

Problem Set 5

PHYSICS 443
October 7, 2020

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1. (a) The systems matrix can be characterized as $\mathbf{R}_2 \mathbf{T} \mathbf{R}_1$,

$$\begin{aligned}\mathbf{S} &= \begin{pmatrix} 1 & 0 \\ \lim_{r \rightarrow \infty} \frac{n-1}{r} & n \end{pmatrix} \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{1-n}{nR} & \frac{1}{n} \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 \\ 0 & n \end{pmatrix} \begin{pmatrix} 1 + \frac{d(1-n)}{\frac{1-n}{nR}} & \frac{d}{\frac{1}{n}} \\ \frac{1-n}{nR} & \frac{1}{n} \end{pmatrix} \\ &= \begin{pmatrix} 1 + \frac{d(1-n)}{\frac{1-n}{nR}} & \frac{d}{\frac{1}{n}} \\ \frac{1-n}{nR} & \frac{1}{n} \end{pmatrix}\end{aligned}$$

- (b) At the thin lens limit $d \rightarrow 0$, the systems matrix becomes

$$\mathbf{S} = \begin{pmatrix} 1 & 0 \\ \frac{1-n}{R} & 1 \end{pmatrix}$$

From inspection of the systems matrix, the focal length is

$$f = \frac{R}{n-1}$$

2. Hecht 5.26. Assuming a thin lens, the systems matrix will be

$$\begin{aligned}\mathbf{S} &= \begin{pmatrix} 1 & 0 \\ \frac{n_2-n_1}{n_1} \left(\frac{1}{R_2} - \frac{1}{R_1} \right) & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 \\ \frac{2(n_2-n_1)}{n_1 R} & 1 \end{pmatrix} \\ f &= \frac{n_1 R}{2(n_2 - n_1)}\end{aligned}$$

For $n_2 = 1.5$ and $R = 12.5$ cm,

$$\begin{aligned}f &= -\frac{12.5 \text{ cm}}{2(1.5 - 1)} \\ &= -12.5 \text{ cm}\end{aligned}$$

For $n_1 = 1.628$,

$$\begin{aligned}f &= -\frac{1.628 (12.5 \text{ cm})}{2(1.5 - 1.628)} \\ &= 79.5 \text{ cm}\end{aligned}$$

3. Hecht 5.47. Using the thin lens equation, $s_{i1} = \infty$, as the object is at the first lens' focal point.

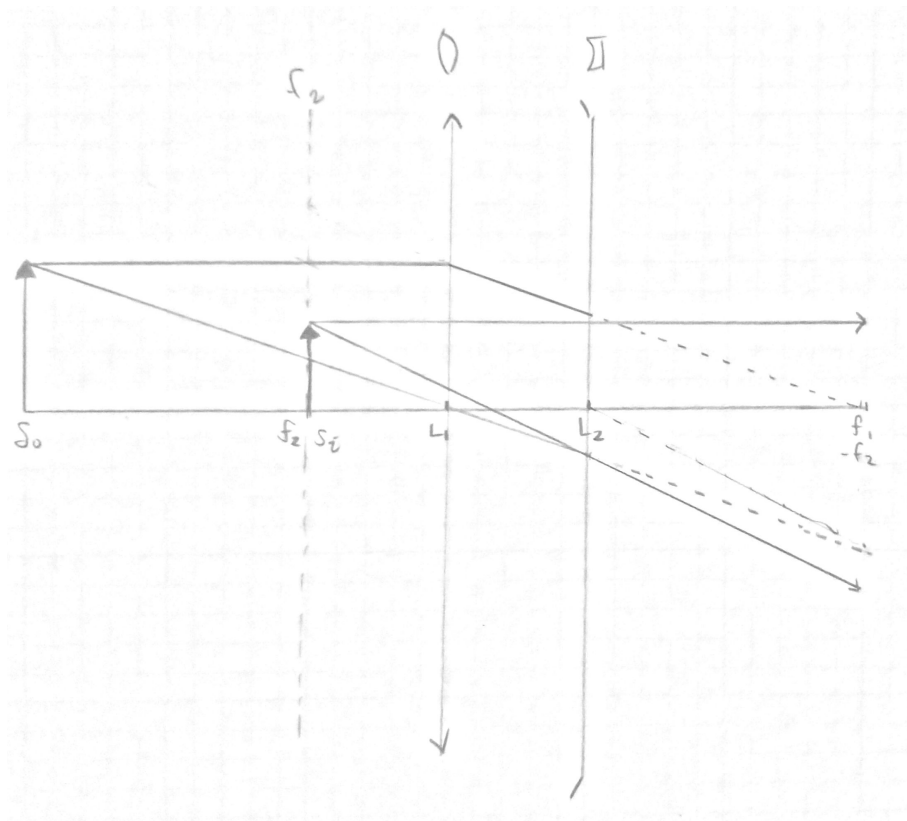
$$\frac{1}{f_2} = \frac{1}{s_{o2}} + \frac{1}{s_{i2}}$$

$$s_{i2} = f_2 = -20 \text{ cm} \quad (\text{rel. to } L_2)$$

Since the image from the first lens is at $+\infty$, it will be at $-\infty$ relative to the second lens, and the total transverse magnification will have a positive sign,

$$M_T = M_{T1} M_{T2} = \frac{\cancel{s_{i1}}}{s_{o1}} \frac{s_{i2}}{\cancel{s_{o2}}}$$

$$= \frac{20}{30} = 0.667$$



4. (a) The critical angle within the fiber is

$$\theta_c = \sin^{-1}(1.46/1.6) = 65.9^\circ$$

- (b) From (5.61),

$$\text{NA} = (n_f^2 - n_c^2)^{1/2}$$

$$= (1.6^2 - 1.46^2)^{1/2}$$

$$= 0.655$$