also observes only the first purchase year for the time frame and so generates costs associated with this first year but does not take into account other years down the line, which might modify the choice of generators. After all, a higher initial investment might mean lower long-term fuel costs or lower operation and maintenance costs.

Strategies used in the model:

The model in theory isn't too complex, but there are a few additions the authors made to help generate the best results. The branch and bound method is used in the model to find the optimal generator array to minimize costs. Branch and bound approach is based on the principle that the optimal solution from a relaxed linear program can be partitioned to find the best integer solution from the available possibilities surrounding a non-integer value. The approach continues until the best feasible solution with integer values for all relevant decision variables is found. Essentially, a linear program is first used to find an optimal solution and then different integer values are tested for each decision variable until a feasible solution with proper integer values is reached. This paper showed convergence in 27 iterations and 69 iterations for test cases 1 and 2, respectively.

Implementing our model:

Our new problem formulation looks like this:

Set:

GEN: All available generator models

Parameters:

Cap: Capacity of each generator model i, [MW], $i \in$ GEN

Cc: Fuel cost coefficient for each generator model i, [$/MW/year], $i \in$ GEN
Cf: Fixed cost for operation and maintenance for each generator model i, [$/MW/year], $i \in$ GEN
C: Investment cost coefficient for each generator model i, [$/year], $i \in$ GEN
Cv: Variable cost coefficient for each generator model i, [$/MW/year], $i \in$ GEN
Dav: Average load demand [MW]
Dmax: Maximum load demand [MW]
n: Maximum number of generators available per array type
Cun: Unavailability cost coefficient for each generator model i, [$/MW/year], $i \in$ GEN

Decision Variables:

x$_i$: Number of generator i in the array, $i \in$ GEN
y$_i$: Binary variable, installation of the array of generator type i, $i \in$ GEN, $\{\frac{1\ if\ installed}{0\ if\ not\ installed}\}$
P$_i$: Total power for each array of generator type i, $i \in$ GEN
USP$_i$: Average unserved power for array of generator type i, $i \in$ GEN

Objective Function:

$$\sum_{i \in GEN} x_i C_i + \mathrm{Cc}_i P_i + \mathrm{Cf}_i x_i + \mathrm{Cv}_i P_i + \mathrm{Cun}_i USP_i = MinCost$$

Constraints:

Non-negativity:

$$x_i, P_i, USP_i \geq 0$$

Meeting the average demand:

$$\sum_{i \in GEN} P_i = Dav$$

Limit on power generation:

$$P_i \leq \mathrm{Cap}_i x_i\ [\forall\ i \in \mathrm{GEN}]$$

Minimum capacity for the generator array:

$$\sum_{i \in GEN} \mathrm{Cap}_i x_i \geq Dmax$$

Connecting generators to generator array:

$$x_i \leq \mathrm{y}_i \mathrm{n}\ [\forall\ i \in \mathrm{GEN}]$$

Only one type of generator array can be chosen:

$$\sum_{i \in GEN} y_i = 1$$

Defining USP:

$$(Dmax + Cap_i)y_i - \mathrm{Cap}_i x_i \leq USP_i \; [\forall\, i \in \mathrm{GEN}]$$

Our model uses x_i, a decision variable associated with the number of generators in the array. Unlike the previous model, which used a binary decision variable with two indices (one for generator type and one for the number of generators) to find this value, this decision variable is an integer value instead and takes out the second index, simplifying the model.

Model Scalability:

The model scales fairly well within the small number of variables that would be required. We multiplied the number of generators by ten to see how the model reacted but noticed no significant change in the run speed. While we could continue to add new generator possibilities, the model is built around small generator arrays with limited options, and the number of decision variables scales linearly with the number of different generator types. Eventually, enough options may cause the model to take more time, but that isn't likely to happen until a large number of different types of generators is introduced.

Data Corrections:

The data tables in the paper were used to populate the data file, though they do not match exactly. Along with fixing the formulation, the units were modified to better reflect the information presented. Firstly, though it no longer matches with the predicted fuel costs, the fuel costs were put into units of $/MW/year, creating a more accurate prediction. Secondly, the variable operations and maintenance costs, previously listed as both 3 $/kW/year and 26.3 $/MW/year, were set to 3000 $/MW/year as this number corresponds to a more reasonable variable cost value at 0.34 $/MWh compared to 0.003 $/MWh (see example variable costs in different industries in 2016, [US DOE Report]. These new values are reflected in the data tables in our code. We also kept unavailability costs in the model as they help to ensure the model chooses a scheme that best takes into account the losses associated with a generator failing unexpectedly. One of the tables for the second test case specifying unavailability costs is missing, so the costs for the generator that is not common for both test cases was checked using the lowest and highest unavailability cost to see if it made a difference to the model.

Results:

The results from our model for both test cases are as follows:

Test Case 1

```
The average power needed will be 14.10 MW for the year.
The generators needed will be 0 GT35 generators, 0 GT10 generators, 2
PG5371 generators, and  0 PG6551 generators.
If unavailability is included, the total cost will be $ 9058470.00 for
the project generator array scheme. The total cost is broken into
investment cost, $ 5586000.00 , fuel cost, $ 2814360.00 , operations
and maintenance cost, $ 83300.00 , and unavailability cost, $
574810.00 .
```

Test Case 2

```
The average power needed will be 9.30 MW for the year.
The generators needed will be 2 PGT10 generators, 0 GT35 generators, 0
GT10 generators, and 0 PG5371 generators.
If unavailability is included, the total cost will be $ 5779631.00 for
the project generator array scheme. The total cost is broken into
investment cost, $ 3395000.00 , fuel cost, $ 1710270.00 , operations
and maintenance cost, $ 44100.00 , and unavailability cost, $
630261.00 .
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```
The average power needed will be 9.30 MW for the year.
The generators needed will be 2 PGT10 generators, 0 GT35 generators, 0
GT10 generators, and 0 PG5371 generators.
If unavailability is included, the total cost will be $ 6040325.50 for
the project generator array scheme. The total cost is broken into
investment cost, $ 3395000.00 , fuel cost, $ 1710270.00 , operations
and maintenance cost, $ 44100.00 , and unavailability cost, $
890955.50 .
```

The values above are printed statements from the pyomo formulation and can be interpreted as follows:

Test case 1 has a power requirement of 14.10 MW and will require 2 PG5371 generators, yielding a total cost of $ 9,058,470.00. This cost is broken up into the four segments specified in the objective function from the original problem formulation, Investment Costs ($ 5,586,000.00), Fuel Costs ($ 2,814,360.00), Operations and Maintenance Costs ($ 83,300.00), and Unavailability Costs ($ 574,810.00).

Test case 2 has a power requirement of 9.30 MW and will require 2 PGT10 generators, yielding a total cost of $ 5,779,631.00 if unavailability costs are set to the minimum value or $ 6040325.50 if unavailability costs are set to the maximum value. This cost is broken up into the four segments specified in the objective function from the original problem formulation, Investment Costs ($ 3,395,000.00), Fuel Costs ($ 1,710,270.00), Operations and Maintenance Costs ($ 44,100.00), and Unavailability Costs ($ 630,261.00 for the minimum value and $890955.50 for the maximum value).

Something important to note is that both of these resultant cases and the cases from the original paper only observe the first year period of time for cost. As investment cost is associated with initial purchases, it would not appear in subsequent years, but maintenance costs and fuel costs are expected to rise as the generators are kept in service. The generator prediction is valid over a one year period, but a larger period of observation might yield more appreciably distinct suggestions as to what generators should be purchased.

Verification and Validation:

To ensure the model was working as desired, we did a few key things. Firstly, we reported the major decision variables: namely, the generator type and unit quantity chosen and the average power used. We manually calculated what the information given to us was telling us and ensured that the manual calculations matched with the information we were seeing by calculating the different costs that should be associated with the generator array chosen and ensuring they matched the results we were given. We modified some of the data temporarily to help test different states and ensure the model still outputted information as desired and continued to test the results by manual calculations.

To validate our results with the authors of the paper, we tried to reach the same optimal solution they reached as best as we could. The authors of the paper matched their first test case, which we were able to replicate, with an older reference. The second test case is not validated, and while our model does predict the same generator type, it does not predict the same number (their model predicts three whereas ours predicts two). From the information provided, we cannot understand the reason for the discrepancy, though it is worth noting the second model does not report unavailability cost coefficients and hours (the table is labeled as such but does not provide the stated information), so we attached both the lowest and highest provided unavailability cost to the generator that did not appear in test case 1 but noticed no change in predicted generator array. The generators that did appear in test case 1 were kept consistent, as all the other information listed with the models was kept consistent except for mild alterations to capacity and, correspondingly, fixed maintenance costs. If unavailability cost coefficients for test case 2 are set to higher values (increasing by an order of magnitude), then our model will predict the same generator number and model as the paper, but without the information we can't say for sure where the discrepancy lies. Additionally, if the unavailability cost coefficients are set to high values in test case 1 in an effort to set the unavailability cost to zero for that model, the model predicts a different generator array, which means we can't confirm this is what they did to modify the values for test case 1 or 2.

Conclusion:

We were able to identify some flaws in the previous formulation and suggest a less complex but more lucid model. The authors could improve their model by modifying how they introduce the decision variables and elaborate on how the branch and bound method was applied. There are several places in the paper itself where clarification or corrections are needed, and without these this is a very difficult model to correctly interpret and yield the same result. A couple of big assumptions made in this model include only one generator can be out of service at a time and unavailability costs are set to zero. While working with smaller generator arrays might help to ensure that there's a lower chance of multiple failures, the chance still exists. It makes the model easier to formulate with this assumption, though, as it drastically limits the different probabilities. The arbitrary setting of the unavailability costs to zero modifies the predicted generators. If unavailability cost is neglected entirely, different generator arrays are predicted that could prove serious detrimental in the long term. Ignoring this cost in the model may not yield the same predictions. If this value was set to zero later after the formulation, which might be true, this is not well explained in the paper. If the value is artificially set to zero by making unavailability costs very high, this, too, changes the predictions. Finally the model only observes a single year, specifically the first purchase year, for these costs. This is not the most valuable way to predict long-term need. Investment costs are only necessary for an initial purchase and the costs of fuel and operation and maintenance will both vary over time. The model would be improved by taking into account a longer period of observation. The paper could also be extended by incorporating selection criteria of the protection schemes and downstream equipment such as transformer, relays, and breakers associated with the generator array selection.

This model can be applied when exploring other situations as well. One example would be emergency generators for smaller towns. Especially in towns in the north where heat can be a serious concern, the lack of power might mean death. Emergency generator units could be purchased and implemented for a town in trouble, and this model could help predict what generators would be cost-effective. There would be changes in the costs associated with this, but it wouldn't be difficult to tailor to this situation. Other industries with needs for small, independent power generation schemes could benefit from a similar model as well. The model itself, at a generic level, represents a selection criteria model and that could be applied to almost any industry related to service.

References:

Paper:
H. M. Khodr, J. F. Gomez, L. Barnique, J. H. Vivas, P. Paiva, J. M. Yusta, and A. J. Urdaneta, "A linear programming methodology for the optimization of electric power-generation schemes," IEEE Transactions on Power Systems, vol. 17, pp. 864-869, 2002.
https://ieeexplore.ieee.org/abstract/document/1033737

Information on Branch and Bound Method:
B. W. Taylor, Module C. Integer Programming: The Branch and Bound Method, 10th ed., 2006.
http://web.tecnico.ulisboa.pt/mcasquilho/compute/_linpro/TaylorB_module_c.pdf

US DOE Cost Estimates for Utility Scale Electricity Generating Plants:
Capital Cost Estimates for Utility Scale Electricity Generating Plants. US Department of Energy, 2016.
https://www.eia.gov/analysis/studies/powerplants/capitalcost/pdf/capcost_assumption.pdf