Homework 2

MATH 301
September 1, 2020

Kevin Evans
ID: 11571810

---

1. (a) If the determinate of a matrix is not zero, then a matrix is invertable.
   (b) If $|r| < 1$, then a geometric series with common ratio r converges.
   (c) If a function is continuous, then it is intergrable.
   (d) If a function is differentiable, then the function is continuous.
   (e) If I'm wearing a hat, then it is sunny.

2. Let $P =$ The Curiosity Rover is on Mars.
   $Q =$ The Curiosity Rover is a good robot.
   $R =$ The Mars Polar Lander is a good robot.

   And we know that P, $Q \vee R$, and $R \implies \neg P$ are all true. From $R \implies \neg P$,

R	$\neg P$	$R \implies \neg P$	
T	T	T	
T	F	F	
F	T	T	
F	F	T	$(*)$

   As $\neg P$ is false, the bottom row $(*)$ intersects with $R \implies P$. So, R must be false.

   (a) True. Since $Q \vee R$ is true and R is false, Q must be true.
   (b) False. Shown above, R is false.

3. If P is false, then $P \wedge Q$ is false. If the original statement is true, then both sides of $\iff$ must be equal, i.e. $(R \implies S)$ must be false too. For an implication to be false, then

   $$R = \text{true}$$
   $$S = \text{false}$$

   It's impossible to know what Q is as P is false and $P \wedge Q$ will always be false regardless of Q's value.

4. For the implication $((P \wedge Q) \vee R) \implies (R \vee S)$ to be false, then

   $$((P \wedge Q) \vee R) = \text{true}$$
   $$(R \vee S) = \text{false}$$

   From the latter, both R and S must be false, so

   $$(P \wedge Q) = \text{true}$$

   This is only the case when both are true, thus

   $$P = Q = \text{true}$$
   $$R = S = \text{false}$$

5. Building out a truth table, we can see both statements are equivalent,

P	Q	$(P \vee Q) \wedge \neg(P \wedge Q)$	$(P \wedge \neg Q) \vee (Q \wedge \neg P)$
T	T	F	F
T	F	T	T
F	T	T	T
F	F	F	F

6. Applying de Morgan's law to the second statement,

$$\neg\left((P \wedge Q) \wedge \neg R\right) = \neg\left(P \wedge Q\right) \vee R$$

For the two original statements to be equivalent, then it would mean

$$\neg\left(P \wedge Q\right) \stackrel{?}{=} (P \implies Q)$$

...which is certainly false. These statements are <u>not</u> equivalent.