Homework 5

PHYSICS 465
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1. (a) The LHS of the equation is the Q of the reaction, it's the available energy that the reaction can yield.

   For this reaction, we know the momentum must be conserved, where

   $$\mathbf{p}_\gamma = \mathbf{p}_{\text{recoil}}.$$

   If we assume non-relativistic motion and use nuclear masses,

   $$\begin{aligned} \mathbf{p}_\gamma = pc &\implies p_\gamma = E_\gamma/c \\ E_D = p^2/2m &= \frac{E_\gamma^2}{2\mathcal{M}_D c^2}. \end{aligned}$$

   We can equate the RHS of the equation as

   $$E_\gamma + E_{\text{recoil}} = E_\gamma + \frac{E_\gamma^2}{2\mathcal{M}_D c^2}.$$

   (b) Atomic masses should be OK to use on the left side for m_p and m_d, as we're dealing with atoms complete with bound electrons.

   Using mass values from the internet,

   $$\begin{aligned} 939.565\,\text{MeV} + 938.272\,\text{MeV} - 1876\,\text{MeV} &= E_\gamma + \frac{E_\gamma^2}{2 \times 1.875\,\text{GeV}} \\ E_\gamma &= 1.84\,\text{MeV}. \end{aligned}$$

   The quadratic term doesn't matter much, as that term is roughly only $900\,\text{eV}$—far less than the MeV range.

   (c) In Williams, $Q_\alpha = 7.834\,\text{MeV}$. We could expect

   $$\begin{aligned} Q_\alpha &= [213.995\,186\,\text{u} - 209.984\,163\,\text{u} - 4.001\,506\,\text{u}]\,c^2 \\ &= 8.87\,\text{MeV}, \end{aligned}$$

   which is pretty close to the $7.8\,\text{MeV}$, I guess...

   I'm not sure how/what to check if it's consistent with the caption of the cloud chamber figure. We see a long-range (high-energy) decay occurring in the cloud chamber. Since the energy of the decay is higher than the other decay mode, shouldn't it be less common?

2. Using Table 5.1 and NIST's Atomic Mass Table, the Q values for each scenario are

   (a) β^- emission.

   $$\begin{aligned} Q_{\beta^-} &= (\mathcal{M}(19,40) - \mathcal{M}(20,40))\,c^2 \\ &= (39.963\,998\,\text{u} - 39.962\,590\,\text{u})\,c^2 \\ &= 1.31\,\text{MeV}. \end{aligned}$$

(b) β^+ emission.

$$\begin{aligned} Q_{\beta+} &= \left(\mathcal{M}(19,40) - \mathcal{M}(18,40) - 2m_e\right)c^2 \\ &= \big(39.963\,998\,\mathrm{u} - 39.962\,383\,\mathrm{u} - 2 \times 5.485\,799 \times 10^{-4}\,\mathrm{u}\big)\,c^2 \\ &= 482\,\mathrm{keV}. \end{aligned}$$

(c) e^- capture.

$$\begin{aligned} Q_{\mathrm{EC}} &= (39.963\,998\,\mathrm{u} - 39.962\,383\,\mathrm{u})\,c^2 \\ &= 1.52\,\mathrm{MeV}. \end{aligned}$$

3. In this sequence from ^{238}U to ^{206}Pb, there are 8 α-decays and 6 β-decays (can we use atomic masses even if there are an uneven number of beta and alpha decays?).

   The Q value can be calculated as

$$\begin{aligned} Q &= (\mathcal{M}(\mathrm{U},938) - \mathcal{M}(\mathrm{Pb},206) - 8\mathcal{M}(\mathrm{He},4))\,c^2 \\ &= (238.050\,788\,\mathrm{u} - 205.974\,46\,\mathrm{u} - 8 \times 4.002\,603\,\mathrm{u})\,c^2 \\ &= 51.7\,\mathrm{MeV} = 8.3 \times 10^{-12}\,\mathrm{J}. \end{aligned}$$

   From room temperature, the total change in temperature required is roughly $1135\,\mathrm{K}$. For one gram, this requires an energy of

$$E = 0.12\,\mathrm{J} \cdot \mathrm{g}^{-1} \cdot \mathrm{K}^{-1} \times 1135\,\mathrm{K} = 136.2\,\mathrm{J}.$$

   In terms of decays, this needs 1.644×10^{13} decays, and in a gram of ^{238}U, there are 2.53×10^{21} atoms. I'm guessing the time required would be solved with

$$\begin{aligned} 1.644 \times 10^{13} &= 2.53 \times 10^{21} \exp(-t/4.5\,\mathrm{Gyr}) \\ t &= 85\,\mathrm{Gyr}? \end{aligned}$$

4. If we consider a particle of energy E tunneling through a potential barrier of energy U, its wavefunction is given by

$$\begin{aligned} \Psi(x) &\propto e^{-kx} \\ \text{where } k &= \frac{1}{\hbar}\sqrt{2m(U-E)}. \end{aligned}$$

   Then, we can assume the rate of tunneling across the spatial interval $[a, b]$ is given by its probability, i.e.

$$\begin{aligned} \text{tunneling rate } \omega &\propto |\Psi|^2 \\ &\propto \exp\left\{-\frac{2}{\hbar}\sqrt{2m}\int_a^b \sqrt{U(x)-E}\,\mathrm{d}x\right\}. \end{aligned}$$

   Then, changing the bounds of the integral and taking the log (not sure if I really understand the reasoning of this part...),

$$\ln\omega = -2\sqrt{\frac{2m}{\hbar}}\int_0^{r=c/E} \sqrt{\frac{c}{r} - E}\,\mathrm{d}r$$

Changing variables to $x = Er/c$,

$$= \dots \frac{c}{\sqrt{E}} \int_0^1 \sqrt{1/x - 1}\,\mathrm{d}x\,.$$

Using an integration table, we can see that

$$\ln \omega \propto -1/\sqrt{E},$$

as depicted in Figure 6.3.