# Introduction

For this project, we used data measuring the height of an undulating spring over 20 seconds and attempted to solve for a differential equation of the form $m\frac{\mathrm{d}^2y}{\mathrm{d}t^2} + b\frac{\mathrm{d}y}{\mathrm{d}t} + ky = 0$ that would accurately represent the given data. Using *Mathematica*, we plotted the given data and adjusted the values of the constants $m$, $b$, and $k$ to find a specific solution for $y$ which accurately modeled the situation.

# Explanation

In order to accurately determine initial values to the solution of our data, we shifted the data to start at a maximum value, which in our case was at $t = 0.2$ with $y = 0.42566$, so that we knew $y'$ at that value was zero. We then vertically shifted the function so that it would be undulating close to the correct sinusoidal axis which we approximated since we did not know the final position of the object. By increasing the values of $m$ and $k$ we were able to make the solution match the frequency and the rate of decrease in amplitude. By changing $b$ we were able to adjust the amplitude. Originally, we used much lower values of $m$ and $k$ which resulted in good representation of the data for a small period of time; however, after increasing both values we were able to accurately model the data much further. For our final solution, we used the values $m = 406$, $b = 65$, and $k = 10000$ leading to the differential equation $406\frac{\mathrm{d}^2y}{\mathrm{d}t^2} + 65\frac{\mathrm{d}y}{\mathrm{d}t} + 10000y = 0$.

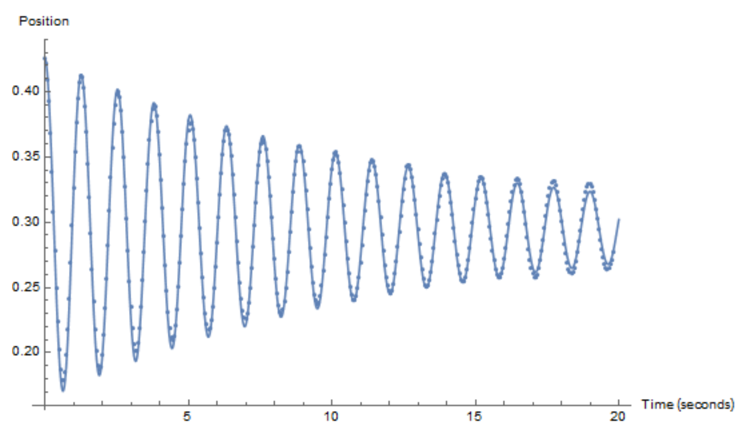


Figure 1: Plot of solution

# Solving IVP

After finding appropriate constants, we needed to find the general solution to the differential equation with the given initial values. We'll start by solving for the general solution assuming that $y = e^{st}$.

$$\begin{aligned}
406\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + 65\frac{\mathrm{d}y}{\mathrm{d}t} + 10000y =& 0 \\
e^{st}\left(406s^2 + 65s + 10000\right) =& 0 \\
s =& \frac{-65 \pm \sqrt{65^2 - 4\left(406 \cdot 10000\right)}}{2 \cdot 406} \\
s =& \frac{-65}{812} \pm \frac{15i\sqrt{72159}}{812} \\
y =& k_1 e^{\frac{-65}{812}t}\cos\left(\frac{15\sqrt{72159}}{812}t\right) + k_2 e^{\frac{-65}{812}t}\sin\left(\frac{15\sqrt{72159}}{812}t\right)
\end{aligned}$$

In order to match the given data, we have to shift $y$ vertically by 0.295. Therefore the general equation results as

$$y = k_1 e^{\frac{-65}{812}t}\cos\left(\frac{15\sqrt{72159}}{812}t\right) + k_2 e^{\frac{-65}{812}t}\sin\left(\frac{15\sqrt{72159}}{812}t\right) + 0.295$$

Given $y(0.2) = 0.42566$ and $y'(0.2) = 0$, we can solve for the specific solution. We can set up a system of equations to find $k_1$ and $k_2$ by first solving for $y(0.2) = 0.42566$ and then solving for $y'(0.2) = 0$.

$$k_1 e^{\frac{-65}{812}\cdot 0.2}\cos\left(4.96 \cdot 0.2\right) + k_2 e^{\frac{-65}{812}\cdot 0.2}\sin\left(4.96 \cdot 0.2\right) + 0.295 = 0.42566$$

$$\frac{-5}{812}e^{-\frac{65\cdot 0.2}{812}}\left(\left(3\sqrt{72159}k_1 + 13k_2\right)\sin\left(4.96 \cdot 0.2\right) + \left(13k_1 - 3\sqrt{72159}k_2\right)\cos\left(4.96 \cdot 0.2\right)\right) = 0$$

We evaluated this system using *Mathematica* and found that $k_1 = 0.0708$ and $k_2 = 0.1123$ leading to the final solution of

$$y = 0.0708 \cdot e^{\frac{-65}{812}t}\cos\left(\frac{15\sqrt{72159}}{812}t\right) + 0.1123 \cdot e^{\frac{-65}{812}t}\sin\left(\frac{15\sqrt{72159}}{812}t\right) + 0.295$$

# Conclusion

This specific solution accurately models the oscillating object at the end of a spring for $0.2 \leq t \leq 20$ seconds. Using different objects and different strings would result in different solutions; however, the shape of the graph is nearly identical in that they are solutions for underdamped oscillators. This same process of modeling can be applied to many physical situations where objects are being acted on by various forces which may be changing over time.