Homework 10

PHYSICS 342
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1. (a) The total energy lost due to bremsstrahlung (assuming $\gamma = 1$) is

$$\begin{aligned} E_{\text{brem}} &= \int_0^\infty P \, \mathrm{d}t \\ &= \frac{\mu_0 q^2 a^2}{6\pi c} \int_0^{v_0/a} \mathrm{d}t \\ &= \frac{\mu_0 e^2 a v_0}{6\pi c} \end{aligned}$$

As a fraction of the initial kinetic energy,

$$\frac{E_{\text{brem}}}{E_{\text{KE}}} = \frac{2\mu_0 e^2 a}{6\pi c m_e v_0^2}$$

(b) From the classical equations of motion,

$$\begin{aligned} v_0^2 &= 2ax \\ \left(10^5 \quad \mathrm{m \cdot s^{-1}}\right)^2 &= 2\,(3.0\,\mathrm{nm})\,a \\ a &= 1.67 \times 10^{19}\,\mathrm{m \cdot s^{-2}} \\ \frac{E_{\text{brem}}}{E_{\text{KE}}} &= 2.2 \times 10^{-15} \end{aligned}$$

The loss is quite small and can be ignored.

2. Since we're doing everything classically,

$$\begin{aligned} E_{\text{Coulomb}} &= E_{\text{KE}} \\ \frac{1}{4\pi\epsilon_0}\frac{e^2}{r_0} &= \frac{1}{2} m_e v^2 \\ v &= \sqrt{\frac{e^2}{2\pi\epsilon_0 m_e r_0}} \\ &\approx 3.1 \times 10^6\,\mathrm{m \cdot s^{-1}} \approx 0.011c \end{aligned}$$

From the Larmor formula, the power radiated is

$$P = \frac{\mu_0 q^2 a^2}{6\pi c}$$

As $P = \frac{\mathrm{d}E}{\mathrm{d}t}$,

$$P = \frac{\mathrm{d}}{\mathrm{d}t}\frac{1}{4\pi\epsilon_0}\frac{e^2}{r} = -\frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2}\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{\mu_0 q^2 a^2}{6\pi c}$$

$$-\int \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2}\,\mathrm{d}r = \int \frac{\mu_0 q^2 (v^2/r)^2}{6\pi c}\,\mathrm{d}t$$

3. (a) The damping factor γ is given by (11.84). For some visible light, $\omega = 10^{15}$ rad/s,

$$\begin{aligned}\gamma &= \omega^2\tau \\ &= \left(6\times10^{-24}\,\mathrm{s}\right)\left(10^{15}\right)^2 \\ &= 6\times10^{6} \\ \gamma &\ll \omega_0\end{aligned}$$

(b) I'm not really sure what to do here, but from the discussion in-class,

$$\begin{aligned}F_{\text{spring}} &= F_{\text{Coulomb}} \\ m\omega^2 x &= \frac{q^2}{4\pi\epsilon_0 x} \\ \omega &= \sqrt{\frac{q^2}{4\pi\epsilon_0 x^2 m}}\end{aligned}$$

4. (a) On each end of the dumbbell, there's $q/2$ charge. From the Abraham-Lorentz formula,

$$F_{\text{rad}} = \frac{\mu_0 q^2}{24\pi c}\dot{a}$$

Adding this to the interaction term results in the expected F_{rad},

$$F_{\text{rad}} = 2\times\frac{\mu_0 q^2}{24\pi c}\dot{a} + \frac{\mu_0 q^2\dot{a}}{12\pi c} = \frac{\mu_0 q^2\dot{a}}{6\pi c}$$

(b)

5. For one charge, the average intensity is

$$I = \frac{\mu_0\ddot{p}^2}{6\pi c}\frac{\sin^2(\theta)}{r^2}$$

The total power over both the charge and its image is then

$$\begin{aligned}P &= \int I\,\mathrm{d}a \\ &= 2\int_0^{2\pi}\int_0^{\pi}\frac{\mu_0\ddot{p}^2}{6\pi c}\sin^3(\theta)\,\mathrm{d}\theta\,\mathrm{d}\phi \\ &= \frac{8\mu_0\ddot{p}^2}{9c} \\ &= \frac{8\mu_0 q\ddot{z}^2}{9c}?\end{aligned}$$