The MTSP optimal model suited for local-cross repeat path

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***Abstract*—Although many people have paid attention to the conventional MTSP problem and doing research on it, its constraint condition ”only one travelling salesman can pass by in each city” suffered from many limitations in reality. On the basis of conventional MTSP model, this article promotes it in adding the ”material transportation capacity” constraint condition and builds a new MTSP material transportation optimal model permitting the existence of local-cross repeat path. We use lingo software to solve the problem combined with an example. The result indicates that the new optimal model accords with fact well. Another advantage of this new model can also avoid the cumbersome process of the GA (Genetic Algorithm) and it is much easier to operate for researchers. Meanwhile, we can bring in other variants and constraint conditions on the basis of this new model and extend to build other models suited for specific needs.**

***Index Terms*—MTSP, local-cross repeat path, material transportation, optimal model, improvement.**

I. Introduction

The major subject of Travelling Salesman Problem(TSP) firstly raised by William Hamilton in the nineteenth century is to seek out the minimum closed loop,which meets the conditions of enabling the travelling salesman to pass by N cities only once from the starting point and finally return back.This is a typical combinatorial optimization problem.For the purpose of being close to real application,the Multiple Travelling Salesman Problem(MTSP) has been put forward during the research process of TSP.The Multiple Travelling Salesman Problem can be explained as finding out the total shortest distance when there are M travelling salesmen setting off from the same city(or different cities) in disparate routes respectively,and only one salesman passes by in each city(except the departure city).Realistic problems like transportation,pipe laying,route alternative,computer network topology design,and so forth can all be abstracted into Travelling Salesman Problem or Multiple Travelling Salesman Problem Researches.The study of these problems is of great practical value especially under the circumstances of high frequency of natural disasters nationwide and worldwide.Enormous efforts including improvement studies of algorithms and solution models have been made by researchers both at home and abroad.

However,there is a significant problem in the practical operation of MTSP model.The constraint condition ”only one travelling salesman can pass by in each city” directly lead to the limit of the existence of local-cross repeat path,which may result in unnecessary distance.If we can effectively remove the limitations,and allow more cars to pass by one city,then the model will be much closer to reality and be of great significance.Nevertheless,recently more attention has been concentrated on how to solve the MTSP problem by the improving genetic algorithm rather than the former method we have mentioned.Meanwhile,considering the disasters and the development of logistics industry,we cancel the constraint condition ”only one travelling salesman can pass by in each city” in the following discussion,a new MTSP material transportation optimal model will be set up according to the practical material transportation.In addition,combined with examples,the corresponding resolution process is to be demonstrated by using the lingo software and the research of model improvement will be explored as well.

II. Propose the problem and symbols hypothesis

There are n affected towns need allotted relief materials,and m vans are arranged to convey materials,whose maximum load is w_k,$k \in [1, m]$.We assume that the transportation starting point is right in the first town,and the distance between town i to town j is d_{ij}. If it is unreachable between two towns,the distance can be marked as $d_{ij} = \infty$.The relief material demand of town j is P_j ,and the deliquty of van k from town i to town j is P_{kij} .Then we try to find out a reasonable transport scheme,which makes the total distance shortest.

III. The establishment of a new MTSP material transportation optimal model permitting the existence of local-cross repeat path

This problem can be designed as a MTSP model in terms of the form.However,the conventional MTSP model has the constraint condition ”only one travelling salesman can pass by in each city”,which directly leads to the limit of the existence of local-cross repeat path.This method usually results in unnecessary distance,and is not consistent with the actual conditions.So the following paper will explore to build up a new MTSP material transportation optimal model permitting the existence of local-cross repeat path.

A. The establishment of the objective function

Because the optimization goal requires the total driving distance to be the shortest, 0,1 is defined as variable

$$\min = \sum_{k=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}X_{kij}$$

B. The establishment of the constraint conditions

1) The deliquty of each van to different towns should meet the constraint of $P_{kij} \geq 0$, and only when the van passes by route ij can it provide relief materials to town j. In contrast,it won't realize it,and the quantity of deliquty is bound to be smaller than the maximum loading capacity. The mathematics type can be written as

$$w_kX_{kij} \geq P_{kij} \geq 0; i = 1, 2, \ldots, n; j = 1, 2, \ldots, n$$

2) For every disaster point,the total quantity of deliquty of all vans should be equivalent to the demand of the disaster point.The mathematics type can be written as

$$\sum_{i=1}^{n}\sum_{k=1}^{m} P_{kij} = P_j; j = 1, 2, \ldots, n$$

3) For every disaster point,there will be at least one van passing by,to provide relief materials.

$$\sum_{k=1}^{m}\sum_{i=1}^{n} X_{kij} \geq 1; i \neq j; j = 1, 2, \ldots, n$$

4) For every disaster point,the number of vans entering it should equal to the one of exiting from it.

$$\sum_{i=1}^{n} X_{kij} = \sum_{i=1}^{n} X_{kji}$$

$$i \neq j; j = 1, 2, \ldots, n; k = 1, 2, \ldots, m$$

5) When $i = j$,X_{kij} is supposed to meet the requirements as

$$X_{kji} = 0; i = j; k = 1, 2, \ldots, m$$

6) In order to prevent the emergence of subcycle,we bring in a constraint condition(reference[2]).

$$U_{ki} - U_{kj} + nX_{kij} \leq n - 1; 0 \leq U_{ki} \leq n - 1; U_{kj} \geq 0$$

$$i, j = 1, 2, \ldots, m; i \neq j$$

7) The amount of relief materials in each van is not allowed to surpass the maximum load.

$$\sum_{i=1}^{n}\sum_{j=1}^{n} P_{kij} \leq w_k; k = 1, 2, \ldots, m$$

8) Instructions of 0,1 variables

$$X_{kij} = 0 \ or \ 1$$

C. The establishment of 0-1 programming model

According to the above analysis,we can build the ultimate 0-1 programming model as

$$\min = \sum_{k=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}X_{kij}$$

$$st. \begin{cases} w_kX_{kij} \geq P_{kij} \geq 0, i, j = 1, 2, \cdots, n; \\ \sum\limits_{i=1}^{n}\sum\limits_{k=1}^{m} P_{kij} = P_j, j = 1, 2, \cdots, n; \\ \sum\limits_{k=1}^{m}\sum\limits_{i=1}^{n} X_{kij} \geq 1, j = 1, 2, \cdots, n; \\ \sum\limits_{i=1}^{n} X_{kij} = \sum\limits_{i=1}^{n} X_{kji}, i \neq j; \\ (j = 1, 2, \cdots, n; k = 1, 2, \cdots, m) \\ X_{kji} = 0, (i = j), k = 1, 2, \cdots, m; \\ U_{ki} - U_{kj} + nX_{kij} \leq n - 1, \\ 0 \leq U_{ki} \leq n - 1; U_{kj} \geq 0; \\ (i, j = 1, 2, \ldots, n; i \neq j) \\ \sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n} P_{kij} \leq w_k, k = 1, 2, \ldots, m; \\ X_{kij} = 0 \ or \ 1 \end{cases}$$

IV. Realistic application

The autocade is arranged to convey relief materials to ten affected towns. Now we assume the dilivery point is right in the first town,and the distance(unit:kilometer) between town i to town j is expressed by the number of position (i, j),$(i, j =$ 1,2,. . . ,10) in the matrix(∞ represents that there is no straight way between two towns).Due to the lack of resources,no big trucks but two small vans are available to accomplish the task.The capacity of each van is 50 units,and the material demand of every town is respectively 81369715105129 units.So which method should we take to distribute materials in order to guarantee a shortest total distance from the starting point and finally return to the origin of the two vans?

$$\begin{array}{c c} & \begin{array}{cccccccccc} 01 & 02 & 03 & 04 & 05 & 06 & 07 & 08 & 09 & 10 \end{array} \\ \begin{array}{c} 01 \\ 02 \\ 03 \\ 04 \\ 05 \\ 06 \\ 07 \\ 08 \\ 09 \\ 10 \end{array} & \begin{bmatrix} 0 & 50 & \infty & 40 & 25 & \infty & 30 & \infty & 50 & \infty \\ 50 & 0 & 30 & \infty & 35 & 50 & \infty & 60 & \infty & \infty \\ \infty & 30 & 0 & 15 & \infty & 30 & 50 & 25 & \infty & 60 \\ 40 & \infty & 15 & 0 & 45 & 30 & 55 & 20 & 40 & 65 \\ 25 & 15 & \infty & 45 & 0 & 60 & 10 & 30 & \infty & 55 \\ \infty & 50 & 30 & 30 & 60 & 0 & 25 & 55 & 35 & \infty \\ 30 & \infty & 50 & \infty & 10 & 25 & 0 & 30 & 45 & 60 \\ \infty & 60 & 25 & 20 & 30 & 55 & 30 & 0 & 10 & \infty \\ 20 & \infty & \infty & 40 & \infty & 15 & 25 & 45 & 0 & 20 \\ 35 & 20 & 10 & 45 & 20 & \infty & 60 & \infty & 30 & \infty \end{bmatrix} \end{array}$$

We refer to 2.3 to build the programming model

$$\min = \sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}(X_{1ij} + X_{2ij})$$

$$st.\begin{cases}\sum\limits_{i=1}^{n} P_{1ij}+\sum\limits_{i=1}^{n} P_{2ij}=P_j, j=1,2,\cdots,n;\\ 50X_{1ij}\geq P_{1ij}\geq 0, 50X_{2ij}\geq P_{2ij}\geq 0;\\ \sum\limits_{i=1}^{n} X_{1ij}+\sum\limits_{i=1}^{n} X_{2ij}\geq 1, i\neq j, j=1,2,\cdots,n;\\ \sum\limits_{i=1}^{n} X_{1ij}=\sum\limits_{i=1}^{n} X_{1ji}, \sum\limits_{i=1}^{n} X_{2ij}=\sum\limits_{i=1}^{n} X_{2ji};\\ (i\neq j; j=1,2,\ldots,n)\\ \sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n} P_{1ij}\leq 50, \sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n} P_{2ij}\leq 50;\\ \left.\begin{array}{l}U_{1i}-U_{1j}+nX_{1ij}\leq n-1,\\ U_{2i}-U_{2j}+nX_{2ij}\leq n-1,\end{array}\right\}\begin{array}{l}U_{ki}, U_{kj}\geq 0; k=1,2;\\ i,j=1,2,\ldots,n; i\neq j\end{array}\\ U_{1i}\leq n-1, U_{2j}\leq n-1;\\ (i,j=1,2,\ldots,n; i\neq j)\\ X_{kij}=0\ or\ 1\end{cases}$$

This model can be easily solved with the use of lingo11 software,and the result can be seen in Table 1.

Table 1

Van number and loadage	Routes and corresponding affected points (quantity of supplies)	Route distance
van1 (44)	1(7)→7(10)→6(15)→9(12)→1	110
van2 (50)	1(7)→5(7)→2(13)→3(6)→4(9)→8(5)→9(0)→10(9)→1	170
Total distance: 280		

The results of table 1 indicate the respective driving routes of van 1 and van 2.Van 1 sets off from the starting point,successively passing by town 7,town 6,town 9,and returns to the original point.The quantity of supplies to each town is 7,10,15,12,with a total driving distance of 110 kilometers.Van 2 sets off from the starting point,successively passing by town 5,town 2,town 3,town 4,town 8,town 9,town 10,and returns to the original point.The quantity of supplies to each town is 7,7,13,6,9,5,0,9,with a total driving distance of 170 kilometers.In the course of driving,both vans pass by 9 towns,with overlapping places on the path,and successfully prove the optimization model of breaking down the constraint condition "only one travelling salesman can pass by in each city" of the conventional MTSP.

V. Results and discussion

Lots of researches related to the conventional MTSP problem have been conducted,but much attention has been concentrated on the genetic algorithm improvement and the neural network method for its solutions,like literature[3,4,5].While the study of unreasonable condition "only one travelling salesman can pass by in each city" is relatively less made.So this paper cancels the foregoing condition,combined with the material transportation,to establish a new MTSP material transportation optimal model permitting the existence of local-cross repeat path. It performs following advantages:

Firstly,the establishment of this new model can effectively avoid the redundant distance due to the limit number of vans passing by towns in the MTSP model which is closer to the practice. This cannot be achieved by conventional models.

Secondly,the successful establishment of this optimization model capacitates the convenient use of optimization software like lingo,etc.Compared with conventional approaches to solve with intelligent algorithms, it avoids complicated process of programming like Genetic Algorithms, which is readily to operate. Meanwhile, the speed and accuracy of solving problems is much higher than conventional Genetic Algorithms.

Thirdly,in the above research,the objective function mainly investigate the condition— the minimum of the total dis-

Table 2 Extended models on the basis of the original model

Scheme	Added variables or constraints	Target	Objective function
1	v_k:the speed of van k	Shortest total time	$\min=\sum\limits_{k=1}^{m}\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n} d_{ij}X_{kij}/v_k$
2	df:reasonable distance constant $\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n} d_{ij}X_{kij}\leq df;$ $k=1,2,\cdots,m$	Shortest total distance on condition of the longest transportation distance being the shorest	$\min=\sum\limits_{k=1}^{m}\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n} d_{ij}X_{kij}$
3	v_k:the speed of van k df:reasonable distance constant $\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n}\frac{d_{ij}X_{kij}}{v_k}\leq df;$ $k=1,2,\cdots,m$	Shortest total time on condition of the longest transportation time being the shorest	$\min=\sum\limits_{k=1}^{m}\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n} d_{ij}X_{kij}/v_k$
4	c_1,c_2:reasonable distance constant $c_1\leq\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n} d_{ij}X_{kij}\leq c_2;$ $k=1,2,\cdots,m$	Shortest total distance on condition of nearly consistent respective delivery distance	$\min=\sum\limits_{k=1}^{m}\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n} d_{ij}X_{kij}$
5	v_k:the speed of van k t_1,t_2:reasonable distance constant $t_1\leq\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n}\frac{d_{ij}X_{kij}}{v_k}\leq t_2;$ $k=1,2,\cdots,m$	Shortest total time on condition of nearly consistent respective delivery time	$\min=\sum\limits_{k=1}^{m}\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n} d_{ij}X_{kij}/v_k$
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tance.While reasonable changes can be made to further develop other models suited for specific needs in practical application,please look at table 2.If time is regarded as measurement unit to seek for the scheme of the minimum of total time,then the variable v_k can be introduced to define the speed of each van.The corresponding changed objective function can lead to the corresponding model(scheme 1).On the condition that all drivers work almost the same time,if fairness needs to be taken into consideration,the transport scheme—the minimum of the total time is aimed to achieved.Defining the rational lowest and highest working time and controlling the gap range between two time points to build an optimization model,can help to limit the working time into a small range,and to guarantee the fairness of the scheme(scheme 5).Here we wont further our discussion about other extended models suited for different applications.

At the same time,the optimization model can not only be applied in the material transportation fields,but also has significant research and extension value on many other realms like pipe laying,public traffic line construction,transport,etc. The promising effects on further researches and study will produce more benefits.

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