Homework 3

PHYSICS 461
February 9, 2021

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1. (a) Equating its potential to the electron's rest mass,

$$\frac{e^2}{4\pi\epsilon_0 r_c} = mc^2$$
$$r_c = 2.84 \times 10^{-15}\,\mathrm{m}$$

(b) For a spin of $1/2$, we can equate its angular momentum to the classical definition as

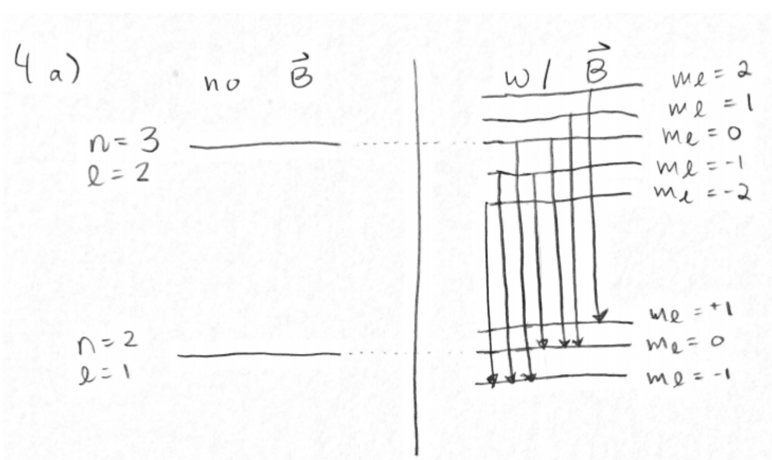
$$S^2 = \hbar^2 s(s+1) = (mvr)^2$$
$$v^2 = \frac{3\hbar^2}{4{m_e}^2{r_c}^2}$$
$$= \frac{3\left(1.054\times10^{-34}\,\mathrm{J\cdot s}\right)^2}{4\left(9.11\times10^{-31}\,\mathrm{kg}\right)^2\left(2.84\times10^{-15}\,\mathrm{kg}\right)^2}$$
$$= 1.24\times10^{21}\,\mathrm{m\cdot s^{-1}}$$

It's roughly 100 times c, which doesn't make sense.

2. (a) The $\ell = 2$ exceeds the maximum for $n = 2$.
   (b) The $m_\ell = 2$ exceeds the maximum for $l = 1$.
   (c) The spin m_s can only be $\pm 1/2$.
   (d) Can't have a negative ℓ value.

3. Since we're only looking at the valance electrons, the number of lines is determined by unpaired spins
   (a) 2, as it's basically an H atom.
   (b) 1 as the s orbital is filled.
   (c) 5 since the p orbital is partially filled and there is only one paired. This means $j = 1 + \frac{1}{2} + \frac{1}{2} = 2$, and it would have five lines.
   (d) 5 for the same reasoning as (c).

4. (a, b)



(c) As $m_\ell = 0$ or $m_\ell = \pm 1$, the allowable energies are either $\Delta E = 0$ or $\Delta E = \pm\mu_B \Delta m B$

5. Between $n = 3$ and $n = 2$, the

(a) Without a magnetic field, the energy between the $n = 3 \to 2$ transition is

$$E_{3\to 2} = -13.6\,\mathrm{eV}\left(3^{-1} - 2^{-1}\right) = 1.9\,\mathrm{eV}$$

Since the selection rules dictate $m_l = 0, \pm 1$,

$$\begin{aligned}\lambda &= \frac{hc}{E \pm \mu_B B m_l} \\ &= 0.197\,\mathrm{eV}\cdot\mu\mathrm{m} \times \frac{1}{1.9\,\mathrm{eV} \pm 5.78\times 10^{-5}\,\mathrm{eV/T} \times 3.5\,\mathrm{T} \times m_l} \\ &= 103.68\,\mathrm{nm}, 103.67\,\mathrm{nm}, 103.70\,\mathrm{nm}\end{aligned}$$

(b) The same as (a)?

6. (a) For $n = 4$,
   - $l = 0$: $4\,\mathrm{S}_{1/2}$
   - $l = 1$: $4\,\mathrm{P}_{1/2}$, $4\,\mathrm{P}_{3/2}$
   - $l = 2$: $4\,\mathrm{D}_{3/2}$, $4\,\mathrm{S}_{5/2}$
   - $l = 3$, $4\,\mathrm{F}_{5/2}$, $4\,\mathrm{F}_{7/2}$

   (b) 2 for each level?

7. (a) $j = \frac{3}{2}, \frac{5}{2}$

   (b) $|J| = \hbar\sqrt{j(j+1)} = \frac{\hbar\sqrt{15}}{2}, \frac{\hbar\sqrt{35}}{2}$

   (c) $J_z = \begin{cases} \left\{-\frac{3\hbar}{2}, -\frac{\hbar}{2}, \frac{\hbar}{2}, \frac{3\hbar}{2}\right\} & j = \frac{3}{2} \\ \left\{-\frac{5\hbar}{2}, -\frac{3\hbar}{2}, -\frac{\hbar}{2}, \frac{\hbar}{2}, \frac{3\hbar}{2}, \frac{5\hbar}{2}\right\} & j = \frac{5}{2} \end{cases}$

8. (a) $\ell = 2, 1, 0$

   (b) $s = 1, 0$

   (c) $j = 3, 2, 1, 0$

   (d) $j_1, j_2 = \frac{3}{2}, \frac{1}{2}$

   (e) $j = j_1 + j_2, \ldots, |j_1 - j_2| = 3, 2, 1, 0$. It's the same as (c)

9. (a) For $589.0\,\mathrm{nm}$ and $589.6\,\mathrm{nm}$,

$$\begin{aligned}E_{1/2} &= \frac{1240\,\mathrm{eV}\cdot\mathrm{nm}}{589.6\,\mathrm{nm}} = 2.103\,\mathrm{eV} \\ E_{3/2} &= \frac{1240\,\mathrm{eV}\cdot\mathrm{nm}}{589.0\,\mathrm{nm}} = 2.105\,\mathrm{eV}\end{aligned}$$

   (b) $\Delta E = 0.002\,14\,\mathrm{eV}$ (or is it half of this?)

(c) The strength of the magnetic field is

$$\begin{aligned}
\Delta E &= 2\mu_B B \\
B &= \frac{\Delta E}{2\mu_B} \\
&= \frac{0.002\,14\,\mathrm{eV}}{2 \times 5.788 \times 10^{-5}\,\mathrm{eV} \cdot \mathrm{T}^{-1}} \\
&= 18.51\,\mathrm{T}
\end{aligned}$$

10. (a) As there is 9 peaks, we can use $2f + 1 = 9$, leading to a total spin of $f = 4$. The lower energy level would result in $f = 3$. Since $s = 1/2$, the nuclear spin must be $i = 7/2$.

(b) Energy is added to drive the m_j to their maximum/minimum state. This can be done by laser light incident on the Cesium beam.