Homework 3

PHYSICS 342
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1. Assuming the electric field is uniform and radially outward between the two cylinders,

$$E = V/(b-a)$$

The magnetic field is given by Ampère's law and is in the azimuthal direction as

$$B = \frac{\mu_0 I}{2\pi s}$$

As the two fields are perpendicular, the magnitude of the Poynting vector is

$$S = \frac{1}{\mu_0} EB = \frac{VI}{2\pi s(b-a)}$$

The Poynting vector is in the direction of the cable's length, so the respective $\mathrm{d}a$ is over the annular face, $s\,\mathrm{d}\phi\,\mathrm{d}s$. Integrating this to find the total power,

$$\begin{aligned} P &= \int_S \frac{VI}{2\pi s(b-a)}\,\mathrm{d}a \\ &= \frac{VI}{2\pi(b-a)} \int_a^b \mathrm{d}s \int_0^{2\pi} \mathrm{d}\phi \\ &\boxed{P = \frac{VI}{(b-a)}(b-a) = VI} \end{aligned}$$

2. For each wire of density λ, the electric field is

$$\mathbf{E} = \frac{\lambda}{2\pi\epsilon_0 s}\,\hat{\mathbf{s}}$$

Taking the surface to be the xy plane, the electric field along the plane is directed radially outward. Along the surface, the net electric field is

$$\begin{aligned} \mathbf{E} &= \frac{2\lambda}{2\pi\epsilon_0 s}\cos\theta\,\hat{\mathbf{s}} = \frac{\lambda}{\pi\epsilon_0 s}\left(\frac{x}{(x^2+a^2)^{1/2}}\right)\hat{\mathbf{x}} \\ &= \frac{\lambda}{\pi\epsilon_0}\left(\frac{x}{x^2+a^2}\right)\hat{\mathbf{x}} \end{aligned}$$

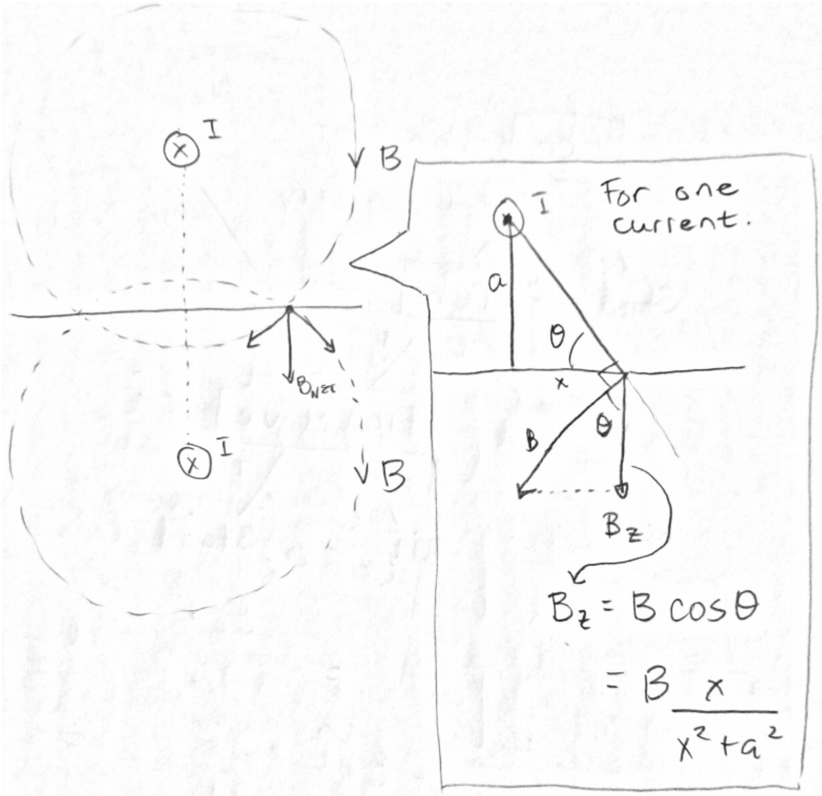
As the electric field is only in the x direction, the tensor can be simplified where only the T_{zz} component remains,

$$T_{zz} = \frac{\epsilon_0 {E_x}^2}{2} = \frac{\lambda^2}{2\pi^2\epsilon_0}\frac{x^2}{(x^2+a^2)^2}$$

Thus, the force per unit length is

$$\begin{aligned} \mathbf{f} &= \int_{-\infty}^{\infty} T_{zz}\,\mathrm{d}x\;\hat{\mathbf{x}} \\ &= \frac{\lambda^2}{2\pi^2\epsilon_0}\int_{-\infty}^{\infty}\frac{x^2}{(x^2+a^2)^2}\,\mathrm{d}x\;\hat{\mathbf{x}} \\ &= \frac{\lambda^2}{4\pi\epsilon_0 a}\,\hat{\mathbf{x}} \end{aligned}$$

3. The diagram would look something like this:



From the diagram and Ampère's law, the vertical component of the magnetic field for both wires is

$$
\begin{aligned}
B_z &= 2 \times \frac{\mu_0 I}{2\pi s}\frac{x}{s} \\
&= \frac{\mu_0 I}{\pi}\frac{x}{x^2+a^2}
\end{aligned}
$$

The only non-zero component of the tensor is T_{zz} acting on the xy plane is

$$
\begin{aligned}
T_{zz} &= -\frac{{B_z}^2}{2\mu_0} \\
&= -\frac{\mu_0 I^2}{2\pi^2}\frac{x^2}{(x^2+a^2)^2}
\end{aligned}
$$

Integrating over all x will give the force per unit length,

$$
\begin{aligned}
\mathbf{f} &= -\frac{\mu_0 I^2}{2\pi^2}\int_{-\infty}^{\infty}\frac{x^2}{(x^2+a^2)^2}\,\mathrm{d}x\;\hat{\mathbf{z}} \\
&= -\frac{\mu_0 I^2}{4\pi a}
\end{aligned}
$$

4. For a solenoid of radius a, current I, and n turns per length, within the solenoid the magnetic field is

$$\mathbf{B} = \mu_0 n I\,\hat{\mathbf{z}}$$

And the electric field is zero as the current is assumed to be constant. So the stress tensor is

$$\overleftrightarrow{\mathbf{T}} = -\frac{\mu_0 n^2 I^2}{2}\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$$

5. (a) Similar to Problem 1 from the last homework, the magnetic field and flux through the ring is

$$B = \mu_0 n I$$
$$\Phi = \int B \, \mathrm{d}a = \mu_0 n I_s \pi a^2$$

Then the emf and current is

$$\mathcal{E} = -\dot{\Phi} = -\mu_0 n \pi a^2 \frac{\mathrm{d}I_s}{\mathrm{d}t}$$
$$I_r = -\frac{\mu_0 n \pi a^2}{R} \frac{\mathrm{d}I_s}{\mathrm{d}t}$$

(b) The induced electric field due to the changing flux is

$$\oint \mathbf{E} \cdot \mathrm{d}\mathbf{l} = \mathcal{E}$$
$$E = \mathcal{E}/2\pi a$$

The magnetic field from I_r is given by Ampère's law,

$$B = \frac{\mu_0 I_r}{2\pi b}$$

The magnitude of the Poynting vector is

$$S = \frac{1}{\mu_0} E B = \frac{\mathcal{E} I_r}{4\pi^2 a b}$$

Integrating over the cylindrical area of the solenoid,

$$P = \int_0^{2\pi} \int_0^L \frac{\mathcal{E} I_r}{4\pi a b} s \, \mathrm{d}z \, \mathrm{d}\phi \bigg|_{s=a}$$
$$= \frac{\mathcal{E} I_r L}{2\pi b}$$

...not really sure where to go from here