Symbols, Translations, and Truth Tables

Today's class marked the transition from Categorical logic to Propositional logic. Categorical logic was concerned (primarily) with the categorical terms (components of propositions), whereas Propositional logic will be (primarily) concerned with whole sentences and connectives.

Atomic / Simple Statement: A statement devoid of the five connectives / operators.

Complex / Compound Statement: A statement that contains at least one of the five connectives / operators.

When you have simple statements you can simply name them with a capital letter. Typically you will choose a letter which corresponds to a prominent word in the statement. You can choose, however, whatever letter you want as long as you make clear what statement the letter corresponds to. For example:

Tom is nice = T	The bluefin tuna is facing extinction = B
James Joyce wrote *Ulysses* = U	The Huskers are a football team = H

Connectives / Operators: We have five operators which are diagramed below:

Operator	Name	Logical function	Used to translate
$\sim$	tilde	negation	not, it is not the case that
$\bullet$	dot	conjunction	and, also, moreover
$\vee$	wedge	disjunction	or, unless
$\supset$	horseshoe	implication	if . . . then . . . , only if
$\equiv$	triple bar	equivalence	if and only if

Note: that there are a number of English phrases the which correspond to our five connectives. These phrases may not be intuitive and so it is usually a good idea to keep the following list handy when translating from English into our logic.

Summary	Operator
not, it is not the case that, it is false that	$\sim$
and, yet, but, however, moreover, nevertheless, still, also, although, both, additionally, furthermore	$\cdot$
or, unless	$\vee$
if . . . then, only if, implies, given that, in case, provided that, on condition that, sufficient condition for, necessary condition for (Note: Do not confuse antecedent with consequent!)	$\supset$
if and only if, is equivalent to, sufficient and necessary condition for	$\equiv$

When you properly translate an English atomic or complex sentence into our logic, then you will have created a well-formed formula.

> **Well-formed formula / wff (pronounce: woof)**: A syntactically allowable arrangement of symbols (i.e. you are properly creating statements according to the grammatical rules of our logic). Atomic and complex statements are both wffs.

You now have the tools to translate complex statements into our symbolic system. For the sentence: **It is not the case that cats are helpful**, we will isolate the atomic component of this compound statement. The atomic component is **cats are helpful**. I will give this the letter **C**. The unary connective ~ is the corresponding symbol for the phrase **it is not the case that**. Hence, our translated sentence is: **~C**. Pretty simple! The same procedure applies to each of the connectives.

Each of the binary connectives (two-place: $\cdot$, $\vee$, $\supset$, $\equiv$) have to be flanked on either side by either simple or compound wffs. The unary connective (one-place: ~) is placed on the left hand side of wffs. Any string of symbols which does not adhere to these placement rules is not a wff.

Example of WFFs	Example of non-WFFs
$A \supset B$ $A \vee C$ $(A \supset B) \equiv (A \vee C)$ $\sim B \cdot C$	$AB \cdot$ $\sim BA$ $A \supset B \equiv A \vee C$[1] $\equiv A \vee C$

[1] This particular example is not a wff because there is a lack of disambiguating parentheses. We will discuss the introduction rules for parentheses during the next class.

Truth Tables:
Now that we have the basic grammar explained we can begin assessing under what conditions a wff receives a truth value of true, and under what conditions a wff receives a truth value of false.

We begin with the most basic entity in our logic: atomic sentences. Atomic sentences just have the truth value that they have. What I mean is that if A is true, then A's truth value is true. This is pretty obvious (I hope) since no operation is being performed on the atomic sentence.

Atomic Statement	p	p
	T F	T F

The conditions for complex statements, however, is more complicated.

Negation

p	$\sim p$
T	F
F	T

Conditional
(material implication)

p	q	$p \supset q$
T	T	T
T	F	F
F	T	T
F	F	T

Conjunction

p	q	$p \bullet q$
T	T	T
T	F	F
F	T	F
F	F	F

Biconditional
(material equivalence)

p	q	$p \equiv q$
T	T	T
T	F	F
F	T	F
F	F	T

Disjunction

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Now that we understand the truth tables for all of our atomic sentences and our connectives, we can compute the truth value of longer propositions. Take the following wff: $(A \supset B) \equiv (A \vee C)$. In the case where we are told the truth values for each sentence letter, we can just plug in T's for the true atomic sentences, and F's for false atomic sentences. So, if you are told A and C are true, and B is false, then the wff can be changed to: $(T \supset F) \equiv (T \vee T)$. We then compute the truth value of wffs in parentheses. For $(T \supset F)$ we check the above tables, and see that this wff is gets an F. Now our wff looks like: $F \equiv (T \vee T)$. We then compute the truth value for the other wff in parense. $(T \vee T)$ according to the above table gets a T. Now our wff looks like: $F \equiv T$. We check the table for this and see that it gets an F. Hence, the wff is false when those assigned truth values are stated.

The $\equiv$ in the above wff was what is called the **main / dominant connective**.

> **Main / dominant connective**: The connective which has the widest scope (i.e. the truth of the wff depends upon it *more* than any other connectives in the wff).

These statements are all **negations**. The main operator is a tilde.

$\sim B$
$\sim (G \supset H)$
$\sim [(A \equiv F) \cdot (C \equiv G)]$

These statements are all **conjunctions**. The main operator is a dot.

$K \cdot \sim L$
$(E \vee F) \cdot \sim (G \vee H)$
$[(R \supset T) \vee (S \supset U)] \cdot [(W \equiv X) \vee (Y \equiv Z)]$

These statements are all **disjunctions**. The main operator is a wedge.

$\sim C \vee \sim D$
$(F \cdot H) \vee (\sim K \cdot \sim L)$
$\sim [S \cdot (T \supset U)] \vee \sim [X \cdot (Y \equiv Z)]$

These statements are all **conditionals** (material implications). The main operator is a horseshoe.

$H \supset \sim J$
$(A \vee C) \supset \sim (D \cdot E)$
$[K \vee (S \cdot \sim T)] \supset [\sim F \vee (M \cdot O)]$

These statements are all **biconditionals** (material equivalences).
The main operator is a triple bar.

$M \equiv \sim T$

$\sim(B \vee D) \equiv \sim(A \cdot C)$

$[K \vee (F \supset I)] \equiv [\sim L \cdot (G \vee H)]$

However, you don't need to be told what the sentence letters have as their truth values in order to know when the wff is either true or false. You can figure out every possible truth value for the sentence letters and thereby compute the truth value of the entire wff.

Suppose, for example, that we are given this proposition: $[(A \vee B)\ (B \supset A)] \supset B$. We would begin by constructing columns for the simple propositions A and B, writing them to the left of the given proposition:

A	B	$[(A \vee B) \cdot (B \supset A)] \supset B$
T	T	
T	F	
F	T	
F	F	

We would then use the columns on the left to derive the truth values of the compound propositions. First, we need to determine the dominant connective. In this case, it is the furthest right horseshoe. Thus, we would compute first the truth values of the expressions in parentheses, then the dot, and finally the right-hand horseshoe:

A	B	$[(A \vee B)$	$\cdot$	$(B \supset A)]$	$\supset B$
T	T	T	T	T	$\boxed{T}$
T	F	T	T	T	$\boxed{F}$
F	T	T	F	F	$\boxed{T}$
F	F	F	F	T	$\boxed{T}$