Final problem assignment, Phys 304

Problem 1. (a) What is the minimum possible total energy of five (non-interacting) spin-$\frac{1}{2}$ particles of mass m in a one-dimensional box of length L?

Problem 2. Same as the previous problem, except that the particles are spin-1.

Problem 3. What happens If we shoot a neutron through a Stern-Gerlach apparatus? A neutron is a spin-½ particle which has a magnetic moment; however, it has no charge.

Problem 4. Identify the different total angular momentum states (j, m_j) that are available for a $3d$ electron in a hydrogen atom.

Problem 5. (a) Derive the formulas

$$\text{(a)} \ \sum_{n=0}^{\infty} x^n = \frac{1}{1-x}, \qquad \text{(b)} \ \sum_{n=0}^{\infty} n x^n = \frac{x}{(1-x)^2}$$

Problem 6. Using the results of the previous problem, derive the average energy of a harmonic oscillator, by starting with the formula

$$\bar{E} \ = \ \frac{\sum_n E_n e^{-E_n/k_\mathrm{B}T}}{\sum_n e^{-E_n/k_\mathrm{B}T}}$$

Your answer should be in terms of the oscillator frequency ω_0.

Problem 7. Using $g(E) = \text{const} \times E^2$ for photons in a box, (e.g. Serway *et al* §10.4) show that the total energy of a box of photons is proportional to T^4 (times other quantities that you do not need to derive). HINT: you will need to evaluate

$$U \ = \ \bar{E} \ = \ \int_0^\infty E\, g(E) f(E)\, dE$$

Problem 8. How can the analysis of the rotational spectrum of a molecule give information about the size of that molecule?

Problem 9. Reexpress

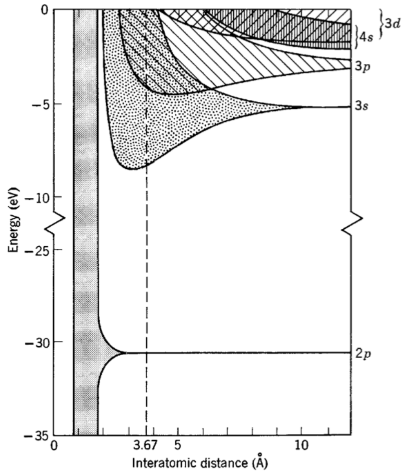
$$dU_\mathrm{phot} \ = \ \frac{hf^3}{e^{hf/k_\mathrm{B}T} - 1} \frac{8\pi V}{c^3}\, df$$

in terms of wavelength λ.

Problem 10. The density of the electron states in a metal can be written $g(E) = AE^{1/2}$, where A is a constant and E is measured from the bottom of the conduction band.

Show that the total number of states is $(2/3)A(E_F)^{3/2}$.

Problem 11. Assume the energy diagram below is that of pure beryllium (two $3s$ electrons). Is beryllium an insulator or conductor, and *why*?



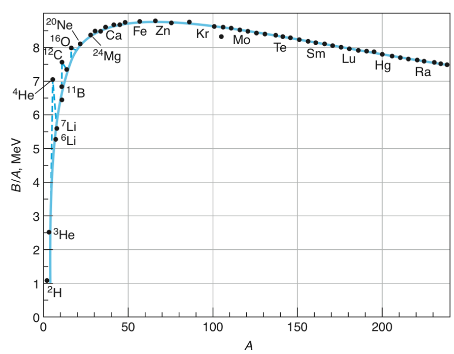
Problem 12. Why is diamond transparent, whereas silicon (which has the same crystal structure) opaque? (Band gaps are roughly 5 eV for diamond and 1 eV for silicon.)

Problem 13. Starting with

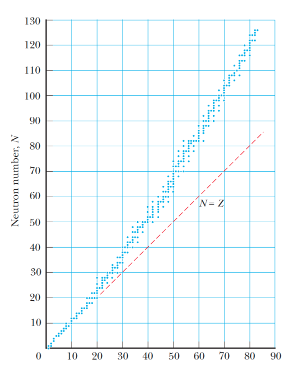
$$dU_\mathrm{phot} \ = \ \frac{hf^3}{e^{hf/k_\mathrm{B}T} - 1} \frac{8\pi V}{c^3}\, df$$

reproduce Einstein's argument regarding his A and B coefficients.

Problem 14. An untrained but perceptive friend exclaims, "They say that nuclear energy can be released by sticking nuclei together and by breaking them apart. That doesn't make sense!" Straighten out your friend's confusion, with reference to the following figure.



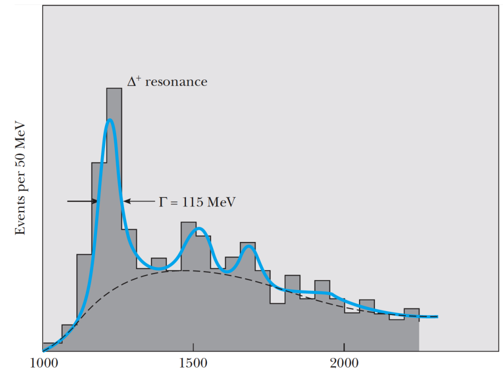
Problem 15. Why do nearly all the naturally occurring isotopes lie above the $N = Z$ line in the figure below?



Problem 16. A star ending its life with a mass of two times the mass of the Sun is expected to collapse, combining its protons and electrons to form a neutron star. Such a star could be thought of as a gigantic atomic nucleus. If a star of mass $2M_S = 2 \times 1.99 \times 10^{30}$ kg collapsed into neutrons ($m_n = 1.67 \times 10^{-27}$ kg), what would its radius be? (Assume that $r = r_0 A^{1/3}$.)

Problem 17. Motivate the time-energy uncertainty relation, and use it to write a formula for the range of the nuclear force, assumed to arise through the exchange of mesons between nucleons.

Problem 18. This graph shows detected energies and momenta of π^+/n pairs that were coincident, i.e. produced in a single event.

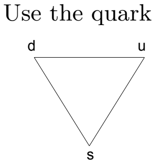


The bottom axis shows the quantity $\sqrt{E^2_{\pi+n} - \mathbf{p}^2_{\pi+n}}$ in MeV. Using the concepts of 4-momentum and the uncertainty principle, and clearly explaining what you are doing, estimate the (a) mass and (b) very short lifetime of the Δ^+ particle in the reaction sequence

$$e^- + p \ \rightarrow \ e^- + \Delta^+$$
$$\Delta^+ \ \rightarrow \ \pi^+ + n$$

are deduced from the graph above, showing detection of a π^+ and n produced in the same event. There is a background of detections of $e^- + p \rightarrow \pi^+ + n +$ (other stuff), where no Δ^+ particle is produced.

Problem 19. Use the quark triplet diagram



to construct the baryon decuplet that includes the following members:

qqq	Q	S	Baryon
uuu	2	0	Δ^{++}
uud	1	0	Δ^{+}
udd	0	0	Δ^{0}
ddd	−1	0	Δ^{-}
uus	1	−1	Σ^{*+}
uds	0	−1	Σ^{*0}
dds	−1	−1	Σ^{*-}
uss	0	−2	Ξ^{*0}
dss	−1	−2	Ξ^{*-}
sss	−1	−3	Ω^{-}

Problem 20. A ball is dropped from a height y on Earth, falling straight down and hitting the floor ($y = 0$) in a time t. By drawing a picture of this motion in space and time, calculate the radius of curvature R_c of spacetime at the Earth's surface.