More Rules!

Today we finalized our list of inference rules.[1] The final rules are:

14. Transposition (Trans):
$(p \supset q) :: (\sim q \supset \sim p)$

15. Material implication (Impl):
$(p \supset q) :: (\sim p \vee q)$

16. Material equivalence (Equiv):
$(p \equiv q) :: [(p \supset q) \cdot (q \supset p)]$
$(p \equiv q) :: [(p \cdot q) \vee (\sim p \cdot \sim q)]$

17. Exportation (Exp):
$[(p \cdot q) \supset r] :: [p \supset (q \supset r)]$

18. Tautology (Taut):
$p :: (p \vee p)$
$p :: (p \cdot p)$

Example problem 1:

1. $F \supset G$	
2. $F \vee G$	/ G
3. $\sim\sim F \vee G$	2, DN
4. $\sim F \supset G$	3, Impl
5. $\sim F \supset \sim\sim G$	4, DN
6. $\sim G \supset F$	5, Trans
7. $\sim G \supset G$	1, 6, HS
8. $\sim\sim G \vee G$	7, Impl
9. $G \vee G$	8, DN
10. G	9, Taut

Example problem 2:

1. $P \supset Q$	
2. $R \supset (S \cdot T)$	
3. $\sim R \supset \sim Q$	
4. $S \supset (T \supset P)$	/ $P \equiv R$
5. $Q \supset R$	3, Trans
6. $P \supset R$	1, 5, HS
7. $(S \cdot T) \supset P$	4, Exp
8. $R \supset P$	2, 7, HS
9. $(P \supset R) \cdot (R \supset P)$	6, 8, Conj
10. $P \equiv R$	9, Equiv

[1] Again, if you want to see why these are called *exchange rules* you need only do a truth table, and you will see that the left hand side of the four dots is equivalent to the right side, and vice versa.

Example problem 3:

1.	$K \supset M$	
2.	$L \supset M$	/ $(K \vee L) \supset M$
3.	$(K \supset M) \cdot (L \supset M)$	1, 2, Conj
4.	$(\sim K \vee M) \cdot (L \supset M)$	3, Impl
5.	$(\sim K \vee M) \cdot (\sim L \vee M)$	4, Impl
6.	$(M \vee \sim K) \cdot (\sim L \vee M)$	5, Com
7.	$(M \vee \sim K) \cdot (M \vee \sim L)$	6, Com
8.	$M \vee (\sim K \cdot \sim L)$	7, Dist
9.	$(\sim K \cdot \sim L) \vee M$	8, Com
10.	$\sim(K \vee L) \vee M$	9, DM
11.	$(K \vee L) \supset M$	10, Impl