Homework 4

PHYSICS 461
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1. From the frame of the electron, the proton is orbiting the electron, creating a magnetic field at the origin. With this magnetic field, the electron spin moment will have an energy $U = -\mu \cdot \mathbf{B}$, with a negative sign due to the electron's charge.

2. If we take $\mathbf{J}^2$ and substitute in $\mathbf{L}$ and $\mathbf{S}$,

$$\begin{aligned} \mathbf{J}^2 &= (\mathbf{L} + \mathbf{S})^2 \\ &= \mathbf{L}^2 + \mathbf{S}^2 + 2\mathbf{L} \cdot \mathbf{S} \\ \Rightarrow \mathbf{L} \cdot \mathbf{S} &= \frac{1}{2}\left(\mathbf{J}^2 - \mathbf{L}^2 - \mathbf{S}^2\right) \\ \alpha \mathbf{L} \cdot \mathbf{S} &= \frac{\alpha \hbar^2}{2}\left[j(j+1) - \ell(\ell+1) - s(s+1)\right] \end{aligned}$$

3. It'd look something like this:

	m_j	m_ℓ	m_s	$m_\ell + 2m_s$
$j = \frac{3}{2}$	$\frac{3}{2}$	1	$\frac{1}{2}$	2
	$\frac{1}{2}$	0	$\frac{1}{2}$	1
	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0
	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	-2
$j = \frac{1}{2}$	$\frac{1}{2}$	1	$-\frac{1}{2}$	0
	$-\frac{1}{2}$	0	$-\frac{1}{2}$	-1

4. For an electron with an orbital angular momentum and spin,

$$\boldsymbol{\mu} = \frac{\mu}{\hbar}\left(g_\ell \mathbf{L} + g_s \mathbf{S}\right)$$

But also, since ℓ and s couple to the total angular momentum j,

$$\begin{aligned} \frac{\mu_B}{\hbar}\left(g_\ell \mathbf{L} + g_s \mathbf{S}\right) &= \frac{\mu_B}{\hbar} g_j \mathbf{J} \\ g_\ell \mathbf{L} + g_s \mathbf{S} &= g_j \mathbf{J} \\ g_\ell \mathbf{L} \cdot \mathbf{J} + g_s \mathbf{S} \cdot \mathbf{J} &= g_j J^2 \end{aligned}$$

And from $\mathbf{L} + \mathbf{S} = \mathbf{J}$, we can find the dot products above in terms of L^2, S^2, and J^2,

$$\begin{aligned} J^2 - 2\mathbf{S} \cdot \mathbf{J} + S^2 &= L^2 \\ J^2 - 2\mathbf{L} \cdot \mathbf{J} + L^2 &= S^2 \end{aligned}$$

Solving for the dot products and replacing it in the earlier equation,

$$g_\ell \frac{J^2 + L^2 - S^2}{2} + g_s \frac{J^2 + S^2 - L^2}{2} = g_j J^2$$

Then replacing these with their expectations,

$$\frac{g_\ell}{2}\left[j(j+1)+\ell(\ell+1)-s(s+1)\right]+\frac{g_s}{2}\left[j(j+1)+s(s+1)-\ell(\ell+1)\right]=g_j j(j+1)$$

Solving for g_j,

$$g_j = g_\ell \frac{j(j+1)+\ell(\ell+1)-s(s+1)}{2j(j+1)} + g_s \frac{j(j+1)+s(s+1)-\ell(\ell+1)}{2j(j+1)}$$

5. The Landé g_j factor for an electron simplifies to

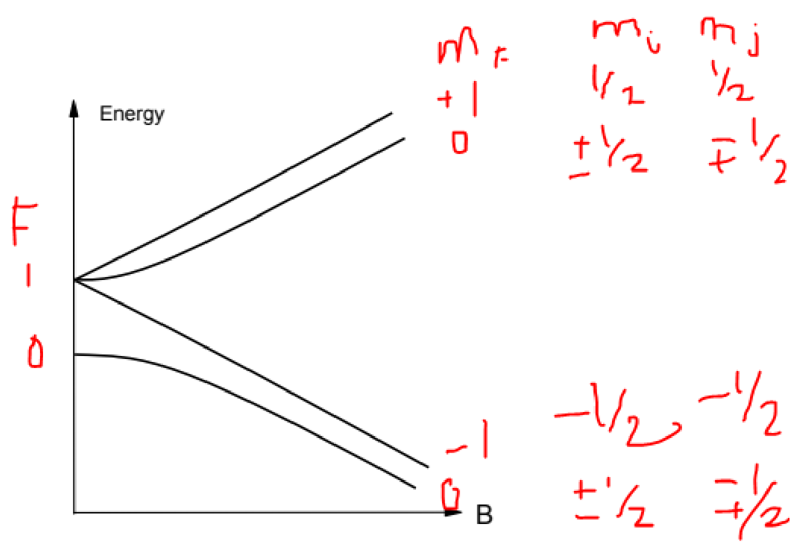
$$g_j(j,\ell) = 1 + \frac{j(j+1)-\ell(\ell+1)+3/4}{2j(j+1)}$$

So copypastaing the table from 3 with the g factor results in

	m_j	g_j	$g_j m_j$	m_ℓ	m_s	$m_\ell + 2m_s$
$j=\frac{3}{2}$	$\frac{3}{2}$	$4/3$	2	1	$\frac{1}{2}$	2
	$\frac{1}{2}$	$4/3$	$2/3$	0	$\frac{1}{2}$	1
	$-\frac{1}{2}$	$4/3$	$-2/3$	-1	$\frac{1}{2}$	0
	$-\frac{3}{2}$	$4/3$	-2	-1	$-\frac{1}{2}$	-2
$j=\frac{1}{2}$	$\frac{1}{2}$	$2/3$	$1/3$	1	$-\frac{1}{2}$	0
	$-\frac{1}{2}$	$2/3$	$-1/3$	0	$-\frac{1}{2}$	-1

If we compare the slopes for the weak field ($g_j m_j$) and the strong field $m_\ell + 2m_s$, it's clear why some of the lines will have an unchanged slope and some will have to curve to match the strong field case.

6. (a) Since the top has 3 lines and bottom has 1 line, $F=1$ and $F=0$. Then $J=I=1/2$.
   (b) It'd look something like this:



Although I'm a little unsure of what the g factors would be and how to rank the $m_f = 0$ states.

7. (a) If $J = 1/2$, then $I = 3/2$ and $F = 2$

   (b) Maybe something like this?