Chaos in Bose-Einstein Condensates

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Introduction

- Classical physics is chaotic but the Schrödinger equation is linear and cannot exhibit chaos
- If classical mechanics is chaotic and if quantum mechanics reaches classical mechanics in the classical limit, where does chaos stem from?
- This is an unsolved problem in physics.

Bose-Einstein Condensates

- Bose-Einstein condensates (BECs) are a purely quantum phenomena where bosons occupy the lowest quantum state
- We can realize BECs using superfluid helium-4 (1938) and rubidium-87 (1995)

Chaos

- Chaos is the apparent disorder of a dynamical system
- We can characterize chaos using Lyapunov exponents
- In a chaotic system, similar initial conditions separate at an exponent rate
- The Lyapunov exponent of a system can be described as

$$\lambda = \lim_{t\to\infty} \lim_{|\delta \mathbf{Z}_0|\to 0} \frac{1}{t} \ln \frac{|\delta \mathbf{Z}(t)|}{|\delta \mathbf{Z}_0|}$$

Gross-Pitaevskii Equation

- The Gross-Pitaevskii equation (GPE) is a nonlinear Schrödinger equation, capable of modeling BECs
- There is a nonlinear coupling term g which may give rise to chaos
- When $g = 0$, the GPE returns the regular Schrödinger equation and should exhibit no chaos
- The GPE is described as

$$i\hbar\frac{\partial \Psi}{\partial t} = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x,t) + g|\Psi|^2\right]\Psi$$

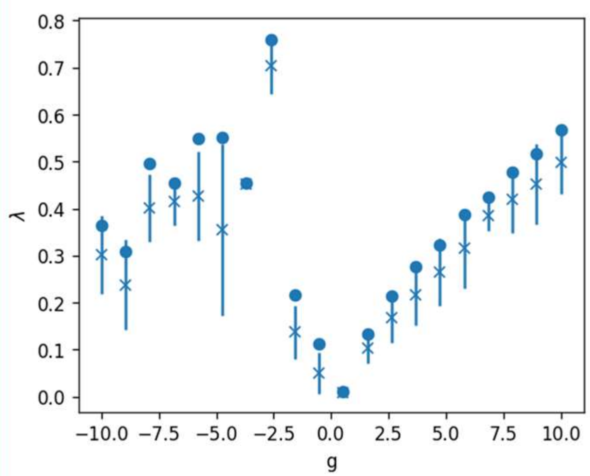
Methods

- Simulated the GPE using numerical methods in Python using NumPy and SciPy
- Perturbed an initial state of sinusoidal noise and calculated the Lyapunov exponent
- Ensured error was minimized by using optimal discretization spacings and checked time-reverse convergence

Results

- The Schrödinger equation is non-chaotic, as expected, and returns a zero exponent
- The GPE exhibits chaos for positive g
- There is a linear relationship between the coupling constant g and the Lyapunov exponent

Effect of *g* on the Lyapunov exponent



For positive g, the relationship appears to be linear between the Lyapunov exponent, indicating a correspondence between the nonlinearity of the GPE and chaos.

Conclusion and Discussion

- The GPE has chaotic motion for non-zero g
- The nonlinearity of the GPE leads to chaotic dynamics
- BECs should exhibit chaos
- Additional research is necessary to understand the roots of chaos in BECs