Adaptive multiple-band CFAR detection

link https://drive.google.com/file/d/1umrHB0QxCTjLPm43422dgQXe-toP-uzT/view

TL;DR

- algorithm that generalizes CFAR for detecing wether a subimage has object of interest or pure clutter
- Uses Maximum likelihood Estimator

Summary

- clutters are modeled as Gaussian processes with space-varying Mean. Covariance is assumed to be static or slowly varying
- The latter is not true in practice, if consider the image as a whole. But it can be true if break image into smaller subimages.
- image has N pixels
- image $\mathbf{X} : (m, N)$ matrix, intensity at each N pixel is measured in m signal-plus-noise bands. Usually m<=12.
- clutter model: $\bar{X} = \frac{1}{w^2}[X \circledast W]$, where $W \in \mathbb{R}^{w,w}$ is all-one kernel of which size has to be chosen to minimze 3rd moment.
- $\mathbf{s} = [s(1), ..., s(N)]^T$ array of N-column vectors of known signal pattern
- $\mathbf{b} = [b(1), ..., b(m)]^T$ array of m-column vectors of unknown signal intensities
- residual (removal of clutter): $X_0 = X - \bar{X}$, $s_0 = s - \bar{s}$
- The detection algorithms test hypotesis, on $\begin{array}{l} H_0 : X = X_0 \text{ (clutter only)} \\ H_A : X = X_0 + b s_0^T \text{ (clutter plus signal)} \end{array}$
- Generalzied likelihood-ratio (GLR) test on each subimage q,
  - test statistic $r(q) = \frac{c_q^T A_q^{-1} c_q}{\alpha_q}$
    where, $\mathbf{c}_q = X_0^{(q)} s_0^{(q)} \in \mathbb{R}^m$
    $\mathbf{A}_q = X_0^{(q)T} X_0^{(q)} \in \mathbb{R}^{m*m}$,
    $\alpha_q = s_0^{(q)T} s_0^{(q)}$
  - for each subimage q, if $r_q \geq r_0$ then H_A else H_0. To know r_0 detection threshold, we meeed PDF of r(q) given H_0.
  - "Probability of a False Alarm (PFA)": the probability that r(q) lies within the rejection region, is the CFAR.
  - $PFA = \int_{r_0}^{1} f(r|H_0) dr$
    where $f(r|H_0) = \frac{\Gamma(\frac{n}{2})}{\Gamma(\frac{n-m}{2})\Gamma(\frac{m}{2})}(1-r)^{(n-m-2)/2} r^{(m-2)/2}$
  - normally PFA is predetermined. Closed interval [r_0, 1] is the rejection region under H_0
  - Hyp Test: If r(q) < r_0, we accept H_0 with (1-PFA) certainty, otherwise reject H_0 and accept H_A