

Practice Final Answer Key

18 Rules:

Use the 18 rules of inference to derive the conclusion of the following arguments. Do **not** use either conditional proof or indirect proof.

c. (10 pts)

1. $\sim(O \supset R) \supset S$
2. $O \supset \sim S$ $/ \sim O \vee R$

| | | | | |
|----|------------------------------------|---------------|---------------------------------------|---------|
| 1. | $\sim(O \supset R) \supset S$ | | 1 $\sim(O \supset R) \supset S$ | |
| 2. | $O \supset \sim S$ | 1- $O \vee R$ | 2. $O \supset \sim S$ | |
| 3. | $\sim S \supset \sim(O \supset R)$ | 1 Trans | 3. $\sim S \supset \sim(O \supset R)$ | 1 Trans |
| 4. | $\sim S \supset (O \supset R)$ | 3 DN | 4. $\sim S \supset (O \supset R)$ | 3 DN |
| 5. | $O \supset (O \supset R)$ | 2, 4 HS | 5. $O \supset (O \supset R)$ | 2, 4 HS |
| 6. | $\sim O \vee (O \supset R)$ | 5 Impl | 6. $(O \cdot O) \supset R$ | 5 Exp |
| 7. | $\sim O \vee (\sim O \vee R)$ | 6 Impl | 7. $O \supset R$ | 6 Taut |
| 8. | $(\sim O \vee \sim O) \vee R$ | 7 Assoc | 8. $\sim O \vee R$ | 7 Imp |
| 9. | $\sim O \vee R$ | 8 Taut | | |

d. (20 pts)

1. $\sim I \supset (K \supset \sim R)$
2. $I \supset (K \supset A)$
3. $A \supset M$
4. $K \cdot \sim M$ $/ \sim(R \vee I)$

(2)

| | | | |
|-----|-------------------------------------|---------------------------|--|
| 1. | $\sim I \supset (K \supset \sim R)$ | | |
| 2. | $I \supset (K \supset A)$ | | |
| 3. | $A \supset M$ | | |
| 4. | $K \cdot \sim M$ | $\neg(R \vee I)$ | |
| 5. | $(I \cdot K) \supset A$ | 2, Exp | |
| 6. | $(\sim I \cdot K) \supset \sim R$ | 1, Exp | |
| 7. | $\sim M \cdot K$ | 4, Comm | |
| 8. | $\sim M$ | 7, Simp | |
| 9. | $\sim A$ | 3, 8, MT | |
| 10. | $\sim(I \cdot K)$ | 5, 9, MT | |
| 11. | $\sim I \vee \sim K$ | 10, DM | |
| 12. | $\sim K \vee \sim I$ | 11, Comm | |
| 13. | $K \cdot \sim M$ | 7, Comm | |
| 14. | K | 13, Simp | |
| 15. | $\sim \sim K$ | 14, DN | |
| 16. | $\sim I$ | 12, 15, DS | |
| 17. | $K \supset \sim R$ | 1, 16, MP | |
| 18. | $\sim R$ | 14, 17, MP | |
| 19. | $\sim R \cdot \sim I$ | 16, 18, Conj ² | |
| 20. | $\sim(R \vee I)$ | 19, DM | |

Al's version on back \Rightarrow

(Neater version)

| | | |
|-----|-----------------------|--------------|
| 9. | $K \cdot \sim M$ | 7, Comm |
| 10. | K | 9, Simp |
| 11. | $\sim A$ | 3, 8, MT |
| 12. | $\sim(I \cdot K)$ | 5, 11, MT |
| 13. | $\sim I \vee \sim K$ | 12, DM |
| 14. | $\sim K \vee \sim I$ | 13, Comm |
| 15. | $\sim \sim K$ | 10, DN |
| 16. | $\sim I$ | 14, 15, DS |
| 17. | $K \supset \sim R$ | 1, 16, MP |
| 18. | $\sim R$ | 10, 17, MP |
| 19. | $\sim R \cdot \sim I$ | 16, 18, Conj |
| 20. | $\sim(R \vee I)$ | 19, DM |

Conditional Proof:

Use the 18 rules of inference and the conditional proof method to derive the conclusion of the following arguments. Do **not** use indirect proof.

e. (10 pts)

1. $P \supset (I \supset W)$ 2. $I \supset (W \supset \sim S)$ $\therefore P \supset (I \supset \sim S)$

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|-----|--------------------------------|--------|--|-----|--------------------------------|--------|--|
| 1. | $P \supset (I \supset W)$ | | | 1. | $P \supset (I \supset W)$ | | |
| 2. | $I \supset (W \supset \sim S)$ | | | 2. | $I \supset (W \supset \sim S)$ | | |
| 3. | P | ACP | | 3. | P | ACP | |
| 4. | $I \supset W$ | 1,3 MP | | 4. | $I \supset W$ | 1,3 | |
| 5. | I | ACP | | 5. | I | ACP | |
| 6. | $W \supset \sim S$ | 2,5 MP | | 6. | $W \supset \sim S$ | 2,5 MP | |
| 7. | W | 4,5 MP | | 7. | $I \supset \sim S$ | 4,6 HS | |
| 8. | $\sim S$ | 6,7 MP | | 8. | $\sim S$ | 5,7 MP | |
| 9. | $I \supset \sim S$ | 5-8 CP | | 9. | $I \supset \sim S$ | 5-8 CP | |
| 10. | $P \supset (I \supset \sim S)$ | 3-9 CP | | 10. | $P \supset (I \supset \sim S)$ | 3-9 CP | |

f. (25 pts)

- $(G \cdot \sim P) \supset K$
- $E \supset Z$
- $P \supset \sim Z$
- $G \supset (L \vee E)$

 $\therefore (G \cdot \sim L) \supset K$

| | | | | | |
|-----|------------------------------|------------|--|---|--|
| 1. | $(G \cdot \sim P) \supset K$ | | | | |
| 2. | $E \supset Z$ | | | | |
| 3. | $P \supset \sim Z$ | | | | |
| 4. | $G \supset (L \vee E)$ | | | $\therefore (G \cdot \sim L) \supset K$ | |
| 5. | $G \cdot \sim L$ | ACP | | | |
| 6. | G | 5, SIMP | | | |
| 7. | $\sim L \cdot G$ | 5, COMM | | | |
| 8. | $\sim L$ | 7, SIMP | | | |
| 9. | $L \vee E$ | 4,6 MP | | | |
| 10. | E | 8,9 DS | | | |
| 11. | Z | 2,10 MP | | | |
| 12. | $\sim \sim Z$ | 11, DN | | | |
| 13. | $\sim P$ | 3,12 MT | | | |
| 14. | $G \cdot \sim P$ | 6,13 Conj. | | | |
| 15. | K | 1,14 MP | | | |
| 16. | $(G \cdot \sim L) \supset K$ | 5-15 CP | | | |

Indirect Proof:

Use the 18 rules of inference and the indirect proof method to derive the conclusion of the following arguments. Do **not** use conditional proof.

g. (25 pts)

- $C \supset (D \supset H)$
- $D \cdot \sim H$
- $H \vee T$

 $\therefore \sim C \cdot T$

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|-----|---|---------------|-----|---------------------------|---------------|
| 1. | $C \supset (D \supset H)$ | | | | |
| 2. | $D \cdot \sim H$ | | | | |
| 3. | $H \vee T$ | $I-C \cdot T$ | | | |
| 4. | $\sim(\sim C \cdot T)$ | AIP | 4 | $\sim(\sim C \cdot T)$ | AIP |
| 5. | D | 2, Simp | 5 | $\sim C \vee \sim T$ | 4 DN |
| 6. | $\sim H \cdot D$ | 2, COMM | 6 | $\sim H \cdot D$ | 6 Simp 2 comm |
| 7. | $\sim H$ | 6, Simp | 7. | $\sim H$ | 6 Simp |
| 8. | $(C \cdot D) \supset H$ | 1, Exp | 8. | T | 3, 7 DS |
| 9. | $\sim(C \cdot D)$ | 7, 8 MT | 9. | $\sim T \vee \sim \sim C$ | 5 comm |
| 10. | $\sim C \vee \sim D$ | 9, DM | 10. | $\sim \sim T$ | 8 DN |
| 11. | $\sim D \vee \sim C$ | 10, COMM | 11. | $\sim \sim C$ | 9, 10 DS |
| 12. | $\sim \sim D$ | 5, DN | 12. | C | 11 DN |
| 13. | $\sim C$ | 11, 12, DS | 13. | $D \supset H$ | 1, 12 MP |
| 14. | T | 3, 7, DS | 14. | D | 2 Simp |
| 15. | $\sim C \cdot T$ | 13, 14, Conj. | 15. | H | 13, 14 MP |
| 16. | $(\sim C \cdot T) \cdot \sim(\sim C \cdot T)$ | 4, 15 Conj. | 16. | $H \cdot \sim H$ | T, H Conj. |
| 17. | $\sim \sim(\sim C \cdot T)$ | 4-16 IP | | | |
| 18. | $\sim C \cdot T$ | 17, DN | 17 | $\sim C \cdot T$ | 4-16 IP |