4. **Proposition:** If n is an odd integer, then $n^2 + 4n + 6$ is odd.

   Proof. Let n be an odd integer, then n can be expressed as

$$
\begin{aligned}
n &= 2a + 1 && \text{...where } a \in \mathbb{Z} \\
n^2 + 4n + 6 &= (2a + 1)^2 + 4(2a + 1) + 6 && \text{Substition for } n \\
&= 4a^2 + 4a + 1 + 8a + 4 + 6 && \text{Expanding the terms} \\
&= 2(\underbrace{2a^2 + 6a + 5}_{b}) + 1 && \text{From closure, } b \in \mathbb{Z} \\
&= 2b + 1 && \text{The result is odd.} \quad \square
\end{aligned}
$$