Homework 9

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1. Assuming all the operators are Hermitian,

$$\begin{aligned}[A, BC] &= (ABC - BCA + BAC - BAC) \\ &= [(AB - BA)C + B(AC - CA)] \\ &= B[A, C] + [A, B]C. \quad \square\end{aligned}$$

2. From the Hamiltonian relation from in class (October 27), for any non-time-dependent operator Q,

$$\sigma_H \sigma_Q \geq \frac{\hbar}{2} \left| \frac{\mathrm{d}}{\mathrm{d}t} \langle Q \rangle \right|.$$

Then using the position operator x for Q,

$$\begin{aligned}\sigma_H \sigma_x &\geq \frac{\hbar}{2} \left| \frac{\mathrm{d} \langle x \rangle}{\mathrm{d}t} \right| \\ &\geq \frac{\hbar}{2m} |\langle p \rangle|. \quad \square\end{aligned}$$

3. (a) From (4.10),

$$[p_i, r_j] = -i\hbar\delta_{ij},$$

Then from (4.96),

$$\begin{aligned}[L_z, x] &= [xp_y - yp_x, x] \\ &= [xp_y, x] - [yp_x, x] = -(-i\hbar y) = i\hbar y. \quad \square\end{aligned}$$

For the other dimensions,

$$\begin{aligned}&[L_z, y] = [xp_y - yp_x, y] = x[p_y, y] = -i\hbar x. \quad \square \\ &[L_z, z] = 0, \text{ as there's no } p_z \text{ in the commutator.} \quad \square\end{aligned}$$

$$\begin{aligned}&[L_z, p_x] = [xp_y - yp_x, p_x] = p_y[x, p_x] - p_x[y, p_x] = i\hbar p_y. \quad \square \\ &[L_z, p_y] = p_y[x, p_y] - p_x[y, p_y] = -i\hbar p_x. \quad \square \\ &[L_z, p_z] = 0, \text{ as there's no } z\text{'s.} \quad \square\end{aligned}$$

(b) From (4.96),

$$\begin{aligned}[L_z, L_x] &= [L_z, yp_z - zp_y] = p_z[L_z, y] - z[L_z, p_y] \\ &= -i\hbar xp_z + iz\hbar p_x \\ &= i\hbar(zp_x - xp_z) = i\hbar L_y. \quad \square\end{aligned}$$

(c) For the position commutator and by the identity in Problem 1,

$$\begin{aligned}\left[L_z, r^2\right] &= \left[L_z, x^2 + y^2 + z^2\right] \\ &= [L_z, x]x + x[L_z, x] + [L_z, y]y + y[L_z, y] + 0 \\ &= i2\hbar yx - i2\hbar yx = 0.\end{aligned}$$

Similarly for the momentum operator,

$$\begin{aligned}\left[L_z, p^2\right] &= \left[L_z, p_x^2 + p_y^2 + p_z^2\right] \\ &= [L_z, p_x]p_x + p_x[L_z, p_x] + [L_z, p_y]p_y + p_y[L_z, p_y] + 0 \\ &= i\hbar p_y p_x + i\hbar p_x p_y - i\hbar p_x p_y - i\hbar p_y p_x = 0.\end{aligned}$$

(d) For the Hamiltonian $H = p^2/2m + V$,

$$\begin{aligned}[L_x, H] &= \frac{1}{2m}\left[p^2, L_x\right] + [L_x, V(r)] \\ &= 0 + 0, \text{ by part (c).}\end{aligned}$$

4. Study Chapter 4.3.