Final Exam

PHYSICS 465
April 29, 2022

Kevin Evans
ID: 11571810

---

1. Given the charge distribution
$$\rho(r) = \rho_0 e^{-r/a}/r,$$
and using Table 3.1 (since it's spherically symmetric), the form factor is
$$\begin{aligned} F(q^2) &= \frac{4\pi\hbar}{Zeq} \int \rho(r) r \sin(qr/\hbar)\,\mathrm{d}r \\ &= \frac{4\pi\hbar\rho_0}{Zeq} \int_0^\infty e^{-r/a} \sin(qr/\hbar)\,\mathrm{d}r \end{aligned}$$
Using a definite integration table,
$$\begin{aligned} F(q^2) &= \frac{4\pi\hbar\rho_0}{Zeq} \left[\frac{q/\hbar}{1/a^2 + (q/\hbar)^2} \right] \\ &= \frac{4\pi\rho_0}{Ze} \frac{1}{1/a^2 + (q/\hbar)^2}. \end{aligned}$$

2. (a) In this problem, we've got:
$$\pi \to \nu + \mu.$$
In terms of the 4-momenta,
$$\begin{aligned} \mathbb{P}_\pi &= \mathbb{P}_\nu + \mathbb{P}_\mu \\ \implies \mathbb{P}_\nu{}^2 &= \mathbb{P}_\pi^2 - \mathbb{P}_\mu^2 - 2\mathbb{P}_\pi \cdot \mathbb{P}_\mu \\ 0 &= m_\pi^2 + m_\mu^2 - 2(m_\pi, 0) \cdot (E_\mu, \mathbf{p}_\mu) = m_\pi^2 - m_\mu^2 - 2E_\mu m_\pi \\ \implies E_\mu &= \frac{m_\pi^2 + m_\mu^2}{2m_\pi}. \end{aligned}$$

   (b) From the energy-momentum relation,
$$\begin{aligned} E_\mu^2 &= p_\mu^2 + m_\mu^2 \\ p_\mu^2 &= E_\mu^2 - m_\mu^2 \\ &= \frac{m_\pi^2 + m_\mu^2}{2m_\pi} - \frac{2m_\pi^2 m_\mu^2}{2m_\pi^2} \\ &= \frac{(m_\pi - m_\mu)^2}{2m_\pi}. \end{aligned}$$

   (c) Assuming nonrelativistic speeds,
$$\begin{aligned} p_\mu &= m_\mu v_\mu \\ v_\mu &= p_\mu / m_\mu \\ &= \frac{m_\pi - m_\mu}{\sqrt{2m_\pi m_\mu^2}}. \end{aligned}$$

3. (a) For the generic reaction to three products,

$$m_1 + m_2 \to m_3 + m_4 + m_5$$
$$\mathbb{P}_{\text{initial}} = \mathbb{P}_{\text{final}}.$$

We can assume there's a frame where m_1 is moving and m_2 is stationary, so the 4-momenta are

$$(E_1, p_1) + (m_2, 0) = \mathbb{P}_{\text{total}}$$
$$\implies \mathbb{P}^2_{\text{total}} = m_1^2 + m_2^2 + 2E_1 m_1. \tag{1}$$

In the threshold kinetic energy case, we can assume that the products are stationary and can be treated as a single mass, so

$$\mathbb{P}^2_{\text{total}} = (m_3 + m_4 + m_5)^2 \tag{2}$$

Equating the 4-momenta (1) and (2),

$$m_1^2 + m_2^2 + 2E_1 m_1 = (m_3 + m_4 + m_5)^2$$

The threshold energy is

$$E_1 = \frac{(m_3 + m_4 + m_5)^2 - m_1^2 - m_2^2}{2m_1}.$$

For the KE,

$$KE_{\text{threshold}} = \frac{(m_3 + m_4 + m_5)^2 - m_1^2 - m_2^2}{2m_1} - m_1.$$

(b) For the case of $p + p \to p + p + \pi^0$,

$$E_1 = \frac{(2 \times 938\,\mathrm{MeV/c} + 135\,\mathrm{MeV/c})^2 - 2 \times (938\,\mathrm{MeV/c})^2}{2 \times 938\,\mathrm{MeV/c}}$$
$$= 1217\,\mathrm{MeV}.$$

In terms of the proton's kinetic energy,

$$KE = E - m_1 = 279\,\mathrm{MeV}.$$

No, it requires an additional energy beyond the rest mass to create the pion.

4. (a) It violates the Pauli exclusion principle (antisymmetry under exchange). The correct wavefunction would have an antisymmetric spatial part, which could be

$$\Psi(x_1, x_2) = [\psi_{1s}(x_1)\psi_{2s}(x_2) - \psi_{2s}(x_1)\psi_{1s}(x_2)] \left|\uparrow\uparrow\right\rangle.$$

(b) There are two other options for a different total state, $S_z = -1, 0$. For -1, the spins would flip,

$$\Psi(x_1, x_2) = [\psi_{1s}(x_1)\psi_{2s}(x_2) - \psi_{2s}(x_1)\psi_{1s}(x_2)] \left|\downarrow\downarrow\right\rangle.$$

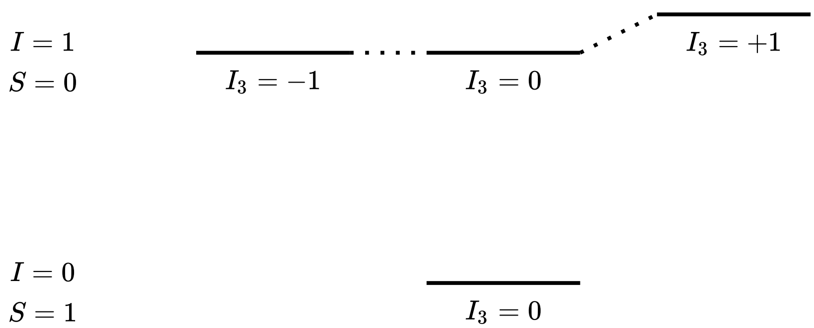
This corresponds to the upper state in the diagram.

(c) They have equal energy, since the spins are aligned in both cases.

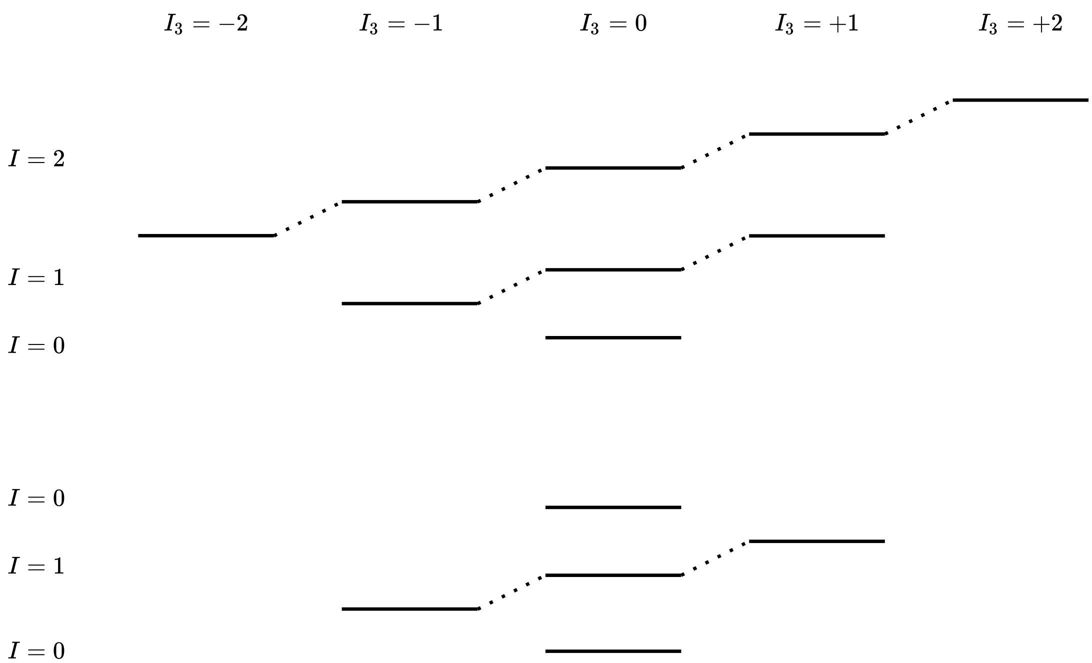
(d) The spinfunctions must be antisymmetric now, as the spatial function is symmetric,

$$\Psi(x_1, x_2) = \psi_{1s}(x_1)\psi_{2s}(x_2)\left[|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right].$$

5. (a) The upper states are in a triplet state with isospin $I = 1$. Since this is a symmetric state, the spin must be antisymmetric, with $S = 0$. Similarly, the lower states have isospin $I = 0$ (antisymmetric), so the spin must be symmetric with $S = 1$. This results in the diagram below.

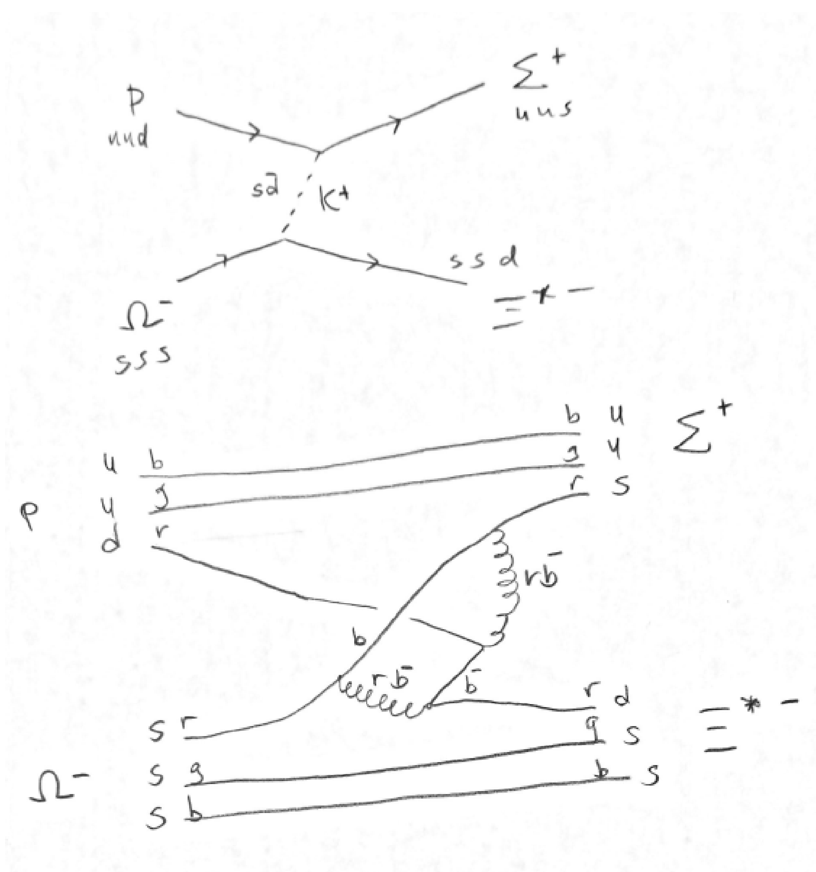


(b) Similarly,



(c) For ^{14}N, we have 7 neutrons/protons, so we're in an unpaired spin state, so total spin is $1/2$. For ^{14}C and ^{14}O, we have paired neutrons/protons with a magic number, so the total spin is $S = 0$. The lowest I singlet ($I = 0$) would have spin $S = 1$, and the triplet ($I = 1$) would have $S = 0$.

6. One possibility is Σ^+,



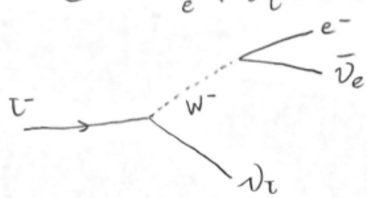
7. I think there is a typo on the μ^--decay, since charge wasn't conserved, so I'm assuming it should've gone to an e^-.

$$\pi^+ \to \pi^0 + e^+ + \nu_e$$
$$\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$$
$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$$
$$K^+ \to \pi^0 + e^- + n\bar{u}_e$$
$$\bar{K}^0 \to \pi^0 + e^- + \bar{\nu}_e$$
$$\Sigma^- \to n + \mu^- + \bar{\nu}_\mu$$
$$\Sigma^+ \to \Lambda^0 + e^+ + \nu_e$$
$$D^0 \to K^- + \pi^0 + e^+ + \nu_e$$

$$\bar{\nu}_e + p \to n + e^+$$
$$\nu_e + {}^{37}\mathrm{Cl} \to {}^{37}\mathrm{Ar} + e^-$$
$$\nu_\mu + p \to \mu^- + p + \pi^+$$
$$\nu_e + n \to e^- + p$$
$${}^3\mathrm{H} \to {}^3\mathrm{He} + e^- + \bar{\nu}_e$$
$$\pi^+ \to \mu^+ + \nu_\mu$$
$$\pi^- \to e^- + \bar{\nu}_e$$
$$\tau^- \to \pi^- + \pi^0 + \nu_\tau$$

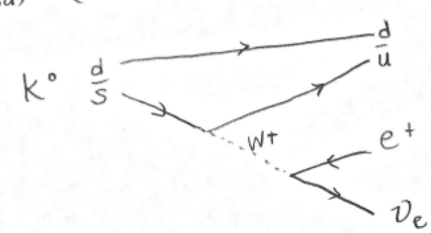
8. (a)

① $\tau^- \to e^- + \bar{\nu}_e + \nu_\tau$



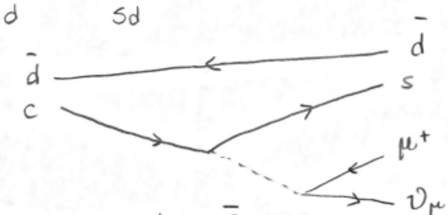
② $K^0 \to \pi^- + e^+ + \nu_e$ (hadron)

$(d\bar{s}, s\bar{d})$ $(d\bar{u})$

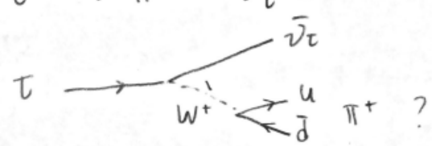


③ $D^+ \to \bar{K}^0 + \mu^+ + \nu_\mu$

$c\bar{d}$ $s\bar{d}$

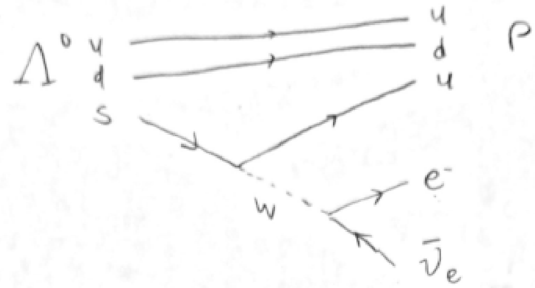


④ $\tau^+ \to \pi^+ + \bar{\nu}_\tau$



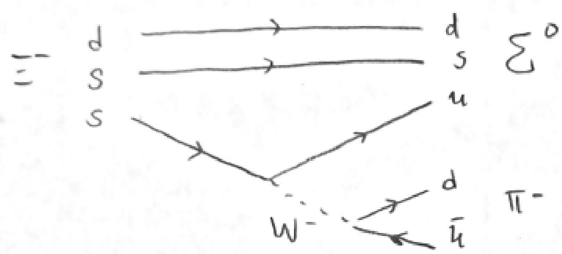
⑤ $\Lambda^0 \to p + e^- + \bar{\nu}_e$

uds uud



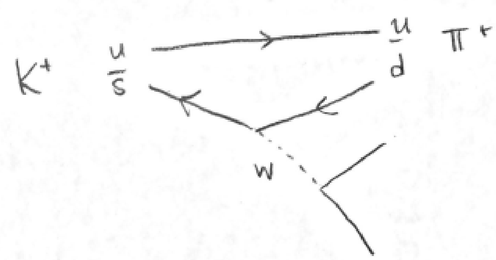
⑥ $\Xi^- \to \Sigma^0 + \pi^-$

dss uds $d\bar{u}$

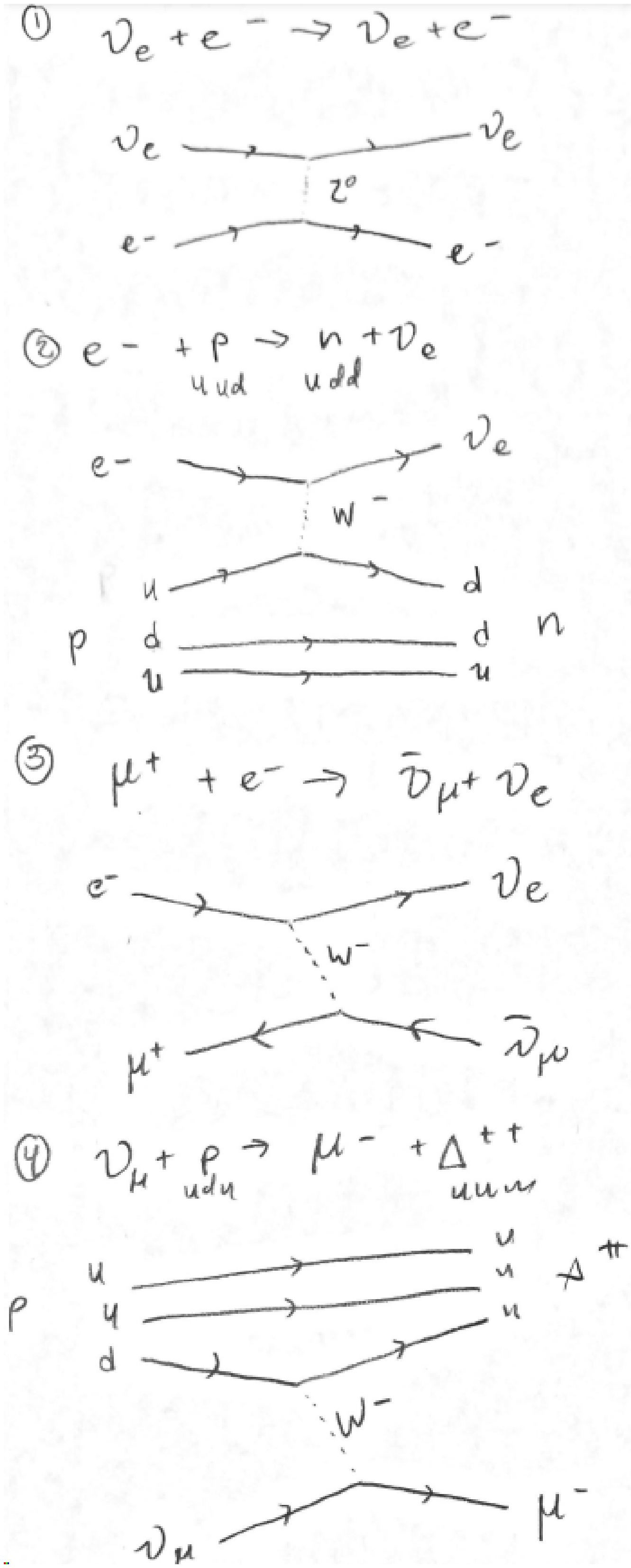


⑦ $K^+ \to \pi^+ + \pi^- + \pi^+$

$u\bar{s}$ $u\bar{d}$ $d\bar{u}$ $u\bar{d}$



(b)


① ν_e + e⁻ → ν_e + e⁻
ν_e
ν_e
Z⁰
e⁻
e⁻
② e⁻ + p → n + ν_e
uud
udd
e⁻
ν_e
W⁻
u
d
p
d
d
n
u
u
③ μ⁺ + e⁻ → ν̄_μ + ν_e
e⁻
ν_e
W⁻
μ⁺
ν̄_μ
④ ν_μ + p → μ⁻ + Δ⁺⁺
udu
uuu
u
u
Δ⁺⁺
p
u
u
u
d
W⁻
ν_μ
μ⁻

9. (a) Under quark exchange, ζ_{flavor} and χ_{spin} are symmetric, so ϕ_{color} *must* be antisymmetric to fulfill the Pauli exclusion principle.

   (b) It expands to that since it must includes all combinations of the symmetric quark ordering.

   (c) For the Σ^{*0} particle, we have to include all permutations, so omitting all the spins (all spin $\uparrow$) and the normalization coefficient,

$$|uds\rangle + |sud\rangle + |dsu\rangle - |sdu\rangle - |dus\rangle - |usd\rangle$$

   (d) The flavor-spin term can't be antisymmetric since the color term is already antisymmetric. This would mean the overall wavefunction is symmetric.

   (e) Flavor and spin can't treated as separate functions and has to be treated as a single function of *both* flavor and spin.

   (f) No, as the flavor part for uuu, sss, ddd is symmetric and the spin part would be symmetric for a spin-1/2 baryon, as $S = 0$. This would mean the total wavefunction is symmetric, which cannot be the case.

10. (a) For the time-dependence, we can use $e^{-iEt/\hbar}$ on each state,

$$\psi_\mu(x,t) = \psi_1 e^{-iE_1 t/\hbar} \cos\theta + \psi_2 e^{-iE_2 t/\hbar} \sin\theta.$$

   (b) (I worked with Wyatt on this part.) For the orthogonal electron neutrino wavefunction,

$$\psi_e(x,t) = -\psi_1 e^{-iE_1 t/\hbar} \sin\theta + \psi_2 e^{-iE_2 t/\hbar} \cos\theta.$$

   Solving for each ψ_i in terms of ψ_μ and ψ_e,

$$\begin{aligned}\psi_1 &= [\psi_\mu \cos\theta - \psi_e \sin\theta]\, e^{-iE_1 t/\hbar} \\ \psi_2 &= [\psi_\mu \sin\theta + \psi_e \sin\theta]\, e^{-iE_1 t/\hbar}.\end{aligned}$$

   Then, the muon wavefunction can be written

$$\begin{aligned}\psi_\mu &= \psi_1 e^{-iE_1 t/\hbar} \cos\theta + \psi_2 e^{-iE_2 t/\hbar} \sin\theta \\ &= [\psi_\mu \cos\theta - \psi_e \sin\theta]\, e^{-i2E_1 t/\hbar} \cos\theta + [\psi_\mu \sin\theta + \psi_e \sin\theta]\, e^{-i2E_1 t/\hbar} \sin\theta\end{aligned}$$

   Looking at just the electron coefficients in the muon wavefunction, the probability is

$$\begin{aligned}P(\mu \to e) &= \left[-\sin\theta\cos\theta e^{-i2E_1 t/\hbar} + \sin\theta\cos\theta e^{-i2E_2 t/\hbar}\right]^2 \\ &= \sin^2(\theta)\cos^2(\theta)\left(e^{-i2E_2 t/\hbar} - e^{-i2E_1 t/\hbar}\right)^2.\end{aligned}$$