Homework 11

PHYSICS 461
April 15, 2021

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1. The absorption coefficient is given by

$$\alpha(\nu_{12}) = [N_1 - (g_1/g_2)N_2]\,\sigma(\nu_{12})$$

where in the $2 \to 1$ transition, the statistical weights are 5 and 3 respectively. As the rotational energy is defined as $E_J = B_e J(J+1)$ where B_e is the rotational constant specific to the molecule, then $\Delta E = hcB_e\left[(J_2(J_2+1) - J_1(J_1+1))\right] =$. Additionally, the populations of the two levels are related as

$$\begin{aligned} \frac{N_2}{N_1} &= \frac{g_2}{g_1} e^{-h\nu/kT} \\ \implies \frac{N_2}{N_1} - \frac{g_1}{g_2} N_2 &= N_1 - \frac{g_1}{g_2}\left[\frac{g_2}{g_1}(1 - \Delta E/kT)\right] N_1 \end{aligned}$$

2. The Fourier transform of $S(t)$ can be written

$$I(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} S(t) e^{i\omega t}\,\mathrm{d}t$$

From the relation $e^{i\omega t} = \cos(\omega t) + i\sin(\omega t)$, we can convert this to a one-sided Fourier transform

$$I(\omega) = \frac{2}{\sqrt{2\pi}} \int_{0}^{\infty} S(t) \cos(\omega vt/c)\,\mathrm{d}t$$

3. For 1200 grooves per mm, the distance between grooves is

$$d = 833.3\,\mathrm{nm}$$

For $m = 1$, the angle of each reflection is

$$\begin{aligned} \beta_1 &= \sin^{-1}\left(\frac{\lambda}{d} - \sin\alpha\right) \\ &= 0.208\,209\,\mathrm{rad} \\ \beta_2 &= 0.208\,945\,\mathrm{rad} \\ \Delta\beta &= 0.000\,735\,98\,\mathrm{rad} \\ \implies \Delta s &= \beta f \\ &= 0.736\,\mathrm{mm} \end{aligned}$$

5. From Beere's law, the transmitted power is

$$P = P_0 e^{-\alpha x} \approx P_0(1 - \alpha x)$$

Then the absorbed power per length is

$$\Delta P = \alpha P_0 = 10^{-7} \quad \mathrm{W \cdot cm^{-1}}$$

Therefore, the power absorption rate is

$$\begin{aligned} N &= \frac{\Delta P}{E} = \frac{\Delta P}{h\nu} \\ &\approx 2.5 \times 10^{11} \text{ photons per s} \end{aligned}$$

For the given angle, the detector receives the fraction

$$\frac{\Delta\Omega}{4\pi} N = \frac{0.2}{4\pi} 2.5 \times 10^{11} = 4 \times 10^{9} \quad \text{photons per s}$$

Then for the given efficiency, the current is just the fraction of photons per second and

$$I = (\eta G) I_0 = 0.13\,\mathrm{mA}$$