Homework 2

PHYSICS 304
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Problems

0. For three spin-1/2, the total angular momenta are $j_s = s_1 + s_2 + s_3 = 3/2$ and $1/2$.

j_s	s_1	s_2	s_3
$1/2$	-	-	+
$1/2$	-	+	-
$1/2$	+	-	-
$1/2$	+	+	-
$3/2$	+	-	+
$3/2$	-	+	+
$3/2$	+	+	+
$3/2$	-	-	-

10. For $n = 2$, $\ell \in \{0, 1\}$. Then, $j \in \{1/2, 3/2\}$.

$$j = \frac{1}{2} \implies m_j \in \left\{-\frac{1}{2}, \frac{1}{2}\right\}$$
$$j = \frac{3}{2} \implies m_j \in \left\{-\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}\right\}$$

11. For a d-electron, $l = 2$ and $j \in \left\{\frac{1}{2}, \frac{3}{2}, \frac{5}{2}\right\}$.

$$j = \frac{1}{2} \implies m_j \in \left\{\pm\frac{1}{2}\right\}$$
$$j = \frac{3}{2} \implies m_j \in \left\{\pm\frac{1}{2}, \pm\frac{3}{2}\right\}$$
$$j = \frac{5}{2} \implies m_j \in \left\{\pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}\right\}$$

12. (a) $7\mathrm{G}_{9/2}$
    (b) $6\mathrm{H}_{11/2}$, $6\mathrm{H}_{9/2}$, $6\mathrm{H}_{7/2}$, $6\mathrm{H}_{5/2}$, $6\mathrm{H}_{3/2}$, $6\mathrm{H}_{1/2}$

17. With no potential inside the cube of side length L,

$$-\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r}) = E\psi(\mathbf{r})$$

The solutions for ψ would have the form

$$\psi(e_i) = \sin(k_i \pi e_i) \implies k_i = \frac{n_i \pi}{L}$$

Then the momentum in each direction and total energy is:

$$p_i = \hbar k_i = \frac{\pi\hbar}{L} n_i$$
$$E_{\text{total}} = \frac{\pi^2\hbar^2}{2mL^2}\left(n_x^2 + n_y^2 + n_z^2\right) = \frac{\pi^2\hbar^2 n^2}{2mL^2}$$

Applying the dimensions given in the problem,

$$\begin{aligned} E_{\text{total}} &\approx \frac{\pi^2 \left(0.1973\,\text{eV}\cdot\mu\text{m}\right)^2}{2\left(0.511\times 10^6\,\text{eV}/\text{c}^2\right)\left(0.2\times 10^{-3}\,\mu\text{m}\right)^2}\, n^2 \\ &\approx 9.398 n^2 \quad [\text{eV}] \end{aligned}$$

(a) For electrons, each state can contain two electrons. In the $n_i = 1$ ($n^2 = 3$) state, 6 electrons would be contained, each with energy

$$E_{111} = 3 \times 9.398$$

The remaining two electrons would be in the threefold-degenerate state with $n^2 = 6$,

$$E_{112} = E_{121} = E_{211} = 6 \times 9.398$$

The lowest energy possible is then

$$\begin{aligned} E_{\text{total}} &= 6 \times 3 \times 9.398 + 2 \times 6 \times 9.398 \\ &= 281.7\,\text{eV}? \end{aligned}$$

Not sure what part I'm not understanding as the answer given is $395\,\text{eV}$.

(b) For bosons, each particle would fall to the ground state,

$$\begin{aligned} E_{\text{total}} &= 8E_{111} \\ &= 225.6\,\text{eV} \end{aligned}$$

21. (a) $1s^2 2s^2 2p^4$

(b)

n	ℓ	m_ℓ	m_s
1	0	0	$-1/2$
1	0	0	$1/2$
2	0	0	$-1/2$
2	0	0	$1/2$
2	1	0	$-1/2$
2	1	0	$1/2$
2	1	± 1	$-1/2$
2	1	± 1	$1/2$

Questions

1. After $Z = 83$, the repulsive Coulomb force between the protons is greater than the nuclear force holding the protons and neutrons together.

2. The frequency also doubles as $\omega \propto |\mathbf{B}|$.

3. Because there's an uneven number of protons and neutrons.

4. Y would be more unstable. If X has a higher binding energy, that would mean it there's more energy holding it together.