Homework 3

MATH 301
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1. (a) All integers are a multiple of 2.
   (b) It's false, since $1 \in \mathbb{Z}$ and 1 is not a multiple of 2.
   (c) There is an integer that is not a multiple of 2.
   (d) $\exists\, x \in \mathbb{Z} : (x \text{ is not a multiple of } 2)$

2. (a) $\left(\sqrt{2} < x\right) \wedge \left(x < \sqrt{3}\right)$
   (b) $\neg \left[\exists x \in \mathbb{Q}, \left(\sqrt{2} < x\right) \wedge \left(x < \sqrt{3}\right)\right]$
   $= \forall x \in \mathbb{Q}, \left(\sqrt{2} \geq x\right) \vee \left(x \geq \sqrt{3}\right)$
   (c) All rational numbers are equal to or less than $\sqrt{2}$, or equal to or greater than $\sqrt{3}$.

3. (a) $E(x) = x$ is even.
   $O(x) = x$ is odd.
   $\forall x \in \mathbb{Z}, E(x) \oplus O(x)$ where $\oplus$ is an XOR operator.
   (b) $\exists x \in \mathbb{Z}, (E(x) \wedge O(x)) \vee (\neg E(x) \wedge \neg O(x))$
   (c) There is an integer that is either: both an odd and even integer, or is neither an odd or even integer.

4. (a) $\forall x \in \mathbb{Z}, E(x) \implies O(x+1)$
   (b) $\exists x \in \mathbb{Z}, E(x) \wedge E(x+1)$
   (c) There is an integer that even and if you add one to that integer, it's also even.

5. (a) ...$\forall\, x_1, x_2 \in \mathbb{R}, x_1 \leq x_2 \implies f(x_1) \geq f(x_2)$
   (b) ...$\exists\, x_1, x_2 \in \mathbb{R}, x_1 \leq x_2 \wedge f(x_1) < f(x_2)$

6. I'm going to define $\mathbb{R}^+$ as the set of positive real integers.
   (a) ...$\forall x \in \mathbb{R}, \varepsilon \in \mathbb{R}^+, \exists \delta \in \mathbb{R}^+, (|x - a| < \delta) \implies (|f(x) - f(a)| < \varepsilon)$
   (b) ...$\exists x \in \mathbb{R}, \varepsilon \in \mathbb{R}^+, \forall \delta \in \mathbb{R}^+, (|x - a| < \delta) \wedge (|f(x) - f(a)| \geq \varepsilon)$