Homework 12

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9.1.1 Determine the payoff matrices for the following two-person zero-sum game.

(d) P_1 selects a number n from $\{1, 2, 3\}$, and P_2 is given two guesses. (P_2's guesses must be from $\{1, 2, 3\}$ but need not be distinct.) After P_2 makes her two guesses, P_1 reveals his selected number n. If P_2 did not guess n, P_1 wins $2n$ from P_2; if P_2 did guess n, P_2 wins from P_1 an amount equal to P_2's guess.

Solution. Considering the distinct permutations, the payoff matrix will look like:

	$(1,2)$	$(2,3)$	$(3,1)$
1	-1	2	-1
2	-2	-2	-4
3	-6	-3	-3

9.2.1 Find strategy pairs that satisfy Principles I and II for the games with the following payoff matrices:

(a) $\begin{bmatrix} 3 & 1 & 2 \\ 1 & 0 & 5 \end{bmatrix}$

Solution. Player 1's security level is maximized in row 1. Player 2's security level is maximized with column 2. The corresponding strategy pair is $\boxed{(s_1, t_2).}$

(b) $\begin{bmatrix} 7 & 1 & 5 & 9 \\ 1 & 0 & 3 & 2 \\ 6 & 3 & 6 & 4 \end{bmatrix}$

Solution. Similarly, $\boxed{(s_2, t_2).}$

9.3.2 For each of the following payoff matrices, determine the set of values of x for which game has a saddle point, and for x in this set, determine the saddle point.

(c) $\begin{bmatrix} x & 1 \\ 3 & x \end{bmatrix}$

Solution. For Player 1, the minimum of the rows are $\min(1, x)$ and $\min(3, x)$. For Player 2, the maximum of the columns are $\max(x, 3)$ and $\max(1, x)$.
We can iterate over the first few integers (not sure if this is the best way to do this...),

x	u_1	u_2	Saddle?
0	0	1	no
1	1	1	yes
2	2	2	yes
3	3	3	yes
4	3	4	no

For a saddle point to occur, $x \in \{1, 2, 3\}$ and the saddle point occurs on the x.

9.4.2 For the matrix game A,

$$A = \begin{bmatrix} -1 & 1 & 2 & 0 \\ 4 & -2 & -3 & 2 \\ 0 & 3 & 1 & -2 \end{bmatrix}$$

(a) Compute P_1's security level for $X_1 = (2/3, 1/3, 0)$ and $X_2 = (1/3, 1/3, 1/3)$.

Solution. Player 1's security level is given by

$$u_1^{(1)} = \min_{Y \in T} \begin{pmatrix} 2/3 \\ 1/3 \\ 0 \end{pmatrix}^T A \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix}$$
$$= \min_{Y \in T} \begin{pmatrix} 2/3 & 0 & 1/3 & 2/3 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix}$$

This is minimized for $Y = (0, 1, 0, 0)$ with $\boxed{u_1^{(1)} = 0.}$ For $X_2 = (1/3, 1/3, 1/3)$, we find

$$u_1^{(2)} = \min_{Y \in T} \begin{pmatrix} 1 & 2/3 & 0 & 0 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix}$$

This is minimized with either $Y = (0, 0, 1, 0)$, $Y = (0, 0, 0, 1)$, or a linear combination, with $\boxed{u_1^{(2)} = 0.}$

(b) Compute P_2's security level for $Y_1 = (1/4, 1/4, 1/4, 1/4)$ and $Y_2 = (0, 1/2, 0, 1/2)$.

Solution. Player 2's security level can be found with

$$u_2^{(1)} = \max_{X \in S} \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} \begin{pmatrix} 1/2 \\ 1/4 \\ 1/2 \end{pmatrix}$$

This is maximized for either $X = (1, 0, 0)$, $X = (0, 0, 1)$ or a linear combination of these two with $\boxed{u_2^{(1)} = 1/2.}$ For the other mixed strategy, the security level is

$$u_2^{(2)} = \max_{X \in S} \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} \begin{pmatrix} 1/2 \\ 0 \\ 1/2 \end{pmatrix}$$

This is maximized in the same way with $\boxed{u_2^{(2)} = 1/2.}$

(c) What can you now conclude about v_1 and v_2?

Solution. We can say that $v_1 \geq 0$ and $v_2 \leq 1/2$, or

$$\boxed{0 \leq v_1 \leq v_2 \leq 1/2.}$$