Homework 8

PHYSICS 461

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Kevin Evans

ID: 11571810

1. Starting from the radial Schrodinger equation

$$-\frac{\hbar^2}{2m_e}\left(R'' + \frac{2}{r}R\right) - \frac{\ell(\ell+1)}{r^2}R - \frac{\not{e}^2}{r}R(r) = ER$$

   Letting the constants go to 1,

$$\frac{1}{2}R'' + \frac{1}{2}R' + \left[\frac{1}{r} - \frac{\ell(\ell+1)}{2r^2}\right]R = -ER$$

   For $R(r) = e^{-r}$ and $\ell = 0$ and $E = -1/2$, the differential equation becomes

$$\frac{1}{2}R(r) - \frac{1}{r}R(r) + \frac{1}{r}R(r) = \frac{R(r)}{2}$$

   ...which is true.

2. (a) The Rydberg constant is $207\times$ larger, so the wavelength is

$$\lambda = \frac{1}{207 \times R_\infty} = 440.2\,\text{pm}$$

   This is in the x-ray to gamma ray region.

   (b) Using the reduced mass,

$$R_\mu = \left(\frac{m_\mu m_p}{m_\mu + m_p}\right)\frac{e^4}{8\epsilon_0^2 h^3 c}$$
$$\lambda = R^{-1} = 481.9\,\text{pm}$$

   It's pretty close to the infinite nuclear mass approximation.

3. (a) Using R_μ from Problem 2, we can equate the energy to the Coulomb potential as

$$E = R_\mu hc\frac{Z^2}{n^2} = \frac{Ze^2}{4\pi\epsilon_0 r}$$
$$r = \frac{e^2}{4\pi\epsilon_0 RZhc} = 6.89\,\text{fm}$$

   (b) For muonic hydrogen, $Z = 1$ and then from the equation used in (a), the radius is $564\,\text{fm}$.

4. (a) The energy levels are given by

$$E_n = -\frac{\mu e^4}{8h^2\epsilon_0^2}\frac{1}{n^2} = -\frac{-6.8\,\text{eV}}{n^2}$$

   For $n = 1, 2, 3$,

$$E_1 = -6.8\,\text{eV}$$
$$E_2 = -1.7\,\text{eV}$$
$$E_3 = -0.75\,\text{eV}$$

(b) For the α and β lines,

$$E_\alpha = E_2 - E_1 = 5.1\,\mathrm{eV}$$
$$\lambda_\alpha = \frac{hc}{E_2 - E_1} = 243\,\mathrm{nm}$$
$$\lambda_\beta = \frac{hc}{E_3 - E_1} = 204\,\mathrm{nm}$$

5. (a) The core electrons shield (p 218) the nuclear charge completely.

(b) We can equate the energy to the Coulomb potential,

$$\frac{Ry}{2000^2} = \frac{e^2}{4\pi\epsilon_0 r}$$
$$r = \frac{e^2 2000^2}{4\pi\epsilon_0 Ry}$$
$$= 427\,\mu\mathrm{m}$$

The H atom is 5×10^{-11} m, so this is much larger.

(c) From Wikipedia,

$$mvr = n\hbar$$
$$v = 1158\,\mathrm{m} \cdot \mathrm{s}^{-1}$$

(d) For $n = 1$, $v = 2 \times 10^6\,\mathrm{m/s}$, which makes sense as the electron is much closer in orbit.

6. (a) For the inner $2s$ electron, the energy is $E_{2s} = Ry\frac{Z^2}{n^2} = Ry$. For the outer $4p$ electron, the effective charge is 1 and the energy is $E_{4p} = Ry/16$. Adding these, the energy is

$$E_{2s4p} = 1 + 13.61\,\mathrm{eV}\left(1 + \frac{1}{16}\right) = 14.5\,\mathrm{eV}$$

The ground state of He (from Wikipedia) is $-79\,\mathrm{eV}$, so the change in energy is $\Delta E = 64.5\,\mathrm{eV}$, corresponding to $\lambda = 19\,\mathrm{nm}$.

(b) Doing this classically, we can take the energy from (a) and set it to $mv^2/2$,

$$\Delta E = \frac{mv^2}{2}$$
$$v = \sqrt{\frac{2\Delta E}{m}} = \sqrt{\frac{2 \times 64.5\,\mathrm{eV}}{0.510\,\mathrm{MeV}}}$$
$$= 4.8 \times 10^6\,\mathrm{m} \cdot \mathrm{s}^{-1}$$

7. From the L shell, the electron drops of $L \to K$ corresponding to the K_α line $3.69\,\mathrm{keV}$. Similarly for the M and N shells, the energies are $0.341\,\mathrm{keV}$ and $0.024\,\mathrm{keV}$ respectively.

8. The emitted electron is roughly $300\,\mathrm{eV}$, which is roughly the L_α line, which makes sense.

9. I can't get the generalized way to work out, so I'm just going to give an example with the $2p3s$ state. Under LS coupling, we get four terms: $^1\mathrm{P}_1$, $^3\mathrm{P}_0$, $^3\mathrm{P}_1$, $^3\mathrm{P}_2$. Under jj coupling, there's four terms as well: for $j_1 = j_2 = 1/2$, we get two states $j = 0, 1$. For $j_1 = 3/2, j_2 = 1/2$, we get two other states, $j = 1, 2$.

10. (a) The combined $L = 1$ and $S = 1$, so J goes from $0, 1, 2$, and the terms are ${}^1\mathrm{P}_1$, ${}^3\mathrm{P}_0$, ${}^3\mathrm{P}_1$, ${}^3\mathrm{P}_2$.

    (b) For $j_1 = 1 + \frac{1}{2} = \frac{1}{2}, \frac{3}{2}$, $j_2 = \frac{1}{2}$, we can couple these with $J = j_1 + j_2$ to $|j_1 - j_2|$. For $j_1 = j_2 = 1/2$, we get two states $j = 0, 1$. For $j_1 = 3/2, j_2 = 1/2$, we get two other states, $j = 1, 2$.

    (c) I think this can be explained by the Paschen-Back effect, except instead of an external field, it's related to the moment from the nucleus?