

Inference Rules 1

Constructive dilemma (CD)

$$(p \supset q) \cdot (r \supset s)$$

$$\frac{p \vee r}{q \vee s}$$

If Oscar is a dog, then you'll have fleas, and
if Oscar is a cat, then you'll have fur balls.
Oscar is either a dog or a cat.
You'll have either fleas or fur balls.

Simplification (Simp)

$$\frac{p \cdot q}{p}$$

Eliza has long legs and runs fast.
Eliza has long legs.

Conjunction (Conj)

$$\frac{p}{p \cdot q}$$

Roxy has big eyes.
Roxy has a tail.
Roxy has big eyes and a tail.

Addition (Add)

$$\frac{p}{p \vee q}$$

Theo has spots.
Theo has either spots or stripes.

Remember: the Ps, Qs, and Rs can stand for simple statements *and* complex statements.

These arguments are both instances of **constructive dilemma** (CD):

$$\frac{\sim M \vee N \quad [(K \supset T) \supset (A \cdot B)] \cdot [(H \supset P) \supset (A \cdot C)]}{(\sim M \supset S) \cdot (N \supset \sim T) \quad (K \supset T) \vee (H \supset P)} \quad \frac{}{S \vee \sim T \quad (A \cdot B) \vee (A \cdot C)}$$

These arguments are all instances of **simplification** (Simp):

$$\frac{\sim F \cdot (U \equiv E)}{\sim F} \quad \frac{(M \vee T) \cdot (S \supset R)}{M \vee T} \quad \frac{[(X \supset Z) \cdot M] \cdot (G \supset H)}{(X \supset Z) \cdot M}$$

These arguments are all instances of **conjunction** (Conj):

$$\frac{\sim E \quad C \supset M}{\sim E \cdot \sim G} \quad \frac{D \supset N}{(C \supset M) \cdot (D \supset N)} \quad \frac{R \supset (H \cdot T) \quad K \supset (H \cdot O)}{[R \supset (H \cdot T)] \cdot [K \supset (H \cdot O)]}$$

These arguments are all instances of **addition** (Add):

$$\frac{S}{S \vee \sim T} \quad \frac{(C \cdot D)}{(C \cdot D) \vee (K \cdot \sim P)} \quad \frac{W \equiv Z}{(W \equiv Z) \vee [A \supset (M \supset O)]}$$

Example 1:

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|--|--------------|
| 1. $K \supset L$ | |
| 2. $(M \supset N) \cdot S$ | |
| 3. $N \supset T$ | |
| 4. $K \vee M$ | / $L \vee T$ |
| 5. $M \supset N$ | 2, Simp |
| 6. $M \supset T$ | 3, 5, HS |
| 7. $(K \supset L) \cdot (M \supset T)$ | 1, 6, Conj |
| 8. $L \vee T$ | 4, 7, CD |

Example 2:

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|---------------------------------|--------------|
| 1. $\sim M \cdot N$ | |
| 2. $P \supset M$ | |
| 3. $Q \cdot R$ | |
| 4. $(\sim P \cdot Q) \supset S$ | / $S \vee T$ |
| 5. $\sim M$ | 1, Simp |
| 6. $\sim P$ | 2, 5, MT |
| 7. Q | 3, Simp |
| 8. $\sim P \cdot Q$ | 6, 7, Conj |
| 9. S | 4, 8, MP |
| 10. $S \vee T$ | 9, Add |