Homework 1

PHYSICS 465
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1. (a) We can just use the area of a circle, i.e. πb^2,

$$\begin{aligned}\sigma(\theta) &= \pi b(\theta)^2 \\ &= \pi \left(\frac{k_e Z z e^2}{2T}\right)^2 \cot^2(\theta/2).\end{aligned}$$

It's the cross-sectional area of an impact event occurring, sort of the probability that the incident particle will be deflected. For $\theta = \pi$, $\sigma = 0$ as the incident particle is fully reflected, so the impact parameter is tiny.

(b) For $\theta = 0$, $\sigma \to \infty$ since the cross section must be huge for the incident particle to "pass through."

(c) The differential cross section $\mathrm{d}\sigma/\mathrm{d}\Omega$ is the "cross-section per unit solid angle located at angle θ." I think it's like: given a certain angle θ, how much cross-sectional area does that correspond to?

Given the relation between the solid angle and the azimuthal and radial angle $\mathrm{d}\Omega = 2\pi \sin\theta \, \mathrm{d}\theta$, and rearranging,

$$\begin{aligned}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} &= \frac{1}{2\pi \sin\theta} \frac{\mathrm{d}\sigma}{\mathrm{d}\theta} \\ &= \frac{1}{2\pi \sin\theta}\left[-\pi \left(\frac{k_e Z z e^2}{2T}\right)^2 \frac{\sin\theta}{2} \csc^4(\theta/2)\right] && \text{(WolframAlpha)} \\ &= \frac{1}{4}\left(\frac{k_e Z z e^2}{2T}\right)^2 \csc^4(\theta/2). && (\csc^4 \text{ is even})\end{aligned}$$

2. (a) For two protons separated by 1 fermi,

$$\begin{aligned}U &= \frac{1}{4\pi\epsilon_0} \frac{e^2}{1\,\mathrm{fm}} \\ &= \frac{1}{4\pi(55.263\,\mathrm{GeV}^{-1}\cdot\mathrm{fm}^{-1})} \frac{1}{1\,\mathrm{fm}} \\ &= 1.44\,\mathrm{MeV}.\end{aligned}$$

(b) Similarly, for a gold nucleus and an α-particle separated by $10\,\mathrm{fm}$,

$$\begin{aligned}U &= \frac{1}{4\pi \times 55.263\,\mathrm{GeV}^{-1}\cdot\mathrm{fm}^{-1}} \frac{79\times 2}{10\,\mathrm{fm}} \\ &= 22.75\,\mathrm{MeV}.\end{aligned}$$

(c) And for two $Z = 46$, $A = 115$ nuclei with radius $R = 1.2 \times A^{1/3}\,\mathrm{fm}$, then the distance between the two atoms is twice the radii. The energy is then

$$\begin{aligned}U &= \frac{1}{4\pi \times 55.263\,\mathrm{GeV}^{-1}\cdot\mathrm{fm}^{-1}} \frac{46^2}{2\times 1.2\times 115^{1/3}\,\mathrm{fm}} \\ &= 33.56\,\mathrm{GeV}?\end{aligned}$$

3. Using an average pion mass of $137.275\,\mathrm{MeV}\cdot c^{-2}$, the energies of each particle are

$$E_1 = \sqrt{528^2 + 137.275^2}$$
$$E_2 = \sqrt{2607^2 + 137.275^2}.$$

The total energy is then

$$E = E_1 + E_2 = 3156.2\,\mathrm{MeV}$$

Taking the total momentum in the x-direction as

$$\begin{aligned} p_x &= 528\cos(30^\circ) + 2166\cos(7^\circ) \quad \mathrm{MeV}\cdot c^{-1} \\ &= 2607\,\mathrm{MeV}\cdot c^{-1}, \end{aligned}$$

Then, we can find the resonance particle mass using

$$\begin{aligned} (E_1 + E_2)^2 &= (m_R c^2)^2 + (pc)^2 \\ m_R &\approx 1778\,\mathrm{MeV}... \end{aligned}$$

This seems pretty high, so I'm thinking there's a mistake somewhere else too...

Taking a similar approach for (b), I'm finding $-61\,\mathrm{MeV}$. There is something again that I'm missing...

4. (a) I'm assuming we should find the energy from Problem 3 should coincide with the peak found in Problem 4. However, because my answer from Problem 3 is seemingly wrong, I am not finding that.

   (b) The approximate width of the resonance is $300\,\mathrm{MeV}$ to $400\,\mathrm{MeV}$. From the uncertainty principle, the approximate lifetime of the particle will be

$$\begin{aligned} \Delta E \Delta t &= \hbar/2 \\ (300\,\mathrm{MeV})\Delta t &= \hbar/2 \\ \Delta t &\approx 1\times 10^{-24}\,\mathrm{s}. \end{aligned}$$