Homework 12

PHYSICS 341
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1. (a) From the cylinder, its magnetic field is

$$\begin{aligned}\int \mathbf{B}\cdot \mathrm{d}\mathbf{l} &= \mu_0 I_{\text{enc}} \\ &= \mu_0 \int \mathbf{J}\cdot \mathrm{d}\mathbf{a} \\ &= \mu_0 J_0 \pi R^2 \\ \mathbf{B} &= \frac{\mu_0 J_0 R^2}{2s}\,\hat{\boldsymbol{\phi}}\end{aligned}$$

Then the torque on the dipole becomes

$$\begin{aligned}\mathbf{N} &= \mathbf{m}\times\mathbf{B} \\ &= m_0 \frac{\mu_0 J_0 R^2}{2s}(\,\hat{\mathbf{z}}\times\,\hat{\boldsymbol{\phi}}) \\ &= -m_0 \frac{\mu_0 J_0 R^2}{2s}\,\hat{\mathbf{s}}\end{aligned}$$

(b) $\mathbf{F} = 0$, since $\mathbf{m}\perp\mathbf{B}$.

(c) It's not true for the magnetic analogs because it requires that $\nabla\times\mathbf{E} = 0 \iff \nabla\times\mathbf{B} = 0$, but this is not true. It's only true in the electostatic case, but for magnetics, this is equal to $\mu_0\mathbf{J}$.

2. (a) For the bound volume current,

$$\begin{aligned}\mathbf{J}_b &= \nabla\times\mathbf{M} \\ &= 2k\,\hat{\mathbf{z}}\end{aligned}$$

From Ampere's law, $\oint B\,\mathrm{d}l = \int J\,\mathrm{d}a$

$$B = \begin{cases}\mu_0 ks & s<R \\ \frac{\mu_0 kR^2}{s} & s>R\end{cases}$$

And the surface charge, $\mathbf{K}_b = \mathbf{M}\times\,\hat{\mathbf{n}}$, but the normal direction $(\,\hat{\mathbf{s}})$ is perpindicular to $\mathbf{M}$, so there is no bound surface current. So the B contribution is zero.

(b) There is no free current, so $\mathbf{H} = 0$? Then the magnetic field is

$$\begin{aligned}B &= \mu\mathbf{H} + \mu\mathbf{M} \\ &= \begin{cases}\mu ks & s<R \\ 0? & s>R\end{cases}\end{aligned}$$

3. Between the tubes, the free current enclosed is I, so the azimuthal H and B field is

$$H = \frac{I}{2\pi s}$$
$$B = \frac{\mu_0(1+\chi_m)I}{2\pi s}$$

To check, the magnetization is

$$\begin{aligned}\mathbf{M} &= \chi_m \mathbf{H} \\ &= \frac{\chi_m I}{2\pi s}\,\hat{\boldsymbol{\phi}}\end{aligned}$$

The bound currents are then

$$\begin{aligned}\mathbf{J}_b &= \boldsymbol{\nabla} \times \mathbf{M} = 0 \\ \mathbf{K}_b &= \mathbf{M} \times (\pm\,\hat{\mathbf{s}}) \\ &= \begin{cases} \frac{\chi_m I}{2\pi a}\,\hat{\mathbf{z}} & s = a \\ \frac{\chi_m I}{2\pi b}\,\hat{\mathbf{z}} & s = b \end{cases}\end{aligned}$$

Between the cylinders, the total current is then

$$\begin{aligned}I_{\text{enc}} &= I + \int K_b \,\mathrm{d}l \\ &= I + \frac{\chi_m I}{2\pi a}(2\pi a) = I(1+\chi_m)\end{aligned}$$

From Ampere's law, the magnetic field is

$$B = \frac{\mu_0 I (1+\chi_m)}{2\pi s}$$

4. For a (free) current $J_z = ks$, within the wire, the H field is

$$\begin{aligned}\mathbf{H} &= \frac{k}{2\pi s}\int_0^s s^2 \,\mathrm{d}\phi\;\hat{\boldsymbol{\phi}} \\ &= \frac{k}{2\pi s}\left(\frac{2\pi s^3}{3}\right)\hat{\boldsymbol{\phi}} \\ &= \frac{ks^2}{3}\,\hat{\boldsymbol{\phi}}\end{aligned}$$

Outside the wire, it's the same thing but bounded at $s = a$,

$$\mathbf{H} = \begin{cases} \frac{ks^2}{3}\,\hat{\boldsymbol{\phi}} & s < a \\ \frac{ka^3}{3s}\,\hat{\boldsymbol{\phi}} & s > a \end{cases}$$

As it's a linear medium,

$$\mathbf{B} = \begin{cases} \mu_0(1+\chi_m)\frac{ks^2}{3}\,\hat{\boldsymbol{\phi}} & s < a \\ \mu_0\frac{ka^3}{3s}\,\hat{\boldsymbol{\phi}} & s > a \end{cases}$$

For the bound charges, the magnetization $\mathbf{M} = \chi_m \mathbf{H}$, so *within* the medium,

$$\begin{aligned} \mathbf{J}_b &= \boldsymbol{\nabla} \times \mathbf{M} = 2k/3\,\hat{\mathbf{z}} \\ \mathbf{K}_b &= \mathbf{M} \times \hat{\mathbf{n}} \\ &= -\frac{ks^2}{3}\,\hat{\mathbf{z}} \end{aligned}$$

5. Heating it up to its Curie temperature.