Homework 6

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1. (a) For ${}^{15}_{7}\mathrm{N}_{8}$, we have 8 neutrons (magic, non-contributing) and 7 protons (unfilled $1\,\mathrm{p}_{1/2}$). The ground state spin is $\frac{1}{2}$ with odd parity, $\boxed{1/2-}$.

   (b) For ${}^{17}_{8}\mathrm{O}_{9}$, the protons are magic and the 9 neutrons lead to an unfilled $1\,\mathrm{d}_{5/2}$ shell. This means we have $\boxed{5/2+}$ spin and parity.

   (c) For ${}^{39}_{19}\mathrm{K}_{20}$, the 19 protons in the unfilled $1\,\mathrm{d}_{3/2}$ shell lead to $\boxed{3/2+}$ spin and parity.

   (d) For ${}^{207}_{82}\mathrm{Pb}_{125}$, the 125 neutrons lead to an unfilled $1\,\mathrm{i}_{13/2}$ shell, with $\boxed{13/2+}$ spin and parity.

   (e) An electron from the $3\,\mathrm{p}_{1/2}$ shell could've jumped up to fill the $1\,\mathrm{i}_{13/2}$ shell, leading to a vacancy in the $3\,\mathrm{p}_{1/2}$ shell.

2. (a) From conservation of energy,

$$\begin{aligned} 2E_m &= E_M \\ 2\gamma m &= M \\ \implies M &= 2\left(1-(3/5)^2\right)^{-1/2} m \\ &= 2.5m. \end{aligned}$$

   (b) Similarly, by conservation of energy again,

$$\begin{aligned} 2\gamma m &= M \\ \gamma &= M/2m \\ 1-v^2/c^2 &= M/2m \\ v/c &= \sqrt{1-(M/2m)}. \end{aligned}$$

   (c) For a 4-vector $\mathbb{P} = (E/c, p)$, its dot product is

$$\begin{aligned} \mathbb{P}^2 &= \mathbb{P}\cdot\mathbb{P} \\ &= \frac{E^2}{c^2} - p^2 = m^2c^2. \end{aligned}$$

   Without the c's,

$$\mathbb{P}^2 = E^2 - p^2 = m^2.$$

   (d) For different vectors $\mathbb{P}_1$ and $\mathbb{P}_2$,

$$\begin{aligned} \mathbb{P}_1\cdot\mathbb{P}_2 &= \frac{E_1E_2}{c^2} - p_1p_2 \\ \implies &= E_1E_2 - p_1p_2. \end{aligned}$$

3. For moving particle m_1 with p_1 and stationary particle m_2, we can begin deriving the CM momenta by considering a frame where m_1 and m_2 have equal momentum p. In this frame moving at velocity v_c, the momentum of each particle will be

$$\begin{aligned} p_c &= m_1(v_1 - v_c) = m_2 v_c. \\ \implies p_1 &= m_1 v_1 = (m_1 + m_2) v_c. \\ \implies v_c &= \frac{m_1}{m_1 + m_2} v_1. \end{aligned}$$

The CM momentum is

$$\begin{aligned} p_c = m_2 v_c &= \frac{m_1 m_2}{m_1 + m_2} v_1 \\ &= \frac{m_2}{m_1 + m_2} p_1. \end{aligned}$$

4. To begin by deriving the CM momentum relativistically, we can consider the same system as Problem 3, with a frame moving at v_c (with Lorentz factor γ_c). In this frame, we can use the Lorentz transformation to find the CM momenta p,

$$\begin{aligned} p &= \gamma_c(p_1 - v_c E_1) && \text{(particle 1)} \\ -p &= \gamma_c(p_2 - v_c E_2) = \gamma_c(0 - v_c m_2 c^2) && \text{(particle 2)} \\ \implies p\ &= \gamma_c v - c m_2 \\ \implies v_c &= p/\gamma_c m. \end{aligned}$$

Substituting v_c in, the CM momentum is

$$\begin{aligned} p &= \gamma_c \left(p_1 - \frac{p}{\gamma_c m_2} E_1 \right)\ gamma_c p_1 - \frac{E_1}{m_2} p \\ &= \frac{\gamma_c}{1 + E_1/m_2} p_1 \\ &\boxed{p = \frac{\gamma_c m_2}{E_1 + m_2} p_1.} \end{aligned}$$

5. From Problem 3, the CM momentum is given by

$$p_c = \frac{m_2}{m_1 + m_2} p_1.$$

For both particles, the total kinetic energy is given by

$$\begin{aligned} T_c &= T_{c1} + T_{c2} \\ &= \frac{p_c^2}{2m_1} + \frac{p_c^2}{2m_2} = \frac{p_c^2}{2} \left(\frac{1}{m_1} + \frac{1}{m_2} \right) = \frac{p_c^2}{2} \frac{m_1 + m_2}{m_1 m_2} \\ &= \frac{m_2^2}{(m_1 + m_2)^2} \frac{(m_1 + m_2)}{m_1 m_2} \frac{p_1^2}{2} \\ &= \frac{m_2}{2m_1(m_1 + m_2)} p_1^2. \end{aligned}$$

In terms of $T_1 = p_1^2/2m_1$,

$$\boxed{T_c = \frac{m_2}{m_1 + m_2} T_1.}$$

6. (a) For the two particles, their 4-momenta are

$$\begin{aligned}\mathbb{P}_1 &= (E_1, p_1)\\ \mathbb{P}_2 &= (m_2, 0). \quad \text{(at rest in lab frame)}\end{aligned}$$

The total 4-momentum is

$$\mathbb{P}_{\text{tot}} = \mathbb{P}_1 + \mathbb{P}_2$$

Squaring this and using stuff from Problem 2,

$$\begin{aligned}\mathbb{P}_{\text{tot}}^2 &= (\mathbb{P}_1 + \mathbb{P}_2)^2 = \mathbb{P}_1^2 + \mathbb{P}_2^2 + 2\mathbb{P}_1 \cdot \mathbb{P}_2\\ &= m_1^2 + m_2^2 + 2\mathbb{P}_1 \cdot \mathbb{P}_2 = m_1^2 + m_2^2 + (E_1, p_1) \cdot (m_2, 0)\\ &= m_1^2 + m_2^2 + E_1 m_2.\end{aligned}$$

Since $\mathbb{P}_{\text{tot}}^2 = E_c^2$, as $\mathbb{P}_{\text{tot}} = (E_c, 0)$,

$$E_c^2 = m_1^2 + m_2^2 + E_1 m_2.$$

(b) I don't really understand this problem, so I might be way off here. In the CM frame, the total energy is given by the masses and total kinetic energy,

$$E_c = m_1 + m_2 + T_c.$$

We can square this to fit the stuff from (a),

$$\begin{aligned}E_c^2 &= (m_1 + m_2 + T_c)^2 = (m_1 + m_2)^2 + T_c^2 + 2(m_1 + m_2)T_c\\ &= m_1^2 + m_2^2 + E_1 m_2 = m_1^2 + m_2^2 + (m_1 + T_1)m_2.\end{aligned}$$

Here's the part that I'm not seeing (where is this coming from?):

$$T_c^2 = 2T_c(m_1 + m_2)$$

The condition when we can neglect T_c^2 is given by

$$T_c \ll 2(m_1 + m_2).$$