5. **Proposition:** if two integers have the opposite parity, their product is even.
Proof. Let a have even parity and b have odd parity, then a and be can be expressed as

$$\begin{aligned} a &= 2n \\ b &= 2m + 1 && \text{where } n, m \in \mathbb{Z} \end{aligned}$$

The product ab becomes

$$\begin{aligned} ab &= (2n)(2m + 1) \\ &= 4nm + 2n \\ &= 2(\underbrace{2nm + n}_{c}) && \text{Closure, } c \in \mathbb{Z} \\ &= 2c && \text{The result is always even.} \quad \square \end{aligned}$$

Two cases are not needed since multiplication is commutative, i.e. $ab = ba$.