Optimal Downlink Power Allocation and Scheduling for MIMO-Based WiMAX Access Networks

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Abstract

Recently, mobile WiMAX equipped with MIMO technology has attracted much attention and has emerged as a leading solution for broadband wireless access networks. However, study on optimal resource management for such networks is still in its infancy and results in this area remain limited. In this paper, we study the joint optimization of power allocation and scheduling for MIMO-based WiMAX networks. Specifically, we consider the maximum weighted sum rate (MWSR) problem for the downlinks. First, for the ideal case where full channel distribution information (CDI) is available, we design an off-line algorithm for the MWSR problem to obtain an optimal solution. For the practical case where full CDI is not available, we use the off-line algorithm as a baseline and design an online adaptive algorithm (OAA) based on subgradient stochastic approximation. For OAA, when the feedback of channel state information (CSI) is error free, we show that it is able to converge with probability one to the same optimal solution obtained by the off-line algorithm. In the case where there is estimation error in CSI, we show that OAA can still converge with probability one to the same optimal solution if the resultant subgradients are unbiased, or to some neighborhood of the optimal solution when the resultant subgradients are biased.

1 Introduction

The last decade has witnessed an explosive demand for high-speed wireless access networks. For broadband access, MIMO-based WiMAX has attracted substantial attention in recent years. Current standards of MIMO-based WiMAX include IEEE 802.16-2004 air interface [1] and IEEE 802.16e mobile amendment [2], which employ orthogonal frequency division multiple access (OFDMA) and MIMO techniques. The use of MIMO technology improves the reception and allows for a better coverage and higher transmission rate. The potential of providing high spectral efficiency in

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multi-path non-line-of-sight environment with MIMO and QoS assurance with WiMAX channelization scheduling have rendered MIMO-based WiMAX as a leading "last mile" access solution for broadband wireless access networks.

However, to achieve high capacity for MIMO-based WiMAX access networks, there remain many challenges that must be addressed. One important challenge is how to determine optimal power allocation among users, subcarriers, and antennas (at physical layer) and optimal scheduling (at MAC layer) to maximize the throughput of downlinks under frequency-selective fading, while simultaneously providing QoS guarantee and assuring fairness among all users. This problem is considerably more challenging than that in conventional single antenna-based WiMAX access networks. This is because, compared to the simple scalar channels in single antenna case, power allocations are now performed over complex matrix channels in time, frequency, and space domains. In 802.16e amendment, scheduling and power allocation algorithms for both uplink and downlink are left undefined. So far, results on how to fully exploit channel and multiuser diversity to design dynamic subcarrier scheduling and adaptive power allocation remain scarce in the literature. In this paper, we address this problem by designing algorithms for optimal joint subcarrier scheduling and power allocation in space, time and frequency domains.

More specifically, we consider the so-called maximum weighted sum rate (MWSR) as the objective in our cross-layer optimization for downlinks in MIMO-based WiMAX networks. The choice of this objective is motivated by recent results in [3–6], where it has been shown that an adaptive policy based on solving a MWSR problem in each time slot can be used to stabilize the transmission buffers for any set of arrive rates in the ergodic capacity region. Further, it has been shown in [7] that by appropriately updating the weights from time to time in the MWSR objective, a proportionally fair scheduler can be designed. Hence, there is enough merit to choose MWSR as the objective function in our cross-layer optimization.

The main contributions of this paper are the following.

1. For the ideal case where channel distribution information (CDI) is available, we design an off-line algorithm for the MWSR problem to obtain an optimal solution. Our approach is

to exploit the special structure of MWSR and transform it to its Lagrangian dual domain. We decompose the Lagrangian dual into an inner power allocation subproblem and an outer scheduling subproblem. This decomposition simplifies the complex original cross-layer optimization and enables us to derive closed-form solutions for both scheduling and power allocation in the dual domain. As a result, the problem of finding an optimal scheduling and matrix-valued power allocation solution is reduced to finding an optimal dual solution, which is in simple scalar form.

2. For the practical case where CDI is unavailable, we use the off-line algorithm as a baseline and design an *online adaptive algorithm* (OAA). Our approach is to employ subgradient stochastic approximation, which uses the instantaneous feedback of channel state information (CSI) to compute an approximation of the true subgradient. OAA can adapt to unknown fading distribution, which is a desirable feature for implementing practical protocols. When the feedback of CSI is error free, we show that OAA can converge with probability one to the same optimal solution obtained by the off-line algorithm, regardless of the underlying fading distribution.

3. When the feedback of CSI has estimation error, we investigate the impact of such error on the performance of OAA. We consider two cases, depending on whether the resultant stochastic subgradients are biased or unbiased. We show that if the stochastic subgradients are unbiased, OAA can still converge with probability one to the same optimal solution obtained by the off-line method. On the other hand, when the stochastic subgradients are biased, OAA will converge with probability one to some neighborhood of the optimal solution. The size of the neighborhood is determined by the channel estimation errors.

The remainder of this paper is organized as follows. In Section 2, we introduce the system architecture and problem formulation. Section 3 presents an off-line algorithm when full CDI is available. In Section 4, we introduce OAA and establish its convergence theorem. The impacts on OAA due to channel estimation errors in CSI feedback are discussed in Section 5. Section 6 presents some numerical results to provide further insights to our theoretical analysis. In Section 7,

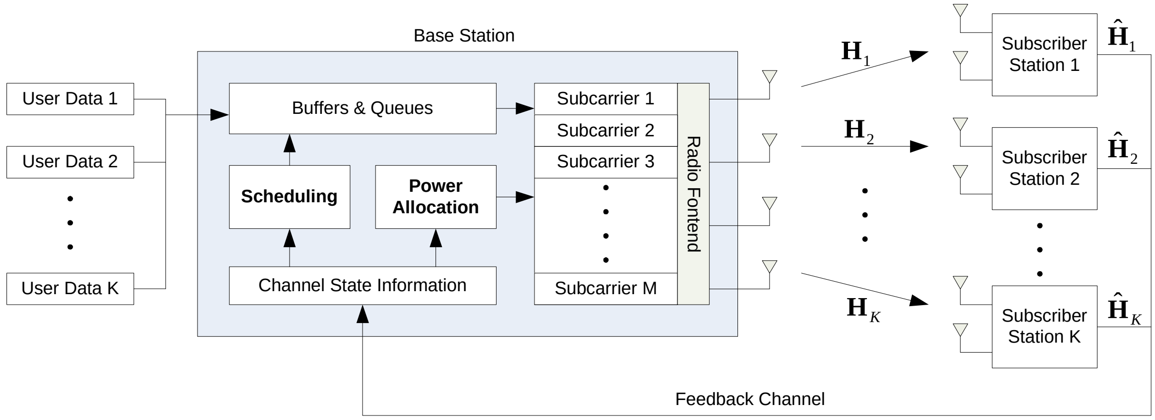


Figure 1: System architecture of a MIMO-based WiMAX downlink.

we review related work and Section 8 concludes this paper.

2 System Architecture and Problem Formulation

2.1 System Architecture

We consider the downlinks of a MIMO-based WiMAX broadband access network consisting of K users and M subcarriers. The system architecture is shown in Figure 1. The base station and subscriber stations are equipped with multiple antennas. The total bandwidth of the system is B. As a result, the bandwidth of each subcarrier is $\frac{B}{M}$. We assume that the bandwidth of each subcarrier is sufficiently small such that the fading on each subcarrier can be considered as flat fading. Base on the training symbols sent from the base station, each subscriber station estimates its channel gain matrix $\mathbf{H}_i$, $i = 1, 2, \ldots, K$, and convey the estimation $\hat{\mathbf{H}}_i$ back to the base station through a feedback channel in its uplink. Based on channel state information (CSI) feedback, the base station schedules an optimal time interval and allocate an optimal amount of power for each user on each subcarrier so that the weighted sum rate of all users in the downlinks is maximized, while satisfying the minimum data rate constraint for each user.

Note that an alternative way to collect CSI is by estimating the uplinks based on channel reciprocity if the system operates in TDD mode. Therefore, in Figure 1, the feedback channel could represent either type of these two different CSI collection mechanisms.

2.2 System Model

In this paper, we use boldface to denote matrices and vectors. For a matrix $\mathbf{A}$, $\mathbf{A}^\dagger$ denotes the conjugate transpose, $\mathrm{Tr}\{\mathbf{A}\}$ denotes the trace of $\mathbf{A}$, $|\mathbf{A}|$ denotes the determinant of $\mathbf{A}$, and $\|\mathbf{A}\|$ denotes the Frobenius norm of $\mathbf{A}$. We denote $\mathbf{I}$ the identity matrix with dimension determined from the context. $\mathbf{A} \succeq 0$ represents that $\mathbf{A}$ is Hermitian and positive semidefinite (PSD). $\mathbf{0}$ denotes a vector whose elements are all zeros, and its dimension is determined from the context. For a real vector $\mathbf{v}$ and a real matrix $\mathbf{A}$, $\mathbf{v} \geq \mathbf{0}$ and $\mathbf{A} \geq \mathbf{0}$ mean that all entries in $\mathbf{v}$ and $\mathbf{A}$ are nonnegative, respectively. The operator "$\langle \cdot, \cdot \rangle$" represents the inner product operation for vectors or a matrices.

2.2.1 Power Allocation

Suppose that the base station has n_t antennas and each subscriber station has n_r antennas. Let $\mathbf{H}_k^{(m)} \in \mathbb{C}^{n_r \times n_t}$ represent the channel gain matrix from the base station to subscriber station k over subcarrier m. For convenience, we let $\mathbf{H} = [\mathbf{H}_k^{(m)} : k = 1, \ldots, K,\ m = 1, \ldots, M]$ denote the collection of all channel gain matrices. The entries in each channel matrix are assumed to be i.i.d. complex Gaussian distributed (Rayleigh faded). The received complex base-band signal vector at subscriber station k over subcarrier m can be computed as

$$\mathbf{y}_k^{(m)} = \sqrt{\rho_k} \mathbf{H}_k^{(m)} \mathbf{x}_k^{(m)} + \mathbf{n}, \tag{1}$$

where $\mathbf{y}_k^{(m)} \in \mathbb{C}^{n_r}$ and $\mathbf{x}_k^{(m)} \in \mathbb{C}^{n_t}$ represent the received and transmitted signal vectors of user k over subcarrier m, respectively, $\mathbf{n}$ represents the normalized complex additive white Gaussian noise vector with zero mean and unit variance. In (1), $\rho_k \triangleq \frac{P_{\max} G \cdot d_k^{-\alpha}}{N_0 B / M}$ denotes the signal-to-noise ratio (SNR) of subscriber station k, where $P_{\max}$ is the maximum transmission power of the base station, G is some constant that depends upon specific system parameters, d_k is the distance between the base station and subscriber station k, α represents the path-loss index, and N_0 is the power density of the complex additive white Gaussian noise.

Let matrix $\mathbf{Q}_k^{(m)}$ denote the covariance matrix of input symbol vector $\mathbf{x}_k^{(m)}$, i.e., $\mathbf{Q}_k^{(m)} = \mathbb{E}\{\mathbf{x}_k^{(m)} \cdot \mathbf{x}_k^{(m)\dagger}\}$. It is evident from this definition that $\mathbf{Q}_k^{(m)} \succeq 0$ and $\mathrm{Tr}(\mathbf{Q}_k^{(m)}) \leq 1$. Physically, $\mathbf{Q}_k^{(m)}$

represents the power allocation among the n_t antennas when transmitting data to subscriber station k over subcarrier m.

2.2.2 Scheduling

For each subcarrier, all users are scheduled to transmit in a time-sharing fashion such that the time-sharing intervals are non-overlapping across users. As a result, orthogonality is guaranteed in frequency and time domains, which prevents interference among users. Let $t_k^{(m)} \in [0\ 1]$ denote the interval for transmitting data to user k over subcarrier m in a unit time frame. For convenience, the collection of all time-sharing intervals is written in a vector form as

$$\mathbf{t} = \left[t_k^{(m)} : k = 1, \ldots, K,\ m = 1, \ldots, M\right]^T.$$

From the time sharing mechanism , it is evident that the entries of $\mathbf{t}$ must satisfy

$$\sum_{k=1}^{K} t_k^{(m)} \leq 1, \qquad \text{for all } m. \tag{2}$$

If user k is scheduled to transmit over subcarrier m, then $t_k^{(m)} > 0$. In this case, let $\mathbf{Q}_k^{(m)}$ be the power allocation for user k over subcarrier m. For convenience, we let $\mathbf{Q} = [\mathbf{Q}_k^{(m)} : k = 1, \ldots, K,\ m = 1, \ldots, M]$ denote the collection of all power allocation variables for all users over all subcarriers.

The ergodic capacity of user k being transmitted over subcarrier m for a time interval $t_k^{(m)}$ can be computed as [8]

$$C_k^{(m)} = E_{\mathbf{H}}\left[t_k^{(m)} \log_2 \left|\mathbf{I} + \rho_k \mathbf{H}_k^{(m)} \mathbf{Q}_k^{(m)} \mathbf{H}_k^{(m)\dagger}\right|\right], \tag{3}$$

where $E_{\mathbf{H}}[\cdot]$ represents the expectation taken over the distribution of $\mathbf{H}$.

2.2.3 Power Constraint

The power consumed by transmitting data to user k over subcarrier m for a time interval $t_k^{(m)}$ is $t_k^{(m)}\mathrm{Tr}(\mathbf{Q}_k^{(m)})$. Since the total power for the transmitter is finite, we have the following average power constraint

$$E_{\mathbf{H}}\left[\sum_{k=1}^{K}\sum_{m=1}^{M} t_k^{(m)}\mathrm{Tr}(\mathbf{Q}_k^{(m)})\right] \leq 1, \tag{4}$$

which represents the fact that the average consumed power cannot be greater than the maximum transmit power of the transmitter.

2.2.4 Minimum Data Rate Constraints

To ensure fairness, let $R_{\min,k}$ be the minimum data rate constraint for user k. Then for user k, we have

$$E_{\mathbf{H}}\left[\sum_{m=1}^{M} C_k^{(m)}\right] = E_{\mathbf{H}}\left[\sum_{m=1}^{M} t_k^{(m)} \log_2\left|\mathbf{I}+\rho_k\mathbf{H}_k^{(m)}\mathbf{Q}_k^{(m)}\mathbf{H}_k^{(m)\dagger}\right|\right] \geq R_{\min,k}. \tag{5}$$

2.3 Problem Formulation

Putting together the power constraint, minimum data rate constraints, time-sharing constraints, and the definitions of $\mathbf{t}$ and $\mathbf{Q}$, the maximum weighted sum rate (MWSR) problem can be formulated as follows:

$$\begin{array}{rll} \text{MWSR: Maximize} & \sum_{k=1}^{K} w_k E_{\mathbf{H}}\left[\sum_{m=1}^{M} t_k^{(m)} \log_2\left|\mathbf{I}+\rho_k\mathbf{H}_k^{(m)}\mathbf{Q}_k^{(m)}\mathbf{H}_k^{(m)\dagger}\right|\right] & \\ \text{subject to} & E_{\mathbf{H}}\left[\sum_{m=1}^{M} t_k^{(m)} \log_2\left|\mathbf{I}+\rho_k\mathbf{H}_k^{(m)}\mathbf{Q}_k^{(m)}\mathbf{H}_k^{(m)\dagger}\right|\right] \geq R_{\min,k} & \forall k \\ & E_{\mathbf{H}}\left[\sum_{k=1}^{K}\sum_{m=1}^{M} t_k^{(m)}\mathrm{Tr}(\mathbf{Q}_k^{(m)})\right] \leq 1 & \\ & \sum_{k=1}^{K} t_k^{(m)} \leq 1 & \forall m \\ & t_k^{(m)} \geq 0,\ \mathbf{Q}_k^{(m)} \geq 0 & \forall k,\ \forall m \end{array}, \tag{6}$$

where w_i, $i = 1, 2, \ldots, K$, are some given weights.

3 An Off-Line Algorithm

In this section, we consider how to solve MWSR under the ideal case where full channel distribution information (CDI) is available. In practice, the knowledge of CDI requires an off-line stage to estimate from the channel stat information (CSI) feedback. Thus, an algorithm based on full CDI is an off-line algorithm. Although the off-line algorithm proposed in this section is not suitable for practical protocol design, it can serve as the baseline that provides insights and an important framework for the online adaptive method that we will design in the next section.

It is readily verifiable that MWSR is a convex optimization problem, which means that it can be solved by standard convex optimization methods, e.g., interior point method [9]. However, even though interior point method is of polynomial-time complexity theoretically, its direct application on MWSR is very cumbersome due to the complex data rate expressions in (6).

Thus, in this paper, we will exploit the special structure of MWSR and design a more efficient algorithm based on Lagrangian dual decomposition. The nice feature of this decomposition approach is that it enables us to derive closed-form solutions for scheduling and power allocation, respectively. As a result, the problem of finding an optimal scheduling and matrix-valued power allocation solution is translated into finding an optimal dual solution, which is in simple scalar form.

Toward this end, associate a dual variable u with the maximum power constraint and associate a dual variable v_k with the k^{th} minimum data rate constraint. For convenience, denote vector $\mathbf{v} \triangleq [v_1\, v_2\, \ldots\, v_K]^T$ the collection of dual variables associated with all minimum data rate constraints. Then, the Lagrangian dual function can be written as

$$\Theta(u, \mathbf{v}) \triangleq \max_{\mathbf{t},\mathbf{Q}} \left\{ L(\mathbf{t}, \mathbf{Q}, u, \mathbf{v}) | (\mathbf{t}, \mathbf{Q}) \in \Psi \right\}, \tag{7}$$

where $\mathbf{t}$ and $\mathbf{Q}$ denotes the collections of all scheduling and power allocation variables, respectively, and the Lagrangian $L(\mathbf{t}, \mathbf{Q}, u, \mathbf{v})$ is written as

$$\begin{aligned} L(\mathbf{t}, \mathbf{Q}, u, \mathbf{v}) \;&=\; \sum_{k=1}^{K} w_k E_{\mathbf{H}} \left[\sum_{m=1}^{M} t_k^{(m)} \log_2 \left| \mathbf{I} + \rho_k \mathbf{H}_k^{(m)} \mathbf{Q}_k^{(m)} \mathbf{H}_k^{(m)\dagger} \right| \right] \\ &+ \sum_{k=1}^{K} v_k \left[E_{\mathbf{H}} \left[\sum_{m=1}^{M} t_k^{(m)} \log_2 \left| \mathbf{I} + \rho_k \mathbf{H}_k^{(m)} \mathbf{Q}_k^{(m)} \mathbf{H}_k^{(m)\dagger} \right| \right] - R_{\min,k} \right] + u \left[1 - E_{\mathbf{H}} \left[\sum_{k=1}^{K} \sum_{m=1}^{M} t_k^{(m)} \mathrm{Tr}(\mathbf{Q}_k^{(m)}) \right] \right] \end{aligned}$$

and Ψ is defined as $\Psi \triangleq \left\{ (\mathbf{t}, \mathbf{Q}) \middle| \sum_{k=1}^{K} t_k^{(m)} \leq 1,\ t_k^{(m)} \geq 0,\ \mathbf{Q}_k^{(m)} \geq 0,\ \forall k,\ \forall m \right\}$. After rearranging terms and interchanging expectation and summations, the Lagrangian can be re-written as

$$L(\mathbf{t}, \mathbf{Q}, u, \mathbf{v}) \;=\; \sum_{k=1}^{K} \sum_{m=1}^{M} E_{\mathbf{H}} \left[t_k^{(m)} F(\mathbf{Q}_k^{(m)}) \right] - \sum_{k=1}^{K} v_k R_{\min,k} + u, \tag{8}$$

where $F(\mathbf{Q}_k^{(m)})$ is defined as

$$F(\mathbf{Q}_k^{(m)}) \triangleq (w_k + v_k) \log_2 \left| \mathbf{I} + \rho_k \mathbf{H}_k^{(m)} \mathbf{Q}_k^{(m)} \mathbf{H}_k^{(m)\dagger} \right| - u \mathrm{Tr}(\mathbf{Q}_k^{(m)}) \tag{9}$$

Then, the Lagrangian dual problem of MWSR can be written as:

$$\mathbf{D}^{\text{MWSR}}: \quad \text{Minimize } \Theta(u, \mathbf{v}) \quad \text{subject to } u \geq 0, \; \mathbf{v} \geq \mathbf{0}. \tag{10}$$

If Slater condition (c.f. [10]) is satisfied, i.e., there exists an interior point in the feasible set Ψ[1], then strong duality holds, which means there is no duality gap between the primal problem in (6) and the dual problem in (10). Therefore, the primal problem in (6) can be equivalently solved by solving its Lagrangian dual problem in (10). Note that for any fixed values of u and $\mathbf{v}$, we need to solve the optimal Lagrangian dual in (7), which itself is a complex nonlinear optimization problem. Toward this end, we first decompose (7) as follows:

$$\Theta(u, \mathbf{v}) = \max_{\mathbf{t}, \mathbf{Q} \in \Psi} L(\mathbf{t}, \mathbf{Q}, u, \mathbf{v}) = \underbrace{\max_{\substack{\sum_{k=1}^{K} t_k^{(m)} \leq 1 \\ t_k^{(m)} \geq 0, \forall k, m}} \left\{ \sum_{k=1}^{K} \sum_{m=1}^{M} \underbrace{\max_{\mathbf{Q}_k^{(m)} \succeq 0} \left\{ E_{\mathbf{H}} \left[t_k^{(m)} F(\mathbf{Q}_k^{(m)}) \right] \right\}}_{\text{Power Allocation Subproblem}} \right\}}_{\text{Scheduling Subproblem}} + u - \sum_{k=1}^{K} v_k R_{\min,k}.$$

As a result, (7) can be solved by first fixing $\mathbf{t}$ to solve the inner power allocation subproblem with respect to $\mathbf{Q}$ and then solving the outer scheduling subproblem with respect to $\mathbf{t}$.

For the inner power allocation subproblem, we have the following proposition and its proof can be found in Appendix A.

Proposition 1. *The optimal solution of the inner maximization problem in (3) is given by*

$$\mathbf{Q}_k^{(m)*} = \mathbf{U}_k^{(m)} \left(\frac{w_k + v_k}{u} \mathbf{I} - (\mathbf{\Lambda}_k^{(m)})^{-1} \right)_+ \mathbf{U}_k^{(m)\dagger}, \tag{11}$$

[1] Slater condition is almost always true in practice.

where $\mathbf{U}_k^{(m)}$ *and* $\mathbf{\Lambda}_k^{(m)}$ *are the unitary matrix and the diagonal matrix achieved by performing eigenvalue decomposition on* $\rho_k \mathbf{H}_k^{(m)\dagger}\mathbf{H}_k^{(m)}$ *such that* $\mathbf{U}_k^{(m)}\mathbf{\Lambda}_k^{(m)}\mathbf{U}_k^{(m)\dagger} = \rho_k \mathbf{H}_k^{(m)\dagger}\mathbf{H}_k^{(m)}$.

In fact, (11) can be interpreted as water-filling over the eigenstructure of channel state $\mathbf{H}_k^{(m)}$ with current dual variables u and $\mathbf{v}$. Next, substituting (11) into (3), the Lagrangian dual function becomes

$$\Theta(u, \mathbf{v}) = \max_{\substack{\sum_{k=1}^K t_k^{(m)} \le 1 \\ t_k^{(m)} \ge 0, \forall k, m}} \left\{ \sum_{k=1}^K \sum_{m=1}^M \left\{ E_{\mathbf{H}} \left[t_k^{(m)} F(\mathbf{Q}_k^{(m)*}) \right] \right\} \right\} + u - \sum_{k=1}^K v_k R_{\min,k}. \tag{12}$$

For the outer scheduling subproblem with respect to $\mathbf{t}$, we have the following proposition and its proof can also be found in Appendix B.

Proposition 2. *The optimal solution of the outer optimization problem in (3) is given by the following scheduling:*

$$t_k^{(m)*} = \begin{cases} 1 & \textit{if } k = k^*, \\ 0 & \textit{otherwise} \end{cases}, \tag{13}$$

where $k^* = \arg\max_k (F(\mathbf{Q}_k^{(m)*}))$.

The scheduling policy in proposition 2 possesses an "opportunistic" interpretation: for current fading realization $\mathbf{H}$, on each subcarrier m, we only schedule the user, denoted as k^*, who has the maximum value of $F(\mathbf{Q}_k^{(m)})$. Thus, $F(\cdot)$ in (9) can be viewed as an indicator function that has the combined effect of both link capacity and power consumption.

So far, we have established that (11) and (13) provide the optimal solution to the Lagrangian dual function for a given dual variable $(u, \mathbf{v})$. From saddle point optimality conditions [10], replacing $(u, \mathbf{v})$ in (11) and (13) with the optimal dual variable $(u^*, \mathbf{v}^*)$ in (10) yields the optimal scheduling and power allocation solution to the original problem in (6). Thus, to solve the original problem, it suffices to determine the optimal Lagrangian dual variables $(u^*, \mathbf{v}^*)$.

It can be observed that the Lagrangian dual problem in (10) is convex but non-differentiable. In general, non-differentiable convex optimization problems can be solved by subgradient algorithm

[10]. For the MWSR problem, the subgradients during the n^{th} iteration can be computed as

$$d_u(n) = 1 - E_{\mathbf{H}}\left[\sum_{k=1}^{K}\sum_{m=1}^{M} t_k^{(m)*}(n)\cdot \mathrm{Tr}(\mathbf{Q}_k^{(m)*}(n))\right], \tag{14}$$

$$d_{v_k}(n) = E_{\mathbf{H}}\left[\sum_{m=1}^{M} t_k^{(m)*}(n)\cdot \log_2\left|\mathbf{I}+\rho_k\mathbf{H}_k^{(m)}\cdot\mathbf{Q}_k^{(m)*}(n)\cdot\mathbf{H}_k^{(m)\dagger}\right|\right] - R_{\min,k}, \tag{15}$$

where $t_k^{(m)*}(n)$ and $\mathbf{Q}_k^{(m)*}(n)$ represent the optimal solutions in (13) and (11) with $(u,\mathbf{v})$ replaced by $(u(n),\mathbf{v}(n))$. The updates of dual variables can be computed as

$$u(n+1) = [u(n) - s_n d_u(n)]_+^U, \quad v_k(n+1) = [v_k(n) - s_n d_{v_k}(n)]_+^V, \quad \forall k, \tag{16}$$

where $[\cdot]_+ = \max(\cdot, 0)$ and s_n is the step size during the n^{th} iteration, U and V are some upper bounds for u and v_k for numerical stability, respectively. It is shown in [10] that if the step size selection satisfies the following three conditions:

$$s_n \to 0, \quad \sum_{n=1}^{\infty} s_n \to \infty, \text{ and } \sum_{n=1}^{\infty} s_n^2 < \infty, \tag{17}$$

then the dual update iterates converge to the optimal solution u^* and $\mathbf{v}^*$. An easy step size selection strategy is the divergent harmonic series: $s_n = \frac{\beta}{n}$, $n = 1, 2, \ldots$, where β is some positive constant. We summarize the subgradient-based off-line algorithm in Algorithm 1.

Algorithm 1 The Subgradient-Based Off-Line Algorithm for Solving MWSR

1. Choose the initial starting points $u(0)$ and $\mathbf{v}(0)$. Compute $\mathbf{Q}^*(1)$ and $\mathbf{t}^*(1)$ according to (11) and (13).
2. In the n^{th} iteration, choose an appropriate step size s_n. Compute the subgradients $d_u(n)$ and $d_{v_k}(n)$ using (14) and (15) with $\mathbf{t}^*(n)$ and $\mathbf{Q}^*(n)$.
3. Update dual variables $u(n)$ and $\mathbf{v}(n)$ using (16) with $d_u(n)$ and $d_{v_k}(n)$.
4. If $|u(n+1) - u(n)| < \epsilon$ and $\|\mathbf{v}_k(n+1) - \mathbf{v}_k(n)\| < \epsilon$, then return $\mathbf{t}^*(n)$ and $\mathbf{Q}^*(n)$ as the final optimal solution and stop. Otherwise, let $n = n+1$ and repeat Step 2.

4 An Online Adaptive Algorithm

In the last section, we designed an off-line algorithm based on subgradient approach to solve MWSR. The limitation of the off-line algorithm is that it requires full CDI before being able to compute the subgradients in (14) and (15). Also, it can be seen from (14) and (15) that the subgradients involves computing expectations over fading distributions, which is computationally expensive. To address this issue, in this section, we use the off-line algorithm as a baseline and design an *online adaptive algorithm* (OAA) by subgradient stochastic approximation.

The basic idea of OAA is that, instead of computing the exact subgradient with full CDI, we use the instantaneous CSI feedback to compute an approximation of the true subgradient. Thus, complete prior knowledge of full CDI is not necessary. We show that the stochastic iterations can adapt to unknown underlying fading distribution and lend itself to an online algorithm, which is a desirable feature in practical protocol design. In this section, we assume that the feedback of CSI is error free. The impact of estimation error in the CSI feedback will be discussed in the next section.

4.1 Subgradient Stochastic Approximation

Similar to the dual iteration of the previous section, each iteration in the stochastic approximation algorithm include the following two phases.

1. We substitute the current dual variables $(u(n), \mathbf{v}(n))$ and the current channel states $\mathbf{H}$ into the power allocation strategy in (11) and the scheduling policy in (13) to obtain $\mathbf{t}^*(n)$ and $\mathbf{Q}^*(n)$.

2. We use $\mathbf{t}^*(n)$ and $\mathbf{Q}^*(n)$ to update the Lagrangian dual variables $(u(n+1), \mathbf{v}(n+1))$ without having to evaluate the expectation over the fading channel distribution.

The way to avoid computing expectations in (14) and (15) is to use the current channel realization in each iteration to compute an approximation of the true subgradient. The stochastic subgradient during the n^{th} iteration can be computed as follows:

$$\hat{d}_u(n) = 1 - \sum_{k=1}^{K} \sum_{m=1}^{M} \hat{t}_k^{(m)*}(n) \cdot \mathrm{Tr}(\hat{\mathbf{Q}}_k^{(m)*}(n)), \tag{18}$$

$$\hat{d}_{v_k}(n) = \sum_{m=1}^{M} t_k^{(m)*}(n) \cdot \log_2 \left| \mathbf{I} + \rho_k \mathbf{H}_k^{(m)} \cdot \hat{\mathbf{Q}}_k^{(m)*}(n) \cdot \mathbf{H}_k^{(m)\dagger} \right| - R_{\min,k}, \tag{19}$$

where $\hat{t}_k^{(m)*}(n)$ and $\hat{\mathbf{Q}}_k^{(m)*}(n)$ denote the suboptimal scheduling and power allocation decisions in the n-th iteration under OAA, respectively. The updates on the Lagrangian dual variables can be computed as

$$\hat{u}(n+1) = \Big[\hat{u}(n) - s_n \hat{d}_u(n)\Big]_+^U, \quad \hat{v}_k(n+1) = \Big[\hat{v}_k(n) - s_n \hat{d}_{v_k}(n)\Big]_+^V, \ \forall k. \tag{20}$$

We summarize OAA in Algorithm 2. Comparing with Algorithm 1, we see that the basic structure of OAA remains the same as that of the off-line algorithm, which means that OAA can also be implemented in a similar fashion. The only difference is that the expectation computation is no longer needed. In the next section, we will prove that the stochastic iterates generated by OAA converges to the optimal solution of (6) with probability one.

Algorithm 2 An Online Algorithm Based on Subgradient Stochastic Approximation

1. Choose the initial starting points $\hat{u}(0)$ and $\hat{\mathbf{v}}(0)$. Compute $\mathbf{Q}^*(1)$ and $\mathbf{t}^*(1)$ according to (11) and (13).
2. In the n^{th} iteration, choose an appropriate step size s_n. Update the subgradients $\hat{d}_u(n)$ and $\hat{d}_{v_k}(n)$ using (18) and (19) with $\mathbf{t}^*(n)$ and $\mathbf{Q}^*(n)$.
3. Update dual variables $\hat{u}(n)$ and $\hat{\mathbf{v}}(n)$ using (16) with $\hat{d}_u(n)$ and $\hat{d}_{v_k}(n)$.
4. If $|\hat{u}(n+1) - \hat{u}(n)| < \epsilon$ and $\|\hat{\mathbf{v}}_k(n+1) - \hat{\mathbf{v}}_k(n)\| < \epsilon$, then return $\mathbf{t}^*(n)$ and $\mathbf{Q}^*(n)$ as the final optimal solution and stop. Otherwise, let $n = n+1$ and repeat Step 2.

4.2 Convergence Theorem

In this section, we show that OAA converges with probability one to the optimal solution of MWSR. The proof of this results hinges on a supermartingale convergence lemma [11], which is restated as follows.

Lemma 1 (Supermartingale Convergence Lemma)**.** *Let $\{X_n\}$ be an $\mathbb{R}^r$-valued stochastic process, and $V(\cdot)$ be a real-valued non-negative function in $\mathbb{R}^r$. Suppose that $\{Y_n\}$ is a sequence of random variables satisfying that $E_n|Y_n| < \infty$ with probability one. Let $\{\mathcal{F}_n\}$ be a sequence of σ-algebra generated by $\{X_i, Y_i, i \le n\}$. Suppose that there exists a compact set $B \subset \mathbb{R}^r$ such that for all n,*

$$E_n[V(X_n)] - V(X_n) \le -s_n\delta + Y_n, \ \textit{for } X_n \notin B,$$

where s_n satisfies $s_n \to 0$, $\sum_{n=1}^{\infty} s_n \to \infty$, $\sum_{n=1}^{\infty} s_n^2 < \infty$, and δ is a positive constant. Then the set B is recurrent for $\{X_n\}$, i.e., $X_n \in B$ for infinitely large n with probability one.

Intuitively, this lemma states that as long as the martingale noise $\{Y_n\}$ is bounded and the step size λ_n is diminishing, then $\{Y_n\}$ cannot drive the iterates $\{X_n\}$ out of B as $n \to \infty$. Now we state the convergence theorem as follows.

Theorem 1. *If the step size selection of s_n satisfies (17), the stochastic iterations in (20) converge to the optimal solution $(u^*, \mathbf{v}^*)$ of (10) with probability one.*

Proof. To prove the convergence theorem, we first examine the structure of the stochastic subgradients during the n-th iteration. It is not difficult to verify that these stochastic subgradients can be decomposed as follows:

$$\begin{aligned} \hat{d}_u(n) &= d_u(n) + \xi_u(n) \\ \hat{d}_{v_k}(n) &= d_{v_k}(n) + \xi_{v_k}(n), \end{aligned}$$

where $\xi_u(n)$ and $\xi_{v_k}(n)$ represents the martingale noise terms, which are defined as

$$\begin{aligned} \xi_u(n) &\triangleq \hat{d}_u(n) - E_{\mathbf{H}}\left[\hat{d}_u(n)\right] \\ &= E_{\mathbf{H}}\left[\hat{t}_k^{(m)*}(n) \cdot \mathrm{Tr}(\hat{\mathbf{Q}}_k^{(m)*}(n))\right] - \hat{t}_k^{(m)*}(n) \cdot \mathrm{Tr}(\hat{\mathbf{Q}}_k^{(m)*}(n)), \end{aligned} \tag{21}$$

$$\begin{aligned} \xi_{v_k}(n) \triangleq \hat{d}_{v_k}(n) - E_{\mathbf{H}}\left[\hat{d}_{v_k}(n)\right] &= \sum_{m=1}^{M} \hat{t}_k^{(m)*}(n) \cdot \log_2 \left|\mathbf{I} + \rho_k \mathbf{H}_k^{(m)} \cdot \hat{\mathbf{Q}}_k^{(m)*}(n) \cdot \mathbf{H}_k^{(m)\dagger}\right| \\ &- E_{\mathbf{H}}\left[\sum_{m=1}^{M} \hat{t}_k^{(m)*}(n) \cdot \log_2 \left|\mathbf{I} + \rho_k \mathbf{H}_k^{(m)} \cdot \hat{\mathbf{Q}}_k^{(m)*}(n) \cdot \mathbf{H}_k^{(m)\dagger}\right|\right]. \end{aligned} \tag{22}$$

Then, the stochastic dual updates can be written as

$$\begin{aligned} \hat{u}(n+1) &= \hat{u}(n) - s_n\left[d_u(n) + \xi_u(n)\right] + z_u(n), \\ \hat{v}_k(n+1) &= \hat{v}_k(n) - s_n\left[d_{v_k}(n) + \xi_{v_k}(n)\right] + z_{v_k}(n), \quad \forall k, \end{aligned}$$

where $z_u(n)$ and $z_{v_k}(n)$ are the correction terms that projects the stochastic subgradients back to the non-negative orthant. It then follows that, for $\hat{u}(n)$, we have

$$|\hat{u}(n+1) - u^*|^2 \le |\hat{u}(n) - u^*|^2 - 2s_n(\hat{u}(n) - u^*)\left[d_u(n) + \xi_u(n)\right] + s_n^2\left[d_u(n) + \xi_u(n)\right]^2,$$

where the inequality holds due to the fact that the correction term $\xi_u(n)$ is non-expansive [12]. Since that $\Theta(u, \mathbf{v})$ is twice-differentiable with respect to u, $F(\mathbf{Q}^{(n)})$ is bounded. Also, it is evident that $|\hat{u}(n) - u^*|$ is bounded and $E_{\mathbf{H}}[\xi_u] = 0$. From the iteration update in (20), it can be concluded that $|\xi_u(n)|$ is bounded. From the iteration update in (16), it can be concluded that $|d_u(n)|$ is bounded. Thus, we have

$$\begin{aligned} E_{\mathbf{H}}\left[|\hat{u}(n+1) - u^*|^2\right] &\leq |\hat{u}^{(n)} - u^*|^2 - 2s_n(\hat{u}(n) - u^*)d_u(n) - 2s_n(\hat{u}(n) - u^*)E_{\mathbf{H}}[\xi_u(n)] + O(s_n^2) \\ &= |\hat{u}^{(n)} - u^*|^2 - 2s_n(\hat{u}(n) - u^*)d_u(n) + O(s_n^2). \end{aligned} \tag{23}$$

Similarly, we can also show that

$$\begin{aligned} E_{\mathbf{H}}\left[\|\hat{\mathbf{v}}(n+1) - \mathbf{v}^*\|^2\right] &\leq \|\hat{\mathbf{v}}^{(n)} - \mathbf{v}^*\|^2 - 2s_n\langle\hat{\mathbf{v}}(n) - \mathbf{v}^*, d_{\mathbf{v}}(n)\rangle - 2s_n\langle\hat{\mathbf{v}}(n) - \mathbf{v}^*, E_{\mathbf{H}}[\xi_{\mathbf{v}}(n)]\rangle \\ &+O(s_n^2) = \|\hat{\mathbf{v}}^{(n)} - \mathbf{v}^*\|^2 - 2s_n(\hat{\mathbf{v}}(n) - \mathbf{v}^*)d_{\mathbf{v}}(n) + O(s_n^2). \end{aligned} \tag{24}$$

Adding (23) and (24), we have

$$\begin{aligned} & E_{\mathbf{H}}\left[|\hat{u}(n+1) - u^*|^2\right] + E_{\mathbf{H}}\left[\|\hat{\mathbf{v}}(n+1) - \mathbf{v}^*\|^2\right] \\ \leq\ & |\hat{u}^{(n)} - u^*|^2 + \|\hat{\mathbf{v}}^{(n)} - \mathbf{v}^*\|^2 - 2s_n\left[(\hat{u}(n) - u^*)d_u(n) + \langle\hat{\mathbf{v}}(n) - \mathbf{v}^*, d_{\mathbf{v}}(n)\rangle\right] + O(s_n^2). \end{aligned} \tag{25}$$

We now define a Lyapunov function as follows:

$$V(\hat{u}, \hat{\mathbf{v}}) \triangleq |\hat{u} - u^*|^2 + \|\hat{\mathbf{v}} - \mathbf{v}^*\|^2, \tag{26}$$

and define a ball centered at $(u^*, \mathbf{v}^*)$ as $\mathbb{B}_\epsilon = \{(\hat{u}, \hat{\mathbf{v}}) : V(\hat{u}, \hat{\mathbf{v}}) \leq \epsilon\}$ for some given $\epsilon > 0$. According to Lemma 1, the proof of recurrence of $\{(\hat{u}(n), \hat{\mathbf{v}}(n))\}$ boils down to showing that

$$(\hat{u}(n) - u^*)d_u(n) + \langle\hat{\mathbf{v}}(n) - \mathbf{v}^*, d_{\mathbf{v}}(n)\rangle \geq 0. \tag{27}$$

The above fact holds because for $(\hat{u}(n), \hat{\mathbf{v}}(n))$, we have

$$\begin{aligned} & (\hat{u}(n) - u^*)d_u(n) + \langle\hat{\mathbf{v}}(n) - \mathbf{v}^*, d_{\mathbf{v}}(n)\rangle \\ \geq\ & [\Theta(\hat{u}(n), \mathbf{v}^*) - \Theta(u^*, \mathbf{v}^*)] + [\Theta(u^*, \hat{\mathbf{v}}(n)) - \Theta(u^*, \mathbf{v}^*)] && (28) \\ \geq\ & 0, && (29) \end{aligned}$$

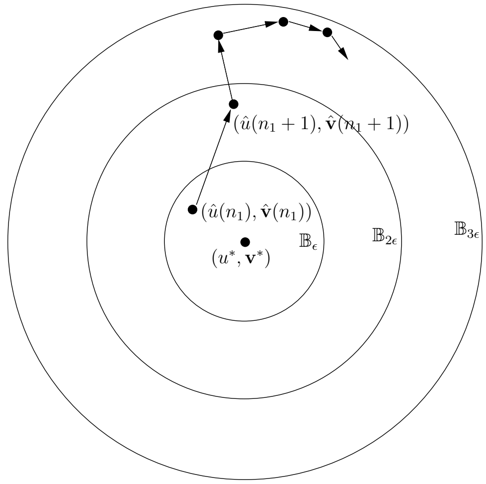


Figure 2: The sketch of the basic idea in the proof of Theorem 1

where the (28) follows from the convexity of $\Theta(u, \mathbf{v})$ and (29) follows from the fact that $(u^*, \mathbf{v}^*)$ is the global minimizer of $\Theta(u, \mathbf{v})$. This shows that there exists $\delta_\epsilon > 0$ such that $(\hat{u}(n) - u^*)d_u(n) + \langle \hat{\mathbf{v}}(n) - \mathbf{v}^*, d_{\mathbf{v}}(n) \rangle > \delta_\epsilon$ when $(\hat{u}, \hat{\mathbf{v}}) \notin \mathbb{B}_\epsilon$. It then follows from Lemma 1 that $\{\hat{u}(n), \hat{\mathbf{v}}(n)\}$ returns to $\mathbb{B}_\epsilon$ infinitely often with probability one, i.e., $\{\hat{u}(n), \hat{\mathbf{v}}(n) : n = 1, 2, \ldots\}$ is recurrent to $\mathbb{B}_\epsilon$.

Having shown the recurrence of $\{(\hat{u}(n), \hat{\mathbf{v}}(n))\}$, our next step to prove the convergence theorem is to show that for any small $\epsilon > 0$, there exists n_1 such that for $n \geq n_1$, even if the martingale noise terms $\xi_u(n_1)$ and $\xi_{\mathbf{v}}(n)$ drive the $(\hat{u}(n+1), \hat{\mathbf{v}}(n))$ away from $\mathbb{B}_\epsilon$, the trajectory of $\{\hat{u}(n), \hat{\mathbf{v}}(n) : n > n_1\}$ is still bounded by the contraction region $\mathbb{B}_{3\epsilon}$ almost surely. We sketch the basic idea of this proof in Figure 2.

Toward this end, we first show that if n_1 is sufficiently large and $(\hat{u}(n_1+1), \hat{\mathbf{v}}(n_1+1))$ moves out of $\mathbb{B}_\epsilon$, then $(\hat{u}(n_1+1), \hat{\mathbf{v}}(n_1+1))$ resides in $\mathbb{B}_{2\epsilon}$ with probability one. This fact holds because by Chebyshev's inequality, we have

$$\Pr\left(s_{n_1}|\xi_u(n_1)| > \frac{\epsilon}{2}\right) \leq \frac{4s_{n_1}^2 E_{\mathbf{H}}[|\xi_u(n_1)|^2]}{\epsilon^2}, \tag{30}$$

$$\Pr\left(s_{n_1}\|\xi_{\mathbf{v}}(n_1)\| > \frac{\epsilon}{2}\right) \leq \frac{4s_{n_1}^2 E_{\mathbf{H}}[\|\xi_{\mathbf{v}}(n_1)\|^2]}{\epsilon^2}. \tag{31}$$

Adding (30) and (31), we have that

$$\Pr\left(s_{n_1}\left[|\xi_u(n_1)| + \|\xi_{\mathbf{v}}(n_1)\|\right] > \epsilon\right) \leq \frac{4s_{n_1}^2 E_{\mathbf{H}}[|\xi_u(n_1)|^2 + \|\xi_{\mathbf{v}}(n_1)\|^2]}{\epsilon^2}. \tag{32}$$

This means that when n_1 is large, the change in one step is almost surely no greater than ϵ. Thus, $(\hat{u}(n_1+1), \hat{\mathbf{v}}(n_1+1))$ resides in $\mathbb{B}_{2\epsilon}$ with probability one.

Next, we show that if $(\hat{u}(n_1+1), \hat{\mathbf{v}}(n_1+1)) \in \mathbb{B}_{2\epsilon} \cap \bar{\mathbb{B}}_\epsilon$, where $\bar{\mathbb{B}}_\epsilon$ denotes the complement of $\mathbb{B}_\epsilon$, then for all $n > n_1 + 1$, $(\hat{u}(n), \hat{\mathbf{v}}(n)) \in \mathbb{B}_{3\epsilon}$ with probability one. That is, the trajectory of the series starting from $(\hat{u}(n_1+1), \hat{\mathbf{v}}(n_1+1))$ resides in $\mathbb{B}_{3\epsilon}$ almost surely. By using the martingale inequality [13, Eq. (1.4)], we have that,

$$\Pr\left\{\sup\left|\sum_{n=n_1+1}^{m} |s_n \xi_u^{(n)}|\right| \geq \frac{\epsilon}{2}\right\} \leq \frac{4\bar{E}_{\mathbf{H}}[\|\xi_u^{(n)}\|^2]}{\epsilon^2} \sum_{n=n_1+1}^{\infty} s_n^2, \quad \text{for } m \geq n_1 + 1,$$

where $\bar{E}_{\mathbf{H}}[|\xi_u(n)|^2] \triangleq \limsup_n E_{\mathbf{H}}[|\xi_u(n)|^2]$. It then follows from the conditions of s_n and the boundedness of $|\xi_u(n)|$ that

$$\lim_{m\to\infty} \Pr\left\{\sup\left|\sum_{n=n_1+1}^{m} s_n |\xi_u(n)|\right| \geq \frac{\epsilon}{2}\right\} = 0, \ \forall \epsilon > 0. \tag{33}$$

Similarly, we can also derive that

$$\lim_{m\to\infty} \Pr\left\{\sup\left|\sum_{n=n_1+1}^{m} s_n \|\xi_{\mathbf{v}}(n)\|\right| \geq \frac{\epsilon}{2}\right\} = 0, \ \forall \epsilon > 0. \tag{34}$$

Combining (33) and (34), we have that

$$\lim_{m\to\infty} \Pr\left\{\sup\left|\sum_{n=n_1+1}^{m} s_n\left[|\xi_u(n)| + \|\xi_{\mathbf{v}}(n)\|\right]\right| \geq \epsilon\right\} = 0, \ \forall \epsilon > 0. \tag{35}$$

That is to say, the accumulated distance deviating from $(\hat{u}(n_1+1), \hat{\mathbf{v}}(n_1+1))$ is almost surely less than ϵ. As a result, the trajectory resides in $\mathbb{B}_{3\epsilon}$ with probability one. Finally, since ϵ can be arbitrary small, it then follows that $\{(\hat{u}(n), \hat{\mathbf{v}}(n)) : n = 1, 2, \ldots\}$ converge to $(u^*, \mathbf{v}^*)$ with probability one. $\square$

5 Impact of Channel Estimation Error on Online Adaptive Algorithm

In the last section, we assumed that CSI feedback, i.e., $\mathbf{H}$, is free of estimation error. In practice, the knowledge of $\mathbf{H}$ is learned by sending a number of training symbols from the base station. Then, the subscriber stations estimate their channel gains and send back their estimations through the uplinks. As a result, the error free assumption may not hold if the training and estimation phase is not long enough and/or the uplinks are noisy. Also, the resolution used in CSI feedback can introduce error. In this section, we will study the impact of the error in CSI feedback on the convergence of OAA.

Denote the channel gain matrices with estimation errors as $\tilde{\mathbf{H}} = [\tilde{\mathbf{H}}_k^{(m)} : k = 1, \ldots, K, m = 1, \ldots, M]$. Let $\tilde{\mathbf{Q}}_k^{(m)}$ and $\tilde{t}_k^{(m)}$ be the resultant power allocation and scheduling decisions from (11) and (13) when $\mathbf{H}_k^{(m)}$ is replaced by $\tilde{\mathbf{H}}_k^{(m)}$. The Lagrangian dual variables are updated as follows:

$$\tilde{u}(n+1) = \left[\tilde{u}(n) - s_n \tilde{d}_u(n)\right]_+, \quad \tilde{v}_k(n+1) = \left[\tilde{v}_k(n) - s_n \tilde{d}_{v_k}(n)\right]_+, \tag{36}$$

where the stochastic subgradients with channel estimation errors can be written as:

$$\tilde{d}_u(n) = 1 - \sum_{k=1}^{K} \sum_{m=1}^{M} \tilde{t}_k^{(m)*}(n) \cdot \mathrm{Tr}(\tilde{\mathbf{Q}}_k^{(m)*}(n)), \tag{37}$$

$$\tilde{d}_{v_k}(n) = \sum_{m=1}^{M} \tilde{t}_k^{(m)*}(n) \cdot \log_2 \left| \mathbf{I} + \rho_k \tilde{\mathbf{H}}_k^{(m)} \cdot \tilde{\mathbf{Q}}_k^{(m)*}(n) \cdot \tilde{\mathbf{H}}_k^{(m)\dagger} \right| - R_{\min,k}. \tag{38}$$

It is readily verifiable that these stochastic subgradients can be decomposed as follows:

$$\tilde{d}_u(n) = d_u(n) + \mu_u(n) + \zeta_u(n) \tag{39}$$

$$\tilde{d}_{v_k}(n) = d_{v_k}(n) + \mu_{v_k}(n) + \zeta_{v_k}(n), \tag{40}$$

where $\mu_u(n)$ and $\mu_{v_k}(n)$ are biased estimation error of the stochastic subgradients, and are defined as

$$\begin{aligned}\mu_u(n) &\triangleq E_{\mathbf{H}}\left[\tilde{d}_u(n)\right] - d_u(n) \\ &= \sum_{k=1}^{K}\sum_{m=1}^{M} E_{\mathbf{H}}\left[\tilde{t}_k^{(m)*}(n)\cdot \mathrm{Tr}(\tilde{\mathbf{Q}}_k^{(m)*}(n)) - t_k^{(m)*}(n)\cdot \mathrm{Tr}(\mathbf{Q}_k^{(m)*}(n))\right], \end{aligned} \tag{41}$$

$$\begin{aligned}\mu_{v_k}(n) &\triangleq E_{\mathbf{H}}\left[\tilde{d}_{v_k}(n)\right] - d_{v_k}(n) = \sum_{m=1}^{M} \mathbb{E}_{\mathbf{H}}\left[\tilde{t}_k^{(m)*}(n)\cdot \log_2\left|\mathbf{I} + \rho_k \mathbf{H}_k^{(m)}\cdot \tilde{\mathbf{Q}}_k^{(m)*}(n)\cdot \mathbf{H}_k^{(m)\dagger}\right|\right. \\ &\left. - t_k^{(m)*}(n)\cdot \log_2\left|\mathbf{I} + \rho_k \mathbf{H}_k^{(m)}\cdot \mathbf{Q}_k^{(m)*}(n)\cdot \mathbf{H}_k^{(m)\dagger}\right|\right], \end{aligned} \tag{42}$$

and $\zeta_u(n)$ and $\zeta_{v_k}(n)$ are martingale noise terms, which are defined as

$$\begin{aligned}\zeta_u(n) &\triangleq \tilde{d}_u(n) - E_{\mathbf{H}}\left[\tilde{d}_u(n)\right] \\ &= E_{\mathbf{H}}\left[\tilde{t}_k^{(m)*}(n)\cdot \mathrm{Tr}(\tilde{\mathbf{Q}}_k^{(m)*}(n))\right] - \tilde{t}_k^{(m)*}(n)\cdot \mathrm{Tr}(\tilde{\mathbf{Q}}_k^{(m)*}(n)), \end{aligned} \tag{43}$$

$$\begin{aligned}\zeta_{v_k}(n) &\triangleq \tilde{d}_{v_k}(n) - E_{\mathbf{H}}\left[\tilde{d}_{v_k}(n)\right] = \sum_{m=1}^{M} \tilde{t}_k^{(m)*}(n)\cdot \log_2\left|\mathbf{I} + \rho_k \mathbf{H}_k^{(m)}\cdot \tilde{\mathbf{Q}}_k^{(m)*}(n)\cdot \mathbf{H}_k^{(m)\dagger}\right| \\ &- E_{\mathbf{H}}\left[\sum_{m=1}^{M} \tilde{t}_k^{(m)*}(n)\cdot \log_2\left|\mathbf{I} + \rho_k \mathbf{H}_k^{(m)}\cdot \tilde{\mathbf{Q}}_k^{(m)*}(n)\cdot \mathbf{H}_k^{(m)\dagger}\right|\right]. \end{aligned} \tag{44}$$

These stochastic subgradient decompositions lead naturally to the following two cases.

5.1 Unbiased Case

In unbiased case, $E_{\mathbf{H}}[\mu_u(n)] = 0$ and $E_{\mathbf{H}}[\mu_{\mathbf{v}}(n)] = 0$ for all n. In this case, we have the following theorem.

Theorem 2. *If $E_{\mathbf{H}}[\mu_u(n)] = 0$ and $E_{\mathbf{H}}[\mu_{\mathbf{v}}(n)] = 0$ for all n and if the step size selection of s_n satisfies (17), the stochastic iterations in (36) converge to the optimal solution $(u^*, \mathbf{v}^*)$ of (10) with probability one.*

Proof. Recall that the stochastic dual updates can be written as

$$\begin{aligned}\tilde{u}(n+1) &= \tilde{u}(n) - s_n\left[d_u(n) + \mu_u(n) + \zeta_u(n)\right] + z_u(n),\\ \tilde{v}_k(n+1) &= \tilde{v}_k(n) - s_n\left[d_{v_k}(n) + \mu_{\mathbf{v}}(n) + \zeta_{v_k}(n)\right] + z_{v_k}(n),\end{aligned}$$

where $z_u(n)$ and $z_{v_k}(n)$ are the correction terms that projects the stochastic subgradients back to the non-negative orthant. For $\tilde{u}(n)$, we have that

$$|\tilde{u}(n+1) - u^*|^2 \leq |\tilde{u}(n) - u^*|^2 - 2s_n(\tilde{u}(n) - u^*)\left[d_u(n) + \mu_u(n) + \zeta_u(n)\right] + s_n^2\left[d_u(n) + \mu_u(n) + \xi_u(n)\right]^2,$$

Applying the similar boundedness arguments in the recurrence proof of Theorem 1 and noting that $E_{\mathbf{H}}\left[\mu_u(n)\right] = 0$ and $E_{\mathbf{H}}\left[\zeta_u(n)\right] = 0$, we have

$$E_{\mathbf{H}}\left[|\tilde{u}(n+1) - u^*|^2\right] \leq |\tilde{u}(n) - u^*|^2 - 2s_n(\tilde{u}(n) - u^*)d_u(n) + O(s_n^2). \tag{45}$$

Along the same line, it can be shown that

$$E_{\mathbf{H}}\left[\|\tilde{\mathbf{v}}(n+1) - \mathbf{v}^*\|^2\right] \leq \|\tilde{\mathbf{v}}(n) - \mathbf{v}^*\|^2 - 2s_n\langle\tilde{\mathbf{v}}(n) - \mathbf{v}^*, d_{\mathbf{v}}(n)\rangle + O(s_n^2). \tag{46}$$

Adding (45) and (46), we have

$$\begin{aligned}& E_{\mathbf{H}}\left[|\tilde{u}(n+1) - u^*|^2 + \|\tilde{\mathbf{v}}(n+1) - \mathbf{v}^*\|^2\right]\\ \leq\ & |\tilde{u}(n) - u^*|^2 + \|\tilde{\mathbf{v}}(n) - \mathbf{v}^*\|^2 - 2s_n\left[(\tilde{u}(n) - u^*)d_u(n) + \langle\tilde{\mathbf{v}}(n) - \mathbf{v}^*, d_{\mathbf{v}}(n)\rangle\right] + O(s_n^2). \end{aligned} \tag{47}$$

Define a Lyapunov function as follows:

$$V(\tilde{u}, \tilde{\mathbf{v}}) \triangleq |\tilde{u} - u^*|^2 + \|\tilde{\mathbf{v}} - \mathbf{v}^*\|^2. \tag{48}$$

The remainder of the recurrence proof follows exactly the same as that in the proof of Theorem 1 and is omitted here for brevity.

Similar to the proof of Theorem 1, having shown the recurrence of $\{(\tilde{u}(n), \tilde{\mathbf{v}}(n))\}$, we need to show that for any small $\epsilon > 0$, there exists a subseries of $(\tilde{u}(n), \tilde{\mathbf{v}}(n))$ that resides in $\mathbb{B}_{3\epsilon}$ with probability one. Note that from the boundedness of $\mu_u(n)$ and $\mu_{\mathbf{v}}(n)$, we have that $\sum_{n=1}^{\infty}|\mu_u(n)| \to 0$ and $\sum_{n=1}^{\infty}\|\mu_{\mathbf{v}}(n)\| \to 0$. Thus, the martingale noise terms $\zeta_u(n)$ and $\zeta_{\mathbf{v}}(n)$ are the only force that drive $(\tilde{u}(n), \tilde{\mathbf{v}}(n))$ away from $(u^*, \mathbf{v}^*)$. Thus, we can again follow exactly the same argument in the proof of Theorem 1 to prove the contraction part. This completes the proof. □

5.2 Biased Case

In biased case, we have $E_{\mathbf{H}}\left[\mu_u(n)\right] \neq 0$ or $E_{\mathbf{H}}\left[\mu_{v_k}(n)\right] \neq 0$ for some k. Let $\bar{\mu}_u \triangleq \limsup_{n\to\infty} E_{\mathbf{H}}\left[\mu_u(n)\right]$ and $\bar{\mu}_{v_k} \triangleq \limsup_{n\to\infty} E_{\mathbf{H}}\left[\mu_{v_k}(n)\right]$, respectively. Define a neighborhood $\mathbb{N}_\phi$ around the optimal solution $(u^*, \mathbf{v}^*)$ as follows:

$$\mathbb{N}_\phi \triangleq \left\{ (u, \mathbf{v}) \,\middle|\, \begin{array}{l} \bar{\mu}_u \geq \phi |d_u(u, \mathbf{v})|, \\ \bar{\mu}_{v_k} \geq \phi |d_{v_k}(u, \mathbf{v})|, \text{ for some } k \\ 0 \leq \phi < 1. \end{array} \right\}. \tag{49}$$

In essence, $\mathbb{N}_\phi$ defines a neighborhood around the optimal point $(u^*, \mathbf{v}^*)$ since by KKT conditions we have $d_u(u^*, \mathbf{v}^*) = 0$ and $d_{v_k}(u^*, \mathbf{v}^*) = 0$ for all k, respectively. Also, $d_u(u, \mathbf{v})$ and $d_{v_k}(u, \mathbf{v})$ are continuous at $(u^*, \mathbf{v}^*)$. (49) requires that point in $\mathbb{N}_\phi$ satisfy $|d_u(u, \mathbf{v})| \leq \frac{\bar{\mu}_u}{\phi}$ and $|d_{v_k}(u, \mathbf{v})| \leq \frac{\bar{\mu}_{v_k}}{\phi}$, which means that $\mathbb{N}_\phi$ is a neighborhood around the optimal point. With these definitions, we now present the theorem for the case where $E_{\mathbf{H}}\left[\mu_u(n)\right] \neq 0$ or $E_{\mathbf{H}}\left[\mu_{v_k}(n)\right] \neq 0$ for some k.

Theorem 3. *If $E_{\mathbf{H}}\left[\mu_u(n)\right] \neq 0$ or $E_{\mathbf{H}}\left[\mu_{v_k}(n)\right] \neq 0$ for some k and if the step size selection satisfies (17), then iterates $\{(\tilde{u}(n), \tilde{\mathbf{v}}(n)) : n = 1, 2, \ldots\}$, generated by (36) returns to $\mathbb{N}_\phi$ with probability one.*

Proof. Define a Lyapunov function as $V(\tilde{u}, \tilde{\mathbf{v}}) \triangleq |\tilde{u} - u^*|^2 + \|\tilde{\mathbf{v}} - \mathbf{v}^*\|^2$. Consider a neighborhood around the optimal solution $(u^*, \mathbf{v}^*)$, denoted by $\mathbb{N}_\phi \cup \mathbb{B}_\epsilon$, where $\mathbb{B}_\epsilon = \{(\tilde{u}, \tilde{\mathbf{v}}) : V(\hat{u}, \hat{\mathbf{v}}) \leq \epsilon\}$. Recall that for $\tilde{u}(n)$, we have that

$$|\tilde{u}(n+1) - u^*|^2 \leq |\tilde{u}(n) - u^*|^2 - 2s_n(\tilde{u}(n) - u^*)\left[d_u(n) + \mu_u(n) + \zeta_u(n)\right] + s_n^2\left[d_u(n) + \mu_u(n) + \xi_u(n)\right]^2,$$

Notice that for $(\hat{u}, \hat{\mathbf{v}}) \notin \mathbb{N}_\phi \cup \mathbb{B}_\epsilon$, we have $\limsup_{n\to\infty} E_{\mathbf{H}}\left[\mu_u(n)\right] < \phi |d_u(u, \mathbf{v})|$. Applying the similar boundedness arguments in the recurrence proof of Theorem 1 and noting that $E_{\mathbf{H}}\left[\zeta_u(n)\right] = 0$, we have

$$E_{\mathbf{H}}\left[|\tilde{u}(n+1) - u^*|^2\right] \leq |\tilde{u}(n) - u^*|^2 - 2s_n(1-\phi)(\tilde{u}(n) - u^*)d_u(n) + O(s_n^2). \tag{50}$$

Along the same line, it can be shown that

$$E_{\mathbf{H}}\left[\|\tilde{\mathbf{v}}(n+1) - \mathbf{v}^*\|^2\right] \leq \|\tilde{\mathbf{v}}(n) - \mathbf{v}^*\|^2 - 2s_n(1-\phi)\langle\tilde{\mathbf{v}}(n) - \mathbf{v}^*, d_{\mathbf{v}}(n)\rangle + O(s_n^2). \tag{51}$$

Adding (50) and (51), we have

$$E_{\mathbf{H}}\left[|\tilde{u}(n+1)-u^*|^2+\|\tilde{\mathbf{v}}(n+1)-\mathbf{v}^*\|^2\right]\leq$$

$$|\tilde{u}(n)-u^*|^2+\|\tilde{\mathbf{v}}(n)-\mathbf{v}^*\|^2-2s_n(1-\phi)\left[(\tilde{u}(n)-u^*)d_u(n)+\langle\tilde{\mathbf{v}}(n)-\mathbf{v}^*,d_{\mathbf{v}}(n)\rangle\right]+O(s_n^2). \quad (52)$$

The recurrence of $\{(\hat{u}(n),\hat{\mathbf{v}}(n)) : n=1,2\ldots\}$ to $\mathbb{N}_\phi\cup\mathbb{B}_\epsilon$ follows exactly the same as that in the proof of Theorem 1. Next, by letting $\epsilon\to 0$, we have that $\{(\hat{u}(n),\hat{\mathbf{v}}(n)) : n=1,2\ldots\}$ is recurrent to $\mathbb{N}_\phi$. □

Theorem 3 shows that there exists a neighborhood around the optimal solution where the iterates generated by OAA return infinitely often with probability one. Thus, it is of interests to study how small such a neighborhood could be. In the following theorem, we characterize the relationship between the size of the neighborhood and the channel estimation errors.

Corollary 4. *The neighborhood around the optimal solution $(u^*,\mathbf{v}^*)$ where OAA returns infinitely often is inner bounded by points $(u,\mathbf{v})$ that satisfy*

$$|d_u(u,\mathbf{v})|=\bar{\mu}_u \text{ and } |d_{v_k}(u,\mathbf{v})|=\bar{\mu}_{v_k} \text{ for all } k. \quad (53)$$

Proof. From Theorem 3, we know that OAA returns with probability one to a neighborhood $\mathbb{N}_\phi$ around the optimal solution. Recall that any point $(\tilde{u},\tilde{\mathbf{v}})\in\mathbb{N}_\phi$ satisfies $\phi|d_u(u,\mathbf{v})|\leq\bar{\mu}_u$. Also by definition, $\bar{\mu}_u$ is the supremum limit of $\mu_u(n)$. Thus, combining these facts, we know that there exists an $\phi\in[0,1)$ such that

$$E_{\mathbf{H}}\left[\mu_u(n)\right]\leq\phi|d_u(u,\mathbf{v})|\leq\bar{\mu}_u, \quad \text{for } (u,\mathbf{v})\in\mathbb{N}_\phi. \quad (54)$$

From (54), it follows that

$$\frac{E_{\mathbf{H}}\left[\mu_u(n)\right]}{|d_u(u,\mathbf{v})|}\leq\phi<1. \quad (55)$$

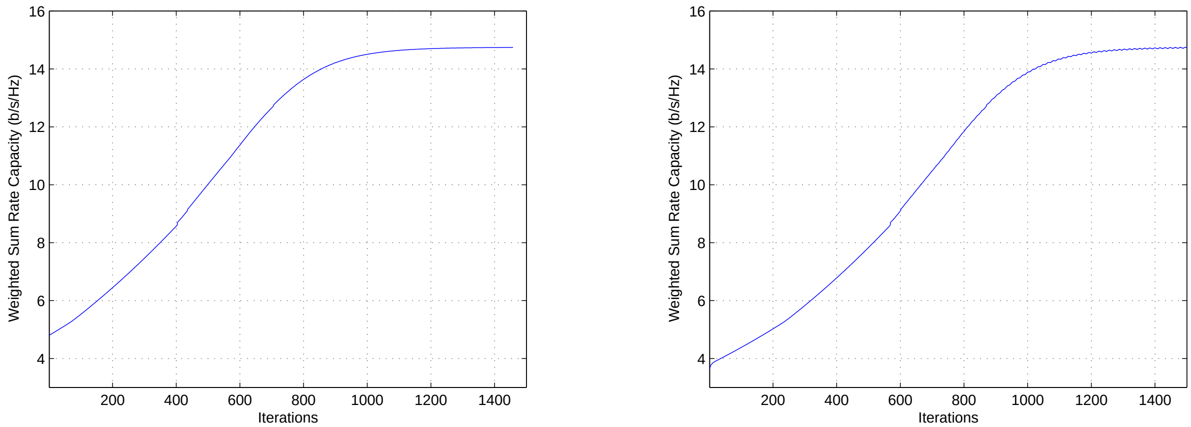
On the other hand, as $n\to\infty$, $|d_u(u,\mathbf{v})|$ decreases because the iterates driven by $d_u(u,\mathbf{v})$ moves toward the optimal point $(u^*,\mathbf{v}^*)$. Thus, as the value of $|d_u(u,\mathbf{v})|$ decreases, ϕ will asymptotically approaches 1. As a result, the iterates generated by OAA are inner bounded by points $(u,\mathbf{v})$ that satisfy $|d_u(u,\mathbf{v})|=\bar{\mu}_u$. Follow the same line, it can also be shown that $|d_{v_k}(u,\mathbf{v})|=\bar{\mu}_{v_k}$. □

6 Numerical Results

In this section, we present some pertinent numerical results to give further insights on our theoretical analysis. In order to show all the details in the cross-layer solution, we use a small 5-user system with 5 subcarriers in the simulation study. Each subcarrier is subject to Rayleigh fading. The mean SNR for these five users are 9.06dB, 7.47dB, 5.53dB, 8.23dB, and 6.78dB, respectively. The base station and subscriber stations are all equipped with 4 antennas. The weights for all five users are all set to 1. Every user has the same minimum rate constraint of 1 b/s/Hz.

We plot the convergence processes of the off-line algorithm and OAA in Figure 3(a) and Figure 3(b), respectively. Figure 3(a) shows that the off-line algorithm takes approximately 1300 iterations to converge to the maximum weighted sum rate, which is 14.745 b/s/Hz for this example. In contrast, it is seen from Figure 3(b) that after approximately 1500 iterations, OAA also converges to the same maximum weighted sum rate. This corroborates our theoretical result in Theorem 1. It is worth pointing out that although the number of iterations of OAA is seemingly larger than that of the off-line algorithm, the running time of OAA is much shorter than that of the off-line algorithm. The reason is because during each iteration, OAA does not evaluate the expectation over the fading distribution, which is computationally expensive. Comparing Figure 3(a) and Figure 3(b), we can also see that, due to using stochastic approximate subgradients, the objective value is not monotonically increasing across iterations. Oscillations can be observed during the convergence process. However, the diminishing step sizes guarantee that the iterates move to a closer distance to the optimal solution after each iteration.

For this example, the optimal data rates for user 1 to user 5 are 3.576, 3.341, 2.447, 2.196, and 3.185, respectively (in b/s/Hz). The optimal dual solution is $u^* = 4.521$ and $\mathbf{v}^* = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$. We have an all-zero solution of $\mathbf{v}$ because the optimal data rates are all greater than the minimum data rate constraint. Since MWSR problem is a convex optimization problem with Slater constraint qualification holds, KKT condition is both necessary and sufficient for optimality. From the complementary slackness property of KKT condition, it is not difficult to see that$\mathbf{v}$ must be all zero at optimality in this case.



(a) Off-line algorithm. (b) OAA.

Figure 3: Convergence behavior comparison between the off-line algorithm and OAA.

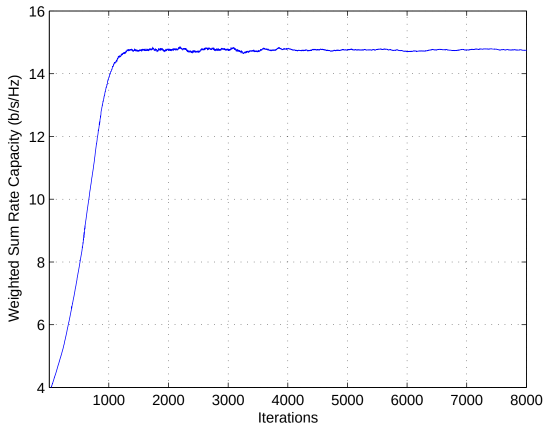
With u^* and $\mathbf{v}^*$, power allocation can be easily computed using (11). For example, suppose that user 1's current channel state over subcarrier 1 is

$$\mathbf{H}_1^{(1)} = \begin{bmatrix} -0.9038 + 0.7245i & 0.0919 - 0.2394i & 0.3681 - 0.4639i & -1.6962 + 0.3020i \\ 0.5010 - 1.7567i & -0.4235 + 0.6095i & -0.1293 + 1.0657i & -0.2720 - 0.9650i \\ -0.6397 - 0.3787i & 0.7217 + 1.1301i & -0.3929 + 0.5060i & 0.0091 - 0.0399i \\ 0.1314 - 0.3127i & -0.6969 + 0.4825i & 0.0652 - 0.3019i & -0.8550 + 0.9631i \end{bmatrix},$$

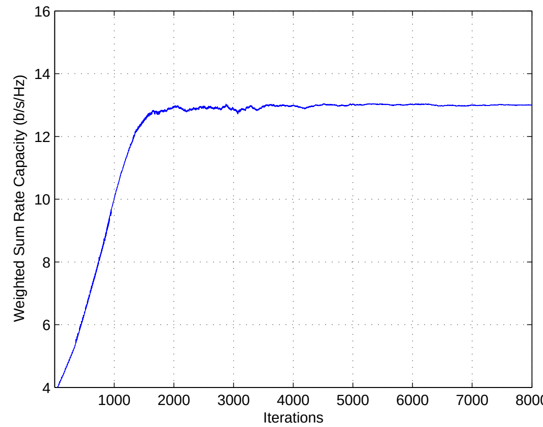
then the optimal power allocation for $\mathbf{H}_1^{(1)}$ is

$$\mathbf{Q}_1^{(1)*} = \begin{bmatrix} 0.1285 & -0.0624 - 0.0328i & -0.0469 + 0.0116i & 0.0322 - 0.0020i \\ -0.0624 + 0.0328i & 0.1154 & 0.0122 + 0.0398i & 0.0369 - 0.0045i \\ -0.0469 - 0.0116i & 0.0122 - 0.0398i & 0.0710 & -0.0320 - 0.0065i \\ 0.0322 + 0.0020i & 0.0369 + 0.0045i & -0.0320 + 0.0065i & 0.1683 \end{bmatrix}.$$

It can be seen that $\mathbf{Q}_1^{(1)*} \succeq 0$ and $\mathrm{Tr}(\mathbf{Q}_1^{(1)*}) = 0.483$. This means that, when user 1 is scheduled to transmit, the optimal power allocation to user 1 for current channel state is approximately one half of the total transmit power and the transmit signal vector $\mathbf{x}$ (corresponding to 4 antennas) should have a covariance equal to $\mathbf{Q}_1^{(1)*}$. As for scheduling, if $\mathbf{Q}_1^{(1)*}$ yields the largest value of $F(\cdot)$ in (9), then the base station will only schedule user 1 to transmit until the channel state varies.



(a) Unbiased case.



(b) Biased case.

Figure 4: OAA with channel estimation error.

Next, we simulate the cases where the CSI is inaccurate and study the impacts of CSI error on OAA's performance. We use the same 5-user network example for comparison. We first simulate the case where the stochastic subgradients are unbiased. In this case, we let $\mu_u(n) \sim \mathcal{N}(0, 0.5)$ and $\mu_{v_k}(n) \sim \mathcal{N}(0, 0.5)$ for all k, where $\mathcal{N}(0, 0.5)$ represents the normal distribution with zero mean and a variance of 0.5. The convergence process of the unbiased case is plot in Figure 4(a). It can be observed that after approximately 7000 iterations, OAA converges to the same maximum weighted sum rate. This corroborates our theoretical analysis in Theorem 2. Next, we simulate the case where the stochastic subgradients are biased. In this case, we assume that $\mu_u(n)$ and $\mu_{v_k}(n)$ are uniformly distributed in [0 0.5]. Figure 4(b) shows the convergence process for this example. It can be seen that, despite the biased estimation error, OAA still converges to a neighborhood of the optimal solution. In this example, OAA converges at 12.954 b/s/Hz. This corroborates our theoretical analysis in Theorem 3.

To see the efficacy of OAA for large-sized networks, we run another example with 50 users. For this example, the running time of the off-line algorithm becomes very long because of the heavy computational load in evaluating expectations over the joint fading distribution of 50 users. In contrast, the running time of OAA is insensitive to the increase of the number of users. In this

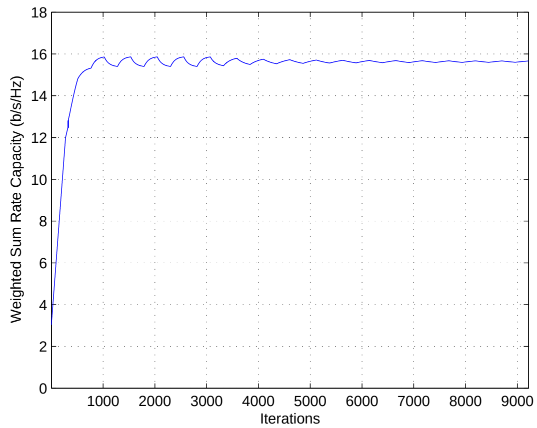


Figure 5: The convergence process of a 50-user example.

example, the mean SNRs for these 50 users are drawn from the uniform distribution on the interval [7 12] (in dB). The base station and subscriber stations are all equipped with 4 antennas. The weights for all users are 1. In this example, every user has the same minimum rate constraint of 0.2 b/s/Hz. For this example, the convergence process of OAA is plotted in Figure 5. It can be seen that OAA takes approximately 9200 iterations to converge, which shows that the efficiency of OAA remains quite satisfactory for large-sized networks.

7 Related Work

In recent years, there has been active research on WiMAX access networks. In particular, after MIMO technology was adopted as an enhancement for WiMAX in IEEE 802.16e specification [2], capacity issues at MIMO-based WiMAX physical layer have received much attention. For example, in [14], Biglieri *et al.* investigated the tradeoffs between spatial diversity, interference cancelation and spatial multiplexing in MIMO-based WiMAX physical layer. In [15], Muquet *et al.* discussed and compared bit error rate performance between Alamouti's space-time block coding [16] and spatial multiplexing [17] through simulations. In [18], Salvekar *et al.* investigated the application of Alamouti's space-time coding scheme and spatial multiplexing in WiMAX systems, and proposed a space-frequency interleaving algorithm for open-loop MIMO in WiMAX . In [19], Mehlführer *et al.*

conducted extensive measurements and evaluated the throughput performance of MIMO WiMAX under various limited feedback settings. However, these works only focused on capacity issues and do not consider resource management such as scheduling and power allocation and their impact on system performance.

Another closely-related line of research is on joint subcarrier scheduling and power allocation for multi-carrier communications systems. Most work in the literature consider single-antenna-based systems. It was shown in [20] that adaptive subcarrier loading and modulation can substantially increase the capacity of a multi-carrier communication system. In [21, 22], Song and Li proposed a framework for subcarrier assignment and power allocation in OFDM wireless networks and designed various optimization algorithms based on this framework. However, their framework did not capture subcarrier scheduling in time domain. In [23] , Liu *et al.* proposed a scheduling algorithm at the MAC layer for multiple connections with different QoS requirements, where each connection employs adaptive modulation and coding schemes at the physical layer over fading channels. The limitation of this work is that the scheduling algorithm assumes no power control and is only valid for block fading channel model. Also, they did not discuss fairness issues and the impact of inaccurate CSI on the performance of their scheduler.

8 Conclusion

In this paper, we investigated the maximum weighted sum rate (MWSR) problem for the downlinks of MIMO-based WiMAX access networks. First, for the ideal case where full channel distribution information (CDI) is available, we designed an off-line algorithm for the MWSR problem to obtain an optimal solution. For the practical case where full CDI is not available, we used the off-line algorithm as a baseline and designed an online adaptive algorithm (OAA) based on subgradient stochastic approximation. For OAA, when channel state information (CSI) is error free, we showed that it is able to converge with probability one to the same optimal solution obtained by the off-line algorithm. In the case when there is estimation error in CSI, we showed that OAA can still converge with probability one to the same optimal solution if the resultant subgradients are unbiased, or to some neighborhood of the optimal solution when the resultant subgradients are biased.

A Proof of Proposition 1

Proof. For convenience, we drop ρ_k since it can be absorbed into $\mathbf{H}_k^{(m)}$. By using $\log_2|\mathbf{I}+\mathbf{AB}| = \log_2|\mathbf{I}+\mathbf{BA}|$, we have $\log_2\left|\mathbf{I}+\mathbf{H}_k^{(m)}\mathbf{Q}_k^{(m)}\mathbf{H}_k^{(m)\dagger}\right| = \log_2\left|\mathbf{I}+(\mathbf{Q}_k^{(m)})^{\frac{1}{2}}(\mathbf{H}_k^{(m)\dagger}\mathbf{H}_k^{(m)})(\mathbf{Q}_k^{(m)})^{\frac{1}{2}}\right|$. Notice that if $(\mathbf{H}_k^{(m)\dagger}\mathbf{H}_k^{(m)})$ is not full rank, we can always construct a full rank matrix by performing eigenvalue decomposition on $(\mathbf{Q}_k^{(m)})^{\frac{1}{2}}(\mathbf{H}_k^{(m)\dagger}\mathbf{H}_k^{(m)})(\mathbf{Q}_k^{(m)})^{\frac{1}{2}}$, discarding all zero eigenvalues and their corresponding eigenvectors, and then multiplying back. Denote this new matrix by $\mathbf{F}$, it can be easily shown that $\log_2\left|\mathbf{I}+(\mathbf{Q}_k^{(m)})^{\frac{1}{2}}(\mathbf{H}_k^{(m)\dagger}\mathbf{H}_k^{(m)})(\mathbf{Q}_k^{(m)})^{\frac{1}{2}}\right| = \log_2|\mathbf{I}+\mathbf{F}|$. Therefore, without loss of generality, we assume that $(\mathbf{H}_k^{(m)\dagger}\mathbf{H}_k^{(m)})$ is full rank.

Construct a Lagrangian function by introducing Lagrangian multiplier $w_k^{(m)}$ associated with constraint $\mathbf{Q}_k^{(m)} \succeq 0$. From KKT optimality conditions, we must have that the derivative of the Lagrangian function with respect to $\mathbf{Q}_k^{(m)}$ is zero and $w_k^{(m)} = 0$. Since $\mathbf{Q}_k^{(m)} = \mathbf{0}$ is obviously not optimal, we must have $w_k^{(m)} = 0$. Therefore, the Lagrangian function would become Θ_{link} itself and we can directly take the derivative of Θ_{link} with respect to $\mathbf{Q}_k^{(m)}$ and set it to zero. By using $\frac{\partial}{\partial\mathbf{X}}\ln\det(\mathbf{A}+\mathbf{BXC}) = [\mathbf{C}(\mathbf{A}+\mathbf{BXC})^{-1}\mathbf{B}]^T$ and $\frac{\partial}{\partial\mathbf{X}}\mathrm{Tr}(X) = \mathbf{I}$ (c.f. [24]), we have

$$
\begin{aligned}
&\frac{\partial}{\partial\mathbf{Q}_k^{(m)}}\left[(w_k+v_k)\log_2\left|\mathbf{I}+\mathbf{H}_k^{(m)\dagger}\mathbf{H}_k^{(m)}\mathbf{Q}_k^{(m)}\right| - u\mathrm{Tr}(\mathbf{Q}_k^{(m)})\right] \\
&= (w_k+v_k)(\mathbf{H}_k^{(m)\dagger}\mathbf{H}_k^{(m)})\left(\mathbf{I}+\mathbf{H}_k^{(m)\dagger}\mathbf{H}_k^{(m)}\mathbf{Q}_k^{(m)}\right)^{-1} - u\mathbf{I} = 0.
\end{aligned}
$$

Solving for $\mathbf{Q}_k^{(m)}$, we have $\mathbf{Q}_k^{(m)*} = \left[\frac{w_k+v_k}{u}\mathbf{I}-(\mathbf{H}_k^{(m)\dagger}\mathbf{H}_k^{(m)})^{-1}\right]_+$, where $[\cdot]_+$ represents the projection onto the positive semidefinite cone. From Moreau decomposition, we know that the positive semidefinite projection can be achieved by performing eigenvalue decomposition on $\frac{w_k+v_k}{u}\mathbf{I}-(\mathbf{H}_k^{(m)\dagger}\mathbf{H}_k^{(m)})^{-1}$, setting all non-positive eigenvalues to zero and keeping all eigenvectors unchanged, and then multiplying back. Thus, letting $(\mathbf{H}_k^{(m)\dagger}\mathbf{H}_k^{(m)}) = \mathbf{U}_k^{(m)}\mathbf{\Lambda}_k^{(m)}\mathbf{U}_k^{(m)\dagger}$ be the eigenvalue decomposition of $(\mathbf{H}_k^{(m)\dagger}\mathbf{H}_k^{(m)})$, where $\mathbf{\Lambda}_k^{(m)}$ is the diagonal matrix containing all eigenvalues and $\mathbf{U}_k^{(m)}$ is the unitary matrix corresponding to all eigenvectors, we have

$$\left[\frac{w_k + v_k}{u}\mathbf{I} - (\mathbf{H}_k^{(m)\dagger}\mathbf{H}_k^{(m)})^{-1}\right]_+ = \mathbf{U}_k^{(m)}\left(\frac{w_k + v_k}{u}\mathbf{I} - (\mathbf{\Lambda}_k^{(m)})^{-1}\right)_+ \mathbf{U}_k^{(m)\dagger}.$$

This completes the proof. $\square$

B Proof of Proposition 2

Proof. Since $\sum_{k=1}^{K} t_k^{(m)} \leq 1\ \forall m$ and $t_k^{(m)} \geq 0$, it follows that

$$\sum_{k=1}^{K}\sum_{m=1}^{M} t_k^{(m)} F(\mathbf{Q}_k^{(m)*}) \leq \sum_{m=1}^{M} F(\mathbf{Q}_k^{(m)*}) \sum_{k=1}^{K} t_k^{(m)} \leq \sum_{m=1}^{M} F(\mathbf{Q}_k^{(m)*}). \tag{56}$$

It is observed that equality in the latter part of (56) holds only when

$$t_k^{(m)*} = \begin{cases} 1 & \text{if } k = k^*, \\ 0 & \text{otherwise} \end{cases},$$

This completes the proof. $\square$

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