Homework 6

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3.5.9 Compute the solution to Problem 7 of Section 2.6. A poultry producer has 112 sq. rods of land on which to raise during the next 12-week period chickens, ducks, and turkeys. The space and labor requirements and the profit—excluding labor costs—from the sale after the 12-week breeding period are as follows:

	Space (sq rod/unit)	Labor (hr/week/unit)	Profit ($/unit)
Chickens	1.2	3	260
Ducks	1.0	2	172
Turkeys	0.8	1	88

The producer has available each week 200 hr of labor at $13/hr and up to 45 hr of overtime at $18/hr. What stock should the producer raise over the 12-week period in order to maximize net income (profits less labor costs)?

Solution. First, we'll need to bring this to the standard format. The decision variables are:

$$\begin{aligned} x_1 &= \text{number of chickens to raise} \\ x_2 &= \text{number of ducks to raise} \\ x_3 &= \text{number of turkeys to raise} \\ x_4 &= \text{hours of overtime to use.} \end{aligned}$$

The constraints are first the non-overtime labor/wk is less than 245 hours ($200 + 45$ hours), and the overtime labor is less than 45 hours/wk,

$$\begin{aligned} 0 \le 3x_1 + 2x_2 + x_3 &\le 245 \\ 0 \le x_4 &\le 45. \end{aligned}$$

The objective function is the net income,

$$\begin{aligned} \text{net income} &= \text{profits} - \text{labor} \\ z &= 260x_1 + 172x_2 + 88x_3 - 12 \times 13(3x_1 + 2x_2 + x_3) - 12 \times (18 - 13)x_4 \\ &= -208x_1 - 140x_2 - 68x_3 - 60x_4. \end{aligned}$$

We can convert this to standard form by adding slack variables, then flipping the sign of the objective function and converting this to a minimization problem,

$$\begin{aligned} \min \quad & z = 208x_1 + 140x_2 + 68x_3 + 60x_4 \\ \text{s.t.} \quad & 3x_1 + 2x_2 + x_3 + x_5 = 245 \\ & x_4 + x_6 = 45 \\ & x \ge 0 \\ & x \in \mathbb{Z}^6 \end{aligned}$$

$\boxed{\text{The solution is to produce no livestock, as the labor costs more than the profit. There will always be a net loss in income.}}$

3.6.3 Using a combination of birdseed mixtures A, B, and C, a blend of minimum cost which is at least 20% thistle and 30% corn is desired. Given the data which follow, determine the percentage of each of the mixtures in the final blend.

	% Thistle	% Corn	Cost (cents/lb)
A	25	40	57
B	0	30	13
C	10	15	20

Solution. The decision variables are the amount of each mixture in the final blend. For ease, we'll be doing this as a batch of 100 lbs.

$$\begin{aligned} \text{Let} \quad x_1 &= \text{lbs of mixture A} \\ x_2 &= \text{lbs of mixture B} \\ x_3 &= \text{lbs of mixture C.} \end{aligned}$$

The constraints are given by the thistle and corn requirements, as well as the 100 lb constraint,

$$\begin{aligned} 0.25x_1 + 0x_2 + 0.10x_3 &\geq 20 \\ 0.40x_1 + 0.30x_2 + 0.15x_3 &\geq 30 \\ x_1 + x_2 + x_3 &= 100. \end{aligned}$$

The objective function is the cost in cents to minimize,

$$\text{Cost } z = 57x_1 + 13x_2 + 20x_3.$$

In standard form, the linear program is

$$\begin{aligned} \min z = {} & 57x_1 + 13x_2 + 20x_3 \\ \text{s.t.} \quad & 0.25x_1 + 0x_2 + 0.10x_3 - x_4 = 20 \\ & 0.40x_1 + 0.30x_2 + 0.15x_3 - x_5 = 30 \\ & x_1 + x_2 + x_3 = 100 \\ & x \geq 0 \\ & x \in \mathbb{R}^5 \end{aligned}$$

We'll have to add three artificial variables (I think) here,

$$\begin{array}{rrrrrrrrcl} 0.25x_1 & & +\,0.10x_3 & -\,x_4 & & +\,x_6 & & & = & 20 \\ 0.40x_1 & +\,0.30x_2 & +\,0.15x_3 & & -\,x_5 & & +\,x_7 & & = & 30 \\ x_1\,+ & x_2\,+ & x_3 & & & & & +\,x_8 & = & 100 \\ 57x_1\,+ & 13x_2\,+ & 20x_3 & & & & & & = & z \\ & & & & & x_6 & +\,x_7 & +\,x_8 & = & w \end{array}$$

The tableaux with the artificial variables is

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8		
0.25	0	0.10	-1	0	1	0	0	20	$= x_6$
0.40	0.30	0.15	0	-1	0	1	0	30	$= x_7$
1	1	1	0	0	0	0	1	100	$= x_8$
57	13	20	0	0	0	0	0	0	$= -z$
0	0	0	0	0	1	1	1	0	$= -w$
0.25	0	0.10	-1	0	1	0	0	20	$= x_6$
$\boxed{0.40}$	0.30	0.15	0	-1	0	1	0	30	$= \boxed{x_7}$
1	1	1	0	0	0	0	1	100	$= x_8$
57	13	20	0	0	0	0	0	0	$= -z$
$\boxed{-1.65}$	-1.30	-1.25	1	1	0	0	0	0	$= -w$
0	-0.1875	0.00625	-1	0.625	1	-0.625	0	1.25	$= x_6$
1	$\boxed{0.75}$	0.375	0	-2.5	0	2.5	0	75	$= \boxed{x_1}$
0	0.25	0.625	0	2.5	0	-2.5	1	25	$= x_8$
0	-29.75	-1.375	0	142.5	0	-142.5	0	-4275	
0	$\boxed{-0.0625}$	-0.63125	1	-3.125	0	4.125	0	123.75	
0.25	0	0.1	-1	0	1	0	0	20	$= x_6$
$4/3$	1	$\boxed{0.5}$	0	$-10/3$	0	$10/3$	0	100	$= \boxed{x_2}$
$-1/3$	0	0.5	0	$10/3$	0	$-10/3$	1	100	$= x_8$
$119/3$	0	13.5	0	$130/3$	0	$-130/3$	0	-1300	
$1/12$	0	$\boxed{-0.6}$	1	$-10/3$	0	$13/3$	0	130	
$-1/60$	-0.2	0	-1	$\boxed{2/3}$	1	$-2/3$	0	0	$= \boxed{x_6}$
$8/3$	2	1	0	$-20/3$	0	$20/3$	0	200	$= x_3$
$-5/3$	-1	0	0	$20/37$	0	$20/3$	1	0	$= x_8$
$20/3$	-27	0	0	$400/3$	0	$-400/3$	0	-4000	
$101/60$	1.2	0	1	$\boxed{-22/3}$	0	$25/3$	0	250	

I think I've made a mistake somewhere, because I'm going in a circular loop after this last tableau.

3.7.3 (b) Determine the optimal value of the objective function, an optimal solution point, and whether or not the system of constraints contains any redundancies.

Maximize $5x_1 + 3x_2 + 3x_3$

subject to
$$\begin{aligned} 2x_1 + \ x_2 + \ \ x_3 &= 12 \\ 3x_1 + \ x_2 + 2x_3 &= 18 \end{aligned}$$

Solution. Putting this in standard form with a couple artificial variables, then sticking it into a tableau,

$$\begin{array}{ccc|cc|ll}
x_1 & x_2 & x_3 & x_4 & x_5 & & \\
\hline
\boxed{2} & 1 & 3 & 1 & 0 & 12 & = \boxed{x_4} \\
3 & 1 & 2 & 0 & 1 & 18 & = x_5 \\
\hline
-5 & -3 & -3 & 0 & 0 & 0 & = -z \\
\boxed{-5} & -2 & -5 & 0 & 0 & 0 & = -w \\
\hline\hline
1 & 0.5 & 1.5 & 0.5 & 0 & 6 & = x_1 \\
0 & -0.5 & -2.5 & -1.5 & 1 & 0 & = x_5 \\
\hline
0 & -0.5 & 4.5 & 2.5 & 0 & 30 & \\
0 & 0.5 & 2.5 & 2.5 & 0 & 30 & \\
\hline\hline
\end{array}$$

Again, I think I've made a mistake somewhere. I've optimized w and am expecting to get $w = 0$, but clearly that is not the case. The book says this indicates there is no solution. However, I'm definitely expecting a solution and I can't figure out where I've went wrong.