Problem Set 5

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1. (a) The systems matrix can be characterized as $\mathbf{R}_2\mathbf{T}\mathbf{R}_1$,

$$\begin{aligned}
\mathbf{S} &= \begin{pmatrix} 1 & 0 \\ \lim\limits_{r\to\infty} \frac{n-1}{r} & n \end{pmatrix} \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{1-n}{nR} & \frac{1}{n} \end{pmatrix} \\
&= \begin{pmatrix} 1 & 0 \\ 0 & n \end{pmatrix} \begin{pmatrix} 1 + \frac{d(1-n)}{nR} & \frac{d}{n} \\ \frac{1-n}{nR} & \frac{1}{n} \end{pmatrix} \\
&= \begin{pmatrix} 1 + \frac{d(1-n)}{nR} & \frac{d}{n} \\ \frac{1-n}{R} & 1 \end{pmatrix}
\end{aligned}$$

(b) At the thin lens limit $d \to 0$, the systems matrix becomes

$$\mathbf{S} = \begin{pmatrix} 1 & 0 \\ \frac{1-n}{R} & 1 \end{pmatrix}$$

From inspection of the systems matrix, the focal length is

$$f = \frac{R}{n-1}$$

2. Hecht 5.26. Assuming a thin lens, the systems matrix will be

$$\begin{aligned}
\mathbf{S} &= \begin{pmatrix} 1 & 0 \\ \frac{n_2-n_1}{n_1}\left(\frac{1}{R_2} - \frac{1}{R_1}\right) & 1 \end{pmatrix} \\
&= \begin{pmatrix} 1 & 0 \\ \frac{2(n_2-n_1)}{n_1 R} & 1 \end{pmatrix} \\
f &= \frac{n_1 R}{2(n_2 - n_1)}
\end{aligned}$$

For $n_2 = 1.5$ and $R = 12.5\,\text{cm}$,

$$\begin{aligned}
f &= -\frac{12.5\,\text{cm}}{2(1.5-1)} \\
&= -12.5\,\text{cm}
\end{aligned}$$

For $n_1 = 1.628$,

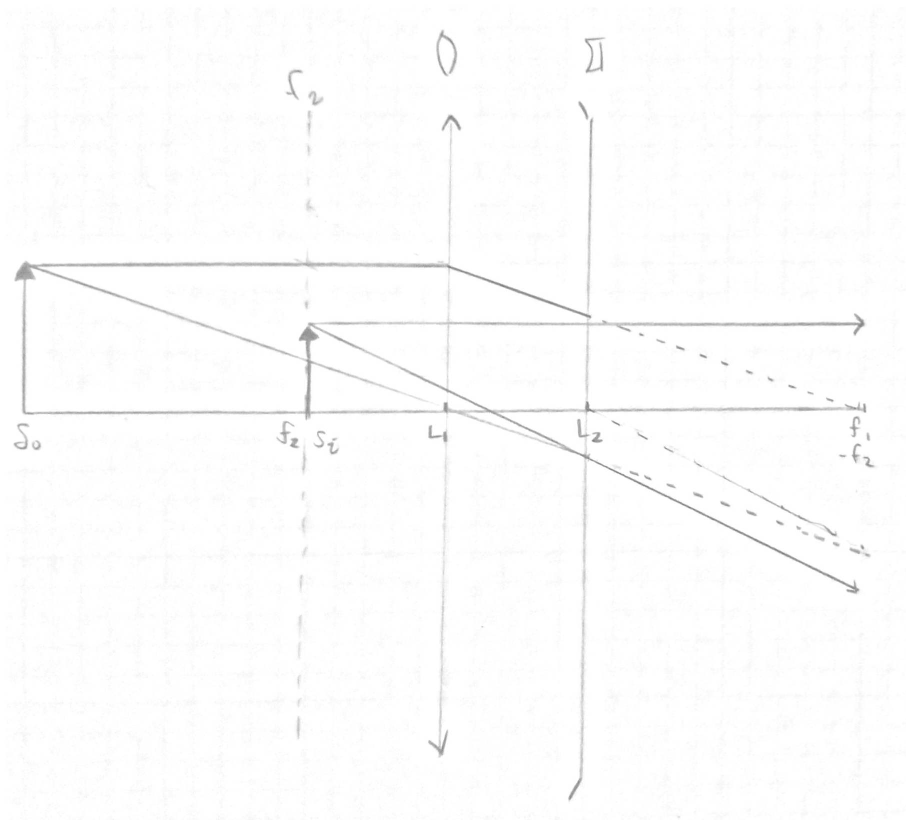
$$\begin{aligned}
f &= -\frac{1.628\,(12.5\,\text{cm})}{2(1.5-1.628)} \\
&= 79.5\,\text{cm}
\end{aligned}$$

3. Hecht 5.47. Using the thin lens equation, $s_{i1} = \infty$, as the object is at the first lens' focal point.

$$\frac{1}{f_2} = \frac{1}{s_{o2}} + \frac{1}{s_{i2}}$$
$$s_{i2} = f_2 = -20\,\text{cm} \quad \text{(rel. to } L_2\text{)}$$

Since the image from the first lens is at $+\infty$, it will be at $-\infty$ relative to the second lens, and the total transverse magnification will have a positive sign,

$$M_T = M_{T1} M_{T2} = \frac{\cancel{s_{i1}}}{s_{o1}} \frac{s_{i2}}{\cancel{s_{o2}}}$$
$$= \frac{20}{30} = 0.667$$



4. (a) The critical angle within the fiber is

$$\theta_c = \sin^{-1}(1.46/1.6) = 65.9°$$

(b) From (5.61),

$$\text{NA} = \left(n_f^2 - n_c^2\right)^{1/2}$$
$$= (1.6^2 - 1.46^2)^{1/2}$$
$$= 0.655$$