Homework 7

PHYSICS 304
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Chapter 10

2. In this problem, it notes to find the peak of $n(v)$. This would occur when $\frac{\mathrm{d}n}{\mathrm{d}v} = 0$.

$$n(v) = \frac{4\pi N}{V}\left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 e^{-mv^2/2k_B T}$$
$$\frac{\mathrm{d}n}{\mathrm{d}v} = \frac{4\pi N}{V}\left(\frac{m}{2\pi k_B T}\right)^{3/2}\left[2ve^{-mv^2/2k_BT} + v^2 e^{-mv^2/2k_BT}\left(-\frac{2mv}{2k_BT}\right)\right] = 0$$

Removing the initial factor,

$$0 = 2ve^{-mv^2/2k_BT} - \frac{-mv^3}{k_BT}e^{-mv^2/2k_BT}$$
$$= 2v - \frac{mv^3}{k_BT} = v\left(1 - \frac{mv^2}{2k_BT}\right)$$

Removing the trivial root,

$$1 = \frac{mv^2}{2k_BT}$$
$$v = \sqrt{\frac{2k_BT}{m}} \quad \square$$

6. (a) We are given: the states have equal weight, so $g_i = g_j \ \forall\, i, j \in \mathbb{N}$, and the energy between the $n = 1$ and $n = 2$ state is

$$\Delta E = 4.86\,\mathrm{eV}$$

Then, we can divide the number per volume of each state to find the ratio between the states,

$$\frac{n_2}{n_1} = e^{\Delta E/k_BT}$$
$$= \exp\left[(-4.86\,\mathrm{eV})/(8.617\times10^{-5}\,\mathrm{eV/K})(1600\,\mathrm{K})\right]$$
$$= 4.91\times10^{-16}$$

If we assume all of the particles are within these two states, then

$$n_1 + n_2 = 10^{20}$$
$$n_1 + (4.91\times10^{-16})n_1 = 10^{20}$$
$$n_1 = \frac{10^{20}}{1 + 4.91\times10^{-16}}$$
$$\approx 10^{20} \leftarrow \text{ My calculator is not precise enough}$$
$$n_2 + \frac{n_2}{4.91\times10^{-16}} = 10^{20}$$
$$n_2 = \frac{10^{20}}{1 + 1/(4.91\times10^{-16})} \approx 49\,160$$

(b) The average power emitted is

$$\begin{aligned} P &= \frac{E_{\text{total}}}{\Delta T} \\ &= \frac{NE_{2\to 1}}{\Delta T} = \frac{(49\,160)(4.86\,\text{eV})}{100\,\text{ns}} \\ &\approx 2.39\times 10^{12}\,\text{eV/s} \\ &\approx 3.83\times 10^{-7}\,\text{W} \end{aligned}$$

9. For a single iron atom, the mean speed would be

$$\begin{aligned} \bar{v} &= \pm\sqrt{\frac{8k_BT}{\pi m}} \\ &= \pm\sqrt{\frac{8\left(1.38\times 10^{-23}\,\text{J}\cdot\text{K}^{-1}\right)(6000\,\text{K})}{\pi\left(55.845\,\text{g}\cdot\text{mol}^{-1}\times\frac{1\,\text{mol}}{6.023\times 10^{23}\text{ atoms}}\times 1\times 10^{-3}\,\text{kg}\cdot\text{g}^{-1}\right)}} \\ &= 1507\,\text{m}\cdot\text{s}^{-1} \end{aligned}$$

The relative change in frequency is

$$\begin{aligned} \frac{\Delta f}{f_0} &= \frac{\Delta v}{c} = \frac{1507\,\text{m}\cdot\text{s}^{-1}}{3\times 10^{8}\,\text{m}\cdot\text{s}^{-1}} \\ &\approx 5.03\times 10^{-6} \end{aligned}$$

Using the relativistic Doppler shift formula is not needed as the iron atoms are not moving relativistically, with the mean speed as only a tiny fraction of c.

12. (a) The average energy would be given as the total energy over the total number of photons,

$$\begin{aligned} \bar{E} = \frac{E}{N} &= \frac{\int_0^\infty Eg(E)f_{BE}(E)\,\mathrm{d}E}{\int_0^\infty g(E)f_{BE}(E)\,\mathrm{d}E} \\ &= \frac{\int_0^\infty E^3\frac{1}{e^{E/k_BT}-1}\,\mathrm{d}E}{\int_0^\infty E^2\frac{1}{e^{E/k_BT}-1}\,\mathrm{d}E} && \text{Removed the constants of } g \end{aligned}$$

Using the hints in the problem,

$$\begin{aligned} z &= E/k_BT \\ \bar{E} &= \frac{(k_BT)^3\int_0^\infty z^3/(e^z-1)\,\mathrm{d}E}{(k_BT)^2\int_0^\infty z^2/(e^z-1)\,\mathrm{d}E} \\ &\approx k_BT\frac{\pi^4}{2.41\times 15} \\ &\approx 2.7k_BT \end{aligned}$$

(b) For a photon's energy at $T = 6000\,\text{K}$,

$$\begin{aligned} \bar{E} &= 2.7\left(8.617\times 10^{-5}\,\text{eV}\cdot\text{K}^{-1}\right)(6000\,\text{K}) \\ &= 1.4\,\text{eV} \end{aligned}$$

14. (a) Using the average energy from Problem 16,

$$\bar{E} = \frac{3(7.05)}{5} = 4.23\,\mathrm{eV}$$

(b) Equating (a) to the energy of an ideal gas,

$$\begin{aligned}\frac{3}{2}k_B T &= 4.23\,\mathrm{eV}\\ T &= \frac{2}{3}\,(4.23\,\mathrm{eV})\left(8.617\times 10^{-5}\,\mathrm{eV}\cdot\mathrm{K}^{-1}\right)\\ &\approx 3.3\times 10^{4}\,\mathrm{K}\end{aligned}$$

16. The average energy is defined as

$$\begin{aligned}\bar{E} &= \frac{\int_0^\infty E g(E) f_{FD}(E)\,\mathrm{d}E}{N/V}\\ &= \frac{\int_0^\infty E g(E) f_{FD}(E)\,\mathrm{d}E}{\int_0^\infty g(E) f_{FD}(E)\,\mathrm{d}E}\end{aligned}$$

As the probability is 0 at energies above E_F at $0\,\mathrm{K}$ and empty at higher energies, we can set the upper bound of both integrals to the Fermi energy and substitute in $g(E)$ using (10.39),

$$\begin{aligned}\bar{E} &= \frac{D\int_0^{E_F} E^{3/2}\,\mathrm{d}E}{D\int_0^{E_F} E^{1/2}\,\mathrm{d}E}\\ &= \frac{\frac{E^{5/2}}{5/2}}{\frac{E^{3/2}}{3/2}} = \frac{3}{5}E_F \quad \square\end{aligned}$$

17. The Fermi energy at $0\,\mathrm{K}$ is given in (10.44) as

$$E_F = \frac{h^2}{2m}\left(\frac{3N}{8\pi V}\right)^{2/3}$$

For the zinc protons, the Fermi energy is found as

$$\begin{aligned}E_F &= \frac{(1240\,\mathrm{MeV}\cdot\mathrm{fm})^2}{2\times 938.28\,\mathrm{MeV}}\left(\frac{3\times 30}{8\pi\times\frac{4}{3}\pi\,(4.8\,\mathrm{fm})^3}\right)^{2/3}\\ &\approx 32.0\,\mathrm{MeV}\end{aligned}$$

Similarily for the zinc neutrons,

$$E_F \approx 34.8\,\mathrm{MeV}$$

Using the average energy from Problem 16,

$$\bar{E} = \frac{3}{5}\,(32.0\,\mathrm{MeV}) \approx 19.2\,\mathrm{MeV}$$

Is that reasonable? Probably?

18. From (10.44), we can pull out n (electrons per volume) and evaluate

$$\begin{aligned} E_F &= \frac{h^2}{2m_e}\left(\frac{3N}{8\pi V}\right)^{2/3} \\ &= \frac{h^2}{2m_e}\left(\frac{3}{8\pi}\right)^{2/3} n^{2/3} \\ &\approx \frac{\left(6.626\times10^{-34}\,\mathrm{J\cdot s}\right)^2}{2\times9.11\times10^{-31}\,\mathrm{kg}} \times \frac{1\,\mathrm{eV}}{1.602\times10^{-19}\,\mathrm{J}} \times 0.2424n^{2/3} \\ &\approx 3.65\times10^{-19} n^{2/3}\mathrm{eV} \quad \square \end{aligned}$$

19. The probability of a copper conduction electron having energy $0.99E_F$ at $300\,\mathrm{K}$ is

$$\begin{aligned} f_{MB} &= \frac{1}{e^{(0.99E_F-E_F)/k_B(300\,\mathrm{K})}+1} \\ &= \frac{1}{\exp\left(-\frac{0.01(7.05\,\mathrm{eV})}{8.62\times10^{-5}\,\mathrm{eV\cdot K^{-1}}\times300\,\mathrm{K}}\right)+1} \\ &= 0.939 \end{aligned}$$

20. At the Fermi energy, the exponential reduces to e^0 and the probability will always be 0.5,

$$f_{MB} = \frac{1}{e^0+1} = \frac{1}{2}$$