Homework 1

PHYSICS 341
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1. For the LHS, if we begin by evaluating $\mathbf{B} \times \mathbf{C}$,

$$
\begin{aligned}
\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) &= \mathbf{A} \times \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} \\
&= \mathbf{A} \times \left[(B_y C_z - B_z C_y)\ \hat{\mathbf{x}} + (B_z C_x - B_x C_z)\ \hat{\mathbf{y}} + (B_x C_y - B_y C_x)\ \hat{\mathbf{z}} \right] \\
&= \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ (B_y C_z - B_z C_y) & (B_z C_x - B_x C_z) & (B_x C_y - B_y C_x) \end{vmatrix} \\
&= (A_y B_x C_y - A_y B_y C_x - A_z B_z C_x + A_z B_x C_z)\ \hat{\mathbf{x}} \\
&\quad + (A_z B_y C_z - A_z B_z C_y - A_x B_x C_y + A_x B_y C_x)\ \hat{\mathbf{y}} \\
&\quad + (A_x B_z C_x - A_x B_x C_z - A_y B_y C_z + A_y B_z C_y)\ \hat{\mathbf{z}} \\
&= (A_y C_y + A_z C_z)\, B_x\, \hat{\mathbf{x}} + (A_z C_z + A_x C_x)\, B_y\, \hat{\mathbf{y}} + (A_x C_x + A_y C_y)\, B_z\, \hat{\mathbf{z}} \\
&\quad - (A_y B_y + A_z B_z)\, C_x\, \hat{\mathbf{x}} - (A_x B_x + A_z B_z)\, C_y\, \hat{\mathbf{y}} - (A_x B_x + A_y B_y)\, C_z\, \hat{\mathbf{z}}
\end{aligned}
$$

Then for the RHS, if we expand the dot products and scale the components of $\mathbf{B}$ and $\mathbf{C}$,

$$
\begin{aligned}
\mathbf{B}\,(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}\,(\mathbf{A} \cdot \mathbf{B}) &= (A_x C_x + A_y C_y + A_z C_z)\, B_x\, \hat{\mathbf{x}} + (\cdots)\, B_y\, \hat{\mathbf{y}} + (\cdots)\, B_z\, \hat{\mathbf{z}} \\
&\quad - (A_x B_x + A_y B_y + A_z B_z)\, C_x\, \hat{\mathbf{x}} - (\cdots)\, C_y\, \hat{\mathbf{y}} - (\cdots)\, C_z\, \hat{\mathbf{z}}
\end{aligned}
$$

Removing the terms that subtract out (all the $A_i B_i C_i\, \hat{\mathbf{e}}_i$), we are left with the LHS result,

$$
\begin{aligned}
&= (A_y C_y + A_z C_z)\, B_x\, \hat{\mathbf{x}} + (A_z C_z + A_x C_x)\, B_y\, \hat{\mathbf{y}} + (A_x C_x + A_y C_y)\, B_z\, \hat{\mathbf{z}} \\
&\quad - (A_y B_y + A_z B_z)\, C_x\, \hat{\mathbf{x}} - (A_x B_x + A_z B_z)\, C_y\, \hat{\mathbf{y}} - (A_x B_x + A_y B_y)\, C_z\, \hat{\mathbf{z}}
\end{aligned}
$$

$\square$

2. (a) $\boldsymbol{\nabla} f(x, y, z) = 2x\, \hat{\mathbf{x}} + 3y^2\, \hat{\mathbf{y}} + 4z^3\, \hat{\mathbf{z}}$
   (b) $\boldsymbol{\nabla} f(x, y, z) = 2xy^3z^4\, \hat{\mathbf{x}} + 3x^2y^2z^4\, \hat{\mathbf{y}} + 4x^2y^3z^3\, \hat{\mathbf{z}}$
   (c) $\boldsymbol{\nabla} f(x, y, z) = e^x \sin(y) \ln(z)\, \hat{\mathbf{x}} + e^x \cos(y) \ln(z)\, \hat{\mathbf{y}} + \frac{e^x \sin(y)}{z}\, \hat{\mathbf{z}}$

3. (a) $\boldsymbol{\nabla} \cdot \mathbf{v}_a = 2x + 3y^2 + 4z^3$
   (b) $\boldsymbol{\nabla} \cdot \mathbf{v}_b = x + y + z$
   (c) $\boldsymbol{\nabla} \cdot \mathbf{v}_c = 2yz - 3y$

4. (a) $$\boldsymbol{\nabla} \times \left[x^2\,\hat{\mathbf{x}} + y^3\,\hat{\mathbf{y}} + z^4\,\hat{\mathbf{z}}\right] = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & y^3 & z^4 \end{vmatrix} = 0$$

(b) $$\boldsymbol{\nabla} \times \left[xy\,\hat{\mathbf{x}} + yz\,\hat{\mathbf{y}} + zx\,\hat{\mathbf{z}}\right] = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & yz & zx \end{vmatrix} = (-y)\,\hat{\mathbf{x}} - z\,\hat{\mathbf{y}} - x\,\hat{\mathbf{z}}$$

(c) $$\boldsymbol{\nabla} \times \left[2z\,\hat{\mathbf{x}} + y^2z\,\hat{\mathbf{y}} - 3yz\,\hat{\mathbf{z}}\right] = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2z & y^2z & -3yz \end{vmatrix} = \left(-3z - y^2\right)\,\hat{\mathbf{x}} + 2\,\hat{\mathbf{y}}$$

5. (a) $$\nabla^2\left[x^2 + y^3 + z^4\right] = 2 + 6y + 12z^2$$

(b) $$\begin{aligned} \nabla^2\left[x^2y^3z^4\right] &= \boldsymbol{\nabla}\cdot\left[2xy^3z^4\,\hat{\mathbf{x}} + 3x^2y^2z^3\,\hat{\mathbf{y}} + 4x^2y^3z^3\,\hat{\mathbf{z}}\right] \\ &= 2y^3z^4 + 6x^2yz^3 + 12x^2y^3z^2 \end{aligned}$$

(c) $$\begin{aligned} \nabla^2[e^x\sin(y)\ln(z)] &= \boldsymbol{\nabla}\cdot\left[e^x\sin(y)\ln(z)\,\hat{\mathbf{x}} + e^x\cos(y)\ln(z)\,\hat{\mathbf{y}} + \frac{e^x\sin(y)}{z}\,\hat{\mathbf{z}}\right] \\ &= e^x\sin(y)\ln(z) - e^x\sin(y)\ln(z) - \frac{e^x\sin(y)}{z^2} \\ &= -\frac{e^x\sin(y)}{z^2} \end{aligned}$$

(d) $\nabla^2[xy\,\hat{\mathbf{x}} + yz\,\hat{\mathbf{y}} + zx\,\hat{\mathbf{z}}] = 0$ (as they're all first order)