Lab report Hilary Term Wk4

Experiment Title: Hubble Law
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Abstract

The aim of this experiment is to find (and visually represent) a relationship between distance and the recessional velocity from two sets of telescope data. The relationship is calculated via linear least squares regression. This relationship is then used to calculate a value for Hubble's constant (H_0).

The first set of data is an older data set with high systematic uncertainty. The calculated value for Hubble's constant serves as a comparison against the more modern and accurate data set.
From the second data set the red-shift from SuperNovas is used to find the recessional velocity. And a distance modulus is used to find the distance values. The resulting Hubble's constant value is a lot more accurate from this data set than the older one.

Data Set 1
Hubble's constant:
Data Set 2
Hubble's constant:
It is observed that the larger distance values in Data Set 2 provide a more accurate H_0 value. By the theory and equations pertaining to Hubble's Law, we can explain:
-A larger distance means systematic errors (from for example the telescope used) have less of an effect on results.

-Over a larger distance the universe is expanding at a greater speed. Ie. there is a larger recessional velocity. Thus as before error is proportionally smaller.

Basic theory & Equations

Hubble's Law:

$v=H_0d$

Where v is recessional velocity, H_0 is Hubble's constant and d is the distance.

The recessional velocity is calculated using redshift from SuperNovas, as the stretch of light wavelengths is caused by the expansion of space.

$$v = c\frac{(z+1)^2-1}{(z+1)^2+1}$$

The redshift is relative meaning the relationship is not strictly linear but for small values of red shift we can say that:

$$v = cz$$

Where $z = \frac{\Delta\lambda}{\lambda}$.

In Data Set 2 the distances are expressed in a distance modulus, as such:

$$d = 10^{(\mu/5)+1}$$

Lastly the relationship between regressional velocity (v) and distance (d) is analysed using a linear least squares fit.

The Goodness of Fit: $\sum_{n=1}^{N}(y_i - y_i^{model})^2$

This is used to measure the accuracy of the best fit line used. This function can be optimised to find the line of best fit or linear regression.

Goodness of fit rewritten: $\sum_{n=1}^{N}(y_i - mx_i - b)^2$

$$\sum_{n=1}^{N}(y_i^{\,2} + m^2x_i^{\,2} + b^2 - 2mx_iy_i + 2bmx_i - 2by_i)$$

This expression can be partially derived to find the y-intercept and slope of the line of best fit. Given as:
N= number of terms

(y-intercept) $b=\frac{\sum y_i - N\sum x_i}{N}$

(Slope) $m=\frac{(N\sum x_iy_i)-(\sum x_i\sum y_i)}{(N\sum x_i^{\,2})-\sum(x_i)^2}$

Apparatus

Logger Pro is used to graph the relationship between Distance and Recessional Velocity.
Both MS Excel and Logger pro were used to create the necessary tables and calculated columns.

Experiment Method:

Data Set 1:
The data set was imported into logger pro. The recessional velocities (kms^{-1}) in Column 2 are plotted against the distances in Column 1 (Mpc). Using the linear fit option in logger pro I found the line of best fit for the data points. This line of best fit is then verified via the goodness of fit and the formulae for the slope and y-intercept. (Derived in ‘Equations and Basic Theory’).

From the line of best fit equation Hubble's constant is found.
The graph with linear fit overlaid is in 'Data Analysis'.

Data Set 2:
The data set was imported into logger pro. Three calculated columns are created for; Distance (Mpc),ΔDistance (Mpc), Recessional Velocity(kms^-1). The functions for Distance (d) and Recessional Velocity (v) are outlined in the lab manual. Respectively:

$d=10^{(\mu/5 +1)}$

v=zc

The ΔDistance (Mpc) function is derived in the 'Error Analysis' section as such:

$$\Delta d = \sqrt{\left(\frac{ln(10)}{5}\left(10^{(\mu/5+1)}\right)(\Delta\mu)\right)^2}$$

Once these columns are created and referenced to the correct original columns, graph Recessional Velocity against Distance. (Using ΔDistance as error bars). The linear fit option in logger pro was used again to find the line of best fit. I did not verify this linear fit using 'Goodness of fit' as we already proved the concept using data set 1.
A more accurate value for Hubble's constant was found using the equation of the best fit line.

-<u>To find the age of the observable universe.</u>

We can say that at one point in time Earth and a distant Supernova/Galaxy etc were in the same position in space. That time can be said to be : t=d/v
d=distance, v=recessional velocity
$t=d/(d*H_0)$
Thus Age of the universe is = $1/H_0$.

-<u>To find the size of the observable universe</u>

We can no longer see any further into the universe when Recessional

Velocity=Speed of light.
Using $v=H_0d$
$c=H_0d$
$d=c/H_0$
This creates a spherical 'boundary' around earth, of the size:
$\frac{4}{3}\pi d^3$= Size of the observable universe.

Experimental data

	hubble(1)			
	Distance (Mpc)	Velocity (kms^-1)	Column 3	Column
1				
2	1.52	650		
3	3.45	1800		
4	2.37	1300		
5	0.52	300		
6	1.15	800		
7	1.42	700		
8	0.57	400		
9	1.24	600		
10	0.79	290		
11	1	500		
12	1.74	940		
13	1.49	810		
14	1.1	600		
15	1.27	730		
16	1.53	800		
17	1.79	800		
18	1.2	580		
19	2.35	1100		
20	2.23	1140		
21	2.05	900		
22	1.73	650		
23				

(1)

Sample of older Hubble Data.

There were no necessary calculated columns and no error is included.

	modern_sn(1)					
	mu	delta_mu	Red-shift (z)	Distance (d) (Mpc)	delta_distance	Recessional Velocity (z*c) (kms^-1)
47	34.37272145	0.308392297	0.016991	74.911	0.003	5093.774
48	35.07969126	0.221390434	0.027865	103.738	0.003	8353.717
49	34.25675484	0.24730267	0.017173	71.015	0.003	5148.336
50	35.96952979	0.223360507	0.029955	156.281	0.004	8980.283
51	34.33595485	0.250405422	0.016559	73.653	0.003	4964.263
52	34.16112422	0.160930618	0.015	67.956	0.002	4496.887
53	36.94828686	0.085185928	0.0544	245.277	0.003	16308.710
54	39.22773689	0.083180324	0.1561	700.725	0.005	46797.603
55	36.32963388	0.098361648	0.0393	184.470	0.003	11781.844
56	38.80893732	0.110959981	0.1241	577.813	0.005	37204.244
57	38.82867186	0.156387994	0.1441	583.088	0.006	43200.093
58	38.97672086	0.129040617	0.1299	624.229	0.006	38943.040
59	37.67843349	0.086376099	0.0784	343.310	0.004	23503.729
60	37.02270819	0.199697932	0.0583	253.829	0.005	17477.900
61	35.91511543	0.176677069	0.0309	152.413	0.004	9263.587
62	36.36355742	0.1646499	0.0406	187.375	0.004	12171.574
63	34.0128062	0.209050001	0.0152	63.469	0.002	4556.845
64	34.96044361	0.232580853	0.0224	98.195	0.003	6715.351
65	34.16651186	0.215338745	0.016	68.124	0.003	4796.679
66	35.98183304	0.164155648	0.0362	157.169	0.003	10852.487
67	34.24816759	0.210066601	0.0173	70.735	0.003	5186.410
68	35.61906681	0.173133568	0.0312	132.988	0.003	9353.525
69	34.88806712	0.183198718	0.0221	94.976	0.003	6625.413
70	33.81426031	0.201710027	0.016	57.923	0.002	4796.679
71	34.78445024	0.18703332	0.0249	90.550	0.003	7464.832
72	35.6227445	0.169350596	0.0303	133.214	0.003	9083.711

(2)

Sample of modern supernova data and the calculated columns.

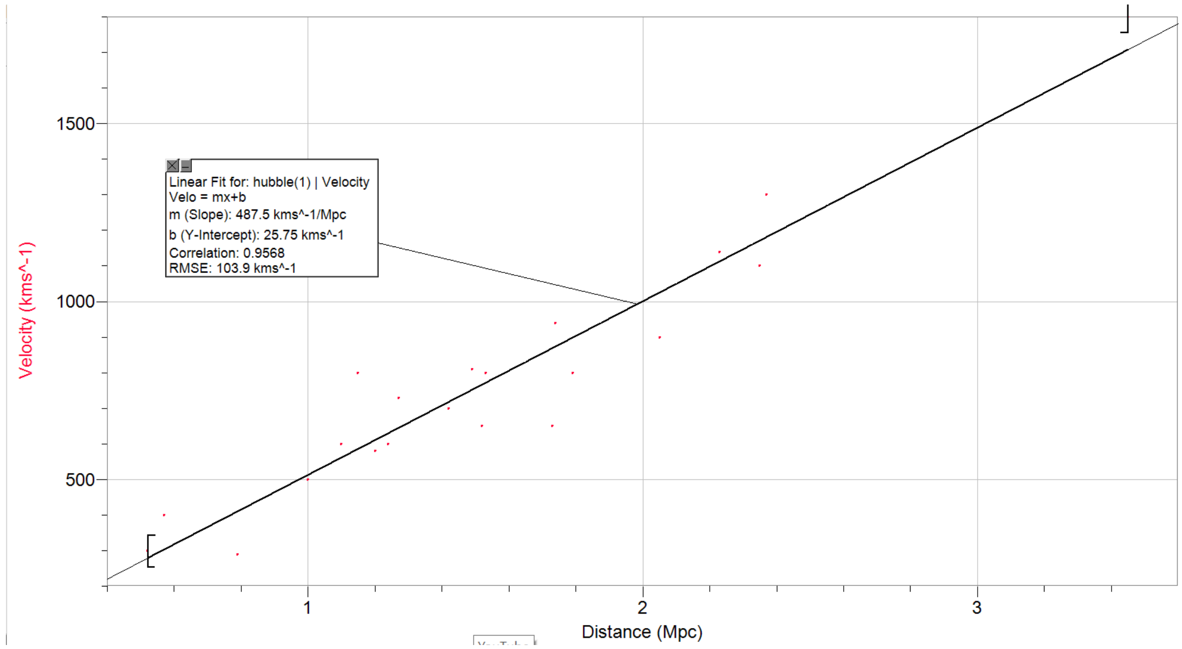
-Distance Column is calculated by: $d = 10^{(\mu/5 + 1)}$, where μis the column titled mu.

-Δd(delta_distance) is calculated via error propagation described in the Error analysis section.

$$\sqrt{\left(\frac{ln(10)}{5}\left(10^{(\mu/5+1)}\right)(\Delta\mu)\right)^{2}}$$

-Recessional Velocity =zc, where c=speed of light and z=red shift.

Data Analysis:



(3)

v=md+c

v=487.5d+25.75

Taking the Hubble Constant as the slope:

H_0=487.5

A	B	C	D	E	F	G	H	I	J
x	y		x^2	y^2		x*y			
1.52	650		2.3104	1300		988			
3.45	1800		11.9025	3600		6210			
2.37	1300		5.6169	2600		3081			
0.52	300		0.2704	600		156			
1.15	800		1.3225	1600		920			
1.42	700		2.0164	1400		994			
0.57	400		0.3249	800		228			
1.24	600		1.5376	1200		744	N=	21	
0.79	290		0.6241	580		229.1			
1	500		1	1000		500	m=	487.5161813	(slope)
1.74	940		3.0276	1880		1635.6			
1.49	810		2.2201	1620		1206.9	b=	25.75471171	(y-intercept)
1.1	600		1.21	1200		660			
1.27	730		1.6129	1460		927.1			
1.53	800		2.3409	1600		1224			
1.79	800		3.2041	1600		1432			
1.2	580		1.44	1160		696			
2.35	1100		5.5225	2200		2585			
2.23	1140		4.9729	2280		2542.2			
2.05	900		4.2025	1800		1845			
1.73	650		2.9929	1300		1124.5			
32.51	16390		59.6721	32780		29928.4			

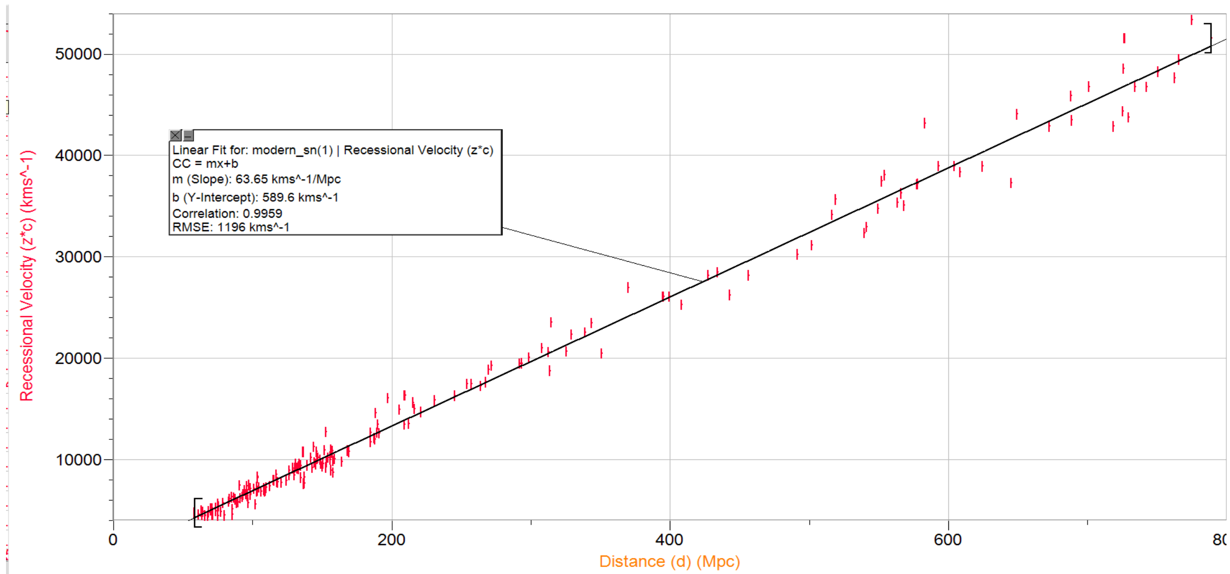
(4)

Using the formulae in excel we can verify this from our initial ‘Goodness of Fit’ Parameter:

(y-intercept) $b=\frac{\sum y_i - N\sum x_i}{N}$

(Slope) $m=\frac{(N\sum x_i y_i)-(\sum x_i \sum y_i)}{(N\sum x_i^{2})-\sum(x_i)^{2}}$

<u>Data Set 2</u>



(5)

Using the best fit line:

v=63.65x+589.6

H_0=63.65

Notice the correlation coefficient is much higher.

(Validity of linear fit proven in Data Set 1 by extension we can accept this best fit)

-Age of the Universe:

Using the most accurate data set where (H_0=63.65)

$1/H_0$ = 1/63.65 (km/s/Mpc)

$\frac{Sec\ Mpc}{Km} * \frac{(3.09*10^{19})}{1\ Mpc}$/3.154*10^+7(sec)

With this all units cancel.

giving :

15.39212*10^10.

Or 15.39 billion years

-Finding size of the observable Universe.

We can no longer see any further into the universe when Recessional Velocity=Speed of light.

Using $v=H_0d$

$c=H_0d$

$d=c/H_0$

This creates a spherical ‘boundary’ around earth, of the size:

$\frac{4}{3}\pi d^3$= Size of the observable universe.

$\frac{4}{3}\pi(\frac{c^3}{H^3})$

(km/s)/(km/s)/(mpc) = mpc

299792/63.65 mpr =1.455*10^23 km

1.455*10^23 km($\frac{4}{3}$π)=8.12844*10^23 km (Size of obv universe)

Analysis of Accuracy and Uncertainty:

Error Propagation:
Derived from

$$\sigma_f^2 = \left(\frac{\partial f}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial f}{\partial y}\right)^2 \sigma_y^2$$

$$\Delta d = \sqrt{\left(\frac{\partial d}{\partial \mu}\Delta\mu\right)^2}$$

$$\frac{\partial d}{\partial \mu} = \left(\frac{\ln(10)}{5}\right)\left(10^{\frac{x}{5}+1}\right)$$

(6)
This function is applied in logger pro.
All other uncertainty probably comes from the telescope or spectrometer.

Final Results and Conclusions:

A large disparity was noticed between the data sets.
H_0=487.5 For Data Set 1
H_0=63.65 For Data Set 2
Data Set 2 Provided a figure much closer to the estimated figure for hubble's constant. (73.8)

Data Set 2 had only 13% error compared to Data Set 1 which had 500% error.
We see the more modern data provided a much more accurate answer. Most likely due to better equipment meaning less systematic error, and longer distances which means error/uncertainty has proportionally much less of an effect on the results.
-Age of the visible universe: 15.39 billion years (14% error)
-Size of visible universe: 8.12844*10^23 km

Appendices:

(1)= Data set 1 Results
(2)=Data Set 2 Results
(3)=Respective Graph
(4)=Proof for goodness of fit
(5)=Respective graphs
(6)=Error Propagation

Hubble's law can answer the questions such as the age and size of the observable universe.
It provides the true distances between points in the universe as it can account for the expansion.

Marker's Comments:

Grade:

Declaration:

I have read and understood the plagiarism provisions as set out in the General Regulations of the College University Calendar and also completed the Online Tutorial on avoiding plagiarism (http://tcd-ie.libguides.com/plagiarism/ready-steady-write). Signature: Kevin Grainger Date: 10/11/2020