Problem Set 9

PHYSICS 463
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1. ***Impurity orbits.*** Indium antimonide has $E_g = 0.23\,\mathrm{eV}$; dielectric constant $\epsilon = 18$; electron effective mass $m_e = 0.015m$.

   (a) Calculate the donor ionization energy.

   Solution. Using (51),

$$\begin{aligned} E_d &= \frac{e^4 m_e}{2(4\pi\epsilon\epsilon_0\hbar)^2} \\ &= \frac{e^4(0.015m)}{2(4\pi 18\epsilon_0\hbar)^2} \\ &= 1.0092 \times 10^{-22}\,\mathrm{J} = 0.629\,\mathrm{meV}. \end{aligned}$$

   (b) Calculate the radius of the ground state orbit.

   Solution. Using (52), the Bohr radius of the donor is

$$\begin{aligned} a_d &= \frac{4\pi\epsilon\epsilon_0\hbar^2}{m_e e^2} \\ &= \frac{4\pi 18\epsilon_0\hbar^2}{0.015me^2} \\ &= 6.35 \times 10^{-8}\,\mathrm{m} = 635\,\text{Å}. \end{aligned}$$

   (c) At what minimum donor concentration will appreciable overlap effects between the orbits of adjacent impurity atoms occur? This overlap tends to produce an impurity band—a band of energy levels which permit conductivity presumably by a hopping mechanism in which electrons move from one impurity site to a neighboring ionized impurity site.

   Solution. We can approximate the concentration $N_d \approxeq 1/R^3$, so

$$N_d \approx 1/(6.35 \times 10^{-8}\,\mathrm{m})^3 = 4 \times 10^{21}\,\mathrm{m}^{-3}.$$

2. ***Ionization of donors.*** In a particular semiconductor, there are 10^{13} donors/cm^3 with ionization energy E_d of $1\,\mathrm{meV}$ and effective mass $0.01m$.

   (a) Estimate the concentration of conduction atoms at $4\,\mathrm{K}$.

   Solution. The conduction concentration can be found using the notes of 9.3. The coefficient is

$$\begin{aligned} N_c &= 2\left(\frac{m_e k_B T}{2\pi\hbar^2}\right)^{3/2} \\ &= 2\left(\frac{0.01 m k_B \times 4\,\mathrm{K}}{2\pi\hbar^2}\right)^{3/2} = 3.86\times 10^{13}\,\mathrm{cm}^{-3}. \end{aligned}$$

   With the approximation $\epsilon_F \approx \frac{1}{2}E_g$, the electron concentration is

$$\begin{aligned} n &\approxeq (N_c N_d)^{1/2} e^{-E_d/2k_BT} \\ &= \left[\left(3.86\times 10^{13}\,\mathrm{cm}^{-3}\right)\left(10^{13}\mathrm{cm}^{-3}\right)\right]^{1/2} \times \exp(-1\,\mathrm{meV}/2k_B\cdot 4\,\mathrm{K}) \\ &= 4.61\times 10^{12}\,\mathrm{cm}^{-3}. \end{aligned}$$

   (b) What is the value of the Hall coefficient? Assume no acceptor atoms are present and that $E_g \gg k_BT$.

   Solution. From (7.55),

$$\begin{aligned} R_H &= -\frac{1}{ne} = -\frac{1}{4.61\times 10^{12}\,\mathrm{cm}^{-3}\times e} \\ &= -1.355\,\mathrm{m}^3\cdot\mathrm{C}^{-1}. \end{aligned}$$

3. ***Magnetoresistance with two carrier types.*** Problem 6.9 shows that in the drift velocity approximation, the motion of charge carriers in electric and magnetic fields do not lead to transverse magnetoresistance. The result is different with two carrier types.

   Consider a conductor with a concentration n of electrons of effective mass m_e and relaxation time τ_e; and a concentration p of holes with effective mass m_h and relaxation time τ_h. Treat the limit of very strong magnetic fields, $\omega_c\tau \gg 1$.

   (a) Show in this limit that

$$\sigma_{yx} = (n-p)ec/B.$$

   Solution. From the in-class discussion, we've showed that the total conductivity of a semiconductor is given by electron and hole parts. We can write the magnetoconductivity tensor as

$$\begin{aligned}
\sigma &= \sigma^e + \sigma^h \\
&= \frac{\sigma_0^e}{1+(\omega_c^e\tau_e)^2}\begin{pmatrix} 1 & -\omega_c^e\tau_e & 0 \\ \omega_c^e\tau_e & 1 & 0 \\ 0 & 0 & 1+(\omega_c^e\tau_e)^2 \end{pmatrix} \\
&+ \frac{\sigma_0^h}{1+(\omega_c^h\tau_h)^2}\begin{pmatrix} 1 & \omega_c^h\tau_h & 0 \\ -\omega_c^h\tau_h & 1 & 0 \\ 0 & 0 & 1+(\omega_c^h\tau_h)^2 \end{pmatrix}
\end{aligned}$$

   Note that above in the hole matrix, we flip sign due to the positive charges flowing opposite of the electrons, in (6.51). Then, the matrix yx element is

$$\begin{aligned}
\sigma_{yx} &= \frac{\sigma_0^e}{1+(\omega_c^e\tau_e)^2}\left(\omega_c^e\tau_e\right) - \frac{\sigma_0^h}{1+(\omega_c^h\tau_h)^2}\left(\omega_c^h\tau_h\right) \\
&\approx \frac{\sigma_0^e}{\omega_c^e\tau_e} - \frac{\sigma_0^h}{\omega_c^h\tau_h} \qquad \text{(in } \omega_c\tau \gg 1\text{)} \\
&= \frac{ne^2\tau_e}{m_e}\frac{m_e c}{eB\tau_e} - \frac{pe^2\tau_p}{m_p}\frac{m_p c}{eB\tau_p} = (n-p)ec/B. \quad \square
\end{aligned}$$

   (b) Show that the Hall field is given by, with $Q \equiv \omega_c\tau$,

$$E_y = -(n-p)\left(\frac{n}{Q_e} + \frac{p}{Q_h}\right)^{-1} E_x,$$

   which vanishes if $n = p$.

   Solution. In the Hall field case, we can set $j_y = 0$. Then from the magnetoconductivity tensor in (a) and using (6.64),

$$\begin{aligned}
j_y = 0 &= \sigma_{yx}E_x + \left(\frac{\sigma_0^e}{Q_e^2} + \frac{\sigma_0^h}{Q_h^2}\right)E_y \\
E_y &= -(n-p)\left(\frac{\sigma_0^e}{Q_e^2} + \frac{\sigma_0^h}{Q_h^2}\right)^{-1}(ec/B)E_x \\
&= -(n-p)\left(\frac{ne^2\tau_e}{m_e Q_e^2} + \frac{pe^2\tau_h}{m_h Q_h^2}\right)^{-1}(ec/B)E_x
\end{aligned}$$

Using $\omega_c = eB/mc \implies Q = eB\tau/mc$, we can bring in the ec/B term as a factor of Q's,

$$E_y = -(n-p)\left(\frac{n}{Q_e} + \frac{p}{Q_h}\right)^{-1} E_x. \quad \square$$

(c) Show that the effective conductivity in the x direction is

$$\sigma_{\text{eff}} = \frac{ec}{B}\left[\left(\frac{n}{Q_e} + \frac{p}{Q_h}\right) + (n-p)^2\left(\frac{n}{Q_e} + \frac{p}{Q_h}\right)^{-1}\right].$$

If $n = p$, $\sigma \propto B^{-2}$. If $n \neq p$, σ saturates in strong fields; that is, it approaches a limit independent of B as $B \to \infty$.

Solution. Similarly, for the x direction, we have

$$\begin{aligned}
j_x &= \sigma_{xx}E_x + \sigma_{xy}E_y \\
&= \left(\frac{\sigma_0^e}{Q_e^2} + \frac{\sigma_0^h}{Q_h^2}\right)E_x - \left[(n-p)ec/B\right]E_y \\
&= \left(\frac{ne^2\tau_e}{m_eQ_e^2} + \frac{pe^2\tau_h}{m_hQ_h^2}\right)E_x + \left[(n-p)ec/B\right]\left[(n-p)\left(\frac{n}{Q_e} + \frac{p}{Q_h}\right)^{-1}\right]E_x \quad \text{from (b)} \\
&= \underbrace{\frac{ec}{B}\left[\left(\frac{n}{Q_e} + \frac{p}{Q_h}\right) + (n-p)^2\left(\frac{n}{Q_e} + \frac{p}{Q_h}\right)^{-1}\right]}_{\sigma_{\text{eff}}}E_x
\end{aligned}$$