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Infinity

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Infinity is a big topic. Most people have some conception of things that have no bound, no boundary, no limit, no end. The rigorous study of infinity began in mathematics and philosophy, but the engagement with infinity traverses the history of cosmology, astronomy, physics, and theology. In the natural and social sciences, the infinite sometimes appears as a consequence of our theories themselves (Barrow 2006, Luminet and Lachièze-Rey 2005) or in the modelling of the relevant phenomena (Fletcher et al. 2019). Mathematics itself has appealed to some form of infinity from its beginning (infinitely many numbers, shapes, iterated addition or division of segments) and its contemporary practice requires infinitary foundations. Any field that employs mathematics at least flirts with infinity indirectly, and in many cases courts it directly.

Philosophy countenances infinity in myriad ways, either directly or indirectly, in most of its sub-fields—here is a tiny sample taken from the contemporary discussion (we shall discuss historical material in Section 1 and in Section 2, and many further examples in later sections). Some metaphysicians contend that there are infinitely many possibilities/possible worlds and canvas how big this infinity is (e.g. Lewis 1986). Philosophers of religion debate whether the divine is infinite, whether the divine creation is infinite, and whether the value of the afterlife is infinite. Epistemologists debate whether there can be an infinite regress of justification, and if so, whether it is problematic (Klein 2000, Peijnenburg 2007, Atkinson and Peijnenburg 2017). Formal epistemologists traffic largely in an infinitary notion of ‘probability’ (more in Section 6). Population ethics for infinite populations is a lively topic, and they are thought to pose distinctive problems for consequentialism (Nelson 1991). Social and political philosophy appeal to the notion of convention, often thought to involve ‘common knowledge’, with a putative infinite hierarchy of mutual knowledge (Lewis 1969). Philosophers of language and mind grapple with problems that infinitary operations such as ‘plus’ create for meaning and rule-following (Kripke 1982), and whether language itself, or minds themselves, can be infinite (Nefdt 2019). Philosophers of mathematics debate whether stipulations that imply the existence of infinitely many objects can be said to be analytic (Boolos 1997, Wright 1999) and whether criteria of identity for infinite numbers must necessarily be Cantorian (Mancosu 2016). See Section 4. Concerns about infinity (and human finitude) appear in continental philosophy, not only in its 19th century historical sources (e.g., among others, Fichte, Schelling, Hegel, Kierkegaard, and Nietzsche) but in contemporary developments as well (e.g., among others, Heidegger 1929, Levinas 1961, Adorno 1966, Foucault 1966, Deleuze 1969, Badiou 2019). This list can be continued, if not *ad infinitum*, then *ad nauseam*.

At this point, one may be tempted to shout three cheers—or perhaps infinitely many of them—for infinity. Indeed, one may get the impression that *we can't live without it*. At the same time, there are various apparent problems with infinity, and it starts to look less congenial. As they pile up, one might get the impression that *we can't live with it*. Infinity, as we shall see, gives rise to numerous paradoxes that have preoccupied philosophers for millennia. Any praise of infinity must be tempered with circumspection and caution.

So we have good reason to want to understand infinity better. Mathematicians and philosophers in particular have done much to enhance our understanding of it. This entry strives to give the reader a sense of some of the main lines of thought regarding infinity.

Our survey begins in section 1, which unpacks some meanings of 'infinite', and traces various philosophical conceptions of infinity from ancient times to the 19th Century. Section 2 turns to the historical development of the mathematics of infinity over a comparable period. This provides background to a presentation in section 3 of modern mathematics' treatment of infinity—some infinite number systems, infinities of measure, of counting, of calculus, and infinitary operations on numbers. This in turn sets the stage for our discussion of mathematical ontology in section 4.

Up to this point, it appears that infinity has been domesticated. This appearance begins to be challenged in section 5, when we canvas some classic paradoxes and puzzles involving infinity. It reappears as both friend and foe in the following sections on some philosophically fecund applications of it. In sections 6 and 7, it is both central to the formulation of probability and decision theory, and the source of more conundrums; we discuss some putative solutions to them. Section

8 presents some problems concerning space and time, as well as some progress that has been made on them—Kant's antinomies, a Zeno-style paradox concerning measure, developments in non-Euclidean geometries and relativistic cosmology, and in determining whether space is finite or infinite. We conclude in section 9, sanguine overall about our relationship with infinity.

Given the magnitude of our topic, we clearly cannot cover all aspects of it, or even a sizable proportion of them. For example, we do not engage much with the many roles infinitude plays in science and the social sciences (except in section 8), retaining our focus on its roles in philosophy. We limit our discussion to what can be understood without highly advanced mathematics, but provide links to a number of supplementary documents that discuss further issues: infinite idealizations, quadratures of the circle, overviews of two recent developments in mathematics that promise to make the infinite realm more tractable (numerosity theories and surreal numbers), further paradoxes (God's lottery, two envelopes), and proofs of theorems. We ask for the forbearance of readers whose favorite topics have been left out. We hope to mitigate this somewhat with our large set of pointers to further topics, the references in our extensive bibliography, and other internet resources.

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1. Infinity in philosophy: some historical remarks

In Greek, 'to apeiron' means 'the infinite': 'a' denotes privation and 'peras' the notion of 'limit' or 'bound'. Etymologically, the English word 'infinite' comes from the Latin word 'infinitas': 'in' = 'not' and 'finis' = 'end', 'boundary', 'limit', 'termination', or 'determining factor'. In contemporary English, there is a range of uses of the word 'infinite':

1. In a loose or hyperbolic sense, 'infinite' means 'indefinitely or exceedingly great', 'exceeding measurement or calculation', 'immense', or 'vast'.
2. In a strict but non-mathematical sense that reflects its etymological history, 'infinite' means 'having no limit or end', 'boundless', 'unlimited', 'endless', 'immeasurably great in extent (or duration, or some other respect)'. This strict, non-mathematical sense is often applied to God and divine attributes, and to space, time and the universe.
3. There is also a strict, mathematical sense, according to which 'infinite' quantities or magnitudes are those that are measurable but that have no finite measure; and 'infinite' lines or surfaces or volumes are measurable lines or surfaces or volumes that have no finite measure.

Related to the distinction between meanings (2) and (3) is a distinction between metaphysical and mathematical meanings of infinity. This has been usefully employed in some of the most encompassing accounts of infinity, such as Moore (1990/2019; for another recent treatment that includes extensive discussion of the history of infinity see Zellini (2005)). Moore sees the metaphysical notion as bound up with the notions of 'totality', 'absoluteness' and 'perfection'. While our entry is focused on the strict mathematical sense of 'infinity', one cannot cleanly separate the various meanings in the historical development of the subject, especially in the first stages. In addition, treating infinity as a 'perfection' in theology from the outset does not mirror the complexity of the historical development; for instance, we find traces in the 13th century of thinkers who attributed finiteness to God or in any case denied God's infinity even when not explicitly stating the finiteness of God (see Coté 2002, 127–144).

The infinite has been of central concern to Western thought since the very first pre-Socratic fragment. It concerned the philosopher Anaximander (who flourished in the 6th century BCE), who identified the principle and origin of existing things as *to apeiron*. In Anaximander, the principle has both an ontological and an ethical significance. The Pythagoreans (6th century BCE) saw the infinite negatively and emphasized the lack of definiteness associated with it; they also gave it spatial connotations. Indeed, in the 5th century BCE the Pythagorean Archytas of Tarentum (see Huffman 2005, 540–550) gave the following argument for the spatial infinitude of the cosmos based on the contradiction that postulating a boundary to it would seem to entail. If the cosmos is bounded, then one could extend one's hand or a stick beyond its boundary to find either empty space or matter. And this would be part of the world, which thus cannot be bounded on pain of contradiction. So the world is *unbounded*. Archytas identified this with the world being *infinite*. Kant similarly identified the unbounded and the infinite in his cosmological antinomy. In Section 8 we will see that these notions should be distinguished, but a mathematically precise articulation of the distinction had to wait until the development of new conceptions of space in the 19th century.

The Eleatics (Parmenides and Melissus, 5th century BCE) held a monist conception of reality, the One, and Melissus declared it to be infinite. Such a monistic conception of reality sees change (or becoming) as appearance, and Zeno's famous paradoxes of infinity (see the entry on Zeno's paradoxes) emerge in this context. Suffice here to say that Zeno's paradoxes (the dichotomy, the Achilles, the arrow, and others) involved the infinitely small and were aimed at buttressing Parmenides' monism. Working across the 5th and the 4th century BCE, Democritus defended an atomistic theory with an infinite void and infinitely many atoms. The infinite by this time had shown some of its major aspects, taken as substance by some and as plurality (of atoms, times, worlds, geometrical points, etc.) by others.

If the urgency of problems related to the infinite reached Greek consciousness with Zeno's paradoxes, the most influential discussion was due to Aristotle. In order to put Aristotle's discussion in perspective, we need to list a number of ways in which mathematical infinity had emerged not only in philosophy, as we have described, but also in mathematics. We have already seen with Archytas the notion of spatial infinitude of the cosmos. But in number theory, the natural numbers were considered infinite, at least in the sense that given any natural number a greater one could be found. In geometry, we find both the infinite by addition (any segment can be extended) and by division (any segment can be halved). Thus, mathematics presented processes of iteration without limit. The most sophisticated technique for dealing with iterated processes in the measurements of plane and solid figures was developed by Eudoxus (4th century BCE), and we discuss it in Section 2.1.

By the time Aristotle (4th century BCE) developed his discussion of the infinite, this concept had thus made its presence felt in philosophy, mathematics, and natural philosophy (including cosmology, astronomy, and physics). It would be hard to exaggerate the role played by Aristotle in the history of infinity. He articulated some essential conceptual distinctions that were to influence all subsequent discussions. He was a finitist in the sense that in his universe, everything is finite. The cosmos is finite, bodies are finite, geometrical segments are finite, each number is finite, etc. However, Rosen (2022) argues hat Aristotle can in principle allow actual infinite multiplicities. In any case, there are processes that can be iterated indefinitely, giving rise to what he called 'potential infinity'. He claimed in fact that "in a sense [the infinite] is and in a sense it is not." (*Phys.* 3.6, 206a13–14).

Any arbitrary segment can be extended in length (subject to cosmological restrictions mentioned below) or halved without limit, but at each stage we remain within the finite. Time is also potentially infinite in both directions and can be divided without limit but it is not a whole.

This conception stands in opposition to that of 'actual infinity', which would result if some infinite processes could be completed, carried out 'all at once', as it were. If actual infinity were real, then one could have infinitely long bodies, infinitely long or infinitely small segments, the totality of natural numbers, an infinite number, infinitely many instants of time, etc. Aristotle rejected the notion of the infinite as a primordial substance, as we have encountered in Anaximander, and most of his discussion of the infinite takes place within a physical context, namely one relating to spatio-temporal features of reality. As a consequence, Aristotle's discussion of the infinite fell squarely in what we have characterized as the 'mathematical' notion of infinity, where infinity applies first of all to magnitudes (continuous or discrete) and what is quantifiable (time, extension, numbers etc.). His *Physics* discusses the infinitely large, excluded because the world is finite; and the infinitely small, excluded because the division of matter can only be potentially infinite and thus finite at each stage, never reaching an infinitesimal quantity—one that is less than any finite quantity, while being something. The exclusion of the infinitely large also has as a consequence that Aristotle cannot allow a potential infinity by addition in an unqualified manner (for otherwise any finite extension could be added to itself sufficiently many times to become larger than the size of the world). Infinity by addition, then, is to be conceptualized as a sort of inverse operation to infinity by division which gives us the primary evidence for the existence of the potential infinite. This is the implicit force of the contrastive "but" in the following quote. Aristotle writes (our emphasis):

> 'To be', then, may mean 'to be **potentially'** or 'to be **actually'**; and the infinite is either in **addition** or in **division**. It has been stated that magnitude is not in actual operation infinite; but it is infinite in division – it is not hard to refute indivisible lines – so that it remains for the infinite to be potentially. (*Physics* 3.6, 206a14–24)

The Aristotelian distinction between potential and actual infinity has had a major influence up to contemporary times. (For further discussion of Aristotle on infinity see Hintikka (1966), Lear (1980), Kouremenos (1995), Coope (2012), Nawar (2015), Cooper (2016), Ugaglia (2018), Rosen (2022), and Hussey's commentary to Aristotle (1983).)

Aristotle's conception had, in addition to issues related to the constitution of the physical continuum, important consequences in cosmology. While he considered the cosmos to be finite, he thought that the movement of the celestial spheres had no beginning and no end. The issue of the "eternity of the world" was to exercise some of the best theological and philosophical minds after Aristotle, especially in connection to theological issues. For instance, Johannes Philoponus (6th century CE; see Philoponus 2004) argued in favor of a beginning of the world by claiming that the contrary thesis would lead to a paradox of infinity (we discuss this in Section 2.4).

Philoponus presented another paradox of infinity concerning infinite time that we will discuss in the version formulated by al-Ghazālī (11th century CE)—see the

Supplement on al-Ghazālī's objection

Of even more pressing significance was the abandonment of Aristotle's view on the finiteness of the cosmos and the Renaissance move from the finite to the infinite universe described in the classic text by Koyré (1957; see also Jammer 1993). While Copernicus (1473–1543) put the sun at the center of the universe, he still worked with a finite model of the universe. Foreshadowed by Epicurus (341–270), Hasdai Crescas (1340–1412), and Nicolaus Cusanus (1401–1464), Giordano Bruno (1548–1600) defended the idea of infinitely many worlds, each of infinite size, existing simultaneously. Bruno is a good example of how mathematical and theological notions of infinity were used simultaneously in the history of the concept. For instance, in *On the Infinite, the Universe, and Worlds* (1584) he argued from God's infinite power to the infinitude of the universe.

By contrast, Kepler and Galileo did not think that the issue of whether the world was infinite in size could be settled either way. Kepler thought that the notion of an infinite universe was a metaphysical one and not founded on empirical evidence. Galileo claimed, in a famous letter to Francesco Ingoli written in 1624, that mankind would never be able to know whether the universe is finite or infinite. The progressive geometrization of space (see De Risi 2015) led to Newton's gravitational theory in which the universe is infinitely extended spatially and temporally. Physical space became identified with the space of Euclid's geometry and in this way physical space was geometrized.

Theological elements were still present when Newton identified space with the "sensorium Dei" ("God's sensorium"). For the next two centuries cosmology was developed according to Newtonian theory: an infinite Euclidean space, flat and absolute, which provides the receptacle for all physical objects whose relations are structured by universal gravitation.

With Riemann in the mid-19th Century, and then with relativistic cosmology, one went back to a finite universe, but cosmologists are now fully aware that the issue of the finitude of the world is very much an open question that depends crucially on the curvature and the topology of space (see Section 8.2).

Our discussion above indicates a few essential aspects of the concept of infinity that will be useful in the later discussion. There are obviously many areas of contact and/or intersection between the more mathematical notion of infinity and the qualitative notion of infinity. Qualitative notions of infinity cannot be easily characterized directly but in general they appeal to features that do not seem to have a clear quantitative aspect. For instance, God might be defined as infinite because it has none of the limitations of finite creatures; this property was accounted for in some Scholastic philosophy by claiming that God, unlike finite creatures, is that unique entity in which essence and existence coincide. Often coupled with this was the claim that God's infinity is incomprehensible, and this might be a good indicator that we cannot achieve a positive account of qualitative infinity. At the same time, claims concerning infinite divine power or goodness offer a possible connection to quantitative conceptions, and this explains why the boundary line between quantitative and qualitative conceptions is not so sharp.

Indeed, according to some authors the qualitative and mathematical conceptions are inextricably tied. Consider for instance Pascal's use of infinite distance both in projective geometry and in his *Pensées* where he muses on the infinite distance (and disproportion) between finite human beings and the infinite God (see Cortese 2015 and 2023). The following passage is representative of the powerful and suggestive role that appeal to finiteness and infinity plays in Pascal's apologetics:

> For in the end what is humanity [l'homme] in nature? A nothingness compared to the infinite, everything [un tout] compared to a nothingness, a mid-point between nothing and everything, infinitely far from understanding the extremes; the end of things and their beginning [principe] are insuperably hidden for him in an impenetrable secret. ⟨What can he therefore imagine? He is⟩ equally incapable of seeing the nothingness from where he came, and the infinite in which he is covered [englouti]. […] (Pascal 2008: 70; we have added the French original where the translation seems less than faithful).

Moreover, Pascal's *pari* (wager) is also intimately tied to the notion of infinity in the form of an infinite reward. (See Section 7.3 on Pascal's wager) These topics are of great importance for philosophy of religion, decision theory, and philosophical anthropology.

However, this entry does not concern those conceptions of infinity that are connected to infinite divine power, infinite modes, and in general about those conceptions of infinity that are not of a mathematical kind. We do not intend to downplay the importance of those aspects of the history of infinity to which giants such as Plotinus, Cusanus, Descartes, Pascal, Spinoza, Fichte, Hegel, and Kierkegaard contributed, among others. Leibniz and Kant also belong to that list, but we will say more about them later on. But our entry would lose focus if we were to try to pursue all these developments even at a superficial level, and the treatment of qualitative infinity is worthy of an article in its own right. Thus, we content ourselves with a list of bibliographical references through which the reader can reconstruct the contributions to the topic.

For overviews of the history of infinity which include both mathematical and metaphysical aspects, see Moore (1990/2019) and Zellini (2005). For further discussion of Aristotle's views on infinity see the entries on: Aristotle; Aristotle and mathematics; and Aristotle and metaphysics. For ancient and medieval conceptions of infinity see Sweeney (1972), Sweeney (1992), Kretzmann (1982), Coté (2002), Biard and Celeyrette (2005), Duhem (1987), Dewender (2002), Davenport (1999), Murdoch (1982), Uckelman (2015), Zarepour (2025); for the early modern period see Nachtomy and Winegar (2018); for infinity in Kant and the idealist period see Kreis (2015); Monnoyeur (1992) spans all periods.

For more on infinity in philosophy of religion, see the following references.

1. on divine infinity: Koetsier and Bergmans (2005), Göcke and Tapp (2018), the papers in the final section of Heller and Woodin (2011), and various entries including God and other ultimates, ontological arguments, Nicolaus Cusanus, Robert Grosseteste, John Duns Scotus, and Ibn Arabi;
2. on infinity in God's creation, apart from our subsequent discussion of whether space and time are infinite: the entries cosmology and theology, cosmological argument, fine-tuning, infinite regress arguments, principle of sufficient reason, and being and becoming in modern physics; and
3. on 'heavenly infinity', apart from our subsequent discussion of Pascal's Wager: the entries on Pascal's wager, the meaning of life, and religion and morality.

It is worth noting that Cantor's development of set theory was influenced by theological considerations: see, for example, Dauben (1990) and Tapp (2005).

As we have said, we are mostly excluding the topic of infinity in science and the social sciences from our purview, although see the

Most working mathematicians don't worry about the existence of infinitely large sets and other objects. There are some other ontological worries about particular infinite sets, related to the Axiom of Choice, and some of the larger cardinalities mentioned above in the section on Cantor. But bigger worries arise in the context of whether there can be physical infinities.

For collections of sources on the classical foundational positions (finitism, intuitionism, predicativism) see van Heijenoort (1967), Ewald (1996), and Mancosu (1998). On finitism and intuitionism see the entries Hilbert's program and intuitionism in mathematics. On predicativity see Feferman (2005). On Paris-Harrington see Katz and Reimann 2018; on Goodstein's theorem see the friendly presentation in Stillwell (2010). Stillwell (2010) also has a chapter on large cardinals; for recent directions see Woodin (2011) and Steel (2015). On the interplay between finite and infinite in recursion theory see Hirschfeldt (2015).

5. Paradise lost? Paradoxes and puzzles involving infinity

The latter part of this entry will explore selected applications of mathematical concepts of infinity in theories of probability, decision, and spacetime, and some associated paradoxes. Before we turn to those theories, we warm up with some paradoxes and puzzles that link mathematics, metaphysical possibility, and physical possibility. There are many different paradoxes and puzzles that we might have included in this section. We consider a small sample of paradoxes and puzzles that some—e.g. Pruss (2018a)—have thought might motivate a return to Aristotle's views on the impossibility of actual infinites.

In the

Supplement on al-Ghazālī's objection,

we discuss a puzzle due to al-Ghazālī that is of historical interest. For more see, for example, Rucker (1982), Moore (1990/2019), Oppy (2006), and Huemer (2016).

5.1 Hilbert's Hotel

Hilbert's Hotel has infinitely many rooms, labelled $1, 2, 3, \ldots$, each of which is currently occupied by a guest. Despite the fact that the hotel is already full, a new guest who turns up at reception is readily accommodated: for each n, the guest in room n is moved to room $n + 1$, and the new guest is installed in room 1. Indeed, despite the fact that the hotel is already full, it can accommodate infinitely many new guests: for each n, the guest in room n is moved to room $2n$, and the new guests are installed in the odd-numbered rooms. Of course, if the infinitely many people in the odd-numbered rooms check out, there are infinitely many people left in Hilbert's Hotel; but if infinitely many people check out from all but the first three rooms, only three people remain.

Some philosophers have thought that Hilbert's Hotel supports an argument against the possibility of physically realized infinities:

1. If there could be physically realized infinities, then there could be a hotel with infinitely many rooms.
2. But if there could be a hotel with infinitely many rooms, then the events described in the preceding paragraph could occur.
3. But it is absurd to suppose that the events described in the preceding paragraph could occur.

So there cannot be physically realized infinities. (See, e.g., Craig (1979).)

This argument faces various challenges, depending on one's views about what physical possibility amounts to. The first premise may be challenged: perhaps some kinds of physical infinities can be realized even though other kinds of physical infinities cannot: for example, perhaps there can be infinitely many stars even though there cannot be a hotel with infinitely many rooms. The second premise may also be challenged: even if there could be a hotel with infinitely many rooms, perhaps the events described in the story could not occur—the story was told at a high level of abstraction, and the details may matter. And the third premise is also questionable: it is not clearly absurd to suppose that there could be an infinite hotel in which guests come and go in the manner described.

The first public discussion of Hilbert's Hotel is Gamow (1946), who cites earlier unpublished lectures by Hilbert and Courant. Gamow's discussion of the aleph numbers (see Section 3.3) erroneously assumes that the Continuum Hypothesis has been established. For further discussion of Hilbert's Hotel, see Huby (1971), Rucker (1982), Moore (1990/2019), Oppy (2006), Kragh (2014), Huemer (2016), and the entries on supertasks and cosmology and theology.