Example

$$\max \quad z = x_1 + 2x_2$$
$$\text{s.t.} \quad x_1 + 3x_2 \le 13$$
$$2x_1 - x_2 \le 6$$
$$x \ge 0, \quad x \in \mathbb{Z}^2$$

- Solve the LP relaxation

1	3	1	0	13	$= x_3$
2	-1	0	1	6	$= x_4$
1	2	0	0	0	$= -z$
1/3	1	1/3	0	13/3	$= x_2$
7/3	0	1/3	1	31/3	$= x_4$
1/3	0	-2/3	0	-26/3	$= -z$
0	1	2/7	-1/7	20/7	$= x_2$
1	0	1/7	3/7	31/7	$= x_1$
0	0	-5/7	-1/7	-71/7	$= -z$

$x^* = \left(\frac{31}{7}, \frac{20}{7}, 0, 0\right)$

$z^* = \frac{71}{7}$

- The solution is non-integral.
- Select any constraint with nonintegral rhs

$$x_2 + \frac{2}{7}x_3 - \frac{1}{7}x_4 = \frac{20}{7}$$

rearrange into integral and nonintegral parts:

$$x_2 + \frac{2}{7}x_3 - x_4 + \frac{6}{7}x_4 = 2 + \frac{6}{7}$$

$$\frac{2}{7}x_3 + \frac{6}{7}x_4 - \frac{6}{7} = \underbrace{-x_2 + x_4 + 2}_{\text{integer}}$$

So, $\frac{2}{7}x_3 + \frac{6}{7}x_4 - \frac{6}{7} \ge 0$ (integer)

$$-\frac{2}{7}x_3 - \frac{6}{7}x_4 + x_5 = -\frac{6}{7}$$

Add this new constraint:

0	1	2/7	-1/7	0	20/7	$= x_2$
1	0	1/7	3/7	0	31/7	$= x_1$
0	0	-2/7	-6/7	1	-6/7	$= x_5$
0	0	-5/7	-1/7	0	-71/7	$= -z$

Apply the Dual Simplex Method

0	1	1/3	0	-1/6	3	$= x_2$
1	0	0	0	1/2	4	$= x_1$
0	0	1/3	1	-7/6	1	$= x_4$
0	0	-2/3	0	-1/6	-10	$= -z$

$\bar{x} = (4, 3)$

$\bar{z} = 10$

Is this really optimal?

- Notice that x^* does not satisfy the new inequality constraint.
- Also, any feasible integer solution still satisfies the new constraint

In short: the new constraint is the negative of the fractional part of a (nonintegral rhs) constraint along with a new slack variable.

* fractional parts are always non-negative

$f(3.14) = 0.14 \quad f(-3.14) = 0.86$