Homework 5

PHYSICS 342
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1. It would look something like:

2. (a) Assuming the form of (1) and ignoring mutual interaction, the potential term can be split into separate potentials for electron 1 and 2. This basically follows (6.1a) but with the last term equal to zero.

$$\begin{aligned} H\psi_{1s2s} &= \hat{p_1}^2\psi_{1s2s} + \hat{p_2}^2\psi_{1s2s} + E_{\text{pot}}\psi_{1s2s} \\ &= \left(\hat{p_1}^2 + E_{\text{pot}}\right)\psi_{1s} + \left(\hat{p_2}^2 + E_{\text{pot}}\right)\psi_{2s} \\ &= H_1 + H_2 \end{aligned}$$

   (b) There is not a way to distinguish these two wavefunctions, presumably because they are degenerate and would have equal energies and since the electrons are indistinguishable.

3. Both (1) and (2) <u>do not</u> abide by the Pauli principle. For (1), if we swap the electrons, we are left with $\psi_{2s}(\mathbf{r}_1)\psi 1s(\mathbf{r}_2)$. For (2), it's the same thing but you would get back (1) after swapping the electrons.

   But (3) does abide by the Pauli principle. Swapping the electrons results in

$$\psi_{2s}(\mathbf{r}_1)\psi_{1s}(\mathbf{r}_2) - \psi_{1s}(\mathbf{r}_1)\psi_{2s}(\mathbf{r}_2) = -\Psi_{1s2s}(\mathbf{r}_2, \mathbf{r}_1)$$

   ...which is just the opposite of the original wavefunction.

4. If they're both in ψ_{1s}, it would just be zero

$$\psi_{1s}\psi_{1s} - \psi_{1s}\psi_{1s} = 0$$

   This is the Pauli exclusion principle.

5. From (3), if both are now in the $1s$ state and we make electron 1 spin up, and electron 2 spin down,

$$\begin{aligned} \Psi_{1s^2}(1,2) &= \psi_{1s}(1)\psi_{1s}(2)\left|\uparrow\right\rangle\left|\downarrow\right\rangle - \psi_{1s}(1)\psi_{1s}(2)\left|\downarrow\right\rangle\left|\uparrow\right\rangle \\ &= \left(\psi_{1s}(1)\psi_{1s}(2) - \psi_{1s}(1)\psi_{1s}(2)\right)\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right) \end{aligned}$$

   It is Pauli approved. The spatial function is symmetric but the spinfunction is antisymmetric.

6. From (3), if we make electron 1 spin up, and electron 2 spin down,

$$\begin{aligned} \Psi_{1s2s}(1,2) &= \psi_{1s}(1)\psi_{2s}(2)\left|\uparrow\right\rangle\left|\downarrow\right\rangle - \psi_{2s}(1)\psi_{1s}(2)\left|\downarrow\right\rangle\left|\uparrow\right\rangle \\ &= \left(\psi_{1s}(1)\psi_{2s}(2) - \psi_{2s}(1)\psi_{1s}(2)\right)\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right) \end{aligned}$$

   (a) It is not Pauli approved.

   (b) Either the spatial function or spin function must be made symmetric:

$$\begin{aligned} \text{Case A:}\quad & \left(\psi_{1s}(1)\psi_{2s}(2) + \psi_{2s}(1)\psi_{1s}(2)\right)\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right) \\ \text{Case B:}\quad & \left(\psi_{1s}(1)\psi_{2s}(2) - \psi_{2s}(1)\psi_{1s}(2)\right)\left(\left|\uparrow\downarrow\right\rangle + \left|\downarrow\uparrow\right\rangle\right) \end{aligned}$$

7. (a) The spatial function must be antisymmetric, since the spin function is now symmetric. So Case B.
   (b) Case B.
8. Yes, since it has multiplicity of 3. The m_s values are $-1, 0, 1$, which is consistent with $S = 1$.
9. Yes, since now it's multiplicity of 1. The m_s value is just 0, since one electron is spin up, the other is spin down.
10. (a) The triplet state. If $r_1 = r_2$, the antisymmetric spatial function would be zero.
    (b) Yes, both figures show the triplet states lower in energy to their singlet state counterparts.