Homework 2

PHYSICS 342
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1. (a) From the last homework, the magnetic field within the solenoid is

$$B = \mu_0 n I_0 \cos \omega t$$

Assuming the loop is normal to B, the flux through a loop of radius $a/2$ is given by

$$\begin{aligned}\Phi &= \int \mathbf{B} \cdot \mathrm{d}\mathbf{a} \\ &= \mu_0 n \pi (a/2)^2 I_0 \cos \omega t\end{aligned}$$

Creating an emf of

$$\mathcal{E} = -\dot{\Phi} = \frac{\mu_0 n \pi a^2 I_0 \omega}{4} \sin \omega t$$

The current induced is

$$I = \frac{\mu_0 n \pi a^2 I_0 \omega}{4R} \sin \omega t$$

(b) Since we have the emf from (a),

$$\begin{aligned}E &= \mathcal{E} \Big/ \int \mathrm{d}\ell \\ &= \frac{\mu_0 n \pi a^2 I_0 \omega}{4} \sin \omega t \, (2\pi a/2)^{-1} \\ &= \frac{\mu_0 n a I_0 \omega}{4} \sin \omega t\end{aligned}$$

2. From equation (7.25) and the mutual inductance of this geometry,

$$\begin{aligned}\mathcal{E}_2 &= -M \frac{\mathrm{d}I_1}{\mathrm{d}t} \\ &= -\left(\mu_0 \pi a^2 n_1 n_2\right) \left(-I_0 \omega \sin \omega t\right) \\ &= \mu_0 \pi a^2 n_1 n_2 I_0 \omega \sin \omega t\end{aligned}$$

3. (a) The flux through the lil loop is

$$\begin{aligned}\Phi &= \int \mathbf{B} \cdot \mathrm{d}a \\ &= BA \leftarrow \text{as it's uniform} \\ &= \frac{\mu_0 I \pi a^2}{2b}\end{aligned}$$

(b) If we treat the little loop as a magnetic dipole, its magnetic field is given by (5.88),

$$\begin{aligned}\mathbf{B}_{\text{dipole}} &= \frac{\mu_0 m}{4\pi r^3}\left(2\cos\theta\,\hat{\mathbf{r}} + \sin\theta\,\hat{\boldsymbol{\theta}}\right) \\ &= \frac{\mu_0 I\pi a^2}{4\pi r^3}\left(2\cos\theta\,\hat{\mathbf{r}} + \sin\theta\,\hat{\boldsymbol{\theta}}\right)\end{aligned}$$

If we consider the $+z$ axis to be normal to the loops, then $\theta = \pi/2$, and only the $\sin\theta\,\hat{\boldsymbol{\theta}}$ term remains. Then using the hint provided in the problem, we can integrate the flux over the outside of the loop and take its opposite,

$$\begin{aligned}\Phi &= -\int \mathbf{B}\cdot \mathrm{d}\mathbf{a} \\ &= -\frac{\mu_0 I a^2}{4}\int_b^\infty r^{-2}\,\mathrm{d}r \int_0^{2\pi}\mathrm{d}\phi \\ &= -\frac{\mu_0 I a^2}{4b}(2\pi) \\ &= -\frac{\mu_0 I\pi a^2}{2b}\end{aligned}$$

(c) Since the flux (fluxes?) are equal and opposite,

$$\begin{aligned}M_{12} = M_{21} &= \frac{\Phi}{I} \\ &= \frac{\mu_0\pi a^2}{2b}\end{aligned}$$

4. (a) From Example 7.11, the self-inductance is provided as

$$L = \frac{\mu_0 N^2 h}{2\pi}\ln(b/a)$$

The energy stored in the coil is

$$W = \frac{LI^2}{2} = \frac{\mu_0 N^2 h I^2}{4\pi}\ln(b/a)$$

(b) Starting from the magnetic field in Example 7.11,

$$\begin{aligned}W &= \frac{1}{2\mu_0}\int_{\mathbb{R}^3} B^2\,\mathrm{d}\tau \\ &= \frac{2\pi h}{2\mu_0}\left(\frac{\mu_0 N I}{2\pi}\right)^2\int_a^b s^{-2}s\,\mathrm{d}s \\ &= \frac{h\mu_0 N^2 I^2}{4\pi}\ln(b/a)\end{aligned}$$

5. (a) The magnetic flux within the solenoid from 1(a) is given by

$$\Phi = \mu_0 n\pi s^2 I_0\cos\omega t$$

Relating this to the electric field,

$$\begin{aligned} E &= -\frac{1}{2\pi s}\frac{\mathrm{d}\Phi}{\mathrm{d}t} \\ &= \frac{\mu_0 n s I_0 \omega}{2}\sin\omega t \end{aligned}$$

The displacement current is given by the time rate-of-change of E,

$$\begin{aligned} \mathbf{J}_d &= \epsilon_0 \frac{\mathrm{d}\mathbf{E}}{\mathrm{d}t} \\ &= \frac{\epsilon_0 \mu_0 n s I_0 \omega^2}{2}\cos\omega t\,\hat{\boldsymbol{\phi}} \end{aligned}$$

(b) The total current per cylinder length is found as

$$\begin{aligned} \mathbf{K}_d &= \int \mathbf{J}_d\,\mathrm{d}\ell \\ &= \int_0^L \left(\frac{\epsilon_0 \mu_0 n s I_0 \omega^2}{2}\cos\omega t\right)_{s=a} \mathrm{d}z\ \hat{\boldsymbol{\phi}} \\ &= \frac{\epsilon_0 \mu_0 n a I_0 \omega^2}{2}\cos\omega t\,\hat{\boldsymbol{\phi}} \end{aligned}$$