Inference Rules 2

Constructive dilemma (CD)

$(p \supset q) \cdot (r \supset s)$

$p \vee r$

$q \vee s$

If Oscar is a dog, then you'll have fleas, and
    if Oscar is a cat, then you'll have fur balls.
Oscar is either a dog or a cat.
You'll have either fleas or fur balls.

Simplification (Simp)

$p \cdot q$

p

Eliza has long legs and runs fast.
Eliza has long legs.

Conjunction (Conj)

p

q

$p \cdot q$

Roxy has big eyes.
Roxy has a tail.
Roxy has big eyes and a tail.

Addition (Add)

p

$p \vee q$

Theo has spots.
Theo has either spots or stripes.

Remember: the Ps, Qs, and Rs can stand for simple statements *and* complex statements.

These arguments are both instances of **constructive dilemma** (CD):

$\sim M \vee N$

$(\sim M \supset S) \cdot (N \supset \sim T)$

$S \vee \sim T$

$[(K \supset T) \supset (A \cdot B)] \cdot [(H \supset P) \supset (A \cdot C)]$

$(K \supset T) \vee (H \supset P)$

$(A \cdot B) \vee (A \cdot C)$

These arguments are all instances of **simplification** (Simp):

$\sim F \cdot (U \equiv E)$

$\sim F$

$(M \vee T) \cdot (S \supset R)$

$M \vee T$

$[(X \supset Z) \cdot M] \cdot (G \supset H)$

$(X \supset Z) \cdot M$

These arguments are all instances of **conjunction** (Conj):

$\sim E$

$\sim G$

$\sim E \cdot \sim G$

$C \supset M$

$D \supset N$

$(C \supset M) \cdot (D \supset N)$

$R \supset (H \cdot T)$

$K \supset (H \cdot O)$

$[R \supset (H \cdot T)] \cdot [K \supset (H \cdot O)]$

These arguments are all instances of **addition** (Add):

S

$S \vee \sim T$

$(C \cdot D)$

$(C \cdot D) \vee (K \cdot \sim P)$

$W \equiv Z$

$(W \equiv Z) \vee [A \supset (M \supset O)]$

Example 1:

1. $K \supset L$
2. $(M \supset N) \cdot S$
3. $N \supset T$
4. $K \vee M$　　　　　　　　/ $L \vee T$
5. $M \supset N$　　　　　　2, Simp
6. $M \supset T$　　　　　　3, 5, HS
7. $(K \supset L) \cdot (M \supset T)$　　1, 6, Conj
8. $L \vee T$　　　　　　　4, 7, CD

Example 2:

1. $\sim M \cdot N$
2. $P \supset M$
3. $Q \cdot R$
4. $(\sim P \cdot Q) \supset S$　　/ $S \vee T$
5. $\sim M$　　　　　　　1, Simp
6. $\sim P$　　　　　　　2, 5, MT
7. Q　　　　　　　　3, Simp
8. $\sim P \cdot Q$　　　　6, 7, Conj
9. S　　　　　　　　4, 8, MP
10. $S \vee T$　　　　　9, Add