CHAOS IN BOSE-EINSTEIN CONDENSATES

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INTRODUCTION

- How does chaos emerge from classical physics if the Schrödinger equation is linear and cannot exhibit chaos?
- Superfluids are well-described by a nonlinear Schrödinger equation
- What can we learn abut chaos from superfluids?
- We hypothesize that the chaos is a function of the nonlinearity of the Gross-Pitaevskii equation, and chaos should vanish when $g = 0$

BOSE-EINSTEIN CONDENSATES

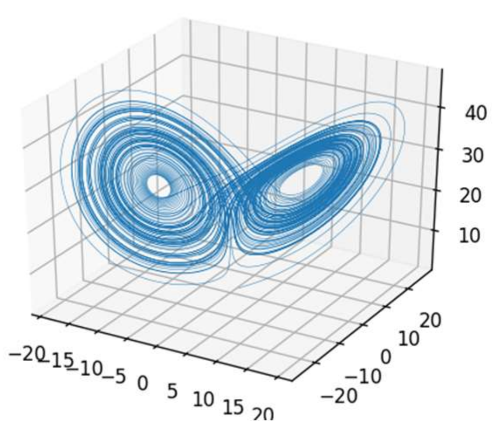
- Bose-Einstein condensates (BECs) are a quantum phenomena where bosons occupy the lowest quantum state
- We can realize BECs using superfluid Helium-4 (1938) and Rubidium-87 (1995)

CHAOS

- Chaos is the sensitivity to initial conditions, characterized by the Lyapunov exponent

$$\lambda = \lim_{t\to\infty} \lim_{|\delta \mathbf{Z}_0|\to 0} \frac{1}{t} \ln \frac{|\delta \mathbf{Z}(t)|}{|\delta \mathbf{Z}_0|}$$

- An example of chaos is seen in the Lorenz system:



GROSS-PITAEVSKII EQUATION

- The Gross-Pitaevskii equation (GPE) is a nonlinear Schrödinger equation, capable of closely approximating BECs

$$i\hbar \frac{\partial \Psi}{\partial t} = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x,t) + g|\Psi|^2 \right] \Psi$$

- There is a nonlinear mean-field term g may give rise to chaos

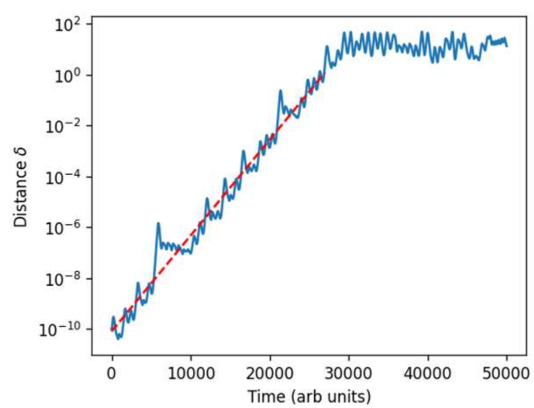
METHODS

- Simulate the GPE numerically in one-dimension
- Evolve two nearby initial states calculated the Lyapunov exponent
- Ensure error was minimized by using optimal discretization spacings and check time-reverse convergence

RESULTS

- For $g = 0$, Schrödinger equation is not chaotic, as expected, and returns a zero exponent
- The GPE exhibits chaos for positive g
- There is a linear relationship between the coupling constant g and the Lyapunov exponent

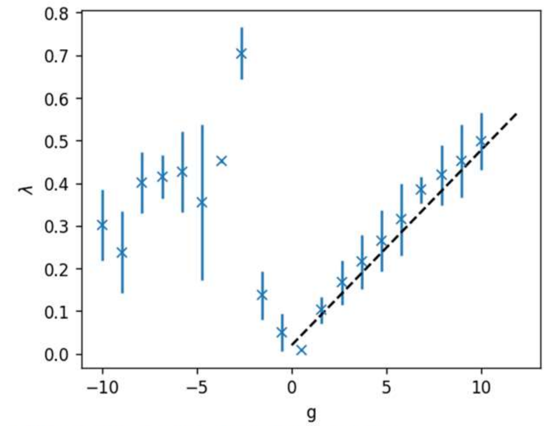
DIVERGENCE OF TRAJECTORIES IN THE LORENZ ATTRACTOR



The Lorenz attractor (left). Two similar initial conditions diverge at an exponential rate, seen by the plot on the right.

LINEARITY WITH NONLINEAR TERM

- For positive g, the dependence is consistent with a linear model



CONCLUSIONS AND DISCUSSION

- We confirm our hypothesis that the nonlinearity of the GPE gives rise to chaos
- We can extend this research to higher dimensions and to systems with quantum turbulence
- It is still unclear whether the chaos is caused by failure of the GPE or if it exists in reality

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