Homework 12

PHYSICS 304
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Chapter 16: Cosmology

3. (a) From Hubble's law,

$$\begin{aligned} v &= H_0 R \\ R &= \frac{0.55 \times 3 \times 10^5 \,\mathrm{km/s}}{23 \times 10^{-6} \,\mathrm{km/s/ly}} \\ &= 7.17 \times 10^9 \,\mathrm{ly} \end{aligned}$$

(b) Assuming a constant speed,

$$\begin{aligned} t &= \frac{R}{v} = \frac{7.17 \times 10^9 \,\mathrm{ly}}{0.55c} \\ &= 13 \,\mathrm{Gyr} \end{aligned}$$

4. (a) For $a(t) = Ae^{bt}$, then $\dot{a}(t) = Abe^{bt} = ba(t)$ and

$$\begin{aligned} H(t) &= \frac{\dot{a}(t)}{a(t)} \\ &= b \quad \square \end{aligned}$$

(b) From (16.21),

$$\begin{aligned} a &= Ct^{2/3} \\ \dot{a} &= \frac{2}{3} Ct^{-1/3} \\ H(t) &= \frac{\dot{a}}{a} = \frac{2t^{-1/3}}{3t^{2/3}} \\ &= \frac{2}{3t} \end{aligned}$$

5. (a) From (16.22) and Hubble's law,

$$\begin{aligned} 1 + Z &= \frac{a(t_0)}{a(t_e)} = \frac{R(t_0)}{R(t_e)} \\ R_0 &= 6R_e \\ &= \frac{6v_e}{H_0} = \frac{6c}{H_0} \left(\frac{Z^2 + 2Z}{Z^2 + 2Z + 2} \right) \\ &= 5.676 \frac{c}{H_0} \\ &= 7.403 \times 10^{10} \,\mathrm{ly} \end{aligned}$$

(b) Using (16.23) and the result of 4(b),

$$\begin{aligned} t_0 &= 6^{3/2} t_e \\ &= 6^{3/2} \left(\frac{2}{3H(t_e)} \right) \\ &= 6^{3/2} \left(\frac{2}{3H(t_e)} \right) \\ &= 426 \times 10^3 \, \mathrm{ly}/(\mathrm{km/s}) \\ &= 1.28 \times 10^{11} \, \mathrm{yr} \end{aligned}$$

7. (a) For that redshift,

$$\begin{aligned} Z &= \frac{\Delta\lambda}{\lambda_0} \approx 0.500 \\ v &= c \left(\frac{0.5^2 + 1}{0.5^2 + 3} \right) = 0.385c \end{aligned}$$

(b) From (16.10),

$$\begin{aligned} R &= \frac{0.385c}{H_0} \\ &\approx 16.7 \times 10^3 \, \mathrm{ly} \end{aligned}$$

13. (a) If the expansion proceeds at a constant rate, from Hubble's law,

$$\begin{aligned} v &= H_0 R \\ \frac{R}{t} &= H_0 R \\ t &= \frac{1}{H_0} \end{aligned}$$

(b) Taking the inverse,

$$\begin{aligned} t &= \frac{3 \times 10^5 \, \mathrm{km/s}}{23 \times 10^{-6} \, (\mathrm{km/s})/\mathrm{ly}} \\ &= 1.3 \times 10^{10} \, \mathrm{yr} \end{aligned}$$