Homework 10

MATH 364
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6.2.8 Formulate integer (or mixed integer) programming models for the following.

(a) *(The Knapsack Problem)* A backpacker's knapsack has a volume of V cu. in. and can hold up to W lb of gear. The backpacker has a choice of n items to carry in it, with the ith item requiring a_i cu. in. of space, weighing w_i lb, and providing c_i units of value for the trip. What items should be taken in the knapsack?

Solution. The decision variables are booleans that represent whether or not to take the item,

$$\text{Let } x_i = \begin{cases} 1 & \text{the } i\text{th item is taken} \\ 0 & \text{the } i\text{th item is not taken.} \end{cases}$$

The objective function is the total value we're trying to minimize,

$$\text{Total value } z = \sum_i^n c_i x_i.$$

And the constraints are given by the volume and weight limitations of the knapsack,

$$\sum_i^n a_i x_i \leq V$$
$$\sum_i^n w_i x_i \leq W.$$

The integer program is then as follows,

$$\boxed{\begin{aligned} \max \quad & z = \sum_i^n c_i x_n \\ \text{s.t.} \quad & \sum_i^n a_i x_i \leq V \\ & \sum_i^n w_i x_i \leq W \\ & x_i \in \{0, 1\} \end{aligned}}$$

(b) Refine part (a) to include the following considerations: Item 1, a can of tuna fish, Item 2, a can of corn, and Item 3, a can of stew, have no value unless Item 4, the can opener is taken; and only one snack, either Item 5, potato chips (light but bulky), or Item 6, unpopped popcorn (small but heavy), is to go. Of course Items 2, 3, and 6 all use Item 7, the cooking pot.

Solution. For the canned food, we must constrain it so Item 4 is in the pack

$$x_4 \geq x_1$$
$$x_4 \geq x_2.$$

For the single snack constraint,

$$x_5 + x_6 = 1.$$

Lastly, for the cooking pot constraint (it's like the first constraint),

$$x_7 \geq x_2$$
$$x_7 \geq x_3$$
$$x_7 \geq x_6.$$

6.2.18 A company must produce weekly either 1500 A's and 1000 B's or 1000 A's and 1500 B's. Three difference processes can be used in production, with input (labor and raw materials M) and output (A's and B's) of 1 hr of operation of each as follows:

	Input		**Output**	
	Labor (hr)	M's (units)	A's (units)	B's (units)
Process 1	20	35	40	42
Process 2	12	12	45	35
Process 3	25	28	36	44

An unlimited number of M's are available weekly at \$15/unit and up to 600 hr of labor at \$12/hr. How many A's and B's should be made, using what production schedule, to minimize weekly costs?

Solution. The decision variables are given by how hours of each process to run,

$$\text{Let } x_i = \text{hours to run the } i\text{th process, where } i = 1, 2, 3,$$

and a boolean to denote whether to produce the 1500/1000 or 1000/1500 outputs,

$$b = \begin{cases} 1 & \text{company produces 1500 } A\text{'s and 1000 } B\text{'s} \\ 0 & \text{company produces 1000 } A\text{'s and 1500 } B\text{'s.} \end{cases}$$

The objective function is the cost to run those processes,

$$\text{Cost } z = 15\left(35x_1 + 12x_2 + 28x_3\right) + 12\left(20x_1 + 12x_2 + 25x_3\right).$$

The constraints are given by the number of outputs to produce and the limitations on labor,

$$\begin{aligned} 40x_1 + 45x_2 + 36x_3 &= 1000 + 500b \\ 42x_1 + 35x_2 + 44x_3 &= 1500 - 500b \\ 20x_1 + 12x_2 + 25x_3 &\leq 600 \end{aligned}$$

The linear program is

$$\begin{aligned} \min \quad & z = 15\left(35x_1 + 12x_2 + 28x_3\right) + 12\left(20x_1 + 12x_2 + 25x_3\right) \\ \text{s.t.} \quad & 40x_1 + 45x_2 + 36x_3 = 1000 + 500b \\ & 42x_1 + 35x_2 + 44x_3 = 1500 - 500b \\ & 20x_1 + 12x_2 + 25x_3 \leq 600 \\ & b \in \{0, 1\} \\ & x \geq 0 \\ & x \in \mathbb{Z}^3 \end{aligned}$$