Homework 4

PHYSICS 304
February 14, 2020

Kevin Evans
ID: 11571810

Chapter 13

26. If the half-life is $8.1\,\mathrm{d}$, then we can determine the decay constant as

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{8.1\,\mathrm{d} \times 86\,400\,\mathrm{s}\cdot\mathrm{d}^{-1}} = 9.904 \times 10^{-7}\,\mathrm{s}^{-1}$$

Then using the current activity, the number of remaining atoms can be found as,

$$\begin{aligned} R &= N\lambda \\ 0.2 \times 10^{-6}\,\mathrm{Ci} \times \left(\frac{3.7 \times 10^{10}\,\mathrm{Bq}}{1\,\mathrm{Ci}}\right) &= N\left(9.904 \times 10^{-7}\,\mathrm{s}^{-1}\right) \\ N &= 7.5 \times 10^{9} \text{ atoms remaining} \end{aligned}$$

28. From eq. (13.10),

$$R = \left|\frac{\mathrm{d}N}{\mathrm{d}t}\right| = N_0 \lambda e^{-\lambda t} = R_0 e^{-\lambda t} \tag{13.10}$$

We can derive the first equation,

$$\begin{aligned} R &= R_0 e^{-\lambda t} \\ \ln\left(\frac{R}{R_0}\right) &= -\lambda t \\ -\left(\ln R - \ln R_0\right) &= \lambda t \\ \ln\left(\frac{R_0}{R}\right) &= \lambda t \\ \lambda &= \frac{1}{t}\ln\left(\frac{R_0}{R}\right) \quad \square \end{aligned}$$

Using the solution above and (13.11),

$$\begin{aligned} \lambda &= \frac{\ln 2}{T_{1/2}} \\ \frac{\ln 2}{T_{1/2}} &= \frac{1}{t}\ln(R_0/R) \\ \frac{T_{1/2}}{\ln 2} &= \frac{t}{\ln(R_0/R)} \\ T_{1/2} &= \frac{(\ln 2)t}{\ln(R_0/R)} \quad \square \end{aligned}$$

35. For a half-life $T_{1/2} = 5730\,\mathrm{yr}$, the rate constant would be

$$\begin{aligned}\lambda &= \frac{\ln 2}{5730} \approx 1.21 \times 10^{-4}\,\mathrm{yr}^{-1} \\ &= \frac{\ln 2}{5730\,\mathrm{yr} \times 365.25\,\mathrm{d/yr} \times 1440\,\mathrm{min}\,/\mathrm{d}} \approx 2.3 \times 10^{-10}\,/\,\mathrm{min}\end{aligned}$$

Then for a gram of Carbon,

$$R = N_0 \lambda e^{-\lambda t} \tag{13.10}$$

$$\begin{aligned}R &\approx \left(1\text{ g Carbon} \times \frac{1\,\mathrm{mol}}{12.011\,\mathrm{g}} \times \frac{6.022 \times 10^{-23}\text{ atoms}}{1\,\mathrm{mol}} \times \frac{1.3 \times 10^{-12}\text{ atoms }{}^{14}\mathrm{C}}{1\text{ atom }{}^{12}\mathrm{C}}\right) \\ &\quad \times \left(2.3 \times 10^{-10}\,/\,\mathrm{min}\right) \\ &\quad \times \exp\left(-1.21 \times 10^{-4}\,\mathrm{yr}^{-1} \times 2000\,\mathrm{yr}\right) \\ &\approx 11.769\ \ \mathrm{disintegrations/min \cdot g}\end{aligned}$$

41. From Table 13.6, the energy released during the decay is the change in mass between the parent and daughter nuclei,

$$\begin{aligned}\Delta m &= 238.050785 - 234.043593 - 4.002603 = 0.004\,589\,\mathrm{u} \\ Q &= \Delta m c^2 = 0.004\,589\,\mathrm{u} \times 931.494\,\mathrm{MeV/u} \\ &\approx 4.27\,\mathrm{MeV}\end{aligned}$$

42. (a) For a photon with energy ΔE, we can use the energy-momentum relation and find the photon's momentum,

$$p = \frac{\Delta E}{c}$$

Then, for any non-relativistic particle of mass M, its kinetic energy can be written as

$$E = \frac{p^2}{2M}$$

As the momentum is conserved, the particle must have an equal momentum (in magnitude),

$$E_r = \frac{(\Delta E)^2}{2Mc^2}$$

(b) For a ${}^{57}\mathrm{Fe}$ nucleus and $14.4\,\mathrm{keV}$ γ-emission,

$$\begin{aligned}E_r &= \frac{\left(14.4 \times 10^{-3}\,\mathrm{MeV}\right)^2}{2\,(57\,\mathrm{u} \times 931.494\,\mathrm{MeV/u})} \\ &= 28.11\,\mu\mathrm{eV}\end{aligned}$$

44. For ${}^{220}_{86}\mathrm{Rn} \to {}^{216}_{84}\mathrm{Po} + {}^{4}_{2}\alpha$, if we assume all of the disintegration energy goes into the α particle's kinetic energy,

$$\begin{aligned}\Delta m &= (220.011368 - 216.001888 - 4.002603) = 0.006\,877\,\mathrm{u} \\ Q &= \Delta m c^2 = 0.006\,877\,\mathrm{u} \times 931.494\,\mathrm{MeV/u} = 6.4059\,\mathrm{MeV}\end{aligned}$$

46. (a) It's forbidden because the mass/energy of the free proton is less than the mass of a neutron.
    (b) It's possible since the mass/energy of the proton bound within a nucleus is greater than its resultants.
    (c) For the reaction in (b) and assuming the ν-energy is negligible,

$$\begin{aligned} Q &= (13.005739 - 13.003355 - 2 \times 0.000549) \times 931.494 \\ &= 1.1979\,\mathrm{MeV} \end{aligned}$$

47. For the nucleus of $^{13}\mathrm{N}$ (from the last problem), its radius is given as

$$\begin{aligned} r &\approx (1.2\,\mathrm{fm})\, 13^{1/3} \\ &\approx 2.82\,\mathrm{fm} \end{aligned}$$

From the uncertainty principle, for an electron exists within the nucleus, the minimum uncertainty in its momentum can be determined as

$$\begin{aligned} \Delta p &\approx h/\Delta x \\ &\approx \left(6.626 \times 10^{-34}\,\mathrm{J \cdot s}\right) / \left(2.82\,\mathrm{fm}\right) \\ &\approx 2.35 \times 10^{-19}\,\mathrm{J \cdot s \cdot m^{-1}} \end{aligned}$$

Using the relativistic energy-momentum relation,

$$\begin{aligned} E^2 &\approx (pc)^2 + (m_e c^2)^2 \\ &= \left(2.35 \times 10^{-19}\,\mathrm{J \cdot s \cdot m^{-1}} \times c\right)^2 + \left(9.11 \times 10^{-31}\,\mathrm{kg} \times c^2\right)^2 \\ E &\approx 7.1 \times 10^{-11}\,\mathrm{J} \\ &\gtrsim 400\,\mathrm{MeV} \end{aligned}$$

The energy of an electron within the nucleus would far exceed the energy of electrons emitted during beta decay.

I definitely looked at the answers in the back of the book on that last part. I'm not sure I would've gotten the connection to that energy and the usual energy of electrons during β decay.

49. I'm going to omit the neutrino mass and disregard the c^2 term (and just add it on at the end).

    Since we're looking at the Q-values for the nuclei energies of mass M_N, we can relate that to the mass of the atom of mass M as,

$$M({}^{\mathrm{A}}_{\mathrm{Z}}\mathrm{X}) = M_N({}^{\mathrm{A}}_{\mathrm{Z}}\mathrm{X}) + Zm_e$$

Or, in terms of a nucleon M_N,

$$M_N({}^{\mathrm{A}}_{\mathrm{Z}}\mathrm{X}) = M({}^{\mathrm{A}}_{\mathrm{Z}}\mathrm{X}) - Zm_e$$

The Q value for β-decay is

$$Q = M_N({}^{\mathrm{A}}_{\mathrm{Z}}\mathrm{X}) - M_N({}^{\mathrm{A}}_{\mathrm{Z+1}}\mathrm{X}) + m_e + m_\nu$$

Substituting the nuclei masses for the atomic masses and neglecting the ν mass,

$$\begin{aligned} Q &= \left[M({}^{\mathrm{A}}_{\mathrm{Z}}\mathrm{X}) - Zm_e\right] - \left[M({}^{\mathrm{A}}_{\mathrm{Z+1}}\mathrm{Y}) - (Z+1)m_e\right] - m_e \\ &= \left[M({}^{\mathrm{A}}_{\mathrm{Z}}\mathrm{X}) - M({}^{\mathrm{A}}_{\mathrm{Z+1}}\mathrm{Y})\right] c^2 \quad \square \end{aligned}$$

Using a similar approach for positron emission,

$$\begin{aligned} Q &= M_N({}^{\mathrm{A}}_{\mathrm{Z}}\mathrm{X}) - M_N({}^{\mathrm{A}}_{\mathrm{Z-1}}\mathrm{Y}) - m_e \\ &= \left[M({}^{\mathrm{A}}_{\mathrm{Z}}\mathrm{X}) - Zm_e\right] - \left[M({}^{\mathrm{A}}_{\mathrm{Z-1}}\mathrm{Y}) - (Z-1)m_e\right] - m_e \\ &= \left[M({}^{\mathrm{A}}_{\mathrm{Z}}\mathrm{X}) - Zm_e\right] - \left[M({}^{\mathrm{A}}_{\mathrm{Z-1}}\mathrm{Y}) - Zm_e + m_e\right] - m_e \\ &= \left[M({}^{\mathrm{A}}_{\mathrm{Z}}\mathrm{X}) - M({}^{\mathrm{A}}_{\mathrm{Z-1}}\mathrm{Y}) - 2m_e\right] c^2 \quad \square \end{aligned}$$

For electron capture, the Q value would be

$$\begin{aligned} Q &= M_N({}^{\mathrm{A}}_{\mathrm{Z}}\mathrm{X}) + m_e - M_N({}^{\mathrm{A}}_{\mathrm{Z-1}}\mathrm{Y}) \\ &= \left[M({}^{\mathrm{A}}_{\mathrm{Z}}\mathrm{X}) - Zm_e\right] + m_e - \left[M({}^{\mathrm{A}}_{\mathrm{Z-1}}\mathrm{Y}) - (Z-1)m_e\right] \\ &= \left[M({}^{\mathrm{A}}_{\mathrm{Z}}\mathrm{X}) - M({}^{\mathrm{A}}_{\mathrm{Z-1}}\mathrm{Y})\right] c^2 \quad \square \end{aligned}$$

50.