Problem Set 2

PHYSICS 443
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1. (a) Linear polarization, $63°$ from the x axis.
   (b) Right elliptical polarization.
   (c) Left circular polarization.
2. (a) If we add a $\pi/2$ phase shift,
   $$\begin{bmatrix} -2 \\ 1 \end{bmatrix}$$
   It's still linearly polarized, just out of phase.
   (b) Using a left elliptical polarization, these matrices are orthogonal
   $$\begin{bmatrix} 2 \\ i \end{bmatrix} \text{ or } \begin{bmatrix} -2i \\ 1 \end{bmatrix}$$
   (The relative difference is the same, right?)
   (c) Using a right circular polarization,
   $$\begin{bmatrix} 1 \\ -i \end{bmatrix}$$
3. (a) Reusing the diagram and conventions from in class, where
   $$\begin{aligned} n_1 &= \text{higher/slower index } (y) \\ n_2 &= \text{lower/faster index } (x) \end{aligned}$$
   Then piggybacking from the quarter-wave derivation,
   $$\begin{aligned} \Delta\phi = k_y d - k_x d &= \frac{2\pi d}{\lambda}(n_1 - n_2) \\ \pi &= \frac{2\pi d}{\lambda}(n_1 - n_2) \\ d(n_1 - n_2) &= \frac{\lambda}{2} \\ &= \frac{\lambda}{2} + m\lambda \\ &\text{(where } m \in \mathbb{Z}\text{, for higher orders)} \end{aligned}$$
   (b) Finding the Eigenvalues λ,
   $$\begin{aligned} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \mathbf{M} &= \lambda \mathbf{M} \\ \begin{vmatrix} 1-\lambda & 0 \\ 0 & -1-\lambda \end{vmatrix} &= 0 \\ (1-\lambda)(-1-\lambda) &= 0 \\ \lambda &= \pm 1 \end{aligned}$$

Substituting the found λ values into the original equation, the Eigenvectors can be found.

$$\begin{bmatrix} 1-\lambda & 0 \\ 0 & -1-\lambda \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = 0$$
$$(1-\lambda)\, A = 0$$
$$(-1-\lambda)\, B = 0$$

For $\lambda = 1$, $B = 0$ and the relative Jones vector is $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

Similarly for $\lambda = -1$, the Jones vector is $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

(c) A linearly polarized wave at $45°$ can be represented with a Jones vector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$, and using the Jones matrix from (b),

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

The x-component remains the same, but now the y-component is "flipped." The wave is now phase shifted by a half-wave and is linearly polarized at $225°$.