Homework 5

MATH 364
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3.3.3 Use the simplex method to do the following problem. The problem is stated in cannonical form with basic variables x_2 and x_3. Notice that in the first step in the simplex method, either x_1 or x_4 can enter the basis.

$$\begin{aligned} \text{Minimize} \quad & -x_1 - 2x_4 + x_5 \\ \text{s.t.} \quad & x_1 + x_3 + 6x_4 + 3x_5 = 2 \\ & -3x_1 + x_2 + 3x_4 + x_5 = 3 \\ & x \geq 0. \end{aligned}$$

Solution. Rewriting this to the format used in-class,

$$\begin{aligned} x_1 + x_3 + 6x_4 + 3x_5 &= 2 \\ -3x_1 + x_2 + 3x_4 + x_5 &= 3 \\ -x_1 - 2x_4 + x_5 &= z \end{aligned}$$

We can try letting x_1 enter the basis. This leads to either $x_1 = 2$ (swap x_3) or $x_1 =$ anything (swap x_2). Swapping for x_3 is the more limiting constraint, so we can pivot on the first line. The system becomes

$$\begin{aligned} x_1 + x_3 + 6x_4 + 3x_5 &= 2 \\ x_2 + 3x_3 + 21x_4 + 10x_5 &= 9 \\ x_3 + 4x_4 + 4x_5 &= 2 + z \end{aligned}$$

All the coefficients in the objective constraint are positive, so we have arrived at the optimal solution,

$$\begin{aligned} x^* &= (2, 9, 0, 0, 0) \\ z^* &= -2. \end{aligned}$$

3.4.2 Solve using the simplex method.

(d) Minimize $x_3 - x_4$ subject to

$$\begin{aligned} x_1 - x_4 &= 5 \\ x_2 + 2x_3 &= 10 \\ x &\geq 0 \end{aligned}$$

Solution. Using the format from class,

$$\begin{aligned} x_1 - x_4 &= 5 \\ x_2 + 2x_3 &= 10 \\ x_3 - x_4 &= z. \end{aligned}$$

By Theorem 3.4.2, the objective function is unbounded below, as there is an index $s = 4$, where $c_s \leq 0$ and $a_{is} \leq 0 \,\forall\, i$.

(e) Minimize $-x_3 + x_4$ subject to the constraints of (d).

Solution. Using the format from class,

$$\begin{aligned} x_1 - x_4 &= 5 \\ x_2 + 2x_3 &= 10 \\ -x_3 + x_4 &= z \end{aligned}$$

We can swap x_3 for x_2, only needing to change the objective function,

$$\begin{aligned} x_1 - x_4 &= 5 \\ x_2 + 2x_3 &= 10 \\ x_2 + 2x_4 &= 20 + 2z. \end{aligned}$$

The coefficients are positive, so we're at an optimal solution,

$$\begin{aligned} x^* &= (5, 0, 5, 0) \\ z^* &= -10. \end{aligned}$$