Problem Set 3

PHYSICS 443
September 21, 2020

Kevin Evans
ID: 11571810

---

1. Using Snell's law, for an air-to-glass interface at $45°$, the transmission angle is given as

$$\theta_t = \sin^{-1}\left(\frac{\sin(45°)}{1.5}\right) = 28.13°$$

   (a) For S-polarized light,

$$\begin{aligned} r_\perp &= \frac{n_i \cos\theta - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t} = \frac{\cos(45°) - 1.5\cos(28°)}{\cos(45°) + 1.5\cos(28°)} = -0.304 \\ R_\perp &= r_\perp^2 \approx 0.092 \end{aligned}$$

   (b) For P-polarized light,

$$\begin{aligned} r_\parallel &= \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_i \cos\theta_t + n_t \cos\theta_i} = 0.092 \\ R_\parallel &= r_\parallel^2 \approx 0.0085 \end{aligned}$$

2. For an interface with air, $n_t \approx 1$ and $n_{ti} \approx 1/n_i$. For the critical angles,

$$\begin{aligned} \theta_c &= \sin^{-1}(1/n_i) \\ &= 48.8° \quad \text{for water} \\ &= 34.4° \quad \text{for sapphire} \end{aligned}$$

   For the Brewster angles,

$$\begin{aligned} \theta_p &= \tan^{-1}(1/n_i) \\ &= 36.9° \quad \text{for water} \\ &= 29.5° \quad \text{for sapphire} \end{aligned}$$

3. (a) Assuming $n_t = 1$, the decay constant

$$\begin{aligned} \beta &= \frac{2\pi n_t}{\lambda_0}\left[\left(\frac{n_i}{n_t}\right)^2 \sin^2(\theta_i) - 1\right]^{1/2} \\ &= \frac{2\pi\,(1)}{589\,\text{nm}}\left[\left(\frac{1.6}{1}\right)^2 \sin^2(45°) - 1\right]^{1/2} \\ &= 5.645 \times 10^6\,\text{m}^{-1} \end{aligned}$$

   (b) For $n_t = 1.33$, the critical angle increases to $\theta_c = 56.2°$ and TIR no longer occurs.

4. For this problem, I am assuming:

   - The Brewster window is in air, $n_i = 1$.
   - The incident waves are half s- and p-polarized and the transmitted waves remain s- and p-polarized.

   For s-polarized waves, the transmission coefficient and transmittance are given from the Fresnel equations,

$$t_\perp = \frac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t}$$
$$T_\perp = \frac{n_t \cos\theta_t}{n_i \cos\theta_i} t^2 = \frac{n_t \cos\theta_t}{n_i \cos\theta_i} \left(\frac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t} \right)^2$$

   Substituting $n_i = 1$ and $n_t = n$ and simplifying somewhat,

$$T_\perp = \frac{4n \cos\theta_t}{\cancel{\cos\theta_i}} \frac{\cos^{\cancel{2}}(\theta_i)}{(\cos\theta_i + n\cos\theta_t)^2} = \frac{4n \cos\theta_t \cos\theta_i}{(\cos\theta_i + n\cos\theta_t)^2}$$

   However Brewster's angle, $\theta_i + \theta_t = 90^\circ$, then

$$\tan\theta_i = \frac{n_t}{n_i} = n, \qquad \cos(\theta_t) = \cos(90^\circ - \theta_i) = \sin(\theta_i)$$

   Applying this to transmittance $T_\perp$,

$$\begin{aligned} T_\perp &= \frac{4n \sin\theta_i \cos\theta_i}{(\cos\theta_i + n\sin\theta_i)^2} = \frac{4n \sin\theta_i \cos\theta_i}{(\cos\theta_i + n^2 \cos\theta_i)^2} = \frac{4n \sin\theta_i \cos\theta_i}{\cos^2(\theta_i)\,(1+n^2)} \\ &= \frac{4n \tan\theta_i}{(1+n^2)^2} = \frac{4n^2}{(1+n^2)^2} \end{aligned}$$

   The degree of polarization is given as

$$P = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

   And since the intensity is proportional to the transmittance of the light, and the incident light is of equal parts s- and p-polarization, we can omit the common factors and write the degree of polarization as a function of $T_{\max}$ and $T_{\min}$.

$$P = \frac{T_{\max} - T_{\min}}{T_{\max} + T_{\min}}$$

   As the p-polarization is perfectly transmitted, $T_{\max} = T_\parallel = 1$. Then substituting $T_\perp$,

$$\boxed{P = \frac{T_\parallel - T_\perp}{T_\parallel + T_\perp} = \frac{1 - 4n^2/\left(1+n^2\right)^2}{1 + 4n^2/\left(1+n^2\right)^2}}$$

   For $n = 1.5$, the degree of polarization is $P \approx 0.08$.