

Replacement Rules 2

14. Transposition (Trans):

$$(p \supset q) :: (\sim q \supset \sim p)$$

15. Material implication (Impl):

$$(p \supset q) :: (\sim p \vee q)$$

16. Material equivalence (Equiv):

$$(p \equiv q) :: [(p \supset q) \cdot (q \supset p)]$$

$$(p \equiv q) :: [(p \cdot q) \vee (\sim p \cdot \sim q)]$$

17. Exportation (Exp):

$$[(p \cdot q) \supset r] :: [p \supset (q \supset r)]$$

18. Tautology (Taut):

$$p :: (p \vee p)$$

$$p :: (p \cdot p)$$

Example problem 1:

1. $F \supset G$	
2. $F \vee G$	/ G
3. $\sim \sim F \vee G$	2, DN
4. $\sim F \supset G$	3, Impl
5. $\sim F \supset \sim \sim G$	4, DN
6. $\sim G \supset F$	5, Trans
7. $\sim G \supset G$	1, 6, HS
8. $\sim \sim G \vee G$	7, Impl
9. $G \vee G$	8, DN
10. G	9, Taut

Example problem 2:

1. $P \supset Q$	
2. $R \supset (S \cdot T)$	
3. $\sim R \supset \sim Q$	
4. $S \supset (T \supset P)$	/ $P \equiv R$
5. $Q \supset R$	3, Trans
6. $P \supset R$	1, 5, HS
7. $(S \cdot T) \supset P$	4, Exp
8. $R \supset P$	2, 7, HS
9. $(P \supset R) \cdot (R \supset P)$	6, 8, Conj
10. $P \equiv R$	9, Equiv

Example problem 3:

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|--|--------------------------|
| 1. $K \supset M$ | |
| 2. $L \supset M$ | / $(K \vee L) \supset M$ |
| 3. $(K \supset M) \cdot (L \supset M)$ | 1, 2, Conj |
| 4. $(\sim K \vee M) \cdot (L \supset M)$ | 3, Impl |
| 5. $(\sim K \vee M) \cdot (\sim L \vee M)$ | 4, Impl |
| 6. $(M \vee \sim K) \cdot (\sim L \vee M)$ | 5, Com |
| 7. $(M \vee \sim K) \cdot (M \vee \sim L)$ | 6, Com |
| 8. $M \vee (\sim K \cdot \sim L)$ | 7, Dist |
| 9. $(\sim K \cdot \sim L) \vee M$ | 8, Com |
| 10. $\sim(K \vee L) \vee M$ | 9, DM |
| 11. $(K \vee L) \supset M$ | 10, Impl |