Quality, Quantity, and Bool's Rules

> **Categorical Proposition (slightly different definition)**: A proposition relating two classes; it asserts that either all or part of one class is either included or excluded from the other.

Categorical propositions have additional properties than those explored in the previous handout. You also need to be able to identify a categorical proposition's quality and quantity. Again, these may seem unintuitive at first, but I think you will find that they are fairly graspable when we explore examples.

There are only two quantities that categorical propositions can have:

> **Universal**: Asserts something about **all** members of the subject class.

> **Particular**: Asserts something about **only some** of members of the subject class.

Additionally, there are two qualities that categorical propositions can have:

> **Affirmative**: Affirms class membership.

> **Negative**: Denies class membership.

Since the early Middle Ages the four kinds of categorical propositions have commonly been designated by letter names corresponding to the first four vowels of the Roman alphabet: A, E, I, O.

The universal affirmative is called an **A** proposition.
All S are P.

The universal negative an **E** proposition.
No S are P.

The particular affirmative an **I** proposition.
Some S are P.

The particular negative an **O** proposition.
Some S are not P.

Each of the 4 standard form categorical propositions has a unique combination of quantity, quality, and a letter name:

Standard Form	Quantity	Quality	Letter Name
All S are P	Universal	Affirmative	A
No S are P	Universal	Negative	E
Some S are P	Particular	Affirmative	I
Some S are not P	Particular	Negative	O

Despite this probably being clear, there is a rather problematic (and interesting!) philosophical problem that arises here. Though categorical propositions have a fairly rigid structure, we can manufacture them quite easily. Simply insert something for S and something for P and - poof! - we have a new categorical proposition. What are allowed to infer, however, about the existence of the S's that we invoke?

To bring the point home, what if we talked about something we know doesn't exist, but can say true things about by using universal propositions. For example: *All ringwraiths are not humans*. Ringwraiths, as we all **should** know, are fictional creations from Tolkien's *Lord of the Rings*. Are we allowed to infer the existence of the subject terms or predicate terms that we use in a universal categorical proposition?

If not, why? If so, why?

Regardless of our feelings on these issues, logicians have basically divided into two groups. Those who claim that we must allow **existential import** for some universal categorical propositions, but not others, adhere to the **Aristotelian standpoint**. Those that deny we can have any existential import from any universal categorical propositions adhere to the **Boolean standpoint**.

> **Aristotelian Existential Import (aka: Traditional Interpretation)**: for any universal categorical proposition, you only allowed to infer the existence of things that actually exist.

All pigeons are birds.	Implies the existence of pigeons.
No pine trees are redwoods.	Implies the existence of pine trees.
All ringwraiths are evil.	Does *not* imply the existence of ringwraiths.

Boolean Existential Import (aka: Modern Interpretation): for any universal categorical proposition, you are never allowed to infer the existence of the 'objects' contained within the proposition.

All pigeons are birds.	Does *not* imply the existence of pigeons.
No pine trees are redwoods.	Does *not* imply the existence of pine trees.
All ringwraiths are evil.	Does *not* imply the existence of ringwraiths.

The Aristotelian standpoint differs from the Boolean standpoint only with regard to universal (A and E) propositions. The two standpoints are identical with regard to particular (I and O) propositions. Both the Aristotelian and the Boolean standpoints recognize that **particular** propositions make a **positive** assertion about existence (i.e. that something does exist). For example, from both standpoints, the statement "Some cats are animals" asserts that at least one cat exists that is an animal. Also, from both standpoints, "Some fish are not mammals" asserts that at least one fish exists that is not a mammal. Thus, from both standpoints, the word "some" implies existence.

The Modern Interpretation

First things first, we need a rigorous understanding of *contradiction*. Two statements will be contradictory when they must have opposite truth values.

Contradictory Propositions: propositions which must have opposite truth values (i.e. when one is true, the other is false and vice versa).

There are two pairs of contradictory categorical propositions:

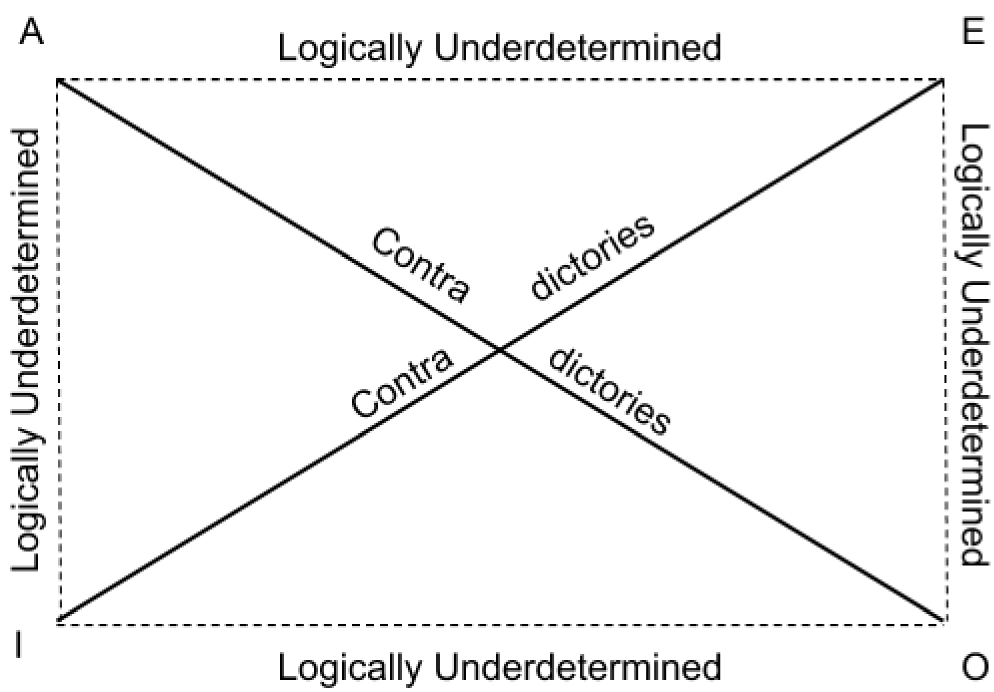
Contradictory Categorical Propositions	
A and O	E and I
All are **vs**. Some are not	None are **vs**. Some are

*Why are A and E not contradictory? Well, since we using the modern (i.e. Boolean) interpretation, A and E do not entail anything about existence. Hence, the following two propositions are **do not** have opposite truth values: All unicorns have a single horn *and* No unicorns have a single horn. Why are they not contradictory? It is simple: because we are not allowed to infer the existence of unicorns. Hence, if unicorns do not exist, then it is true that all and none of them have a single horn. Our universal categorical propositions are not claiming anything about the way the world actually is.

Given all this discussion we now have 8 valid inferences (valid in our technical sense from last week!).

If A true → then O false	If E true → then I false
If O true → then A false	If I true → then E false
If A false → then O true	If E false → then I true
If O false → then A true	If I false → then E true

The Modern Square



Logically Underdetermined: Two propositions are logically underdetermined when the truth value of one proposition does not allow you to validly infer the truth or falsity of the other.