Homework 10

PHYSICS 341 — Kevin Evans

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1. (a) It's stationary, as there's no initial velocity and the electric field is zero.

   (b) $B = B\,\hat{\mathbf{z}}$, $v(0) = v_0\,\hat{\mathbf{y}}$, the velocity can be written

$$\begin{aligned}\mathbf{v} &= \dot{x}\,\hat{\mathbf{x}} + \dot{y}\,\hat{\mathbf{y}} \\ \mathbf{a} &= \ddot{x}\,\hat{\mathbf{x}} + \ddot{y}\,\hat{\mathbf{y}}\end{aligned}$$

Balancing the forces,

$$\begin{aligned}q\mathbf{v} \times \mathbf{B} &= m\dot{\mathbf{v}} \\ qB\left(-\dot{x}\,\hat{\mathbf{y}} + \dot{y}\,\hat{\mathbf{x}}\right) &= m\left(\ddot{x}\,\hat{\mathbf{x}} + \ddot{y}\,\hat{\mathbf{y}}\right)\end{aligned}$$

Equating each compoment, we're left with these coupled equations,

$$\begin{aligned}-qB\dot{x} &= m\ddot{y} \\ qB\dot{y} &= m\ddot{x}\end{aligned}$$

Letting $\omega = qB/m$, then letting $u = \dot{x}$,

$$\begin{aligned}-\omega^2\dot{x} &= \dddot{x} \\ -\omega^2 u &= \ddot{u}\end{aligned}$$

The solution for $u(t)$ is sinusoidal and we can integrate to find $x(t)$,

$$\begin{aligned}u(t) &= A\cos\omega t + B\sin\omega t \\ x(t) &= C_1\cos\omega t + C_2\sin\omega t + C_3\end{aligned}$$

Then for y,

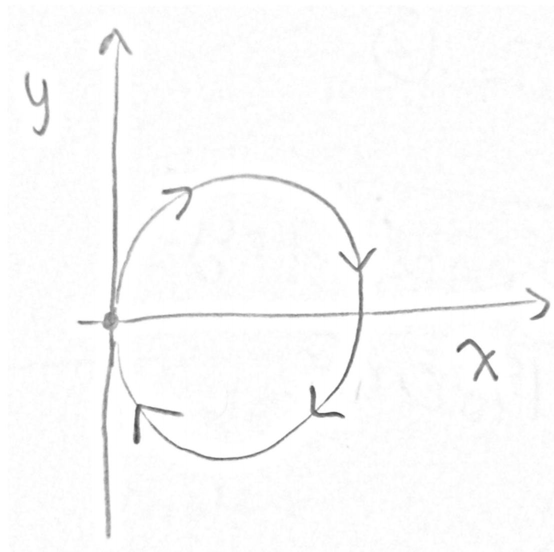
$$\begin{aligned}\ddot{x} &= -C_1\omega^2\cos\omega t - C_2\omega^2\sin\omega t \\ \dot{y} &= \frac{\ddot{x}}{\omega} = -C_1\omega\cos\omega t - C_2\omega\sin\omega t \\ y(t) &= -C_1\sin\omega t + C_2\cos\omega t + C_4\end{aligned}$$

For the IC $v(0) = v_0\,\hat{\mathbf{y}}$ and assuming the particle starts at $(0, 0)$

$$\begin{aligned}C_1 &= -\frac{v_0}{\omega} \\ C_2 &= 0 \\ C_3 &= -C_1 \\ C_4 &= 0\end{aligned}$$

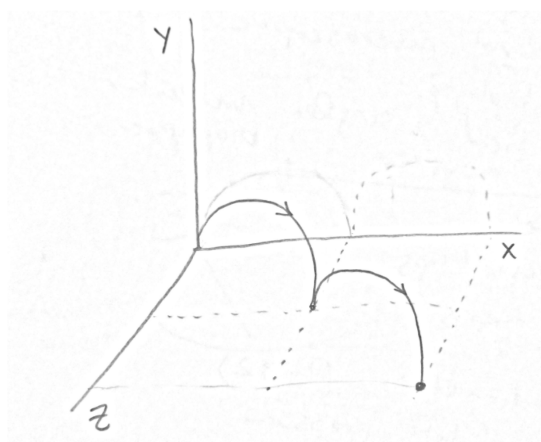
The equations of motion are

$$\begin{aligned}x(t) &= \frac{v_0}{\omega}\left(1 - \cos\omega t\right) \\ y(t) &= \frac{v_0}{\omega}\sin\omega t\end{aligned}$$



(c) Since the velocity is in the direction of the magnetic field and is perpendicular to the electric field, it'll remain constant in the z direction. Taking a similar approach to the Example 5.2 in the book and using (5.7), it's evident that

$$x(t) = \frac{E}{\omega B}(\omega t - \sin \omega t)$$
$$y(t) = \frac{E}{\omega B}(1 - \cos \omega t)$$
$$z(t) = \frac{E}{B}t$$



(d) Starting from the (b), the electric field would appear in the y-force equation as

$$qE - qB\dot{x} = m\ddot{y}$$

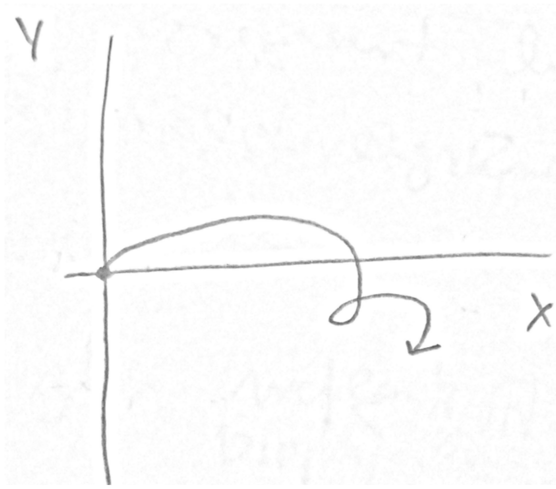
and yields

$$\dddot{x} = \omega^2 \frac{E}{B} - \omega^2 \dot{x}$$
$$x(t) = C_1 \cos \omega t + C_2 \sin \omega t + \frac{E}{B}t + C_3$$
$$y(t) = -C_1 \sin \omega t + C_2 \cos \omega t + C_4$$

Using $v(0) = E/B\,\hat{\mathbf{x}}$ and the particle starting at $(0, 0)$, and solving with a 4×5 matrix,

$$x(t) = \frac{E}{2B}\left(-\cos\omega t + \sin\omega t + t + 1\right)$$
$$y(t) = \frac{E}{2B}\left(\sin\omega t + \cos\omega t - 1\right)$$

Pretty sure this one is not right.



2. We can parameterize the loop's $\mathrm{d}\mathbf{l}$ with an angle θ,

$$\begin{aligned}
y &= a\cos\theta \\
\mathrm{d}y &= -a\sin\theta\,\mathrm{d}\theta \\
z &= a\sin\theta \\
\mathrm{d}z &= a\cos\theta\,\mathrm{d}\theta \\
\mathrm{d}\mathbf{l} &= -a\sin\theta\,\mathrm{d}\theta\ \hat{\mathbf{y}} + a\cos\theta\,\hat{\mathbf{z}}\,\mathrm{d}\theta
\end{aligned}$$

Crossing this with $\mathbf{B}$,

$$\begin{aligned}
\mathrm{d}\ell \times \mathbf{B} &= -kz^2 a\cos\theta\,\mathrm{d}\theta\ \hat{\mathbf{y}} - kz^2 a\sin\theta\,\mathrm{d}\theta\ \hat{\mathbf{z}} \\
&= -ka\sin^2(\theta)\left(\cos\theta\,\hat{\mathbf{y}} + \sin\theta\,\hat{\mathbf{z}}\right)\mathrm{d}\theta
\end{aligned}$$

The force can be found by integrating from 0 to 2π,

$$\begin{aligned}
\mathbf{F} &= -kIa\int_0^{2\pi} \sin^2(\theta)\left(\cos\theta\,\hat{\mathbf{y}} + \sin\theta\,\hat{\mathbf{z}}\right)\mathrm{d}\theta \\
&= 0?
\end{aligned}$$

3. The uniform charge density is

$$\rho = \frac{Q}{V} = \frac{3Q}{4\pi R^3}$$

At any point, the instantaneous velocity is

$$\mathbf{v} = r\,\hat{\boldsymbol{\phi}}$$

The volume current density is then

$$\begin{aligned}\mathbf{J} &= \rho\mathbf{v} \\ &= \frac{3Qr}{4\pi R^3}\,\hat{\boldsymbol{\phi}}\end{aligned}$$

4. (a) Using Example 5.5 as a starting point, we can use the angles

$$\begin{aligned}\theta_1 &= -\pi/4 \\ \theta_2 &= \pi/4\end{aligned}$$

And use equation (5.37)

$$\mathbf{B} = \frac{\mu_0 I}{4\pi d}\sqrt{2}\text{ (out of page)}$$

(b) Here, there's a constant radius R, so the Biot-Savart law simplifies

$$\begin{aligned}\mathbf{B} &= \frac{\mu I}{4\pi}\left(\pi R/R^2\right) \\ &= \frac{\mu I}{4R}\text{ (into page)}\end{aligned}$$

5. We can just integrate over the the length of the cylinder, where z now goes from $-L/2$ to $L/2$, and the current $I = \lambda(R\omega)$,

$$\begin{aligned}B &= \frac{\mu_0\lambda\omega R^3}{2}\int_{-L/2}^{L/2}(R^2+z^2)^{-3/2}\,\mathrm{d}z \\ \mathbf{B} &= \frac{\mu_0\lambda\omega R^2 L}{\sqrt{L^2+4R^2}}\,\hat{\mathbf{z}} \quad \text{(WolframAlpha)}\end{aligned}$$

As L tends to infinity, it'll approach a constant

$$\mathbf{B} = \mu_0\lambda\omega R^2\,\hat{\mathbf{z}}$$