Indirect Truth Tables

An indirect truth table saves you much time and effort by avoiding the lines on a given truth table that are irrelevant to what you are attempting to prove.

Let’s look at the following argument.

N ∨ ~O	P ∨ O	P ⊃ Q	(N ∨ Q) ⊃ (R • S)	S ⊃ (R ⊃ T)	O ⊃ (T ⊃ U)	// U

The directions state to use the indirect method for determining whether the argument is valid or invalid. The quickest way to do this is to set up the line(s) on the truth table that we care about. In this case, we are going to prove that it is invalid. Thus, we will set the conclusion to false (in this case, just the sentence letter U).

N ∨ ~O	P ∨ O	P ⊃ Q	(N ∨ Q) ⊃ (R • S)	S ⊃ (R ⊃ T)	O ⊃ (T ⊃ U)	// U
					F	F

Since we want to see if the argument is invalid, we will try to make each premise compute as T because that would be sufficient to show that the argument is invalid (see end of last handout). This is usually the most confusing part since we have *very little* information about the remaining premises. So, let’s begin with the premise we have the most info about: the 6th premise. We want to try everything we can to make it come out as T. So, we need to figure out what values O and T can have to make that happen. We also note that O also plays an important role in the first and second premise - hence, we can get a lot done by focusing on O’s truth value.

Let’s try having O be false since that will guarantee the 6th premise comes out true, but let’s everything else (except U of course) be true. The first premise comes out as true, as does the second, third, fourth, fifth, *and* the sixth premise. Hence we generated a line which has all true premises and a false conclusion. That is EXACTLY what we wanted.

N ∨ ~O	P ∨ O	P ⊃ Q	(N ∨ Q) ⊃ (R • S)	S ⊃ (R ⊃ T)	O ⊃ (T ⊃ U)	// U
T **T** T F	T **T** F	T **T** T	T T T **T** T T T	T **T** T T T	F **T** T F F	F

So, the process for creating an indirect truth table turns out to be very simple.

1. Determine what truth values are needed to make the conclusion false. Those will be the lines we will focus on for the truth tables.
2. Fill in the values for the conclusion everywhere else the sentence letters in the conclusion occur.
3. Then, see if applying either T or all F to the remaining sentence letters makes the dominant operator for the premises come out as true.
4. If step three works, then you are done and the argument is invalid.
5. If step three does not work, ask yourself if there is a minor adjustment to one or more of the sentence letters that will make the premises all true.
6. If step 5 works, then you are done and the argument is invalid.

If this is still unclear, then you can always do a less-indirect-truth-table. Do steps 1 and 2. But, instead of doing 3, create all the lines on truth table by computing all the T / F combinations following our 2^n formula. Then, get rid of the lines which do not match step 1. Use the remaining lines to compute all the truth values for the premises. If there are any lines on which the premises get Ts but the conclusion is an F, then the argument is invalid. Depending on the number of sentence letters in the premises, this could be quite big, but, if the indirect way is not clear, then this is your only option short of a direct truth table.