Homework 4

MATH 301
September 17, 2020

Kevin Evans
ID: 11571810

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1. (a) $A \cup B = \{1, 5, 3, 6, 7, 8\}$

   (b) $$\begin{aligned} A \cap (B \cup C) &= A \cap \{3, 5, 6, 7, 8\} \\ &= \{3, 5, 6, 7\} \end{aligned}$$

   (c) $A - B = \{1, 3\}$

   (d) $$\begin{aligned} C - A &= \{8\} \\ (C - A) \cup B &= \{5, 6, 7, 8\} \\ ((C - A) \cup B) \cap A &= \{3, 5, 6, 7\} \end{aligned}$$

   (e) $\varnothing$ (as both sets are disjoint)

2. (a) $(A \cup C) - B$

   (b) $(A \cup C) - (A \cap B)$

   (c) $(A \cup B \cup C) - (A \cap B \cap C)$

   (d) $(A \cap B) \cup (A \cap C) \cup (B \cap C)$

   (e) $(A \cup B \cup C) - (A \cap B) \cup (A \cap C) \cup (B \cap C)$

3. (a) $A \cup B - (A \cap C) - \left(A \cap B \cap \overline{C}\right)$

   (b) $\left(A \cap \overline{B} \cap \overline{C}\right) \cup (B \cap C)$

   (c) $\overline{(B - (A \cup C)) \cup (C - (A \cup B))}$

   (d) $\overline{(A \cup C) - B}$

   (e) $\overline{B \cup \left(A \cap \overline{C}\right)}$

4. **Proposition:** If n is an odd integer, then $n^2 + 4n + 6$ is odd.

   Proof. Let n be an odd integer, then n can be expressed as

   $$\begin{aligned} n &= 2a + 1 && \text{...where } a \in \mathbb{Z} \\ n^2 + 4n + 6 &= (2a + 1)^2 + 4(2a + 1) + 6 && \text{Substition for } n \\ &= 4a^2 + 4a + 1 + 8a + 4 + 6 && \text{Expanding the terms} \\ &= 2(\underbrace{2a^2 + 6a + 5}_{b}) + 1 && \text{From closure, } b \in \mathbb{Z} \\ &= 2b + 1 && \text{The result is odd.} \quad \square \end{aligned}$$

5. **Proposition:** if two integers have the opposite parity, their product is even. *Proof.* Let a have even parity and b have odd parity, then a and be can be expressed as

   $$\begin{aligned} a &= 2n \\ b &= 2m + 1 && \text{where } n, m \in \mathbb{Z} \end{aligned}$$

The product ab becomes

$$\begin{aligned}
ab &= (2n)\,(2m+1) \\
&= 4nm + 2n \\
&= 2(\underbrace{2nm+n}_{c}) && \text{Closure, } c \in \mathbb{Z} \\
&= 2c && \text{The result is always even.} \quad \square
\end{aligned}$$

Two cases are not needed since multiplication is commutative, i.e. $ab = ba$.