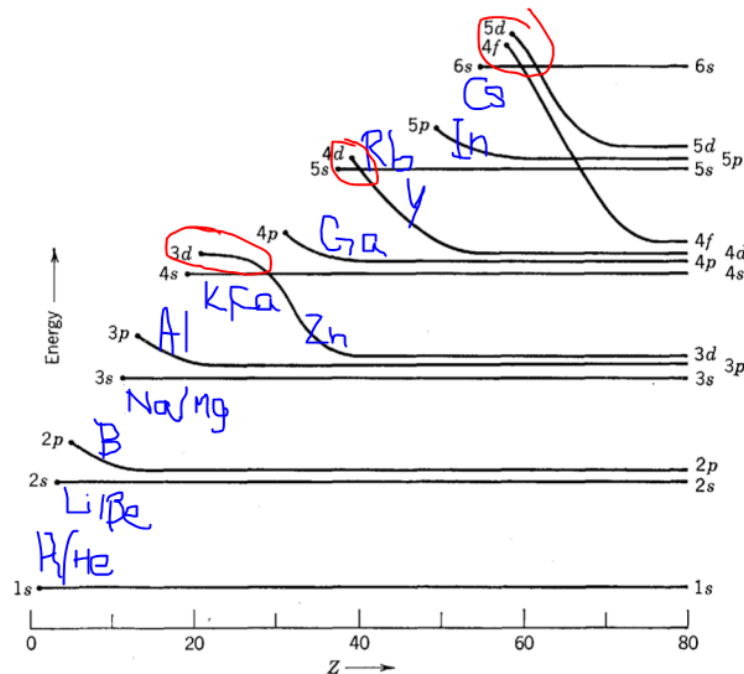


# Homework 7

PHYSICS 461  
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1. (a) Presumably because of electron screening. The  $p$  and  $d$  levels are further than the  $s$ , and the electrons in  $s$  can screen the electric field from the nucleus. As  $z$  increases, the nucleus becomes more positively charged and it reduces the effect of the screening, leading to a plateau.
- (b) Maybe something like this:



2. (a) Because in the experiment, the X-ray emission is coming from the  $K$  series lines, which are the same transitions seen by the sodium atom during absorption spectroscopy.
  - (b) Presumably because tungsten is a high  $Z$  atom, leading to a larger magnetic moment and a larger energy splitting.
  - (c) Because the energy is proportional to  $Z^2$  in hydrogen-like states.
  - (d) The  $K$  series lines are the lowest ones ( $\rightarrow 1s$ ). The  $L$  series are the next lowest,  $\rightarrow 2s/2p$ .
  - (e) No? The upper plot has a higher energy value.
3. (a) There are four states in the Slater basis,

$$\begin{aligned}
 |\alpha\beta\rangle_1 &= \mathcal{A} |\alpha\beta\rangle_a = |\alpha\beta\rangle_a - |\alpha\beta\rangle_{a'} \\
 |\alpha\beta\rangle_2 &= \mathcal{A} |\alpha\beta\rangle_b = |\alpha\beta\rangle_b - |\alpha\beta\rangle_{b'} \\
 |\alpha\beta\rangle_3 &= \mathcal{A} |\alpha\beta\rangle_c = |\alpha\beta\rangle_c - |\alpha\beta\rangle_{c'} \\
 |\alpha\beta\rangle_4 &= \mathcal{A} |\alpha\beta\rangle_d = |\alpha\beta\rangle_d - |\alpha\beta\rangle_{d'}
 \end{aligned}$$

- (b) The singlet state has an antisymmetric spin function, so the total wavefunction will be

$$\begin{aligned} (|\alpha\beta\rangle + |\beta\alpha\rangle)(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) &= |\alpha\uparrow\rangle|\beta\downarrow\rangle - |\alpha\downarrow\rangle|\beta\uparrow\rangle + |\beta\uparrow\rangle|\alpha\downarrow\rangle - |\beta\downarrow\rangle|\alpha\uparrow\rangle \\ &= |\alpha\beta\rangle_b - |\alpha\beta\rangle_c + |\alpha\beta\rangle_{c'} - |\alpha\beta\rangle_{b'} \\ &= |\alpha\beta\rangle_2 - |\alpha\beta\rangle_3 \end{aligned}$$

For the triplet state, the spinfunction is symmetric, so

$$\begin{aligned} (|\alpha\beta\rangle - |\beta\alpha\rangle)(|\uparrow\uparrow\rangle) &= |\alpha\uparrow\rangle|\beta\uparrow\rangle - |\beta\uparrow\rangle|\alpha\uparrow\rangle \\ &= |\alpha\beta\rangle_1 \end{aligned}$$

I'm guessing the other ones will be  $|\alpha\beta\rangle_4$ . The last one will be

$$\begin{aligned} (|\alpha\beta\rangle - |\beta\alpha\rangle)(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) &= |\alpha\uparrow\rangle|\beta\downarrow\rangle - |\beta\uparrow\rangle|\alpha\downarrow\rangle + |\alpha\downarrow\rangle|\beta\uparrow\rangle - |\beta\downarrow\rangle|\alpha\uparrow\rangle \\ &= |\alpha\beta\rangle_2 + |\alpha\beta\rangle_3 \end{aligned}$$

- (c) 4 states, but I don't understand Slater stuff well enough to understand what this question is asking me.

- (d) We need Russell-Saunders coupling because we're dealing with the electron-electron interaction.

Treating them separately results in  $l = l_1 + l_2 = 2$ ,  $s = s_1 + s_2 = 1$ .

$^3D$ ,  $^1D$ ,  $^3P$ ,  $^1P$ ,  $^3S$ ,  $^1S$

For  $^3D$ ,  $l = 2$  and  $s = 1$ ,  $m_l = 2, 1, 0, -1, -2$ ,  $m_s = 1, 0, -1$ , giving 15 unique combinations.

For  $^1D$ ,  $m_s = 0$ , giving 5 combinations.

For the P states, 9 and 3. For the S states, 3 and 1 combinations.

The sum of all this is 36 states.

- (e) This is jj-coupling, right? So this would be needed when there's electron-electron interaction, but for larger atoms with high  $Z$ .

Each  $j_i = \frac{3}{2}, \frac{1}{2}$ . This results in a total  $j = 3, 2, 1, 0$ . For  $j = 3$ ,  $m_j = 0, \pm 1, \pm 2, \pm 3$  with 7 states. For  $j = 2$ , it's 5 states. For  $j = 1$ , it's 3 states. For  $j = 0$ , there's a single state.

This is a total of 16 states.

4. Omitting the  $\hbar$ , then we can factor  $j_c$  as

$$J_C = j_c \sqrt{1 + \frac{1}{j_c}}$$

...not actually sure what to do here. Should I be substituting in  $j_a + j_b$  for  $j_c$ ? Or should I just check the inequality with  $j_a(j_a + 1)$  and  $j_b(j_b + 1)$ ?

5. (a) For the  $p$  subshell,  $\ell = 1$  and the spherical harmonics squared are

$$\begin{aligned} Y_{10}^2 &= \frac{3}{4\pi} \cos^2(\theta) \\ Y_{11}^2 &= \frac{3}{8\pi} \sin^2(\theta) \\ Y_{1(-1)}^2 &= \frac{3}{8\pi} \sin^2(\theta) \end{aligned}$$

Summing these,

$$\begin{aligned}\sum_k Y_{1k}^2 &= \frac{3}{4\pi} \cos^2(\theta) + 2 \left( \frac{3}{8\pi} \right) \sin^2(\theta) \\ &= \frac{3}{8\pi}\end{aligned}$$

(b) For the  $d$  shell,

$$\begin{aligned}Y_{20}^2 &= \frac{5}{16\pi} (3 \cos^2(\theta) - 1)^2 \\ Y_{2\pm 1} &= \frac{15}{8\pi} (\cos \theta \sin \theta)^2 \\ Y_{2\pm 2} &= \frac{15}{32\pi} \sin^4(\theta)\end{aligned}$$

Summing these in WolframAlpha results in

$$\sum_k Y_{2k}^2 = \frac{5}{4\pi}$$

6. (a)  ${}^2S_{3/2}$ :  $s = 1/2, l = 0, j = 3/2$

${}^3D_2$ :  $s = 1, l = 2, j = 2$

${}^5P_3$ :  $s = 2, l = 1, j = 3$

(b)  ${}^2S_{3/2}$  is impossible, since the total  $j$  exceeds  $l + s$ .

7. Gonna omit the  $j$  value to save space on the spectroscopy notation bit,

$L$	$S$	$J$	Spect.
5	0	5	${}^1\text{H}$
5	1	6, 5, 4	${}^3\text{H}$
4	0	4	${}^1\text{G}$
4	1	5, 4, 3	${}^3\text{G}$
3	0	3	${}^1\text{F}$
3	1	4, 3, 2	${}^3\text{F}$
2	0	2	${}^1\text{D}$
2	1	3, 2, 1	${}^3\text{D}$
1	0	1	${}^1\text{P}$
1	1	2, 1, 0	${}^3\text{P}$

8. (a) Since the three negative terms ( $\epsilon_{321} = \epsilon_{132} = \epsilon_{213}$ ) squared are just 1. The other terms squared are 1, so summing this results in

$$\epsilon_{ijk}\epsilon_{ijk} = 6$$

(b) There are only two non-zero components when  $k = l$ , resulting in  $2\delta_{kl}$ .

9. Using the determinate form with the Kronecker deltas,

$$\varepsilon_{ijk}\varepsilon_{lmn} = \begin{vmatrix} \delta_{il} & \delta_{im} & \delta_{in} \\ \delta_{jl} & \delta_{jm} & \delta_{jn} \\ \delta_{kl} & \delta_{km} & \delta_{kn} \end{vmatrix}$$

As  $i = l$ , then the only non-zero term is

$$\varepsilon_{ijk}\varepsilon_{imn} = \delta_{jm}\delta_{kn} - \delta_{jn}\delta_{km}$$

10. Given the rule provided,

$$\begin{aligned} [\mathbf{A} \times (\mathbf{B} \times \mathbf{C})]_i &= \varepsilon_{ijk} A_j (\mathbf{B} \times \mathbf{C})_k \\ &= \varepsilon_{ijk} A_j \varepsilon_{klm} B_l C_m = \varepsilon_{kij} \varepsilon_{klm} A_j B_l C_m \\ &= (\delta_{il} \delta_{jm} - \delta_{im} \delta_{jl}) A_j B_l C_m \\ &= B_i A_m B_m - C_i A_l B_l \quad \square \end{aligned}$$