Inference Rules 2

1. *Modus ponens* (MP)

$p \supset q$	If Su Lin is a panda, then Su Lin is cute.
$\underline{p}$	Su Lin is a panda.
q	Su Lin is cute.

2. *Modus tollens* (MT)

$p \supset q$	If Koko is a koala, then Koko is cuddly.
$\underline{\sim q}$	Koko is not cuddly.
$\sim p$	Koko is not a koala.

3. Pure hypothetical syllogism (HS)

$p \supset q$	If Leo is a lion, then Leo roars.
$\underline{q \supset r}$	If Leo roars, then Leo is fierce.
$p \supset r$	If Leo is a lion, then Leo is fierce.

4. Disjunctive syllogism (DS)

$p \vee q$	Scooter is either a mouse or a rat.
$\underline{\sim p}$	Scooter is not a mouse.
q	Scooter is a rat.

Using these rules is more straightforward than the truth tables were, but will also require your 'seeing' the connections between the stated premises and the needed conclusion. One important thing to note is that the P's and Q's in the above characterization of each rule are **not** meant to imply that only sentence letters can stand in for the P's and Q's. Complex wffs can be in the P and Q positions. For example:

These arguments are all instances of ***modus ponens*** (MP):

$$
\begin{array}{l}
\sim F \supset (G \equiv H) \\
\sim F \\
\hline
G \equiv H
\end{array}
\qquad
\begin{array}{l}
(A \vee B) \supset \sim(C \bullet D) \\
A \vee B \\
\hline
\sim(C \bullet D)
\end{array}
\qquad
\begin{array}{l}
K \bullet L \\
(K \bullet L) \supset [(R \supset S) \bullet (T \supset U)] \\
\hline
(R \supset S) \bullet (T \supset U)
\end{array}
$$

These arguments are all instances of ***modus tollens*** (MT):

$$
\begin{array}{l}
(D \vee F) \supset K \\
\sim K \\
\hline
\sim(D \vee F)
\end{array}
\qquad
\begin{array}{l}
\sim G \supset \sim(M \vee N) \\
\sim\sim(M \vee N) \\
\hline
\sim\sim G
\end{array}
\qquad
\begin{array}{l}
\sim T \\
[(H \vee K) \bullet (L \vee N)] \supset T \\
\hline
\sim[(H \vee K) \bullet (L \vee N)]
\end{array}
$$

These arguments are all instances of **pure hypothetical syllogism** (HS):

$$
\begin{array}{l}
A \supset (D \bullet F) \\
(D \bullet F) \supset \sim H \\
\hline
A \supset \sim H
\end{array}
\qquad
\begin{array}{l}
\sim M \supset (R \supset S) \\
(C \vee K) \supset \sim M \\
\hline
(C \vee K) \supset (R \supset S)
\end{array}
\qquad
\begin{array}{l}
(L \supset N) \supset [(S \vee T) \bullet K] \\
(C \equiv F) \supset (L \supset N) \\
\hline
(C \equiv F) \supset [(S \vee T) \bullet K]
\end{array}
$$

These arguments are all instances of **disjunctive syllogism** (DS):

$U \vee \sim(W \bullet X)$	$\sim(E \vee F)$	$\sim B \vee [(H \supset M) \bullet (S \supset T)]$
$\sim U$	$(E \vee F) \vee (N \supset K)$	$\sim\sim B$
$\sim(W \bullet X)$	$N \supset K$	$(H \supset M) \bullet (S \supset T)$

The structure of our proofs will also be rigorous so that there is no confusion on our part as to what your reasoning is in the proof. Hurley uses the convention of listing the premises in numbered order, and then stating the conclusion you must prove by first using a slashed line, /, followed by the desired wff. The rules you use and the lines you are using the rules on will be listed to the right of the new line you are creating:

1. $A \supset \sim B$	
2. $C \supset B$	
3. A	/ $\sim C$
4. $\sim B$	1, 3, MP

So, in this example, line 4 is the line that was derived from lines 1 and 3 via the rule MP (modus ponens). As usual, practice makes perfect and so you will gain comfort with the rules the more that you use them. A more complicated problem using multiple rules to derive the conclusion is the following problem (which we did in class):

1. $F \supset G$	
2. $F \vee H$	
3. $\sim G$	
4. $H \supset (G \supset I)$	/ $F \supset I$

And the proof is as follows:

1. $F \supset G$	
2. $F \vee H$	
3. $\sim G$	
4. $H \supset (G \supset I)$	/ $F \supset I$
5. $\sim F$	1, 3, MT
6. H	2, 5, DS
7. $G \supset I$	4, 6, MP
8. $F \supset I$	1, 7, HS