Homework 2

PHYSICS 465
January 27, 2021

Kevin Evans
ID: 11571810

---

1. (a) Because the probability involves $\psi_0(t)^*\psi_0(t)$, where the imaginary part cancels out. We're then left with

$$\begin{aligned} P(t) &= |\psi_0^*\psi_0|^2 \\ &= e^{-\Gamma t/\hbar}. \end{aligned}$$

Taking this at $t = 0$, we see that $P(t) = 1$. This makes sense as the state is definite at the start time with a probability of surely existing.
The decay constant is $\alpha = \Gamma/\hbar$.

(b)

$$\begin{aligned} g(\omega) &= \int_0^\infty e^{iE_0 t/\hbar - \frac{1}{2}\Gamma t/\hbar + i\omega t}\,\mathrm{d}t \\ &= \int_0^\infty e^{i(E_0+E)t/\hbar - \frac{1}{2}\Gamma t/\hbar}\,\mathrm{d}t \\ &= \frac{2\hbar}{\Gamma - 2i(E+E_0)}. && \text{(WolframAlpha)} \end{aligned}$$

(c) The probability is given by $g(\omega)^*g(\omega)$, so

$$\begin{aligned} \frac{2\hbar}{\Gamma + 2i(E+E_0)}\frac{2\hbar}{\Gamma - 2i(E+E_0)} &= \frac{4\hbar^2}{\Gamma^2 - 4(E+E_0)^2} \\ &= \frac{\hbar^2}{(\Gamma/2)^2 - (E+E_0)^2}? \end{aligned}$$

2. For an ideal gas at STP, one liter contains

$$\begin{aligned} n = \frac{PV}{RT} &= \frac{1\,\mathrm{bar} \times 1\,\mathrm{L}}{0.083\,145\,\mathrm{L\cdot bar\cdot mol^{-1}\cdot K^{-1}} \times 273\,\mathrm{K}} \\ &= 0.044\,\mathrm{mol}. \end{aligned}$$

Then for the atoms of carbon,

$$\begin{aligned} N_{\text{carbon}} &= nN_A = 0.044\,\mathrm{mol} \times 6.022 \times 10^{23}\,\mathrm{mol^{-1}} \\ &= 2.65 \times 10^{22}\,\text{atoms}. \end{aligned}$$

The transition rate per second is

$$\begin{aligned} \omega &= \tau^{-1} \\ &= 1/\left(8267\,\mathrm{yr} \times 365\,\mathrm{d\cdot yr^{-1}} \times 86\,400\,\mathrm{s\cdot d^{-1}}\right) \\ &= 3.835 \times 10^{-12}\,\mathrm{s^{-1}}. \end{aligned}$$

Then at 5 disintegrations per minute, the atomic fraction of nitrogen to carbon can be found with

$$\begin{aligned} \dot{N} &= -\omega N \\ \frac{N_{N-14}}{N_{\mathrm{C}}} &= \dot{N}_{\mathrm{N}}/\omega N_{\mathrm{C}} \\ &= 8.2 \times 10^{-13}. \end{aligned}$$

3. During the first reaction, 10^{10} neutrons are absorbed per second. The decay constant from Au-198 to Hg-198 is given by

$$\omega = 1/\tau = \frac{1}{3.89\,\mathrm{d} \times 86\,400\,\mathrm{s}\cdot\mathrm{d}^{-1}} = 2.97 \times 10^{-6}\,\mathrm{s}^{-1}.$$

Then, after six days, the Au-198 atoms present is

$$\begin{aligned} N(t) &= \frac{1 \times 10^{10}\,\mathrm{reactions/s}}{2.97 \times 10^{-6}\,\mathrm{s}^{-1}} \left[1 - e^{-6\,\mathrm{d}/3.89\,\mathrm{d}}\right] \\ &= 2.63 \times 10^{15}\,\mathrm{atoms}. \end{aligned}$$

After six days, the amount of Hg atoms is given by

$$\begin{aligned} N_{\mathrm{Hg}} &= pt - N_{\mathrm{Au}} \\ &= \left(1 \times 10^{10}\,\mathrm{sec}\right)\left(6\,\mathrm{d} \times 86\,400\,\mathrm{s}\cdot\mathrm{d}^{-1}\right) \\ &= 2.5 \times 10^{15}\,\mathrm{atoms}. \end{aligned}$$

The equilibrium number is reached when $t \to \infty$,

$$\begin{aligned} \lim_{t\to\infty} N(t) &= \frac{1 \times 10^{10}\,\mathrm{reactions/s}}{2.97 \times 10^{-6}\,\mathrm{s}^{-1}} \\ &= 3.36 \times 10^{15}\,\mathrm{atoms}. \end{aligned}$$

4. The transition rates are

$$\begin{aligned} \omega_{235} &= 1/\left(1.03 \times 10^{9}\,\mathrm{yr} \times 3.154 \times 10^{7}\,\mathrm{s}\cdot\mathrm{yr}^{-1}\right) = 3.07 \times 10^{-17}\,\mathrm{s}^{-1} \\ \omega_{238} &= 1/\left(6.49 \times 10^{9}\,\mathrm{yr} \times 3.154 \times 10^{7}\,\mathrm{s}\cdot\mathrm{yr}^{-1}\right) = 4.88 \times 10^{-18}\,\mathrm{s}^{-1}. \end{aligned}$$

Then for a multimodal decay,

$$\begin{aligned} N(t) &= N(0)e^{-(\omega_{235}+\omega_{238})t} \\ 7.3 \times 10^{-3} &= e^{-(\omega_{235}+\omega_{238})t} \\ t &\approx 4.38 \times 10^{9}\,\mathrm{yr}. \end{aligned}$$

5. From Problem 2.3, we know the atomic fraction is

$$\frac{N_{14}(t)}{N_{12}} = 8.1 \times 10^{-13}.$$

And this is equal to

$$\begin{aligned} &= \frac{N_{14}(0)}{N_{12}} e^{-t/\tau} \\ &= 10^{-12} e^{-t/\tau} \\ \implies t &= 1742\,\mathrm{yr}. \end{aligned}$$

6. (a) The total cross section is given as

$$\begin{aligned} \sigma &= \sigma_e + \sigma_c + \sigma_f \\ &= 2.7002 \times 10^{-26}\,\mathrm{m}^2. \end{aligned}$$

The attenuation can be found using

$$1 - n\sigma x = \frac{1 \times 10^{-1}\,\mathrm{kg} \cdot \mathrm{m}^{-2}}{0.235\,\mathrm{kg} \cdot \mathrm{mol}^{-1}} \times 6.02 \times 10^{23}\,\mathrm{atoms/mol} \times 2.7002 \times 10^{-26}\,\mathrm{m}^2 = 0.993.$$

(b) The total rate is given by the incident particle beam hitting the cross-section,

$$\begin{aligned} R_{\text{total}} &= 1 \times 10^5\,\mathrm{s}^{-1} \times 0.006\,91 \\ &= 691.7\,\mathrm{s}^{-1}. \end{aligned}$$

The number of fission reactions is then

$$\begin{aligned} R_{\text{f}} &= \frac{\sigma_f}{\sigma} R_{\text{total}} \\ &= 512.3\,\mathrm{s}^{-1}. \end{aligned}$$

(c) Similarly, for elastically scattered particles,

$$\begin{aligned} R_{\text{e}} &= \frac{\sigma_f}{\sigma} R_{\text{total}} \\ &= 0.051. \end{aligned}$$