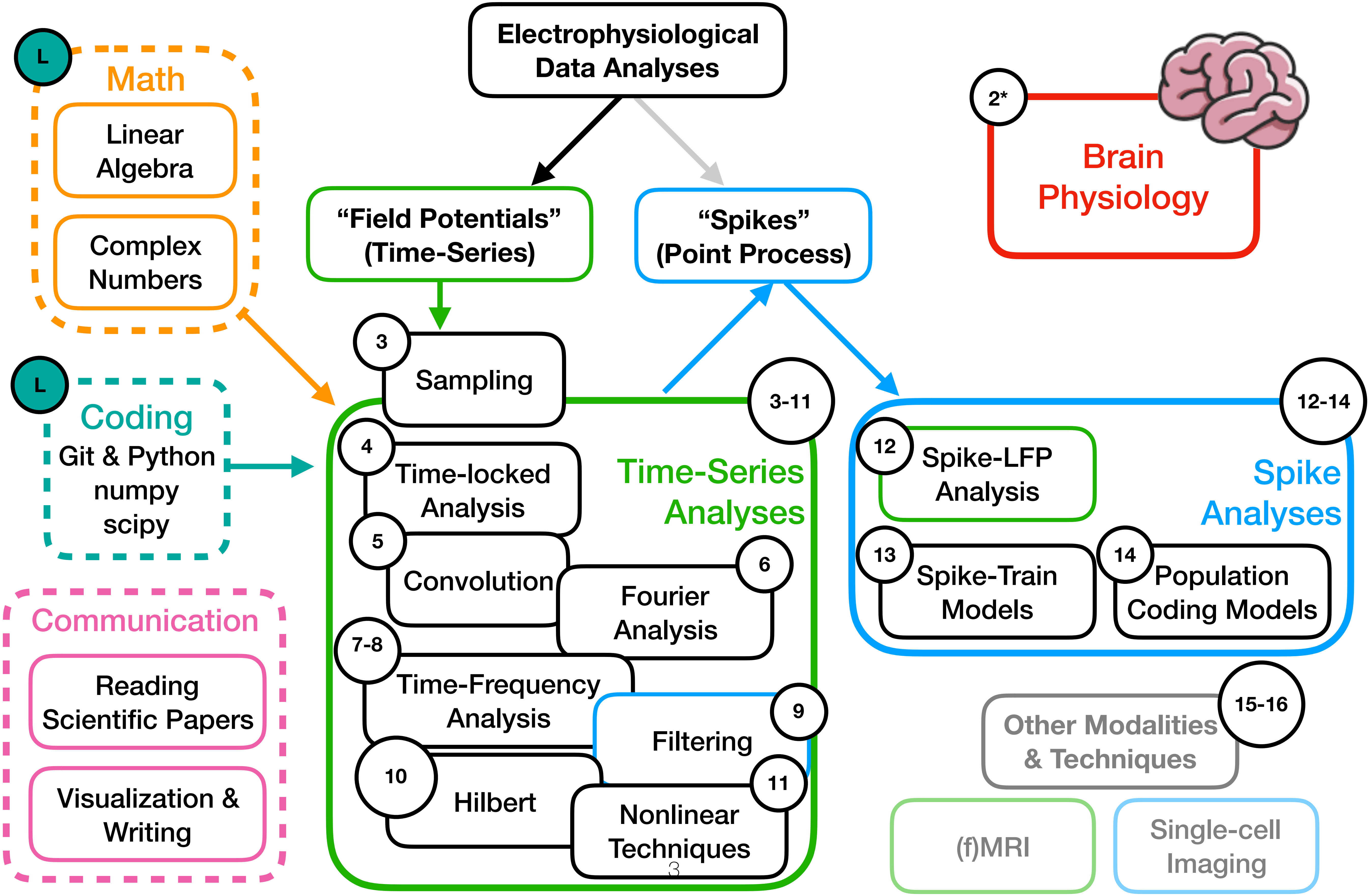
Question of the Day

Are colors of the rainbow truly separate components?

Kernels, Coupling & Non-Linearity

Lecture 11
July 22, 2019

Course Outline: Road Map



Goals for Today

1. Wavelet & similarities between kernel-based TF methods

2. Examples of coupling analyses

3. Motivate & understand nonlinear analyses

Analytic Signal - Hilbert Transform

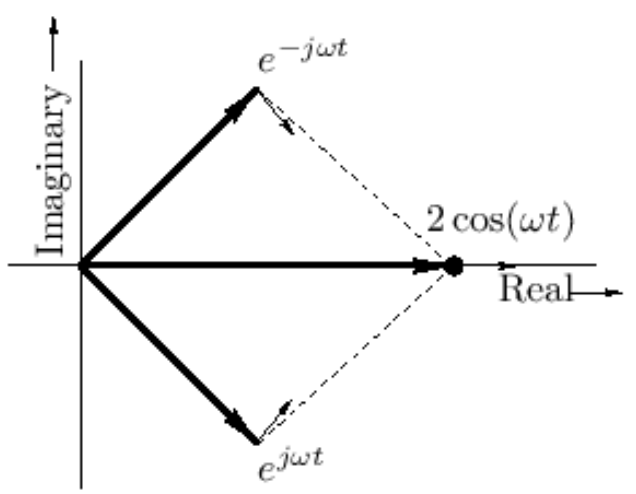
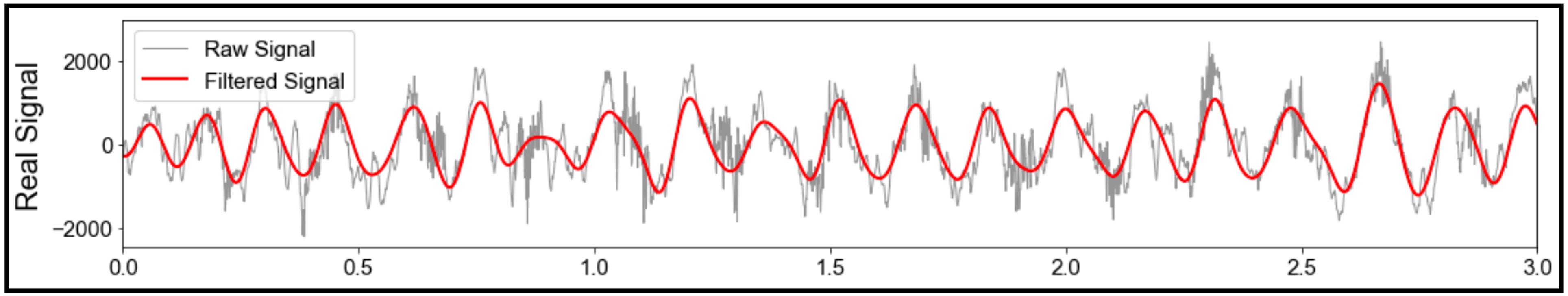
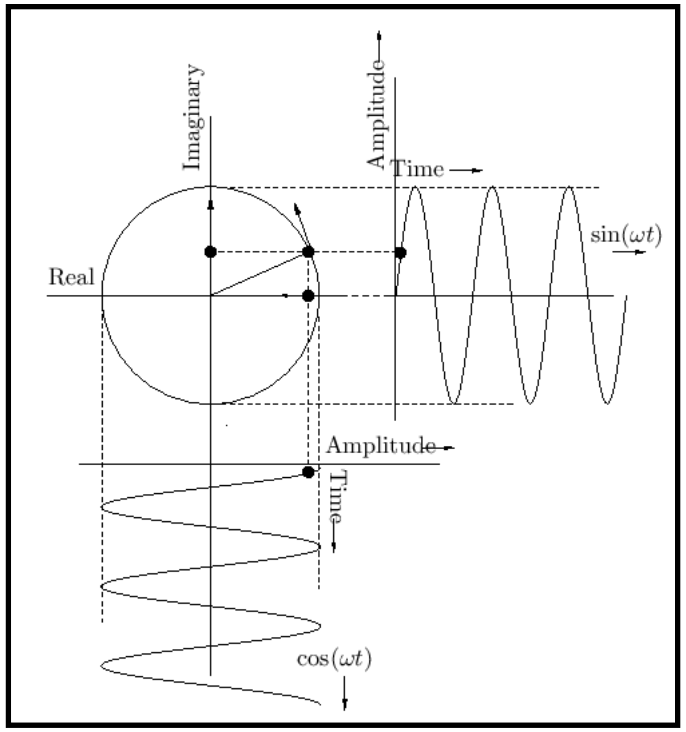


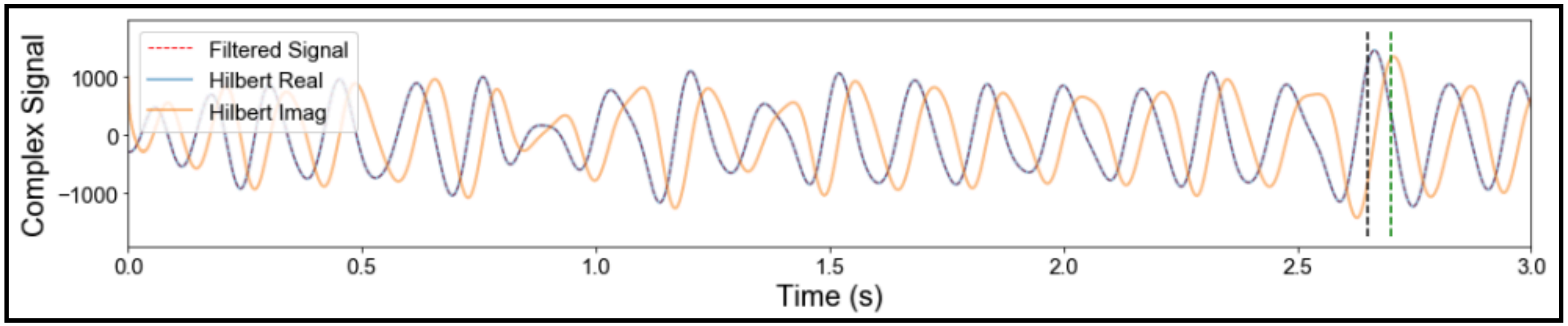
Figure 1.9: Opposite circular motions add to give real sinusoidal motion, $e^{j\omega t} + e^{-j\omega t} = 2\cos(\omega t)$.

Real Signal: sum of 2 complex exponential conjugates



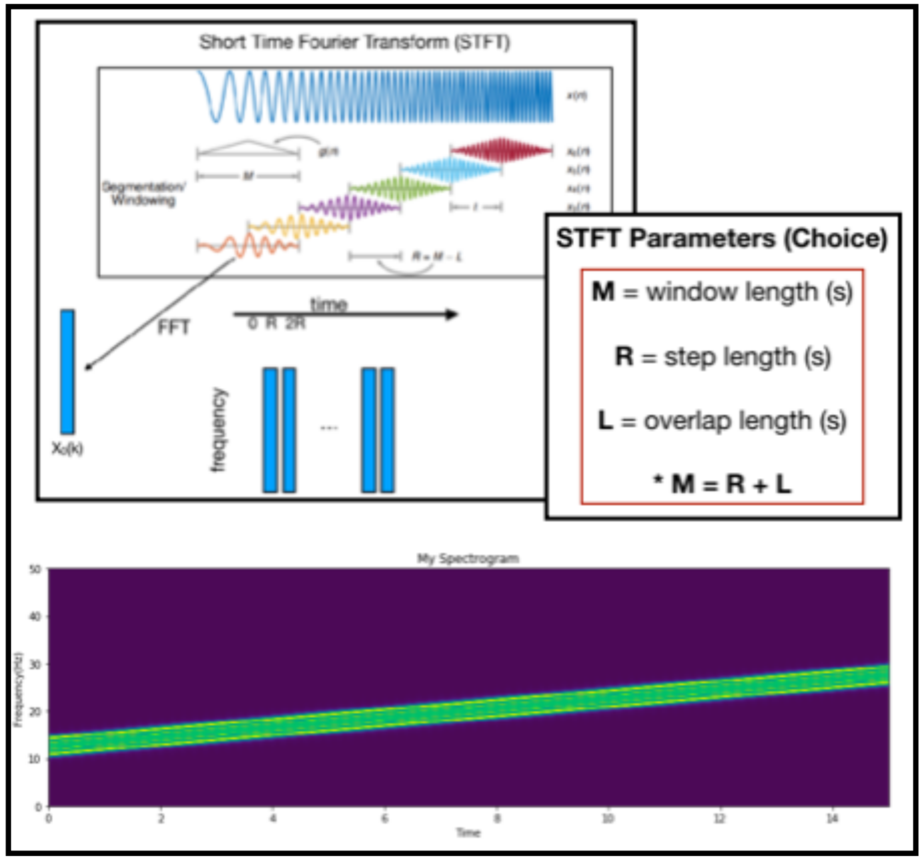


Analytic (Complex) Signal: a single complex exponential

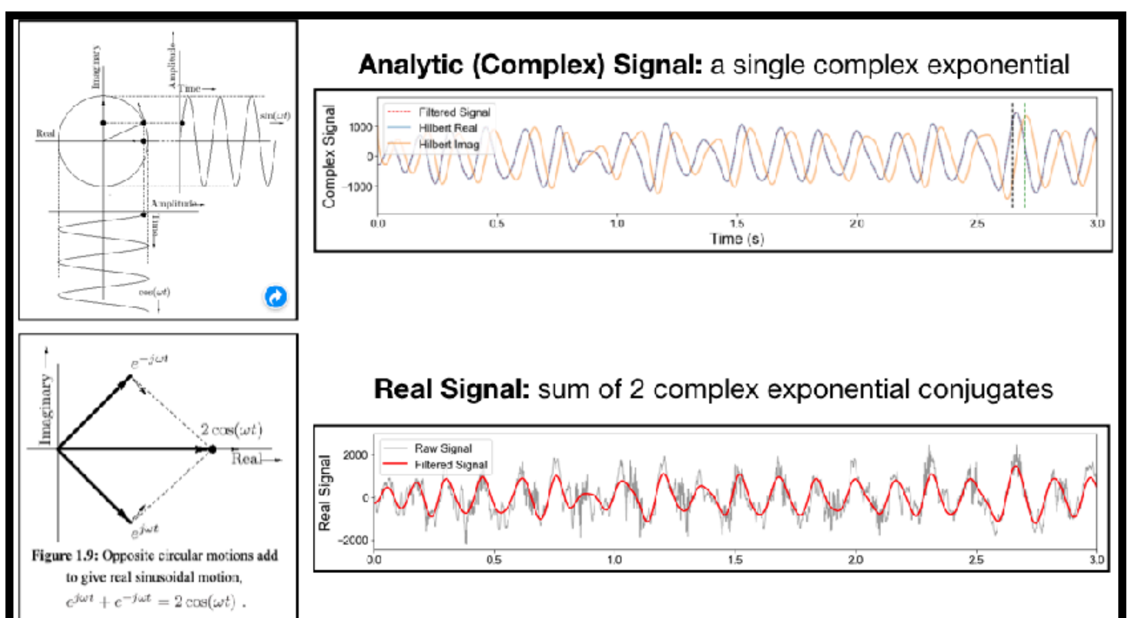


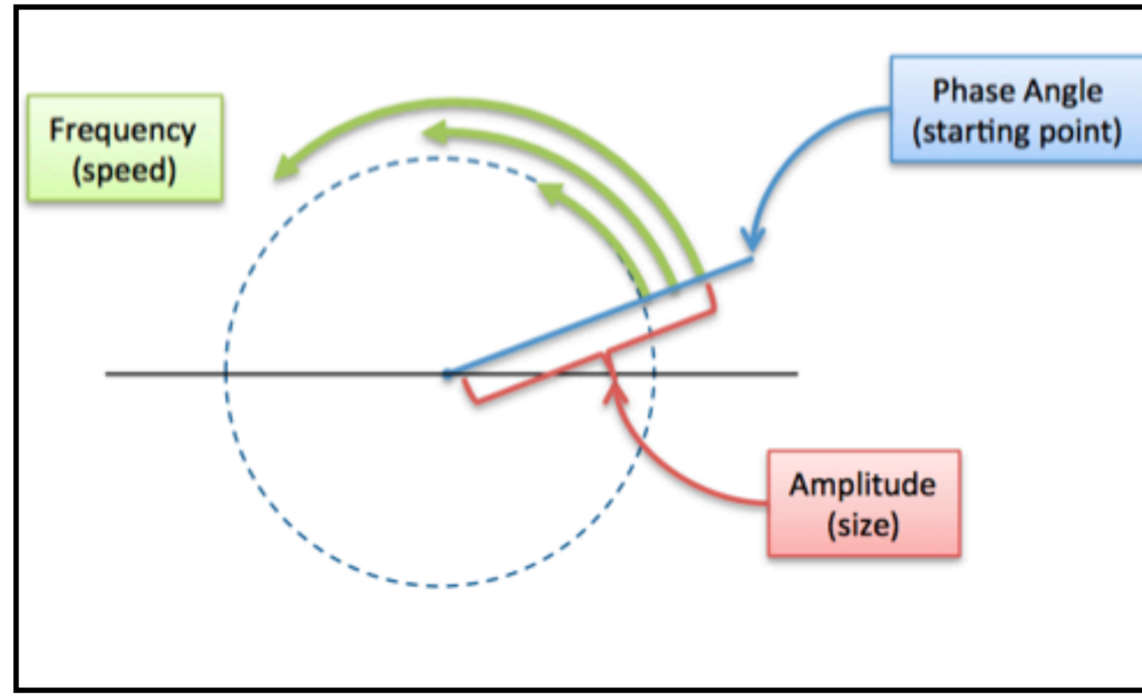
Retrieving Signal Amplitude & Phase

Method 1. Short-Time Fourier Transform



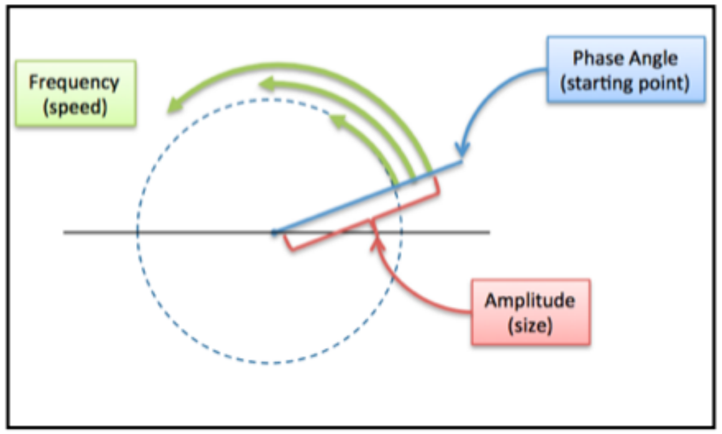
Method 2. Bandpass Filter + Hilbert Transform





In both cases, we want to retrieve the amplitude (power) and phase (frequency) of the signal **as a function of time**.

Retrieving Signal Amplitude & Phase



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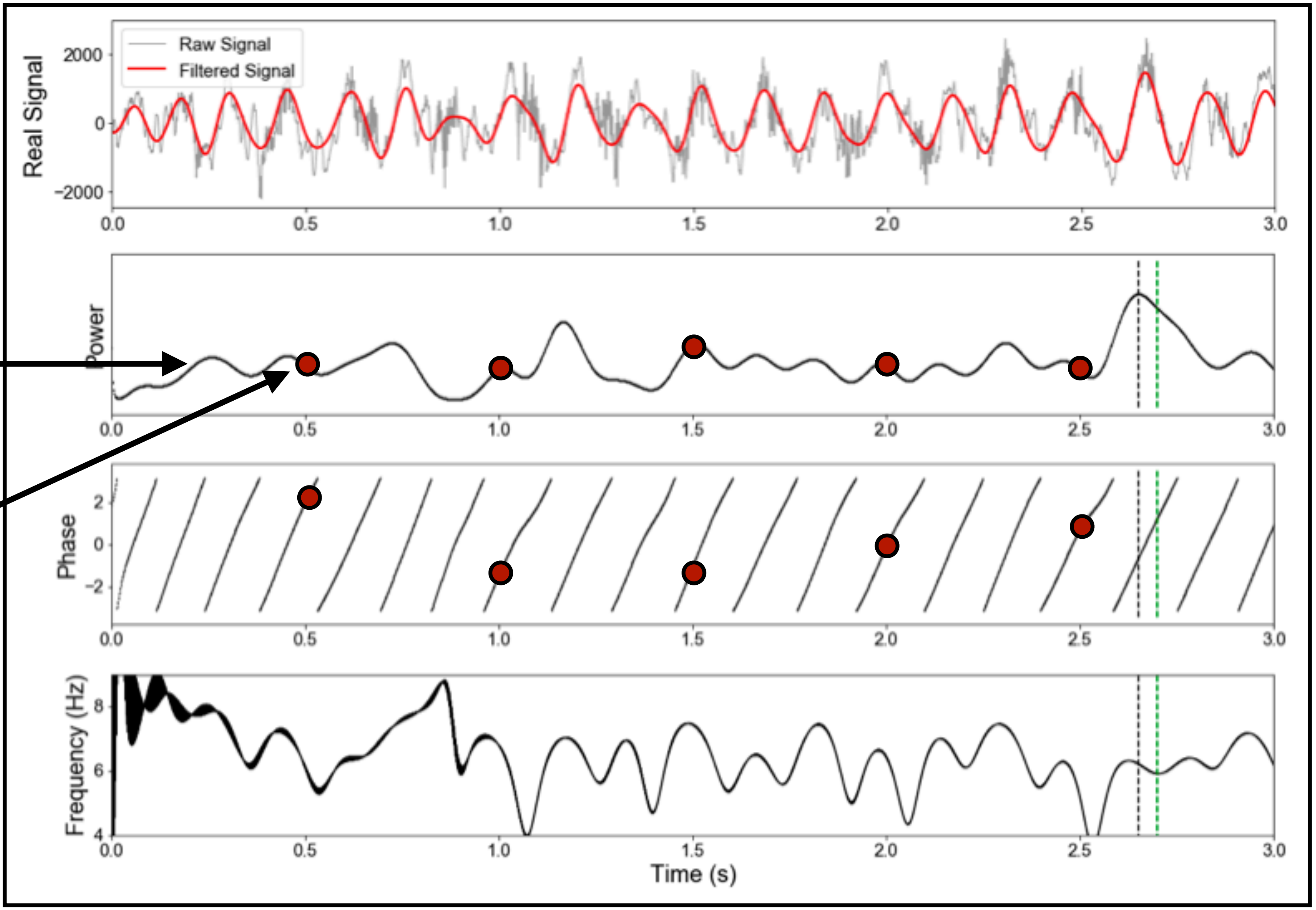
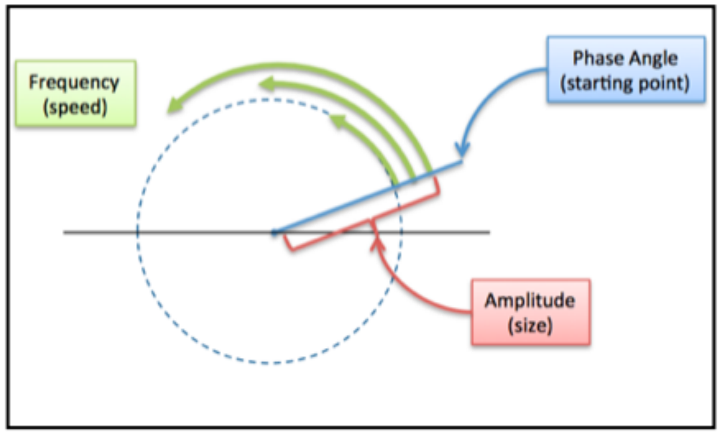
BP Filter + Hilbert

STFT

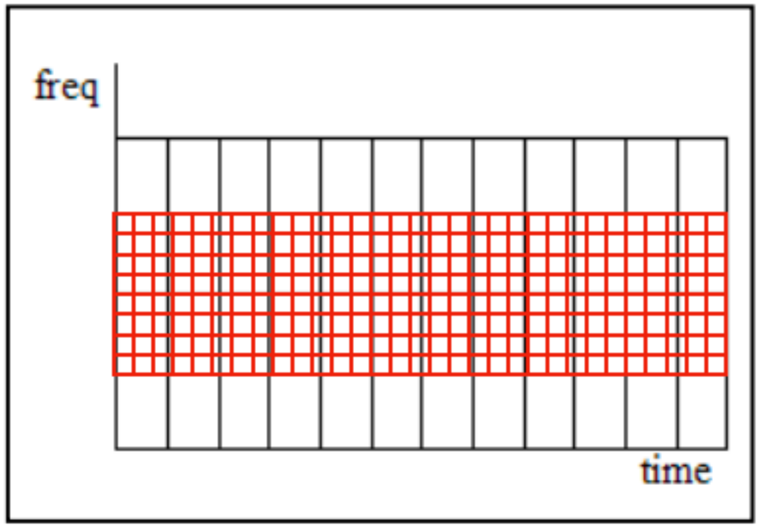


Time-Frequency Resolution Tradeoff

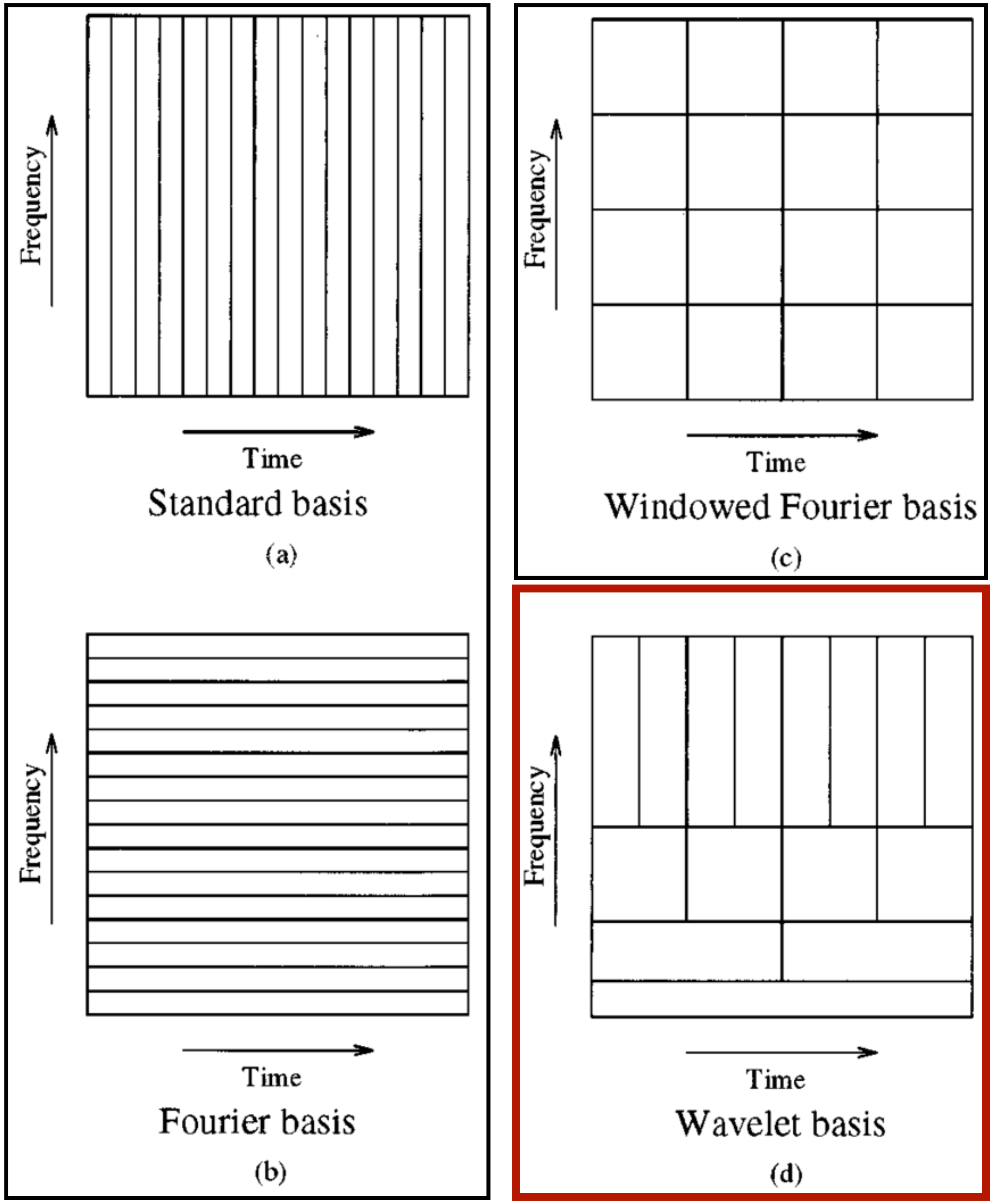


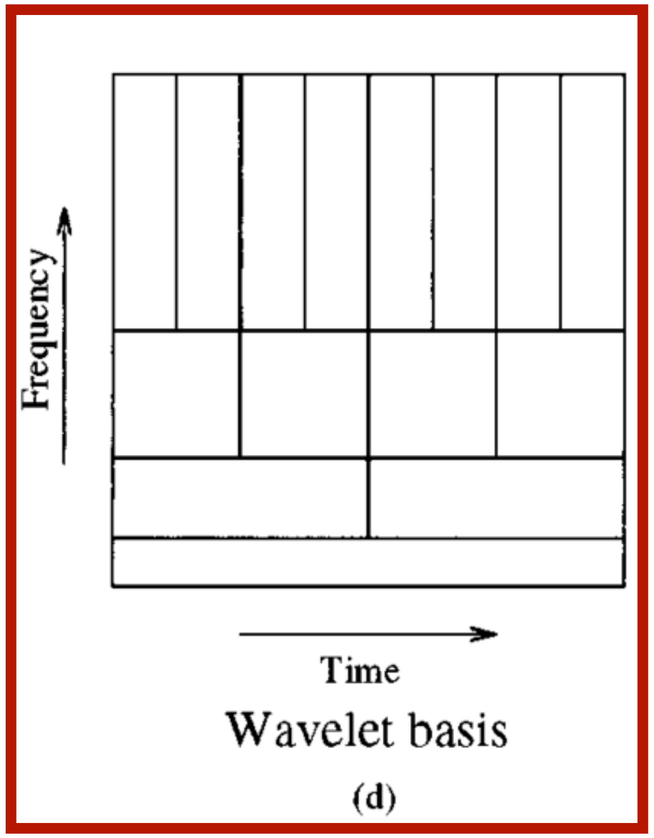
In both cases, we want to retrieve the amplitude (power) and phase (frequency) of the signal **as a function of time**.

Hilbert Transform

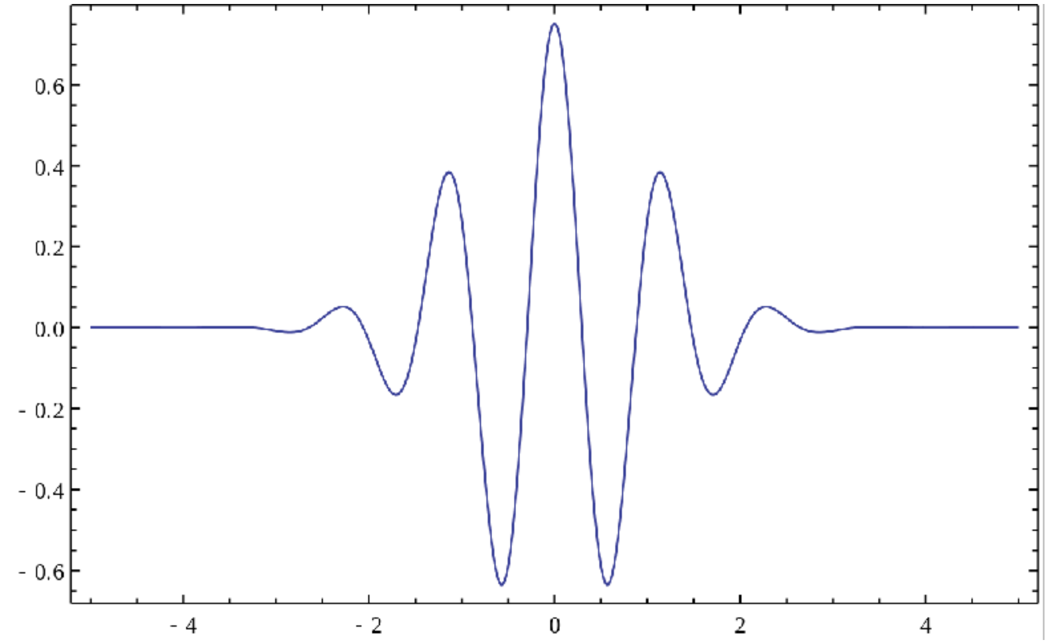


Know the tiling diagrams!



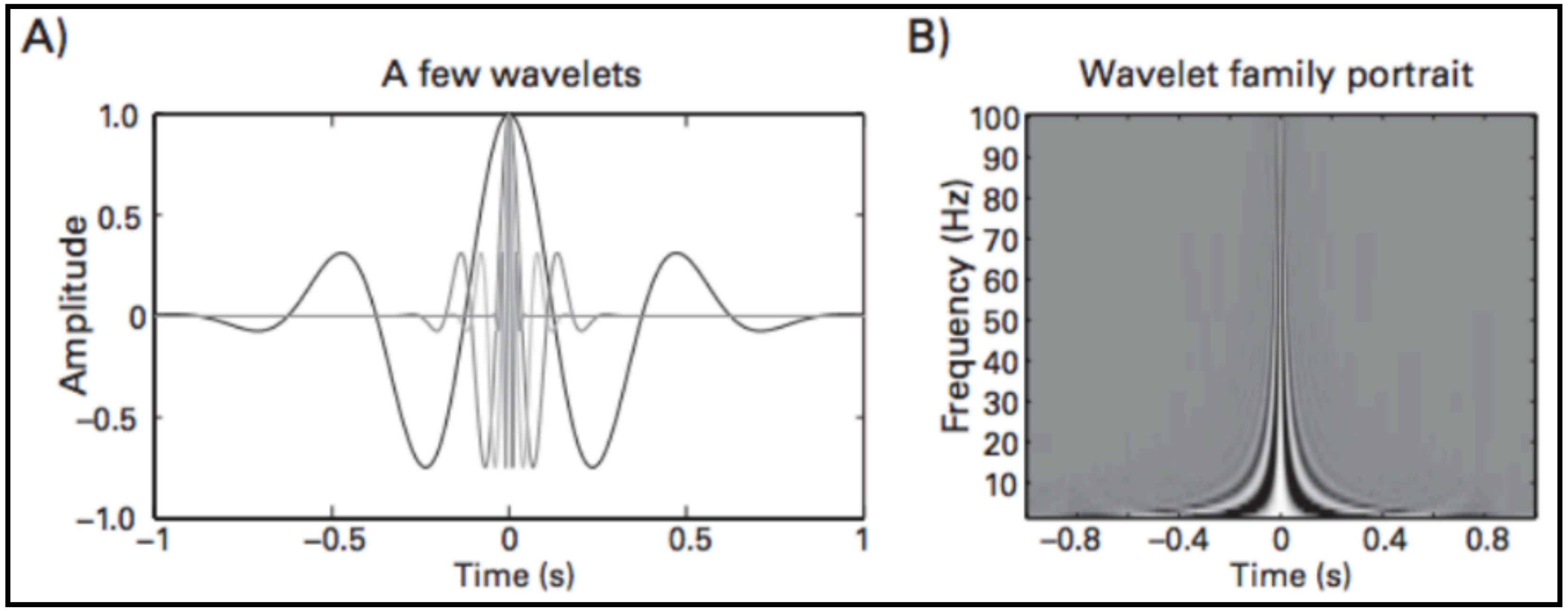


Morlet/Gabor Wavelet (Mother)

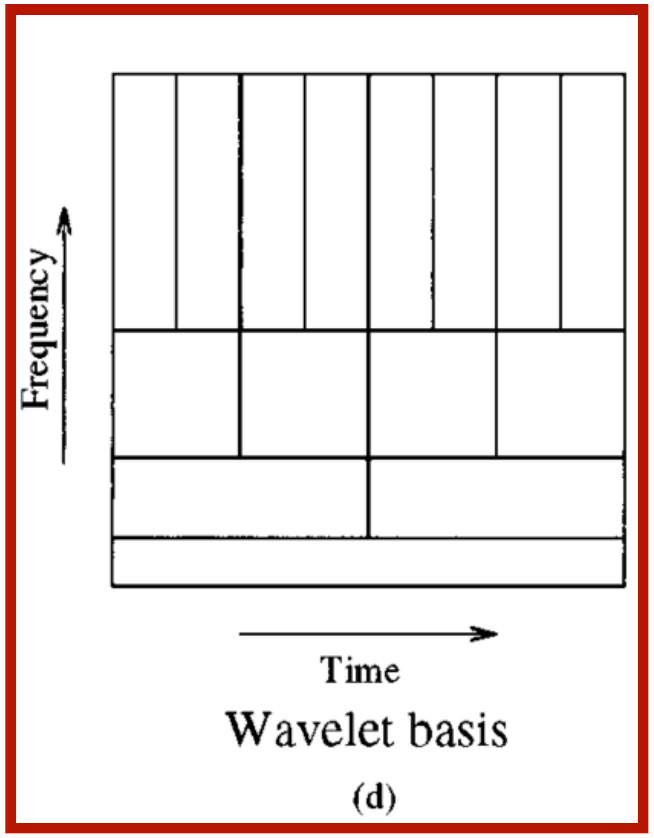


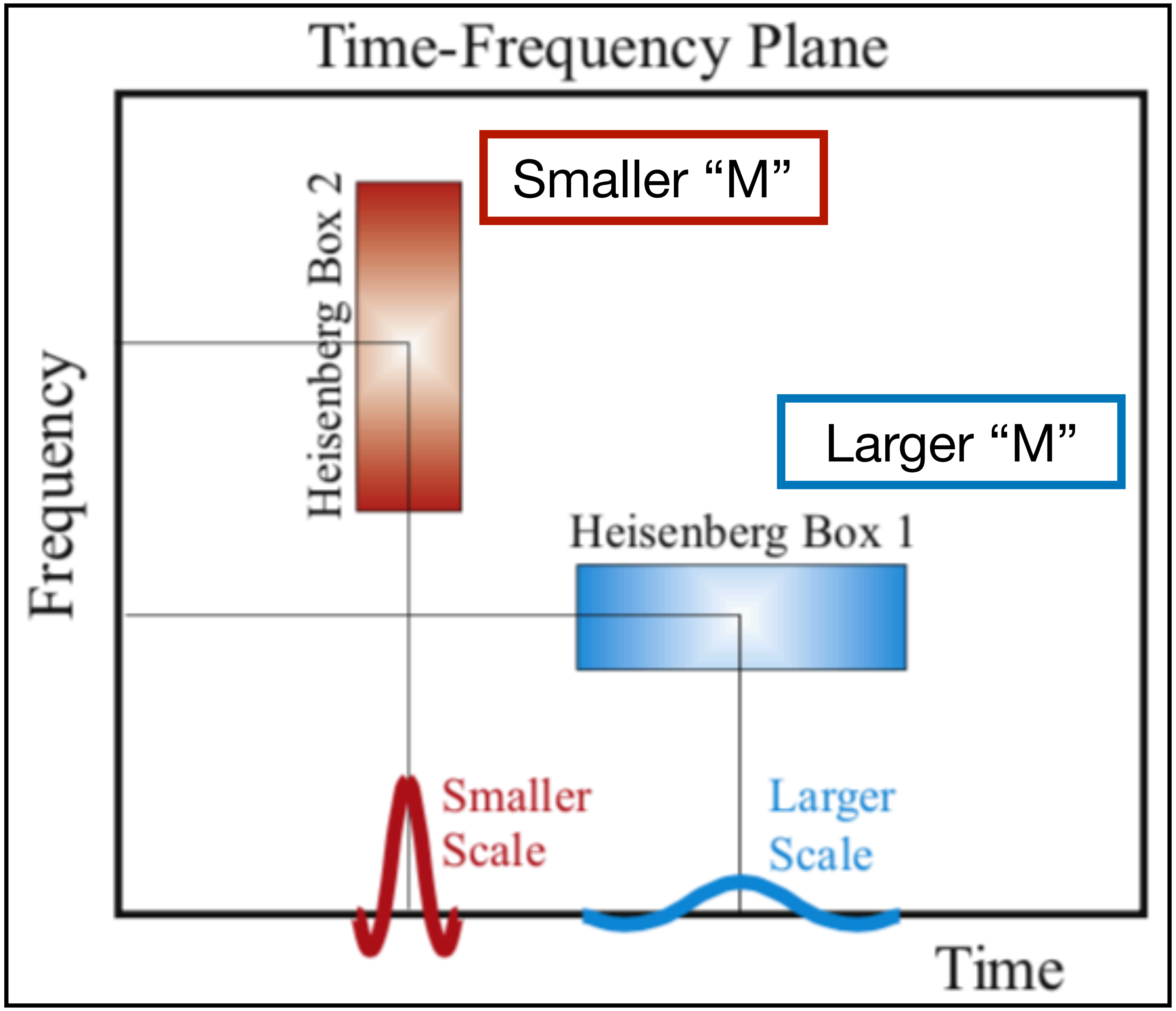
Cosine multiplied by a Gaussian window

Scaling (stretching) the wavelet mother in time gives access to different frequencies, acting as a series of bandpass filter at logarithmic frequency scales.



Wavelet Transform

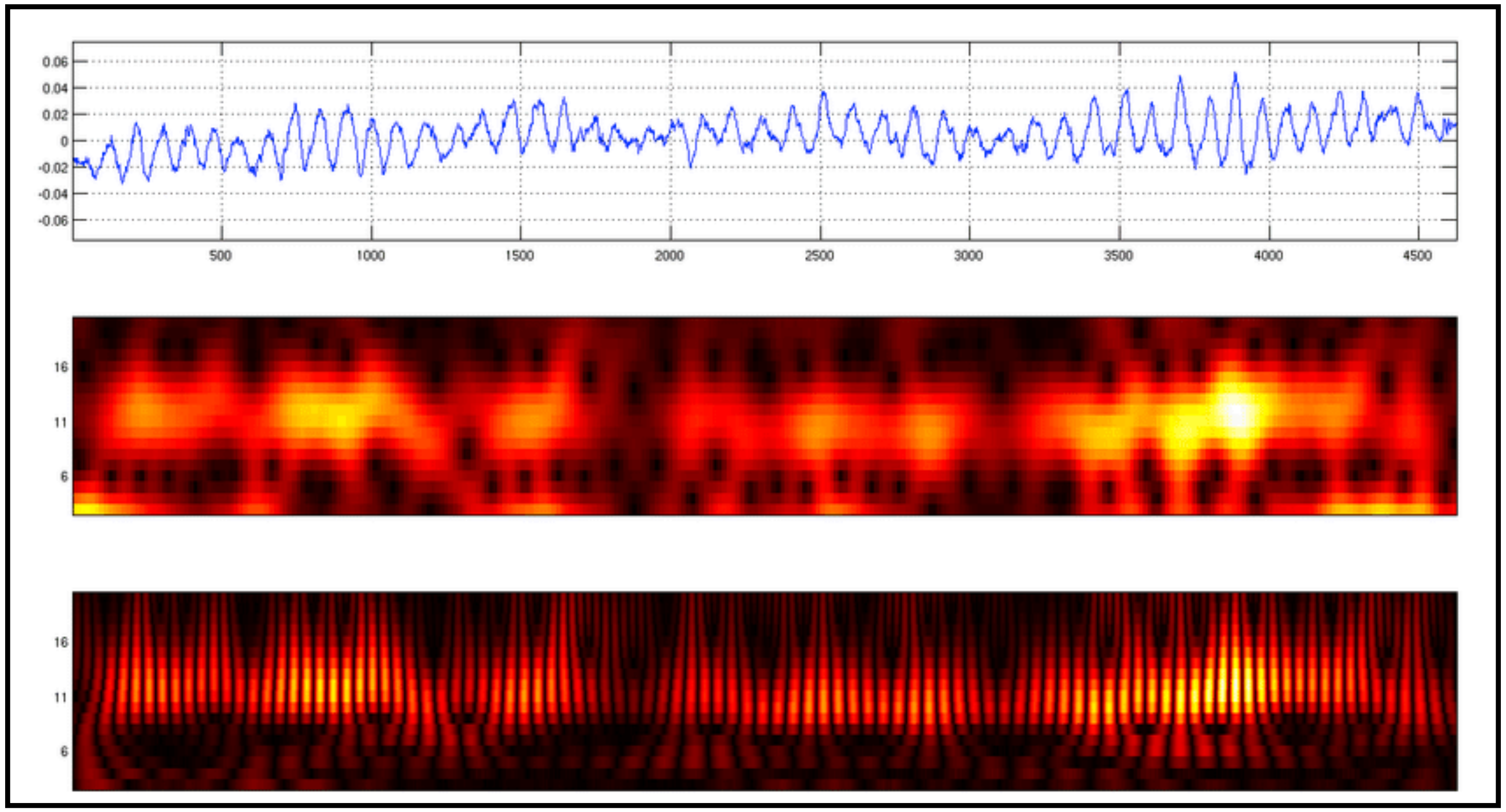




Orthonormal Bases

Why not just bandpass filter for different frequencies and take the Hilbert Transform?

Wavelet series forms an orthonormal basis: picks up **independent and all** components of the signal, conserves signal power.

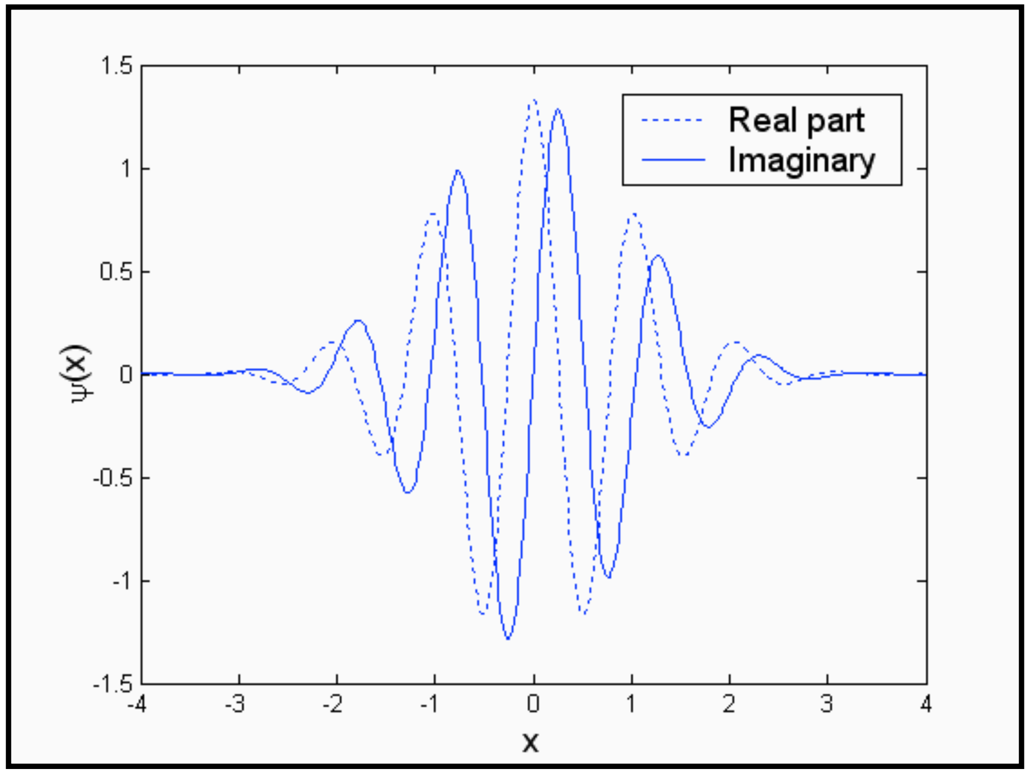


STFT Spectrogram

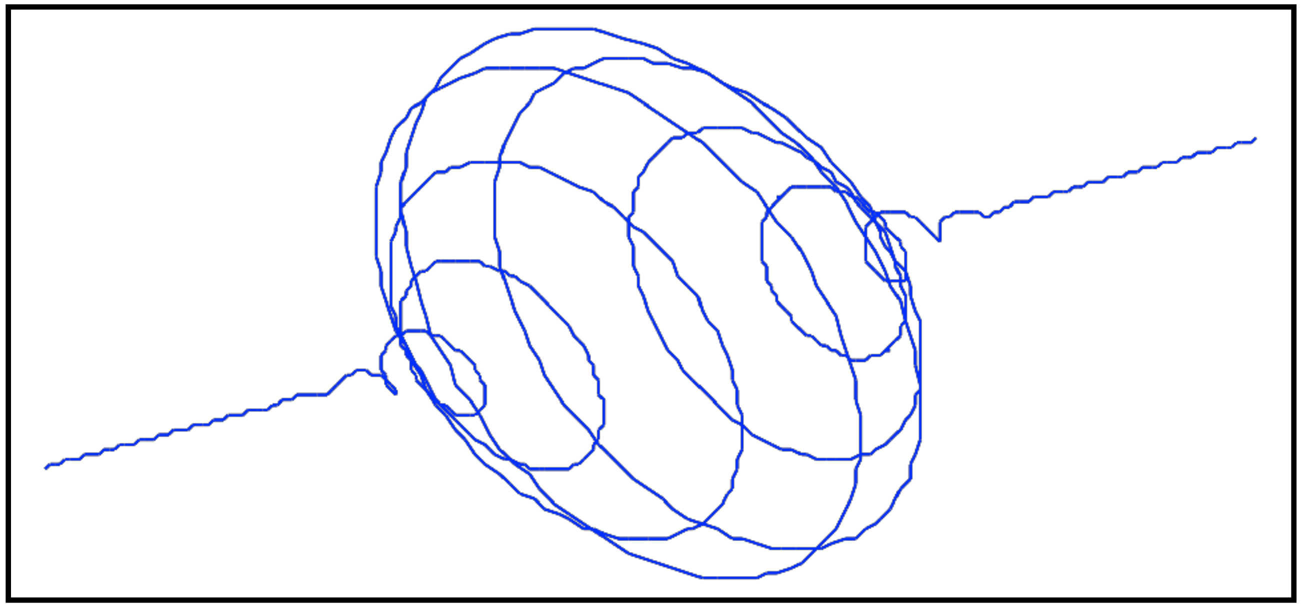
Wavelet Spectrogram

Wavelet is like a halfway point between STFT and Hilbert Transform.

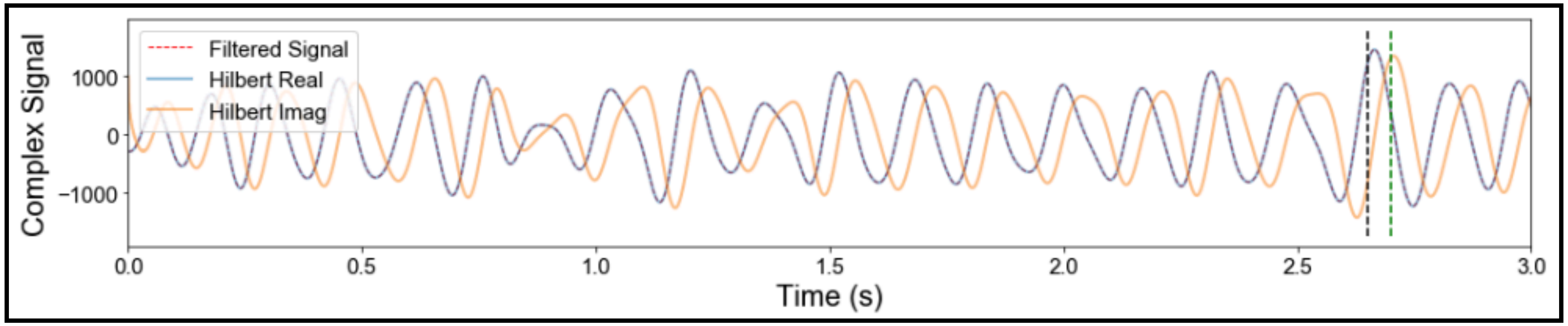
Complex Wavelet Transform



Complex Exponential Multiplied by a Gaussian Window

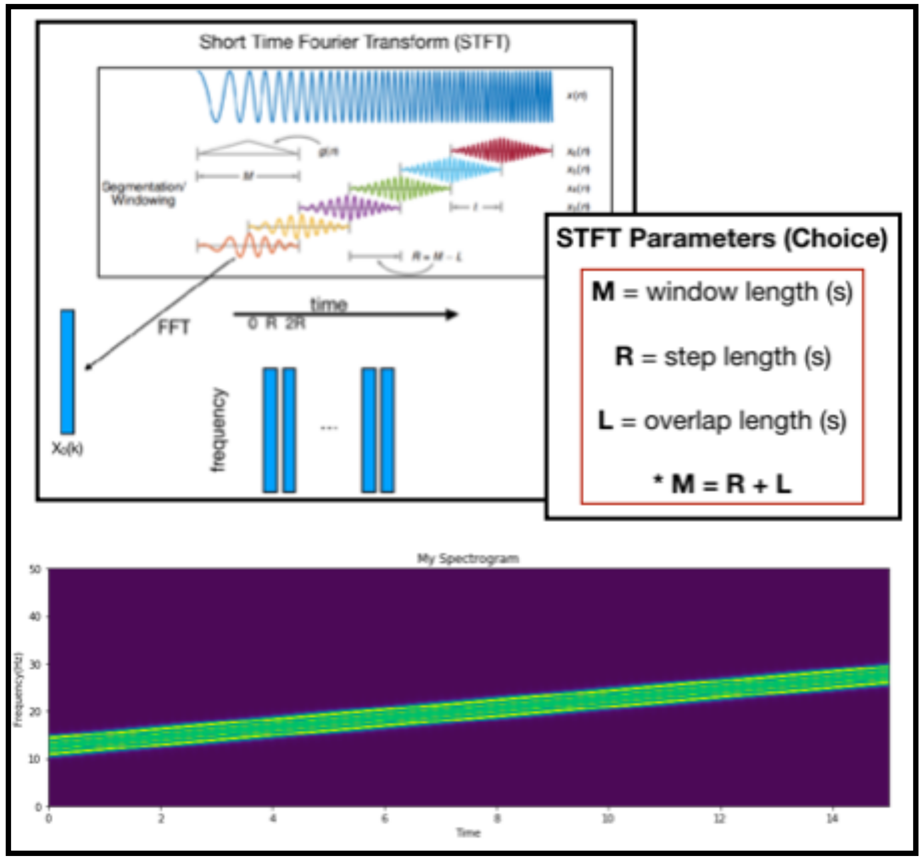


Filtering with complex wavelet directly results in analytic signal.

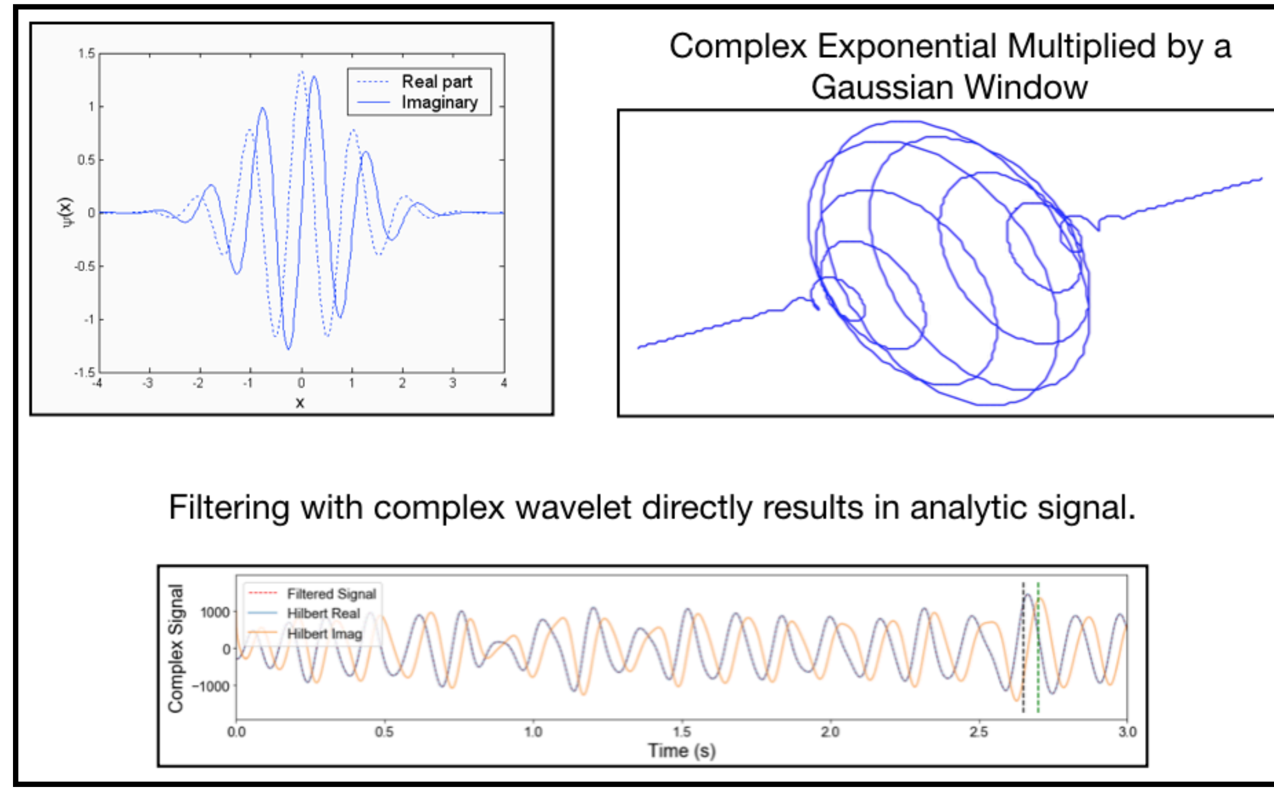


Equivalence of “Kernel” Methods

Method 1. Short-Time Fourier Transform



Method 3. Wavelet Transform



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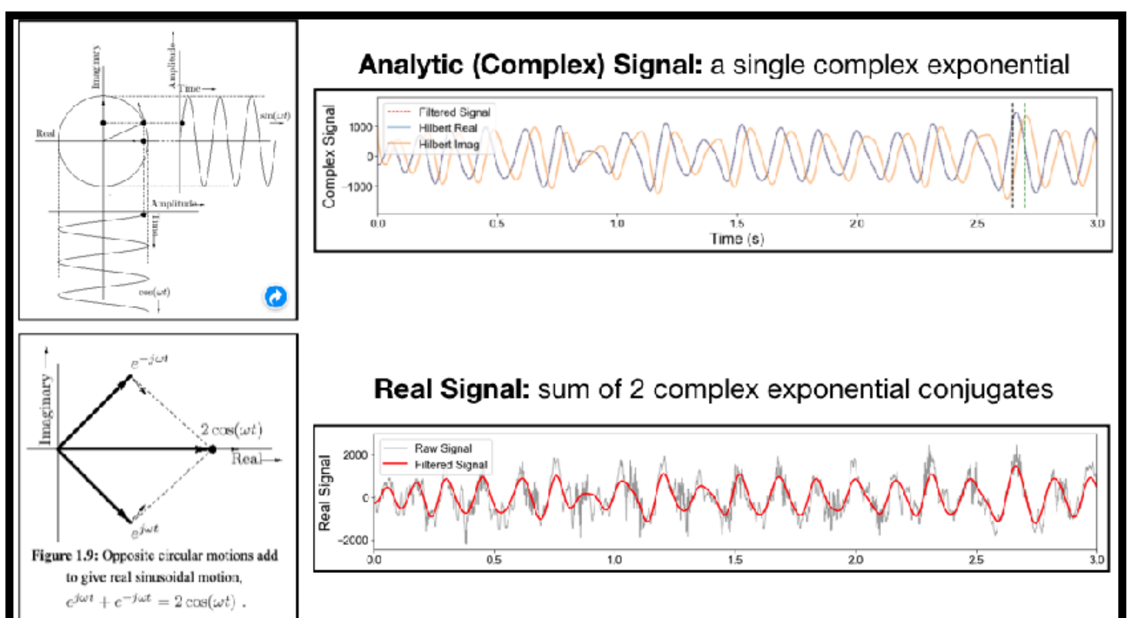
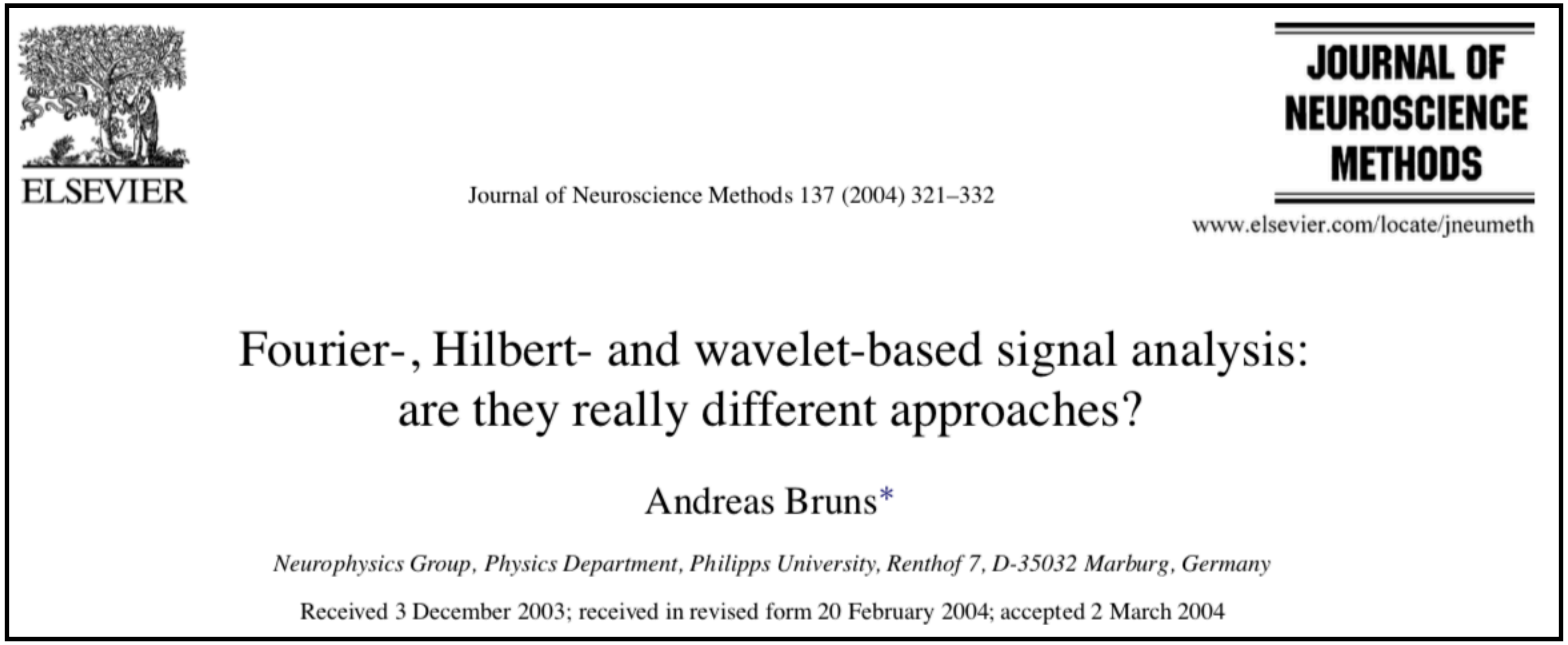


Understand time-frequency resolution requirements and which one to use.

5 minute paper exercise.

e.g., music is logarithmically scaled in frequency, which is more appropriate?

Equivalence of "Kernel" Methods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Journal of Neuroscience Methods 137 (2004) 321–332

JOURNAL OF NEUROSCIENCE METHODS

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Fourier-, Hilbert- and wavelet-based signal analysis: are they really different approaches?

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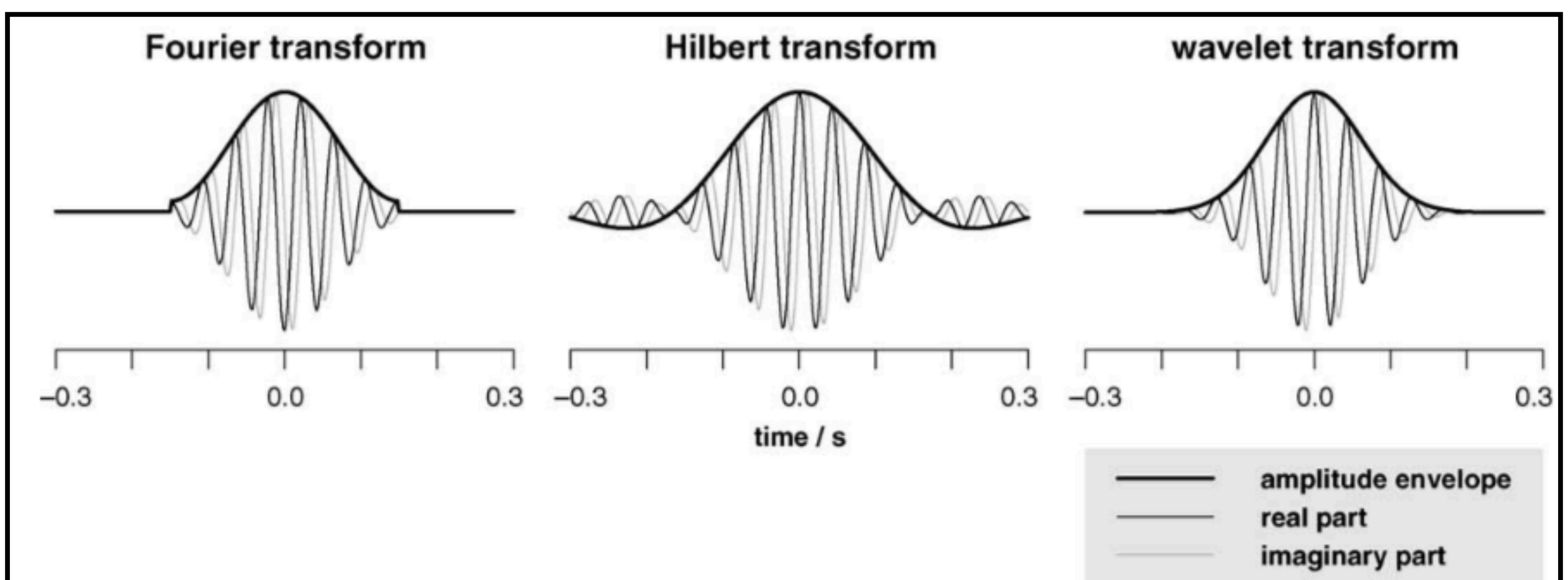
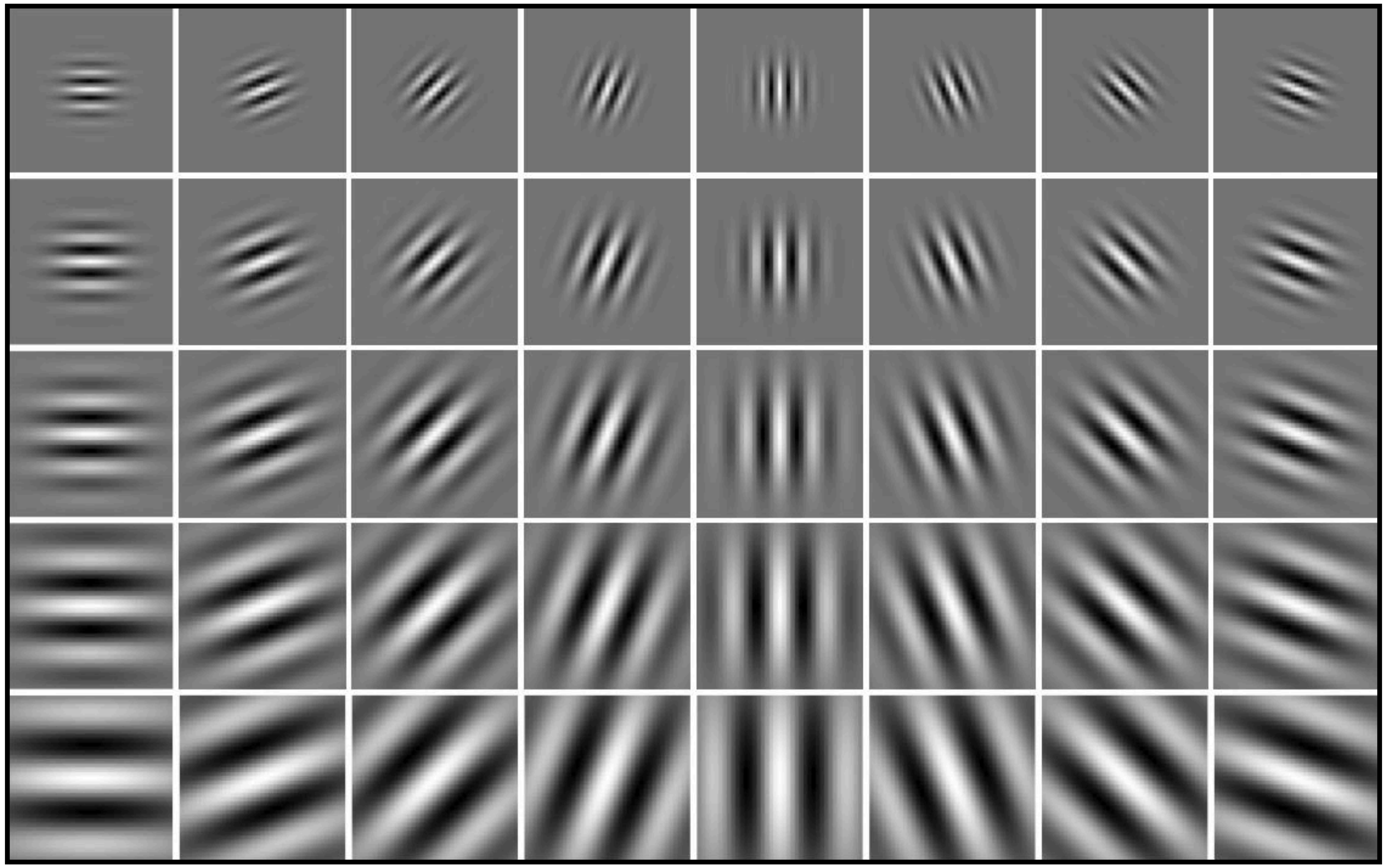
Received 3 December 2003; received in revised form 20 February 2004; accepted 2 March 2004

With appropriately chosen parameters, they produce very similar results.
All convolution of signal with complex sinusoid kernel.



2D Gabor Wavelets

Earlier convolution neural networks use Gabor patches to find image features at different scales.



Some evidence that the early stages of biological visual systems (e.g., in humans) also have “receptive fields” like these filters.

Also used for image compression (JPEG)

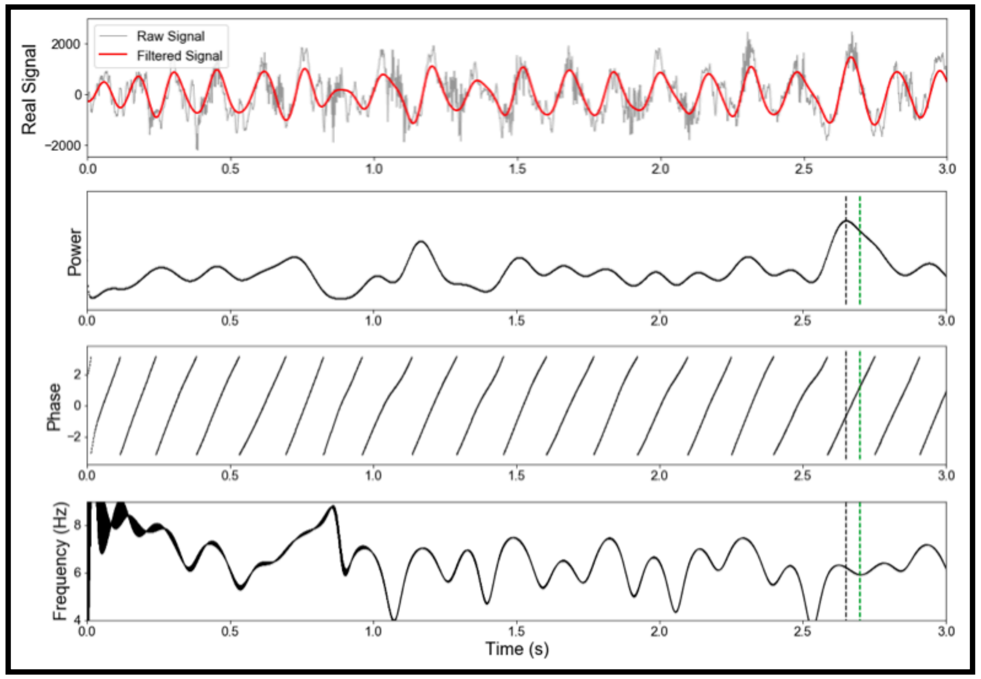
Goals for Today

1. Wavelet & similarities between kernel-based TF methods

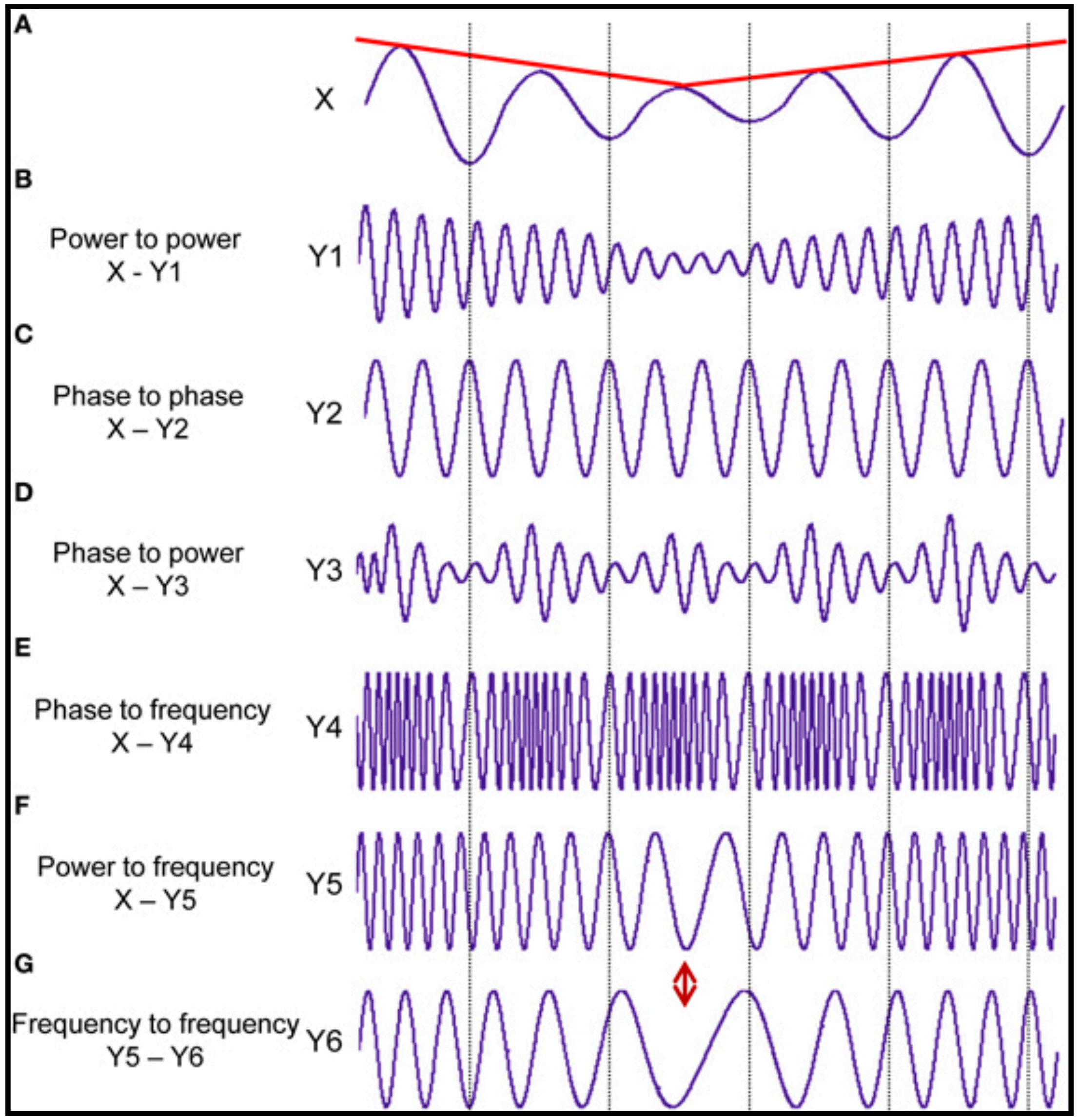
2. Examples of coupling analyses

3. Motivate & understand nonlinear analyses

Coupling of Neural Oscillations



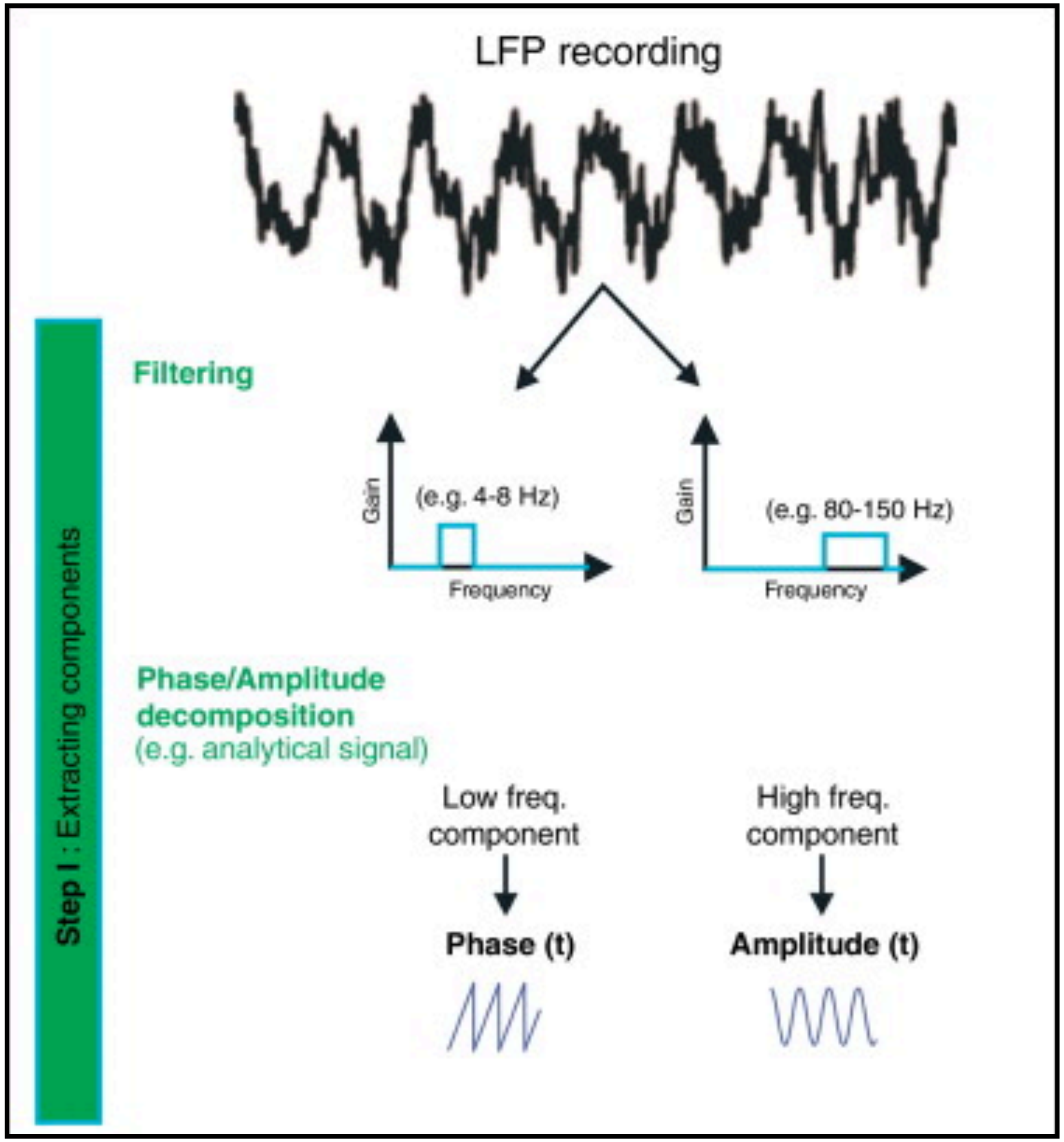
Now we have the instantaneous signal power and phase, what do we do with them?

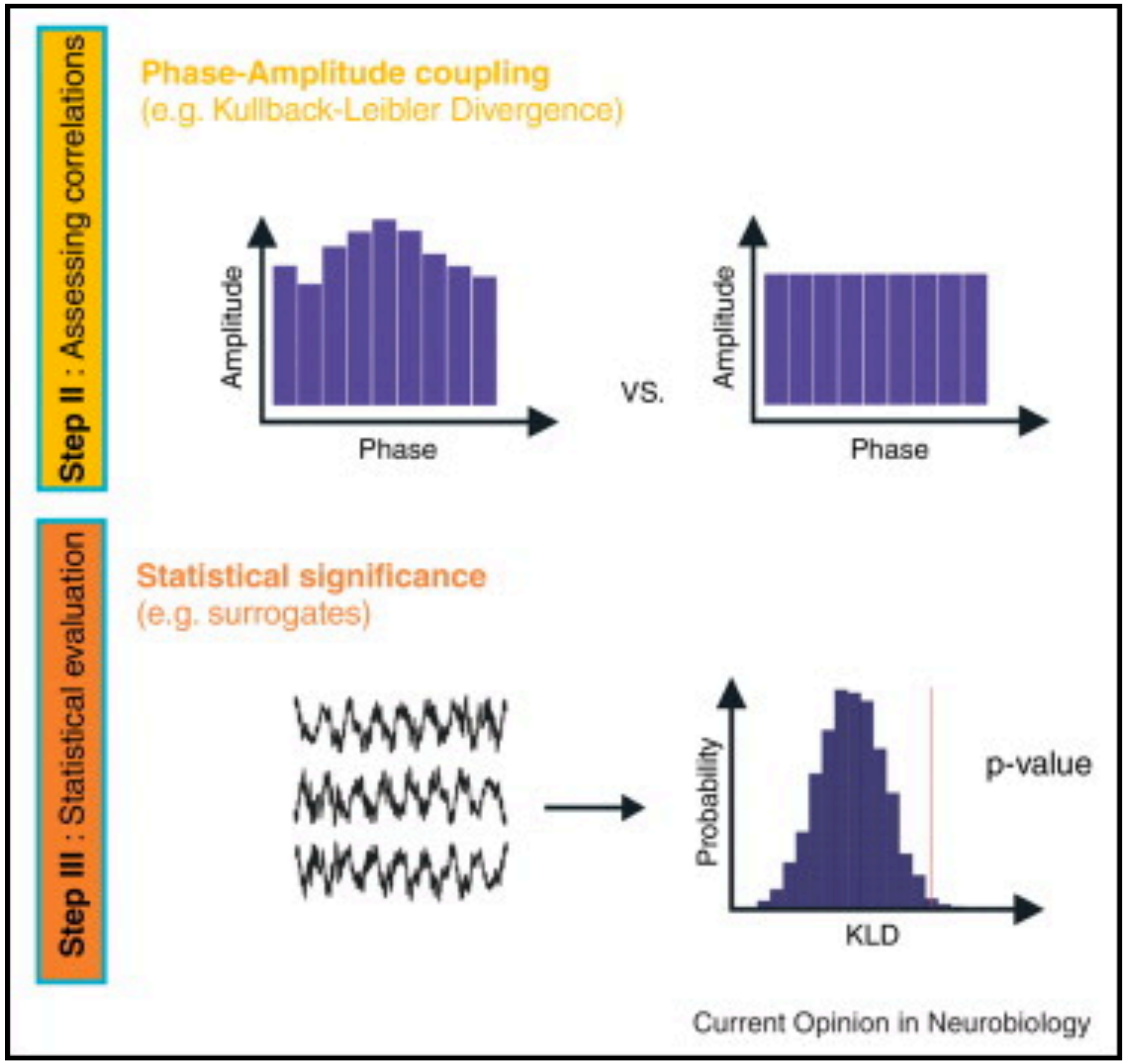


We can look at:

- Coupling between different frequencies in the same signal.
- Coupling between the same frequency in different signals.
- Coupling between the different frequency in different signals.

Cross-Frequency Coupling





Case Study: Phase-Amplitude Coupling

High Gamma Power Is Phase-Locked to Theta Oscillations in Human Neocortex

R. T. Canolty,[1*] E. Edwards,[1,2] S. S. Dalal,[3] M. Soltani,[1,2] S. S. Nagarajan,[3,4] H. E. Kirsch,[5] M. S. Berger,[6] N. M. Barbaro,[5,6] R. T. Knight[1,2,3,5,6]

Why did they run this study (context)?

How did they perform the experiment & analysis (methods)?

What did they find (results)?

What are some possible limitations/improvements?

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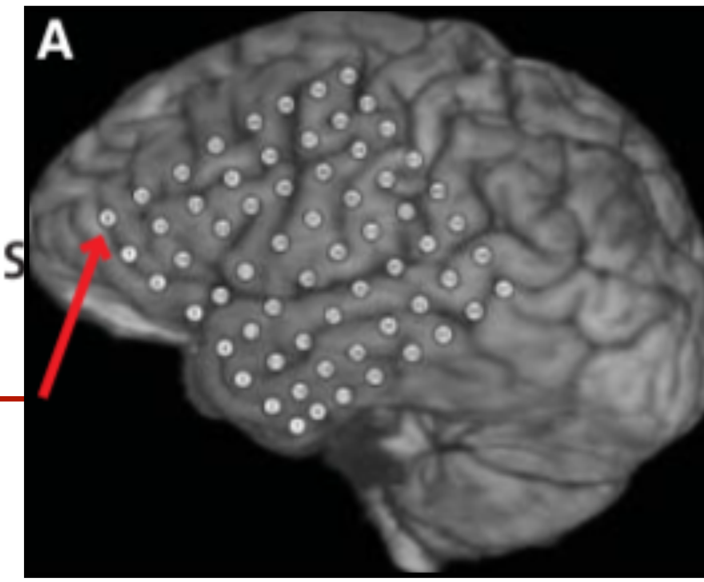
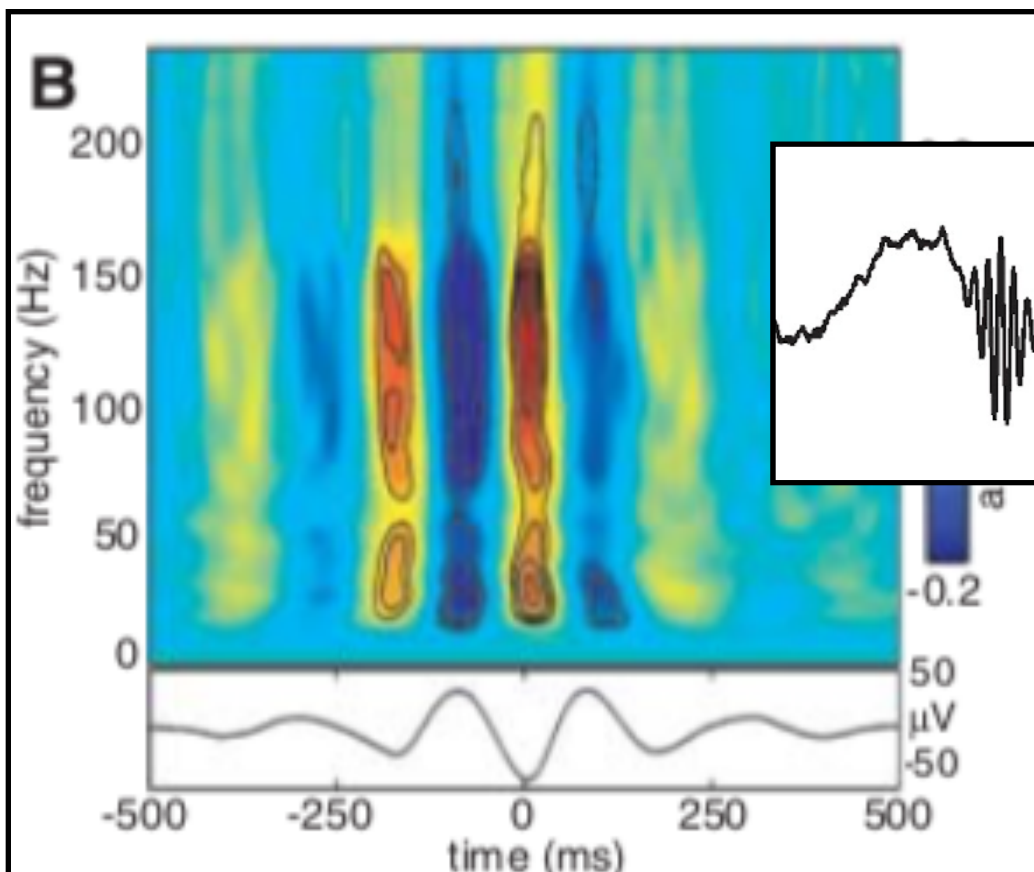
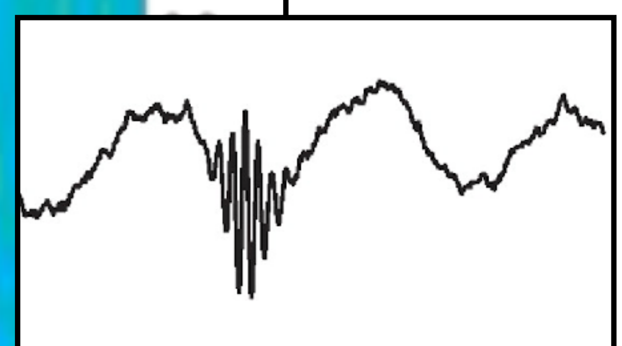
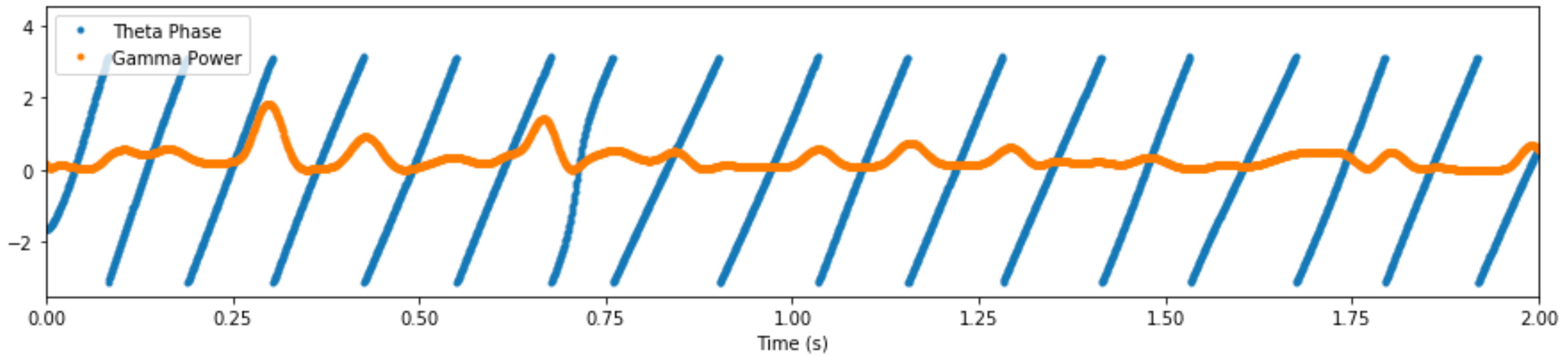






High-frequency power is selectively increased at the trough phase of the low-frequency oscillation.

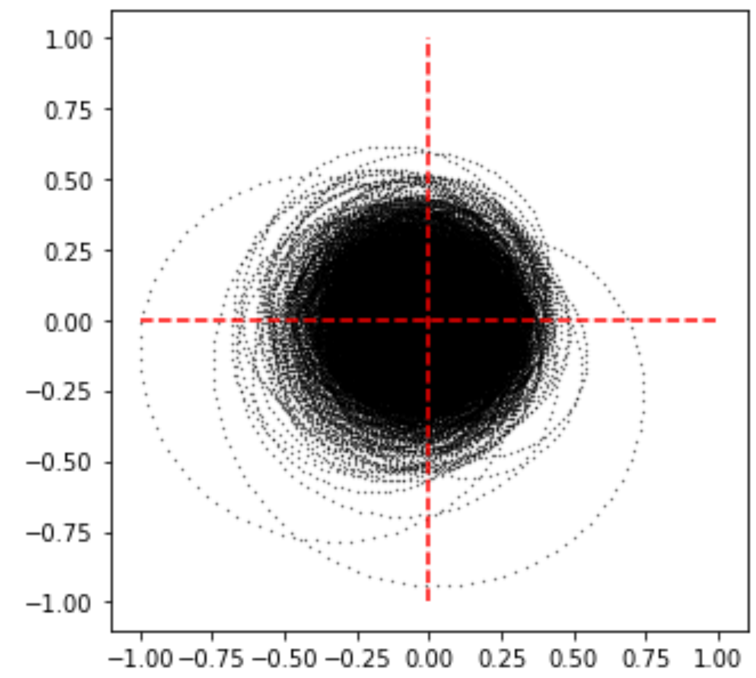
Case Study: Phase-Amplitude Coupling



$$theta\ phase : \phi_\theta(t) \qquad gamma\ power : A_\gamma(t)$$

New complex time-varying exponential

$$z(t) = A_\gamma(t)e^{j\phi_\theta(t)}$$

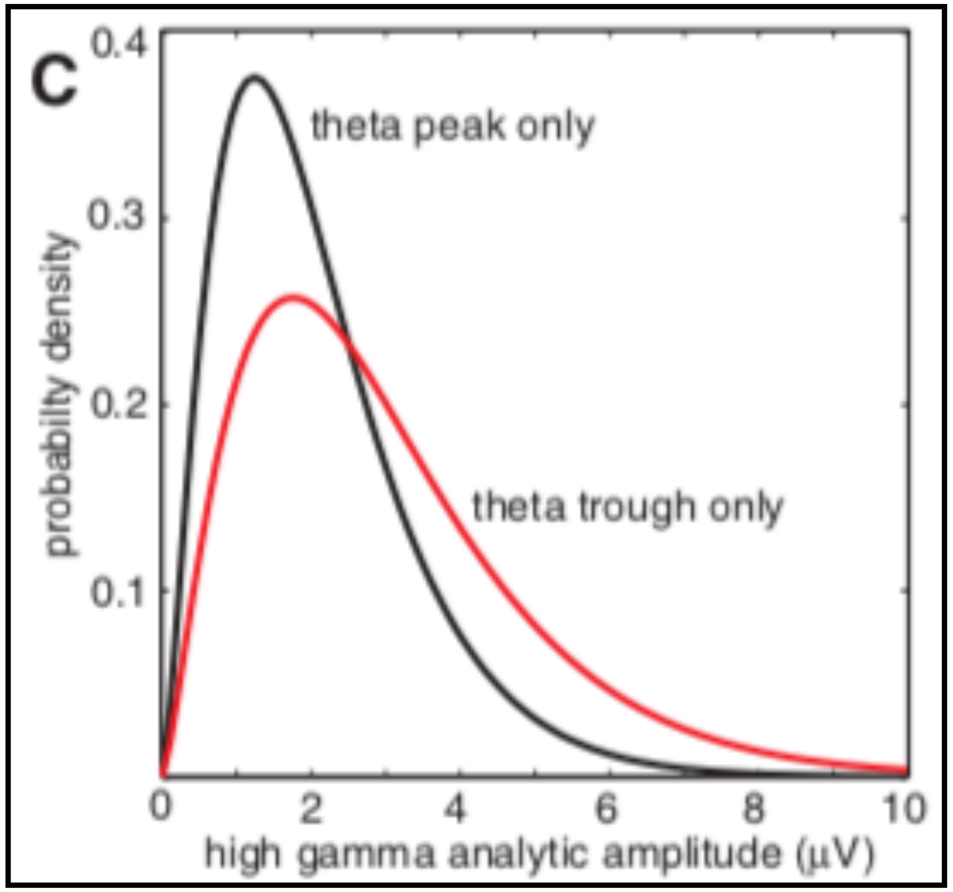


Modulation Index (MI):
vector magnitude of mean(z(t))

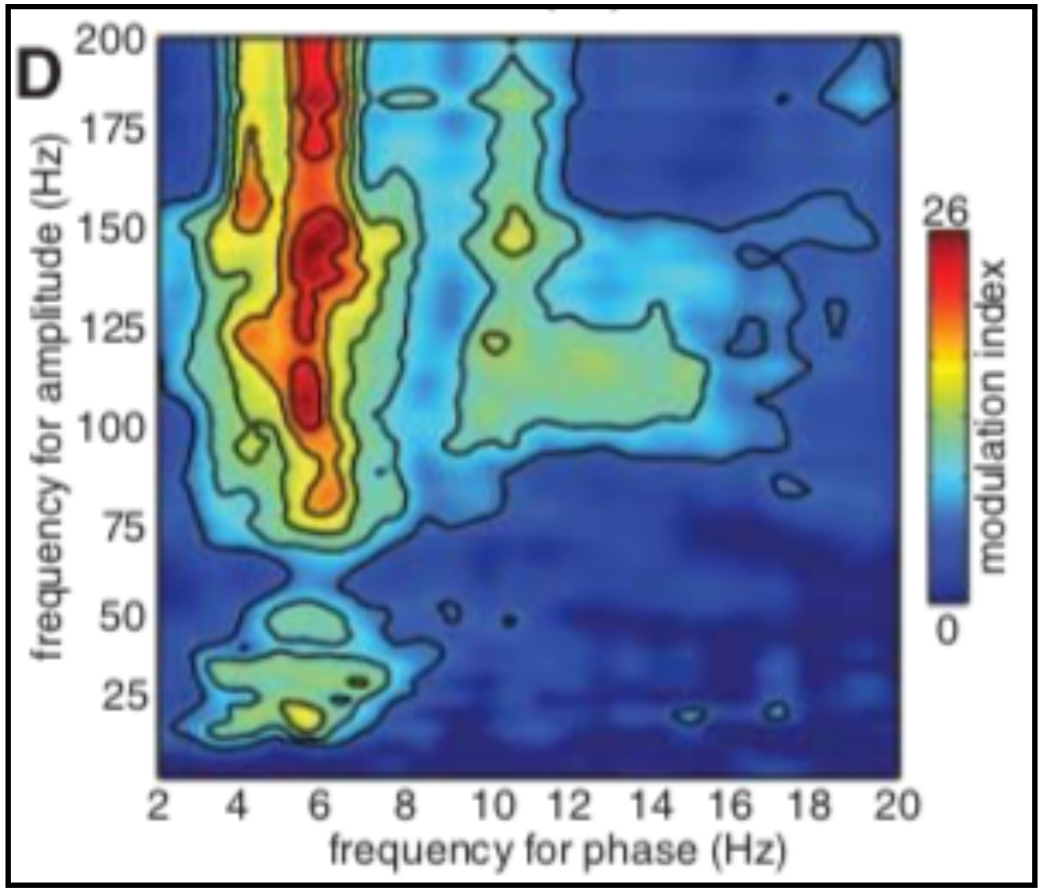
Many other metrics to measure phase-amplitude coupling!

Case Study: Phase-Amplitude Coupling

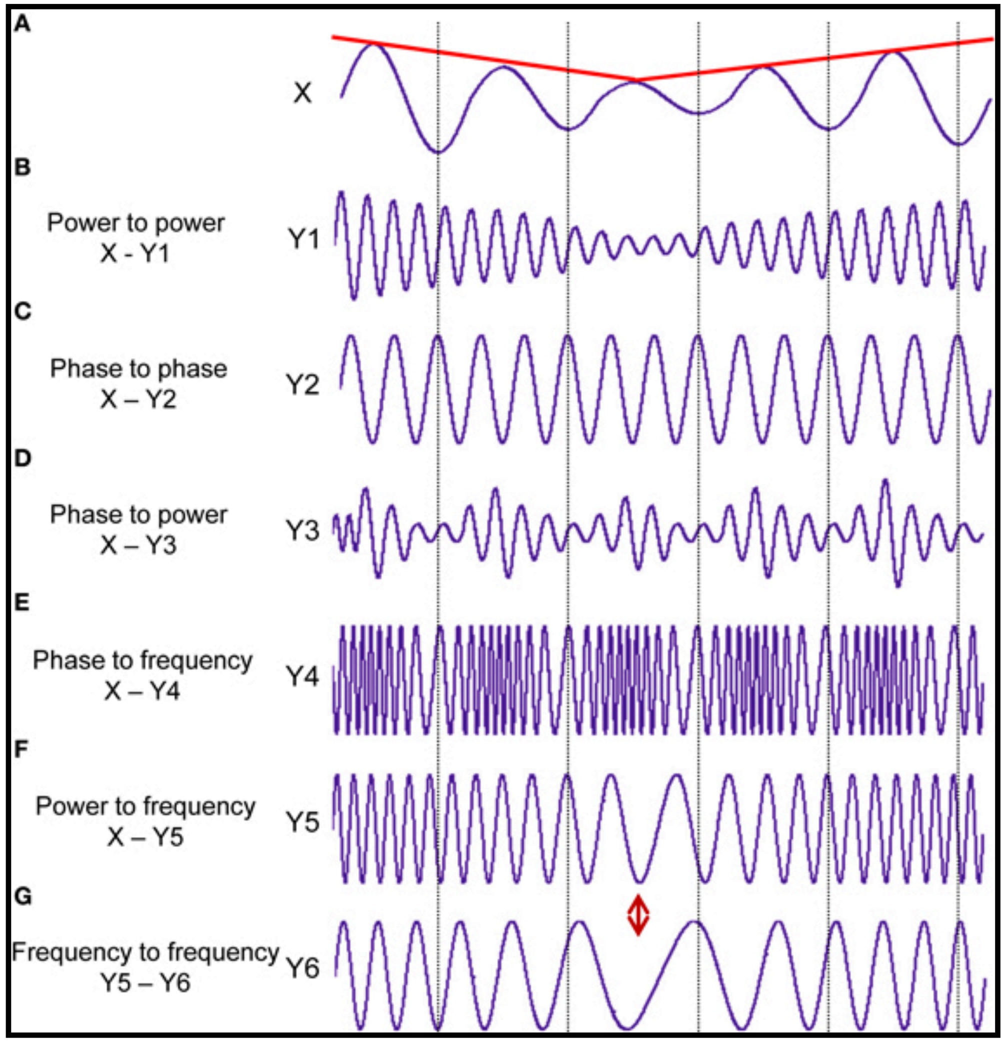
High-frequency power is selectively increased at the trough phase of the low-frequency oscillation.



Coupling is frequency specific.



Cross-Frequency Coupling



Amplitude-Amplitude Coupling (B):
coactivation of network components
$A_1(t) \sim A_2(t)$

Phase-Amplitude Coupling (D):
modulation of one component on the other
$A_1(t) \sim ph_2(t)$

Phase-Phase Coupling (Coherence) (C):
synchronization of two components
$ph_1(t) \sim ph_2(t)$

(typically)

All involve pairwise correlation of some sort between two time series.

Active areas of research!

1. Wavelet & similarities between kernel-based TF methods

2. Examples of coupling analyses

3. Motivate & understand nonlinear analyses

Linearity In Fourier Analysis

Fourier analyses are often referred to as linear systems analysis

For linear systems, a sinusoidal input results in a sinusoidal output, just scaled and/or phase-delayed.

Example: FIR filters (scale and delay some components)

Finding dominant oscillations in different frequency band does not imply that they are different processes!

Nonlinear Signals

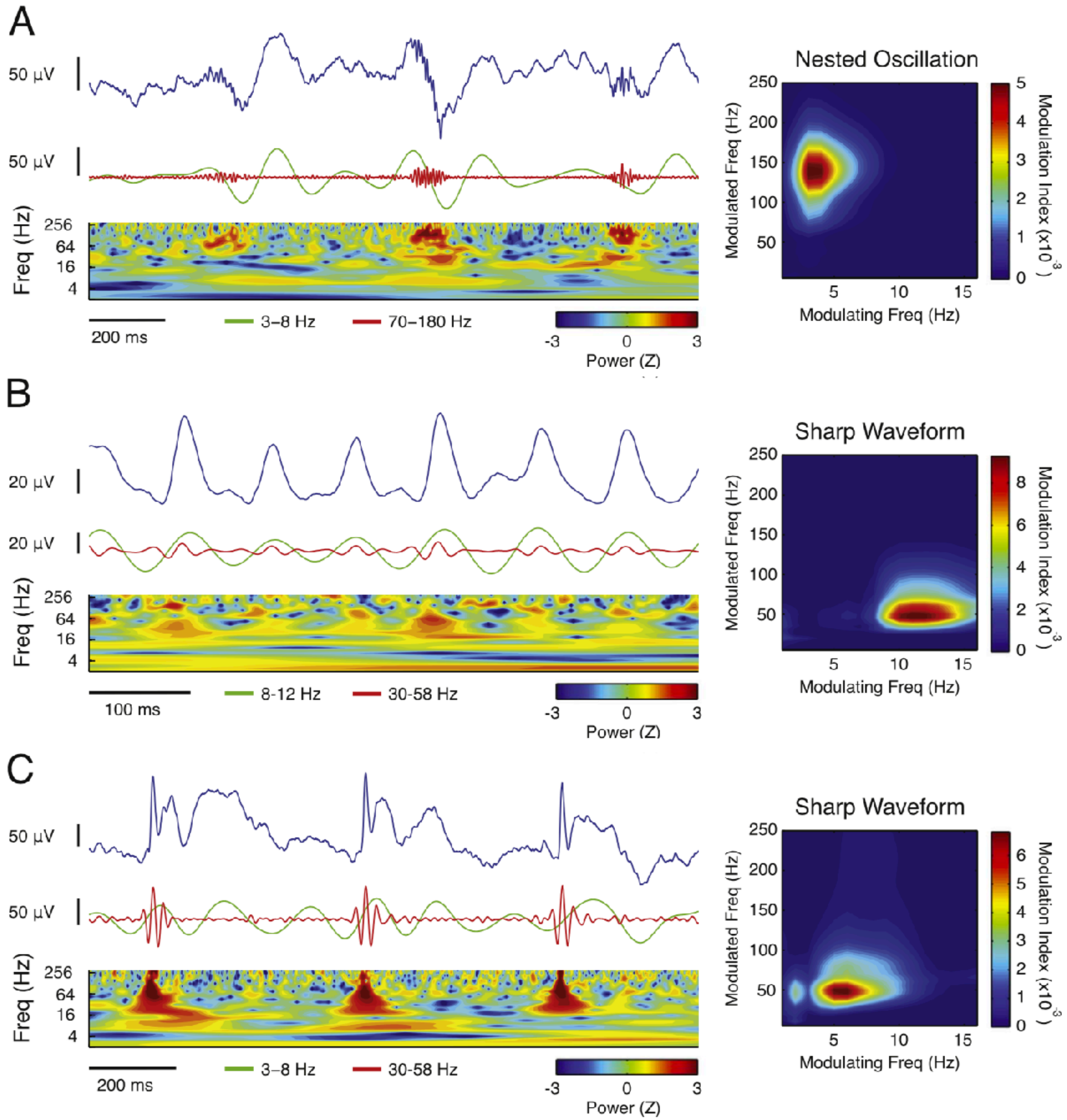
“Nonlinear” systems or signals does not satisfy the sine-in-sine-out assumption.

i.e. systematic relationships across non-independent frequencies (harmonics)

Dual origins of measured phase-amplitude coupling reveal distinct neural mechanisms underlying episodic memory in the human cortex

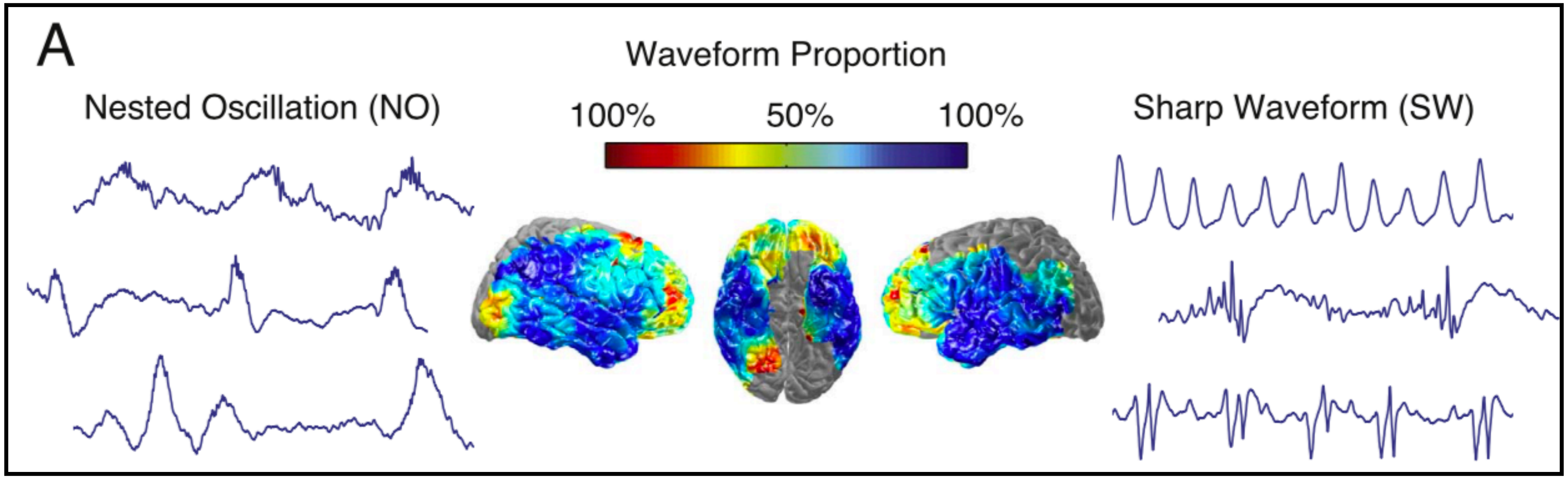
Alex P. Vaz[a,b], Robert B. Yaffe[a,c], John H. Wittig Jr[a], Sara K. Inati[d], Kareem A. Zaghloul[a,*]

[a] *Surgical Neurology Branch, NINDS, National Institutes of Health, Bethesda, MD 20892, USA*
[b] *Medical Scientist Training Program, Duke University School of Medicine, Durham, NC 27710, USA*
[c] *Department of Biomedical Engineering, Johns Hopkins University, Baltimore, MD 21218, USA*
[d] *Office of the Clinical Director, NINDS, National Institutes of Health, Bethesda, MD 20892, USA*



Many ways to produce
“phase-amplitude coupling”

Nonlinear Signals



Final takeaway for Fourier analysis: just because you **can** separate the signal into components using Fourier (frequency decomposition), it does **not** imply that there are independent generators.

Summary

1. Wavelet & similarities between kernel-based TF methods

2. Examples of coupling analyses

3. Motivate & understand **nonlinear** analyses

https://tinyurl.com/cogs118c-att