Homework 7

MATH 364
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2.6.15 Model, but do not solve. A company has plants P_1, P_2, and P_3 which produce sandwiches needed at assembly centers C_1, C_2, C_3, and C_4. The annual output capacities of the plants and demands at the assembly centers are:

Annual Output of Sandwiches		
P_1	P_2	P_3
10,500	18,800	13,200

Annual Demand of Sandwiches			
C_1	C_2	C_3	C_4
7,700	9,900	12,200	11,100

The units can be delivered from the plants to the centers either by truck or rail. However, for each route for which units are delivered by rail, there is an annual route lease fee, independent of the number of units shipped through the route. The data follow.

			To Center			
	From Plant	By	C_1	C_2	C_3	C_4
Delivery Cost ($/unit)	P_1	Truck	60	80	50	30
		Rail	35	62	22	25
	P_2	Truck	95	120	65	75
		Rail	75	89	45	55
	P_3	Truck	110	98	77	88
		Rail	53	35	32	38

		To Center			
	From Plant	C_1	C_2	C_3	C_4
Route Lease Fee (in $1000)	P_1	165	220	175	150
	P_2	250	200	220	210
	P_3	180	190	200	180

Determine a minimum cost delivery schedule for the next year; that is, for each plant and center, determine how many units are to be shipped from plant to the center by truck, and how many by rail, so that the supplies are not exceeded, demands are met, and total cost is minimized.

Solution. Let's begin by determining the decision variables. We'll need to track how many units are shipped for both by truck and by rail, as well as a binary variable to track if one route uses rail. We'll define

$$\begin{aligned}
\text{Let} \quad t_{ij} &= \text{number of units shipped from plant } P_i \text{ to center } C_j \text{ by truck,} \\
r_{ij} &= \text{number of units shipped from plant } P_i \text{ to center } C_j \text{ by rail,} \\
b_{ij} &= \begin{cases} 1 & \text{if the shipments from } P_i \text{ to } C_j \text{ will be via rail} \\ 0 & \text{otherwise} \end{cases}
\end{aligned}$$

We can now figure out the objective function. For this problem, we'll be minimizing the total cost

$$
\begin{aligned}
\text{Cost} \quad & z = T \cdot t + R \cdot r + B \cdot b, \\
\text{where} \quad & T = \begin{pmatrix} 60 & 80 & 50 & 30 \\ 95 & 120 & 65 & 75 \\ 110 & 98 & 77 & 88 \end{pmatrix}, \\
& R = \begin{pmatrix} 35 & 62 & 22 & 25 \\ 75 & 89 & 45 & 55 \\ 53 & 35 & 32 & 38 \end{pmatrix}, \\
& B = \begin{pmatrix} 165 & 220 & 175 & 150 \\ 250 & 200 & 220 & 210 \\ 180 & 190 & 200 & 180 \end{pmatrix} \times 1000.
\end{aligned}
$$

The constraints will be given by the annual output and demand, where

$$
\begin{aligned}
\text{output of plant } i = \text{amount produced by } P_i &= \sum_{j=1}^{4} t_{ij} + r_{ij}, && \text{for } i = 1, 2, 3 \\
\text{demand by center } j = \text{amount sent to } C_j &= \sum_{i=1}^{3} t_{ij} + r_{ij}. && \text{for } j = 1, 2, 3, 4
\end{aligned}
$$

We'll also need a constraint to ensure that we can use the rail only when the fee is charged,

$$r_{ij} \leq M \times b_{ij}.$$

Putting this all together, the integer program becomes

$$
\begin{aligned}
\min \quad & z = T \cdot t + R \cdot r + B \cdot b \\
& \text{where the matrices } T, R, B \text{ are defined above} \\
\text{s.t.} \quad & P_i = \sum_{j=1}^{4} t_{ij} + r_{ij}, && \text{for } i = 1, 2, 3 \\
& C_j = \sum_{i=1}^{3} t_{ij} + r_{ij}. && \text{for } j = 1, 2, 3, 4 \\
& r_{ij} \leq M b_{ij} \\
& M \text{ large} \\
& b_{ij} \in \{0, 1\} \\
& t_{ij}, r_{ij} \geq 0 \\
& t_{ij}, r_{ij} \in \mathbb{Z}
\end{aligned}
$$