Problem Set 1

PHYSICS 443
September 2, 2020

Kevin Evans
ID: 11571810

---

1. (a) From $\lambda = 3.0\,\mathrm{m}$,

$$\begin{aligned}\lambda\nu &= c \\ \nu &= 3\times10^8\,\mathrm{m}\cdot\mathrm{s}^{-1}/3.0\,\mathrm{m} \\ &= 1.0\times10^8\,\mathrm{Hz}\end{aligned}$$

(b) From Faraday's law $\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$, as the wave propagates in $\hat{\mathbf{x}}$ and $\mathbf{E}$ only depends on that variable, the curl reduces to

$$\begin{aligned}\frac{\partial E_y}{\partial x} &= -\frac{\partial B_z}{\partial t} \\ \frac{\partial B_z}{\partial t} &= \frac{\partial}{\partial x}E_0\sin(\omega t - kx) \\ B_z &= E_0 k\int\cos(\omega t - kx)\,\mathrm{d}t \\ &= \frac{E_0 k}{\omega}\sin(\omega t - kx) = \frac{E_0}{c}\sin(\omega t - kx)\end{aligned}$$

The magnitude B_0 is given as

$$\begin{aligned}B_0 &= \frac{E_0}{c} = \frac{300\,\mathrm{V}\cdot\mathrm{m}^{-1}}{3\times10^8\,\mathrm{m}\cdot\mathrm{s}^{-1}} \\ &= 1\,\mu\mathrm{T}\ \text{(in the } \hat{\mathbf{z}} \text{ direction)}\end{aligned}$$

(c) Given the wavelength is $3.0\,\mathrm{m}$, the wavenumber

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{3.0\,\mathrm{m}} \approx 2.1\,\mathrm{rad}\cdot\mathrm{m}^{-1}$$

Similarly, as the frequency was found in (a), the angular frequency

$$\omega = 2\pi\nu \approx 6.3\,\mathrm{rad}\cdot\mathrm{s}^{-1}$$

2. (a) From inspection,

$$\mathbf{k} = -3\,\hat{\mathbf{x}} - 4\,\hat{\mathbf{z}} + 5\,\hat{\mathbf{z}}$$

And as $\hat{\mathbf{k}} \perp \mathbf{E} \implies \mathbf{k} \cdot \mathbf{E} = 0$,

$$\begin{aligned} \mathbf{k} \cdot \mathbf{E} &= 100\,(2\,\hat{\mathbf{x}} + 3\,\hat{\mathbf{y}} + E_z\,\hat{\mathbf{z}}) \cdot (-3\,\hat{\mathbf{x}} - 4\,\hat{\mathbf{z}} + 5\,\hat{\mathbf{z}}) = 0 \\ 5E_z &= 18 \\ E_z &= 3.6 \end{aligned}$$

(b) From Faraday's law, we begin by evaluating the curl,

$$\begin{aligned} -\frac{\partial \mathbf{B}}{\partial t} &= \boldsymbol{\nabla} \times \mathbf{E} \\ &= 100 \cdot \boldsymbol{\nabla} \times [(2\,\hat{\mathbf{x}} + 3\,\hat{\mathbf{y}} + 3.6\,\hat{\mathbf{z}}) \sin(\omega t - 3x - 4y + 5z)] \\ &= 100\,[\,\hat{\mathbf{x}}\,[3.6(-4) - 3(5)] + \,\hat{\mathbf{y}}\,[2(5) - 3.6(-3)] + \,\hat{\mathbf{z}}\,[3(-3) - 2(-4)]] \cos(\ldots) \\ &= 100\,(-29.4\,\hat{\mathbf{x}} + 20.8\,\hat{\mathbf{y}} - \,\hat{\mathbf{z}}) \cos(\omega t - 3x - 4y + 5z) \end{aligned}$$

Integrating with respect to t and simplifying,

$$\mathbf{B} = \frac{100}{\omega}\,(29.4\,\hat{\mathbf{x}} - 20.8\,\hat{\mathbf{y}} + \,\hat{\mathbf{z}}) \sin(\omega t - 3x - 4y + 5z) \qquad [\mathrm{T}]$$

(c) The energy flux vector can be defined as

$$\begin{aligned} \mathbf{S} &= c^2 \epsilon_0 \mathbf{E}_0 \times \mathbf{B}_0 \cos^2(\mathbf{k} \cdot \mathbf{r} - \omega t) \\ &= c^2 \epsilon_0 \frac{100^2}{\omega}\,[(2\,\hat{\mathbf{x}} + 3\,\hat{\mathbf{y}} + 3.6\,\hat{\mathbf{z}}) \times (29.4\,\hat{\mathbf{x}} - 20.8\,\hat{\mathbf{y}} + \,\hat{\mathbf{z}})] \cos^2(\ldots) \\ &= c^2 \epsilon_0 \frac{100^2}{\omega} \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ 2 & 3 & 3.6 \\ 29.4 & -20.8 & 1 \end{vmatrix} \cos^2(\ldots) \\ &\approx \frac{7.965 \times 10^9}{\omega}\,(77.9\,\hat{\mathbf{x}} + 103.8\,\hat{\mathbf{y}} - 129.8\,\hat{\mathbf{z}}) \cos^2(\omega t - 3x - 4y + 5z) \quad \left[\mathrm{W} \cdot \mathrm{m}^{-2}\right] \end{aligned}$$

3. (a) As we're taking the magnitude/modulus first,

$$\begin{aligned} |f| &= A \\ \mathrm{Re}\left\{|f|^2\right\} &= A^2 \end{aligned}$$

(b) The real part of f is given as the cosine component and

$$\begin{aligned} [\mathrm{Re}\{f\}]^2 &= [A \cos(kx - \omega t)]^2 \\ &= A^2 \cos^2(kx - \omega t) \end{aligned}$$

4. From (3.44), as $I \equiv \langle S \rangle_T = \frac{c\epsilon_0}{2} E_0^2$,

$$\begin{aligned} E_0 &= \sqrt{\frac{2I}{c\epsilon_0}} = \sqrt{\frac{2\left(1.34 \times 10^3\,\mathrm{W} \cdot \mathrm{m}^{-2}\right)}{c\epsilon_0}} \\ &\approx 1.00\,\mathrm{kV} \cdot \mathrm{m}^{-1} \end{aligned}$$

From the relation $E = cB$, the magnitude of the magnetic field is given as

$$B_0 = \frac{E_0}{c} = 3.34\,\mu\mathrm{T}$$