Homework 11

MATH 364
November 30, 2021

Kevin Evans
ID: 11571810

6.3.2 Solve using the cutting plane algorithm.

(b) Maximize $3x_1 + 8x_2$
subject to

$$\begin{array}{lcll} x_1 & + & 2x_2 & \leq 9 \\ & & 2x_2 & \leq 5 \end{array}$$

$x \geq 0, x \in \mathbb{Z}^2$

Solution. Putting this in a tableau and applying the simplex method to solve the LP relaxation,

$$\begin{array}{cc|cc|cc}
x_1 & x_2 & x_3 & x_4 & & \\
\hline
\boxed{1} & 2 & 1 & 0 & 9 & \boxed{x_3} \\
0 & 2 & 0 & 1 & 5 & x_4 \\
\hline
\boxed{3} & 8 & 0 & 0 & 0 & -z \\
\hline
1 & 2 & 1 & 0 & 9 & x_1 \\
0 & \boxed{2} & 0 & 1 & 5 & \boxed{x_4} \\
\hline
0 & \boxed{2} & -3 & 0 & -27 & -z \\
\hline
1 & 0 & 1 & -1 & 4 & x_1 \\
0 & 1 & 0 & 1/2 & 5/2 & x_2 \\
\hline
0 & 0 & -3 & -1 & -32 & -z \\
\hline
\end{array}$$

The LP relaxation is optimal at $x^* = (4, 5/2)$, $z^* = 32$. The second constraint has a non-integral coefficient,

$$x_2 + (1/2)x_4 = 5/2.$$

Rearranging this to integral and nonintegral parts,

$$\begin{aligned} x_2 + (1/2)x_4 &= 2 + 1/2 \\ (1/2)x_4 - 1/2 &= -x_2 + 2. \\ \implies (1/2)x_4 - 1/2 &\geq 0 \quad (\in \mathbb{Z}) \end{aligned}$$

We can add a new constraint,

$$\boxed{(-1/2)x_4 + x_5 = -1/2.}$$

The modified tableau is

x_1	x_2	x_3	x_4	x_5		
1	0	1	−1	0	4	x_1
0	1	0	1/2	0	5/2	x_2
0	0	0	$\boxed{-1/2}$	1	$\boxed{-1/2}$	x_5
0	0	−3	−1	0	−32	$-z$
1	0	1	0	−2	5	x_1
0	1	0	0	1	2	x_2
0	0	0	1	−2	1	x_4
0	0	−3	0	−2	−31	$-z$

The optimal solution to the IP is $\bar{x} = (5, 2)$, $\bar{z} = 31$.

6.4.2 Solve using the branch and bound approach.

(a) Maximize $4x_1 + 5x_2 + 3x_3$
subject to

$$\begin{array}{rcrcrcr} 3x_1 & - & 2x_2 & + & x_3 & \leq & 15 \\ x_1 & + & 2x_2 & + & x_3 & \leq & 8 \end{array}$$

$x \leq 0, x \in \mathbb{Z}^3$

Solution. Using a computer solver, the LP relaxation (1) solution is $x^{(1)} = (5.75, 1.125, 0)$, $z^{(1)} = 28.625$.

i. $L = \{(1)\}$
$B = \varnothing$
$\bar{z} = -\infty$
Looking at (1), we can split the solution on $x_1^{(1)} = 5.75$, we can add

$$x_1^{(2)} \leq 5 \tag{2}$$
$$x_1^{(3)} \geq 6. \tag{3}$$

ii. $L = \{(2), (3)\}$
$B = \varnothing$
$\bar{z} = -\infty$
Looking at (2), $x^{(2)} = (5, 0.75, 1.5)$, $z^{(2)} = 28.25$. We'll add constraints

$$x_2^{(4)} = 0 \tag{4}$$
$$x_2^{(5)} \geq 1. \tag{5}$$

iii. $L = \{(3), (4), (5)\}$
$B = \varnothing$
$\bar{z} = -\infty$
Looking at (5), $x^{(5)} = (5, 1, 1)$ and $z^{(5)} = 28$. This is integer and we can add it into B and set $\bar{z}$.

iv. $L = \{(3), (4)\}$ 3 is infeasible and can just be removed from the list.
$B = \{(5)\}$
$\bar{z} = 28$

v. $L = \{(4)\}$
$B = \{(5)\}$
$\bar{z} = 28$
Looking at (4), we'll have $x^(4) = (3.5, 0, 4.5)$ and $z^{(4)} = 27.5$. Since the objective value is worse than (5), we can ignore this one.

vi. $L = \varnothing$
$B = \{(5)\}$
$\bar{z} = 28$
The algorithm is now finished and we're left with an optimal solution from (5):

$$\bar{x} = (5, 1, 1)$$
$$\bar{z} = 28$$