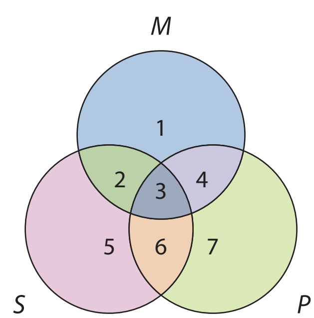
5

5.2 Venn Diagrams

PREVIEW • *You hear on TV that a robbery is in progress at a local bank. The bank employees are being held as hostages in the vault, while the customers are hiding from the robbers in a meeting room. You then discover that a friend of yours was in the bank when the robbery began. You reason that since your friend was a customer, he is in the meeting room—which means that he is not being held hostage. This reasoning process, which assigns classes to three spatial areas, underlies the application of Venn diagrams to syllogisms.*

Venn diagrams provide the most intuitively evident and, in the long run, easiest to remember technique for testing the validity of categorical syllogisms. The technique is basically an extension of the one developed in Chapter 4 to represent the informational content of categorical propositions. Because syllogisms contain three terms, whereas propositions contain only two, the application of Venn diagrams to syllogisms requires three overlapping circles.

These circles should be drawn so that seven areas are clearly distinguishable within the diagram. The second step is to label the circles, one for each term. The precise order of the labeling is not critical, but we will adopt the convention of always assigning the lower-left circle to the subject of the conclusion, the lower-right circle to the predicate of the conclusion, and the top circle to the middle term. This convention is easy to remember because it conforms to the arrangement of the terms in a standard-form syllogism: The subject of the conclusion is on the lower left, the predicate of the conclusion is on the lower right, and the middle term is in the premises, above the conclusion.*



Anything in the area marked "1" is an *M* but neither an *S* nor a *P*, anything in the area marked "2" is both an *S* and an *M* but not a *P*, anything in the area marked "3" is a member of all three classes, and so on.

The test procedure consists of transferring the information content of the premises to the diagram and then inspecting the diagram to see whether it necessarily implies

Some textbooks present three-circle Venn diagrams with two circles on the top and one on the bottom. This textbook presents them the way John Venn himself drew them, with two circles on the bottom and one on the top. See his Symbolic Logic *(London: Macmillan, 1881), 105, 114, 116.*

the truth of the conclusion. If the information in the diagram does do this, the argument is valid; otherwise it is invalid.

The use of Venn diagrams to evaluate syllogisms usually requires a little practice at first. Perhaps the best way of learning the technique is through illustrative examples, but a few pointers are needed first:

1. Marks (shading or placing an X) are entered only for the premises. No marks are made for the conclusion.

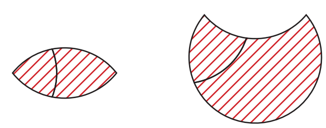
2. If the argument contains one universal premise, this premise should be entered first in the diagram. If there are two universal premises, either one can be done first.

3. When entering the information contained in a premise, one should concentrate on the circles corresponding to the two terms in the statement. While the third circle cannot be ignored altogether, it should be given only minimal attention.

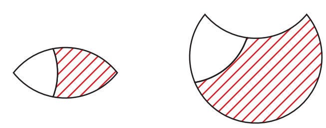
4. When inspecting a completed diagram to see whether it supports a particular conclusion, one should remember that particular statements assert two things. "Some *S* are *P*" means "At least one *S* exists *and* that *S* is a *P*"; "Some *S* are not *P*" means "At least one *S* exists *and* that *S* is not a *P*."

5. When shading an area, one must be careful to shade *all* of the area in question. Examples:

Right:

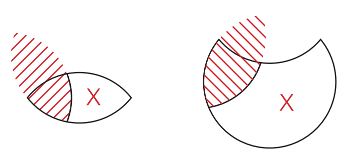


Wrong:



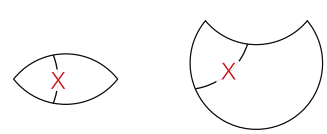
6. The area where an X goes is always initially divided into two parts. If one of these parts has already been shaded, the X goes in the unshaded part. Examples:

Right:



If one of the two parts is not shaded, the X goes on the line separating the two parts. Examples:

Right:



eminentlogicians

John Venn 1834–1923

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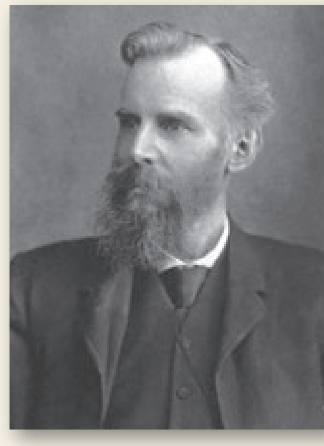
John Venn is known mainly for his circle diagrams, which have contributed to work in many areas of mathematics and logic, including computer science, set theory, and statistics. His book *The Logic of Chance* (1866) advanced probability theory by introducing the relative frequency interpretation of probability; it significantly influenced later developments in statistical theory as well. In *Symbolic Logic* (1881) he defended George Boole against various critics and rendered the new logic intelligible to nonmathematical thinkers. Finally, in *The Principles of Empirical and Inductive Logic* (1889) he criticized Mill's methods of induction as being of limited application as an engine of discovery in science.

John Venn was born in Hull, England, the son of Henry Venn, the Drypool parish rector, and Martha Sykes Venn, who died when John was a child. The Venns were prominent members of the evangelical movement within the Church of England. John Venn's grandfather had been an evangelical leader, as was his father, whom his contemporaries regarded as the head of the evangelical movement. His father served for many years in an administrative capacity for the Church Missionary Society, and John was expected to follow in the family tradition. In 1858, after graduating from Gonville and Caius (pronounced "keys") College, Cambridge, he was ordained and served for a time as a curate in parishes near London.

Perhaps owing to his contact with Henry Sidgwick and other Cambridge agnostics, Venn's confidence in the Thirty-Nine Articles of the Church of England began to erode. Also, as a result of his reading the works of De Morgan, Boole, and Mill, his interest shifted almost totally from theological matters to issues related to logic. At age twenty-eight, Venn returned to Cambridge to become a lecturer in logic and probability theory. Five years later, he married Susanna Carnegie Edmonstone, the daughter of an Anglican cleric, and they had one child, John Archibald Venn. In 1883, at age forty-nine, Venn became a fellow of the Royal Society and received the degree of Doctor of Science.

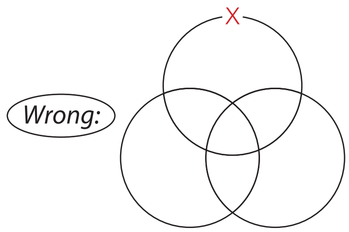


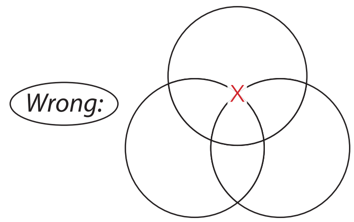
The greater part of Venn's life centered completely on his association with Cambridge. In 1857 he became a fellow of Caius, and he remained a member of the college foundation for sixty-six years, until his death. During the last twenty years of his life he served as college president, during which time he wrote a history of the college. Also, in collaboration with his son, he completed Part I of the massive *Alumni Cantabrigienses*, which contains short biographies of 76,000 graduates and officeholders ranging from the university's earliest days through 1751.

John Venn's son said of his father that he was a "fine walker and mountain climber." Also, in keeping with his view that abstract subjects such as logic and mathematics ought to serve practical utility, Venn loved to use this knowledge to build machines. He invented a cricket bowling machine that was used against the best batsman of an Australian team. The machine "clean bowled" this batsman four times. Today Venn is memorialized by a stained-glass window in the dining hall of Caius College that contains a representation of a Venn diagram.

This means that the X may be in either (or both) of the two areas—but it is not known which one.

7. An X should never be placed in such a way that it dangles outside of the diagram, and it should never be placed on the intersection of two lines.





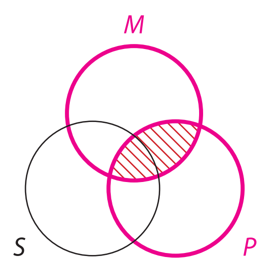
Boolean Standpoint

Because the Boolean standpoint does not recognize universal premises as having existential import, its approach to testing syllogisms is simpler and more general than that of the Aristotelian standpoint. Hence, we will begin by testing syllogisms from that standpoint and later proceed to the Aristotelian standpoint. Here is an example:

1. No *P* are *M*. **EAE**-2
   All *S* are *M*.
   No *S* are *P*.

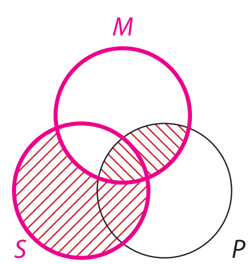
Since both premises are universal, it makes no difference which premise we enter first in the diagram. To enter the major premise, we concentrate our attention on the *M* and *P* circles, which are highlighted with color:

No *P* are *M*.



We now complete the diagram by entering the minor premise. In doing so, we concentrate our attention on the *S* and *M* circles, which are highlighted with color:

All *S* are *M*.



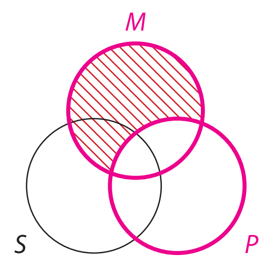
The conclusion states that the area where the *S* and *P* circles overlap is shaded. Inspection of the diagram reveals that this area is indeed shaded, so the syllogistic form is valid. Because the form is valid from the Boolean standpoint, it is *unconditionally valid.* In other words, it is valid regardless of whether its premises are recognized as having existential import.

Here is another example:

2. All *M* are *P*. **AEE**-1
   No *S* are *M*.
   No *S* are *P*.

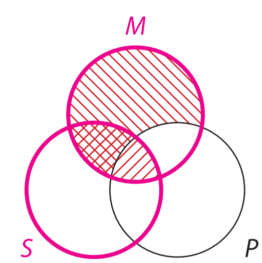
Again, both premises are universal, so it makes no difference which premise we enter first in the diagram. To enter the major premise, we concentrate our attention on the *M* and *P* circles:

All *M* are *P*.



To enter the minor premise, we concentrate our attention on the *M* and *S* circles:

No *S* are *M*.



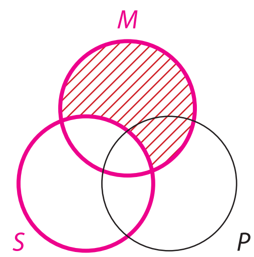
Again, the conclusion states that the area where the *S* and *P* circles overlap is shaded. Inspection of the diagram reveals that only part of this area is shaded, so the syllogistic form is invalid.

Another example:

3. Some *P* are *M*. **IAI**-4
   All *M* are *S*.
   Some *S* are *P*.

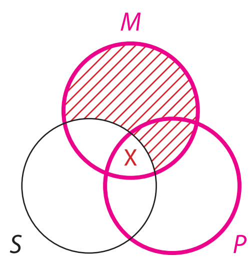
We enter the universal premise first. To do so, we concentrate our attention on the *M* and *S* circles:

All *M* are *S*.



To enter the particular premise, we concentrate our attention on the *M* and *P* circles. This premise tells us to place an X in the area where the *M* and *P* circles overlap. Because part of this area is shaded, we place the X in the remaining area:

Some *P* are *M*.

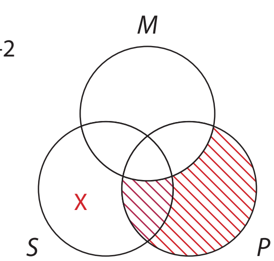


The conclusion states that there is an X in the area where the *S* and *P* circles overlap. Inspection of the diagram reveals that there is indeed an X in this area, so the syllogistic form is valid.

The examples that follow are done in a single step.

4. All *P* are *M*.
   Some *S* are not *M*.
   Some *S* are not *P*.

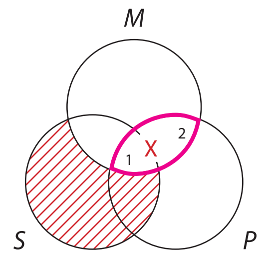
AOO-2



The universal premise is entered first. The particular premise tells us to place an X in the part of the *S* circle that lies outside the *M* circle. Because part of this area is shaded, we place the X in the remaining area. The conclusion states that there is an X that is inside the *S* circle but outside the *P* circle. Inspection of the diagram reveals that there is indeed an X in this area, so the syllogistic form is valid.

5. Some *M* are *P*.
All *S* are *M*.
Some *S* are *P*.

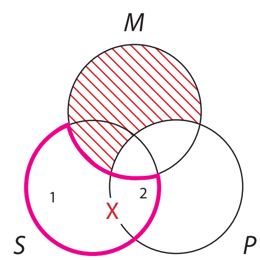
IAI-1



As usual, we enter the universal premise first. In entering the particular premise, we concentrate on the area where the *M* and *P* circles overlap. (For emphasis, this area is colored in the diagram.) Because this overlap area is divided into two parts (the areas marked "1" and "2"), we place the X on the line (arc of the *S* circle) that separates the two parts. The conclusion states that there is an X in the area where the *S* and *P* circles overlap. Inspection of the diagram reveals that the single X is dangling outside of this overlap area. We do not know if it is in or out. Thus, the syllogistic form is invalid.

6. All *M* are *P*.
Some *S* are not *M*.
Some *S* are not *P*.

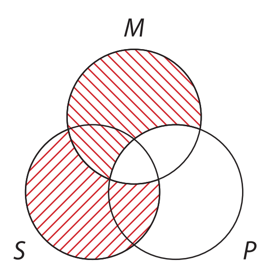
AOO-1



In entering the particular premise, we concentrate our attention on the part of the *S* circle that lies outside the *M* circle (colored area). Because this area is divided into two parts (the areas marked "1" and "2"), we place the X on the line (arc of the *P* circle) separating the two areas. The conclusion states that there is an X that is inside the *S* circle but outside the *P* circle. There *is* an X in the *S* circle, but we do not know whether it is inside or outside the *P* circle. Hence, the syllogistic form is invalid.

7. All *M* are *P*.
All *S* are *M*.
All *S* are *P*.

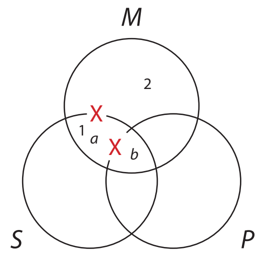
AAA-1



This is the "Barbara" syllogism. The conclusion states that the part of the *S* circle that is outside the *P* circle is empty. Inspection of the diagram reveals that this area is indeed empty. Thus, the syllogistic form is valid.

8. Some *M* are not *P*.
   Some *S* are *M*.
   Some *S* are not *P*.

OIO-1



In this diagram no areas have been shaded, so there are two possible areas for each of the two X's. The X from the first premise goes on the line (arc of the *S* circle) separating areas 1 and 2, and the X from the second premise goes on the line (arc of the *P* circle) separating areas *a* and *b*. The conclusion states that there is an X that is inside the *S* circle but outside the *P* circle. We have no certainty that the X from the first premise is inside the *S* circle, and while the X from the second premise is inside the *S* circle, we have no certainty that it is outside the *P* circle. Hence, the syllogistic form is invalid.

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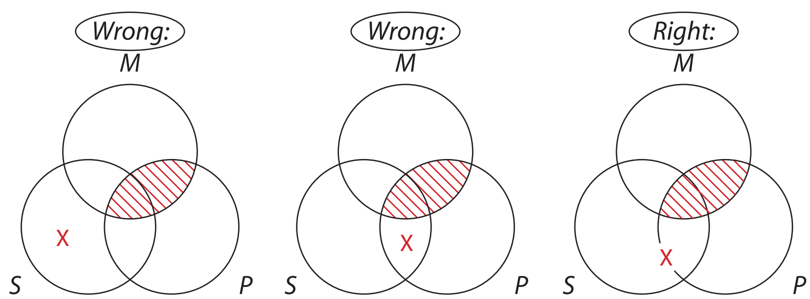
We have yet to explain the rationale for placing the X on the boundary separating two areas when neither of the areas is shaded. Consider this syllogistic form:

No *P* are *M*.
Some *S* are not *M*.
Some *S* are *P*.



In each of the three diagrams the content of the first premise is represented correctly. The problem concerns placing the X from the second premise. In the first diagram the X is placed inside the *S* circle but outside both the *M* circle and the *P* circle. This diagram asserts: "At least one *S* is not an *M* and it is also not a *P*." Clearly the diagram says more than the premise does, and so it is incorrect. In the second diagram the X is placed inside the *S* circle, outside the *M* circle, and inside the *P* circle. This diagram asserts: "At least one *S* is not an *M*, but it is a *P*." Again, the diagram says more than the premise says, and so it is incorrect. In the third diagram, which is done correctly, the X is placed on the boundary between the two areas. This diagram asserts: "At least one *S* is not an *M*, and it may or may not be a *P*." In other words, nothing at all is said about *P*, and so the diagram represents exactly the content of the second premise.

Aristotelian Standpoint

For the syllogistic forms tested thus far, we have adopted the Boolean standpoint, which does not recognize universal premises as having existential import. We now shift to the Aristotelian standpoint, where existential import can make a difference to validity. To test a syllogism from the Aristotelian standpoint, we follow basically the same procedure we followed in Section 4.6 to test immediate inferences:

1. Reduce the syllogism to its form and test it from the Boolean standpoint. If the form is valid, proceed no further. The syllogism is valid from both standpoints.
2. If the syllogistic form is invalid from the Boolean standpoint and has universal premises and a particular conclusion, then adopt the Aristotelian standpoint and look to see if there is a Venn circle that is completely shaded except for one area. If there is, enter a circled X in that area and retest the form.
3. If the syllogistic form is conditionally valid, determine if the circled X represents something that exists. If it does, the condition is fulfilled, and the syllogism is valid from the Aristotelian standpoint.

In regard to step 2, if the diagram contains no Venn circle completely shaded except for one area, then the syllogism is invalid from the Aristotelian standpoint. However, if it does contain such a Venn circle and the syllogism has a particular conclusion, then we place a circled X in the one unshaded area. This circled X represents the temporary assumption that the Venn circle in question is not empty.

In regard to step 3, if the circled X does not represent something that exists, then the syllogism is invalid. As we will see in Section 5.3, such syllogisms commit the existential fallacy from the Aristotelian standpoint.

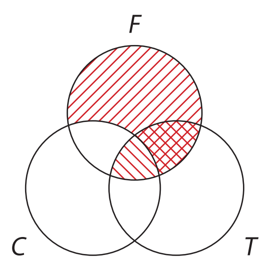
The table of conditionally valid syllogistic forms presented in Section 5.1 names nine forms that are valid from the Aristotelian standpoint if a certain condition is fulfilled. The following syllogism has one of those forms:

9. No fighter pilots are tank commanders.
   All fighter pilots are courageous individuals.
   Therefore, some courageous individuals are not tank commanders.

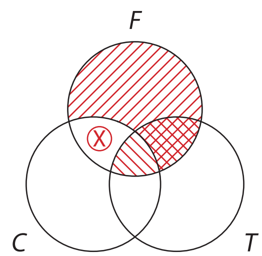
First, we replace the terms with letters and test the syllogism from the Boolean standpoint:

No *F* are *T*.
All *F* are *C*.
Some *C* are not *T*.

EAO-3



The conclusion asserts that there is an X that is inside the *C* circle but outside the *T* circle. Inspection of the diagram reveals no X's at all, so the syllogism is invalid from the Boolean standpoint. Proceeding to step 2, we adopt the Aristotelian standpoint and, noting that the conclusion is particular and that the *F* circle is all shaded except for one area, we enter a circled X in that area:



The diagram now indicates that the syllogism is conditionally valid, so we proceed to step 3 and determine whether the circled X represents something that actually exists. Since the circled X represents an *F*, and since *F* stands for fighter pilots, the circled X does represent something that exists. Thus, the condition is fulfilled, and the syllogism is valid from the Aristotelian standpoint.

The *F* circle in this diagram represents the critical term, first mentioned in connection with the table of conditionally valid forms in Section 5.1. The critical term is the term listed in the farthest right-hand column of that table. As the diagram shows, if the circle for the critical term has at least one member, then this member is necessarily also in the circle representing the subject of the conclusion (in this case *C*). This fact provides the basis for a definition of *critical term*. The **critical term** is the term in a categorical syllogism which, when it denotes at least one existing thing, guarantees that the subject of the conclusion denotes at least one existing thing.

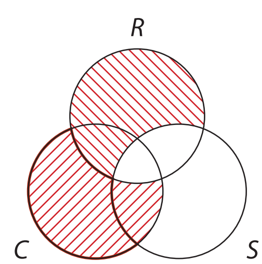
Here is another example:

10. All reptiles are scaly animals.
    All currently living tyrannosaurs are reptiles.
    Therefore, some currently living tyrannosaurs are scaly animals.

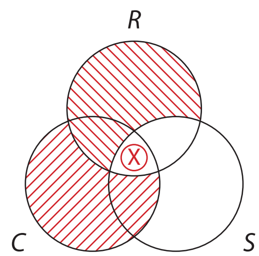
First we test the syllogism from the Boolean standpoint:

All *R* are *S*.
All *C* are *R*.
Some *C* are *S*.

AAI-1



The conclusion asserts that there is an X in the area where the *C* and *S* circles overlap. Since the diagram contains no X's at all, the syllogism is invalid from the Boolean standpoint. Proceeding to step 2, we adopt the Aristotelian standpoint. Then, after noticing that the conclusion is particular and that the *C* circle is all shaded except for one area, we enter a circled X in that area:



The diagram now indicates that the syllogism is conditionally valid, so we proceed to the third step and determine whether the circled X represents something that actually exists. Since the circled X represents a *C,* and *C* stands for currently living tyrannosaurs, the circled X does not represent something that actually exists. Thus, the condition is not fulfilled, and the syllogism is invalid. As we will see in the next section of this chapter, the syllogism commits the existential fallacy from the Aristotelian standpoint.

In determining whether the circled X stands for something that exists, we always look to the Venn circle that is all shaded except for one area. If the term corresponding to that circle denotes existing things, then the circled X represents one of those things. In some diagrams, however, there may be two Venn circles that are all shaded except for one area, and each may contain a circled X in the unshaded area. In these cases we direct our attention only to the circled X needed to draw the conclusion. If that circled X stands for something that exists, the argument is valid; if not, it is invalid.

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EXERCISE **5.2**

I. Use Venn diagrams to determine whether the following standard-form categorical syllogisms are valid from the Boolean standpoint, valid from the Aristotelian standpoint, or invalid. Then, identify the mood and figure, and cross-check your answers with the tables of valid syllogisms found in Section 5.1.

★**1.** All corporations that overcharge their customers are unethical businesses.
Some unethical businesses are investor-owned utilities.
Therefore, some investor-owned utilities are corporations that overcharge their customers.

2. No AIDS victims are people who pose an immediate threat to the lives of others.
Some kindergarten children are AIDS victims.
Therefore, some kindergarten children are not people who pose an immediate threat to the lives of others.

3. No individuals truly concerned with the plight of suffering humanity are people motivated primarily by self-interest.
All television evangelists are people motivated primarily by self-interest.

Therefore, some television evangelists are not individuals truly concerned with the plight of suffering humanity.

★4. All high-fat diets are diets high in cholesterol.
Some diets high in cholesterol are not healthy food programs.
Therefore, some healthy food programs are not high-fat diets.

5. No engineering majors are candidates for nightly hookups.
No candidates for nightly hookups are deeply emotional individuals.
Therefore, no deeply emotional individuals are engineering majors.

6. All impulse buyers are consumers with credit cards.
All shopaholics are impulse buyers.
Therefore, all shopaholics are consumers with credit cards.

★7. No pediatricians are individuals who jeopardize the health of children.
All faith healers are individuals who jeopardize the health of children.
Therefore, no faith healers are pediatricians.

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8. Some individuals prone to violence are not men who treat others humanely.
Some police officers are individuals prone to violence.
Therefore, some police officers are not men who treat others humanely.

9. Some ATM locations are places criminals lurk.
All places criminals lurk are places to avoid at night.
Therefore, some places to avoid at night are ATM locations.

★10. No corporations that defraud the government are organizations the government should deal with.
Some defense contractors are not organizations the government should deal with.
Therefore, some defense contractors are not corporations that defraud the government.

11. All circular triangles are plane figures.
All circular triangles are three-sided figures.
Therefore, some three-sided figures are plane figures.

12. All supernovas are objects that emit massive amounts of energy.
All quasars are objects that emit massive amounts of energy.
Therefore, all quasars are supernovas.

★13. No people who profit from the illegality of their activities are people who want their activities legalized.
All drug dealers are people who profit from the illegality of their activities.
Therefore, no drug dealers are people who want their activities legalized.

14. Some individuals who risk heart disease are people who will die young.
Some smokers are individuals who risk heart disease.
Therefore, some smokers are people who will die young.

15. Some communications satellites are rocket-launched failures.
All communications satellites are devices with antennas.
Therefore, some devices with antennas are rocket-launched failures.

★16. All currently living dinosaurs are giant reptiles.
All giant reptiles are ectothermic animals.
Therefore, some ectothermic animals are currently living dinosaurs.

17. All survivalists are people who enjoy simulated war games.
No people who enjoy simulated war games are soldiers who have tasted the agony of real war.
Therefore, all soldiers who have tasted the agony of real war are survivalists.

18. No spurned lovers are Valentine's Day fanatics.
Some moonstruck romantics are Valentine's Day fanatics.
Therefore, some moonstruck romantics are not spurned lovers.

★19. No theocracies are regimes open to change.
All theocracies are governments that rule by force.
Therefore, some governments that rule by force are not regimes open to change.

20. Some snowflakes are not uniform solids.
All snowflakes are six-pointed crystals.
Therefore, some six-pointed crystals are not uniform solids.

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II. Use Venn diagrams to obtain the conclusion that is validly implied by each of the following sets of premises. If no conclusion can be validly drawn, write "no conclusion."

★1. No *P* are *M*.
All *S* are *M*.

2. Some *P* are not *M*.
Some *M* are *S*.

3. Some *M* are *P*.
All *S* are *M*.

★4. Some *M* are not *P*.
All *M* are *S*.

5. Some *P* are *M*.
All *M* are *S*.

6. No *M* are *P*.
Some *S* are not *M*.

★7. All *M* are *P*.
All *S* are *M*.

8. All *P* are *M*.
All *S* are *M*.

9. No *P* are *M*.
Some *M* are *S*.

★10. No *P* are *M*.
No *M* are *S*.

III. Answer "true" or "false" to the following statements:

1. In the use of Venn diagrams to test the validity of syllogisms, marks are sometimes entered in the diagram for the conclusion.
2. When an X is placed on the arc of a circle, it means that the X could be in either (or both) of the two areas that the arc separates.
3. If an X lies on the arc of a circle, the argument cannot be valid.
4. When representing a universal statement in a Venn diagram, one always shades two of the seven areas in the diagram (unless one of these areas is already shaded).
5. If a completed diagram contains two X's, the argument cannot be valid.
6. If the conclusion asserts that a certain area is shaded, and inspection of the diagram reveals that only half that area is shaded, the argument is valid.

7. If the conclusion asserts that a certain area contains an X and inspection of the diagram reveals that only half an X appears in that area, the argument is valid.
8. If the conclusion is in the form "All *S* are *P*," and inspection of the diagram reveals that the part of the *S* circle that is outside the *P* circle is shaded, then the argument is valid.
9. If, in a completed diagram, three areas of a single circle are shaded, and placing a circled X in the one remaining area would make the conclusion true, then the argument is valid from the Aristotelian standpoint but not from the Boolean standpoint.
10. If, in a completed diagram, three areas of a single circle are shaded, but the argument is not valid from the Boolean standpoint, then it must be valid from the Aristotelian standpoint.

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5.3 Rules and Fallacies

PREVIEW • *Your college or university probably specifies rules that a student must satisfy in order to graduate. You must earn a certain number of course units, the courses must be in designated subject areas, you must maintain a certain GPA, and so on. If you follow all of the rules, you will reach your goal—which is graduation. Likewise, there are rules that determine whether the premises of a categorical syllogism support the conclusion. If all the rules are followed, the syllogism is valid.*

The idea that valid syllogisms conform to certain rules was first expressed by Aristotle. Many such rules are discussed in Aristotle's own account, but logicians of today generally settle on five or six.* If any one of these rules is violated, a specific formal fallacy is committed and, accordingly, the syllogism is invalid. Conversely, if none of the rules is broken, the syllogism is valid. These rules may be used as a convenient cross-check against the method of Venn diagrams. We will first consider the rules as they apply from the Boolean standpoint, and then shift to the Aristotelian standpoint.

Boolean Standpoint

Of the five rules presented in this section, the first two depend on the concept of distribution, the second two on the concept of quality, and the last on the concept of quantity. In applying the first two rules, you may want to recall either of the two mnemonic devices presented in Chapter 4: "Unprepared Students Never Pass" and "Any

**Some texts include a rule stating that the three terms of a categorical syllogism must be used in the same sense throughout the argument. In this text this requirement is included as part of the definition of* standard-form categorical syllogism *and is subsequently incorporated into the definition of* categorical syllogism. *See Section 5.1.*