More rules

We added four more rules to our set today. Those rules, and some example problems are produced below. As usual, the best way to get comfortable and gain proficiency with this proof method is to practice, practice, practice, sleep, eat, and practice some more. If you need a reminder about why we are using the rules that we are, then see the previous handout.

Constructive dilemma (CD)

$(p \supset q) \bullet (r \supset s)$

$p \vee r$

$q \vee s$

If Oscar is a dog, then you'll have fleas, and if Oscar is a cat, then you'll have fur balls.

Oscar is either a dog or a cat.

You'll have either fleas or fur balls.

Simplification (Simp)

$p \bullet q$

p

Eliza has long legs and runs fast.

Eliza has long legs.

Conjunction (Conj)

p

q

$p \bullet q$

Roxy has big eyes.

Roxy has a tail.

Roxy has big eyes and a tail.

Addition (Add)

p

$p \vee q$

Theo has spots.

Theo has either spots or stripes.

Remember: the Ps, Qs, and Rs can stand for simple statements *and* complex statements.

These arguments are both instances of **constructive dilemma** (CD):

$\sim M \vee N$
$(\sim M \supset S) \bullet (N \supset \sim T)$
$S \vee \sim T$

$[(K \supset T) \supset (A \bullet B)] \bullet [(H \supset P) \supset (A \bullet C)]$
$(K \supset T) \vee (H \supset P)$
$(A \bullet B) \vee (A \bullet C)$

These arguments are all instances of **simplification** (Simp):

$\sim F \bullet (U \equiv E)$ / $\sim F$

$(M \vee T) \bullet (S \supset R)$ / $M \vee T$

$[(X \supset Z) \bullet M] \bullet (G \supset H)$ / $(X \supset Z) \bullet M$

These arguments are all instances of **conjunction** (Conj):

$\sim E$
$\sim G$
$\sim E \bullet \sim G$

$C \supset M$
$D \supset N$
$(C \supset M) \bullet (D \supset N)$

$R \supset (H \bullet T)$
$K \supset (H \bullet O)$
$[R \supset (H \bullet T)] \bullet [K \supset (H \bullet O)]$

These arguments are all instances of **addition** (Add):

$$\frac{S}{S \vee \sim T} \qquad \frac{(C \bullet D)}{(C \bullet D) \vee (K \bullet \sim P)} \qquad \frac{W \equiv Z}{(W \equiv Z) \vee [A \supset (M \supset O)]}$$

Example 1:

1.	$K \supset L$	
2.	$(M \supset N) \bullet S$	
3.	$N \supset T$	
4.	$K \vee M$	/ $L \vee T$
5.	$M \supset N$	2, Simp
6.	$M \supset T$	3, 5, HS
7.	$(K \supset L) \bullet (M \supset T)$	1, 6, Conj
8.	$L \vee T$	4, 7, CD

Example 2:

1.	$\sim M \bullet N$	
2.	$P \supset M$	
3.	$Q \bullet R$	
4.	$(\sim P \bullet Q) \supset S$	/ $S \vee T$
5.	$\sim M$	1, Simp
6.	$\sim P$	2, 5, MT
7.	Q	3, Simp
8.	$\sim P \bullet Q$	6, 7, Conj
9.	S	4, 8, MP
10.	$S \vee T$	9, Add