Homework 4

PHYSICS 342
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Kevin Evans
ID: 11571810

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1. (a) The momentum is

$$
\begin{aligned}
\mathbf{p} &= \mu_0\epsilon_0 \int_V \mathbf{S}\,\mathrm{d}\tau \\
&= \epsilon_0 EB\,(\hat{\mathbf{z}} \times \hat{\mathbf{x}})\,Ad \\
&= \epsilon_0 AEBd\,\hat{\mathbf{y}}
\end{aligned}
$$

   (b) From the magnetic force on the wire,

$$
\begin{aligned}
F &= IdB \\
p &= \int_0^t F\,\mathrm{d}t \\
&= dB\int_0^t I(t)\,\mathrm{d}t = dBQ_{\text{tot}}?
\end{aligned}
$$

2. (a) By Gauss's law,

$$
\begin{aligned}
\mathbf{E} &= \frac{Q_{\text{enc}}}{2\pi\epsilon_0 sL}\,\hat{\mathbf{s}} \\
&= -\frac{\lambda}{2\pi\epsilon_0 s}\,\hat{\mathbf{s}}
\end{aligned}
$$

   (b) The total charge must be zero outside of the cylinder, so

$$
\begin{aligned}
Q_{\text{enc}} &= \sigma A = \lambda L \\
\sigma &= \frac{\lambda L}{2\pi aL} \\
&= \frac{\lambda}{2\pi a}
\end{aligned}
$$

   (c) Omitting the z axis from the integral, it should give the momentum per unit length as

$$
\begin{aligned}
\mathbf{p}/L &= \mu_0\epsilon_0 \iint_A \mathbf{S}s\,\mathrm{d}s\,\mathrm{d}\phi \\
&= \mu_0\epsilon_0 B_{\text{ext}}\left(-\frac{\lambda}{2\pi\epsilon_0}\right)(\hat{\mathbf{s}} \times \hat{\mathbf{z}})\int_0^a s^2\,\mathrm{d}s\int_0^{2\pi}\mathrm{d}\phi \\
&= \frac{\mu_0 B_{\text{ext}}\lambda a^3}{3}\,\hat{\boldsymbol{\phi}}
\end{aligned}
$$

3. For $f(z,t) = A\sin^2(\alpha z + \beta t)$,

$$
\begin{aligned}
\frac{\partial^2 f}{\partial z^2} &= 2Aa^2\cos(2(\alpha z + \beta t)) \\
\frac{\partial^2 f}{\partial t^2} &= 2A\beta^2\cos(2(\alpha z + \beta t))
\end{aligned}
$$

Applying the wave equation and removing terms on both sides,

$$\alpha^2 = \frac{1}{v^2}\beta^2$$
$$v = \beta/\alpha$$

4. (a) -1
   (b) i
   (c) $\frac{1}{\sqrt{2}}(1+i)$

5. Proposition: $\sin(u+v) = \sin u \cos v + \cos u \sin v$ and $\cos(u+v) = \cos u \cos v - \sin u \sin v$.

   Proof. By Euler's formula,

$$\begin{aligned} e^{i(u+v)} &= e^{iu}e^{iv} = \cos(u+v) + i\sin(u+v) \\ &= (\cos u + i \sin u)(\cos v + i \sin v) \\ &= \cos u \cos v + i \cos u \sin v + i \sin u \cos v - \sin u \sin v \end{aligned}$$

   Separating the real and imaginary parts,

$$\begin{aligned} \sin(u+v) &= \cos u \sin v + \sin u \cos v \\ \cos(u+v) &= \cos u \cos v - \sin u \sin v \end{aligned}$$

$\square$