

Replacement Rules 1

The rules we will be looking at today are not so much inference rules (like the last 8 have been) where we go from a set of premises to a conclusion. The rules for today are known as replacement rules because we will be allowed to re-state a given line in our proof with another wff that is logically equivalent to the original line (remember our definition of equivalent).

9. De Morgan's rule (DM):

$$\sim(p \cdot q) :: (\sim p \vee \sim q)$$

$$\sim(p \vee q) :: (\sim p \cdot \sim q)$$

10. Commutativity (Com):

$$(p \vee q) :: (q \vee p)$$

$$(p \cdot q) :: (q \cdot p)$$

11. Associativity (Assoc):

$$[p \vee (q \vee r)] :: [(p \vee q) \vee r]$$

$$[p \cdot (q \cdot r)] :: [(p \cdot q) \cdot r]$$

12. Distribution (Dist):

$$[p \cdot (q \vee r)] :: [(p \cdot q) \vee (p \cdot r)]$$

$$[p \vee (q \cdot r)] :: [(p \vee q) \cdot (p \vee r)]$$

13. Double negation (DN):

$$p :: \sim\sim p$$

Some examples of how to use these replacement rules in proofs are provided below:

Example 1:

| | |
|--------------------------------|-------------------|
| 1. $A \supset \sim(B \cdot C)$ | |
| 2. $A \cdot C$ | / $\sim B$ |
| 3. A | 2, Simp |
| 4. $\sim(B \cdot C)$ | 1, 3, MP |
| 5. $\sim B \vee \sim C$ | 4, DM |
| 6. C | 2, Com, Simp |
| 7. $\sim B$ | 5, 6, Com, DN, DS |

Example 2:

| | |
|-----------------------------------|---------------|
| 1. $D \cdot (E \vee F)$ | |
| 2. $\sim D \vee \sim F$ | / $D \cdot E$ |
| 3. $(D \cdot E) \vee (D \cdot F)$ | 1, Dist |
| 4. $(D \cdot F) \vee (D \cdot E)$ | 3, Com |
| 5. $\sim(D \cdot F)$ | 2, DM |
| 6. $D \cdot E$ | 4, 5, DS |