Replacement Rules

The rules we will be looking at today are not so much inference rules (like the last 8 have been) where we go from a set of premises to a conclusion. The rules for today are known as replacement rules because we will be allowed to re-state a given line in our proof with another wff that is logically equivalent to the original line (remember our definition of equivalent).

9. De Morgan's rule (DM):
   $\sim(p \bullet q) :: (\sim p \vee \sim q)$
   $\sim(p \vee q) :: (\sim p \bullet \sim q)$
10. Commutativity (Com):
    $(p \vee q) :: (q \vee p)$
    $(p \bullet q) :: (q \bullet p)$
11. Associativity (Assoc):
    $[p \vee (q \vee r)] :: [(p \vee q) \vee r]$
    $[p \bullet (q \bullet r)] :: [(p \bullet q) \bullet r]$
12. Distribution (Dist):
    $[p \bullet (q \vee r)] :: [(p \bullet q) \vee (p \bullet r)]$
    $[p \vee (q \bullet r)] :: [(p \vee q) \bullet (p \vee r)]$
13. Double negation (DN):
    $p :: \sim\sim p$

Some examples of how to use these replacement rules in proofs are provided below:

Example 1:

1. $A \supset \sim(B \bullet C)$	
2. $A \bullet C$	/ $\sim B$
3. A	2, Simp
4. $\sim(B \bullet C)$	1, 3, MP
5. $\sim B \vee \sim C$	4, DM
6. C	2, Com, Simp
7. $\sim B$	5, 6, Com, DN, DS

Example 2:

1. $D \bullet (E \vee F)$	
2. $\sim D \vee \sim F$	/ $D \bullet E$
3. $(D \bullet E) \vee (D \bullet F)$	1, Dist
4. $(D \bullet F) \vee (D \bullet E)$	3, Com
5. $\sim(D \bullet F)$	2, DM
6. $D \bullet E$	4, 5, DS