More on Truth Tables

So, as we saw in today's class, our use of truth tables can help us understand the ways in which wffs can, as a whole, be either true or false. Certain wffs are always assessed as true regardless of what truth value their sentence letter's truth values are; certain wffs are always assessed as false regardless of what truth value their sentence letter's truth values are; and certain wffs change truth value depending on the truth value of their sentence letter's truth values.

Tautologous: A wff is tautologous iff the dominant operator is assigned a T under every truth value assignment of the wff's components. (example to the right)

[(G	⊃	H)	•	G]	⊃	H
T	T	T	T	T	T	T
T	F	F	F	T	T	F
F	T	T	F	F	T	T
F	T	F	F	F	T	F

Self-Contradictory: A wff is self-contradictory iff the dominant operator is assigned an F under every truth value assignment of the wff's components. (example to the right)

(G	∨	H)	≡	(~	G	•	~	H)
T	T	T	F	F	T	F	F	T
T	T	F	F	F	T	F	T	F
F	T	T	F	T	F	F	F	T
F	F	F	F	T	F	T	T	F

Contingent: A wff is contingent iff the dominant operator is assigned an F under some truth value assignments of the wff's components, and assigned a T under other truth value assignments of the wff's components. (example to the right

A	B	[(A ∨ B)	•	(B ⊃ A)]	⊃ B
T	T	T	T	T	T
T	F	T	T	T	F
F	T	T	F	F	T
F	F	F	F	T	T

Additionally, some wffs share relationships to one another which is expressible using the truth tables.

Logically Equivalent: Two wffs are logically equivalent iff their truth tables have the same truth value assignments for the dominant operators.

K	⊃	L
T	T	T
T	F	F
F	T	T
F	T	F

~	L	⊃	~	K
F	T	T	F	T
T	F	F	F	T
F	T	T	T	F
T	F	T	T	F

Contradictory: Two wffs are contradictory iff their truth tables have opposite truth value assignments for the dominant operators.	K ⊃ L T T T T F F F T T F T F	K • ~ L T F F T T T T F F F F T F F T F
Consistent: Two wffs are consistent iff there is at least one line in the truth table on which the truth value assignment of the dominant operator's are both true.	K ∨ L T T T T T F F T T F F F	K • L T T T T F F F F T F F F
Inconsistent: Two wffs are inconsistent iff there is no line in the truth table on which the truth value assignments of the dominant operator's are both true.	K ≡ L T T T T F F F F T F T F	K • ~ L T F F T T T T F F F F T F F T F

Now that we have a more robust understanding of the truth tables and some relations that are expressible between wffs by using those tables, we can begin to test wffs for validity. There are two methods for doing this. The first is the direct truth table.

The direct truth table is exactly what we've been doing so far. A fully exhaustive and filled out truth table for each premise, and the conclusion is the direct method. Hurley's way to organize the table is stated on pages 351 - 352. We simply put each sentence letter to the left, create our list of all possible truth value combinations using our 2^n formula, and then insert each line's value into the premises/conclusion portions of the table. The resulting table expresses every way the dominant operators of the premises, and conclusions can be assigned a T or an F. This tells us what it would take for the arguments to be valid.

Remembering that our definition of a valid argument is one where *if* the premises were true, *then* the conclusion would have to be true, we focus only on the lines where the dominant operator of the premises are **all** assigned a T. Then, we look at the truth value assignment of those lines for the conclusion. If they are all T, the argument is valid. If even **one** of them is assigned an F, then the argument is invalid. **This testing method must be clear** (i.e. you can expect it showing up on the problem set). Let's put this testing method to work by looking at the following argument:

J	⊃	E	/	~	J	//	~	E
T	T	T		F	T		F	T
T	F	F		F	T		T	F
F	T	T		T	F		F	T
F	T	F		T	F		T	F

After filling in the entire truth table with the computed values of the dominant operators, we isolate only the lines which assign both premises a T. There is only one line like this in our table (line 3). Then, we look at the conclusion of that line and see that it is assigned an F. We now know that this argument can have the premises be true without the conclusion being true. This violates our definition of validity, and so this argument is invalid. If the conclusion was assigned a T, then it would be valid. The next argument is valid:

O	⊃	~	T	/	~	T	⊃	B	//	O	⊃	B
T	F	F	T		F	T	T	T		T	T	T
T	F	F	T		F	T	T	F		T	F	F
T	T	T	F		T	F	T	T		T	T	T
T	T	T	F		T	F	F	F		T	F	F
F	T	F	T		F	T	T	T		F	T	T
F	T	F	T		F	T	T	F		F	T	F
F	T	T	F		T	F	T	T		F	T	T
F	T	T	F		T	F	F	F		F	T	F

The other method when using truth tables is called the indirect method. The indirect method is so named because it does not require you to fully compute every truth value in the table. Rather, you isolate those parts of the table that are relevant to what you are trying to prove. For example, and as should be obvious by now, not every line on the truth table matters when testing validity. Only those lines where the premises' dominant operator are assigned a T matter. So, if we just fill in those lines, we would save quite a bit of time and still get the right answer!

We can use this indirect method to also test for other conditions obtain between wffs. We can test whether two or more wffs are consistent and inconsistent using the indirect method. We will go over this in the next class, though you are welcome to attempt to do so now!