# Homework 2

MATH 364
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2.3.15 Problem. Using carnations and roses, a florist can make up to three different floral arrangements for the Mother's Day trade. The composition (number of flowers of each type) and selling price ($) of a single arrangement of each type are as follows:

| | *Carnations* | *Roses* | *Price ($)* |
|---|---|---|---|
| Type A | 5 | 2 | 2.75 |
| Type B | 12 | 4 | 6.50 |
| Type C | 3 | 6 | 5.25 |

The florist can purchase from a local wholesaler up to 85 doz carnations at $1.80/doz and up to 75 roses at $4.80/doz. The florist can also purchase up to an additional 65 doz carnations at $3/doz from a distant dealer. Assuming that all arrangements made can be sold, how many of each type should the florist make to maximize income?

*Solution.* First, we can identify the decision variables. It seems like the obvious choices would be:

$$\begin{aligned}
\text{Let } x_k &= \text{the number of type } k \text{ arrangements made, where } k \in \{A, B, C\} \\
\ell_C &= \text{dozens of carnations bought locally} \\
\ell_R &= \text{dozens of roses bought locally} \\
r_C &= \text{dozens of carnations bought from the distant dealer.}
\end{aligned}$$

Next, we can identify the objective function. Here, we'll be trying to maximize the income, which is determined by the number of arrangements sold and the costs of flowers,

$$\begin{aligned}
\text{Income } z &= \text{profits} - \text{costs} \\
&= 2.75x_a + 6.50x_b + 5.25x_c - 1.80\ell_C - 4.80\ell_R - 3r_C.
\end{aligned}$$

The constraints are given by the compositions of each arrangement and the purchase limits per dozen of flowers:

$$\begin{aligned}
x_a &= \frac{1}{12}\left[5\left(\ell_C + r_C\right) + 2\ell_R\right] \\
x_b &= \frac{1}{12}\left[12\left(\ell_C + r_C\right) + 4\ell_R\right] \\
x_c &= \frac{1}{12}\left[3\left(\ell_C + r_C\right) + 6\ell_R\right] \\
0 &\le \ell_C \le 85 \\
0 &\le \ell_R \le 75 \\
0 &\le r_C \le 65 \\
x_k&, \ell_C, \ell_R, r_C \in \mathbb{Z}^+.
\end{aligned}$$

The linear program in the standard notation is

$$
\begin{aligned}
\max \quad & z = 2.75x_a + 6.50x_b + 5.25x_c - 1.80\ell_C - 4.80\ell_R - 3r_C \\
\text{s.t.} \quad & x_a = \frac{1}{12}\left[5\left(\ell_C + r_C\right) + 2\ell_R\right] \\
& x_b = \frac{1}{12}\left[12\left(\ell_C + r_C\right) + 4\ell_R\right] \\
& x_c = \frac{1}{12}\left[3\left(\ell_C + r_C\right) + 6\ell_R\right] \\
& 0 \le \ell_C \le 85 \\
& 0 \le \ell_R \le 75 \\
& 0 \le r_C \le 65 \\
& x_k, \ell_C, \ell_R, r_C \in \mathbb{Z}^+.
\end{aligned}
$$

2.4.6 Problem. Two sources supply three destinations with a commodity. Each source has a supply of 80 units and each destination has a demand for 50 units. Shipping costs in dollars per unit are:

| | | Destinations | | |
|---|---|---|---|---|
| | | *1* | *2* | *3* |
| Sources | *1* | 8 | 17 | 19 |
| | *2* | – | 21 | 22 |

The transportation costs from Source 2 to Destination 1 vary. The first 20 units shipped on this route cost \$10/unit, and each unit over 20 cost \$13/unit. Determine a minimal-cost shipping schedule.

*Solution.* The decision variables can be the unit shipments from source $i$ to destination $j$, represented by a matrix $x$ and an additional variable is needed for the number of units from source 2 to destination 1 over 20,

$$
\begin{aligned}
\text{Let } x_{ij} &= \text{number of units shipped from source } i \text{ to destination } j \\
&\quad \text{where } i \in \{1,2\}, j \in \{1,2,3\} \\
y_{21} &= \text{number of units shipped } 2 \to 1 \text{ at the higher cost}
\end{aligned}
$$

The objective function is the total cost of the shipping schedule,

$$
\begin{aligned}
\text{Cost } z &= c^T x + 13y_{21} \\
c^T &= \begin{pmatrix} 8 & 17 & 19 \\ 10 & 21 & 22 \end{pmatrix}
\end{aligned}
$$

Lastly the constraints are given by the source supplies, destination demand, and the $2 \to 1$ limitations,

$$
\begin{aligned}
0 \le \sum_j x_{ij} &\le 80 \qquad \text{for } i \in \{1,2\} \\
\sum_i x_{ij} &= 50 \qquad \text{for } j \in \{1,2,3\} \\
0 \le x_{21} &\le 20 \\
x_{ij}, y_{ij} &\in \mathbb{Z}^+
\end{aligned}
$$

The linear program in the standard notation is

$$\begin{aligned}
&\min_{x_{ij}, y_{21} \in \mathbb{Z}^+} z = c^T x + 13 y_{21} \\
&\text{where } c^T = \begin{pmatrix} 8 & 17 & 19 \\ 10 & 21 & 22 \end{pmatrix} \\
&\text{s.t.} \qquad 0 \leq \sum_j x_{ij} \leq 80 \qquad \text{for } i \in \{1, 2\} \\
&\qquad\qquad \sum_i x_{ij} = 50 \qquad \text{for } j \in \{1, 2, 3\} \\
&\qquad\qquad 0 \leq x_{21} \leq 20 \\
&\qquad\qquad x_{ij}, y_{ij} \in \mathbb{Z}^+
\end{aligned}$$

2.5.6 (a) Problem. Suppose the agent in (TK) Problem 5 can also by and sell Commodity B at the following prices per unit:

| *B* | Buy ($) | Sell ($) |
|---|---|---|
| *Month 1* | 80 | 95 |
| *Month 2* | 85 | 110 |
| *Month 3* | 95 | 125 |

The dealer can buy at most 200 units of B and sell at most 250 units during any one month and can also store B at the local warehouse, but space is limited. Assume the warehouse has 30 cu. yd of space available at $2/cu. yard and that a unit of A requires 1 cu. yd and a unit of B requires 2 cu. yd. Again, the dealer has no stock on hand and wants none at the end of the 3 months. Determine an optimal buying, selling, and storing program utilizing both commodities.

*Solution.* Let's begin by first looking at the decision variables. For each month $k$ and for commodity $i$, we'll let

$$\begin{aligned}
b_{ik} &= \text{commodity } i \text{ bought in month } k \\
s_{ik} &= \text{commodity } i \text{ sold in month } k \\
w_{ik} &= \text{commodity } i \text{ stored in month } k, \\
&\text{where } i \in \{A, B\}, k \in \{1, 2, 3\}.
\end{aligned}$$

The objective function is the income generated by both commodities,

$$\begin{aligned}
\text{Income } z &= \text{selling income} - \text{purchasing costs} - \text{storage costs} \\
&= 40 s_{A1} + 44 s_{A2} + 48 s_{A3} \\
&\quad + 95 s_{B1} + 110 s_{B2} + 125 s_{B3} \\
&\quad - 31 b_{A1} - 33 b_{A2} - 36 b_{A3} \\
&\quad - 80 b_{B1} - 85 b_{B2} - 95 b_{B3} \\
&\quad - 2 w_{A1} - 2 w_{A2} - 4 w_{B1} - 4 w_{B2}
\end{aligned}$$

The constraints are given by the commodity flow,

$$\begin{aligned}\text{prev. in storage} + \text{bought} &= \text{sold} + \text{stored}\\ w_{ik-1} + b_{ik} &= s_{ik} + w_{ik}\end{aligned}$$

For $i \in \{A, B\}$,

$$\begin{aligned}\cancel{w_{i0}} + b_{i1} &= s_{i1} + w_{i1}\\ w_{i1} + b_{i2} &= s_{i2} + w_{i2}\\ w_{i2} + b_{i3} &= s_{i3} + \cancel{w_{i3}}.\end{aligned}$$

And the per-month constraints, for month $k \in \{1, 2, 3\}$,

$$\begin{aligned}0 \le b_{Ak} &\le 450 & 0 \le b_{Bk} &\le 200\\ 0 \le s_{Ak} &\le 600 & 0 \le s_{Bk} &\le 250\\ 0 \le w_{Ak} + 2w_{Bk} &\le 30.\end{aligned}$$

$$\begin{aligned}\max z &= c^T x\\ \text{where } x^T &= \begin{pmatrix} b_{A1} & b_{A2} & b_{A3} & s_{A1} & s_{A2} & s_{A3} & w_{A1} & w_{A2}\\ b_{B1} & b_{B2} & b_{B3} & s_{B1} & s_{B2} & s_{B3} & w_{B1} & w_{B2}\end{pmatrix}\\ c^T &= \begin{pmatrix} -31 & -33 & -36 & 40 & 44 & 48 & -2 & -2\\ -80 & -85 & -95 & 95 & 110 & 125 & -4 & -4\end{pmatrix}\\ \text{s.t.} \quad & 0 \le b_{Ak} \le 450 \qquad 0 \le b_{Bk} \le 200\\ & 0 \le s_{Ak} \le 600 \qquad 0 \le s_{Bk} \le 250\\ & 0 \le w_{Ak} + 2w_{Bk} \le 30\\ & b_{i1} = s_{i1} + w_{i1}\\ & w_{i1} + b_{i2} = s_{i2} + w_{i2}\\ & w_{i2} + b_{i3} = s_{i3}\end{aligned}$$

(b) Problem. In the above problem, any units stored represent an investment of capital. Reconsider the problem, assuming that a maximum of \$10,000 can be borrowed each month for this purpose, with an accompanying 2% interest rate.

*Solution.*[1] In this scenario, we can add an additional variable for borrowed capital,

$$\text{Let } c_k = \text{capital borrowed for storing units in month } k.$$

We can modify the objective function as

$$\begin{aligned}
\text{Income } z &= \text{selling income} + \text{borrowed} - \text{purchasing costs} - \text{storage costs} - \text{interest} \\
&= 40s_{A1} + 44s_{A2} + 48s_{A3} \\
&\quad + 95s_{B1} + 110s_{B2} + 125s_{B3} \\
&\quad + c_1 + c_2 \\
&\quad - 31b_{A1} - 33b_{A2} - 36b_{A3} \\
&\quad - 80b_{B1} - 85b_{B2} - 95b_{B3} \\
&\quad - 2w_{A1} - 2w_{A2} - 4w_{B1} - 4w_{B2} \\
&\quad - 1.02c_2 - 1.02c_1.
\end{aligned}$$

Additionally, we'll have to add constraints ensuring the borrowed money at the end is zero and so we cannot borrow more than \$10,000 each month:

$$\begin{aligned}
0 \leq c_k &\leq 10000 \\
c_3 &= 0.
\end{aligned}$$

The modified linear program is then

$$\begin{aligned}
&\max z = c^T x \\
&\text{where } x^T = \begin{pmatrix} b_{A1} & b_{A2} & b_{A3} & s_{A1} & s_{A2} & s_{A3} & w_{A1} & w_{A2} & c_1 & c_2 \\ b_{B1} & b_{B2} & b_{B3} & s_{B1} & s_{B2} & s_{B3} & w_{B1} & w_{B2} & 0 & 0 \end{pmatrix} \\
&\qquad c^T = \begin{pmatrix} -31 & -33 & -36 & 40 & 44 & 48 & -2 & -2 & -0.02 & -0.02 \\ -80 & -85 & -95 & 95 & 110 & 125 & -4 & -4 & 0 & 0 \end{pmatrix} \\
&\text{s.t.} \quad \text{for } k \in \{1,2,3\}: \\
&\qquad 0 \leq b_{Ak} \leq 450 \qquad 0 \leq b_{Bk} \leq 200 \\
&\qquad 0 \leq s_{Ak} \leq 600 \qquad 0 \leq s_{Bk} \leq 250 \\
&\qquad 0 \leq w_{Ak} + 2w_{Bk} \leq 30 \\
&\qquad 0 \leq c_k \leq 10000 \\
&\qquad b_{i1} = s_{i1} + w_{i1} \\
&\qquad w_{i1} + b_{i2} = s_{i2} + w_{i2} \\
&\qquad w_{i2} + b_{i3} = s_{i3} \\
&\qquad c_3 = 0
\end{aligned}$$

[1] I'm 75% sure I'm not understanding this part completely. I'm assuming we can both borrow capital *and* use our liquid money to rent storage.