Homework 9

MATH 301
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1. **Proposition.** $\{12n : n \in \mathbb{Z}\} \subseteq \{2n : n \in \mathbb{Z}\} \cap \{3n : n \in \mathbb{Z}\}$

   Proof. Suppose $a \in \{12n : n \in \mathbb{Z}\}$. Then a can be expressed as $12n$ for some $n \in \mathbb{Z}$, and can be written as $a = 2 \cdot 3(2n)$. This means $a = 2k$ and $a = 3m$ for a $k, m \in \mathbb{Z}$ and therefore belongs in the intersection of the two latter sets. This then implies

   $$\{12n : n \in \mathbb{Z}\} \subseteq \{2n : n \in \mathbb{Z}\} \cap \{3n : n \in \mathbb{Z}\}$$

   ■

2. **Proposition.** *If* $m, n \in \mathbb{Z}$, *then* $\{x \in \mathbb{Z} : mn \mid x\} \subseteq \{x \in \mathbb{Z} : m \mid x\} \cap \{x \in \mathbb{Z} : n \mid x\}$.

   Proof. Suppose $m, n \in \mathbb{Z}$ and $y \in \{x \in \mathbb{Z} : mn \mid x\}$. Then y can be expressed as $y = kmn$ for a $k \in \mathbb{Z}$. This also means $m \mid y$ and $n \mid y$. Therefore, all elements of the first set are also elements of the intersection of the latter two sets, i.e. $\{x \in \mathbb{Z} : mn \mid x\} \subseteq \{x \in \mathbb{Z} : m \mid x\} \cap \{x \in \mathbb{Z} : n \mid x\}$. ■

3. *Disproof.* Let $A = \{1\}$, $B = \{2\}$, $X = \{1, 2\}$. Then $X \subseteq A \cup B$, but $X \nsubseteq A$ and $X \nsubseteq B$. ■

4. **Proposition.** $\{9^n : n \in \mathbb{Z}\} \subseteq \{3^n : n \in \mathbb{Z}\}$, *but* $\{9^n : n \in \mathbb{Z}\} \neq \{3^n : n \in \mathbb{Z}\}$.

   Proof. Let $x \in \{9^n : n \in \mathbb{Z}\}$, then $x = 9^m$ for an $m \in \mathbb{Z}$. This can also be written $x = (3^2)^m = 3^{2m}$. Since $2m \in \mathbb{Z}$, $x \in \{3^n : n \in \mathbb{Z}\}$ too. Therefore $\{9^n : n \in \mathbb{Z}\} \subseteq \{3^n : n \in \mathbb{Z}\}$.
   However, $3 \in \{3^n : n \in \mathbb{Z}\}$, but $3 \notin \{9^n : n \in \mathbb{Z}\}$. Therefore these two sets are not equal. ■

5. **Proposition.** *For sets* A, B, *and* C,

   $$(A \cup B) - C = (A - C) \cup (B - C)$$

   Proof. If we let $x \in (A \cup B) - C$, this means $x \in A \cup B$ but $x \notin C$. And since $x \in A \cup B$, this means $(x \in A \vee x \in B) \wedge (x \notin C)$. This can also be expressed as $(x \in A \wedge x \notin C) \vee (x \in B \wedge x \notin C)$, or $(A \cup B) - C \subseteq (A - C) \cup (B - C)$.
   Then, if we let $x \in (A - C) \cup (B - C)$. This means $x \in A - C \vee x \in B - C$, or equivalently, $(x \in A \wedge x \notin C) \vee (x \in B \wedge x \notin C)$. Since $x \notin C$ is common to both terms, this is also equivalent to $(A \cup B) - C$. ■

6. **Proposition.** *For sets* A *and* B, $A \subseteq B \iff A - B = \varnothing$.

   Proof. Suppose $A \subseteq B$. Then let $x \in A$, then $x \in B$ $(\forall x \in A)$. Therefore $\nexists y \in A \wedge y \notin B$, and $A - B = \varnothing$.
   Next, suppose $A - B = \varnothing$. Then $\forall x \in A$, $x \in B$, therefore $A \subseteq B$. ■