Computer Vision

3th November 2020

1 Homography

Genereral perspective projection:

$$\begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} \sim \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \end{bmatrix} \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \end{bmatrix} = KR[I| - t]$$

where K is the camera matrix

$$K = \begin{bmatrix} f_u & 0 & u_0 \\ 0 & f_v & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

Problem: projection cannot be inverted.
However, for special cases, it can be. There are two cases.
(1) Plane-plane transformation.

$$\begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} \sim \begin{bmatrix} P_{11} & P_{12} & P_{14} \\ P_{21} & P_{22} & P_{24} \\ P_{31} & P_{32} & P_{34} \end{bmatrix} \begin{bmatrix} X_i \\ Y_i \\ 1 \end{bmatrix}$$

The inverse:

$$\begin{bmatrix} P_{11} & P_{12} & P_{14} \\ P_{21} & P_{22} & P_{24} \\ P_{31} & P_{32} & P_{34} \end{bmatrix}^{-1} \begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} \sim \begin{bmatrix} X_i \\ Y_i \\ 1 \end{bmatrix}$$

$$H = \begin{bmatrix} P_{11} & P_{12} & P_{14} \\ P_{21} & P_{22} & P_{24} \\ P_{31} & P_{32} & P_{34} \end{bmatrix}$$

is the homography.
(2) Panoramic images: $t = [0, 0, 0]^T$

$$\begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} \sim KR \begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix}$$

The inverse:

$$R^T K^{-1} \begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} \sim \begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix}$$

2 Homography Estimation

$$\begin{bmatrix} u_i' \\ v_i' \\ 1 \end{bmatrix} \sim \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix}$$

Basic equations for homography:

$$u_i' = \frac{h_{11}u_i + h_{12}v_i + h_{13}}{h_{31}u_i + h_{32}v_i + h_{33}}$$

$$v_i' = \frac{h_{21}u_i + h_{22}v_i + h_{23}}{h_{31}u_i + h_{32}v_i + h_{33}}$$

Linearize the problem:

$$0 = h_{11}u_i + h_{12}v_i + h_{13} - (h_{31}u_i + h_{32}v_i + h_{33})\, u_i'$$

$$0 = h_{21}u_i + h_{22}v_i + h_{23} - (h_{31}u_i + h_{32}v_i + h_{33})\, v_i'$$

Write the equations into matrix-vector form.

$$\begin{bmatrix} u_i & v_i & 1 & 0 & 0 & 0 & -u_iu_i' & -v_iu_i' & -u_i' \\ 0 & 0 & 0 & u_i & v_i & 1 & -u_iv_i' & -v_iv_i' & -v_i' \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = 0$$

It is a homogeneous linear system of equations.

2.1 Data normalization

$$T' \begin{bmatrix} u_i' \\ v_i' \\ 1 \end{bmatrix} \sim \hat{H} T \begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix}$$

where T and T' are affine transformations.
Modify the equation:

$$\begin{bmatrix} u_i' \\ v_i' \\ 1 \end{bmatrix} \sim T'^{-1} \hat{H} T \begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix}$$

then

$$H = T'^{-1} \hat{H} T$$