Assignment - by Kevin George

Part1

Abstract Formulation: -

SET:-

Job : Set of Job {1,2,3,4,5)
Machine: Set of Machine {1,2,3,4,5)

PARAMETER:-

$Setup_i$: setup time for machine i ∀ i ∈ Machine
$Time_{ij}$: operating time for machine i to do job j ∀ i ∈ Machine , j ∈ Job
P_{ij}: presence of machine i for job j ∀ i ∈ Machine , j ∈ Job

DECISION VARIABLE:-

X_{ij}: whether machine i used for job j ->1 if used, 0 if not ∀ i ∈ Machine , j ∈ Job
Y_i: whether machine i is used -> 1 if used, 0 if not ∀ i ∈ Machine
K: binary 1 or 0

OBJECTIVE FUNCTION:-

Minimize Time = $\sum_{\text{i} \in \text{Machine}} Setup_i * Y_i + \sum_{\text{i} \in \text{Machine,j} \in \text{Job}} Time_{ij} * X_{ij} * P_{ij}$

CONSTRAINTS:-

$X_{ij} = 0\ or\ 1$ ∀ i ∈ Machine , j ∈ Job {*Binary Constraint*}
$Y_i = 0\ or\ 1$ ∀ i ∈ Machine {*Binary Constraint*}
$K = 0\ or\ 1$ {*Binary Constraint*}
$\sum_{\text{i} \in \text{Machine}} X_{ij} * P_{ij}$ = 1 ∀ j ∈ Job {*One job done by one machine constraint* }
$\sum_{\text{j} \in \text{Job}} X_{ij} * P_{ij}$ ≤ 3*Y_i ∀ i ∈ Machine { *equivalent to which all machine is selected - if machine i operates the* Y_i *is one* }
X_{11} ≤ 1-K {*if machine 1 is used for job1 then it is used for job3 -part2*}
1-X_{13} ≤ K {*if machine 1 is used for job1 then it is used for job3 -part1*}

Concrete Formulation: -

DECISION VARIABLE: -

X_{11}- binary variable(1 or 0)- whether machine 1 used for job 1
X_{12}- binary variable(1 or 0)- whether machine 1 used for job 2
X_{13}- binary variable(1 or 0)- whether machine 1 used for job 3
X_{22}- binary variable(1 or 0)- whether machine 2 used for job 2
X_{23}- binary variable(1 or 0)- whether machine 2 used for job 3
X_{31}- binary variable(1 or 0)- whether machine 3 used for job 1
X_{34}- binary variable(1 or 0)- whether machine 3 used for job 4
X_{41}- binary variable(1 or 0)- whether machine 4 used for job 1
X_{43}- binary variable(1 or 0)- whether machine 4 used for job 3
X_{45}- binary variable(1 or 0)- whether machine 4 used for job 5
X_{52}- binary variable(1 or 0)- whether machine 5 used for job 2
X_{54}- binary variable(1 or 0)- whether machine 5 used for job 4
Y_1- binary variable(1 or 0)- whether machine 1 is used
Y_2- binary variable(1 or 0)- whether machine 2 is used
Y_3- binary variable(1 or 0)- whether machine 3 is used
Y_4- binary variable(1 or 0)- whether machine 4 is used
Y_5- binary variable(1 or 0)- whether machine 5 is used
K- binary variable(1 or 0)

OBJECTIVE FUNCTION:-

Minimize Time =30*Y_1 +40*Y_2 +50*Y_3 $+ 60 * Y_4$+20*Y_5 + 42*$X_{11} + 70 * X_{12} + 93 * X_{13} + 85 * X_{22} + 45 * X_{23} + 58 * X_{31} + 37 * X_{34} + 58 * X_{41} + 55 * X_{43} + 38 * X_{45} + 60 * X_{52} + 54 * X_{54}$

CONSTRAINTS:-

$X_{11} + X_{31} + X_{41} = 1$ { *job 1 done by one machine constraint*.}
$X_{12} + X_{22} + X_{52} = 1$ { *job 2 done by one machine constraint*.}
$X_{13} + X_{23} + X_{43} = 1$ { *job 3 done by one machine constraint*.}
$X_{34} + X_{54} = 1$ { *job 4 done by one machine constraint*.}
$X_{45} = 1$ { *job 5 done by one machine constraint*.}
$X_{11} + X_{12} + X_{13} \leq 3 * Y_1$ {*if machine 1 operates the* Y_1 *is one*}
$X_{22} + X_{23} \leq 3 * Y_2$ { *if machine 2 operates the* Y_2 *is one* }
$X_{31} + X_{34} \leq 3 * Y_3$ { *if machine 3 operates the* Y_3 *is one*.}
$X_{41} + X_{43} + X_{45} \leq 3 * Y_4$ { *if machine 4 operates the* Y_4 *is one*.}
$X_{52} + X_{54} \leq 3 * Y_5$ { *if machine 5 operates the* Y_5 *is one* }
$X_{11} \leq 1-K$ {*if machine 1 is used for job1 then it is used for job3 -part2*}
1-$X_{13} \leq$ K {*if machine 1 is used for job1 then it is used for job3 -part1*}
$K = 0 \text{ or } 1$ {*Binary Constraint*}
$Y_1, Y_2, Y_3, Y_4, Y_5 = 0 \text{ or } 1$ {*Binary Constraint*}
$X_{11}, X_{12}, X_{13}, X_{22}, X_{23}, X_{31}, X_{34}, X_{41}, X_{43}, X_{45}, X_{52}, X_{54} = 0 \text{ or } 1$ {*Binary Constraint*}

OPTIMAL SOLUTION & INTERPRETATION.

$Y_1, Y_2 Y_3$=0
Y_4=1, Y_5=1
$X_{41}, X_{43}, X_{45}, X_{52}, X_{54}$= 1
$X_{11}, X_{12}, X_{13}, X_{22}, X_{23}, X_{31}, X_{34}$ =0
K=1
Objective function=345

1) For the problem optimal solution says that only machine 4 and 5 is only on and rest are off. $(from\ the\ Y_i\ values\ we\ got\ 1\ if\ "on"\ 0\ if\ "off")$
2) Also the job 1 ,job 3 and job 5 is done by machine 4 . And Job 2 and Job 4 is done by machine 5.
3) We got the least time for operating and setting up of machine . The value 345 is the summation of least operating time for each job and setting up of each machine which is operating

Part2

y_1 =0
y_2 =0
y_3 =1
z_1=0
z_2 =0
z_3 = 0.59
z_4 = 0.41

Part2

$$c(p) = \begin{cases} 0.25p & (0 \le p \le 500) \\ 0.20p + 25 & (500 \le p \le 1000) \\ 0.15p + 75 & (1000 \le p \le 1500) \end{cases}$$

$z_1 \le y_1$

$z_2 \le y_1 + y_2$

$z_3 \le y_2 + y_3$

$z_4 \le y_3$

$y_1 + y_2 + y_3 = 1$

$z_1 + z_2 + z_3 + z_4 = 1$

$x = z_1 \times 0 + z_2 \times 500 + z_3 \times 1000 + z_4 * 1500$

$y_i = 0 \text{ or } 1 \quad \forall i \in 0 \cdots 3$

$z_i \ge 0 \quad \forall i \in 0 \cdots 4$

$\max \text{Profit} = 0.12(x_{11} + x_{21}) + 0.14(x_{12} + x_{22}) - z_1 \times 0 - z_2(125) - z_3(225) - z_4(300)$

If $x = 1205$

$\therefore y_3 = 1 \Rightarrow y_1, y_2 = 0$

$z_1 = 0, z_2 = 0 \therefore z_3 = ?\ z_4 = ?$ where $z_3 \le 1, z_4 \le 1$

$1205 = z_3 \times 1000 + z_4 \times 1500$

$1 = z_3 + z_4$

$\therefore z_3 = 0.59, z_4 = 0.41$

$1205 = z_3 \times 1000 + (1 - z_3) * 1500$

$500 * z_3 = 295$

$z_3 = \frac{295}{500} = 0.59$

$z_4 = 1 - 0.59 = 0.41$

or

$1205 = \overset{(1-z_4)}{\cancel{z_3}} \times 1000 + z_4(1500)$

$205 = 500 z_4$

$\therefore z_4 = 0.41$

~~Not if $y_1 = 1 \Rightarrow y_2 = y_3 = 0$~~

$\therefore \cancel{x \le z_1 + z_2 = 1}$

$\cancel{1205 = 0z_1 + 500z_2}$ $\rightarrow$

Note if $y_1 = 1 \Rightarrow y_2, y_3 = 0$

$z_1 \le 1$

$z_2 \le 1$

$z_1 + z_2 = 1$

$0z_1 + 500z_2 = 1205$

$z_3 = 0, z_4 = 0$

$\rightarrow$ Infeasible Soln $\left[\begin{array}{l} z_2 = 2.41 \\ z_2 \not< 1 \\ z_1 = -1.4 \end{array}\right]$

Note if $y_2 = 1 \Rightarrow y_1, y_3 = 0$

$z_2 \le 1$

$z_3 \le 1$

$z_2 + z_3 = 1$

$z_2 * 500 + z_3 \times 1000 = 1205$

$z_1 = 0, z_4 = 0$

Infeasible $\left[\begin{array}{l} z_2 = -0.41 \\ z_3 = 1.41 \end{array}\right]$

Part3

PROBLEM FORMULATION

DECISION VARIABLE:-

H- Number of Single Home Sold
D- Number of Duplex Sold
Z1- Piecewise function variable1
Z2- Piecewise function variable2
Z3- Piecewise function variable3
Z4 - Piecewise function variable4
Y1 - Piecewise function binary variable1
Y2 - Piecewise function binary variable2
Y3 - Piecewise function binary variable3

OBJECTIVE FUNCTION:-

Maximize Profit =0* Z1+3000000* Z2+7050000* Z3+15800000*Z4+ 140000*H -85000* D-90000* H

CONSTRAINTS:-

D +H <=120 {max unit to be built.}
3* D >= H {*Duplex to house ratio*}
D+1.75*H<=150 {*acres of land constraint*}
Z1<=Y1 {*Piecewise function constraint for selling price of Duplex*}
Z2<=Y1+Y2 { *Piecewise function constraint for selling price of Duplex* }
Z3<=Y2+Y3 { *Piecewise function constraint for selling price of Duplex* }
Z4<= Y3 { *Piecewise function constraint for selling price of Duplex* }
Y1+ Y2+ Y3=1 { *Piecewise function constraint for selling price of Duplex* }
Z1+ Z2+ Z3+ Z4=1 { *Piecewise function constraint for selling price of Duplex* }
0*.Z1+20* Z2+50* Z3+120* Z4 = D { *Piecewise function constraint for selling price of Duplex* }
H ,D > = 0 {*Non Negative Integer Constraint*}
Z1,Z2,Z3,Z4 > = 0 {*Non Negative Real Constraint*}
Y1,Y2,Y3 = 0 or 1 {*Binary Constraint*}

//The commented PYOMO model is attached

OPTIMAL SOLUTION & INTERPRETATION.

H- Number of Single Home Sold = 40
D- Number of Duplex Sold =80
Z1- Piecewise function variable1=0
Z2- Piecewise function variable2=0

Z3- Piecewise function variable3=0.571
Z4 - Piecewise function variable4=0.428
Y1 - Piecewise function binary variable1=0
Y2 - Piecewise function binary variable2=0
Y3 - Piecewise function binary variable3=1

Maximized Profit is $ 6000000.

Hint:- Selling Price(SP) for Duplex (D) equation:-

SP= 150000*D for ($0 \leq D \leq 20$)
SP=135000*D + 300000 for ($20 \leq D \leq 50$)
SP=125000*D + 800000 for ($50 \leq D \leq 120$)