them. Can you think of a way to grasp the dilemma by the horns? Here is a counterdilemma:

> If we prosecute suspected terrorists, then we discourage terrorism; but if we release them, then we avoid the risk of retaliation by other terrorists. We must either prosecute or release suspected terrorists. Therefore, either we will discourage terrorism or we will avoid the risk of retaliation by other terrorists.

V.

1. If human organs are given first to registered donors, then more people will register as donors. If more people register as donors, then the supply of organs will increase. Therefore, if human organs are given first to registered donors, then the supply of organs will increase. (HS)
4. If group problem solving is important, then we should not emphasize individual testing. Group problem solving is important. Therefore, we should not emphasize individual testing. (MP)

   If we should not emphasize individual testing, then the national math test is a mistake. We should not emphasize individual testing. Therefore, the national math test is a mistake. (MP)
7. If we close the library at Central Juvenile Hall, then delinquents will be deprived of an opportunity to read. If delinquents are deprived of an opportunity to read, then they will not have access to ideas, dreams, and alternative ways of living. Therefore, if we close the library at Central Juvenile Hall, then delinquents will not have access to ideas, dreams, and alternative ways of living. (HS)

   If we close the library at Central Juvenile Hall, then delinquents will not have access to ideas, dreams, and alternative ways of living. Delinquents must have access to ideas, dreams, and alternative ways of living. Therefore, we must not close the library at Central Juvenile Hall. (MT)
10. If viewing adult videos led to violent sex crimes, then there would be over a million violent sex crimes per week. It is not the case that there are over a million violent sex crimes per week. Therefore, viewing adult videos does not lead to violent sex crimes. (MT)

Exercise 7.1

I.

1. $\sim G$	1, 2, MT
4. C	1, 2, DS
7. $F \supset D$	1, 3, HS
10. $G \supset A$	1, 4, HS
13. $\sim\sim C$	1, 3, MT
16. $\sim P$	1, 2, MP
19. $\sim(S \vee C)$	1, 3, MT

II.

1. $\sim B$	1, 2, DS
4. $R \supset C$	1, 2, HS
7. Q	2, 3, MP
10. $\sim A$	1, 4, MT
13. $\sim\sim S$	3, 4, MT
16. $\sim Z$	3, 4, MP
19. $H \vee G$	2, 4, MP

III.

(1)	1.	$\sim C \supset (A \supset C)$	
	2.	$\sim C$	/ $\sim A$
	3.	$A \supset C$	1, 2, MP
	4.	$\sim A$	2, 3, MT
(4)	1.	$P \supset (G \supset T)$	
	2.	$Q \supset (T \supset E)$	
	3.	P	
	4.	Q	/ $G \supset E$
	5.	$G \supset T$	1, 3, MP
	6.	$T \supset E$	2, 4, MP
	7.	$G \supset E$	5, 6, HS
(7)	1.	$\sim S \supset D$	
	2.	$\sim S \vee (\sim D \supset K)$	
	3.	$\sim D$	/ K
	4.	$\sim\sim S$	1, 3, MT
	5.	$\sim D \supset K$	2, 4, DS
	6.	K	3, 5, MP
(10)	1.	$N \supset (J \supset P)$	
	2.	$(J \supset P) \supset (N \supset J)$	
	3.	N	/ P
	4.	$J \supset P$	1, 3, MP
	5.	$N \supset J$	2, 4, MP
	6.	$N \supset P$	4, 5, HS
	7.	P	3, 6, MP
(13)	1.	$R \supset (G \vee \sim A)$	
	2.	$(G \vee \sim A) \supset \sim S$	
	3.	$G \supset S$	
	4.	R	/ $\sim A$
	5.	$G \vee \sim A$	1, 4, MP
	6.	$\sim S$	2, 5, MP
	7.	$\sim G$	3, 6, MT
	8.	$\sim A$	5, 7, DS
(16)	1.	$(B \supset \sim M) \supset (T \supset \sim S)$	
	2.	$B \supset K$	
	3.	$K \supset \sim M$	
	4.	$\sim S \supset N$	/ $T \supset N$
	5.	$B \supset \sim M$	2, 3, HS
	6.	$T \supset \sim S$	1, 5, MP
	7.	$T \supset N$	4, 6, HS
(19)	1.	$\sim G \supset [G \vee (S \supset G)]$	
	2.	$(S \vee L) \supset \sim G$	
	3.	$S \vee L$	/ L

	4.	$\sim G$	2, 3, MP
	5.	$G \vee (S \supset G)$	1, 4, MP
	6.	$S \supset G$	4, 5, DS
	7.	$\sim S$	4, 6, MT
	8.	L	3, 7, DS
(22)	1.	$(C \supset M) \supset (N \supset P)$	
	2.	$(C \supset N) \supset (N \supset M)$	
	3.	$(C \supset P) \supset \sim M$	
	4.	$C \supset N$	/ $\sim C$
	5.	$N \supset M$	2, 4, MP
	6.	$C \supset M$	4, 5, HS
	7.	$N \supset P$	1, 6, MP
	8.	$C \supset P$	4, 7, HS
	9.	$\sim M$	3, 8, MP
	10.	$\sim C$	6, 9, MT
(25)	1.	$\sim N \supset [(B \supset D) \supset (N \vee \sim E)]$	
	2.	$(B \supset E) \supset \sim N$	
	3.	$B \supset D$	
	4.	$D \supset E$	/ $\sim D$
	5.	$B \supset E$	3, 4, HS
	6.	$\sim N$	2, 5, MP
	7.	$(B \supset D) \supset (N \vee \sim E)$	1, 6, MP
	8.	$N \vee \sim E$	3, 7, MP
	9.	$\sim E$	6, 8, DS
	10.	$\sim D$	4, 9, MT

IV.

(1)	1.	$W \supset (P \vee C)$	
	2.	$\sim P$	
	3.	W	/ C
	4.	$P \vee C$	1, 3, MP
	5.	C	2, 4, DS
(4)	1.	$(R \supset L) \supset (L \supset \sim F)$	
	2.	$\sim F \vee (R \supset L)$	
	3.	$\sim\sim F$	/ $\sim R$
	4.	$R \supset L$	2, 3, DS
	5.	$L \supset \sim F$	1, 4, MP
	6.	$\sim L$	3, 5, MT
	7.	$\sim R$	4, 6, MT
(7)	1.	$H \supset (D \equiv A)$	
	2.	$V \vee (R \supset V)$	
	3.	$R \vee H$	
	4.	$\sim V$	/ $D \equiv A$
	5.	$R \supset V$	2, 4, DS
	6.	$\sim R$	4, 5, MT
	7.	H	3, 6, DS
	8.	$D \equiv A$	1, 7, MP
(10)	1.	$\sim C \supset [C \vee (J \supset D)]$	
	2.	$C \supset (C \bullet U)$	
	3.	$\sim (C \bullet U)$	
	4.	$\sim D$	/ $\sim J$
	5.	$\sim C$	2, 3, MT
	6.	$C \vee (J \supset D)$	1, 5, MP
	7.	$J \supset D$	5, 6, DS
	8.	$\sim J$	4, 7, MT

Exercise 7.2

I.

1.	B	2
4.	$H \vee F$	1
7.	$Q \vee K$	1
10.	$\sim L \vee M$	1, 2

II.

1.	G	2, Simp 3, Add
4.	$T \vee U$	1, Add 3, 4, MP
7.	$\sim F$	2, 3, MT 1, 4, Conj
10.	$M \bullet E$	1, 3, Conj 2, 4, MP

III.

(1)	1.	$\sim M \supset Q$	
	2.	$R \supset \sim T$	
	3.	$\sim M \vee R$	/ $Q \vee \sim T$
	4.	$(\sim M \supset Q) \bullet (R \supset \sim T)$	1, 2, Conj
	5.	$Q \vee \sim T$	3, 4, CD
(4)	1.	$(H \vee \sim B) \supset R$	
	2.	$(H \vee \sim M) \supset P$	
	3.	H	/ $R \bullet P$
	4.	$H \vee \sim B$	3, Add
	5.	R	1, 4, MP
	6.	$H \vee \sim M$	3, Add
	7.	P	2, 6, MP
	8.	$R \bullet P$	5, 7, Conj
(7)	1.	$(\sim F \vee X) \supset (P \vee T)$	
	2.	$F \supset P$	
	3.	$\sim P$	/ T
	4.	$\sim F$	2, 3, MT
	5.	$\sim F \vee X$	4, Add
	6.	$P \vee T$	1, 5, MP
	7.	T	3, 6, DS
(10)	1.	$(D \vee E) \supset (G \bullet H)$	
	2.	$G \supset \sim D$	
	3.	$D \bullet F$	/ M
	4.	D	3, Simp
	5.	$D \vee E$	4, Add
	6.	$G \bullet H$	1, 5, MP
	7.	G	6, Simp
	8.	$\sim D$	2, 7, MP
	9.	$D \vee M$	4, Add
	10.	M	8, 9, DS
(13)	1.	$(C \supset N) \bullet E$	
	2.	$D \vee (N \supset D)$	
	3.	$\sim D$	/ $\sim C \vee P$

	4.	$N \supset D$	2, 3, DS
	5.	$\sim N$	3, 4, MT
	6.	$C \supset N$	1, Simp
	7.	$\sim C$	5, 6, MT
	8.	$\sim C \vee P$	7, Add
(16)	1.	$(C \vee \sim G) \supset (\sim P \bullet L)$	
	2.	$(\sim P \bullet C) \supset (C \supset D)$	
	3.	$C \bullet \sim R$	/ $D \vee R$
	4.	C	3, Simp
	5.	$C \vee \sim G$	4, Add
	6.	$\sim P \bullet L$	1, 5, MP
	7.	$\sim P$	6, Simp
	8.	$\sim P \bullet C$	4, 7, Conj
	9.	$C \supset D$	2, 8 MP
	10.	D	4, 9, MP
	11.	$D \vee R$	10, Add
(19)	1.	$(U \bullet \sim\sim P) \supset Q$	
	2.	$\sim O \supset U$	
	3.	$\sim P \supset O$	
	4.	$\sim O \bullet T$	/ Q
	5.	$\sim O$	4, Simp
	6.	U	2, 5, MP
	7.	$\sim\sim P$	3, 5, MT
	8.	$U \bullet \sim\sim P$	6, 7, Conj
	9.	Q	1, 8, MP
(22)	1.	$(\sim K \bullet \sim N) \supset [(\sim P \supset K) \bullet (\sim R \supset G)]$	
	2.	$K \supset N$	
	3.	$\sim N \bullet B$	
	4.	$\sim P \vee \sim R$	/ G
	5.	$\sim N$	3, Simp
	6.	$\sim K$	2, 5, MT
	7.	$\sim K \bullet \sim N$	5, 6, Conj
	8.	$(\sim P \supset K) \bullet (\sim R \supset G)$	1, 7, MP
	9.	$K \vee G$	4, 8, CD
	10.	G	6, 9, DS
(25)	1.	$(\sim M \bullet N) \supset [(\sim M \vee H) \supset (K \bullet L)]$	
	2.	$\sim M \bullet (C \supset D)$	
	3.	$\sim N \bullet (F \equiv G)$	/ $K \bullet \sim N$
	4.	$\sim M$	2, Simp
	5.	$\sim N$	3, Simp
	6.	$\sim M \bullet \sim N$	4, 5, Conj
	7.	$(\sim M \vee H) \supset (K \bullet L)$	1, 6, MP
	8.	$\sim M \vee H$	4, Add
	9.	$K \bullet L$	7, 8, MP
	10.	K	9, Simp
	11.	$K \bullet \sim N$	5, 10, Conj
(28)	1.	$(D \supset B) \bullet (C \supset D)$	
	2.	$(B \supset D) \bullet (E \supset C)$	
	3.	$B \vee E$	/ $D \vee B$
	4.	$D \vee C$	2, 3, CD
	5.	$B \vee D$	1, 4, CD
	6.	$B \supset D$	2, Simp
	7.	$D \supset B$	1, Simp
	8.	$(B \supset D) \bullet (D \supset B)$	6, 7, Conj
	9.	$D \vee B$	5, 8, CD

IV.

(1)	1.	$T \supset (Q \bullet F)$	
	2.	$T \bullet C$	/ $Q \vee O$
	3.	T	2, Simp
	4.	$Q \bullet F$	1, 3, MP
	5.	Q	4, Simp
	6.	$Q \vee O$	5, Add
(4)	1.	$M \vee P$	
	2.	$(P \vee S) \supset (R \bullet D)$	
	3.	$\sim M$	/ R
	4.	P	1, 3, DS
	5.	$P \vee S$	4, Add
	6.	$R \bullet D$	2, 5, MP
	7.	R	6, Simp
(7)	1.	$(\sim C \vee \sim M) \supset (\sim C \supset T)$	
	2.	$C \vee \sim T$	
	3.	$\sim C$	/ B
	4.	$\sim C \vee \sim M$	3, Add
	5.	$\sim C \supset T$	1, 4, MP
	6.	T	3, 5, MP
	7.	$T \vee B$	6, Add
	8.	$\sim T$	2, 3, DS
	9.	B	7, 8, DS
(10)	1.	$(V \bullet \sim E) \supset (P \supset E)$	
	2.	$V \supset \sim E$	
	3.	$V \bullet I$	
	4.	$\sim E \supset (P \vee J)$	/ $J \bullet \sim E$
	5.	V	3, Simp
	6.	$\sim E$	2, 5, MP
	7.	$V \bullet \sim E$	5, 6, Conj
	8.	$P \supset E$	1, 7, MP
	9.	$\sim P$	6, 8, MT
	10.	$P \vee J$	4, 6, MP
	11.	J	9, 10, DS
	12.	$J \bullet \sim E$	6, 11, Conj

Exercise 7.3

I.

1.	$\sim N \bullet \sim G$	2
4.	$A \bullet S$	3
7.	$\sim G \vee \sim\sim Q$	1
10.	$\sim (R \bullet P)$	1
13.	$H \supset \sim (L \vee D)$	2

II.

1.	$C \vee K$	1, Com
		2, 3, DS
4.	$L \bullet (S \bullet F)$	1, Assoc
		2, Simp
7.	$D \bullet (M \vee N)$	1, Dist
		2, Simp

	10.	$(D \vee N) \bullet (D \vee H)$	1, Dist 2, Simp
	13.	$M \vee (G \vee T)$	1, Assoc 2, 3, DS

III.

(1)	1.	$(\sim M \supset P) \bullet (\sim N \supset Q)$	
	2.	$\sim(M \bullet N)$	/ $P \vee Q$
	3.	$\sim M \vee \sim N$	2, DM
	4.	$P \vee Q$	1, 3, CD
(4)	1.	$\sim(N \bullet T)$	
	2.	T	/ $\sim N$
	3.	$\sim N \vee \sim T$	1, DM
	4.	$\sim T \vee \sim N$	3, Com
	5.	$\sim\sim T$	2, DN
	6.	$\sim N$	4, 5, DS
(7)	1.	$T \supset (B \vee E)$	
	2.	$\sim E \bullet T$	/ B
	3.	$T \bullet \sim E$	2, Com
	4.	T	3, Simp
	5.	$B \vee E$	1, 4 MP
	6.	$E \vee B$	5, Com
	7.	$\sim E$	2, Simp
	8.	B	6, 7, DS
(10)	1.	$(K \bullet H) \vee (K \bullet L)$	
	2.	$\sim L$	/ H
	3.	$K \bullet (H \vee L)$	1, Dist
	4.	$(H \vee L) \bullet K$	3, Com
	5.	$H \vee L$	4, Simp
	6.	$L \vee H$	5, Com
	7.	H	2, 6, DS
(13)	1.	$(E \bullet I) \vee (M \bullet U)$	
	2.	$\sim E$	/ $\sim(E \vee \sim M)$
	3.	$\sim E \vee \sim I$	2, Add
	4.	$\sim(E \bullet I)$	3, DM
	5.	$M \bullet U$	1, 4, DS
	6.	M	5, Simp
	7.	$\sim\sim M$	6, DN
	8.	$\sim E \bullet \sim\sim M$	2, 7, Conj
	9.	$\sim(E \vee \sim M)$	8, DM
(16)	1.	$(Q \bullet N) \vee (N \bullet T)$	
	2.	$(Q \vee C) \supset \sim N$	/ T
	3.	$(N \bullet Q) \vee (N \bullet T)$	1, Com
	4.	$N \bullet (Q \vee T)$	3, Dist
	5.	N	4, Simp
	6.	$\sim\sim N$	5, DN
	7.	$\sim(Q \vee C)$	2, 6, MT
	8.	$\sim Q \bullet \sim C$	7, DM
	9.	$\sim Q$	8, Simp
	10.	$(Q \vee T) \bullet N$	4, Com
	11.	$Q \vee T$	10, Simp
	12.	T	9, 11, DS
(19)	1.	$[(I \vee M) \vee G] \supset \sim G$	
	2.	$M \vee G$	/ M
	3.	$(M \vee G) \vee I$	2, Add
	4.	$I \vee (M \vee G)$	3, Com
	5.	$(I \vee M) \vee G$	4, Assoc
	6.	$\sim G$	1, 5, MP
	7.	$G \vee M$	2, Com
	8.	M	6, 7, DS
(22)	1.	$S \vee (I \bullet \sim J)$	
	2.	$S \supset \sim R$	
	3.	$\sim J \supset \sim Q$	/ $\sim(R \bullet Q)$
	4.	$(S \vee I) \bullet (S \vee \sim J)$	1, Dist
	5.	$(S \vee \sim J) \bullet (S \vee I)$	4, Com
	6.	$S \vee \sim J$	5, Simp
	7.	$(S \supset \sim R) \bullet (\sim J \supset \sim Q)$	2, 3, Conj
	8.	$\sim R \vee \sim Q$	6, 7, CD
	9.	$\sim(R \bullet Q)$	8, DM
(25)	1.	$E \vee \sim(D \vee C)$	
	2.	$(E \vee \sim D) \supset C$	/ E
	3.	$E \vee (\sim D \bullet \sim C)$	1, DM
	4.	$(E \vee \sim D) \bullet (E \vee \sim C)$	3, Dist
	5.	$E \vee \sim D$	4, Simp
	6.	C	2, 5, MP
	7.	$(E \vee \sim C) \bullet (E \vee \sim D)$	4, Com
	8.	$E \vee \sim C$	7, Simp
	9.	$\sim C \vee E$	8, Com
	10.	$\sim\sim C$	6, DN
	11.	E	9, 10, DS
(28)	1.	$P \vee (I \bullet L)$	
	2.	$(P \vee I) \supset \sim(L \vee C)$	
	3.	$(P \bullet \sim C) \supset (E \bullet F)$	/ $F \vee D$
	4.	$(P \vee I) \bullet (P \vee L)$	1, Dist
	5.	$P \vee I$	4, Simp
	6.	$\sim(L \vee C)$	2, 5, MP
	7.	$\sim L \bullet \sim C$	6, DM
	8.	$\sim L$	7, Simp
	9.	$(P \vee L) \bullet (P \vee I)$	4, Com
	10.	$P \vee L$	9, Simp
	11.	$L \vee P$	10, Com
	12.	P	8, 11, DS
	13.	$\sim C \bullet \sim L$	7, Com
	14.	$\sim C$	13, Simp
	15.	$P \bullet \sim C$	12, 14, Conj
	16.	$E \bullet F$	3, 15, MP
	17.	$F \bullet E$	16, Com
	18.	F	17, Simp
	19.	$F \vee D$	18, Add
(31)	1.	$(\sim R \vee D) \supset \sim(F \bullet G)$	
	2.	$(F \bullet R) \supset S$	
	3.	$F \bullet \sim S$	/ $\sim(S \vee G)$
	4.	$\sim S \bullet F$	3, Com
	5.	$\sim S$	4, Simp
	6.	$\sim(F \bullet R)$	2, 5, MT
	7.	$\sim F \vee \sim R$	6, DM
	8.	F	3, Simp
	9.	$\sim\sim F$	8, DN
	10.	$\sim R$	7, 9, DS
	11.	$\sim R \vee D$	10, Add
	12.	$\sim(F \bullet G)$	1, 11, MP
	13.	$\sim F \vee \sim G$	12, DM
	14.	$\sim G$	9, 13, DS

	15.	$\sim S \bullet \sim G$	5, 14, Conj
	16.	$\sim(S \vee G)$	15, DM
(34)	1.	$(M \bullet N) \vee (O \bullet P)$	
	2.	$(N \vee O) \supset \sim P$	/ N
	3.	$[(M \bullet N) \vee O] \bullet [(M \bullet N) \vee P]$	1, Dist
	4.	$(M \bullet N) \vee O$	3, Simp
	5.	$O \vee (M \bullet N)$	4, Com
	6.	$(O \vee M) \bullet (O \vee N)$	5, Dist
	7.	$(O \vee N) \bullet (O \vee M)$	6, Com
	8.	$O \vee N$	7, Simp
	9.	$N \vee O$	8, Com
	10.	$\sim P$	2, 9, MP
	11.	$[(M \bullet N) \vee P] \bullet [(M \bullet N) \vee O]$	3, Com
	12.	$(M \bullet N) \vee P$	11, Simp
	13.	$P \vee (M \bullet N)$	12, Com
	14.	$M \bullet N$	10, 13, DS
	15.	$N \bullet M$	14, Com
	16.	N	15, Simp

IV.

(1)	1.	$(S \bullet D) \vee (S \bullet H)$	
	2.	$S \supset (I \bullet R)$	/ $S \bullet R$
	3.	$S \bullet (D \vee H)$	1, Dist
	4.	S	3, Simp
	5.	$I \bullet R$	2, 4, MP
	6.	$R \bullet I$	5, Com
	7.	R	6, Simp
	8.	$S \bullet R$	4, 7, Conj
(4)	1.	$G \vee (R \bullet E)$	
	2.	$(G \vee E) \supset \sim R$	/ $G \vee M$
	3.	$(G \vee R) \bullet (G \vee E)$	1, Dist
	4.	$(G \vee E) \bullet (G \vee R)$	3, Com
	5.	$G \vee E$	4, Simp
	6.	$\sim R$	2, 5, MP
	7.	$G \vee R$	3, Simp
	8.	$R \vee G$	7, Com
	9.	G	6, 8, DS
	10.	$G \vee M$	9, Add
(7)	1.	$R \supset (C \vee M)$	
	2.	$\sim(I \vee C)$	
	3.	$\sim(A \vee M)$	/ $\sim R$
	4.	$\sim I \bullet \sim C$	2, DM
	5.	$\sim A \bullet \sim M$	3, DM
	6.	$\sim C \bullet \sim I$	4, Com
	7.	$\sim C$	6, Simp
	8.	$\sim M \bullet \sim A$	5, Com
	9.	$\sim M$	8, Simp
	10.	$\sim C \bullet \sim M$	7, 9, Conj
	11.	$\sim(C \vee M)$	10, DM
	12.	$\sim R$	1, 11, MT
(10)	1.	$\sim E \vee (B \bullet P)$	
	2.	$\sim E \vee (G \bullet W)$	
	3.	$\sim P \vee \sim W$	/ $\sim E$
	4.	$(\sim E \vee B) \bullet (\sim E \vee P)$	1, Dist
	5.	$(\sim E \vee P) \bullet (\sim E \vee B)$	4, Com
	6.	$\sim E \vee P$	5, Simp
	7.	$(\sim E \vee G) \bullet (\sim E \vee W)$	2, Dist
	8.	$(\sim E \vee W) \bullet (\sim E \vee G)$	7, Com
	9.	$\sim E \vee W$	8, Simp
	10.	$(\sim E \vee P) \bullet (\sim E \vee W)$	6, 9, Conj
	11.	$\sim E \vee (P \bullet W)$	10, Dist
	12.	$(P \bullet W) \vee \sim E$	11, Com
	13.	$\sim(P \bullet W)$	3, DM
	14.	$\sim E$	12, 13, DS

Exercise 7.4

I.

1.	$G \supset Q$	3
4.	$B \equiv N$	1
7.	$\sim\sim C \vee \sim F$	1
10.	$S \supset G$	3
13.	$W \equiv \sim T$	2

II.

1.	$J \supset M$	1, Impl
		2, 3, HS
4.	$K \vee K$	1, 2, CD
		3, Taut
7.	$H \supset (C \supset R)$	1, Impl
		2, Exp
10.	$\sim H \vee \sim H$	1, Impl
		2, Taut
13.	$(N \supset A) \bullet (A \supset N)$	1, Trans
		2, Equiv

III.

(1)	1.	$(S \bullet K) \supset R$	
	2.	K	/ $S \supset R$
	3.	$(K \bullet S) \supset R$	1, Com
	4.	$K \supset (S \supset R)$	3, Exp
	5.	$S \supset R$	2, 4, MP
(4)	1.	$S \equiv Q$	
	2.	$\sim S$	/ $\sim Q$
	3.	$(S \supset Q) \bullet (Q \supset S)$	1, Equiv
	4.	$(Q \supset S) \bullet (S \supset Q)$	3, Com
	5.	$Q \supset S$	4, Simp
	6.	$\sim Q$	2, 5, MT
(7)	1.	$(B \supset M) \bullet (D \supset M)$	
	2.	$B \vee D$	/ M
	3.	$M \vee M$	1, 2, CD
	4.	M	3, Taut
(10)	1.	$(B \supset G) \bullet (F \supset N)$	
	2.	$\sim(G \bullet N)$	/ $\sim(B \bullet F)$
	3.	$\sim G \vee \sim N$	2, DM
	4.	$(\sim G \supset \sim B) \bullet (F \supset N)$	1, Trans
	5.	$(\sim G \supset \sim B) \bullet (\sim N \supset \sim F)$	4, Trans
	6.	$\sim B \vee \sim F$	3, 5, CD
	7.	$\sim(B \bullet F)$	6, DM

(13) 1. $K \supset (B \supset {\sim}M)$
2. $D \supset (K \bullet M)$ / $D \supset {\sim}B$
3. $K \supset ({\sim}{\sim}M \supset {\sim}B)$ 1, Trans
4. $K \supset (M \supset {\sim}B)$ 3, DN
5. $(K \bullet M) \supset {\sim}B$ 4, Exp
6. $D \supset {\sim}B$ 2, 5, HS

(16) 1. $T \supset R$
2. $T \supset {\sim}R$ / ${\sim}T$
3. ${\sim}{\sim}R \supset {\sim}T$ 2, Trans
4. $R \supset {\sim}T$ 3, DN
5. $T \supset {\sim}T$ 1, 4, HS
6. ${\sim}T \vee {\sim}T$ 5, Impl
7. ${\sim}T$ 6, Taut

(19) 1. ${\sim}R \vee P$
2. $R \vee {\sim}P$ / $R \equiv P$
3. $R \supset P$ 1, Impl
4. ${\sim}P \vee R$ 2, Com
5. $P \supset R$ 4, Impl
6. $(R \supset P) \bullet (P \supset R)$ 3, 5, Conj
7. $R \equiv P$ 6, Equiv

(22) 1. $S \supset (L \bullet M)$
2. $M \supset (L \supset R)$ / $S \supset R$
3. $(M \bullet L) \supset R$ 2, Exp
4. $(L \bullet M) \supset R$ 3, Com
5. $S \supset R$ 1, 4, HS

(25) 1. $T \supset G$
2. $S \supset G$ / $(T \vee S) \supset G$
3. ${\sim}T \vee G$ 1, Impl
4. ${\sim}S \vee G$ 2, Impl
5. $G \vee {\sim}T$ 3, Com
6. $G \vee {\sim}S$ 4, Com
7. $(G \vee {\sim}T) \bullet (G \vee {\sim}S)$ 5, 6, Conj
8. $G \vee ({\sim}T \bullet {\sim}S)$ 7, Dist
9. $({\sim}T \bullet {\sim}S) \vee G$ 8, Com
10. ${\sim}(T \vee S) \vee G$ 9, DM
11. $(T \vee S) \supset G$ 10, Impl

(28) 1. $P \supset ({\sim}E \supset B)$
2. ${\sim}(B \vee E)$ / ${\sim}P$
3. ${\sim}(E \vee B)$ 2, Com
4. ${\sim}({\sim}{\sim}E \vee \mathrm{B})$ 3, DN
5. ${\sim}({\sim}E \supset B)$ 4, Impl
6. ${\sim}P$ 1, 5, MT

(31) 1. $K \equiv R$
2. $K \supset (R \supset P)$
3. ${\sim}P$ / ${\sim}R$
4. $(K \bullet R) \vee ({\sim}K \bullet {\sim}R)$ 1, Equiv
5. $(K \bullet R) \supset P$ 2, Exp
6. ${\sim}(K \bullet R)$ 3, 5, MT
7. ${\sim}K \bullet {\sim}R$ 4, 6, DS
8. ${\sim}R \bullet {\sim}K$ 7, Com
9. ${\sim}R$ 8, Simp

(34) 1. $(F \bullet H) \supset N$
2. $F \vee S$
3. H / $N \vee S$
4. $(H \bullet F) \supset N$ 1, Com
5. $H \supset (F \supset N)$ 4, Exp
6. $F \supset N$ 3, 5, MP
7. ${\sim}N \supset {\sim}F$ 6, Trans
8. ${\sim}{\sim}F \vee S$ 2, DN
9. ${\sim}F \supset S$ 8, Impl
10. ${\sim}N \supset S$ 7, 9, HS
11. ${\sim}{\sim}N \vee S$ 10, Impl
12. $N \vee S$ 11, DN

(37) 1. $(D \supset E) \supset (E \supset D)$
2. $(D \equiv E) \supset {\sim}(G \bullet {\sim}H)$
3. $E \bullet G$ / $G \bullet H$
4. E 3, Simp
5. $E \vee {\sim}D$ 4, Add
6. ${\sim}D \vee E$ 5, Com
7. $D \supset E$ 6, Impl
8. $E \supset D$ 1, 7, MP
9. $(D \supset E) \bullet (E \supset D)$ 7, 8, Conj
10. $D \equiv E$ 9, Equiv
11. ${\sim}(G \bullet {\sim}H)$ 2, 10, MP
12. ${\sim}G \vee {\sim}{\sim}H$ 11, DM
13. ${\sim}G \vee H$ 12, DN
14. $G \bullet E$ 3, Com
15. G 14, Simp
16. ${\sim}{\sim}G$ 15, DN
17. H 13, 16, DS
18. $G \bullet H$ 15, 17, Conj

(40) 1. $A \equiv W$
2. ${\sim}A \vee {\sim}W$
3. $R \supset A$ / ${\sim}(W \vee R)$
4. $(A \bullet W) \vee ({\sim}A \bullet {\sim}W)$ 1, Equiv
5. ${\sim}(A \bullet W)$ 2, DM
6. ${\sim}A \bullet {\sim}W$ 4, 5, DS
7. ${\sim}A$ 6, Simp
8. ${\sim}R$ 3, 7, MT
9. ${\sim}W \bullet {\sim}A$ 6, Com
10. ${\sim}W$ 9, Simp
11. ${\sim}W \bullet {\sim}R$ 8, 10, Conj
12. ${\sim}(W \vee R)$ 11, DM

(43) 1. $O \supset (Q \bullet N)$
2. $(N \vee E) \supset S$ / $O \supset S$
3. ${\sim}O \vee (Q \bullet N)$ 1, Impl
4. $({\sim}O \vee Q) \bullet ({\sim}O \vee N)$ 3, Dist
5. $({\sim}O \vee N) \bullet ({\sim}O \vee Q)$ 4, Com
6. ${\sim}O \vee N$ 5, Simp
7. $O \supset N$ 6, Impl
8. ${\sim}(N \vee E) \vee S$ 2, Impl
9. $({\sim}N \bullet {\sim}E) \vee S$ 8, DM
10. $S \vee ({\sim}N \bullet {\sim}E)$ 9, Com
11. $(S \vee {\sim}N) \bullet (S \vee {\sim}E)$ 10, Dist
12. $S \vee {\sim}N$ 11, Simp
13. ${\sim}N \vee S$ 12, Com
14. $N \supset S$ 13, Impl
15. $O \supset S$ 7, 14, HS

(45) 1. $P \supset A$
2. $Q \supset B$ / $(P \vee Q) \supset (A \vee B)$
3. ${\sim}P \vee A$ 1, Impl
4. ${\sim}Q \vee B$ 2, Impl

5. $(\sim P \vee A) \vee B$ 3, Add
6. $(\sim Q \vee B) \vee A$ 4, Add
7. $\sim P \vee (A \vee B)$ 5, Assoc
8. $(A \vee B) \vee \sim P$ 7, Com
9. $\sim Q \vee (B \vee A)$ 6, Assoc
10. $\sim Q \vee (A \vee B)$ 9, Com
11. $(A \vee B) \vee \sim Q$ 10, Com
12. $[(A \vee B) \vee \sim P] \bullet [(A \vee B) \vee \sim Q]$ 8, 11, Conj
13. $(A \vee B) \vee (\sim P \bullet \sim Q)$ 12, Dist
14. $(\sim P \bullet \sim Q) \vee (A \vee B)$ 13, Com
15. $\sim (P \vee Q) \vee (A \vee B)$ 14, DM
16. $(P \vee Q) \supset (A \vee B)$ 15, Impl

IV.

(1) 1. $D \supset C$
2. $\sim (C \bullet \sim S)$ / $D \supset S$
3. $\sim C \vee \sim\sim S$ 2, DM
4. $C \supset \sim\sim S$ 3, Impl
5. $C \supset S$ 4, DN
6. $D \supset S$ 1, 5, HS

(4) 1. $D \supset P$ / $(I \bullet D) \supset P$
2. $\sim D \vee P$ 1, Impl
3. $(\sim D \vee P) \vee \sim I$ 2, Add
4. $\sim I \vee (\sim D \vee P)$ 3, Com
5. $(\sim I \vee \sim D) \vee P$ 4, Assoc
6. $\sim (I \bullet D) \vee P$ 5, DM
7. $(I \bullet D) \supset P$ 6, Impl

(7) 1. $G \supset A$
2. $G \supset L$ / $G \supset (A \bullet L)$
3. $\sim G \vee A$ 1, Impl
4. $\sim G \vee L$ 2, Impl
5. $(\sim G \vee A) \bullet (\sim G \vee L)$ 3, 4, Conj
6. $\sim G \vee (A \bullet L)$ 5, Dist
7. $G \supset (A \bullet L)$ 6, Impl

(10) 1. $(A \bullet U) \equiv \sim R$
2. $\sim (\sim R \vee \sim A)$ / $\sim U$
3. $[(A \bullet U) \supset \sim R] \bullet [\sim R \supset (A \bullet U)]$ 1, Equiv
4. $(A \bullet U) \supset \sim R$ 3, Simp
5. $\sim\sim R \bullet \sim\sim A$ 2, DM
6. $\sim\sim R$ 5, Simp
7. $\sim (A \bullet U)$ 4, 6, MT
8. $\sim A \vee \sim U$ 7, DM
9. $\sim\sim A \bullet \sim\sim R$ 5, Com
10. $\sim\sim A$ 9, Simp
11. $\sim U$ 8, 10, DS

Exercise 7.5

I.

(1) 1. $N \supset O$
2. $N \supset P$ / $N \supset (O \bullet P)$
   3. N ACP
   4. O 1, 3, MP
   5. P 2, 3, MP
   6. $O \bullet P$ 4, 5, Conj
7. $N \supset (O \bullet P)$ 3–6, CP

(4) 1. $(G \vee H) \supset (S \bullet T)$
2. $(T \vee U) \supset (C \bullet D)$ / $G \supset C$
   3. G ACP
   4. $G \vee H$ 3, Add
   5. $S \bullet T$ 1, 4, MP
   6. $T \bullet S$ 5, Com
   7. T 6, Simp
   8. $T \vee U$ 7, Add
   9. $C \bullet D$ 2, 8, MP
   10. C 9, Simp
11. $G \supset C$ 3–10, CP

(7) 1. $M \vee (N \bullet O)$ / $\sim N \supset M$
   2. $\sim M$ ACP
   3. $N \bullet O$ 1, 2, DS
   4. N 3, Simp
5. $\sim M \supset N$ 2–4, CP
6. $\sim N \supset \sim\sim M$ 5, Trans
7. $\sim N \supset M$ 6, DN

(10) 1. $C \supset (A \bullet D)$
2. $B \supset (A \bullet E)$ / $(C \vee B) \supset A$
   3. $C \vee B$ ACP
   4. $[C \supset (A \bullet D)] \bullet [B \supset (A \bullet E)]$ 1, 2, Conj
   5. $(A \bullet D) \vee (A \bullet E)$ 3, 4, CD
   6. $A \bullet (D \vee E)$ 5, Dist
   7. A 6, Simp
8. $(C \vee B) \supset A$ 3–7, CP

(13) 1. $R \supset B$
2. $R \supset (B \supset F)$
3. $B \supset (F \supset H)$ / $R \supset H$
   4. R ACP
   5. B 1, 4, MP
   6. $B \supset F$ 2, 4, MP
   7. F 5, 6, MP
   8. $F \supset H$ 3, 5, MP
   9. H 7, 8, MP
10. $R \supset H$ 4–9, CP

(16) 1. $Q \supset (R \supset S)$
2. $Q \supset (T \supset \sim U)$
3. $U \supset (R \vee T)$ / $Q \supset (U \supset S)$
   4. Q ACP
      5. U ACP
      6. $R \supset S$ 1, 4, MP
      7. $T \supset \sim U$ 2, 4, MP
      8. $\sim\sim U$ 5, DN
      9. $\sim T$ 7, 8, MT
      10. $R \vee T$ 3, 5, MP
      11. $T \vee R$ 10, Com
      12. R 9, 11, DS
      13. S 6, 12, MP
   14. $U \supset S$ 5–13, CP
15. $Q \supset (U \supset S)$ 4–14, CP

(19) 1. $P \supset [(L \vee M) \supset (N \bullet O)]$
2. $(O \vee T) \supset W$ / $P \supset (M \supset W)$
   3. P ACP
      4. M ACP

5. $(L \vee M) \supset (N \bullet O)$ 1, 3, MP
6. $M \vee L$ 4, Add
7. $L \vee M$ 6, Com
8. $N \bullet O$ 5, 7, MP
9. $O \bullet N$ 8, Com
10. O 9, Simp
11. $O \vee T$ 10, Add
12. W 2, 11, MP

13. $M \supset W$ 4–12, CP

14. $P \supset (M \supset W)$ 3–13, CP

II.

(1) 1. $H \supset D$
2. $U \supset S$ / $(H \bullet U) \supset (S \bullet D)$
3. $H \bullet U$ ACP
4. H 3, Simp
5. D 1, 4, MP
6. $U \bullet H$ 3, Com
7. U 6, Simp
8. S 2, 7, MP
9. $S \bullet D$ 5, 8, Conj

10. $(H \bullet U) \supset (S \bullet D)$ 3–9, CP

(4) 1. $J \supset D$
2. $(J \bullet D) \supset C$
3. $(N \bullet C) \supset I$ / $J \supset (N \supset I)$
4. J ACP
5. N ACP
6. D 1, 4, MP
7. $J \bullet D$ 4, 6, Conj
8. C 2, 7, MP
9. $N \bullet C$ 5, 8, Conj
10. I 3, 9, MP

11. $N \supset I$ 5–10, CP

12. $J \supset (N \supset I)$ 4–11, CP

Exercise 7.6

I.

(1) 1. $(S \vee T) \supset \sim S$ / $\sim S$
2. S AIP
3. $S \vee T$ 2, Add
4. $\sim S$ 1, 3, MP
5. $S \bullet \sim S$ 2, 4, Conj

6. $\sim S$ 2–5, IP

(4) 1. $H \supset (L \supset K)$
2. $L \supset (K \supset \sim L)$ / $\sim H \vee \sim L$
3. $H \bullet L$ AIP
4. H 3, Simp
5. $L \supset K$ 1, 4, MP
6. $L \bullet H$ 3, Com
7. L 6, Simp
8. $K \supset \sim L$ 2, 7, MP
9. K 5, 7, MP
10. $\sim L$ 8, 9, MP
11. $L \bullet \sim L$ 7, 10, Conj

12. $\sim (H \bullet L)$ 3–11, IP
13. $\sim H \vee \sim L$ 12, DM

(7) 1. $(E \vee F) \supset (C \bullet D)$
2. $(D \vee G) \supset H$
3. $E \vee G$ / H
4. $\sim H$ AIP
5. $\sim (D \vee G)$ 2, 4, MT
6. $\sim D \bullet \sim G$ 5, DM
7. $\sim D$ 6, Simp
8. $\sim D \vee \sim C$ 7, Add
9. $\sim C \vee \sim D$ 8, Com
10. $\sim (C \bullet D)$ 9, DM
11. $\sim (E \vee F)$ 1, 10, MT
12. $\sim E \bullet \sim F$ 11, DM
13. $\sim E$ 12, Simp
14. G 3, 13, DS
15. $\sim G \bullet \sim D$ 6, Com
16. $\sim G$ 15, Simp
17. $G \bullet \sim G$ 14, 16, Conj

18. $\sim\sim H$ 4–17, IP
19. H 18, DN

(10) 1. K / $S \supset (T \supset S)$
2. S ACP
3. $S \vee \sim T$ 2, Add
4. $\sim T \vee S$ 3, Com
5. $T \supset S$ 4, Impl

6. $S \supset (T \supset S)$ 2–5, CP

(13) 1. $[C \supset (D \supset C)] \supset E$ / E
2. C ACP
3. $C \vee \sim D$ 2, Add
4. $\sim D \vee C$ 3, Com
5. $D \supset C$ 4, Impl

6. $C \supset (D \supset C)$ 2–5, CP
7. E 1, 6, MP

(16) 1. $(N \vee O) \supset (C \bullet D)$
2. $(D \vee K) \supset (P \vee \sim C)$
3. $(P \vee G) \supset \sim (N \bullet D)$ / $\sim N$
4. N AIP
5. $N \vee O$ 4, Add
6. $C \bullet D$ 1, 5, MP
7. $D \bullet C$ 6, Com
8. D 7, Simp
9. $D \vee K$ 8, Add
10. $P \vee \sim C$ 2, 9, MP
11. C 6, Simp
12. $\sim C \vee P$ 10, Com
13. $\sim\sim C$ 11, DN
14. P 12, 13, DS
15. $P \vee G$ 14, Add
16. $\sim (N \bullet D)$ 3, 15, MP
17. $\sim N \vee \sim D$ 16, DM
18. $\sim\sim N$ 4, DN
19. $\sim D$ 17, 18, DS
20. $D \bullet \sim D$ 8, 19, Conj

21. $\sim N$ 4–20, IP

(19)

1.	$A \supset [(N \vee \sim N) \supset (S \vee T)]$	
2.	$T \supset \sim(F \vee \sim F)$	/ $A \supset S$
3.	$A \bullet \sim S$	AIP
4.	A	3, Simp
5.	$(N \vee \sim N) \supset (S \vee T)$	1, 4, MP
6.	N	ACP
7.	$N \vee N$	6, Add
8.	N	7, Taut
9.	$N \supset N$	6–8, CP
10.	$\sim N \vee N$	9, Impl
11.	$N \vee \sim N$	10, Com
12.	$S \vee T$	5, 11, MP
13.	$\sim S \bullet A$	3, Com
14.	$\sim S$	13, Simp
15.	T	12, 14, DS
16.	$\sim(F \vee \sim F)$	2, 15, MP
17.	$\sim F \bullet \sim\sim F$	16, DM
18.	$\sim(A \bullet \sim S)$	3–17, IP
19.	$\sim A \vee \sim\sim S$	18, DM
20.	$\sim A \vee S$	19, DN
21.	$A \supset S$	20, Impl

II.

(1)

1.	$(C \bullet R) \supset (I \bullet D)$	
2.	$R \supset \sim D$	/ $\sim C \vee \sim R$
3.	$C \bullet R$	AIP
4.	$I \bullet D$	1, 3, MP
5.	$D \bullet I$	4, Com
6.	D	5, Simp
7.	$R \bullet C$	3, Com
8.	R	7, Simp
9.	$\sim D$	2, 8, MP
10.	$D \bullet \sim D$	6, 9, Conj
11.	$\sim(C \bullet R)$	3–10, IP
12.	$\sim C \vee \sim R$	11, DM

(4)

1.	$(Z \supset C) \supset B$	
2.	$(V \supset Z) \supset B$	/ B
3.	$\sim B$	AIP
4.	$\sim(Z \supset C)$	1, 3, MT
5.	$\sim(\sim Z \vee C)$	4, Impl
6.	$\sim\sim Z \bullet \sim C$	5, DM
7.	$\sim\sim Z$	6, Simp
8.	$\sim(V \supset Z)$	2, 3, MT
9.	$\sim(\sim V \vee Z)$	8, Impl
10.	$\sim\sim V \bullet \sim Z$	9, DM
11.	$\sim Z \bullet \sim\sim V$	10, Com
12.	$\sim Z$	11, Simp
13.	$\sim Z \bullet \sim\sim Z$	7, 12, Conj
14.	$\sim\sim B$	3–13, IP
15.	B	14, DN

Exercise 7.7

(1)

		/ $P \supset [(P \supset Q) \supset Q]$
1.	P	ACP
2.	$P \supset Q$	ACP
3.	Q	1, 2, MP
4.	$(P \supset Q) \supset Q$	2–3, CP
5.	$P \supset [(P \supset Q) \supset Q]$	1–4, CP

(4)

		/ $(P \supset Q) \supset [(P \bullet R) \supset (Q \bullet R)]$
1.	$P \supset Q$	ACP
2.	$P \bullet R$	ACP
3.	P	2, Simp
4.	Q	1, 3, MP
5.	$R \bullet P$	2, Com
6.	R	5, Simp
7.	$Q \bullet R$	4, 6, Conj
8.	$(P \bullet R) \supset (Q \bullet R)$	2–7, CP
9.	$(P \supset Q) \supset [(P \bullet R) \supset (Q \bullet R)]$	1–8, CP

(7)

		/ $(P \supset Q) \vee (\sim Q \supset P)$
1.	$\sim[(P \supset Q) \vee (\sim Q \supset P)]$	AIP
2.	$\sim(P \supset Q) \bullet \sim(\sim Q \supset P)$	1, DM
3.	$\sim(P \supset Q)$	2, Simp
4.	$\sim(\sim P \vee Q)$	3, Impl
5.	$\sim\sim P \bullet \sim Q$	4, DM
6.	$P \bullet \sim Q$	5, DN
7.	P	6, Simp
8.	$\sim(\sim Q \supset P) \bullet \sim(P \supset Q)$	2, Com
9.	$\sim(\sim Q \supset P)$	8, Simp
10.	$\sim(\sim\sim Q \vee P)$	9, Impl
11.	$\sim(Q \vee P)$	10, DN
12.	$\sim Q \bullet \sim P$	11, DM
13.	$\sim P \bullet \sim Q$	12, Com
14.	$\sim P$	13, Simp
15.	$P \bullet \sim P$	7, 14, Conj
16.	$\sim\sim[(P \supset Q) \vee (\sim Q \supset P)]$	1–15, IP
17.	$(P \supset Q) \vee (\sim Q \supset P)$	16, DN

(10)

		/ $[\sim(P \bullet \sim Q) \bullet \sim Q] \supset \sim P$
1.	$\sim(P \bullet \sim Q) \bullet \sim Q$	ACP
2.	$\sim(P \bullet \sim Q)$	1, Simp
3.	$\sim P \vee \sim\sim Q$	2, DM
4.	$\sim P \vee Q$	3, DN
5.	$\sim Q \bullet \sim(P \bullet \sim Q)$	1, Com
6.	$\sim Q$	5, Simp
7.	$Q \vee \sim P$	4, Com
8.	$\sim P$	6, 7, DS
9.	$[\sim(P \bullet \sim Q) \bullet \sim Q] \supset \sim P$	1–8, CP

(13)

		/ $(P \supset Q) \supset [(P \supset \sim Q) \supset \sim P]$
1.	$P \supset Q$	ACP
2.	$P \supset \sim Q$	ACP
3.	$\sim\sim Q \supset \sim P$	2, Trans
4.	$Q \supset \sim P$	3, DN
5.	$P \supset \sim P$	1, 4, HS
6.	$\sim P \vee \sim P$	5, Impl
7.	$\sim P$	6, Taut
8.	$(P \supset \sim Q) \supset \sim P$	2–7, CP
9.	$(P \supset Q) \supset [(P \supset \sim Q) \supset \sim P]$	1–8, CP

(16)

		/ $\sim[(P \supset \sim P) \bullet (\sim P \supset P)]$
1.	$(P \supset \sim P) \bullet (\sim P \supset P)$	AIP
2.	$(\sim P \vee \sim P) \bullet (\sim P \supset P)$	1, Impl
3.	$\sim P \bullet (\sim P \supset P)$	2, Taut
4.	$\sim P \bullet (\sim\sim P \vee P)$	3, Impl

	5. $\sim P \bullet (P \vee P)$	4, DN
	6. $\sim P \bullet P$	5, Taut
	7. $P \bullet \sim P$	6, Com
	8. $\sim[(P \supset \sim P) \bullet (\sim P \supset P)]$	1–7, IP
(19)	/ $P \equiv [P \vee (Q \bullet \sim Q)]$	
	1. P	ACP
	2. $P \vee (Q \bullet \sim Q)$	1, Add
	3. $P \supset [P \vee (Q \bullet \sim Q)]$	1–2, CP
	4. $P \vee (Q \bullet \sim Q)$	ACP
	5. $\sim P$	AIP
	6. $Q \bullet \sim Q$	4, 5, DS
	7. $\sim\sim P$	5–6, IP
	8. P	7, DN
	9. $[P \vee (Q \bullet \sim Q)] \supset P$	4–8, CP
	10. {line 3} • {line 9}	3, 9, Conj
	11. $P \equiv [P \vee (Q \bullet \sim Q)]$	10, Equiv

Exercise 8.1

1. Ce
4. $Jr \vee Nr$
7. $(x)(Mx \supset Tx)$
10. $(\exists x)(Hx \bullet \sim Rx)$
13. $(\exists x)Tx$
16. $(\exists x)(Sx \bullet \sim Gx)$
19. $(x)(Sx \supset Vx)$
22. $(x)(Cx \supset \sim Hx)$
25. $(x)(Tx \supset Hx)$
28. $(x)(Hx \supset \sim Ex)$
31. $(\exists x)[Cx \bullet \sim(Sx \vee Bx)]$
34. $(\exists x)[Dx \bullet (Bx \equiv Tx)]$
37. $(\exists x)[Cx \bullet (Ax \supset Tx)]$
40. $(x)[(Wx \bullet Cx) \supset Rx]$
43. $(x)[(Vx \vee Cx) \supset (Sx \bullet Ix)]$
46. $(\exists x)[(Fx \bullet Rx) \bullet Ex]$
49. $Gt \equiv (x)(Wx \supset Cx)$
52. $(\exists x)(Ix \bullet Mx) \supset Ir$
55. $(x)[(Bx \bullet Mx) \supset Sx] \supset Sc$
58. $(\exists x)(Ex \bullet Rx) \equiv (\exists x)(Mx \bullet Ox)$

Exercise 8.2

I.

(1)	1.	$(x)(Ax \supset Bx)$	
	2.	$(x)(Bx \supset Cx)$	/ $(x)(Ax \supset Cx)$
	3.	$Ax \supset Bx$	1, UI
	4.	$Bx \supset Cx$	2, UI
	5.	$Ax \supset Cx$	3, 4, HS
	6.	$(x)(Ax \supset Cx)$	5, UG
(4)	1.	$(x)(Ax \supset Bx)$	
	2.	$(y)(Ay \vee \sim By)$	/ $(x)(Ax \equiv Bx)$
	3.	$Ax \supset Bx$	1, UI
	4.	$Ax \vee \sim Bx$	2, UI
	5.	$\sim Bx \vee Ax$	4, Com
	6.	$Bx \supset Ax$	5, Impl
	7.	$Ax \equiv Bx$	3, 6, Equiv
	8.	$(x)(Ax \equiv Bx)$	7, UG
(7)	1.	$(x)[Jx \supset (Kx \bullet Lx)]$	
	2.	$(\exists y) \sim Ky$	/ $(\exists z) \sim Jz$
	3.	$\sim Km$	2, EI
	4.	$Jm \supset (Km \bullet Lm)$	1, UI
	5.	$\sim Km \vee \sim Lm$	3, Add
	6.	$\sim(Km \bullet Lm)$	5, DM
	7.	$\sim Jm$	4, 6, MT
	8.	$(\exists z) \sim Jz$	7, EG
(10)	1.	$(x)(Ax \supset Bx)$	
	2.	$Am \vee An$	/ $Bm \vee Bn$
	3.	$Am \supset Bm$	1, UI
	4.	$An \supset Bn$	1, UI
	5.	$(Am \supset Bm) \bullet (An \supset Bn)$	3, 4, Conj
	6.	$Bm \vee Bn$	2, 5, CD
(13)	1.	$(x)[(Ax \bullet Bx) \supset Cx]$	
	2.	$(\exists x)(Bx \bullet \sim Cx)$	/ $(\exists x) \sim Ax$
	3.	$Bm \bullet \sim Cm$	2, EI
	4.	$(Am \bullet Bm) \supset Cm$	1, UI
	5.	$\sim Cm \bullet Bm$	3, Com
	6.	$\sim Cm$	5, Simp
	7.	$\sim(Am \bullet Bm)$	4, 6, MT
	8.	$\sim Am \vee \sim Bm$	7, DM
	9.	Bm	3, Simp
	10.	$\sim\sim Bm$	9, DN
	11.	$\sim Bm \vee \sim Am$	8, Com
	12.	$\sim Am$	10, 11, DS
	13.	$(\exists x) \sim Ax$	12, EG
(16)	1.	$(\exists x)Ax \supset (x)Bx$	
	2.	$(\exists x)Cx \supset (\exists x)Dx$	
	3.	$An \bullet Cn$	/ $(\exists x)(Bx \bullet Dx)$
	4.	An	2, Simp
	5.	$(\exists x)Ax$	4, EG
	6.	$(x)Bx$	1, 5, MP
	7.	$Cn \bullet An$	3, Com
	8.	Cn	7, Simp
	9.	$(\exists x)Cx$	8, EG
	10.	$(\exists x)Dx$	2, 9, MP
	11.	Dm	10, EI
	12.	Bm	6, UI
	13.	$Bm \bullet Dm$	11, 12, Conj
	14.	$(\exists x)(Bx \bullet Dx)$	13, EG
(19)	1.	$(\exists x)Ax \supset (x)(Cx \supset Bx)$	
	2.	$(\exists x)(Ax \vee Bx)$	
	3.	$(x)(Bx \supset Ax)$	/ $(x)(Cx \supset Ax)$
	4.	$Am \vee Bm$	2, EI
	5.	$Bm \supset Am$	3, UI
	6.	$\sim\sim Am \vee Bm$	4, DN
	7.	$\sim Am \supset Bm$	6, Impl
	8.	$\sim Am \supset Am$	5, 7, HS
	9.	$\sim\sim Am \vee Am$	8, Impl
	10.	$Am \vee Am$	9, DN
	11.	Am	10, Taut
	12.	$(\exists x)Ax$	11, EG
	13.	$(x)(Cx \supset Bx)$	1, 12, MP