Homework 5

PHYSICS 304
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Chapter 14

6. The Q value for ${}^{10}_{5}\mathrm{B}\,(\alpha, \mathrm{p})\,{}^{13}_{6}\mathrm{C}$ is

$$Q_{B\to C} = (M_B + M_\alpha - M_C - M_p)c^2$$

For its inverse,

$$\begin{aligned} Q_{C\to B} &= (M_C + M_p - M_B - M_\alpha)c^2 \\ &= -Q_{B\to C} \\ \therefore \quad |Q_{B\to C}| &= |Q_{C\to B}| \end{aligned}$$

10. Using (14.5), if N_0 particles are incident to the first layer, then the particles that make it through to the second layer would be

$$N_1 = N_0 e^{-n_1 \sigma x_1}$$

Since these are now incident to the second layer, the number that then emerge past both layers is

$$\begin{aligned} N &= N_1 e^{-n_2\sigma x_2} = N_0 e^{-n_1\sigma x_1} e^{-n_2 \sigma x_2} \\ &= N_0 e^{-\sigma(n_1 x_1 + n_2 x_2)} \end{aligned}$$

For an arbitrary number of layers k with the same cross section σ,

$$N_k = N_0 e^{-\sigma \sum\limits_k n_k x_k}$$

11. For a density of $70\,\mathrm{kg}\cdot\mathrm{m}^{-3}$ vat of liquid hydrogen, the number of particles per cubic meter is

$$\begin{aligned} n &= 70\,\mathrm{kg} \times \frac{1000\,\mathrm{g}}{1\,\mathrm{kg}} \times \frac{1.008\,\mathrm{mol}}{1\,\mathrm{g}\ \mathrm{H}\ (\ell)} \times \frac{6.022\times 10^{23}\ \text{particles}}{1\,\mathrm{mol}} \\ &= 4.25\times 10^{28}\ \text{particles H per cubic meter} \end{aligned}$$

Then if 20% reacts with the hydrogen after $2\,\mathrm{m}$, then 80% of the particles emerge past $x = 2\,\mathrm{m}$,

$$\begin{aligned} \frac{N}{N_0} &= 0.80 \\ 0.80 &= e^{-n\sigma x} \\ \ln 0.80 &= -4.25\times 10^{28}\,\mathrm{m}^{-3} \times 2\,\mathrm{m}\times\sigma \\ \sigma &= 2.63\times 10^{-30}\,\mathrm{m}^2 \\ &= 0.0263\,\mathrm{b} \end{aligned}$$

19. For an energy E_0 that is halved each collision, then its energy after n collisions can be described as

$$E_n = E_0 \left(\frac{1}{2}\right)^n$$

For the neutron of initially $1\,\mathrm{MeV}$, it will reach the thermal energy at:

$$\begin{aligned} 0.039\,\mathrm{eV} &= \frac{1 \times 10^6\,\mathrm{eV}}{2^n} \\ 2^n &= \frac{1 \times 10^6}{0.039} \\ n &= \left\lceil \log_2 \left(\frac{1 \times 10^6}{0.039}\right) \right\rceil = \lceil 24.6 \rceil \\ &= 25 \text{ collisions} \end{aligned}$$

(Taking the ceil there since $n \in \mathbb{Z}$ and it's needs to exceed that number to reach that energy.)

20. (a) For a thermal neutron at $300\,\mathrm{K}$, its average kinetic energy is

$$\begin{aligned} \langle K \rangle &= \frac{3}{2} k_B T = \frac{3}{2} \times 8.617 \times 10^{-5}\,\mathrm{eV} \cdot \mathrm{K}^{-1} \times 300\,\mathrm{K} \\ &= 0.0388\,\mathrm{eV} \end{aligned}$$

If we use the classical momentum-energy relationship, then

$$\begin{aligned} \frac{p^2}{2m_n} &= K \\ p &= \sqrt{2m_n K} = \sqrt{2\left(939.6 \times 10^6\,\mathrm{eV}/c^2\right)(0.0388\,\mathrm{eV})} \\ &= 8538.9\,\mathrm{eV}/c \end{aligned}$$

(b) For the de Broglie wavelength,

$$\begin{aligned} \lambda &= \frac{h}{p} \\ &= \frac{1240\,\mathrm{eV} \cdot \mathrm{nm}}{8538.9\,\mathrm{eV}} \\ &= 0.1452\,\mathrm{nm} \\ &= 145.2\,\mathrm{pm} \end{aligned}$$

Which is fairly huge, relative to nuclei,

$$\frac{\text{de Broglie wavelength}}{\text{nucleus diameter}} = \frac{145.2\,\mathrm{pm}}{1.2\,\mathrm{fm}} \approx 10^5$$

It's approximately equal to the van der Waals radii of elements,

$$\begin{aligned} 145.2\,\mathrm{pm} &\approx 1.5\,\text{Å} \\ &\approx 1.2\,\text{Å} && \leftarrow \text{Radius of H} \\ &\approx 1.4\,\text{Å} && \leftarrow \text{Radius of He} \end{aligned}$$

21. In the reaction, the change in mass governs the energy released,

$$\begin{aligned}\Delta M &= (1.008665 + 235.043915) - (140.9139 + 91.8973 + 3 \times 1.008665) \\ &= 0.215\,385\,\mathrm{u} \\ Q &= \Delta M c^2 = 0.215\,385\,\mathrm{u} \times 931.494\,\mathrm{MeV}/c^2 \\ &= 200.63\,\mathrm{MeV}\end{aligned}$$

22. (a) From Example 14.4 (p. 513), the energy released during a single $^{235}\mathrm{U}$ fission event is

$$Q = 208\,\mathrm{MeV}$$

If we need to generate $1000\,\mathrm{MW}$ of power in a day,

$$\begin{aligned}\langle P\rangle &= \frac{\Delta W}{\Delta T} \\ 1000 \times 10^6\,\mathrm{J\cdot s^{-1}} &= \frac{\Delta W}{86\,400\,\mathrm{s}} \\ Q = \Delta W &= 86.4 \times 10^{13}\,\mathrm{J} \\ &= 86.4 \times 10^{13}\,\mathrm{J} \times \frac{1\,\mathrm{eV}}{1.602 \times 10^{-19}\,\mathrm{J}} \times \frac{1\,\mathrm{MeV}}{1 \times 10^6\,\mathrm{eV}} \\ &= 5.393 \times 10^{26}\,\mathrm{MeV}\end{aligned}$$

Relative to $^{235}\mathrm{U}$ fission events,

$$N = \frac{5.393 \times 10^{26}\,\mathrm{MeV}}{208\,\mathrm{MeV}} = 2.593 \times 10^{24} \text{ fission events needed}$$

If we assume every $^{235}\mathrm{U}$ atom in a pure sample reacts, we can calculate the minimum mass needed to generate that power

$$\begin{aligned}m &\geq 2.593 \times 10^{24}\,\mathrm{nuclei} \times \frac{235\,\mathrm{g/mol}}{6.022 \times 10^{23}\,\mathrm{nuclei/mol}} \\ &\geq 1011.8\,\mathrm{g} \text{ of } {}^{235}\mathrm{U}\end{aligned}$$

(b) Given the density of uranium, the volume would be

$$V = 1011.8\,\mathrm{g} \times \frac{1\,\mathrm{cm}^3}{18.7\,\mathrm{g}} = 54.1\,\mathrm{cm}^3$$

As a sphere,

$$\begin{aligned}V &= \frac{4}{3}\pi r^3 \\ r &= \left(\frac{3}{4\pi}V\right)^{1/3} \\ &= 2.35\,\mathrm{cm}\end{aligned}$$

The sphere would have a radius of $2.35\,\mathrm{cm}$.

30. (a) For the given reaction, the approximate energy released per fusion event is $25\,\mathrm{MeV}$ (p. 518, under *Thermonuclear Reactions*).

Using a similar process as the last problem,

$$\begin{aligned}\frac{N}{\Delta T} &= 4\times 10^{26}\,\mathrm{J\cdot s^{-1}} \times \frac{1\times 10^{-6}\,\mathrm{MeV}}{1.602\times 10^{-19}\,\mathrm{J}} \times \frac{1\text{ fusion event}}{25\,\mathrm{MeV}} \\ &\approx 1\times 10^{38}\,\mathrm{s^{-1}}\end{aligned}$$

(b) The change in mass during the fusion is

$$\Delta M = 4(1.007825) - 4.002603 - 5.486\times 10^{-4} = 0.028\,15\,\mathrm{u}$$

Using the rate from (a), the mass to energy rate is

$$\begin{aligned}\frac{\Delta M}{\Delta T} &= 2.815\times 10^{36}\,\mathrm{u\cdot s^{-1}} \\ &= 4.67\,\mathrm{kg\cdot s^{-1}}\end{aligned}$$

34. (a) For $1\,\mathrm{kg}$ of ^{239}Pu undergoing fission,

$$\begin{aligned}N &= 1000\,\mathrm{g}\text{ Pu} \times \frac{1\,\mathrm{mol}}{239\,\mathrm{g}} \times \frac{6.022\times 10^{23}\text{ nuclei}}{1\,\mathrm{mol}} \\ &= 2.52\times 10^{24}\text{ Pu nuclei} \\ Q_{\text{total}} &= NQ_{\text{single}} \\ &= 2.52\times 10^{24}\times 200\,\mathrm{MeV}\times \frac{1.602\times 10^{-22}\,\mathrm{kJ}}{1\times 10^{-6}\,\mathrm{MeV}} \times \frac{1}{3600\,\mathrm{s}} \\ &= 22.43\,\mathrm{kW\cdot h}\end{aligned}$$

(b) The energy per fission event is given by the change in mass

$$\begin{aligned}Q &= (2.014102 + 3.016049 - 4.002603 - 1.009)\times 931.494 \\ &= 17.278\,\mathrm{MeV}\end{aligned}$$

(c) For a kilogram of deuterium,

$$\begin{aligned}Q_{\text{total}} &= 1000\,\mathrm{g}\times\frac{1\,\mathrm{mol}}{2\,\mathrm{g}}\times\frac{6.022\times 10^{23}\text{ nuclei}}{1\,\mathrm{mol}}\times 17.278\,\mathrm{MeV}\text{ per nucleus} \\ &\quad\times \frac{1.602\times 10^{-22}\,\mathrm{kJ}}{1\times 10^{-6}\,\mathrm{MeV}}\times\frac{1}{3600\,\mathrm{s}} \\ &= 2.32\times 10^{8}\,\mathrm{kWh}\end{aligned}$$

(d) For a kilogram of coal,

$$\begin{aligned}Q &= \frac{1000\,\mathrm{g}}{12\,\mathrm{g\cdot mol^{-1}}}\times\frac{6.022\times 10^{23}\text{ carbon atoms}}{1\,\mathrm{mol}}\times 4.2\times 10^{-6}\,\mathrm{MeV} \\ &\quad\times\frac{1.602\times 10^{-22}\,\mathrm{kJ}}{1\times 10^{-6}\,\mathrm{MeV}}\times\frac{1}{3600\,\mathrm{s}} \\ &= 9.38\,\mathrm{kWh}\end{aligned}$$

(e) **Fission**

Pros: generates lots of energy per gram; not terrible for the environment.

Cons: public perceives nuclear power as scary; generates harmful waste.

Fusion

Pros: generates lots of energy, waste is less harmful, currently expensive.

Cons: literally impossible to do economically.

Combustion

Pros: cheap and (currently) readily-abundant, simple to extract energy from

Cons: terrible for the environment, contributes to income inequality, generates little energy relative to nuclear options, mining for coal and other fuels destroys the environment, oil and coal reserves depleting.

53. As a rad increases a kilogram by $1 \times 10^{-2}\,\mathrm{J}$,

$$E = 25 \times 10^{-2}\,\mathrm{J} \cdot \mathrm{kg}^{-1} \times 75\,\mathrm{kg} = 18.75\,\mathrm{J}$$