Excerpts from

An Open Introduction to Logic

Selections for Introduction to Logic (PHIL 110), Summer 2017 - University of Nebraska-Lincoln

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This is version 0.1 of An Open Introduction to Logic. It is current as of July 7, 2017.

J. Robert Loftis compiled this edition and wrote original material for it. He takes full responsibility for any mistakes remaining in this version of the text.

Typesetting was carried out entirely in LaTeX2ε. The style for typesetting proofs is based on fitch.sty (v0.4) by Peter Selinger, University of Ottawa.

"When you come to any passage you don't understand, *read it again*: if you *still* don't understand it, *read it again*: if you fail, even after *three* readings, very likely your brain is getting a little tired. In that case, put the book away, and take to other occupations, and next day, when you come to it fresh, you will very likely find that it is *quite* easy."
– Charles Dodgson (Lewis Carroll) *Symbolic Logic* (1896)

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About this Book

This book was created by combining two previous books on logic and critical thinking, both made available under a Creative Commons license, and then adding some material so that the coverage matched that of commonly used logic textbooks.

Cathal Woods' *Introduction to Reasoning* (2014) formed the basis of most of Part II: Critical Thinking and Part IV: Inductive and Scientific Reasoning of the complete version of this text. Some of *Introduction to Reasoning*: Propositional and Categorical Reasoning was incorporated into Part III: Formal Logic. Parts of the section "The Venn Diagram Method" were folded into sections 12.1 and 12.3.

P.D. Magnus' *For All* X (2008) formed the basis of Part 3: Formal Logic, but the development was slightly more indirect. I began using *For All* X in my own logic classes in 2009, but I quickly realized I needed to make changes to make it appropriate for the community college students I was teaching. In 2010 I began developing *For All* X *: The Lorain County Remix* and using it in my classes. The main change I made was to separate the discussions of sentential and quantificational logic and to add exercises. It is this remixed version that became the basis for Part III: Formal Logic complete version of this text.

Complete version information is available at [url for textbook homepage].

J. Robert Loftis
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Acknowledgments

Thanks first of all go to the authors of the textbooks here stitched together: P.D. Magnus for *For All* X and Cathal Woods for *Introduction to Reasoning*. My thanks go to them for writing the excellent textbooks that have been incorporated into this one, for making those publicly available under Creative Commons licenses, and for giving their blessing to this derivative work.

In general, this book would not be possible without a culture of sharing knowledge. The book was typeset using LaTeX2ε developed by Leslie Lamport. Lamport was building on TeXby Donald Knuth. Peter Selinger built on what Lamport made by developing the Fitch typesetting format that the proofs were laid out in. Diagrams were made in PikZ by Till Tantu. All of these are coding systems are not only freely available online, they have extensive user support communities. Add-on packages are designed, manuals are written, questions are answered in discussion forums, all by people who are donating their time and expertise.

The culture of sharing isn't just responsible for the typesetting of this book; it was essential to the content. Essential background information comes from the free online *Stanford Encyclopedia of Philosophy*. Primary sources from the history of logic came from *Project Gutenberg*. Logicians, too, can and should create free knowledge.

Many early adopters of this text provided invaluable feedback, including Jeremy Dolan, Terry Winant, Benjamin Lennertz, Ben Sheredos, and Michael Hartsock. Lennertz, in particular, provided useful edits. Helpful comments were also made by Ben Cordry, John Emerson, Andrew Mills, Nathan Smith, Vera Tobin, Cathal Woods, and many more that I have forgot to mention, but whose emails are probably sitting on my computer somewhere.

I would also like to thank Lorain County Community College for providing the sabbatical leave that allowed me to write the sections of this book on Aristotelian logic. Special thanks goes to all the students at LCCC who had to suffer through earlier versions of this work and provided much helpful feedback. Most importantly, I would like to thank Molly, Caroline and Joey for their incredible love and support.

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Intellectual debts too great to articulate are owed to scholars too many to enumerate. At different points in the work, readers might detect the influence of various works of Aristotle, Toulmin (especially 1958), Fisher and Scriven (1997), Walton (especially 1996), Epstein (2002), Johnson-Laird (especially 2006), Scriven (1962), Giere (1997) and the works of the Amsterdam school of pragma-dialectics (Eemeren van et al., 2002).

Thanks are due to Virginia Wesleyan College for providing me with Summer Faculty Development funding in 2008 and 2010 and a Batten professorship in 2011. These funds, along with some undergraduate research funds (also provided by VWC), allowed me to hire students Gaby Alexander (2008), Ksera Dyette (2009), Mark Jones (2008), Andrea Medrano (2011), Lauren Perry (2009), and Alan Robertson (2010). My thanks to all of them for their hard work and enthusiasm.

For feedback on the text, thanks are due to James Robert (Rob) Loftis (Lorain County Community College) and Bill Roche (Texas Christian University). Answers (to exercises) marked with (JRL) are by James Robert Loftis.

Particular thanks are due to my (once) Ohio State colleague Bill Roche. The book began as a collection of lecture notes, combining work by myself and Bill.

Cathal Woods
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(Taken from *Introduction to Reasoning* (2014))

The author would like to thank the people who made this project possible. Notable among these are Cristyn Magnus, who read many early drafts; Aaron Schiller, who was an early adopter and provided considerable, helpful feedback; and Bin Kang, Craig Erb, Nathan Carter, Wes McMichael, and the students of Introduction to Logic, who detected various errors in previous versions of the book.

P.D. Magnus
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(Taken from *For All X* (2008))

Part I

Basic Concepts

Chapter 1

What Is Logic?

1.1 Introduction

Many of you in this class will have heard, and perhaps used, the term 'logic' in a number of ways and in a variety of circumstances. Perhaps you've heard someone say one thing at one time, and later a very different thing at another time. You might claim that this person is being illogical. Additionally, you may have caught an episode or two of Star Trek, and heard the term 'logic' used by a race of hyper rational beings known as Vulcans. Regardless of where you've heard the term used or if you've used the term, we can ask the following question: what does logic mean?

Sadly, many of the *usual* ways of thinking and talking about logic will not be precise enough for our purposes. In this class we will be, to a very large degree, stating definitions and then seeing what follows from those stated definitions by using rules which will, themselves, be defined. How are we, then, to define logic?

As a first stab, logic will be defined as the systematic analysis of arguments. Each of the terms in that definition also need defining. A logical argument is structured to give someone a reason to believe some conclusion. Here is a kind of argument which is structured in a way to help make clear how arguments link evidence and conclusions.

Example: 1.1

P_1: If the Huskers lose, then a riot will occur.
P_2: The Huskers lose.

C: Therefore, a riot will occur.

In the argument above, statements P_1-P_2 are the evidence. We call these the *premises*. The word "therefore" indicates that the final statement, marked with a

C, is the *conclusion* of the argument. If the premises are true, then the argument provides you with a reason to believe the conclusion. You've probably used a reasoning process like this, even if you've never done so explicitly (i.e. you've never listed your premises and conclusion in a structured fashion).

Logic, as the study of argument, has been pursued for thousands of years by people from civilizations all over the globe. The ancient Greeks, Roman, medieval Europeans and Arabs, and many, many more have all contributed to the discipline of logic. The initial motivation for studying logic is generally practical. Given that we use arguments and make inferences all the time, it only makes sense that we would want to learn to do these things better. However, as we will soon see in this class, the study of logic ends up being a lot more like mathematics than in debating topics with friends and family. It is *not* required that you have any sophisticated mathematical abilities to do well in this class. What is required is that you be able to follow rules in a rigorous fashion.

What we will see as we proceed through this course is that we will care less and less about the what the actual arguments are *about*. Instead we will be focusing on the structure of the arguments. Look at the argument above about the Huskers and riots. What if instead of the Huskers, we instead inserted the Hawkeyes. Would this change that force of the argument? It seems to many that the argument would have the same logical force. Compare them next to each other:

Example: (1.1)

versus

P_1: If the Hawkeyes lose, then a riot will occur.
P_2: The Hawkeyes lose.

C: Therefore, a riot will occur.

The arguments look the same - in *some* sense - and that actually points us to something *very* interesting. If you abstract away from the English, we can generate a formal way to represent why both arguments seem equally forceful.

P_1: If _____1_____, then _____2_____.
P_2: _____1_____.

C: Therefore, _____2_____.

You can plug anything you want in for 1 and 2 and the new argument will have the same logical force as any other. This gets us closer to understanding our definition of logic. Our definition contained the word 'systematic' and we can now understand systematic as being the kind of process that has a method. What will our method be? It will depend on the logic! Yes, there are different logics! The important thing to take note of is that when we abstract away and use variables, we are analyzing arguments in a *content neutral* manner.

Whether we are pursuing logic for practical or theoretical reasons, our focus is on argument. The key to studying argument is to set aside the subject being argued about and to focus on the *way* it is argued *for*. Suppose there are six murder suspects and no one else could have committed the murder. If you can rule out five of the suspects, then the last one is your criminal. We could give an argument from the premise that one of the six must be the murderer plus the premise that five of them are not the murder to the conclusion that the remaining one is the murderer. Suppose a group of friends is deciding which restaurant to eat at, and there are six restaurants in town. If you could rule out five of the possibilities, you would use an argument just like the one above to decide where to eat. Because logic sets aside what an argument is about, and just looks at how it works rationally, logic is said to have **content neutrality**. If we say an argument is good, then the same kind of argument applied to a different topic will also be good. If we say an argument is good for solving murders, we will also say that the same kind of argument is good for deciding where to eat, what kind of disease is destroying your crops, or whom to vote for.

When logic is studied for theoretical reasons, it typically is pursued as **formal logic**. In formal logic we get content neutrality by replacing parts of the argument we are studying with abstract symbols. For instance, we could turn the argument above into a formal argument like this:

P_1: There are six possibilities: A, B, C, D, E, and F.
P_2: A is false.
P_3: B, D, E, and F are also false.

C: $\therefore$ The correct answer is C.

Here we have replaced the concrete possibilities in the first argument with abstract letters that could stand for anything. We have also replaced the English word "therefore" with the symbol "$\therefore$," which means therefore. This lets us see the formal structure of the argument, which is why it works in any domain you can think of. In fact, we can think of formal logic as the method for studying argument that uses abstract notation to identify the formal structure of argument. Formal logic is closely allied with mathematics, and studying formal logic often has the sort of puzzle-solving character one associates with mathematics. You will see this when we get to Parts II and III, which cover formal logic.

When logic is studied for practical reasons, it is typically called critical thinking. It is the study of reasoning with the goal of improving our reasoning in the real world. Sometimes people use the term "critical thinking" to simply refer to any time one is reasoning well. However, we will be using the term **critical thinking** more narrowly to refer to the use of metareasoning to improve our reasoning in practical situations. Critical thinking is generally pursued as **informal logic**, rather than formal logic. This means that we will keep arguments in ordinary language and draw extensively on your knowledge of the world to evaluate them. In contrast to the clarity and rigor of formal logic, informal logic is suffused with ambiguity and vagueness. There are problems with multiple correct answers, or where reasonable people can disagree with what the correct answer is. This is because you will be dealing with reasoning in the real world, which is messy.

You can think of the difference between formal logic and informal logic as the difference between a laboratory science and a field science. If you are studying, say, mice, you could discover things about them by running experiments in a lab, or you can go out into the field where mice live and observe them in their natural habitat. Informal logic is the field science for arguments: you go out and study arguments in their natural habitats, like newspapers, courtrooms, and scientific journal articles. Like studying mice scurrying around a meadow, the process takes patience, and often doesn't yield clear answers but it lets you see how things work in the real world. Formal logic takes arguments out of their natural habitat and performs experiments on them to see what they are capable of. The arguments here are like lab mice. They are pumped full of chemicals and asked to perform strange tasks, as it were. They live lives very different than their wild cousins. Some of these arguments will be huge, awkward, and completely unable to survive in the wild. But they will tell us a lot about the limits of logic as a process.

Our main goal in studying arguments is to separate the good ones from the bad ones. The argument about Clue we saw earlier is a good one, based on the process of elimination. It is good because it leads to truth. If I've got all the premises right, the conclusion will also be right. The textbook *Logic: Techniques of Formal Reasoning* (Kalish et al., 1980) had a nice way of capturing the meaning of logic: "logic is the study of virtue in argument." This textbook will accept this definition, with the caveat that an argument is virtuous if it helps us get to the truth.

Logic is different from **rhetoric**, which is the study of effective persuasion. Rhetoric does not look at virtue in argument. It only looks at the power of arguments, regardless of whether they lead to truth. An advertisement might convince you to buy a new truck by having a gravelly voiced announcer tell you it is "ram tough" and showing you a picture of the truck on top of a mountain, where it no doubt actually had to be airlifted. This sort of persuasion is often more effective at getting people to believe things than logical argument, but it has nothing to do with whether the truck is really the right thing to buy. In this textbook we will only be interested in rhetoric to the extent that we need to learn to defend ourselves against the misleading rhetoric of others. This will not, however, be anything close to a full treatment of the study of rhetoric.

1.2 Statement, Argument, Premise, Conclusion

So far we have defined logic as the study of argument and outlined its relationship to related fields. To go any further, we are going to need a more precise definition of what exactly an argument is. We have said that an argument is not simply two people disagreeing; it is an attempt to prove something using evidence. More specifically, an argument is composed of statements. In logic, we define a **statement** as a unit of language that can be true or false. To put it another way, it is some combination of words or symbols that have been put together in a way that lets someone agree or disagree with it. All of the items below are statements.

(a) *Tyrannosaurus rex* went extinct 65 million years ago.

(b) *Tyrannosaurus rex* went extinct last week.

(c) On this exact spot, 100 million years ago, a *T. rex* laid a clutch of eggs.

(d) George W. Bush is the king of Jupiter.

(e) Murder is wrong.

(f) Abortion is murder.

(g) Abortion is a woman's right.

(h) Lady Gaga is pretty.

(i) Murder is the unjustified killing of a person.

(j) The murder of logician Richard Montague was never solved.

Because a statement is something that can be true *or* false, statements include truths like (a) and falsehoods like (b). A statement can also be something that that must either be true or false, but we don't know which, like (c). A statement can be something that is completely silly, like (d). Statements in logic include statements about morality, like (e), and things that in other contexts might be called "opinions," like (f) and (g). People disagree strongly about whether (f) or (g)are true, but it is definitely possible for one of them to be true. The same is true about (h), although it is a less important issue than (f) and (g). A statement in logic can also simply give a definition, like (i). This sort of statement announces that we plan to use words a certain way, which is different from statements that describe the world, like (a), or statements about morality, like (f). All of this relates back to the content neutrality of logic. The statements we study can be about dinosaurs, abortion, Lady Gaga, and even the history of logic itself, as in statement (j), which is true.

We are treating statements primarily as units of language or strings of symbols, and most of the time the statements you will be working with will just be words printed on a page. However, it is important to remember that statements are also what philosophers call "speech acts." They are actions people take when they speak (or write). If someone makes a statement they are typically telling other people that they believe the statement to be true, and will back it up with evidence if asked to. When people make statements, they always do it in a context—they make statements at a place and a time with an audience. Often the context statements are made in will be important for us, so when we give examples, statements, or arguments we will sometimes include a description of the context. When we do that, we will give the context in *italics.* See Table 1.1 for examples. For the most part, the context for a statement or argument will be important in the chapters on critical thinking, when we are pursuing the study of logic for practical reasons. In the chapters on formal logic, context is less important, and we will be more likely to skip it.

"Statements' in this text does *not* include questions, commands, exclamations, or sentence fragments. Someone who asks a *question* like "Does the grass need to be mowed?" is typically not claiming that anything is true or false. (Sometimes people make statements and disguise them as questions, for instance if they were trying to hint that the lawn needs to be mowed. These are generally called rhetorical questions, and we will leave the study of them to the rhetoricians.) Generally, *questions* will not count as statements, but *answers* will. "What is this course

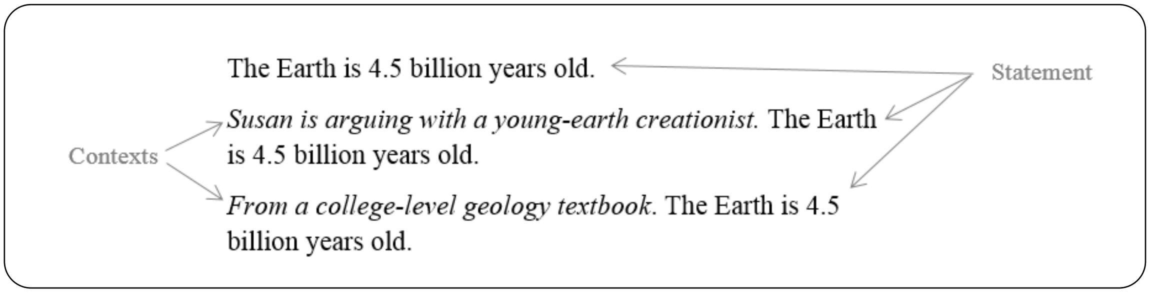


Table 1.1: A statement in different contexts, or no context.

about?" is not a statement. "No one knows what this course is about," is a statement.

For the same reason *commands* do not count as statements for us. If someone bellows "Mow the grass, now!" they are not saying whether the grass has been mowed or not. You might infer that they believe the lawn has not been mowed, but then again maybe they think the lawn is fine and just want to see you exercise. Note, however, that commands are not always phrased as imperatives. "You will respect my authority" *is* either true or false—either you will or you will not—and so it counts as a statement in the logical sense.

An exclamation like "Ouch!" is also neither true nor false. On its own, it is not a statement. We will treat "Ouch, I hurt my toe!" as meaning the same thing as "I hurt my toe." The "ouch" does not add anything that could be true or false.

Finally, a lot of possible strings of words will fail to qualify as statements simply because they don't form a complete sentence. In your composition classes, these were probably referred to as sentence fragments. This includes strings of words that are parts of sentences, such as noun phrases like "The tall man with the hat" and verb phrases, like "ran down the hall." Phrases like these are missing something they need to make a claim about the world. The class of sentence fragments also includes completely random combinations of words, like "The up if blender route," which don't even have the form of a statement about the world.

Other logic textbooks describe the components of argument as "propositions," or "assertions," and we will use these terms sometimes as well. There is actually a great deal of disagreement about what the differences between all of these things are and which term is best used to describe parts of arguments. However, none of that makes a difference for this textbook. We could have used any of the other terms in this text, and it wouldn't change anything. Some textbooks will also use the term "sentence" here. We will not use the word "sentence" to mean the same thing as "statement." Instead, we will use "sentence" the way it is used in ordinary grammar, to refer generally to statements, questions, and commands.

We use statements to build arguments. An **argument** is a connected series of statements designed to convince an audience of another statement. Here an audience might be a literal audience sitting in front of you at some public speaking engagement. Or it might be the readers of a book or article. The audience might even be yourself as you reason your way through a problem. Let's start with an example of an argument given to an external audience. This passage is from an essay by Peter Singer called "Famine, Affluence, and Morality" in which he tries to convince people in rich nations that they need to do more to help people in poor nations who are experiencing famine.

> *A contemporary philosopher writing in an academic journal* If it is in our power to prevent something bad from happening, without thereby sacrificing anything of comparable moral importance, we ought, morally, to do so. Famine is something bad, and it can be prevented without sacrificing anything of comparable moral importance. So, we ought to prevent famine. (Singer, 1972)

Singer wants his readers to work to prevent famine. This is represented by the last statement of the passage, "we ought to prevent famine," which is called the conclusion of the passage. The **conclusion** of an argument is the statement that the argument is trying to convince the audience of. The statements that do the convincing are called the **premises**. In this case, the argument has three premises: (1) "If it is in our power to prevent something bad from happening, without thereby sacrificing anything of comparable moral importance, we ought, morally, to do so"; (2) "Famine is something bad"; and (3) "it can be prevented without sacrificing anything of comparable moral importance."

Now let's look at an example of internal reasoning.

> *Jack arrives at the track, in bad weather.* There is no one here. I guess the race is not happening.

In the passage above, the words in *italics* explain the context for the reasoning, and the words in regular type represent what Jack is actually thinking to himself. This passage again has a premise and a conclusion. The premise is that no one is at

Premise Indicators:	because, as, for, since, given that, for the reason that
Conclusion Indicators:	therefore, thus, hence, so, consequently, it follows that, in conclusion, as a result, then, must, accordingly, this implies that, this entails that, we may infer that

Table 1.3: Premise and Conclusion Indicators.

the track, and the conclusion is that the race was canceled. The context gives another reason why Jack might believe the race has been canceled, the weather is bad. You could view this as another premise–it is very likely a reason Jack has come to believe that the race is canceled. In general, when you are looking at people's internal reasoning, it is often hard to determine what is actually working as a premise and what is just working in the background of their unconscious.

When people give arguments to each other, they typically use words like "therefore" and "because." These are meant to signal to the audience that what is coming is either a premise or a conclusion in an argument. Words and phrases like "because" signal that a premise is coming, so we call these **premise indicators**. Similarly, words and phrases like "therefore" signal a conclusion and are called **conclusion indicators**. The argument from Peter Singer (on page 9) uses the conclusion indicator word, "so." Table 1.3 is an incomplete list of indicator words and phrases in English.

The two passages we have looked at in this section so far have been simply presented as quotations. But often it is extremely useful to rewrite arguments in a way that makes their logical structure clear. One way to do this is to use something called "canonical form." An argument written in **canonical form** has each premise numbered and written on a separate line. Indicator words and other unnecessary material should be removed from the premises. Although you can shorten the premises and conclusion, you need to be sure to keep them all complete sentences with the same meaning, so that they can be true or false. The argument from Peter Singer, above, looks like this in canonical form:

P_1: If we can stop something bad from happening, without sacrificing anything of comparable moral importance, we ought to do so.
P_2: Famine is something bad.
P_3: Famine can be prevented without sacrificing anything of comparable moral importance.

C: We ought to prevent famine.

Each statement has been written on its own line and given a number. The statements have been paraphrased slightly, for brevity, and the indicator word "so" has been removed. Also notice that the "it" in the third premise has been replaced by the word "famine," so that statement reads naturally on its own.

Similarly, we can rewrite the argument Jack gives at the racetrack, on page 9, like this:

P: There is no one at the race track.

C: The race is not happening.

Notice that we did not include anything from the part of the passage in italics. The italics represent the context, not the argument itself. Also, notice that the "I guess" has been removed. When we write things out in canonical form, we write the content of the statements, ignore information about the speaker's mental state, like "I believe" or "I guess."

One of the first things you have to learn to do in logic is to identify arguments and rewrite them in canonical form. This is a foundational skill for everything else we will be doing in this text, so we are going to run through a few examples now, and there will be more in the exercises. The passage below is paraphrased from the ancient Greek philosopher Aristotle.

> *An ancient philosopher, writing for his students* Again, our observations of the stars make it evident that the earth is round. For quite a small change of position to south or north causes a manifest alteration in the stars which are overhead. (Aristotle, c.350 BCE/1c. 350 BCE/1984c, 298a2-10)

The first thing we need to do to put this argument in canonical form is to identify the conclusion. The indicator words are the best way to do this. The phrase "make it evident that" is a conclusion indicator phrase. He is saying that everything else is *evidence* for what follows. So we know that the conclusion is that the earth is round. "For" is a premise indicator word—it is sort of a weaker version of "because." Thus the premise is that the stars in the sky change if you move north or south. In canonical form, Aristotle's argument that the earth is round looks like this.

P: There are different stars overhead in the northern and southern parts of the earth.

C: The earth is spherical in shape.

That one is fairly simple, because it just has one premise. Here's another example of an argument, this time from the book of Ecclesiastes in the Bible. The speaker in this part of the bible is generally referred to as The Preacher, or in Hebrew, Koheleth. In this verse, Koheleth uses both a premise indicator and a conclusion indicator to let you know he is giving reasons for enjoying life.

> *The words of the Preacher, son of David, King of Jerusalem* There is something else meaningless that occurs on earth: the righteous who get what the wicked deserve, and the wicked who get what the righteous deserve. . . . So I commend the enjoyment of life, because there is nothing better for a person under the sun than to eat and drink and be glad. (Ecclesiastes 8:14-15, New International Version)

Koheleth begins by pointing out that good things happen to bad people and bad things happen to good people. This is his first premise. (Most Bible teachers provide some context here by pointing that the ways of God are mysterious and this is an important theme in Ecclesiastes.) Then Koheleth gives his conclusion, that we should enjoy life, which he marks with the word "so." Finally he gives an extra premise, marked with a "because," that there is nothing better for a person than to eat and drink and be glad. In canonical form, the argument would look like this.

P_1: Good things happen to bad people and bad things happen to good people.
P_2: There is nothing better for people than to eat, to drink and to enjoy life.

C: You should enjoy life.

Notice that in the original passages, Aristotle put the conclusion in the first sentence, while Koheleth put it in the middle of the passage, between two premises. In ordinary English, people can put the conclusion of their argument where ever they want. However, when we write the argument in canonical form, the conclusion goes last.

Unfortunately, indicator words aren't a perfect guide to when people are giving an argument. Look at this passage from a newspaper:

> *From the general news section of a national newspaper* The new budget underscores the consistent and paramount importance of tax cuts in the Bush philosophy. His first term cuts affected more money than any other initiative undertaken in his presidency, including the costs thus far of the war in Iraq. All told, including tax incentives for health care programs and the extension of other tax breaks that are likely to be taken up by Congress, the White House budget calls for nearly $300 billion in tax cuts over the next five years, and $1.5 trillion over the next 10 years. (Toner, 2006)

Although there are no indicator words, this is in fact an argument. The writer wants you to believe something about George Bush: tax cuts are his number one priority. The next two sentences in the paragraph give you reasons to believe this. You can write the argument in canonical form like this.

P_1: Bush's first term cuts affected more money than any other initiative undertaken in his presidency, including the costs thus far of the war in Iraq.

P_2: The White House budget calls for nearly $300 billion in tax cuts over the next five years, and $1.5 trillion over the next 10 years.

C: Tax cuts are of consistent and paramount importance of in the Bush philosophy.

The ultimate test of whether something is an argument is simply whether some of the sentences provide reason to believe another one of the sentences. If some sentences support others, you are looking at an argument. The speakers in these two cases use indicator phrases to let you know they are trying to give an argument.

A final bit of terminology for this section. An INFERENCE is the act of coming to believe a conclusion on the basis of some set of premises. When Jack in the example above saw that no one was at the track, and came to believe that the race was not on, he was making an inference. We also use the term inference to refer to the connection between the premises and the conclusion of an argument. If your mind moves from premises to conclusion, you make an inference, and the premises and the conclusion are said to be linked by an inference. In that way inferences are like argument glue: they hold the premises and conclusion together.

Practice Exercises

Throughout the book, you will find a series of practice problems that review and explore the material covered in the chapter. There is no substitute for actually working through some problems, because logic is more about a way of thinking than it is about memorizing facts.

Part A Decide whether the following passages are statements in the logical sense and give reasons for your answers.

Example: Did you follow the instructions?
Answer: Not a statement, a question.

(1) England is smaller than China.
(2) Greenland is south of Jerusalem.
(3) Is New Jersey east of Wisconsin?
(4) The atomic number of helium is 2.
(5) The atomic number of helium is π.
(6) I hate overcooked noodles.
(7) Blech! Overcooked noodles!
(8) Overcooked noodles are disgusting.
(9) Take your time.
(10) This is the last question.

Part B Decide whether the following passages are statements in the logical sense and give reasons for your answers.

(1) Is this a question?
(2) Nineteen out of the 20 known species of Eurasian elephants are extinct.
(3) The government of the United Kingdom has formally apologized for the way it treated the logician Alan Turing.
(4) Texting while driving
(5) Texting while driving is dangerous.
(6) Insanity ran in the family of logician Bertrand Russell, and he had a life-long fear of going mad.

(7) For the love of Pete, put that thing down before someone gets hurt!

(8) Don't try to make too much sense of this.

(9) Never look a gift horse in the mouth.

(10) The physical impossibility of death in the mind of someone living

Part C Rewrite each of the following arguments in canonical form. Be sure to remove all indicator words and keep the premises and conclusion as complete sentences. Write the indicator words and phrases separately and state whether they are premise or conclusion indicators.

Example: *An ancient philosopher writes* There is no reason to fear death. Why? Because death is a time when you will not exist, just like the time before you were born. You are not troubled by the fact that you didn't exist before you were born, and the time that you won't exist after you are dead is no different. (Based on Lucretius [50 BCE])

Answer:
P_1: Death is a time when you will not exist.
P_2: Death is not different than the time before you were born.
P_3: You are not troubled by the fact that you didn't exist before you were born.

C: There is no reason to fear death.

Premise indicator: Because

(1) *A detective is speaking:* Henry's finger-prints were found on the stolen computer. So, I infer that Henry stole the computer.

(2) *Monica is wondering about her co-workers political opinions* You cannot both oppose abortion and support the death penalty, unless you think there is a difference between fetuses and felons. Steve opposes abortion and supports the death penalty. Therefore Steve thinks there is a difference between fetuses and felons.

(3) *The Grand Moff of Earth defense considers strategy* We know that whenever people from one planet invade another, they always wind up being killed by the local diseases, because in 1938, when Martians invaded the Earth, they were defeated because they lacked immunity to Earth's diseases. Also, in 1942, when Hitler's forces landed on the Moon, they were killed by Moon diseases.

(4) If you have slain the Jabberwock, my son, it will be a frabjous day. The Jabberwock lies there dead, its head cleft with your vorpal sword. This is

truly a fabjous day.

(5) *A detective trying to crack a case thinks to herself* Miss Scarlett was jealous that Professor Plum would not leave his wife to be with her. Therefore she must be the killer, because she is the only one with a motive.

Part D Rewrite each of the following arguments in canonical form. Be sure to remove all indicator words and keep the premises and conclusion as complete sentences. Write the indicator words and phrases separately and state whether they are premise or conclusion indicators.

1) *A pundit is speaking on a Sunday political talk show* Hillary Clinton should drop out of the race for Democratic Presidential nominee. For every day she stays in the race, McCain gets a day free from public scrutiny and the members of the Democratic party get to fight one another.

2) You have to be smart to understand the rules of Dungeons and Dragons. Most smart people are nerds. So, I bet most people who play D&D are nerds.

3) Any time the public receives a tax rebate, consumer spending increases. Since the public just received a tax rebate, consumer spending will increase.

4) Isabelle is taller than Jacob. Kate must also be taller than Jacob, because she is taller than Isabelle.

1.3 Arguments and Nonarguments

We just saw that arguments are made of statements. However, there are lots of other things you can do with statements. Part of learning what an argument is involves learning what an argument is not, so in this section and the next we are going to look at some other things you can do with statements besides make arguments.

The list below of kinds of nonarguments is not meant to be exhaustive: there are all sorts of things you can do with statements that are not discussed. Nor are the items on this list meant to be exclusive. One passage may function as both, for instance, a narrative and a statement of belief. Right now we are looking at real world reasoning, so you should expect a lot of ambiguity and imperfection. If your class is continuing on into the critical thinking portions of this textbook, you will quickly get used to this.

Simple Statements of Belief

An argument is an attempt to persuade an audience to believe something, using reasons. Often, though, when people try to persuade others to believe something, they skip the reasons, and give a **simple statement of belief**. This is a kind of nonargumentative passage where the speaker simply asserts what they believe without giving reasons. Sometimes simple statements of belief are prefaced with the words "I believe," and sometimes they are not. A simple statement of belief can be a profoundly inspiring way to change people's hearts and minds. Consider this passage from Dr. Martin Luther King's Nobel acceptance speech.

> I believe that even amid today's mortar bursts and whining bullets, there is still hope for a brighter tomorrow. I believe that wounded justice, lying prostrate on the blood-flowing streets of our nations, can be lifted from this dust of shame to reign supreme among the children of men. I have the audacity to believe that peoples everywhere can have three meals a day for their bodies, education and culture for their minds, and dignity, equality and freedom for their spirits. (King, 1964)

This actually is a part of a longer passage that consists almost entirely of statements that begin with some variation of "I believe."It is incredibly powerful oration, because the audience, feeling the power of King's beliefs, comes to share in those beliefs. The language King uses to describe how he believes is important, too. He says his belief in freedom and equality requires audacity, making the audience feel his courage and want to share in this courage by believing the same things.

These statements are moving, but they do not form an argument. None of these statements provide evidence for any of the other statements. In fact, they all say roughly the same thing, that good will triumph over evil. So the study of this kind of speech belongs to the discipline of rhetoric, not of logic.

Expository Passages

Perhaps the most basic use of a statement is to convey information. Often if we have a lot of information to convey, we will sometimes organize our statements around a theme or a topic. Information organized in this fashion can often appear like an argument, because all of the statements in the passage relate back to some central statement. However, unless the other statements are given as reasons to believe the central statement, the passage you are looking at is not an argument. Consider this passage:

> *From a college psychology textbook.* Eysenck advocated three major behavior techniques that have been used successfully to treat a variety of phobias. These techniques are modeling, flooding, and systematic desensitization. In **modeling** phobic people watch nonphobics cope successfully with dreaded objects or situations.In **flooding** clients are exposed to dreaded objects or situations for prolonged periods of time in order to extinguish their fear. In contrast to flooding, **systematic desensitization** involves gradual, client-controlled exposure to the anxiety eliciting object or situation. (Adapted from Ryckman 2007)

We call this kind of passage an expository passage. In an **expository passage** , statements are organized around a central theme or topic statement. The topic statement might look like a conclusion, but the other statements are not meant to be evidence for the topic statement. Instead, they elaborate on the topic statement by providing more details or giving examples. In the passage above, the topic statement is "Eysenck advocated three major behavioral techniques . . . ." The statements describing these techniques elaborate on the topic statement, but they are not evidence for it. Although the audience may not have known this fact about Eysenk before reading the passage, they will typically accept the truth of this statement instantly, based on the textbook's authority. Subsequent statements in the passage merely provide detail.

The name "expository passage" might make it sound like we are only talking about formal writing, but really people give information organized around topic sentences all the time. Consider this:

> *Your friend Bea is on the phone:* Kelly is driving me insane. First she told Michael that I was out when I was right there in my room, and then she ate the leftover food I was keeping for lunch today.

In this passage, "Kelly is driving me insane" acts as a topic sentence, and the other two statements provide details illustrating the topic sentence. This doesn't really count as an argument, though, because Bea probably doesn't need to convince you that Kelly is driving her insane. You take your friend's word for it as soon as she says it. Most human communication is actually like this. We assume people are telling the truth until we are given reason to doubt them. Societies that lack this basic level of trust can quickly break down because no one can cooperate enough to get basic tasks accomplished.

Deciding whether a passage is an argument or an expository passage is complicated by the fact that sometimes people argue by example:

> **Steve:** Kenyans are better distance runners than everyone else.
>
> **Monica:** Oh come on, that sounds like an exaggeration of a stereotype that isn't even true.
>
> **Steve:** What about Dennis Kimetto, the Kenyan who set the world record for running the marathon? And you know who the previous record holder was: Emmanuel Mutai, also Kenyan.

Here Steve has made a general statement about all Kenyans. Monica clearly doubts this claim, so Steve backs it up with some examples that seem to match his generalization. This isn't a very strong way to argue: moving from two examples to statement about all Kenyans is probably going to be a kind of bad argument known as a hasty generalization. (This mistake is covered in the complete version of this text in the chapter on induction) The point here however, is that Steve is just offering it as an argument.

The key to telling the difference between expository passages and arguments by example is whether there is a conclusion that the audience needs to be convinced of. In the passage from the psychology textbook, "Eysenck advocated three major behavioral techniques" doesn't really work as a conclusion for an argument. The audience, students in an introductory psychology course, aren't likely to challenge this assertion, the way Monica challenges Steve's overgeneralizing claim.

Context is very important here, too. The Internet is a place where people argue in the ordinary sense of exchanging angry words and insults. In that context, people are likely to actually give some arguments in the logical sense of giving reasons to believe a conclusion.

Narratives

Statements can also be organized into descriptions of events and actions, as in this snippet from book V of *Harry Potter*.

> But she [Hermione] broke off; the morning post was arriving and, as

> usual, the *Daily Prophet* was soaring toward her in the beak of a screech owl, which landed perilously close to the sugar bowl and held out a leg. Hermione pushed a Knut into its leather pouch, took the newspaper, and scanned the front page critically as the owl took off again. (Rowling, 2003)

We will use the term **narrative** loosely to refer to any passage that gives a sequence of events or actions. A narrative can be fictional or nonfictional. It can be told in regular temporal sequence or it can jump around, forcing the audience to try to reconstruct a temporal sequence. A narrative can describe a short sequence of actions, like Hermione taking a newspaper from an owl, or a grand sweep of events, like this passage about the rise and fall of an empire in the ancient near east:

> The Guti were finally expelled from Mesopotamia by the Sumerians of Erech (*c.* 2100), but it was left to the kings of Ur's famous third dynasty to re-establish the Sargonoid frontiers and write the final chapter of the Sumerian History. The dynasty lasted through the twenty first century at the close of which the armies of Ur were overthrown by the Elamites and Amorites (McEvedy, Woodcock, 1967).

This passage does not feature individual people performing specific actions, but it is still united by character and action. Instead of Hermione at breakfast, we have the Sumerians in Mesopotamia. Instead of retrieving a message from an owl, the Sumerians conquer the Guti, but then are conquered by the Elamites and Amorites. The important thing is that the statements in a narrative are not related as premises and conclusion. Instead, they are all events which are united common characters acting in specific times and places.

Practice Exercises

Part A Identify each passage below as an argument or a nonargument, and give reasons for your answers. If it is a nonargument, say what kind of nonargument you think it is. If it is an argument, write it out in canonical form.

Example: *One student speaks to another student who has missed class:* The instructor passed out the syllabus at 9:30. Then he went over some basic points about reasoning, arguments and explanations. Then he said we should call it a day.

Answer: Not an argument, because none of the statements provide any support for any of the others. This is probably better classified as a narration because the events are in temporal sequence.

(1) *An anthropology teacher is speaking to her class* Different gangs use different colors to distinguish themselves. Here are some illustrations: biologists tend to wear some blue, while the philosophy gang wears black.

(2) The economy has been in trouble recently. And it's certainly true that cell phone use has been rising during that same period. So, I suspect increasing cell phone use is bad for the economy.

(3) *At Widget-World Corporate Headquarters:* We believe that our company must deliver a quality product to our customers. Our customers also expect first-class customer service. At the same time, we must make a profit.

(4) *Jack is at the breakfast table and shows no sign of hurrying. Gill says:* You should leave now. It's almost nine a.m. and it takes three hours to get there.

(5) *In a text book on the brain:* Axons are distinguished from dendrites by several features, including shape (dendrites often taper while axons usually maintain a constant radius), length (dendrites are restricted to a small region around the cell body while axons can be much longer), and function (dendrites usually receive signals while axons usually transmit them).

Part B Identify each passage below as an argument or a nonargument, and give reasons for your answers. If it is a nonargument, say what kind of nonargument you think it is. If it is an argument, write it out in canonical form.

(1) *Suzi doesn't believe she can quit smoking. Her friend Brenda says* Some people have been able to give up cigarettes by using their will-power. Everyone can draw on their will-power. So, anyone who wants to give up cigarettes can do so.

(2) *The words of the Preacher, son of David, King of Jerusalem* I have seen something else under the sun: The race is not to the swift or the battle to the strong, nor does food come to the wise or wealth to the brilliant or favor to the learned; but time and chance happen to them all. (Ecclesiastes 9:11, New International Version)

(3) *An economic development expert is speaking.* The introduction of cooperative marketing into Europe greatly increased the prosperity of the farmers, so we may be confident that a similar system in Africa will greatly increase the prosperity of our farmers.

(4) *From the CBS News website, US section.* Headline: "FBI nabs 5 in alleged plot to blow up Ohio bridge." Five alleged anarchists have been arrested after a months-long sting operation, charged with plotting to blow up a bridge in the Cleveland area, the FBI announced Tuesday. CBS News senior correspondent John Miller reports the group had been involved in a series of escalating plots that ended with their arrest last night by FBI agents. The sting operation supplied the anarchists with what they thought were explosives and bomb-making materials. At no time during the case was the public in danger, the FBI said. (CBS News, 2012)

(5) *At a school board meeting.* Since creationism can be discussed effectively as a scientific model, and since evolutionism is fundamentally a religious philosophy rather than a science, it is unsound educational practice for evolution to be taught and promoted in the public schools to the exclusion or detriment of special creation. (Kitcher 1982, p. 177, citing Morris 1975.)

1.4 Arguments and Explanations

Explanations are not arguments, but they share important characteristics with arguments, so we should devote a separate section to them. Both explanations and arguments are parts of reasoning, because both feature statements that act as reasons for other statements. The difference is that explanations are not used to convince an audience of a conclusion.

Let's start with workplace example. Suppose you see your co-worker, Henry, removing a computer from his office. You think to yourself "Gosh, is he stealing from work?" But when you ask him about it later, Henry says, "I took the computer because I believed that it was scheduled for repair." Henry's statement looks like an argument. It has the indicator word "because" in it, which would mean that the statement "I believed it was scheduled for repairs" would be a premise. If it was, we could put the argument in canonical form, like this:

P: I believed the computer was scheduled for repair

C: I took the computer from the office.

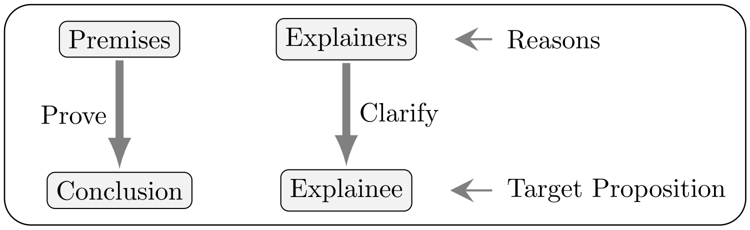


Figure 1.1: Arguments vs. Explanations.

But this would be awfully weird as an argument. If it were an argument, it would be trying to convince us of the conclusion, that Henry took the computer from the office. But you don't need to be convinced of this. You already know it—that's why you were talking to him in the first place.

Henry is giving reasons here, but they aren't reasons that try to *prove* something. They are reasons that *explain* something. When you explain something with reasons, you increase your understanding of the world by placing something you already know in a new context. You already knew that Henry took the computer, but now you know *why* Henry took the computer, and can see that his action was completely innocent (if his story checks out).

Both arguments and explanations involve giving reasons, but the reasons function differently in each case. An **explanation** is defined as a kind of reasoning where reasons are used to provide a greater understanding of something that is already known.

Because both arguments and explanations are parts of reasoning, we will use parallel language to describe them. In the case of an argument, we called the reasons "premises." In the case of an explanation, we will call them **explainers**. Instead of a "conclusion," we say that the explanation has an **explainee**. We can use the generic term **reasons** to refer to either premises or explainers and the generic term **target proposition** to refer to either conclusions or explainees. Figure 1.1 shows this relationship.

We can put explanations in canonical form, just like arguments, but to distinguish the two, we will simply number the statements, rather than writing Ps and Cs, and we will put an E next to the line that separates explainers and exaplainee, like this:

1. Henry believed the computer was scheduled for repair

——————————————————— E

2. Henry took the computer from the office.

Often the same piece of reasoning can work as either an argument or an explanation, depending on the situation where it is used. Consider this short dialogue

Monica visits Steve's cubical.

Monica: All your plants are dead.

Steve: It's because I never water them.

In the passage above, Steve uses the word "because," which we've seen in the past is a premise indicator word. But if it were a premise, the conclusion would be "All Steve's plants are dead." But Steve can't possibly be trying to convince Monica that all his plants are dead. It is something that Monica herself says, and that they both can see. The "because" here indicates a reason, but here Steve is giving an explanation, not an argument. He takes something that Steve and Monica already know—that the plants are dead—and puts it in a new light by explaining how it came to be. In this case, the plants died because they didn't get water, rather than dying because they didn't get enough light or were poisoned by a malicious co-worker. The reasoning is best represented like this:

1. Steve never waters his plants.

———————————— E

2. All the plants are dead.

But the same piece of reasoning can change from an explanation into an argument simply by putting it into a new situation:

Monica and Steve are away from the office.

Monica: Did you have someone water your plants while you were away?

Steve: No.

Monica: I bet they are all dead.

Here Steve and Monica do not know that Steve's plants are dead. Monica is inferring this idea based on the premise which she learns from Steve, that his plants are not being watered. This time "Steve's plants are not being watered" is a premise and "The plants are dead" is a conclusion. We represent the argument like this:

P. Steve never waters his plants.

————————————

C. All the plants are dead.

In the example of Steve's plants, the same piece of reasoning can function either as an argument or an explanation, depending on the context in which it is given. This is because the reasoning in the example of the plants is causal: the *causes* of the plants dying are given as reasons for the death, and we can appeal to causes either to explain something that we know happened or to predict something that we think might have happened.

Not all kinds of reasoning are flexible like that, however. Reasoning from authority can be used in some kinds of argument, but often makes a lousy explanation. Consider another conversation between Steve and Monica:

Monica: I saw on a documentary last night that the universe is expanding and probably will keep expanding forever.

Steve: Really?

Monica: Yeah, Stephen Hawking said so.

There aren't any indicator words here, but it looks like Monica is giving an argument. She states that the universe is expanding, and Steve gives a skeptical "really?" Monica then replies by saying that she got this information from the famous physicist Steven Hawking. It looks like Steve is supposed to believe that the universe will expand indefinitely because Hawking, an authority in the relevant field, said so. This makes for an ok argument:

P: Steven Hawking said that the universe is expanding and will continue to do so indefinitely.

C: The universe is expanding and will continue to do so indefinitely.

Arguments from authority aren't very reliable, but for very many things they are all we have to go on. We can't all be experts on everything. But now try to imagine this argument as an explanation. What would it mean to say that the expansion of the universe can be *explained* by the fact that Steven Hawking said that it should expand. It would be as if Hawking were a god, and the universe obeyed his commands! Arguments from authority are acceptable, but not ideal. Explanations from authority, on the other hand, are completely illegitimate.

In general, arguments that appeal to how the world works are more satisfying than ones which appeals to the authority or expertise of others. Compare the following pair of arguments:

(a) Jack says traffic will be bad this afternoon. So, traffic will be bad this afternoon.

(b) Oh no! Highway repairs begin downtown today. And a bridge lift is scheduled for the middle of rush hour. Traffic is going to be terrible

Even though the second passage is an argument, the reasons used to justify the conclusion could be used in an explanation. Someone who accepts this argument will also have an explanation ready to offer if someone should later ask "Traffic was terrible today! I wonder why?". This is not true of the first passage: bad traffic is not explained by saying "Jack said it would be bad." The argument that refers to the drawbridge going up is appealing to a more powerful sort of reason, one that works in both explanations and arguments. This simply makes for a more satisfying argument, one that makes for a deeper understanding of the world, than one that merely appeals to authority.

Although arguments based on explanatory premises are preferred, we must often rely on other people for our beliefs, because of constraints on our time and access to evidence. But the other people we rely on should hopefully hold the belief on the basis of an empirical understanding. And if *those* people are just relying on authority, then we should hope that at some point the chain of testimony ends with someone who is relying on something more than mere authority.

We just have seen that the same set of statements can be used as an argument or an explanation depending on the context. This can cause confusion between speakers as to what is going on. Consider the following case:

Bill and Henry have just finished playing basketball.

Bill:	Man, I was terrible today.
Henry:	I thought you played fine.
Bill:	Nah. It's because I have a lot on my mind from work.

Bill and Henry disagree about what is happening—arguing or explaining. Henry doubts Bill's initial statement, which should provoke Bill to argue. But instead, he appears to plough ahead with his explanation. What Henry can do in this case, however, is take the reason that Bill offers as an explanation (that Bill is preoccupied by issues at work) and use it as a premise in an argument for the conclusion "Bill played terribly." Perhaps Henry will argue (to himself) something like this: "It's true that Bill has a lot on his mind from work. And whenever a person is preoccupied, his basketball performance is likely to be degraded. So, perhaps he did play poorly today (even though I didn't notice)."

In other situations, people can switch back and forth between arguing and explaining. Imagine that Jones says "The reservoir is at a low level because of several releases to protect the down-stream ecology." Jones might intend this as an explanation, but since Smith does not share the belief that the reservoir's water level is low, he will first have to be given reasons for believing that it is low. The conversation might go as follows:

Jones: The reservoir is at a low level because of several releases to protect the down-stream ecology.

Smith: Wait. The reservoir is low?

Jones: Yeah. I just walked by there this morning. You haven't been up there in a while?

Smith: I guess not.

Jones: Yeah, it's because they've been releasing a lot of water to protect the ecology lately.

When challenged, Smith offers evidence from his memory: he saw the reservoir that morning. Once Smith accepts that the water level is low, Jones can restate his explanation.

Some forms of explanation overlap with other kinds of nonargumentative passages. We are dealing right now with thinking in the real world, and as we mentioned on page 5 the real world is full of messiness and ambiguity. One effect of this is that all the categories we are discussing will wind up overlapping. Narratives and expository passages, for instance, can also function as explanations. Consider this passage

> *From an article on espn.go.com* Duke beat Butler 61-59 for the national championship Monday night. Gordon Hayward's half-court, 3-point heave for the win barely missed to leave tiny Butler one cruel basket short of the Hollywood ending.

On the one hand, this is clearly a narrative—retelling a sequence of events united by time, place, and character. But it also can work as an explanation about how Duke won, if the audience immediately accepts the result. 'The last shot was a miss and then Duke won' can be understood as 'the last shot was a miss and so Duke won'.

Practice Exercises

Part A Identify each of the passages below as an argument, an explanation, or neither, and justify your answer. If the passage is an argument write it in canonical form, with premises marked P_1 etc., then a line, and then the conclusion marked with a C. If the argument is an explanation, write it in the canonical form for an explanation, with the explainers numbered and an "E" after the line that separates the explainers and the explainee. If the argument is neither an argument nor an explanation, state what kind of nonargument you think it is, such as a narrative or an expository passage.

Example: *Henry arrives at work late:* Bill is not here. He very rarely arrives late. So, he is not coming in today.

Answer: *Argument* You can tell Henry is giving an argument to himself here because the conclusion is something that he did not already believe.

P_1: Bill is not here.
P_2: Bill very rarely arrives late.

C: Bill is not coming in today

(1) *Jack is reading a popular science magazine. It reads:* Recent research has shown that people who rate themselves as "very happy" are less successful financially than those who rate themselves as "moderately happy." *He says,* "Huh! It seems that a little unhappiness is good in life."

(2) *An anthropologist is speaking.* People get nicknames based on some distinctive feature they possess. And so, Mark, for example, who is 6'6" is (ironically) called "Smalls", while Matt, who looks young, is called "Baby Face." John looks just like his dad, and is called "Chip."

(3) *Henry is lamenting to his friend Bill.* I can't stand it any more. I'll tell you why: I'm tired of living all alone. No one ever calls me on the phone. And my

landlady tried to hit me with a mop. (Based on Lou Reed's "I Can't Stand It," from *Lou Reed*.)

(4) *Two teenaged friends are talking. Analyze Saida's reasoning.*

Saida: I can't go to the show tonight.

Jordan: Bummer.

Saida: I know! My mother wouldn't let me go out when I asked.

(5) *A mother is speaking to her teenage son.* You should always listen to your mother. I say "no." So, you have to stay in tonight.

(6) *An economist is speaking.* Any time the public receives a tax rebate, consumer spending increases. Since the public just received a tax rebate, consumer spending will increase.

(7) *In a letter to the editor.* Today's kids are all slackers. American society is doomed.

(8) *On Monday, Jack is told that his unit ships to Iraq in two days:* I was hoping to go to Henry's birthday party next weekend. But I'm shipping out on Wednesday. So, I will miss it.

(9) *A student is speaking to her instructor:* I was late for class because the battery in my mobile phone, which I was using as an alarm clock, ran out.

(10) There is a lot of positive talk concerning parenthood because people tend to think about the positive effects that have a child brings and they tend to exclude the numerous negatives that it brings.

Part B Identify each of the passages below as an argument, an explanation, or neither, and justify your answer. If the passage is an argument write it in canonical form, with premises marked P_1 etc., then a line, and then the conclusion marked with a C. If the argument is an explanation, write it in the canonical form for an explanation, with the explainers numbered and an "E" after the line that separates the explainers and the explainee. If the argument is neither an argument nor an explanation, state what kind of nonargument you think it is, such as a narrative or an expository passage.

(1) You have to be smart to understand the rules of Dungeons and Dragons. Most smart people are nerds. So, I bet most people who play D&D are nerds.

(2) *A coach is emailing parents in a neighborhood youth soccer league.* The game is canceled since it is raining heavily.

(3) *At the market.* You know, granola bars generally aren't healthy. The

ingredients include lots of processed sugars.

(4) *At the pet store.*

Salesman: A small dog makes just as effective a guard dog for your home as a big dog.

Henry: No way!

Salesman: It might seem strange. But smaller "yappy" dogs bark readily and they also generate distinctive higher-pitched sounds. Most of a dog's effectiveness as a guard is due to making a sound, not physical size.

(5) *A child is thinking out loud.* I think my cat must be dead. It isn't in any of its usual places. And when I asked my mother if she had seen it, she couldn't look me in the eyes.

(6) **Smith:** I can solve any puzzle more quickly than you.

Jones: Get out of here.

Smith: It's true! I'm a member of MENSA, and you're not.

(7) *In the comments on a biology blog:* According to Darwin's theory, my ancestors were monkeys. But since that's ridiculous, Darwin's theory is false.

(8) If you believe in [the Christian] God and turn out to be incorrect, you have lost nothing. But if you don't believe in God and turn out to be incorrect, you will go to hell. Believing in God is better in both cases. One should therefore believe in God. (A formulation of "Pascal's Wager" by Blaise Pascal.)

(9) *Bill and Henry are in Columbus.*

Bill: Good news—I just accepted a job offer in Omaha.

Henry: That's great. Congratulations! I suppose this means you'll be leaving us, then?

Bill: Yes, I'll need to move sometime before September.

(10) You already know that God kicked humanity out of Eden before they could eat of the tree of life but only after they had eaten of the tree of knowledge of good and evil. That was because Satan wanted to take over God's throne and was responsible for their eating from the tree. If humans had eaten of both trees they could have been a threat to God.

Key Terms

Argument

Canonical form

Conclusion

Conclusion indicator

Content neutrality

Critical thinking

Explainee

Explainer

Explanation

Expository passage

Formal logic

Inference

Informal logic

Logic

Metacognition

Metareasoning

Narrative

Premise

Premise indicator

Reason

Rhetoric

Simple statement of belief

Statement

Target proposition

Chapter 2

The Basics of Evaluating Argument

2.1 Two Ways an Argument Can Go Wrong

Arguments are supposed to lead us to the truth, but they don't always succeed. There are two ways they can fail in their mission. First, they can simply start out wrong, using false premises. Consider the following argument.

P_1: It is raining heavily.
P_2: If you do not take an umbrella, you will get soaked.

C: You should take an umbrella.

If premise (1) is false—if it is sunny outside—then the argument gives you no reason to carry an umbrella.The argument has failed its job. Premise (2) could also be false: Even if it is raining outside, you might not need an umbrella. You might wear a rain poncho or keep to covered walkways and still avoid getting soaked. Again, the argument fails because a premise is false.

Even if an argument has all true premises, there is still a second way it can fail. Suppose for a moment that both the premises in the argument above are true. You do not own a rain poncho. You need to go places where there are no covered walkways. Now does the argument show you that you should take an umbrella?

Not necessarily. Perhaps you enjoy walking in the rain, and you would like to get soaked. In that case, even though the premises were true, the conclusion would be false. The premises, although true, do not *support* the conclusion. Back on page 13 we defined an inference, and said it was like argument glue: it holds the premises and conclusion together. When an argument goes wrong because the premises do not support the conclusion, we say there is something wrong with the inference.

Consider another example:

P_1: You are reading this book.
P_2: This is a logic book.

C: You are a logic student.

This is not a terrible argument. Most people who read this book are logic students. Yet, it is possible for someone besides a logic student to read this book. If your roommate picked up the book and thumbed through it, they would not immediately become a logic student. So the premises of this argument, even though they are true, do not guarantee the truth of the conclusion. Its inference is less than perfect.

Again, for any argument, there are two ways that it could fail. First, one or more of the premises might be false. Second, the premises might fail to support the conclusion. Even if the premises were true, the form of the argument might be weak, meaning the inference is bad.

2.2 Valid, Sound

In logic, we are mostly concerned with evaluating the quality of inferences, not the truth of the premises. The truth of various premises will be a matter of whatever specific topic we are arguing about, and, as we have said, logic is content neutral.

The strongest inference possible would be one where the premises, if true, would somehow force the conclusion to be true. This kind of inference is called valid. There are a number of different ways to make this idea of the premises forcing the truth of the conclusion more precise. Here are a few:

An argument is valid if and only if...

P_1: Lady Gaga is from Mars.
P_2: Mars is the fourth planet from our sun.

C: Lady Gaga is from the fourth planet from our sun.

Figure 2.1: A **valid** argument.

(a) it is impossible to consistently both (i) accept the premises and (ii) reject the conclusion

(b) it is impossible for the premises to be true and the conclusion false

(c) the premises, if true, would necessarily make the conclusion true.

(d) the conclusion is true in every imaginable scenario in which the premises are true

(e) it is impossible to write a consistent story (even fictional) in which the premises are true and the conclusion is false

In the glossary, we formally adopt item (b) as the definition for this textbook: an argument is VALID if and only if it is impossible for the premises to be true and the conclusion false. However, nothing will really ride on the differences between the definitions in the list above, and we can look at all of them in order to give us a sense of what logicians mean when they use the term "valid".

The important thing to see is that all the definitions in the list above try to get at what *would* happen if the premises were true. None of them assert that the premises actually *are* true. This is why definitions (d) and (e) talk about what would happen if you somehow *pretend* the premises are true, for instance by telling a story. The argument is valid if, when you pretend the premises are true, you also have to pretend the conclusion is true. Consider the argument in Figure 2.1

The American pop star Lady Gaga is not from Mars. (She's from New York City.) Nevertheless, if you imagine she's from Mars and that Mars is the fourth planet from our sun (the second part is true), then you must imagine that Lady Gaga is from the fourth planet from our sun. Therefore this argument is valid.

This way of understanding validity is based on what you can imagine, but not everyone is convinced that the imagination is a reliable tool in logic. That is why definitions like (c) and (b) talk about what is necessary or impossible. If the premises are true, the conclusion necessarily must be true. Alternately, it is

P$_1$: Oranges are either fruits or musical instruments.
P$_2$: Oranges are not fruits.

C: Oranges are musical instruments.

Figure 2.2: A **valid** argument

impossible for the premises to be true and the conclusion false. The idea here is that instead of talking about the imagination, we will just talk about what can or cannot happen at the same time. The fundamental notion of validity remains the same, however: the truth of the premises would simply guarantee the truth of conclusion.

So, assessing validity means wondering about whether the conclusion would be true *if* the premises were true. This means that valid arguments can have false conclusions. This is important to keep in mind because people naturally tend to think that any argument must be good if they agree with the conclusion. And the more passionately people believe in the conclusion, the more likely we are to think that any argument for it must be brilliant. Conversely, if the conclusion is something we don't believe in, we naturally tend to think the argument is poor. And the more we don't like the conclusion, the less likely we are to like the argument.

But this is not the way to evaluate inferences at all. The quality of the inference is entirely independent of the truth of the conclusion. You can have great arguments for false conclusions and horrible arguments for true conclusions. An argument is valid if it is impossible for the premises to be true and the conclusion false. This means that you can have valid arguments with false conclusions, they just have to also have false premises. Consider the example in Figure 2.2

The conclusion of this argument is ridiculous. Nevertheless, it follows validly from the premises. This is a valid argument. *If* both premises were true, *then* the conclusion would necessarily be true.

This shows that a valid argument does not need to have true premises or a true conclusion. Conversely, having true premises and a true conclusion is not enough to make an argument valid. Consider the example in Figure 2.3

The premises and conclusion of this argument are, as a matter of fact, all true. This is a terrible argument, however, because the premises have nothing to do with the conclusion. Imagine what would happen if Paris declared independence from the rest of France. Then the conclusion would be false, even though the premises would both still be true. Thus, it is *logically possible* for the premises of this argument to be true and the conclusion false. The argument is not valid. If an argument is not valid, it is called **invalid**. As we shall see, this term is a little misleading, because less than perfect arguments can be very useful. But before we do that, we need to look more at the concept of validity.

In general, then, the *actual* truth or falsity of the premises, if known, do not tell you whether or not an inference is valid. There is one exception: when the premises are true and the conclusion is false, the inference cannot be valid, because valid reasoning can only yield a true conclusion when beginning from true premises.

Figure 2.4 has another invalid argument:

In this case, we can see that the argument is invalid by looking at the truth of the premises and conclusion. We know the premises are true. We know that the conclusion is false. This is the one circumstance that a valid argument is supposed to make impossible.

Some invalid arguments are hard to detect because they resemble valid arguments. Consider the one in Figure 2.5

This reasoning is not valid since the premises do not *definitively* support the conclusion. To see this, assume that the premises are true and then ask, "Is it possible that the conclusion could be false in such a situation?". There is no inconsistency in taking the premises to be true without taking the conclusion to be true. The first premise says that the stimulus package will allow the U.S. to avoid a depression, but it does not say that a stimulus package is the *only* way to avoid a depression. Thus, the mere fact that there is no stimulus package does not necessarily mean that a depression will occur.

Here is another, trickier, example. I will give it first in ordinary language.

> *A pundit is speaking on a cable news show* If the U.S. economy were in

P_1: London is in England.
P_2: Beijing is in China.

C: Paris is in France.

Figure 2.3: An **invalid** argument.

P$_1$: All dogs are mammals
P$_2$: All dogs are animals

C: All animals are mammals.

Figure 2.4: An **invalid** argument.

> recession and inflation were running at more than 4%, then the value of the U.S. dollar would be falling against other major currencies. But this is not happening — the dollar continues to be strong. So, the U.S. is not in recession.

The conclusion is "The U.S. economy is not in recession." If we put the argument in canonical form, it looks like figure 2.6

The conclusion does not follow necessarily from the premises. It does follow necessarily from the premises that (i) the U.S. economy is not in recession or (ii) inflation is running at more than 4%, but they do not guarantee (i) in particular, which is the conclusion. For all the premises say, it is possible that the U.S. economy is in recession but inflation is less than 4%. So, the inference does not *necessarily* establish that the U.S. is not in recession. A parallel inference would be "Jack needs eggs and milk to make an omelet. He can't make an omelet. So, he doesn't have eggs.".

If an argument is not only valid, but also has true premises, we call it **sound**. "Sound" is the highest compliment you can pay an argument. If logic is the study of virtue in argument, sound arguments are the most virtuous. We said in Section 2.1 that there were two ways an argument could go wrong, either by having false premises or weak inferences. Sound arguments have true premises and undeniable inferences. If someone gives a sound argument in a conversation, you have to believe the conclusion, or else you are irrational.

The argument on the left in Figure 2.7 is valid, but not sound. The argument on

P$_1$: An economic stimulus package will allow the U.S. to avoid a depression.
P$_2$: There is no economic stimulus package

C: The U.S. will go into a depression.

Figure 2.5: An **invalid** argument

P_1: If the U.S. were in a recession with more than 4% inflation, then the dollar would be falling

P_2: The dollar is not falling

C: The U.S. is not in a recession.

Figure 2.6: An **invalid** argument

the right is both valid and sound.

Both arguments have the exact same form. They say that a thing belongs to a general category and everything in that category has a certain property, so the thing has that property. Because the form is the same, it is the same valid inference each time. The difference in the arguments is not the validity of the inference, but the truth of the second premise. People are not carrots, therefore the argument on the left is not sound. People are mortal, so the argument on the right is sound.

Often it is easy to tell the difference between validity and soundness if you are using completely silly examples. Things become more complicated with false premises that you might be tempted to believe, as in the argument in Figure 2.8.

You might have a general sense that the argument in Figure 2.8 is bad—you shouldn't assume that someone drinks Guinness just because they are Irish. But the argument is completely valid (at least when it is expressed this way.) The inference here is the same as it was in the previous two arguments. The problem is the first premise. Not all Irishmen drink Guinness, but if they did, and Smith was an Irishman, he would drink Guinness.

The important thing to remember is that validity is not about the actual truth or falsity of the statements in the argument. Instead, it is about the way the

P_1: Socrates is a person.	P_1: Socrates is a person.
P_2: All people are carrots.	P_2: All people are mortal.
C: Therefore, Socrates is a carrot.	C: Therefore, Socrates is mortal.
Valid, but not sound	**Valid and sound**

Figure 2.7: These two arguments are valid, but only the one on the right is sound

P_1: Every Irishman drinks Guinness
P_2: Smith is an Irishman

C: Smith drinks Guinness.

Figure 2.8: An argument that is **valid** but not *sound*

premises and conclusion are put together. It is really about the *form* of the argument. A valid argument has perfect logical form. The premises and conclusion have been put together so that the truth of the premises is incompatible with the falsity of the conclusion.

A general trick for determining whether an argument is valid is to try to come up with just one way in which the premises could be true but the conclusion false. If you can think of one, the reasoning is *invalid.*

Practice Exercises

Part A Put the following arguments in canonical form and then decide whether they are valid. If the argument is invalid, explain why.

Example: *Monica is looking for her coworker* Jack is in his office. Jack's office is on the second floor. So, Jack is on the second floor.

Answer: P_1: Jack is in his office.
P_2: Jack's office is on the second floor.

C: Jack is on the second floor.

Valid

(1) All dinosaurs are people, and all people are fruit. Therefore all dinosaurs are fruit.

(2) All dogs are mammals. Therefore, Fido is a mammal, because Fido is a dog.

(3) Abe Lincoln must have been from France, because he was either from France or from Luxemborg, and we know was not from Luxemborg.

(4) Love is blind. God is love. Ray Charles is blind. Therefore, Ray Charles is God

(5) If the world were to end today, then I would not need to get up tomorrow morning. I will need to get up tomorrow morning. Therefore, the world will not end today.

(6) All people are mortal. Socrates is mortal. Therefore all people are Socrates.

(7) *A forest ranger is surveying the park* I can tell that bears have been down by the river, because there are tracks in the mud. Tracks like these are made by

bears in almost every case.

(8) If the triceratops were a dinosaur, it would be extinct. Therefore, the triceratops is extinct, because the triceratops was a dinosaur.

(9) If George Washington was assassinated, he is dead. George Washington is dead. Therefore George Washington was assassinated.

(10) Jack prefers Pepsi to Coke. After all, about 52% of people prefer Pepsi to Coke, and Jack is a person.

Part B Put the following arguments in canonical form and then decide whether they are valid. If the argument is invalid, explain why.

(1) Cindy Lou Who lives in Whoville. You can tell because Cindy Lou Who is a Who, and all Whos live in Whoville.

(2) If Frog and Toad like each other, they are friends. Frog and Toad like each other. Therefore, Frog and Toad are friends.

(3) If Cindy Lou Who is no more than two, then she is not five years old. Cindy Lou Who is not five. Therefore Cindy Lou Who is two or more.

(4) *Jack's suspicious house mate is in the kitchen* Jack has moved my leftover slice of pizza. Jack must have moved it, because Jack is the only person who has been in the house, and the pizza is no longer in the fridge.

(5) Jack is Smith's work colleague. So, Jack and Smith are friends.

(6) Abe Lincoln was either born in Illinois or he was once president. Therefore Abe Lincoln was born in Illinois, because he was never president.

(7) Politicians get a generous allowance for transportation costs. Enda Kenny is a politician. Therefore Kenny gets a generous transportation allowance.

(8) Jones is taller than Bill, because Smith is taller than Jones and Bill is shorter than Smith.

(9) If grass is green, then I am the pope. Grass is green. So, I am the pope.

(10) Smith is paid more than Jack. They are both paid weekly. So, Smith has more money than Jack.

Part C Put the following arguments in canonical form and then decide whether they are valid. If the argument is invalid, explain why.

(1) Jack is close to the pond. The pond is close to the playground. So, Jack is close to the playground.

(2) *Jack is at work:* I have up to half an hour to get to the bank, because work ends at 5:00 and the bank closes at 5:30.

(3) Jack and Gill ate at Guadalajara restaurant earlier and both of them feel nauseated now. So, something they ate there is making them sick.

(4) Zhaoquing must be west of Huizhou, because Zhaoquing is west of Guangzhou, which is west of Huizhou.

(5) *Henry can't find his glasses.* I remember I had them when I came in from the car. So, they are in the house somewhere.

(6) I was talking about tall John—the one who is over 6'4"—but Jack was talking about short John, who is at most 5'2". So, we were talking about two different Johns.

(7) Tomorrow's trip to Ensenada will take about 10 hours, because the last time I drove there from here it took 10 hours.

Part D Put the following arguments in canonical form and then decide whether they are valid. If the argument is invalid, explain why.

(1) *Monica is surveying the crowd that showed up for her talk* There must be at least 150 people here. That's how many people the auditorium holds, and every seat is full and people are beginning to sit on the stairs at the side.

(2) The fire bell in the building is ringing. There is sometimes a fire in the building when the alarm goes off. So, there is a fire.

(3) I cannot drive on the motorways yet, because I just passed my driving test and anyone who passes can drive on the roads but not on the motorway for six months.

(4) Yesterday's the temperature reached 91 degrees Fahrenheit. Today it is 94. So, today is warmer than yesterday.

(5) My car is functioning well at the moment. So, all of the parts in my car are functioning well.

(6) It has been sunny every day for the last five days. So, it will be sunny today.

(7) Jack is in front of Gill. So, Gill is behind Jack.

(8) *Gill is returning home*: The door to my house is still locked. So, my possessions are still inside.

2.3 Strong, Cogent, Deductive, Inductive

We have just seen that sound arguments are the very best arguments. Unfortunately, sound arguments are really hard to come by, and when you do find them, they often only prove things that were already quite obvious, like that Socrates (a dead man) is mortal. Fortunately, arguments can still be worthwhile, even if they are not sound. Consider this one:

P_1: In January 1997, it rained in San Diego.
P_2: In January 1998, it rained in San Diego.
P_3: In January 1999, it rained in San Diego.

C: It rains every January in San Diego.

This argument is not valid, because the conclusion could be false even though the premises are true. It is possible, although unlikely, that it will fail to rain next January in San Diego. Moreover, we know that the weather can be fickle. No amount of evidence should convince us that it rains there *every* January. Who is to say that some year will not be a freakish year in which there is no rain in January in San Diego? Even a single counterexample is enough to make the conclusion of the argument false.

Still, this argument is pretty good. Certainly, the argument could be made stronger by adding additional premises: In January 2000, it rained in San Diego. In January 2001. . . and so on. Regardless of how many premises we add, however, the argument will still not be deductively valid. Instead of being valid, this argument is strong. An argument is **strong** if the premises would make the conclusion more likely, were they true. In a strong argument, the premises don't guarantee the truth of the conclusion, but they do make it a good bet. If an argument is strong, and it has true premises, we say that it is **cogent** Cogency is the equivalent of soundness in strong arguments. If an inference is neither valid, nor strong, we say it is **weak**. In a weak argument, the premises would not even make the conclusion likely, even if they were true.

You may have noticed that the word "likely" is a little vague. How likely do the premises have to make the conclusion before we can count the argument as strong? The answer is a very unsatisfying "it depends." It depends on what is at stake in the decision to believe the conclusion. What happens if you are wrong? What happens if you are right? The phrase "make the conclusion a good bet" is really quite apt. Whether something is a good bet depends a lot on how much money is at stake and how much you are willing to lose. Sometimes people feel comfortable taking a bet that has a 50% chance of doubling their money, sometimes they don't.

The vagueness of the word "likely" brings out an interesting feature of strong arguments: some strong arguments are stronger than others. The argument about rain in San Diego, above, has three premises referring to three previous Januaries. The argument is pretty strong, but it can become stronger if we go back farther into the past, and find more years where it rains in January. The more evidence we have, the better a bet the conclusion is. Validity is not like this. Validity is a black-or-white matter. You either have it, and you're perfect, or you don't, and you're nothing. There is no point in adding premises to an argument that is already valid.

Arguments that are valid, or at least try to be, are called **deductive** , and people who attempt to argue using valid arguments are said to be arguing *deductively.* The notion of validity we are using here is, in fact, sometimes called *deductive validity.* Deductive argument is difficult, because, as we said, in the real world sound arguments are hard to come by, and people don't always recognize them as sound when they find them. Arguments that purport to merely be strong rather than valid are called **inductive**. The most common kind of inductive argument includes arguments like the one above about rain in San Diego, which generalize from many cases to a conclusion about all cases.

Deduction is possible in only a few contexts. You need to have clear, fixed meanings for all of your terms and rules that are universal and have no exceptions. One can find situations like this if you are dealing with things like legal codes, mathematical systems or logical puzzles. One can also create, as it were, a context where deduction is possible by imagining a universal, exceptionless rule, even if you know that no such rule exists in reality. In the example above about rain in San Diego, we can change the argument from inductive to deductive by adding a universal, exceptionless premise like "It always rains in January in San Diego." This premise is unlikely to be true, but it can make the inference valid. (For more about trade offs between the validity of the inference and the truth of the premise, see the chapter on incomplete arguments in the complete version of this text.

For an example of deductive reasoning consider a Sudoku puzzle. The rules of Sudoku are that each cell contains a single number from 1 to 9, and each row, each column and each 9-cell square contain one occurrence of each number from 1 to 9. Consider the following partially completed board:

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

The following inference shows that, in the first column, a 9 must be entered below the 7:

> The 9 in the first column must go in one of the open cells in the column. It cannot go in the third cell in the column, because there is already a 9 in that 9-cell square. It cannot go in the eighth or ninth cell because each of these rows already contains a 9, and a row cannot contain two occurrences of the same number. Therefore, since there must be a 9 somewhere in this column, it must be entered in the seventh cell, below the 7.

The reasoning in this inference is valid: if the premises are true, then the conclusion must be true. Logic puzzles of all sorts operate by artificially restricting the available options in various ways. This then means that each conclusion arrived at (assuming the reasoning is correct) is necessarily true.

One can also create a context where deduction is possible by imagining a rule that holds without exception. This can be done with respect to any subject matter at all. Speakers often exaggerate the connecting premise in order to ensure that the justificatory or explanatory power of the inference is as strong as possible. Consider Smith's words in the following passage:

Smith: I'm going to have some excellent pizza this evening.

Jones: I'm glad to hear it. How do you know?

Smith: I'm going to Adriatico's. They always make a great pizza.

Here, Smith justifies his belief that the pizza will be excellent — it comes from Adriatico's, where the pizza, he claims, is *always* great: in the past, present and future.

As stated by Smith, the inference that the pizza will be great this evening is valid. However, making the inference valid in this way often means making the general premise false: it's not likely that the pizza is great *every single* time; Smith is overstating the case for emphasis. Note that Smith does not need to use a universal proposition in order to convince Jones that the pizza will *very likely* be good. The inference to the conclusion would be strong (though not valid) if he had said that the pizza is "almost always" great, or that the pizza has been great on all of the many occasions he has been at that restaurant in the past. The strength of the inference would fall to some extent—it would not be guaranteed to be great this evening—but a slightly weaker inference seems appropriate, given that sometimes things go contrary to expectation.

Sometimes the laws of nature make constructing contexts for valid arguments more reasonable. Now consider the following passage, which involves a scientific law:

> Jack is about to let go of Jim's leash. The operation of gravity makes all unsupported objects near the Earth's surface fall toward the center of the Earth. Nothing stands in the way. Therefore, Jim's leash will fall.

(Or, as Spock said in a Star Trek episode, "If I let go of a hammer on a planet that has a positive gravity, I need not see it fall to know that it has in fact fallen.") The inference above is represented in canonical form as follows:

P_1: Jack is about to let go of Jim's leash.
P_2: The operation of gravity makes all unsupported objects near the Earth's surface fall toward the center of the Earth.
P_3: Nothing stands in the way of the leash falling.

C: Jim's leash will fall toward the center of the Earth.

As stated, this argument is valid. That is, if you pretend that they are true or accept them "for the sake of argument", you would *necessarily* also accept the conclusion. Or, to put it another way, there is no way in which you could hold the premises to be true and the conclusion false.

Although this argument is valid, it involves idealizing assumptions similar to the

A strong argument	A weak argument
P_1: 92% of Republicans from Texas voted for Bush in 2000. P_2: Jack is a Republican from Texas. ——— C: Jack voted for Bush.	P_1: Just over half of drivers are female. P_2: There's a person driving the car that just cut me off. ——— C: The person driving the car that just cut me off is female.

Figure 2.9: Neither argument is valid, but one is strong and one is weak

ones we saw in the pizza example. P_2 states a physical law which is about as well confirmed as any statement about the world around us you care to name. However, physical laws make assumptions about the situations they apply to—they typically neglect things like wind resistance. In this case, the idealizing assumption is just that nothing stands in the way of the leash falling. This can be checked just by looking, but this check can go wrong. Perhaps there is an invisible pillar underneath Jack's hand? Perhaps a huge gust of wind will come? These events are much less likely than Adriatico's making a lousy pizza, but they are still possible.

Thus we see that using scientific laws to create a context where deductive validity is possible is a much safer bet than simply asserting whatever exceptionless rule pops into your head. However, it still involves improving the quality of the inference by introducing premises that are less likely to be true.

So deduction is possible in artificial contexts like logical puzzles. It is also possible in cases where we make idealizing assumptions or imagine exceptionless rules. The rest of the time we are dealing with induction. When we do induction, we try for strong inferences, where the premises, assuming they are true, would make the truth of the conclusion very likely, though not necessary. Consider the two arguments in Figure 2.9

Note that the premises in neither inference *guarantee* the truth of the conclusion. For all the premises in the first one say, Jack could be one of the 8% of Republicans from Texas who did not vote for Bush; perhaps, for example, Jack soured on Bush, but not on Republicans in general, when Bush served as governor. Likewise for the second; the driver could be one of the 49%.

So, neither inference is valid. But there is a big difference between how much support the premises, if true, would give to the conclusion in the first and how much they would in the second. The premises in the first, assuming they are true, would provide very strong reasons to accept the conclusion. This, however, is not the case with the second: if the premises in it were true then they would give only weak reasons for believing the conclusion. thus, the first is strong while the second is weak.

As we said earlier, there there are only two options with respect to validity—valid or not valid. On the other hand, strength comes in degrees, and sometimes arguments will have percentages that will enable you to exactly quantify their strength, as in the two examples in Figure 2.9.

However, even where the degree of support is made explicit by a percentage there is no firm way to say at what degree of support an inference can be classified as strong and below which it is weak. In other words, it is difficult to say whether or not a conclusion is *very likely* to be true. For example, In the inference about whether Jack, a Texas Republican, voted for Bush. If 92% of Texas Republicans voted for Bush, the conclusion, if the premises are granted, would very probably be true. But what if the number were 85%? Or 75%? Or 65%? Would the conclusion very likely be true? Similarly, the second inference involves a percentage greater than 50%, but this does not seem sufficient. At what point, however, would it be sufficient?

In order to answer this question, go back to basics and ask yourself: "If I accept the truth of the premises, would I then have sufficient reason to believe the conclusion?". If you would not feel safe in adopting the conclusion as a belief as a result of the inference, then you think it is weak, that is, you do not think the premises give sufficient support to the conclusion.

Note that the same inference might be weak in one context but strong in another, because the degree of support needed changes. For example, if you merely have a deposit to make, you might accept that the bank is open on Saturday based on your memory of having gone to the bank on Saturday at some time in the past. If, on the other hand, you have a vital mortgage payment to make, you might not consider your memory sufficient justification. Instead, you will want to call up the bank and increase your level of confidence in the belief that it will be open on Saturday.

Most inferences (if successful) are strong rather than valid. This is because they deal with situations which are in some way open-ended or where our knowledge is not precise. In the example of Jack voting for Bush, we know only that 92% of Republicans voted for Bush, and so there is no definitive connection between being a Texas Republican and voting for Bush. Further, we have only statistical information to go on. This statistical information was based on polling or surveying a sample of Texas voters and so is itself subject to error (as is discussed in the chapter on induction in the complete version of this text. A more precise version of the premise might be "92% $\pm$ 3% of Texas Republicans voted for Bush.".

At the risk of redundancy, let's consider a variety of examples of valid, strong and weak inferences, presented in standard form.

P_1: David Duchovny weighs more than 200 pounds.

C: David Duchovny weighs more than 150 pounds.

The inference here is valid. It is valid because of the number system (here applied to weight): 200 is more than 150. It might be false, as a matter of fact, that David Duchovny weighs more than 200 pounds, and false, as a matter of fact, that David Duchovny weighs more than 150 pounds. But if you *suppose* or *grant* or *imagine* that David Duchovny weighs more than 200 pounds, it would then *have* to be true that David Duchovny weighs more than 150 pounds. Next:

P_1: Armistice Day is November 11th, each year.
P_2: Halloween is October 31st, each year.

C: Armistice Day is later than Halloween, each year.

This inference is valid. It is valid because of order of the months in the Gregorian calendar and the placement of the New Year in this system. Next:

P_1: All people are mortal.
P_2: Professor Pappas is a person.

C: Professor Pappas is mortal.

As written, this inference is valid. If you accept for the sake of argument that all men are mortal (as the first premise says) and likewise that Professor Pappas is a man (as the second premise says), then you would have to accept also that Professor Pappas is mortal (as the conclusion says). You could not consistently both (i) affirm that all men are mortal and that Professor Pappas is a man and (ii) deny that Professor Pappas is mortal. If a person accepted these premises but denied the conclusion, that person would be making a mistake in logic.

This inference's validity is due to the fact that the first premise uses the word "all". You might, however, wonder whether or not this premise is true, given that we believe it to be true only on our experience of men *in the past*. This might be a case of over-stating a premise, which we mentioned earlier. Next:

P_1: In 1933, it rained in Columbus, Ohio on 175 days.
P_2: In 1934, it rained in Columbus, Ohio on 177 days.
P_3: In 1935, it rained in Columbus, Ohio on 171 days.

C: In 1936, it rained in Columbus, Ohio on at least 150 days.

This inference is strong. The premises establish a record of days of rainfall that is well above 150. It is possible, however, that 1936 was exceptionally dry, and this possibility means that the inference does not achieve validity. Next:

P_1: The Bible says that homosexuality is an abomination.

C: Homosexuality is an abomination.

This inference is an appeal to a source. Appeals to sources are discussed in the sections on arguments from authority in the complete version of this text. of this book. In brief, you should think about whether the source is reliable, is biased, and whether the claim is consistent with what other authorities on the subject say. You should apply all these criteria to this argument for yourself. You should ask what issues, if any, the Bible is reliable on. If you believe humans had any role in writing the Bible, you can ask about what biases and agendas they might have had. And you can think about what other sources—religious texts or moral experts—say on this issue. You can certainly find many who disagree. Given the controversial nature of this issue, we will not give our evaluation. We will only encourage you to think it through systematically.

P_1: Some professional philosophers published books in 2007.
P_2: Some books published in 2007 sold more than 100,000 copies.

C: Some professional philosophers published books in 2007 that sold more than 100,000 copies.

This reasoning is weak. Both premises use the word "some" which doesn't tell you a lot about many professional philosophers published books and how many books sold more than 100,000 copies in 2007. This means that you cannot be confident that even one professional philosopher sold more than 100,000 copies. Next:

P_1: Lots of Russians prefer vodka to bourbon.

C: George Bush was the President of the United States in 2006.

No one (in her right mind) would make an inference like this. It is presented here as an example only: it is clearly weak. It's hard to see how the premise justifies the conclusion to any extent at all.

To sum up this section, we have seen that there are two styles of reasoning, deductive and inductive. The former tries to use valid arguments, while the latter contents itself to give arguments that are merely strong. The section of this book on formal logic will deal entirely with deductive reasoning. Historically, most of formal logic has been devoted to the study of deductive arguments, although many great systems have been developed for the formal treatment of inductive logic. On the other hand, the sections of this book on informal logic and critical thinking will focus mostly on inductive logic, because these arguments are more readily available in the real world.

Practice Exercises

Part A For each inference, (i) say whether it is valid, strong, or weak and (ii) explain your answer.

Example: The patient has a red rash covering the extremities and head, but not the torso. The only cause of such a rash is a deficiency in vitamin K. So, the patient must have a vitamin K deficiency.

Answer: (i) Valid.
(ii) The word "only" means it must be vitamin K deficiency.

(1) All men are things with purple hair, and all things with purple hair have nine legs. Therefore, all men have nine legs.

(2) Elvis Presley was known as The King. Elvis had 18 songs reach #1 in the Billboard charts. So, The King had 18 #1 hits.

(3) Most philosophers are right-handed. Terence Irwin is a philosopher. So, he is right-handed.

(4) Jack has purple hair, and purple toe nails. Hence, he has toe nails.

(5) The Ohio State football team beat the Miami football team on 2003-01-03 for the college national championship. So, the Ohio State football team was the best team in college football in the 2002-2003 season.

(6) Willie Mosconi made almost all of the pool shots he took from 1940-1945. He took a bunch of shots in 1941. So, he made almost every shot he took in 1941.

(7) Some philosophers are people who are right-handed. Therefore, some people who are right-handed are philosophers.

(8) U.S. President Obama firmly believed that Iran is planning a nuclear attack against Israel. We can conclude that Iran is planning a nuclear attack on Israel.

(9) Since the Spanish American War occurred before the American Civil War, and since the American Civil War occurred after the Korean War, it follows that the Spanish American War occurred before the Korean War.

(10) There are exactly 10 humans in Carnegie Hall right now. Every human in Carnegie Hall right now has exactly ten legs. And, of course, no human in Carnegie Hall shares any legs with another human. Thus, there are at least 100 legs in Carnegie Hall right now.

(11) Amy Bishop is an evolutionary biologist (who shot a number of her colleagues to death in 2010). Evolutionary biology is incompatible with [Christian] scriptural teaching. Scriptural teaching is the only grounding for morality. Thus, evolutionary biologists are immoral.

(12) Corrupt people do harm to those around them, and no one intentionally wants to be done harm. Therefore, I [Socrates] did not corrupt my associates intentionally.

(13) Taxation means paying some of your earned income to the government. Some of this income is distributed to others. Paying so that someone else can benefit is slavery. Therefore, taxation is slavery.

Part B For each inference, (i) say whether it is valid, strong, or weak and (ii) explain your answer.

(1) The sun has come up in the east every day in the past. So, the sun will come up in the east tomorrow.

(2) Jack's dog Jim will die before the age of 73 (in human years). After all, you are familiar with lots of dogs, and lots of different kinds of dogs, and any dog that is now dead died before the age of 73 (in human years).

(3) Any time the public receives a tax rebate, consumer spending increases, and the economy is stimulated. Since the public just received a tax rebate, consumer spending will increase.

(4) 90% of the marbles in the box are blue. So, about 90% of the 20 I pick at random will be blue.

(5) According to the world-renowned physicist Stephen Hawking, quarks are one of the fundamental particles of matter. So, quarks are one of the fundamental particles of matter.

(6) Sean Penn, Susan Sarandon and Tim Robbins are actors, and Democrats. So, most actors are Democrats.

(7) The President's approval rating has now fallen to 53%, employment is at a 10 year high, and he is in charge of two foreign wars. He would not win another term in two years' time, if he were to run.

(8) If Bill Gates owns a lot of gold then Bill Gates is rich, and Bill Gates doesn't own a lot of gold. So, Bill Gates isn't rich.

(9) All birds have wings, and all vertebrates have wings. So, all birds are vertebrates.

(10) U.S. President Obama gave a speech in Berlin shortly after his inauguration. Berlin, of course, is where Hitler gave many speeches. Thus, Obama intends to establish a socialist system in the U.S.

(11) Einstein said that he believed in a god only in the sense of a pantheistic god equivalent with nature. Thus, there is no god in the Judeo-Christian sense.

(12) The United States Congress has more members than there are days in the year. Thus, at least two members of the United States Congress celebrate their birthdays on the same day of the year.

(13) The base at Guantanamo ought to be closed. The continued incarceration of prisoners without any move to try or release them provides terrorist organizations with an effective recruiting tool, perhaps leading to attacks

against Americans overseas.

(14) Smith and Jones surveyed teenagers (13-19 years old) at a local mall and found that 94% of this group owned a mobile phone. Therefore, they concluded, about 94% of all teenagers own mobile phone.

(15) Janice Brooks is an unfit mother. Her Facebook and Twitter records show that in the hour prior to the youngest son's accident she had sent 50 messages — any parent who spends this much time on social media when they have kids is not giving them proper attention.

Key Terms

Cogent

Deductive

Fallacy

Fallacy of mistaking the conclusion for the argument

Inductive

Invalid

Sound

Strong

Valid

Weak

Part II

Categorical Logic

Chapter 3

Categorical Statements

3.1 Quantified Categorical Statements

Back in Chapter 1, we saw that a statement was a unit of language that could be true or false. In this chapter and the next we are going to look at a particular kind of statement, called a quantified categorical statement, and begin to develop a formal theory of how to create arguments using these statements. This kind of logic is generally called "categorical" or "Aristotelian" logic, because it was originally invented by the great logician and philosopher Aristotle in the fourth century BCE. This kind of logic dominated the European and Islamic worlds for 20 centuries afterward, and was expanded in all kinds of fascinating ways, some of which we will look at here.

Consider the following propositions:

(a) All dogs are mammals.

(b) Some physicists are female.

(c) No dogs are cats.

(d) Some Americans are doctors.

(e) Some adults are not logicians.

(f) All humans breathe.

These are all examples of quantified categorical statements. A **quantified categorical statement** is a statement that makes a claim about the inclusion or exclusion of all or some of the members of one category from another category. (Sometimes we will just call these "categorical statements") Statement (a), for example, is about the class of dogs and the class of mammals. These statements make no mention of any particular members of the categories or classes or types they are about. The propositions are also *quantified* in that they state *how many* of the things in one class are also members of the other. For instance, statement (b) talks about *some* physicists, while statement (a) talks about *all* dogs.

Categorical statements can be broken down into four parts: the quantifier, the subject term, the predicate term, and the copula. The **quantifier** is the part of a categorical sentence that specifies how many things the statement is talking about. The quantifiers in the sentences above are all, no, and some. Notice that the "no" in sentence (c) counts as a quantifier, since it says of all of the members of the class of dogs that they are not in the class of cats. The subject and predicate terms are the two classes the statement talks about. The **subject class** is the first class mentioned in a quantified categorical statement, and the **predicate class** is the second. In sentence (d), for instance, the subject class is the class of Americans and the predicate class is the class of doctors. The **copula** is simply the form of the verb "to be" that links subject and predicate. To summarize: The quantifier tells us how much of the subject class is included or excluded from the predicate class.

Sentence (f) is a little different than the others. In sentence (f) the subject is the class of humans and the predicate is the class of things that breathe. That's not quite the way it is written, however. There is no explicit copula, and instead of giving a noun phrase for the predicate term, like "people who breathe," it has a verb phrase, "breathe." If you are asked to identify the copula and predicate for a sentence like this, you should say that the copula is either "are" or "are not," depending on the situation and transform the verb phrase into a noun phrase. We will go into more detail about these issues in Section 3.5.

In formal logic we achieve content neutrality by replacing some or all of the ordinary words in a statement with symbols. For categorical logic, we are only going to be making one such substitution. Sometimes we will replace the classes referred to in a quantified categorical statement with capital letters that act as variables. Typically we will use the letter S when referring to the class in the subject term and P when referring to the predicate term, although sometimes more letters will be needed. Thus the sentence "Some Americans are doctors," above, will sometimes become "Some S are P." The sentence "No dogs are cats" will

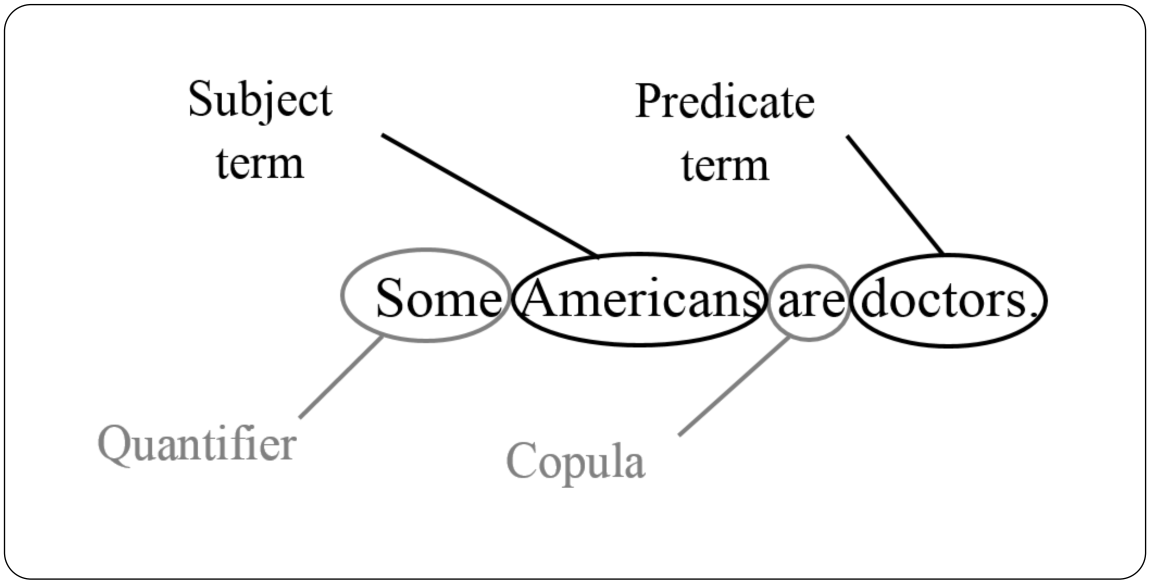


Figure 3.1: Parts of a quantified categorical statement.

sometimes become"No S are P."

Practice Exercises

Part A For each of the following sentences identify the quantifier, the subject term, the predicate term, and the copula. Some of these are like the example "Thirty percent of Canadians speak French" where the copula is implicit and the predicate needs to be transformed into a noun phrase.

Example: Some dinosaurs had feathers

Answer: Quantifier: Some
Subject term: Dinosaurs
Copula: Implicit
Predicate term: Things with feathers

(1) Some politicians are not members of tennis clubs.

(2) All dogs go to heaven.

(3) Some birds do not fly.

(4) No elephants are pocket-sized.

(5) All applicants must submit to a background check.

(6) All handguns are weapons.

Part B For each of the following sentences identify the quantifier, the subject term, the predicate term, and the copula. Some of these are like the example "Thirty percent of Canadians speak French" where the copula is implicit and the predicate needs to be transformed into a noun phrase.

(1) No dog has been to Mars.

(2) All human beings are mortal.

(3) Some spears are six feet long.

(4) All squids are cephalopods.

(5) No fish can sing.

(6) Some songs are sad.

3.2 Quantity, Quality, Distribution, and Venn Diagrams

The quantifier used in a statement is said to give the **quantity** of the statement. Statements with the quantifier "All" or "No" are said to be "**universal**" and those with the quantifier "some" are said to be "**particular**."

Here "some" will just mean "at least one." So, "some people in the room are standing" will be true even if there is only one person standing. Also, because "some" means "at least one," it is compatible with "all" statements. If I say "some people in the room are standing" it might actually be that *all* people in the room are standing, because if all people are standing, then at least one person is standing. This can sound a little weird, because in ordinary circumstances, you wouldn't bother to point out that something applies to some members of a class when, in fact, it applies to all of them. It sounds odd to say "*some* dogs are mammals," when in fact they *all* are. Nevertheless, when "some" means "at least one" it is perfectly true that some dogs are mammals.

In addition to talking about the quantity of statements, we will talk about their **quality**. The quality of a statement refers to whether the statement includes or excludes members of the subject class from the predicate class. Statements that include the words "no" or "not" are **negative**, and statements with the quantifier "all" or particular statements with the copula "are" are **affirmative**. Combining quantity and quality gives us four basic types of quantified categorical statements, which we call the **statement moods** or just "moods." The four moods are labeled with the letters A, E, I, and O. Statements that are universal and affirmative are

Mood	Form	Example
A	All S are P	All dogs are mammals.
E	No S are P	No dogs are reptiles.
I	Some S are P	Some birds can fly.
O	Some S are not P	Some birds cannot fly.

Table 3.2: The four moods of a categorical statement

MOOD-A STATEMENTS. Statements that are universal and negative are MOOD-E STATEMENTS. Particular and affirmative statements are MOOD-I STATEMENTS, and particular and negative statements are MOOD-O STATEMENTS. (See Table 3.2.)

Aristotle didn't actually use those letters to name the kinds of categorical propositions. His later followers writing in Latin came up with the idea. You can remember the labels because the "A" and the "I" were in the Latin word "**a**ff**i**rmo," ("I affirm") and the "E" and the "O" were in the Latin word "n**e**g**o**" ("I deny").

The **distribution** of a categorical statement refers to how the statement describes its subject and predicate class. A term in a sentence is said to be distributed if a claim is being made about the whole class. In the sentence "All dogs are mammals," the subject class, dogs, is distributed, because the quantifier "All" refers to the subject. The sentence is asserting that every dog out there is a mammal. On the other hand, the predicate class, mammals, is not distributed, because the sentence isn't making a claim about all the mammals. We can infer that at least some of them are dogs, but we can't infer that all of them are dogs. So in mood-A statements, only the subject is distributed.

On the other hand, in an I sentence like "Some birds can fly" the subject is not distributed. The quantifier "some" refers to the subject, and indicates that we are not saying something about all of that subject. We also aren't saying anything about all flying things, either. So in mood-I statements, neither subject nor predicate is distributed.

Even though the quantifier always refers to the subject, the predicate class can be distributed as well. This happens when the statement is negative. The sentence "No dogs are reptiles" is making a claim about all dogs: they are all not reptiles. It is also making a claim about all reptiles: they are all not dogs. So mood-E statements distribute both subject and predicate. Finally, negative particular statements (mood-O) have only the predicate class distributed. The statement "some birds cannot fly" does not say anything about all birds. It does, however say something about all flying things: the class of all flying things excludes some birds.

The quantity, quality, and distribution of the four forms of a categorical statement are given in Table 3.3. The general rule to remember here is that universal statements distribute the subject, and negative statements distribute the predicate.

In 1880 English logician John Venn published two essays on the use of diagrams with circles to represent categorical propositions (Venn 1880a, 1880b). Venn noted that the best use of such diagrams so far had come from the brilliant Swiss mathematician Leonhard Euler, but they still had many problems, which Venn felt could be solved by bringing in some ideas about logic from his fellow English logician George Boole. Although Venn only claimed to be building on the long logical tradition he traced, since his time these kinds of circle diagrams have been known as VENN DIAGRAMS.

In this section we are going to learn to use Venn diagrams to represent our four basic types of categorical statement. Later in this chapter, we will find them useful in evaluating arguments. Let us start with a statement in mood A: "All S are P." We are going to use one circle to represent S and another to represent P. There are a couple of different ways we could draw the circles if we wanted to represent "All S are P." One option would be to draw the circle for S entirely inside the circle for P, as in Figure 3.2

It is clear from Figure 3.2 that all S are in fact P. And outside of college logic classes, you may have seen people use a diagram like this to represent a situation where one group is a subclass of another. You may have even seen people call concentric circles like this a Venn diagram. But Venn did not think we should put one circle entirely inside the other if we just want to represent "All S is P." Technically speaking Figure 3.2 shows Euler circles.

Mood	Form	Quantity	Quality	Terms Distributed
A	All S are P	Universal	Affirmative	S
E	No S are P	Universal	Negative	S and P
I	Some S are P	Particular	Affirmative	None
O	Some S are not P	Particular	Negative	P

Table 3.3: Quantity, quality, and distribution.

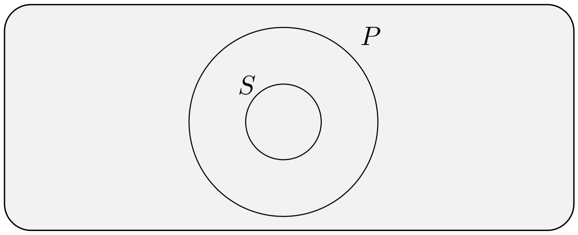


Figure 3.2: Euler Circles

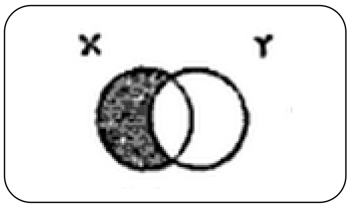
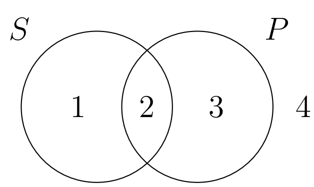


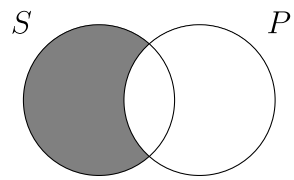
Figure 3.3: Venn's original diagram for an mood-A statement (Venn 1880a).

Venn pointed out that the circles in Figure 3.2 don't just say that "All S are P." They also says that "All P are S" is false. But we don't necessarily know that if we have only asserted "All S are P." The statement "All S are P" leaves it open whether the S circle should be smaller than or the same size as the P circle.

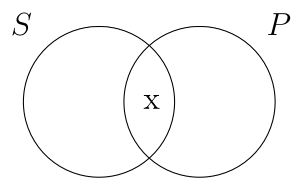
Venn suggested that to represent just the content of a single proposition, we should always begin by drawing partially overlapping circles. This means that we always have spaces available to represent the four possible ways the terms can combine:



Area 1 represents things that are S but not P; area 2, things that are S and P; area 3, things that are just P; and area 4 represents things that are neither S nor P. We can then mark up these areas to indicate whether something is there or could be there. We shade a region of the diagram to represent the claim that nothing can exist in that region. For instance, if we say "All S are P," we are asserting that nothing can exist that is in the S circle unless it is also in the P circle. So we shade out the part of the S circle that doesn't overlap with P.



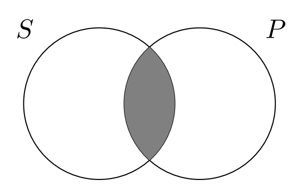
If we want to say that something does exist in a region, we put an "x" in it. This is the diagram for "Some S are P":



If a region of a Venn diagram is blank, if it is neither shaded nor has an x in it, it could go either way. Maybe such things exist, maybe they do not.

The Venn diagrams for all four basic forms of categorical statements are in Figure 3.4. Notice that when we draw diagrams for the two universal forms, A and E, we do not draw any x's. For these forms we are only ruling out possibilities, not asserting that things actually exist. This is part of what Venn learned from Boole, and we will see its importance in Section 3.4.

Finally, notice that so far, we have only been talking about categorical statements involving the variables S and P. Sometimes, though, we will want to represent statements in regular English. To do this, we will include a dictionary saying what the variables S and P represent in this case. For instance, this is the diagram for "No dogs are reptiles."



S: Dogs
P: Reptiles

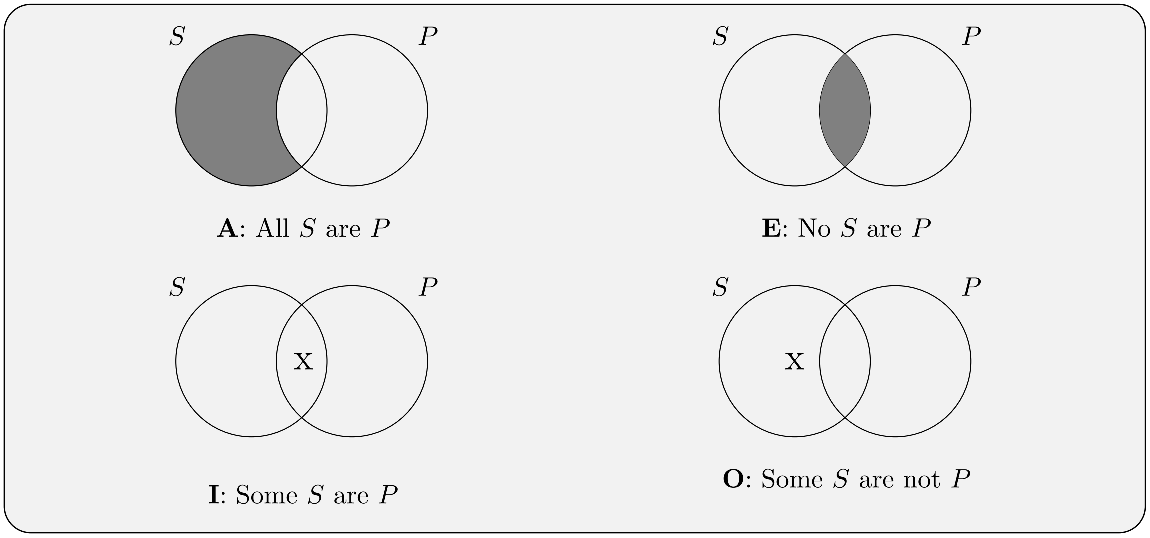


Figure 3.4: Venn Diagrams for the Four Basic Forms of a Categorical Statement

Practice Exercises

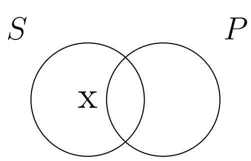
Part A Identify each of the following sentences as A, E, I, or O; state its quantity and quality; and state which terms are distributed. Then draw the Venn Diagram for each.

Example: Some dinosaurs are not herbivores

Answer: Form: O

Quantity: particular

Quality: negative



S: Dinosaurs

P: Herbivores

(1) All gerbils are rodents.

(2) Some planets do not have life.

(3) Some manatees are not rappers.

(4) All rooms have televisions.

(5) All stores are closed.

(6) Some dancers are graceful.

(7) No extraterrestrials are in Cleveland.

(8) Some crates are empty.

(9) No customers are mistaken.

(10) All cats love catnip.

Part B Identify each of the following sentences as A, E, I, or O; state its quantity

(3) Some employees are late.

(4) All forgeries are discovered eventually.

(5) Some shirts are purple.

(6) Some societies are matriarchal.

(7) No sunflowers are blue.

(8) Some appetizers are filling.

(9) Some jokes are funny.

(10) Some arguments are invalid.

Part C Transform the following sentences by switching their quantity, but not their quality.

> **Example**: Some dogs have fleas.
> **Answer**: All dogs have fleas.

(1) Some trees are not evergreen.

(2) All smurfs are blue.

(3) Some swords are sharp.

(4) Some sweaters are not soft.

(5) All snails are invertebrates.

Part D Transform the following sentences by switching their quantity, but not their quality.

(1) Some puffins are not large.

(2) Some Smurfs are female.

(3) All guitars are stringed instruments.

(4) No lobsters are extraterrestrial.

(5) Some metals are alloys

Part E Transform the following sentences by switching their quality, but not their quantity.

> **Example**: Some elephants are in zoos.
> **Answer**: Some elephants are not in zoos.

(1) Some lobsters are white.

(2) Some responsibilities are onerous.

(3) No walls are bridges.

(4) Some riddles are not clever.

(5) All red things are colored.

Part F Transform the following sentences by switching their quality, but not their quantity.

(1) All drums are musical instruments.

(2) No grandsons are female.

(3) Some crimes are felonies.

(4) Some airplanes are not commercial.

(5) All scorpions are arachnids.

Part G Transform the following sentences by switching both their quality and quantity.

Example: No sharks are virtuous.
Answer: Some sharks are virtuous.

(1) No lobsters are vertebrates.

(2) Some colors are not pastel.

(3) All tents are temporary structures.

(4) No goats are bipeds.

(5) Some shirts are plaid.

Part H Transform the following sentences by switching both their quality and quantity.

(1) No shirts are pants.

(2) All ducks are birds.

(3) Some possibilities are not likely events.

(4) Some raincoats are blue.

(5) Some days are holidays.

3.3 The Traditional Square of Opposition

The original investigation made by the Aristotelian philosophers made an assumption that logicians no longer make. To help you understand all sides of the issue, we will begin by looking at things in the traditional Aristotelian fashion, and then in the next section move on to the modern way of looking at things.

When Aristotle was first investigating these four kinds of categorical statements, he noticed they they conflicted with each other in different ways. If you are just thinking casually about it, you might say that “No S is P” is somehow “the opposite” of “All S is P.” But isn’t the real “opposite” of “All S is P” actually “Some S is not P”?

Aristotle, in his book *On Interpretation* (c. 350 BCE/1c. 350 BCE/1984a), notes that the real opposite of A is O, because one must always be true and the other false. If we know that “All dogs are mammals” is true, then we know “some dog is not a mammal” is false. On the other hand, if “All dogs are mammals” is false then “some dog is not a mammal” must be true. When two propositions must have opposite truth values they are called **contradictories**. Aristotle noted that A and O sentences are contradictory in this way. Forms E and I also form a contradictory pair. If “Some dogs are mammals” then “No dogs are mammals” is false, and if “Some dogs are mammals” is false, then “No dogs are mammals” is true.

Mood-A and mood-E statements are opposed to each other in a different way. Aristotle claimed that they can’t both be true, but could both be false. Take the statements “All dogs are strays” and “No dogs are strays.” We know that they are both false, because some dogs are strays and others aren’t. However, it is also clear that they could not both be true. When a pair of statements cannot both be true, but might both be false, the Aristotelian tradition says they are **contraries**.

These distinctions, plus a few other comments from Aristotle, were developed by his later followers into an idea that came to be known as the **square of opposition** . The square of opposition is simply the diagram you see in Figure 3.5. It is a way of representing the four basic propositions and the ways they relate to one another. As we said before, this way of picturing the proposition turned out to make a problematic assumption. To emphasize that this is no longer the way logicians view things, we will call this diagram the traditional square of opposition.

The traditional square of opposition begins by picturing a square with A, E, I, and O at the four corners. The lines between the corners then represent the ways

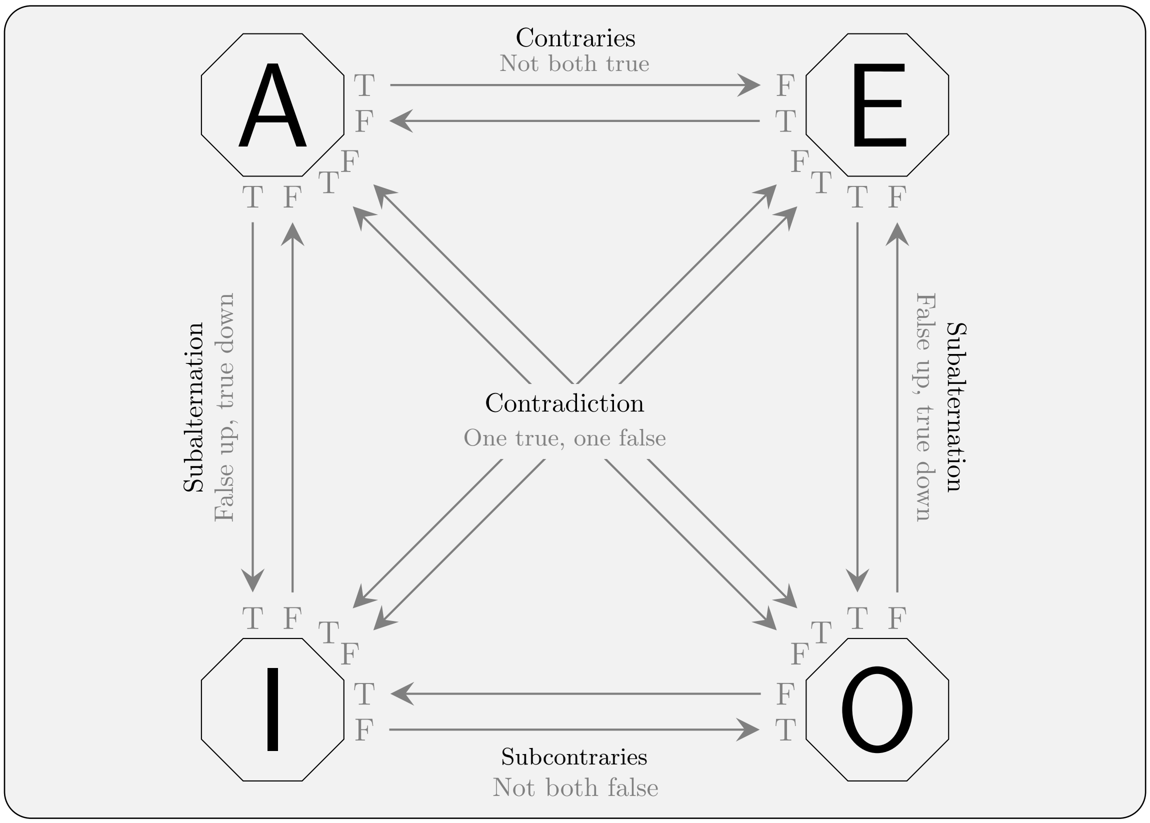


Figure 3.5: The Traditional square of opposition

that the kinds of propositions can be opposed to each other. The diagonal lines between A and O and between E and I represent contradiction. These are pairs of propositions where one has to be true and the other false. The line across the top represents contraries. These are propositions that Aristotle thought could not both be true, although they might both be false.

In Figure 3.5, we have actually drawn each relationship as a pair of lines, representing the kinds of inferences you can make in that relationship. Contraries cannot both be true. So we know that if one is true, the other must be false. This is represented by the two lines going from a T to an F. Notice that there aren't any lines here that point from an F to something else. This is because you can't infer anything about contrary statements if you just know that one is false. For the contradictory statements, on the other hand, we have drawn double-headed arrows. This is because we know both that the truth of one statement implies that the other is false and that the falsity of one statement implies the truth of the other.

Contraries and contradictories just give us the diagonal lines and the top line of the square. There are still three other sides to investigate. Form I and form O are called **subcontraries**. In the traditional square of opposition, their situation is reversed from that of A and E. Statements of forms A and E cannot both be true, but they can both be false. Statements of forms I and O cannot both be false, but they can both be true. Consider the sentences "Some people in the classroom are paying attention" and "Some people in the classroom are not paying attention." It is possible for them both to be true. Some people are paying attention and some aren't. But the two sentences couldn't both be false. That would mean that everyone in the room was neither paying attention nor not paying attention. But they have to be doing one or the other!

This means that there are two inferences we can make about subcontraries. We know that if I is false, O must be true, and vice versa. This is represented in Figure 3.5 by arrows going from Fs on one side to Ts on the other. This is reversed from the way things were on the top of the square with the contraries. Notice that this time there are no arrows going away from a T. This is because we can't infer anything about subcontraries if all we know is that one is true

The trickiest relationship is the one between universal statements and their corresponding particulars. We call this **subalternation**. Both of the statements in these pairs could be true, or they could both be false. However, in the traditional square of opposition, if the universal statement is true, its corresponding particular statement must also be true. For instance, "All dogs are mammals" implies that some dogs are mammals. Also, if the particular statement is false, then the

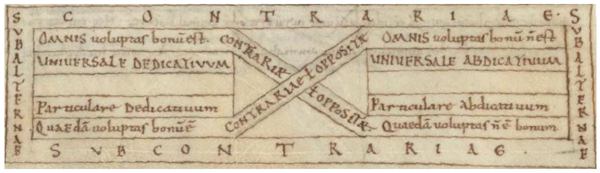


Figure 3.6: One of the earliest surviving versions of the square of opposition, from a 9th century manuscript of a commentary by Apuleius of Madaura on Aristotle's *On Interpretation*. Digital image from `www.logicmuseum.com`, curated by Edward Buckner.

universal statement must also be false. Consider the statement "Some dinosaurs had feathers." If that statement is false, if no dinosaurs had feathers, then "All dinosaurs have feathers" must also be false. Something like this seems to be true on the negative side of the diagram as well. If "No dinosaurs have feathers" is true, then you would think that "some dinosaurs do not have feathers" is true. Similarly, if "some dinosaurs do not have feathers" is false, then "No dinosaurs have feathers" cannot be true either.

In our diagram for the traditional square of opposition, we represent subalternation by a downward arrow for truth and an upward arrow for falsity. We can infer something here if we know the top is true, or if we know the bottom is false. In other situations, there is nothing we can infer.

Note, by the way, that the language of subalternation works a little differently than the other relationships. With contradiction, we say that each sentence is the "contradictory" of the other. The relationship is symmetrical. With subalternation, we say that the particular sentence is the "subaltern" of the universal one, but not the other way around.

People started using diagrams like this as early as the second century CE to explain Aristotle's ideas in *On Interpretation* (See Parsons 1997). Figure 3.6 shows one of the earliest surviving versions of the square of opposition, from a 9th century manuscript of a commentary on Aristotle attributed to the Roman writer Apuleius of Madaura. Although this particular manuscript dates from the 9th century, the commentary itself was written in the 2nd century, and copied by hand many times over before this one was made. Figure 3.7 shows a later illustration of the square, from a 16th century book by the Scottish philosopher and logician Johannes de Magistris.

We can use the traditional square of opposition to evaluate arguments written in canonical form. It will help us here to introduce the phrase "It is false that" to

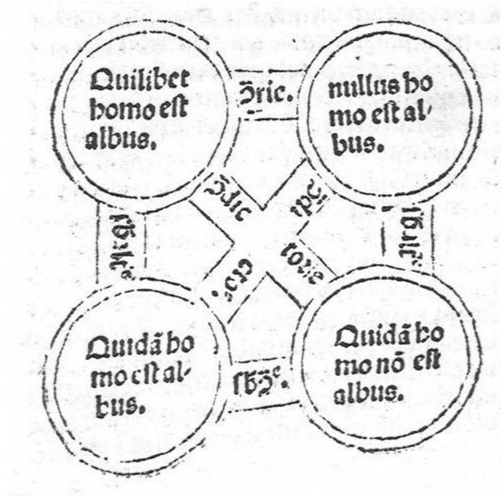


Figure 3.7: A 16th century illustration of the square of opposition from *Summulae Logicales* by Johannes de Magistris, digital image by Peter Damian and uploaded to Wikimedia Commons: tinyurl.com/kmzmvzn. Public Domain-U.S.

some of our statements, so that we can make inferences from the truth of one proposition to the falsity of another. This, for instance, is a valid argument, because A and O statements are contradictories:

P_1: All humans are mortal.

C: It is false that some humans are not mortal.

The argument above is an immediate inference because it only has one premise. In fact, the premise and the conclusion are logically equivalent. Two statements are **logically equivalent** if they always have the same truth value. When one is true the other one is and when one is false the other one is. This will not be the case for all immediate inferences based on the square of opposition, however. For example, below is a valid argument based on the subaltern relationship, but the premise and the conclusion are not logically equivalent.

P_1: It is false that some humans are dinosaurs.

C: It is false that all humans are dinosaurs.

Practice Exercises

Part A For each pair of sentences say whether they are contradictories, contraries, subcontraries, or one is the subaltern of the other.

Example: Some peppers are spicy.
No peppers are spicy.

Answer: Contradictory

(1) No quotations are spurious.
Some quotations are not spurious.

(2) Some children are not picky eaters.
All children are picky eaters.

(3) Some joys are not fleeting.
Some joys are fleeting.

(4) All fires are hot.
Some fires are not hot.

(5) Some diseases are not fatal.
No diseases are fatal.

(6) Some planets are not habitable.
Some planets are habitable.

(7) Some toys are plastic.
No toys are plastic.

(8) No transfats are healthy.
All transfats are healthy.

(9) No superheroes are invincible.
Some superheroes are invincible.

(10) Some villains are deplorable.
Some villains are not deplorable.

Part B For each pair of sentences say whether they are contradictories, contraries, subcontraries, or one is the subaltern of the other.

(1) No pants are headgear.
All pants are headgear.

(2) Some dietitians are not qualified.

All dietitians are qualified.

(3) Some monkeys are curious.
No monkeys are curious.

(4) All dolphins are intelligent.
Some dolphins are intelligent.

(5) No manuscripts are accepted.
All manuscripts are accepted.

(6) Some hijinks are wacky.
No hijinks are wacky.

(7) All clowns are terrifying.
No clowns are terrifying.

(8) No cupcakes are nutritious.
Some cupcakes are not nutritious.

(9) "Some kinds of love are mistaken for vision." –Lou Reed
All kinds of love are mistaken for vision.

(10) All sharks are cartilaginous.
No sharks are cartilaginous.

Part C For each sentence write its contradictory, contrary, subcontrary, or the corresponding sentence in subalternation as directed.

Example: Write the subcontrary of "Some jellyfish sting."
Answer: Some jellyfish do not sting.

(1) Write the contrary of "No hashtags are symbols."

(2) Write the contradictory of "All elephants are social."

(3) Write the subcontrary of "Some children are well behaved."

(4) Write the contradictory of "All eggplants are purple."

(5) Write the sentence that "Some guitars are electric" is a subaltern of.

(6) Write the contradictory of "Some arches are not crumbling."

(7) Write the contrary of "No resolutions are unsatisfying."

(8) Write the contradictory of "All flags are flying."

(9) Write the subaltern of "No pains are chronic."

(10) Write the contradictory of "No puffins are mammals.'

Part D For each sentence write its contradictory, contrary, subcontrary, or the corresponding sentence in subalternation as directed.

(1) Write the subaltern of "No libraries are unfunded."
(2) Write the contrary of "All hooks are sharp."
(3) Write the contradictory of "Some tankers are not seaworthy."
(4) Write the sentence that "Some positions are not tenable" is the subaltern of.
(5) Write the contradictory of "Some haircuts are unfortunate."
(6) Write the contradictory of "No violins are worthless."
(7) Write the subcontrary of "Some missiles are not nuclear."
(8) Write the contrary of "All animals are lifeforms."
(9) Write the contradictory of "All animals are lifeforms."
(10) Write the subaltern of "All animals are lifeforms."

Part E Given a sentence and its truth value, evaluate the truth of a second sentence, according to the traditional square of opposition. If the truth value cannot be determined, just write "undetermined."

Example:If "Some S are P" is true, what is the truth value of "Some S are not P"?
Answer: Undetermined

(1) If "Some S are not P" is true, what is the truth value of "All S are P"?
(2) If "Some S are not P" is false, what is the truth value of "Some S are P"?
(3) If "All S are P" is true, what is the truth value of "No S are P"?
(4) If "Some S are not P" is false, what is the truth value of "No S are P"?
(5) If "No S are P" is true, what is the truth value of "Some S are not P"?
(6) If "Some S are not P" is true, what is the truth value of "All S are P"?
(7) If "Some S are P" is true, what is the truth value of "All S are P"?
(8) If "All S are P" is false, what is the truth value of "Some S are P"?
(9) If "No S are P" is false, what is the truth value of "All S are P"?
(10) If "No S are P" is true, what is the truth value of "Some S are P"?

Part F Given a sentence and its truth value, evaluate the truth of a second sentence, according to the traditional square of opposition. If the truth value cannot be determined, just write "undetermined."

(1) If "Some S are not P" is true, what is the truth value of "All S are P"?

(2) If "Some S are not P" is false, what is the truth value of "Some S are P"?

(3) If "All S are P" is true, what is the truth value of "No S are P"?

(4) If "Some S are not P" is false, what is the truth value of "No S are P"?

(5) If "No S are P" is true, what is the truth value of "Some S are not P"?

(6) If "Some S are not P" is true, what is the truth value of "All S are P"?

(7) If "Some S are P" is true, what is the truth value of "All S are P"?

(8) If "All S are P" is false, what is the truth value of "Some S are P"?

(9) If "No S are P" is false, what is the truth value of "All S are P"?

(10) If "No S are P" is true, what is the truth value of "Some S are P"?

Part G Evaluate the following arguments using the traditional square of opposition. If the argument is valid, say which relationship in the square of opposition makes it valid.

Example:No S are P.Therefore, some S are not P.
Answer: Valid, because the conclusion is the subaltern of the premise.

(1) No S are P. Therefore, it is false that some S are P.

(2) It is false that no S are P. Therefore, it is false that all S are P.

(3) All S are P. Therefore, it is false that no S are P.

(4) It is false that no S are P. Therefore, it is false that some S are not P.

(5) It is false that all S are P. Therefore, some S are not P.

(6) It is false that no S are P. Therefore, some S are P.

(7) It is false that all S are P. Therefore, some S are P.

(8) Some S are P. Therefore, all S are P.

(9) It is false that some S are P. Therefore, some S are P.

(10) It is false that some S are not P. Therefore, it is false that no S are P.

Part H Evaluate the following arguments using the traditional square of opposition. If the argument is valid, say which relationship in the square of opposition makes it valid.

(1) Some S are not P. Therefore, it is false that some S are P

(2) It is false that some S are not P. Therefore, some S are P

(3) It is false that all S are P. Therefore, some S are not P.

(4) It is false that all S are P. Therefore, no S are P.

(5) Some S are P. Therefore, it is false that no S are P.

(6) Some S are P. Therefore, it is false that some S are not P

(7) No S are P. Therefore, it is false that all S are P.

(8) It is false that no S are P. Therefore, all S are P.

(9) It is false that all S are P. Therefore, it is false that some S are P.

(10) All S are P. Therefore, some S are P.

3.4 Existential Import and the Modern Square of Opposition

The traditional square of opposition seems straightforward and fairly clever. Aristotle made an interesting distinction between contraries and contradictories, and subsequent logicians developed it into a nifty little diagram. So why did we have to keep saying things like "Aristotle thought" and "according to the traditional square of opposition." What is wrong here?

The traditional square of opposition goes awry because it makes assumptions about the existence of the things being talked about. Remember that when we drew the Venn diagram for "All S are P," we shaded out the area of S that did not overlap with P to show that nothing could exist there. We pointed out, though, that we did not put a little x in the intersection between S and P. Statements of the form A ruled out the existence of one kind of thing, but they did not assert the existence of another. The A proposition, "All dogs are mammals," denies the existence of any dog that is not a mammal, but it does not assert the existence of some dog that is a mammal. But why not? Dogs obviously do exist.

The problem comes when you start to consider categorical statements about things that don't exist, for instance "All unicorns have one horn." This seems like a true statement, but unicorns don't exist. Perhaps what we mean by "All unicorns have one horn" is that *if* a unicorn existed, *then* is would have one horn. But if we interpret the statement about unicorns that way, shouldn't we also interpret the

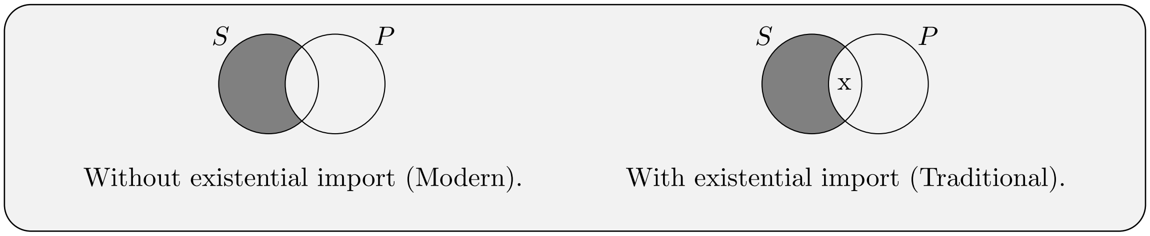


Figure 3.8: Interpretations of A: “All S are P.”

statement about dogs that way? Really all we mean when we say “All dogs are mammals” is that if there were dogs, then they would be mammals. It takes an extra assertion to point out that dogs do, in fact, exist.

The issue we are discussing here is called existential import. A sentence is said to have **existential import** if it asserts the existence of the things it is talking about. Figure 3.8 shows the two ways you could draw Venn diagrams for an A statement, with the x, as in the traditional interpretation, and without, as in our interpretation. If you interpret A statements in the traditional way, they are always false when you are talking about things that don’t exist. So, “All unicorns have one horn” is false in the traditional interpretation. On the other hand, in the modern interpretation all statements about things that don’t exist are true. “All unicorns have one horn” simply asserts that there are no multi-horned unicorns, and this is true because there are no unicorns at all. We call this **vacuous truth**. Something is vacuously true if it is true simply because it is about things that don’t exist. Note that *all* statements about nonexistent things become vacuously true if you assume they have no existential import, even a statement like “All unicorns have more than one horn.” A statement like this simply rules out the existence of unicorns with one horn or fewer, and these don’t exist because unicorns don’t exist. This is a complicated issue that will come up again starting in Chapter 5 when we consider conditional statements. For now just assume that this makes sense because you can make up any stories you want about unicorns.

Any statement can be read with or without existential import, even the particular ones. Consider the statements “Some unicorns are rainbow colored” and “Some unicorns are not rainbow colored.” You can argue that both of these statements are true, in the sense that if unicorns existed, they could come in many colors. If you say these statements are true, however, you are assuming that particular statements do not have existential import. As Terence Parsons (1997) points out, you can change the wording of particular categorical statements in English to make them seem like they do or do not have existential import. “Some unicorns are not rainbow colored" might have existential import, but "not every unicorn is rainbow colored" doesn't seem to.

So what does this have to do with the square of opposition? A lot of the claims made in the traditional square of opposition depend on assumptions about which statements have existential import. For instance, Aristotle's claim that contrary statements cannot both be true requires that A statements have existential import. Think about the sentences "All dragons breathe fire" and "no dragons breathe fire." If the first sentence has no existential import, then both sentences could actually be true. They are both ruling out the existence of certain kinds of dragons and are correct because no dragons exist.

In fact, the entire traditional square of opposition falls apart if you assume that all four forms of a categorical statement have existential import. Parsons (1997) shows how we can derive a contradiction in this situation. Consider the I statement "Some dragons breathe fire." If you interpret it as having existential import, it is false, because dragons don't exist. But then its contradictory statement, the E statement "No dragons breathe fire" must be true. And if that statement is true, and has existential import, then its subaltern, "Some dragon does not breathe fire" is true. But if it has existential import, it can't be true, because dragons don't exist. In logic, the worst thing you can ever do is contradict yourself, but that is what we have just done. So we have to change the traditional square of opposition.

The way some textbooks talk about the problem, you'd think that for two thousand years logicians were simply ignorant about the problem of existential import and thus woefully confused about the square of opposition, until finally George Boole wrote *The Laws of Thought* (1854) and found the one true solution to the problem. In fact, there was an extensive discussion of existential import from the 12th to the 16th centuries, mostly under the heading of the "supposition" of a term. Very roughly, we can say that the supposition of a term is the way it refers to objects, or what we now call the "denotation" of the term (Read 2002). So in "All people are mortal" the supposition of the subject term is all of the people out there in the world. Or, as the medievals sometimes put it, the subject term "supposits" all the people in the world.

At least some medieval thinkers had a theory of supposition that made the traditional square of opposition work. Terrance Parsons (1997, 2008) has argued for the importance of one solution, found most clearly in the writings of William of Ockham. Under this theory, affirmative forms A and I had existential import, but the negative forms E and O did not. We would say that a statement has existential import if it would be false whenever the subject or predicate terms refer to things that don't exist. To put the matter more precisely, we would say that the statement would be false whenever the subject or predicate terms "fail to refer." Linguistic philosophers these days prefer say that a term "fails to refer" rather than saying that it "refers to something that doesn't exist," because referring to things that don't exist seems impossible.

In any case, Ockham describes the supposition of affirmative propositions the same way we would describe the reference of terms in those propositions. Again, if the proposition supposes the existence of something in the world, the medievals would say it "supposits." Ockham says "In affirmative propositions a term is always asserted to supposit for something. Thus, if it supposits for nothing the proposition is false" (1343/1974). On the other hand, failure to refer or to supposit actually supports the truth of negative propositions: "in negative propositions the assertion is either that the term does not supposit for something or that it supposits for something of which the predicate is truly denied. Thus a negative proposition has two causes of truth" (1343/1974).

So, for Ockham, affirmative statements about nonexistent objects are false. "All unicorns have one horn" and "Some unicorns are rainbow colored" are false, because there are no unicorns. Negative statements, on the other hand, are vacuously true. "No unicorns are rainbow colored" and "No unicorns have one horn" are both true. There are no rainbow colored unicorns out there, and no one horned unicorns out there, because there are no unicorns out there. The O statement "Some unicorns are not rainbow colored" is also vacuously true. This might be harder to see, but it helps to think of the statement as saying "It is not the case that every unicorn is rainbow colored."

This way of thinking about existential import leaves the traditional square of opposition intact, even in cases where you are referring to nonexistent objects. Contraries still cannot both be true when you are talking about nonexistent objects, because the A proposition will be false, and the E vacuously true. "All dragons breathe fire" is false, because dragons don't exist, and "No dragons breathe fire" is vacuously true for the same reason. Similarly, subcontraries cannot both be false when talking about dragons and whatnot, because the I will always be false and the O will always be true. You can go through the rest of the relationships and show that similar arguments hold.

Boole proposed a different solution, which is now taken as the standard way to do things. Instead of looking at the division between positive and negative statements, Boole looked at the division between singular and universal propositions. The universal statements A and E do not have existential import, but the particular statements I and O do have existential import. Thus all particular statements about nonexistent things are false and all universal statements about nonexistent things are vacuously true.

John Venn was building on the work of George Boole. His diagrams avoided the problems that Euler had by using a Boolean interpretation of mood-A statements, where they really just assert that something is impossible. In fact, the whole system of Venn diagrams embodies Boole's assumptions about existential import, as you can see in Figure 3.4. The particular forms I and O have you draw an x, indicating that something exists. The other two forms just have us shade in regions to indicate that certain combinations of subject and predicate are impossible. Thus A and E statements like "All dragons breathe fire" or "No dragons are friendly" can be true, even though no dragons exist.

Venn diagrams don't even have the capacity to represent Ockham's understanding of existential import. We can represent A statements as having existential import by adding an x, as we did on the right hand side of Figure 3.8. However, we have no way to represent the O form without existential import. We have to draw the x, indicating existence. We don't have a way of representing O form statements about nonexistent objects as vacuously true.

The Boolean solution to the the question of existential import leaves us with a greatly restricted form of the square of opposition. Contrary statements are both vacuously true when you refer to nonexistent objects, because neither have existential import. Subcontrary statements are both false when you refer to nonexistent objects, because they do have existential import. Finally, the subalterns of vacuously true statements are false, while on the traditional square of opposition they had to be true. The only thing remaining from the traditional square of opposition is the relationship of contradiction, as you can see in Figure 3.9.

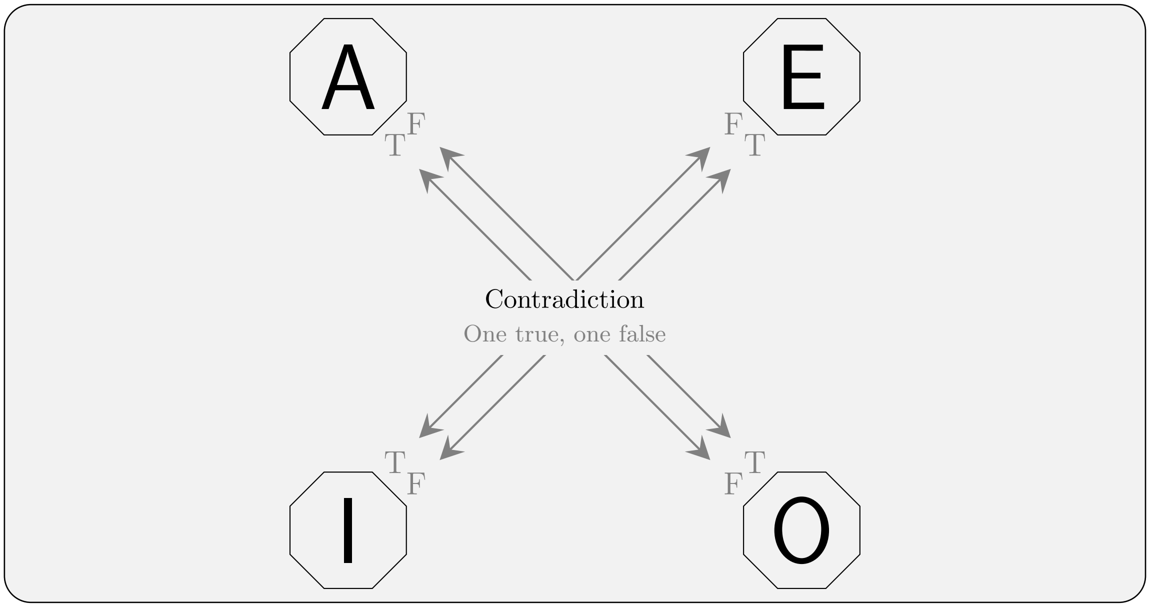


Figure 3.9: The Modern square of opposition

Practice Exercises

Part A Evaluate each of the following arguments twice. First, evaluate it using the traditional square of opposition. If the argument is valid, state which relationship makes it valid (contradictories, contraries, etc.) Second, evaluate the argument using the modern square of opposition.

(1) It is false that some S are P. Therefore, it is false that all S are P.

(2) All S are P. Therefore, some S are not P.

(3) It is false that some S are P. Therefore no S are P.

(4) No S are P. Therefore, it is false that all S are P.

(5) Some S are not P. Therefore, it is false that all S are P.

(6) It is false that some S are not P. Therefore, it is false that no S are P.

(7) It is false that some S are P. Therefore, it is false that no S are P.

(8) It is false that some S are P. Therefore, some S are not P.

(9) Some S are P. Therefore, it is false that some S are not P

(10) Some S are P. Therefore all S are P.

Part B See the instructions for Part A.

(1) Some S are P. Therefore, it is false that no S are P.

(2) It is false that some S are not P. Therefore, some S are P

(3) Some S are not P. Therefore, it is false that some S are P

(4) Some S are not P. Therefore, it is false that all S are P

(5) All S are P. Therefore, some S are P

(6) Some S are not P. Therefore, no S are P

(7) Some S are P. Therefore, it is false that no S are P

(8) It is false that all S are P. Therefore, it is false that no S are P

(9) No S are P. Therefore some S are not P

(10) It is false that some S are P. Therefore, no S are P.

3.5 Transforming English into Logically Structured English

Because the four basic forms are stated using variables, they have a great deal of generality. We can expand on that generality by showing how many different kinds of English sentences can be transformed into sentences in our four basic forms. We already touched on this a little in section 3.1, when we look at sentences like "Thirty percent of Canadians speak French." There we saw that the predicate was not explicitly a class. We needed to change "speak French" to "people who speak French." In this section, we are going to expand on that to show how ordinary English sentences can be transformed into something we will call "logically structured English." LOGICALLY STRUCTURED ENGLISH is English that has been put into a standardized form that allows us to see its logical structure more clearly and removes ambiguity. Doing this is a step towards the creation of formal languages, which we will start doing in Chapter 5.

Transforming English sentences into logically structured English is fundamentally a matter of understanding the meaning of the English sentence and then finding the logically structured English statements with the same or similar meaning. Sometimes this will require judgment calls. English, like any natural language, is fraught with ambiguity. One of our goals with logically structured English is to reduce the amount of ambiguity. Clarifying ambiguous sentences will always require making judgments that can be questioned. Things will only get harder when we start using full blown formal languages in Chapter 5, which are supposed to be completely free of ambiguity.

To transform a quantified categorical statement into logically structured English, we have to put all of its elements in a fixed order and be sure they are all of the right type. All statements must begin with the quantifies "All" or "Some" or the negated quantifier "No." Next comes the subject term, which must be a plural noun, a noun phrase, or a variable that stands for any plural noun or noun phrase. Then comes the copula "are" or the negated copula "are not." Last is the predicate term, which must also be a plural noun or noun phrase. We also specify that you can only say "are not" with the quantifier "some," that way the universal negative statement is always phrased "No S are P," not "All S are not P." Taken together, these criteria define the STANDARD FORM FOR A CATEGORICAL STATEMENT in logically structured English.

The subsections below identify different kinds of changes you might need to make to put a statement into logically structured English. Sometimes translating a sentence will require using multiple changes.

Nonstandard Verbs

In section 3.1 we saw that "Some Canadians speak French" has a verb phrase "speaks French" instead of a copula and a plural noun phrase. To transform these sentences into logically structured English, you need to add the copula and turn all the terms into plural nouns or plural noun phrases.

Below are some examples

English	Logically Structured English
No cats bark.	No cats are animals that bark.
All birds can fly.	All birds are animals that can fly.
Some thoughts should be left unsaid.	Some thoughts are things that should be left unsaid.

Adding a plural noun phrase means you have to come up with some category, like "people" or "animals." When in doubt, you can always use the most general category, "things."

Implicit Noun Phrases

Sometimes you just have an adjective for the predicate, and you need to turn it into a noun, as in the examples below.

English	Logically Structured English
Some roses are red.	Some roses are red flowers.
Football players are strong.	All football players are strong persons.
Some names are hurtful.	Some names are hurtful things.

Again, you will have to come up with a category for the predicate, and when it doubt, you can just use "things."

Unexpressed Quantifiers

Sometimes categorical generalizations come without an explicit quantifier, which you need to add.

English	Logically Structured English
Boots are footwear.	All boots are footwear.
Giraffes are tall.	All giraffes are tall things.
A dog is not a cat.	No dogs are cats.
A lion is a fierce creature.	All lions are fierce creatures.

Notice that in the second sentence we had to make two changes, adding both the words "All" and "things."

In the last two sentences, the indefinite article "a" is being used to create a kind of generic sentence. Not all sentences using the indefinite article work this way. If a story begins "A man is walking down the street," it is not talking about all men generically. It is introducing some specific man. For this kind of statement, see the subsection on singular propositions. You will have to use your good judgment and understanding of context to know how the indefinite article is being used.

Nonstandard Quantifiers

English has many alternate ways of saying "all" and "some." You need to change these when translating to logically structured English.

English	Logically Structured English
Every day is a blessing.	All days are blessings.
Whatever is a dog is not a cat.	No dogs are cats.
Not a single dog is a cat.	No dogs are cats.
There are Americans that are doctors.	Some Americans are doctors.
Someone in America is a doctor.	Some Americans are doctors.
At least a few Americans are doctors.	Some Americans are doctors.
Not everyone who is an adult is a logician.	Some adults are not logicians.
Most people with a PhD in psychology are female.	Some people with a PhD in psychology are female.
Among the things that Sylvia inherited was a large mirror	Some things that Sylvia inherited were large mirrors

Notice in the last case we are losing quite a bit of information when we transform the sentence into logically structured English. "Most" means more that fifty percent, while "some" could be any percentage less than a hundred. This is simply a price we have to pay in creating a standard logical form. As we will see when we move to constructing artificial languages in Chapter 5, no logical language has the expressive richness of a natural language.

Singular Propositions

Aristotle treated sentences about individual things, like specific people, differently than either general or particular categorical statements. A statement like "Socrates is mortal," for Aristotle, was neither A, E, I, nor O. We can expand the power of logically structured English by bringing these kind of singular propositions into our system of categorical propositions. Using phrases like "All things identical to. . . " we can turn singular terms into general ones.

English	Logically Structured English
Socrates is mortal.	All persons identical with Socrates are mortal.
The Empire State Building is tall.	All things identical to The Empire State Building are tall things.
Ludwig was not happy.	No persons identical with Ludwig are happy.
A man is walking down the street.	Some men are things that are walking down the street.

Adverbs and Pronouns

In English we use specific adverbs like "everywhere" and "always" to create quantified statements about place and time. We can transform these into logically structured English by talking about "all places" or "all times" and things like that. English also has specific pronouns for quantified statements about people or things, such as "everyone" or "nothing."

English	Logically Structured English
"Whenever you need me, I'll be there." – Michael Jackson	All times that you need me are times that I will be there.
"We are never, ever, ever getting back together." – Taylor Swift	No times are times when we will get back together.
"Whoever fights with monsters should be careful lest he thereby become a monster." –Friedrich Nietzsche	All persons who fight with monsters are persons who should be careful lest they become a monster.
"What does not destroy me, makes me stronger." –Friedrich Nietzsche	All things that do not destroy me are things that make me stronger.

Conditional Statements

A conditional is a statement of the form "If . . . then . . . ." They will become a big focus of our attention starting in Chapter 5 when we begin introducing modern formal languages. They are not given special treatment in the Aristotelian tradition, however. Instead, where we can, we just treat them as categorical generalizations:

English	Logically Structured English
If something is a cat, then it is a feline.	All cats are feline.
If something is a dog, then it's not a cat.	No dogs are cats.

Exclusive Propositions

Phrases like "only," "none but," or "none except" are used in English to create exclusive propositions. They are applied to the predicate term and exclude everything but the predicate term from the subject term.

English	Logically Structured English
Only people over 21 may drink.	All people who drink are over 21.
No one, except those with a ticket, may enter the theater.	All people who enter the theater have a ticket.
None but the strong survive.	All people who survive are strong people.

The important thing to see here is that words like "only" are actually modifying the predicate, and not the subject. So when you translate them into logically structured English, the order of the words often gets turned around. In "only people over 21 may drink" the predicate is actually "people over 21."

"The Only"

Sentences with "The only" are a little different than sentences that just have "only" in them. The sentence "Humans are the only animals that talk on cell phones" should be translated as "All animals who talk on cell phones are humans." In this sentence, "the only" introduces the subject, rather than the predicate.

English	Logically Structured English
Humans are the only animals who talk on cell phones.	All animals who talk on cell phones are human.

English	Logically Structured English
Shrews are the only venomous mammal in North America.	All venomous mammals in North America are shrews.

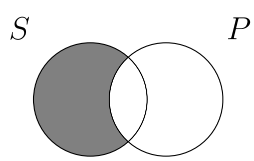
Practice Exercises

Part A Transform the following into logically structured English; identify it as A, E, I, or O; and provide the appropriate Venn diagram.

Example: If you can't stand the heat, get out of the kitchen

Answer: All people who cannot stand the heat are people who should get out of the kitchen.

Form: A



S: People who can't stand the heat

P: People who should get out of the kitchen

(1) Cats are predators.

(2) If something is worth doing, it is worth doing well.

(3) Some chimpanzees know sign language.

(4) All dogs are loyal.

(5) Monotremes are the only egg-laying mammals.

(6) Whenever a bell rings, an angel gets its wings.

(7) At least one person in this room is a liar.

(8) Only natural born citizens can be president of the United States.

(9) Gottlob Frege suffered from severe depression.

(10) "Anyone who ever had a heart, wouldn't turn around and break it." –Lou Reed.

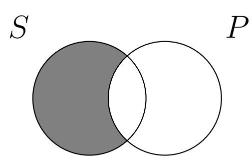
Part B Transform the following into logically structured English, and identify it as A, E, I, or O.

(1) If a muffin has frosting, then it is a cupcake.
(2) "Anyone who ever played a part, wouldn't turn around and hate it." –Lou Reed.
(3) Seahorses are the only fish species in which the male carries the eggs.
(4) Seahorses are animals that mate for life.
(5) Few dogs are fans of classical music.
(6) Ruth Barcan Marcus was a member of the Yale faculty.
(7) Only zombies are brain eaters.
(8) Some logicians are not mentally ill
(9) Some birds eat fish.

Part C Transform the following into logically structured English and identify it as A, E, I, or O. Some problems will require multiple transformations.

Example: Bertrand Russell was married four times.

Answer: All people who are identical to Bertrand Russell are people who were married four times.
Form: A



S: People who are identical to Bertrand Russell

P: People who were married four times

(1) Many logicians work in computer science.
(2) Ludwig Wittgenstein severed in the Austro-Hungarian Army in World War I.
(3) Humans can be found everywhere on Earth.
(4) Cats are crepuscular.
(5) Grover Cleveland was the only president to serve two nonconsecutive terms.

(6) Grover Cleveland was the only president named "Grover."

(7) The band's only singer also plays guitar.

(8) If a dog has a collar, it is someone's pet.

(9) Only the basketball players in the class were tall.

(10) If you study, then you will pass the test.

Part D Transform the following into logically structured English, and identify it as A, E, I, or O. Some problems will require multiple transformations.

(1) People have walked on the moon at least once.

(2) Basketball players are tall.

(3) Most senior citizens vote.

(4) If a bird is a crow, then it is very intelligent.

(5) Whoever ate the last cookie is in trouble.

(6) "Euclid alone has looked on Beauty bare." –Edna St. Vincent Millay.

(7) If something is a dog, then it is man's best friend.

(8) More than a few students will fail the test.

(9) Mercury is the only metal that is liquid at room temperature.

(10) Bertrand Russell was married four times.

3.6 Conversion, Obversion, and Contraposition

Now that we have shown the wide range of statements that can be represented in our four standard logical forms A, E, I, and O, it is time to begin constructing arguments with them. The arguments we are going to look at are sometimes called "immediate inferences" because they only have one premise. We are going to learn to identify some valid forms of these one-premise arguments by looking at ways you can transform a true sentence that maintain its truth value. For instance, "No dogs are reptiles" and "No reptiles are dogs" have the same truth value and basically mean the same thing. On the other hand if you change "All dogs are mammals" into "All mammals are dogs" you turn a true sentence into a false one. In this section we are going to look at three ways of transforming categorical statements—conversion, obversion, and contraposition—and use Venn diagrams to determine whether these transformations also lead to a change in truth value. From

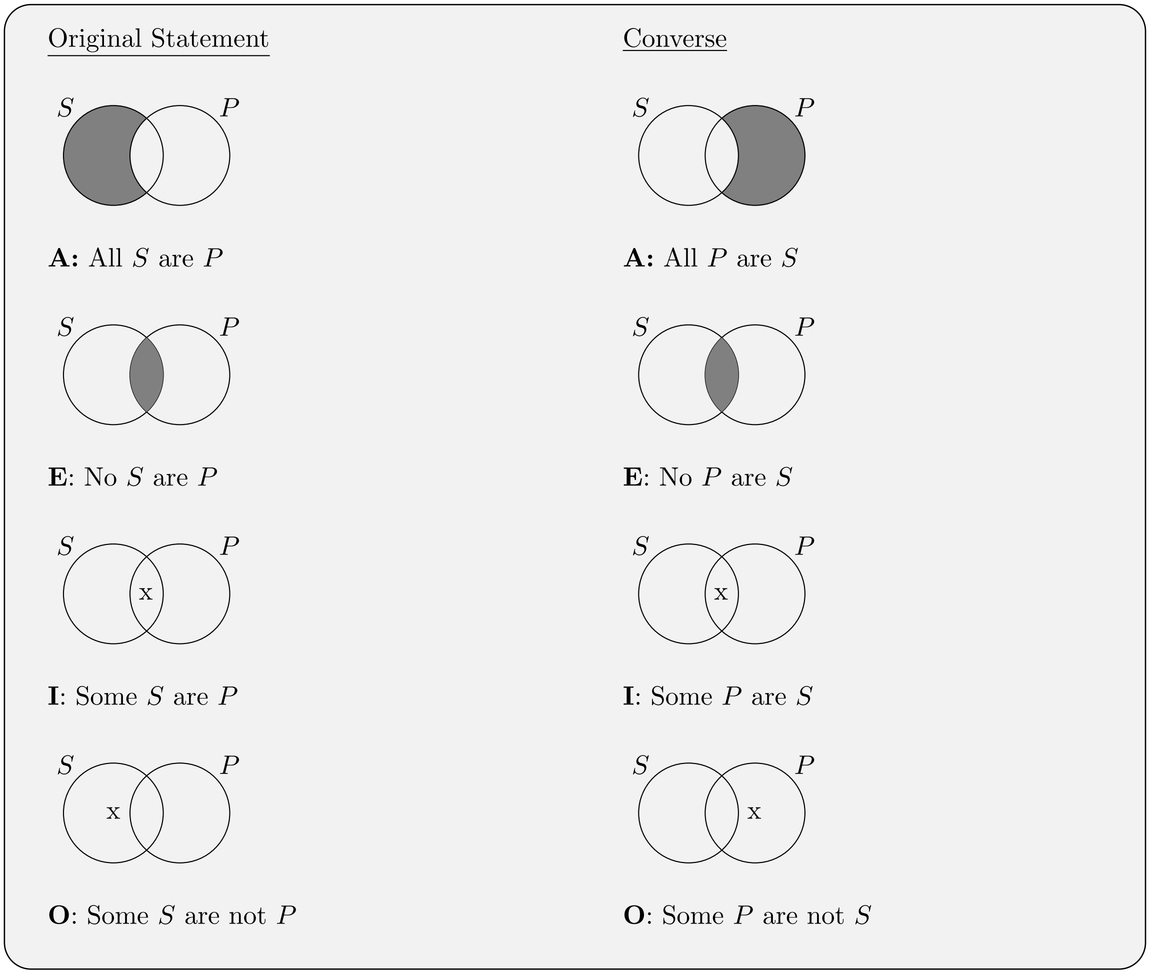


Figure 3.10: Conversions of the Four Basic Forms

there we can identify valid argument forms.

Conversion

The two examples in the last paragraph are examples of conversion. **conversion** is the process of transforming a categorical statement by switching the subject and the predicate. When you convert a statement, it keeps its form—an A statement remains an A statement, an E statement remains an E statement—however it might change its truth value. The Venn diagrams in Figure 3.10 illustrate this.

P. No S are P.	P. Some S are P.
C. No P are S.	C. Some P are S.

Figure 3.11: Valid Arguments by Conversion

As you can see, the Venn diagram for the converse of an E statement is exactly the same as the original E statement, and likewise for I statements. This means that the two statements are logically equivalent. Recall that two statements are logically equivalent if they always have the same truth value.. In this case, that means that if an E statement is true, then its converse is also true, and if an E statement is false, then its converse is also false. For instance, “No dogs are reptiles” is true, and so is “No reptiles are dogs.” On the other hand “No dogs are mammals” is false, and so is “No mammals are dogs.”

Likewise, if an I statement is true, its converse is true, and if an I statement is false, then its converse is false. “Some dogs are pets” is true, and so is “Some pets are dogs.” On the other hand “Some dogs can fly” is false and so is “Some flying things are dogs.”

The converses of A and O statements are not so illuminating. As you can see from the Venn diagrams, these statements are not identical to their converses. They also don’t contradict their converses. If we know that an A or O statement is true, we still don’t know anything about their converses. We say their truth value is undetermined.

Because E and I statements are logically equivalent to their converses, we can use them to construct valid arguments. Recall from Chapter 2 (page 34) that an argument is valid if it is impossible for its conclusion to be false whenever its premises are true. Because E and I are logically equivalent to their converses, the two argument forms in Figure 3.11 are valid.

Notice that these are argument forms, with variables in the place of the key terms. This means that these arguments will be valid no matter what; S and P could be people, or squirrels, or the Gross Domestic Product of industrialized nations, or anything, and the arguments are still valid. While these particular argument forms may seem trivial and obvious, we are beginning to see some of the power of formal logic here. We have uncovered a very general truth about the nature of validity with these two argument forms.

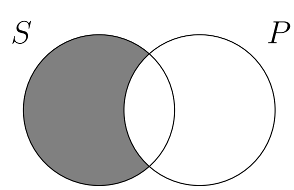
The truth value of the converses of A and O statements, on the other hand, are undetermined by the truth value of the original statements. This means we cannot construct valid arguments from them. Imagine you have an argument with an A or O statement as its premise and the converse of that statement as the conclusion. Even if the premise is true, we know nothing about the truth of the conclusion. So there are no valid argument forms to be found here.

Obversion

Obversion is a more complex process. To understand what an obverse is, we first need to define the complement of a class. The **complement** of a class is everything that is not in the class. So the complement of the class of dogs is everything that is not a dog, including not just cats, but battleships, pop songs, and black holes. In English we can easily create a name for the complement of any class using the prefix "non-". So the complement of the class of dogs is the class of non-dogs. We will use complements in defining both obversion and contraposition.

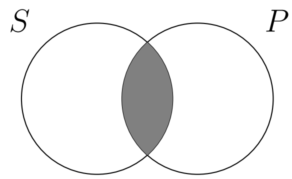
The **obversion** of a categorical proposition is a new proposition created by changing the quality of the original proposition and switching its predicate to its complement. Obversion is thus a two step process. Take, again, the proposition "All dogs are mammals." For step 1, we change its quality, in this case going from affirmative to negative. That gives us "No dogs are mammals." For step 2, we take the complement of the predicate. The predicate in this case is "mammals" so the complement is "non-mammals." That gives us the obverse "No dogs are non-mammals."

We can map this process out using Venn diagrams. Let's start with an A statement.



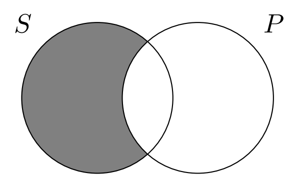
A: All S are P.

Changing the quality turns it into an E statement.



E: No S are P.

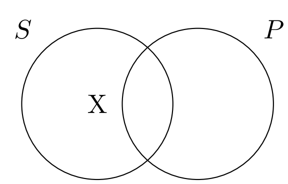
Now what happens when we take the complement of P? That means we will shade in all the parts of S that are non-P, which puts us back where we started.



E: No S are non-P.

The final statement is logically equivalent to the original A statement. It has the same form as an E statement, but because we have changed the predicate, it is not logically equivalent to an A statement. As you can see from Figure 3.12 this is true for all four forms of categorical statement. This in turn gives us four valid argument forms, which are shown in Figure 3.13

One further note on complements. We don't just use complements to describe sentences that come out of obversion and contraposition. We can also perform these operations on statements that already have complements in them. Consider the sentence "Some S are non-P." This is its Venn diagram.



Some S are non-P

How would we take the obverse of this statement? Step 1 is to change the quality, making it "Some S are not non-P." Now how do we take the complement of the predicate? We could write "non-non-P," but if we think about it for a second, we'd

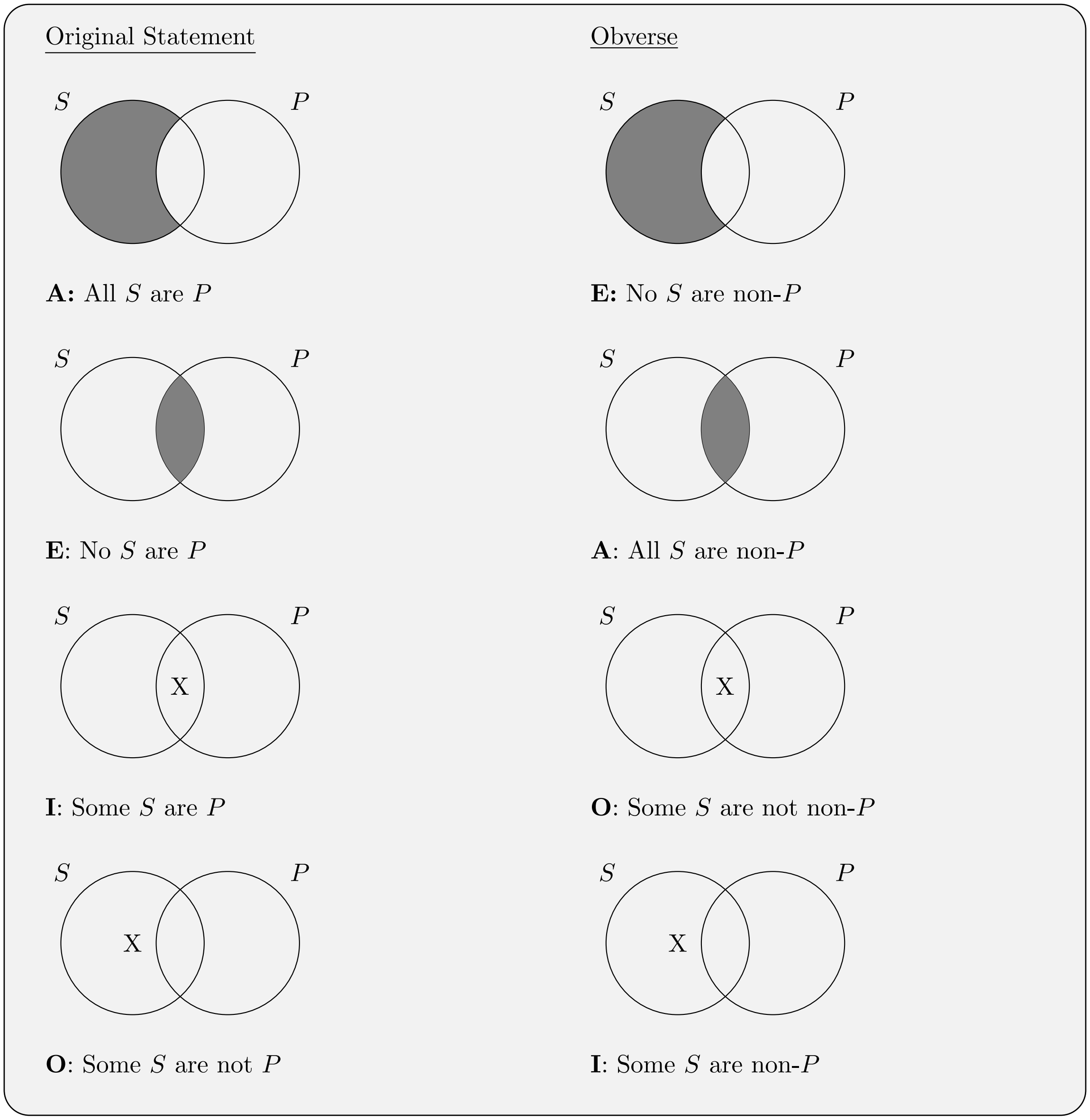


Figure 3.12: Obversions of the Four Basic Forms

P_1: All S are P.

C: No S are non-P.

P_1: No S are P.

C: All S are non-P.

P_1: Some S are P.

C: Some S are not non-P.

P_1: Some S are not P.

C: Some S are non-P.

Figure 3.13: Valid argument forms by obversion

realize that this is the same thing as P. So we can just write "Some S is not P." This is logically equivalent to the original statement, which is what we wanted.

Taking the converse of "Some S are non-P" also takes a moment of thought. We are supposed to reverse subject and predicate. But does that mean that the "non-" moves to the subject position along with the "P"? Or does the "non-" now attach to the S? We saw that E and I statements kept their truth value after conversion, and we want this to still be true when the statements start out referring to the complement of some class. This means that the "non-" has to travel with the predicate, because "Some S are non-P" will always have the same truth value as "Some non-P are S." Another way of thinking about this is that the "non-" is part of the name of the class that forms the predicate of "Some S are non-P." The statement is making a claim about a class, and that class happens to be defined as the complement of another class. So, the bottom line is when you take the converse of a statement where one of the terms is a complement, move the "non-" with that term.

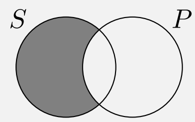
Contraposition

contraposition is a two-step process, like obversion, but it doesn't always lead to results that are logically equivalent to the original sentence. The contrapositive of a categorical sentence is the sentence that results from reversing subject and predicate and then replacing them with their complements. Thus "All S are P" becomes "All non-P are non-S."

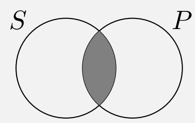
Figure 3.14 shows the corresponding Venn diagrams. In this case, the shading around the outside of the two circles in the contraposed form of E is meant to indicate that nothing can lie outside the two circles. Everything must be S or P or both. Like conversion, applying contraposition to two of the forms gives us statements that are logically equivalent to the original. This time, though, it is forms A and O that come through the process without changing their truth value.

This then gives us two valid argument forms, shown in Figure 3.15. If you have an argument with an A or O statement as its premise and the contraposition of that statement as the conclusion, you know it must be valid. Whenever the premise is true, the conclusion must be true, because the two statements are logically equivalent. On the other hand, if you had an E or an I statement as the premise, the truth of the conclusion is undetermined, so these arguments would not be valid.

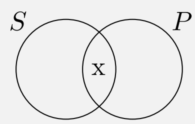
Original Statement



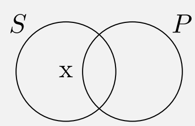
A: All S are P



E: No S are P

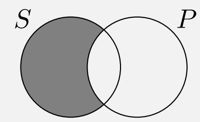


I: Some S are P

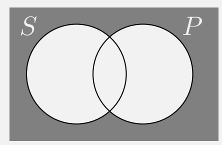


O: Some S are not P

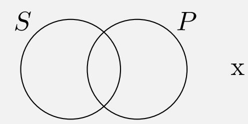
Contrapositive



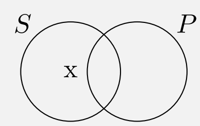
A: All non-P are non-S



E: No non-P are non-S



I: Some non-P are non-S



O: Some non-P are not non-S

Figure 3.14: Contrapositions the Four Basic Forms

P$_1$: All S are P.

C: All non-P are non-S.

P$_1$: Some S are not P

C: Some non-P are not non-S.

Figure 3.15: Valid argument forms from contraposition

Proofs in Categorical Logic

We now have a list of valid argument forms. You have just learned that E propositions and I propositions are logically equivalent to their converses, A and O statements are logically equivalent to their contrapositives, and all kinds of categorical propositions are logically equivalent to their obverses. This was proved to you using venn diagrams. Making an argument out of any of these will give us a valid argument. The same applies to the relations depicted by the square of opposition. If you start an argument with a true A statement you can validly conclude that the corresponding O is also true.

We will now combine the traditional square of opposition with conversion, obversion, and contraposition to prove that other arguments are valid. These are arguments that are not shown to be valid immediately by any of the operations or the square of opposition. This is where logic starts getting fun, if you weren't already having fun. You will need to use some ingenuity to solve these puzzles.

Proofs in categorical logic work by starting with a single premise and altering it one step at a time by appealing to the valid argument forms above. Let's walk through a proof.

1. No S are non-P. (T) /∴ Some P are not non-S. (T)

In this example we are starting with the premise "No S are non-P," and we want to prove that "Some P are not non-S" follows validly from it. To do this we will add new lines that we know follow validly from the premise until we reach the conclusion. Since each line and the one that precedes it will constitute a valid argument, we know that the conclusion will follow validly from the premise.

1. No S are non-P. (T) /∴ Some P are not non-S. (T)
2. Some S are not non-P. (T) Subalternation

After making a new line, we cite the relation or operation that justifies the new line. In this case, since the premise is a true E statement, we can use subalternation to get to the corresponding true O. Now, to get to our conclusion, we need to change both terms to their complements and swap their position, the best move to use would be contraposition, since it does both.

1. No S are non-P. (T) /∴ Some P are not non-S. (T)

2. Some S are not non-P. (T) Subalternation
3. Some P are not non-S. (T) Contraposition

Some people can easily figure out how to get from the premise to the conclusion. However, this kind of puzzle solving is often new to students in an introduction to logic course. If you struggle to what order to apply the operations and relations, you can follow these guidelines:

1. If the premise is (F), use the contradictories relation. If not, skip to 2.
2. Apply conversion, obversion, or contraposition in whatever order is needed to get the subject and predicate terms of your current line and the conclusion to match.
3. Once the terms match, consult the square of opposition to see what relation gets you to your conclusion.

Here is another example of a completed proof. In this one we will follow the guidelines above.

1. Some non-S are P. (F) /∴ Some P are not S. (F)
2. No non-S are P. (T) Contradictories
3. No P are non-S. (T) Conversion
4. All P are S. (T) Obversion
5. Some P are not S. (F) Contradictories

Since our premise is (F) we first use the contradictories relation. We then need to change non-S to S. Since we only want to change one of the terms we need to use obversion. But, since the non-S is the subject and obversion only works on the predicate, we need to use conversion to swap them. Once non-S is in the predicate place, we can use obversion to change it to S. Now the terms are in the right places, so we look at the square of opposition and see that the A statement we have now is the contradictory of our conclusion, so we use contradictories as our final step.

There are two complications with these guidelines. If you have a true E statement and you want to apply contraposition, use subalternation first. If you have a true A and you want to apply conversion, use subalternation first.

Finally, a warning about the square of opposition. The square only tells you what statements with the same terms follow validly from your premise. You cannot use the square when the terms of your premise do not match the terms of your conclusion.

Practice Exercises

Part A The first two columns in the table below give you a statement and a truth value for that statement. The next column gives an operation that can be performed on the statement in the first column, and the final two columns give the new statement and its truth value.

The first row is completed, as an example, but after that there are blanks. In problems 1–5 you must fill in the new statement and its truth value, and in problems 6–10 you must fill in the operation and the final truth value. If the truth value of the resulting statement cannot be determined from the original one, write a "?" for "undetermined." You can check your work with Venn diagrams, or by identifying the logical form of the original statement and seeing if it is one where the named operation changes the truth value.

	Given statement	T/F	Operation	New Statement	T/F
Ex	All S are P	F	Conv.	All P are S	?
1.	Some S are P	F	Obv.	______________	___
2.	Some non-S are P	F	Conv.	______________	___
3.	All S are P	F	Contrap.	______________	___
4.	Some S are P	F	Contrap.	______________	___
5.	Some S are non-P	T	Obv.	______________	___
6.	All S are non-P	T	________	All P are non-S	___
7.	Some non-S are not P	T	________	Some P are not non-S	___
8.	Some S are not P	F	________	Some P are not S	___
9.	All non-S are P	T	________	No non-S are non-P	___
10.	No non-S are non-P	T	________	All non-S are non-non-P	___

Part B See the instructions for Part A.

Given statement	T/F	Operation	New Statement	T/F

1.	All S are P	T	Obv.	______________	___
2.	Some S are not non-P	T	Contrap.	______________	___
3.	No S are non-P	T	Obv.	______________	___
4.	All non-S are P	T	Obv.	______________	___
5.	Some non-S are P	F	Contrap.	______________	___
6.	Some S are P	F	________	Some S are not non-P	___
7.	No non-S are non-P	F	________	No P are S	___
8.	Some non-S are non-P	T	________	Some P are S	___
9.	All S are P	F	________	No S are non-P	___
10.	Some non-S are not non-P	T	________	Some non-S are P	___

Part C For each sentence, write the converse, obvserse, or contrapostive as directed.

Example: Write the contrapositive of "Some sentences are categorical."

Answer: Some non-categorical things are non-sentences.

(1) Write the converse of "No weeds are benign."

(2) Write the converse of "Some minds are not closed."

(3) Write the contraposition of "Some dentists are underpaid."

(4) Write the converse of "All humor is good."

(5) Write the contraposition of "No organizations are self-sustaining."

(6) Write the obverse of "Some dogs have fleas."

(7) Write the converse of "Some things that have fleas are dogs."

(8) Write the obverse of "No detectives are uniformed."

(9) Write the converse of "No monkeys are well-behaved."

(10) Write the contraposition of "No donkeys are obedient."

Part D For each sentence, write the converse, obvserse, or contrapostive as directed.

(1) Write the converse of "No supplies are limited."

(2) Write the obverse of "No knives are toys."

(3) Write the contraposition of "All logicians are rational."

(4) Write the obverse of "All uniforms are clothing."

(5) Write the converse of "All risks are negligible."

(6) Write the contraposition of "No bestsellers are great works of literature."

(7) Write the obverse of "Some descriptions are accurate."

(8) Write the contraposition of "Some ties are not tacky."

(9) Write the obverse of "All spies are concealed."

(10) Write the contraposition of "No valleys are barren."

Part E Prove that the following arguments are valid using the three operations and the square of opposition.

(1) Some non-S are not non-P. (T) /∴ All P are S. (F)

(2) All non-S are P. (T) /∴ No P are non-S. (F)

(3) Some S are not P. (F) /∴ All S are non-P. (F)

(4) All non-S are P. (T) /∴ All non-S are non-P. (F)

(5) Some S are non-P. (T) /∴ Some non-P are not non-S. (T)

(6) No non-S are P. (F) /∴ Some P are not S. (T)

(7) No non-S are non-P. (T) /∴ All P are S. (F)

(8) Some S are not non-P. (F) /∴ All non-P are non-S. (F)

Key Terms

Affirmative

Complement

Contradictories

Contraposition

Contraries

Converse

Copula

Distribution

Existential import

Logically structured English

Mood-A statement

Mood-E statement

Mood-I statement

Mood-O statement

Negative

Obverse

Particular

Predicate class

Quality

Quantified categorical statement

Quantifier

Quantity

Square of opposition

Standard form categorical statement

Subalternation

Subcontraries

Subject class

Truth value

Universal

Vacuous truth

Venn diagram

Chapter 4

Categorical Syllogisms

4.1 Standard Form, Mood, and Figure

So far we have just been looking at very short arguments using categorical statements. The arguments just had one premise and a conclusion that was often logically equivalent to the premise. For most of the history of logic in the West, however, the focus has been on arguments that are a step more complicated called CATEGORICAL SYLLOGISMS. A categorical syllogism is a two-premise argument composed of categorical statements. Aristotle began the study of this kind of argument in his book the *Prior Analytics* (c.350 BCE/1c. 350 BCE/1984b). This work was refined over the centuries by many thinkers in the Pagan, Christian, Jewish, and Islamic traditions until it reached the form it is in today.

There are actually all kinds of two-premise arguments using categorical statements, but Aristotle only looked at arguments where each statement is in one of the moods A, E, I, or O. The arguments also had to have exactly three terms, arranged so that any two pairs of statements will share one term. Let's call a categorical syllogism that fits this more narrow description an ARISTOTELIAN SYLLOGISM Here is a typical Aristotelian syllogism using only mood-A sentences:

P_1: All mammals are vertebrates.
P_2: All dogs are mammals.

C: All dogs are vertebrates.

Notice how the statements in this argument overlap each other. Each statement shares a term with the other two. Premise 2 shares its subject term with the conclusion and its predicate with Premise 1. Thus there are only three terms spread across the three statements. Aristotle dubbed these the major, middle, and minor premises, but there was initially some confusion about how to define them. In the 6th century, the Christian philosopher John Philoponus, drawing on the work of his pagan teacher Ammonius, decided to arbitrarily designate the MAJOR TERM as the predicate of the conclusion, the MINOR TERM as the subject of the conclusion, and the MIDDLE TERM as the one term of the Aristotelian syllogism that does not appear in the conclusion. So in the argument above, the major term is "vertebrate," the middle term is "mammal," and the minor term is "dog." We can also define the MAJOR PREMISE as the one premise in an Aristotelian syllogism that names the major term, and the MINOR PREMISE as the one premise that names the minor term. So in the argument above, Premise 1 is the major premise and Premise 2 is the minor premise.

With these definitions in place, we can now define the STANDARD FORM FOR AN ARISTOTELIAN SYLLOGISM in logically structured English. Recall that in Section 3.2, we started standardizing our language into something we called "logically structured English" in order to remove ambiguity and to make its logical structure clear. The first step was to define the standard form for a categorical statement, which we did on page 82. Now we do the same thing for an Aristotelian syllogism. We say that an Aristotelian syllogism is in standard form for logically structured English if and only if these criteria have been met: (1) all of the individual statements are in standard form, (2) each instance of a term is in the same format and is used in the same sense, (3) the major premise appears first, followed by the minor premise, and then the conclusion.

Once we standardize things this way, we can actually catalog every possible form of an Aristotelian syllogism. To begin with, each of the three statements can take one of four forms: A, E, I, or O. This gives us $4 \times 4 \times 4$, or 64 possibilities. These 64 possibilities are called the SYLLOGISM MOOD, and we designate it just by writing the three letters of the moods of the statements that make it up. So the mood of the argument on page 104 is simply AAA.

In addition to varying the kind of statements we use in an Aristotelian syllogism, we can also vary the placement of the major, middle, and minor terms. There are four ways we can arrange them that fit the definition of an Aristotelian syllogism in standard form, shown in Table 4.1. Here S stands for the major term, P for the minor term, and M for the middle. The thing to pay attention to is the placement

$$\begin{array}{ll}
P_1: & \mathbf{M} \quad P \\
P_2: & S \quad \mathbf{M} \\
\hline
C: & S \quad P
\end{array}$$

Figure 1

$$\begin{array}{ll}
P_1: & P \quad \mathbf{M} \\
P_2: & S \quad \mathbf{M} \\
\hline
C: & S \quad P
\end{array}$$

Figure 2

$$\begin{array}{ll}
P_1: & \mathbf{M} \quad P \\
P_2: & \mathbf{M} \quad S \\
\hline
C: & S \quad P
\end{array}$$

Figure 3

$$\begin{array}{ll}
P_1: & P \quad \mathbf{M} \\
P_2: & \mathbf{M} \quad S \\
\hline
C: & S \quad P
\end{array}$$

Figure 4

Table 4.1: The four figures of the Aristotelian syllogism

of the middle terms. In figure 1, the middle terms form a line slanting down to the right. In figure 2, the middle terms are both pushed over to the right. In figure 3, they are pushed to the left, and in figure 4, they slant in the opposite direction from figure 1.

The combination of 64 moods and 4 figures gives us a total of 256 possible Aristotelian syllogisms. We can name them by simply giving their mood and figure. So this is OAO-3:

$$\begin{array}{rl}
P_1: & \text{Some } M \text{ are not } P. \\
P_2: & \text{All } M \text{ are } S. \\
\hline
C: & \text{Some } S \text{ are not } P.
\end{array}$$

Syllogism OAO-3 is a valid argument. We will be able to prove this with Venn diagrams in the next section. For now just read it over and try to see intuitively why it is valid. Most of the 256 possible syllogisms, however, are not valid. In fact, most of them, like IIE-2, are quite obviously invalid:

$$\begin{array}{rl}
P_1: & \text{Some } P \text{ are } M. \\
P_2: & \text{Some } S \text{ are } M. \\
\hline
C: & \text{No } S \text{ are } P.
\end{array}$$

Given an Aristotelian syllogism in ordinary English, we can transform it into standard form in logically structured English and identify its mood and figure. Consider the following:

> No geckos are cats. I know this because all geckos are lizards, but cats aren't lizards.

The first step is to identify the conclusion, using the basic skills you acquired back in Chapter 1. In this case, you can see that "because" is a premise indicator word, so the statement before it, "No geckos are cats," must be the conclusion.

Step two is to identify the major, middle, and minor terms. Remember that the major term is the predicate of the conclusion, and the minor term is the subject. So here the major term is "cats," the minor term is "geckos." The leftover term, "lizards," must then be the middle term.

We show that we have identified the major, middle, and minor terms by writing a TRANSLATION KEY. A translation key is just a list that assigns English phrases or sentences to variable names. For categorical syllogism, this means matching the English phrases for the terms with the variables S, M, and P.

S**:** Geckos
M**:** Lizards
P**:** Cats

Step three is to write the argument in canonical form using variables for the terms. The last statement, "cats aren't lizards," is the major premise, because it has the major term in it. We need to change it to standard form, however, before we substitute in the variables. So first we change it to "No cats are lizards." Then we write "No S are M." For the minor premise and the conclusion we can just substitute in the variables, so we get this:

P_1: No S are M.
P_2: All P are M.

C: No S are P.

Step four is to identify mood and figure. We can see that this is figure 2, because the middle term is in the predicate of both premises. Looking at the form of the sentences tells us that this is EAE.

Practice Exercises

Part A Put the following English arguments into standard form in logically structured English using variables for the terms. Be sure to include a translation key. Then identify mood and figure of the argument.

Example: No magical creatures are rainbow colored. Therefore, some unicorns are not rainbow colored, because all unicorns are magical creatures.

Answer:

S**:** Unicorns
M**:** Magical Creatures
P**:** Things that are rainbow colored

P_1: No M are P.
P_2: All S are M.
C: Some S are not P.

EAO-1

(1) Some beds are bunk beds, and all bunk beds are tall. So some beds are tall.

(2) No fluffy things are tusked pachyderms, but all tusked pachyderms are elephants. Therefore some elephants are not fluffy.

(3) Some strangers are not dangerous. After all, nothing that is dangerous is also a kitten. But some kittens are strangers.

(4) Some giant monsters are not things to be trifled with. This is because all kaiju are giant monsters and no kaiju is a creature to be trifled with.

(5) All parties are celebrations, because no celebrations are unhappy and no parties are unhappy.

(6) Nothing that is deadly is safe. Therefore, some snakes are not deadly, because some snakes are safe.

(7) Godzilla is a character in a movie by the Toho company. This is because Godzilla is a kaiju, and some kaiju are Toho characters.

(8) No tyrannosaurs are birds. Therefore some pteranodons are not tyrannosaurs, because no pteranodons are birds.

(9) Every carnivorous animal is a college professor. Therefore all logicians are carnivores, because some college professors are not logicians.

(10) Some balderdash is chicanery. Therefore no hogwash is chicanery, because all hogwash is balderdash.

Part B Put the following English arguments into standard form in logically structured English using variables for the terms. Be sure to include a translation key. Then identify mood and figure of the argument.

(1) No chairs are tables. But all tables are furniture. So some furniture are not chairs.

(2) All dogs bark, and some things that bark are annoying. Therefore some dogs are annoying.

(3) Some superheroes are not arrogant. This is because anyone who is arrogant is unpopular. But no superheroes are unpopular.

(4) Some mornings are not free time. But no evenings are mornings. Therefore no evenings are free time.

(5) All veterinarians are doctors. Therefore some veterinarians are well trained, because all doctors are well trained.

(6) No books are valueless. Therefore some books are not nonsense, because all nonsense is valueless.

(7) No battleships are brightly colored, because no brightly colored things are lizards, and no battleships are lizards.

(8) No octagons are curvilinear, because all circles are curvilinear and some octagons are circles.

(9) Some eggs do not come from chickens. You can tell, because no milk comes from chickens, but all eggs are milk.

(10) Some ichthyosaurs are not eoraptors. Therefore some ichthyosaurs are mixosauruses, because some eoraptors are not mixosauruses.

Part C Put the following English arguments into standard form in logically structured English using variables for the terms. Be sure to include a translation key. Then identify mood and figure of the argument.

(1) All spiders make thread, and anything that makes thread makes webs. So for sure, all spiders make webs.

(2) Some children are not afraid to explore. For no one afraid to explore suffers from abandonment issues, and some children don't suffer from abandonment issues.

(3) Every professional baseball player is a professional athlete, and no professional athlete is poor. No professional baseball player, thus, is poor.

(4) No horse contracts scrapie. So, because some animals contracting scrapie lose weight, there are horses that do not lose weight.

(5) Since everyone in this room is enrolled in logic, and since everyone at the college is enrolled in logic, everyone in this room is attending the college.

(6) All arguments are attempts to convince, and some attempts to convince are denials of autonomy. Therefore, some arguments are denials of autonomy.

(7) No one who likes smoked eel is completely reliable. For, everyone who likes smoked eel is a person with odd characteristics, and no one with odd characteristics is completely reliable.

(8) Breaking an addiction requires self-control, and nothing requiring self-control is easy. Thus, breaking an addiction is never easy.

(9) Jack is an American soldier in Iraq, and some American soldiers in Iraq are unable to sleep much. Hence, Jack is unable to sleep much.

(10) All smurfs are blue, and no smurfs are tall. Therefore some tall things are not blue.

Part D Put the following English arguments into standard form in logically structured English using variables for the terms. Be sure to include a translation key. Then identify mood and figure of the argument.

(1) All Old World monkeys are primates. Some Old World monkeys are baboons. Therefore some primates are baboons

(2) All gardeners are schnarf. And all extraterrestrials are gardeners. Therefore all extraterrestrials are schnarf.

(3) No corn chips are potato chips, but all corn chips are snacks, so no snacks are potato chips.

(4) Everything in the attic is old and musty. Moreover, some pieces of furniture are old and musty. So, necessarily, some pieces of furniture are in the attic.

(5) Some offices are pleasant places to work. All friendly places to work are workplaces. Therefore some workplaces are offices.

(6) Some rabbits are not white, but some snowdrifts are white. Therefore some snowdrifts are not rabbits.

(7) No airplanes are submarines, and some submarines are u-boats, so some airplanes are not u-boats.

(8) All rules have exceptions, but no commands from God have exceptions. So no rules are commands from God.

(9) All spies are liars, and some liars are not platypuses. Therefore some

platypuses are spies.

(10) Some bacteria are not harmful, and some harmful things are lions. Therefore some bacteria are lions.

Part E Given the mood and figure, write out the full syllogism, using the term variables S, M, and P.

Example: IAA-2

Answer: P_1: Some P are M.
P_2: All S are M.

C: All S are P.

(1) EEE-4
(2) EIE-1
(3) AII-1
(4) IIA-4
(5) IOO-2
(6) OEI-4
(7) IIO-2
(8) OAI-I
(9) AAA-2
(10) IAA-3

Part F Given the mood and figure, write out the full syllogism, using the term variables S, M, and P

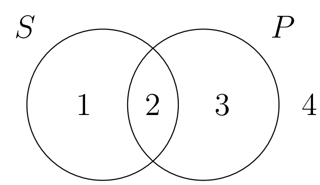
(1) EIO-1
(2) AAI-4
(3) IIO-4
(4) AEA-4
(5) AOE-3
(6) IAO-4
(7) OAI-3
(8) IOE-2
(9) IAE-2
(10) EAO-2

4.2 Testing Validity

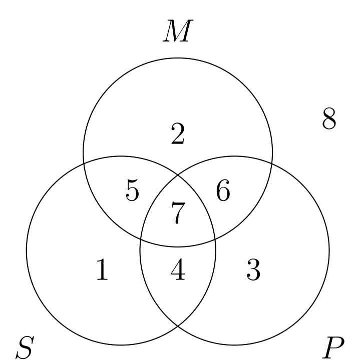
We have seen that there are 256 possible categorical arguments that fit Aristotle's requirements. Most of them are not valid, and as you probably saw in the exercises, many don't even make sense. In this section, we will learn to use Venn diagrams to sort the good arguments from the bad. The method we will use will simply be an extension of what we did in the last chapter, except with three circles instead of two.

Venn Diagrams for Single Propositions

In the previous chapter, we drew Venn diagrams with two circles for arguments that had had two terms. The circles partially overlapped, giving us four areas, each of which represented a way an individual could relate to the two classes. So area 1 represented things that were S but not P, etc.



Now that we are considering arguments with three terms, we will need to draw three circles, and they need to overlap in a way that will let us represent the eight possible ways an individual can be inside or outside these three classes.

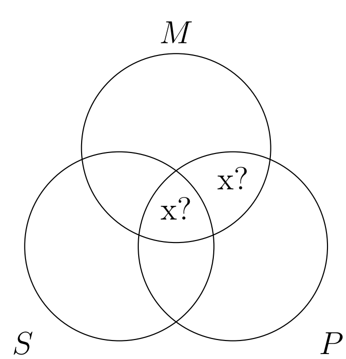


So in this diagram, area 1 represents the things that are S but not M or P, area 2 represents the things that are M but not S or P, etc.

As before, we represent universal statements by filling in the area that the statement says cannot be occupied. The only difference is that now there are more possibilities. So, for instance, there are now four mood-A propositions that can occur in the two premises. The major premise can either be "All P are M" or "All M are P," and the minor premise can be either "All S are M" or "All M are S." The Venn diagrams for those four sentences are given in the top two rows of Figure 4.1.

Similarly, there are four mood-E propositions that can occur in the premises of an Aristotelian syllogism: "No P are M," "No M are P," "No S are M," and "No M are S." And again, we diagram these by shading out overlap between the two relevant circles. In this case, however, the first two statements are equivalent by conversion (see page 3.6), as are the second two. Thus we only have two diagrams to worry about. See the bottom of Figure 4.1

Particular propositions are a bit trickier. Consider the statement "Some M are P." With a two circle diagram, you would just put an x in the overlap between the M circle and the P circle. But with the three circle diagram, there are now two places we can put it. It can go in either area 6 or area 7:



The solution here will be to put the x on the boundary between areas 6 and 7, to represent the fact that it could go in either location.

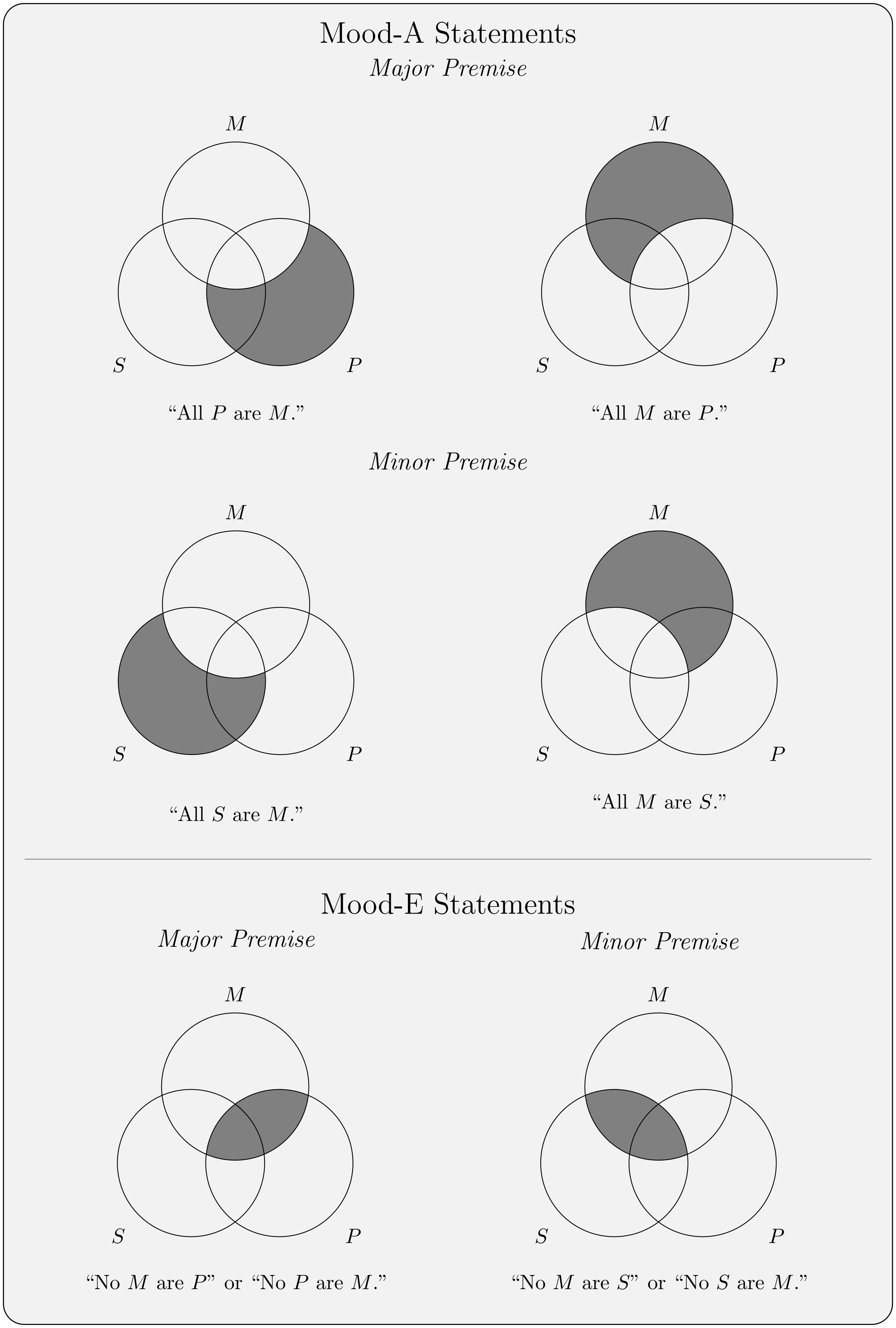
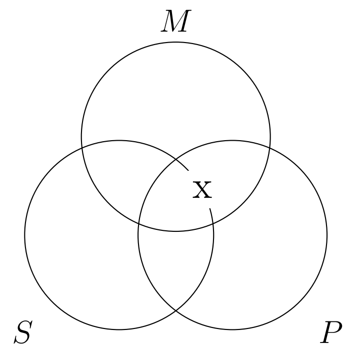
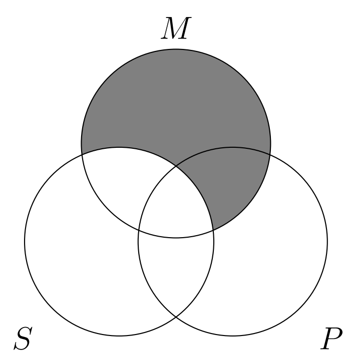


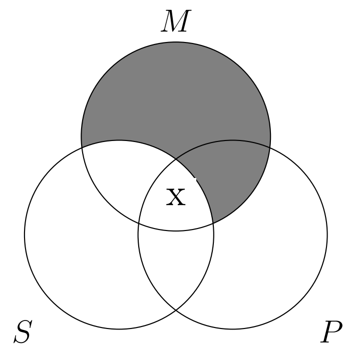
Figure 4.1: Venn diagrams for the eight universal statements that can occur in the premises.



Sometimes, however, you won't have to draw the x on a border between two areas, because you will already know that one of those areas can't be occupied. Suppose, for instance, that you want to diagram "Some M are P," but you already know that all M are S. You would diagram "All M are S" like this:



Then, when it comes time to add the x for "Some M are P," you know that it has to go in the exact center of the diagram:



The Venn diagrams for the particular premises that can appear in Aristotelian syllogisms are given in Figure 4.2. The figure assumes that you are just representing the individual premises, and don't know any other premises that would shade some regions out. Again, some of these premises are equivalent by conversion, and thus share a Venn diagram.

Venn Diagrams for Full Syllogisms

In the last chapter, we used Venn diagrams to evaluate arguments with single premises. It turned out that when those arguments were valid, the conclusion was logically equivalent to the premise, so they had the exact same Venn diagram. This time we have two premises to diagram, and the conclusion won't be logically equivalent to either of them. Nevertheless we will find that for valid arguments, once we have diagrammed the two premises, we will also have diagrammed the conclusion.

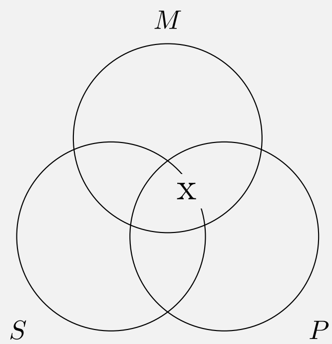
First we need to specify a rule about the order to diagram the premises in: if one of the premises is universal and the other is particular, diagram the universal one first. This will allow us to narrow down the area where we need to put the x from the particular premise, as in the example above where we diagrammed "Some M are P" assuming that we already knew that all M are S.

Let's start with a simple example, an argument with the form AAA-1.

P$_1$: All M are P.
P$_2$: All S are M.
___

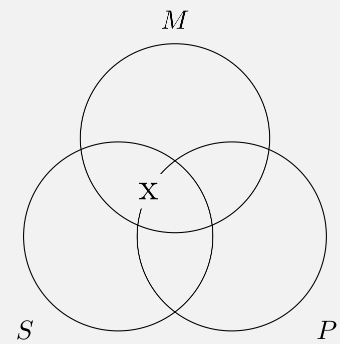
Mood-I Statements

Major Premise



"Some M are P" or "Some P are M."

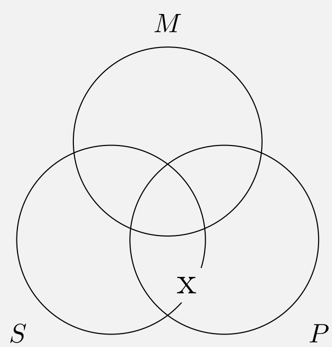
Minor Premise



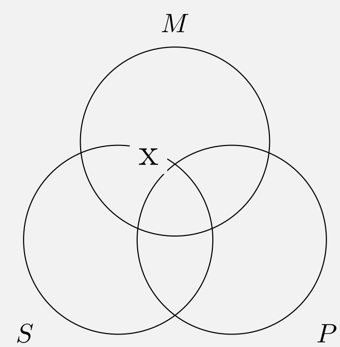
"Some M are S" or "Some S are M."

Mood-O Statements

Major Premise

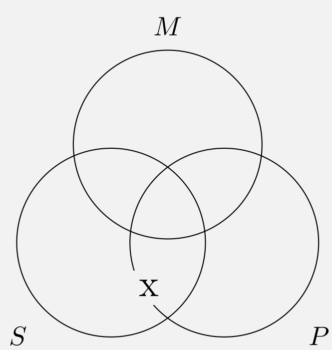


"Some P are not M."

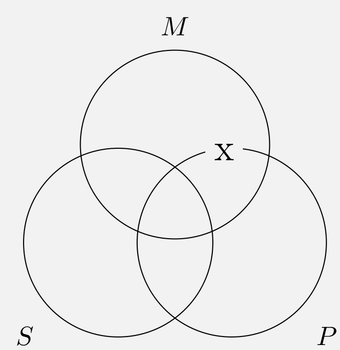


"Some M are not P."

Minor Premise



"Some S are not M."

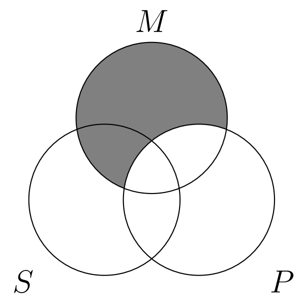


"Some M are not S."

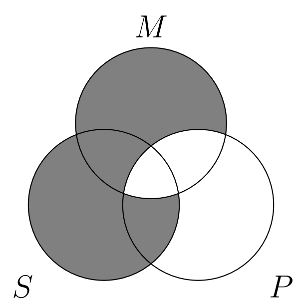
Figure 4.2: Venn diagrams for the eight particular statements that can occur in the premises.

C: All S are P.

Since both premises are universal, it doesn't matter what order we do them in. Let's do the major premise first. The major premise has us shade out the parts of the M circle that don't overlap the P circle, like this:



The second premise, on the other hand, tells us that there is nothing in the S circle that isn't also in the M circle. We put that together with the first diagram, and we get this:



From this we can see that the conclusion must be true. All S are P, because the only space left in S is the area in the exact center, area 7.

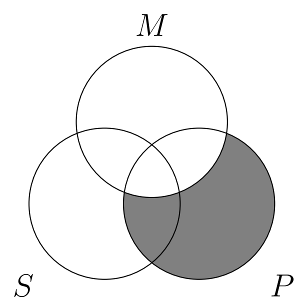
Now let's look at an argument that is invalid. One of the interesting things about the syllogism AAA-1 is that if you change the figure, it ceases to be valid. Consider AAA-2.

P_1: All P are M.

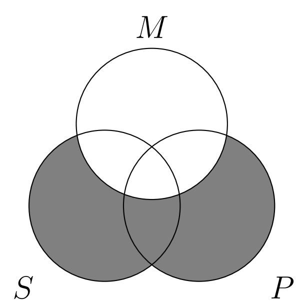
P_2: All S are M.

C: All S are P.

Again, both premises are universal, so we can do them in any order, so we will do the major premise first. This time, the major premise tells us to shade out the part of P that does not overlap M.



The second premise adds the idea that all S are M, which we diagram like this:



Now we ask if the diagram of the two premises also shows that the conclusion is true. Here the conclusion is that all S are P. If this diagram had made this true, we would have shaded out all the parts of S that do not overlap P. But we haven't done that. It is still possible for something to be in area 5. Therefore this argument is invalid.

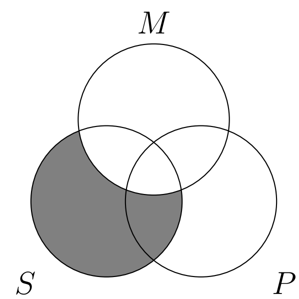
Now let's try an argument with a particular statement in the premises. Consider the argument IAI-1:

P_1: Some M are P.

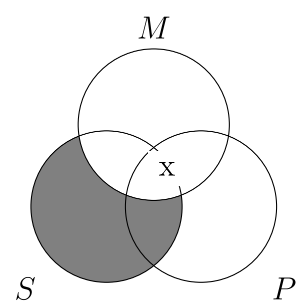
P_2: All S are M.

C: Some S are P.

Here, the second premise is universal, while the first is particular, so we begin by diagramming the universal premise.



Then we diagram the particular premise "Some M are P." This tells us that something is in the overlap between M and P, but it doesn't tell us whether that thing is in the exact center of the diagram or in the area for things that are M and P but not S. Therefore, we place the x on the border between these two areas.



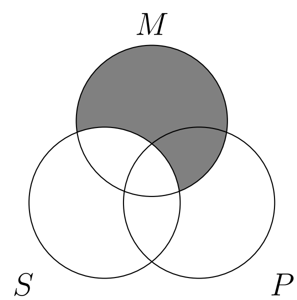
Now we can see that the argument is not valid. The conclusion asserts that something is in the overlap between S and P. But the x we drew does not necessarily represent an object that exists in that overlap. There is something out there that could be in area 7, but it could just as easily be in area 6. The second premise doesn't help us, because it just rules out the existence of objects in areas 1 and 4.

For a final example, let's look at a case of a valid argument with a particular statement in the premises. If we simply change the figure of the argument in the last example from 1 to 3, we get a valid argument. This is the argument IAI-3:

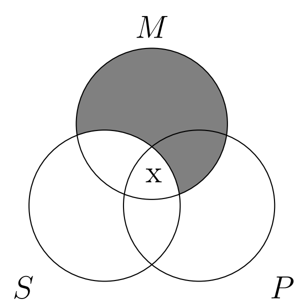
P_1: Some M are P.
P_2: All M are S.

C: Some S are P.

Again, we begin with the universal premise. This time it tells us to shade out part of the M circle.



But now we fill in the parts of M that don't overlap with S, we have to put the x in the exact center of the diagram.



And now this time we see that "Some S are P" has to be true based on the premises, because the X has to be in area 7. So this argument is valid.

Using this method, we can show that 15 of the 256 possible syllogisms are valid. Remember, however, that the Venn diagram method uses Boolean assumptions about existential import. If you make other assumptions about existential import,

Figure 1	**Figure 2**	**Figure 3**	**Figure 4**
Barbara (AAA)	Camestres (AEE)	Disamis (IAI)	Calemes (AEE)
Celarent (EAE)	Cesare (EAE)	Bocardo (OAO)	Dimatis (IAI)
Ferio (EIO)	Festino (EIO)	Ferison (EIO)	Fresison (EIO)
Darii (AII)	Baroco (AOO)	Datisi (AII)	

Table 4.7: The 15 unconditionally valid syllogisms.

you will allow more valid syllogisms, as we will see in the next section. The additional syllogisms we will be able to prove valid in the next section will be said to have CONDITIONAL VALIDITY because they are valid on the condition that the objects talked about in the universal statements actually exist. The 15 syllogisms that we can prove valid using the Venn diagram method have UNCONDITIONAL VALIDITY. These syllogisms are given in Table 4.7.

The names on Table 4.7 come from the Christian part of the Aristotelian tradition, where thinkers were writing in Latin. Students in that part of the tradition learned the valid forms by giving each one a female name. The vowels in the name represented the mood of the syllogism. So B**a**rb**a**r**a** has the mood AAA, Fr**e**s**i**s**o**n has the mood EIO, etc. The consonants in each name were also significant: they related to a process the Aristotelians were interested in called reduction, where arguments in the later figures were shown to be equivalent to arguments in the first figure, which was taken to be more self-evident. We won't worry about reduction in this textbook, however. The names of the valid syllogisms were often worked into a mnemonic poem. The oldest known version of the poem appears in a late 13th century book called *Introduction to Logic* by William of Sherwood (Sherwood c. 1275/1966). Figure 4.3 is an image of the oldest surviving manuscript of the poem, digitized by the Bibliothque Nationale de France.

The columns in Table 4.7 represent the four figures. Syllogisms with the same mood also appear in the same row. So the EIO sisters—Ferio, Festino, Ferison, and Fresison—fill up row 3. Camestres and Calemes share row 1; Celarent and Cesare share row 2; and Darii and Datisi share row 4.

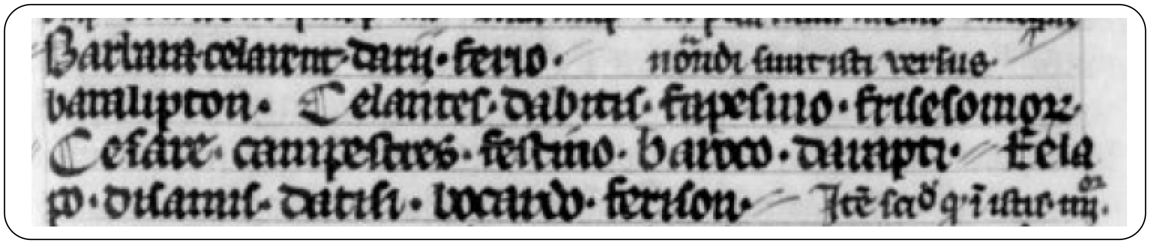


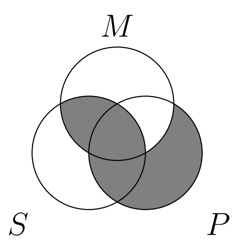
Figure 4.3: The oldest surviving version of the "Barbara, Celarent..." poem, from William of Sherwood (c. 1275/1966)

Practice Exercises

Part A Use Venn diagrams to determine whether the following Aristotelian syllogisms are valid. You can check your answers against Table 4.7.

Example:All P are M and no M are S. Therefore, no S are P.

Answer: Valid (Calemes, AEE-4)



(1) Some P are not M, and no M are S. Therefore, some S are P.

(2) All M are P, and some M are S. Therefore some S are P.

(3) No P are M, and some S are M. Therefore, some S are not P.

(4) No M are P, and all S are M. Therefore, no S are P.

(5) All M are P, and no M are S. Therefore, all S are P.

(6) Some M are not P, and some M are S. Therefore, all S are P.

(7) No P are M, and some S are not M. Therefore, some S are not P.

(8) Some P are M, and some S are M. Therefore, no S are P.

(9) No P are M, and all S are M. Therefore no S are P.

(10) No M are P and all S are M. Therefore some S are not P

Part B Use Venn diagrams to determine whether the following Aristotelian syllogisms are valid. You can check your answers against Table 4.7.

(1) No M are P, and some M are S. Therefore some S are not P.

(2) Some M are not P, and all M are S. Therefore some S are not P.

(3) No M are P, and some S are M. Therefore, some S are not P.

(4) All M are P, and some S are not M. Therefore, some S are P.

(5) Some P are M, and all M are S. Therefore some S are P.

(6) Some M are P, and all S are M. Therefore, all S are P.

(7) All P are M, and all M are S. Therefore, some S are P.

(8) All P are M, and some S are not M. Therefore, some S are not P.

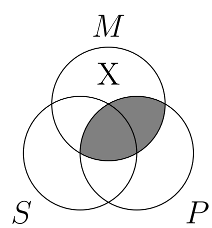
(9) No P are M, and all M are S. Therefore no S are P.

(10) Some P are M, and some S are M. Therefore, some S are P.

Part C The arguments below are missing conclusions. Use Venn diagrams to determine what conclusion can be drawn from the two premises. If no conclusion can be drawn, write "No conclusion."

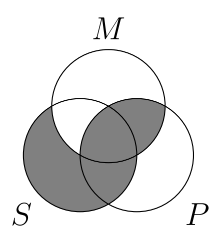
Example 1: No P are M and some M are not S. Therefore __________

Answer: No conclusion



Example 2: No P are M and All S are M. Therefore __________

Answer: No S are P



(1) All M are P, and all S are M. Therefore __________.

(2) All M are P, and some M are S. Therefore __________.

(3) No M are P and some S are not M. Therefore __________.

(4) Some M are P, and some S are M. Therefore __________.

(5) Some P are M, and all M are S. Therefore __________.

(6) All P are M and no M are S. Therefore __________ .

(7) No M are P and all S are M. Therefore __________ .

(8) No P are M, and no M are S. Therefore __________.

(9) No M are P, and some S are M. Therefore __________.

(10) Some P are M, and some S are M. Therefore __________.

Part D The arguments below are missing conclusions. Use Venn diagrams to determine what conclusion can be drawn from the two premises. If no conclusion can be drawn, write "No conclusion."

(1) Some P are not M, and all M are S. Therefore, __________.

(2) All M are P, and some S are M. Therefore __________.

(3) All P are M, and some S are not M. Therefore __________.

(4) Some P are M, and all M are S. Therefore __________.

(5) All P are M, and some M are not S. Therefore __________.

(6) No M are P, and some S are M. Therefore __________.

(7) No P are M, and no S are M. Therefore __________.

(8) Some M are not P, and no M are S. Therefore __________.

(9) No P are M, and all S are M. Therefore __________.

(10) No M are P, and some M are S. Therefore __________.

Part E

(1) Do you think there are any valid arguments in Aristotle's set of 256 syllogisms where both premises are particular? Why or why not?

(2) Do you think there are any valid arguments in Aristotle's set of 256 syllogisms where both premises are negative? Why or why not?

(3) Can a valid argument have a negative statement in the conclusion, but only affirmative statements in the premises? Why or why not?

(4) Can a valid argument have an affirmative statement in the conclusion, but only one affirmative premise?

(5) Can a valid argument have two universal premises and a particular conclusion?

4.3 Existential Import and Conditionally Valid Forms

In the last section, we mentioned that you can prove more syllogisms valid if you make different assumptions about existential import. Recall that a statement has existential import if, when you assert the statement, you are also asserting the existence of the things the statement talks about. (See page 76.) So if you interpret a mood-A statement as having existential import, it not only asserts "All S is P," it also asserts "S exists." Thus the mood-A statement "All unicorns have one horn" is false, if it is taken to have existential import, because unicorns do not exist. It is probably true, however, if you do not imagine the statement as having existential import. If anything is true of unicorns, it is that they would have one horn if they existed.

We saw in Section 3.4 that before Boole, Aristotelian thinkers had all sorts of opinions about existential import, or, as they put it, whether a term "supposits." This generally led them to recognize additional syllogism forms as valid. You can see this pretty quickly if you just remember the traditional square of opposition. The traditional square allowed for many more valid immediate inferences than the modern square. It stands to reason that traditional ideas about existential import will also allow for more valid syllogisms.

Our system of Venn diagrams can't represent all of the alternative ideas about existential import. For instance, it has no way of representing Ockham's belief that mood-O statements do *not* have existential import. Nevertheless, it would be nice if we could expand our system of Venn diagrams to show that some syllogisms are valid if you make additional assumptions about existence.

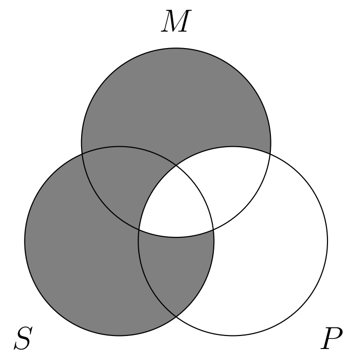
Consider the argument Barbari (AAI-1).

P_1: All M are P.

P_2: All S are M.

C: Some S are P.

You won't find this argument in the list of unconditionally valid forms in Table 4.7. This is because under Boolean assumptions about existence it is not valid. The Venn diagram, which follows Boolean assumptions, shows this.



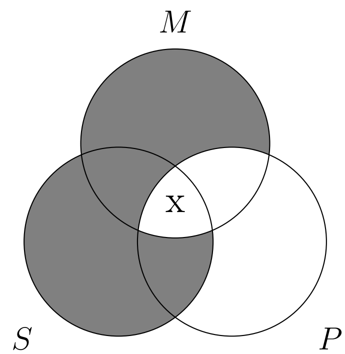
This is essentially the same argument as Barbara, but the mood-A statement in the conclusion has been replaced by a mood-I statement. We can see from the diagram that the mood-A statement "All S are P" is true. There is no place to put an S other than in the overlap with P. But we don't actually know the mood-I statement "Some S is P," because we haven't drawn an x in that spot. Really, all we have shown is that *if* an S existed, it would be P.

But by the traditional square of opposition (p. 67) we know that the mood-I statement is true. The traditional square, unlike the modern one, allows us to infer the truth of a particular statement given the truth of its corresponding universal statement. This is because the traditional square assumes that the universal statement has existential import. It is really two statements, "All S is P" and "Some S exists."

Because the mood-A statement is actually two statements on the traditional interpretation, we can represent it simply by adding an additional line to our argument. It is always legitimate to change an argument by making additional assumptions. The new argument won't have the exact same impact on the audience as the old argument. The audience will now have to accept an additional premise, but in this case all we are doing is making explicit an assumption that the Aristotelian audience was making anyway. The expanded argument will look like this:

P_1: All M are P.
P_2: All S are M.
P_3: Some S exists.*
―――――――――
C: Some S are P

Here the asterisk indicates that we are looking at an implicit premise that has been made explicit. Now that we have an extra premise, we can add it to our Venn diagram. Since there is only one place for the S to be, we know where to put our x.



In this argument S is what we call the "critical term." The critical term is the term that names things that must exist in order for a conditionally valid argument to be actually valid. In this argument, the critical term was S, but sometimes it will be M or P.

We have used Venn diagrams to show that Barbari is valid once you include the additional premise. Using this method we can identify nine more forms, on top of the previous 15, that are valid if we add the right existence assumptions (Table 4.11)

Thus we now have an expanded method for evaluating arguments using Venn diagrams. To evaluate an argument, we first use a Venn diagram to determine whether it is unconditionally valid. If it is, then we are done. If it is not, then we see if adding an existence assumption can make it conditionally valid. If we can add such an assumption, add it to the list of premises and put an x in the relevant part of the Venn diagram. If we cannot make the argument valid by including additional existence assumptions, we say it is completely invalid.

Let's run through a couple examples. Consider the argument EAO-3.

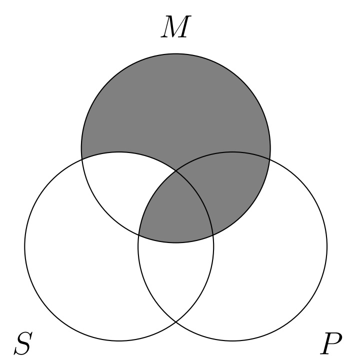
P_1: No M are P.
P_2: All M are S.

C: Some S are not P.

	Figure 1	**Figure 2**	**Figure 3**	**Figure 4**	**Condition**
Unconditional	Barbara (AAA)	Camestres (AEE)	Disamis (IAI)	Calemes (AEE)	
	Celarent (EAE)	Cesare (EAE)	Bocardo (OAO)	Dimatis (IAI)	
	Ferio (EIO)	Festino (EIO)	Ferison (EIO)	Fresison (EIO)	
	Darii (AII)	Baroco (AOO)	Datisi (AII)		
Conditional	Barbari (AAI)	Camestros (AEO)		Calemos (AEO)	S exists
	Celaront (EAO)	Cesaro (EAO)			S exists
			Felapton (EAO)	Fesapo (EAO)	M exists
			Darapti (AAI)		M exists
				Bamalip (AAI)	P exists

Table 4.11: All 24 Valid Syllogisms

First we use the regular Venn digram method to see whether the argument is unconditionally valid.



We can see from this that the argument is not valid. The conclusion says that some S are not P, but we can't tell that from this diagram. There are three possible ways something could be S, and we don't know if any of them are occupied.

Simply adding the premise S exists won't help us, because we don't know whether to put the x in the overlap between S and M, the overlap between S and P, or in the area that is just S. Of course, we would want to put it in the overlap between S and M, because that would mean that there is an S that is not P. However, we can't justifying doing this simply based on the premise that S exists.

The premise that P exists will definitely not help us. The P would either go in the overlap between S and P or in the area that is only P. Neither of these would show "Some S is not P."

The premise "M exists" does the trick, however. If an M exists, it has to also be S but not P. And this is sufficient so show that some S is not P. We can then add this additional premise to the argument to make it valid.

P_1: No M are P.
P_2: All M are S.
P_3: M exists.*

C: Some S are not P.

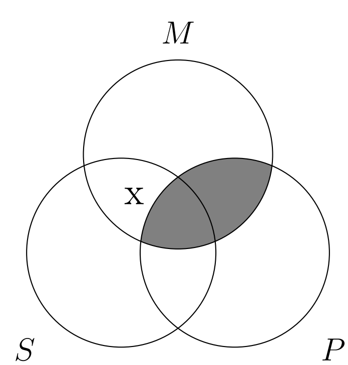
Checking it against Table 4.11, we see that we were right: this is a conditionally valid argument named Felapton.

Now consider the argument EII-3:

P_1: No M are P.
P_2: Some M are S.

C: Some S are P.

First we need to see if it is unconditionally valid. So we draw the Venn diagram.



The conclusion says that some S are P, but we obviously don't know this from the diagram above. There is no x in the overlap between S and P. Part of that region is shaded out, but the rest could go either way.

What about conditional validity? Can we add an existence assumption that would make this valid? Well, the x we have already drawn lets us know that both S and M exist, so it won't help to add those premises. What about adding P? That won't help either. We could add the premise "P exists" but we wouldn't know whether that P is in the overlap between S and P or in the area to the right, which is just P.

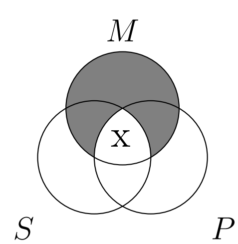
Therefore this argument is invalid. And when we check the argument against Table 4.11, we see that it is not present.

Practice Exercises

Part A Use Venn diagrams to determine whether the following arguments are unconditionally valid, conditionally valid, or invalid. If they are conditionally valid, write out the premise you need to add. You can check your answers against Table 4.11.

Example: All M are P and all M are S. Therefore some S are P

Answer: Added premise: P_3: M exists.



Conditionally valid (Darapti, AAI-3)

(1) No P are M, and all S are M, so some S are not P.

(2) Some P are M, and some S are M. Therefore all S are P.

(3) No M are P, and some S are M. Therefore, some S are not P.

(4) No M are P, and all S are M. Therefore, some S are not P.

(5) No P are M, and some M are S, so some S are not P.

(6) All P are M, and all M are S, so some S are P.

(7) Some P are not M, and some M are S. Therefore all S are P.

(8) No P are M, and all M are S. Therefore some S are not P.

(9) All M are P, and no S are M. Therefore, some S are P.

(10) Some M are P, and some S are not M. Therefore, some S are P.

Part B Use Venn diagrams to determine whether the following arguments are unconditionally valid, conditionally valid, or invalid. If they are conditionally valid, write out the premise you need to add. You can check your answers against Table 4.11.

(1) No M are P, and all M are S. Therefore some S are not P.

(2) Some M are P, and all M are S. Therefore some S are P.

(3) All M are P, and some M are S, so no S are P.

(4) No M are P, and some M are S, so some S are not P.

(5) Some P are M, and some S are not M, so no S are P

(6) All M are P, and all S are M, so some S are P.

(7) All P are M, and no M are S, so no S are P.

(8) No P are M, and all S are M, so some S are not P.

(9) All P are M, and no M are S, so some S are not P.

(10) All M are P, and some S are M, so some S are P.

4.4 Rules and Fallacies

Did you do the exercises in Part E of Section 4.2? If you didn't, go back and think about those questions now, before reading any further. The problem set had five general questions like, "Can a valid argument have only negative premises?" The point of those questions was to get you to think about what patterns might exist among the 256 Aristotelian syllogisms, and how those patterns might single out 24 syllogisms as the only ones that can be valid.

In this section, we are going to answer the questions in Part E in detail by identifying rules that all valid syllogisms amongst the 256 Aristotelian syllogisms must obey. Seeing these rules will help you understand the *structure* of this part of logic. We aren't just assigning the labels "valid" and "invalid" to arguments randomly.

Each of the rules we will identify is associated with a fallacy. If you violate the rule, you commit the fallacy. These fallacies are like the fallacies that are identified in the parts of the complete version of this text on critical thinking, in that they represent mistakes in reasoning. If you make an inference that commits one of these fallacies, you have used reasoning incorrectly. However, unlike the fallacies we looked at in the Critical Thinking section, many of these fallacies are not even tempting. They are not ways the human mind goes naturally off the rails. They are just things that you shouldn't do.

In the next subsection we are going to outline five basic rules and the fallacies that go with them, along with an addition rule/fallacy pair that can be derived from the initial five. All standard logic textbooks these days use some version of these rules, although they might divide them up differently. Some textbooks also include rules that we have built into our definition of an Aristotelian syllogism in standard form. For instance, other textbooks might have a rule here saying valid syllogisms can't have four terms, or have to use terms in the same way each time. All of this is built into our definitions of an Aristotelian syllogism and standard form for such a syllogism, so we don't need to discuss them here.

Six Rules and Fallacies

Rule 1: The middle term in a valid Aristotelian syllogism must be distributed at least once.

Consider these two arguments:

P$_1$: All M are P.	P$_1$: All P are M.
P$_2$: All S are M.	P$_2$: All S are M.
C: All S are P.	C: All S are P.

The syllogism on the left (Barbara) is obviously valid, but if you change it to figure 2, you get the syllogism on the right, which is obviously invalid. What causes this change?

The premises in the second syllogism say that S and P are both parts of M, but they no longer tell us anything about the relationship between S and P. To see why this is the case, we need to bring back a term we saw on page 59, distribution. A term is distributed in a statement if the statement makes a claim about every member of that class. So in "All M are P" the term M is distributed, because the statement tells us something about every single M. They are all also P. The term P is not distributed in this sentence, however. We do not know anything about every single P. We know that M is in P, but not vice versa.

In general mood-A statements distribute the subject, but not the predicate. This means that when we reverse P and M in the first premise, we create an argument where S and P are distributed, but M is not. This means that the argument is always going to be invalid.

This short argument can show us that arguments with an undistributed middle are always invalid: The conclusion of an Aristotelian syllogism tries to say something about the relationship between S and P. It does this using the relationship those two terms have to the third term M. But if M is never distributed, then S and P can be different, unrelated parts of M. Therefore arguments with an undistributed middle are invalid. Syllogisms that violate this rule are said to commit the FALLACY OF THE UNDISTRIBUTED MIDDLE.

Rule 2: If a term is distributed in the conclusion of a valid Aristotelian syllogism, then it must also be distributed in one of the premises.

Suppose, instead of changing Barbara from a figure 1 to a figure 2 argument, we changed it to a figure 4 argument. This is what we'd get.

P$_1$: All P are M.

P_2: All M are S.

C: All S are P.

When we changed the argument from figure 1 to figure 2, it ceased to be valid because the middle became undistributed. But this time the middle is distributed in the second premise, and the argument still doesn't work. You can see this by filling in "animals," "mammals," and "dogs," for S, M, and P.

P_1 :	All dogs are mammals.	$\Leftarrow$ True
P_2 :	All mammals are animals.	$\Leftarrow$ True
C:	All animals are dogs.	$\Leftarrow$ False

This version of the argument has true premises and a false conclusion, so you know the argument form must be invalid. A valid argument form should never be able to take true premises and turn them into a false conclusion. What went wrong here?

The conclusion is a mood-A statement, which means it tries to say something about the entire subject class, namely, that it is completely contained by the predicate class. But that is not what these premises tell us. The premises tell us that the the subject class, animals, is actually the broadest class of the three, containing within it the classes of mammals and dogs.

As with the previous rule, the problem here is a matter of distribution. The conclusion has the subject class distributed. It wants to say something about the entire subject class, animals. But the premises do not have "animals" as a distributed class. Premise 1 distributes the class "dogs" and premise 2 distributes the class "mammals."

Here is another argument that makes a similar mistake:

P_1: All M are P.
P_2: Some S are not M.

C: Some S are not P.

This time the conclusion is a mood-O statement, so the predicate term is distributed. We are trying to say something about the entire class P. But again, the premises do not say something about the entire class P. P is undistributed in the major premise.

These examples illustrate rule 2: If a term is distributed in the conclusion, it must also be distributed in the corresponding premise. Arguments that violate this rule are said to commit the FALLACY OF ILLICIT PROCESS. This fallacy has two versions, depending on which term is not distributed. If the subject term is the one that is not distributed, we say that the argument commits the fallacy of an illicit minor. If the predicate term isn't distributed, we say that the argument commits the fallacy of the illicit major. Some particularly silly arguments commit both.

The justification for this rule is easy enough to see. If the conclusion makes a claim about all of a class, but the premises only make a claim about some of the class, the conclusion clearly says more than what the premises justify.

Rule 3: A valid Aristotelian syllogism cannot have two negative premises.

Back in exercise set C you were asked to determine what conclusion, if any, could be drawn from a given pair of premises. Some of the exercises involved arguments with two negative premises. Problem (8) went like this: "No P are M, and no M are S, therefore __________." If you haven't done so already, try to find a conclusion about S and P that you can draw from this pair of premises.

Hopefully you have convinced yourself that there is no conclusion to be drawn from the premises above using standard Aristotelian format. No matter what mood you put the conclusion is, it will not follow from the premises. The same thing would be true of any syllogism with two negative premises. We could show this conclusively by running through the 16 possible combinations of negative premises and figures. A more intuitive proof of this rule goes like this: The conclusion of an Aristotelian syllogism must tell us about the relationship between subject and predicate. But if both premises are negative then the middle term must be disjoint, either entirely or partially, from the subject and predicate terms. An argument that breaks this rule is said to commit the FALLACY OF EXCLUSIVE PREMISES.

Rule 4: A valid Aristotelian syllogism can have a negative conclusion if and only if it has exactly one negative premise.

Again, let's start with examples, and try to see what is wrong with them.

P$_1$: All M are P.
P$_2$: All P are M.
C: Some S are not P.

P$_1$: No P are M.
P$_2$: All S are M.
C: All S are P.

These arguments are so obviously invalid, you might look at them and say, "Sheesh, is there anything *right* about them?" Actually, these arguments obey all the rules we have seen so far. Look at the left hand argument. Premise 1 ensures that the middle term is distributed. The conclusion is mood O, which means the predicate is distributed, but P is also distributed in the second premise. The argument does not have two negative premises. A similar check will show that the argument on the right also obeys the first three rules.

Actually, these arguments illustrate an important premise that is independent of the previous three. You can't draw a negative conclusion from two affirmative premises, and you cannot drawn an affirmative conclusion if there is a negative premise. Because the previous rule tells us that you can never have two negative premises, we can actually state this rule quite simply: an argument can have a negative conclusion if and only if it has exactly one negative premise. (The phrase "if and only if" will become important when we get to SL in Chapter 5. For now you can just note that "if and only if" means that the rule goes both ways. If you have a negative conclusion, then you must have one negative premise, and if you have one negative premise, you must have a negative conclusion.)

To see why this rule is justified, you need to look at each part of it separately. First, consider the case with the affirmative conclusion. An affirmative conclusion tells us that some or all of S is contained in P. The only way to show this is if some or all of S is in M, and some or all of M is in P. You need a complete chain of inclusion. Therefore if an argument has a negative premise, it cannot have an affirmative conclusion.

On the other hand, if an argument has a negative conclusion, it is saying that S and P are at least partially separate. But if you have all affirmative premises you are never separating classes. Also, a valid argument cannot have two negative premises. Therefore, a valid argument with a negative conclusion must have exactly one negative premise.

There is not a succinct name for the fallacy that goes with violating this rule, because this is not a mistake people commonly make. We will call it the NEGATIVE-AFFIRMATIVE FALLACY.

Rule 5: A valid Aristotelian syllogism cannot have two universal premises and a particular conclusion.

This rule is a little different than the previous ones, because it really only applies if you take a Boolean approach to existential import. Consider Barbari, the sometimes maligned step-sister of Barbara:

P$_1$: All M are P.
P$_2$: All S are M.
C: Some S are P.

This syllogism is not part of the core 15 valid syllogisms we identified with the Venn diagram method using Boolean assumptions about existential import. The premises never assert the existence of something, but the conclusion does. And this is something that is generally true under the Boolean interpretation. Universal statements never have existential import and particular statements always do. Therefore you cannot derive a particular statement from two universal statements.

Some textbooks act as if the ancient Aristotelians simply overlooked this rule. They say things like "the traditional account paid no attention to the problem of existential import" which is simply false. As we have seen, the Latin part of the Aristotelian tradition engaged in an extensive discussion of the issue from the 12th to the 16th centuries, under the heading "supposition of terms" (Read, 2002). And at least some people, like William of Ockham, had consistent theories that show why syllogisms like Barbari were valid (Parsons, 2008).

In this textbook, we handle the existential import of universal statements by adding a premise, where appropriate, which makes the existence assumption explicit. So Barbari should look like this.

P$_1$: All M are P.
P$_2$: All S are M.
P$_3$: Some S exist.*

C: Some S are P.

Adding this premise merely gives a separate line in the proof for an idea that Ockham said was already contained in premise 2. And if we make it a practice of adding existential premises to arguments like these, Rule 5 still holds true. You cannot conclude a particular statement from all universal premises. However in this case, we do have a particular premise, namely, P_3. So if we provide this reasonable accommodation, we can see that syllogisms like Barbari are perfectly good members of the valid syllogism family. We will say, however, that an argument like this that does not provide the extra premise commits the EXISTENTIAL FALLACY.

Proving the Rules

For each rule, we have presented an argument that any syllogism that breaks that rule is invalid. It turns out that the reverse is also true. If a syllogism obeys all five of these rules, it must be valid. In other words, these rules are *sufficient* to characterize validity for Aristotelian syllogisms. It is good practice to actually walk through a proof that these five rules are sufficient for validity. After all, that sort of proof is what formal logic is really all about. The proof below follows Hurley (2014).

Imagine we have a syllogism that obeys the five rules above. We need to show that it must be valid. There are four possibilities to consider: the conclusion is either mood A, mood E, mood I, or mood O.

If the conclusion is in mood A, then we know that S is distributed in the conclusion. If the syllogism obeys rules 1 and 2, then we know that S and M are distributed in the premises. Rule 4 tells us that both premises must be affirmative, so the premises can't be I or O. They can't be E, either, because E does not distribute any terms, and we know that terms are distributed in the premises. Therefore both premises are in mood A. Furthermore, we know that they are in the first figure, because they have to distribute S and M. Therefore the syllogism is Barbara, which is valid.

Now suppose the conclusion is in mood E. By rule 4, we have one negative and one affirmative premise. Because mood-E statements distribute both subject and predicate, rules 1 and 2 tell us that all three terms must be distributed in the premises. Therefore one premise must be E, because it will have to distribute two terms. Since E is negative, the other premise must be affirmative, and since it has to distribute a term, it can't be I. So we know one premise is A and the other E. If all the terms are distributed, this leaves us four possibilities: EAE-1, EAE-2, AEE-2, and AEE-4. These are the valid syllogisms Celarent, Cesare, Camestres, and Calemes.

Next up, consider the case where the conclusion is in mood I. By rule 4, it has two affirmative premises, and by rule 5 both premises cannot be universal. This means that one premise must be an affirmative particular statement, that is, mood I. But we also know that by rule 1 some premise must distribute the middle term. Since this can't be the mood-I premise, it must be the other premise, which then must be in mood A. Again we are reduced to four possibilities: AII-1, AII-2, IAI-3, and IAI-4, which are the valid syllogisms Darii, Datisi, Disamis, and Dimatis.

Finally, we need to consider the case where the conclusion is mood O. Rule 4 tells us that one premise must be negative and the other affirmative, and rule 5 tells us that they can't both be universal. Rules 1 and 2 tell us that M and P are distributed in the premises. This means that the premises can't both be particular, because then one would be I and one would be O, and only one term could be distributed. So one premise must be negative and the other affirmative, and one premise must be particular and the other universal. In other words, our premises must be a pair that goes across the diagonal of the square of opposition, either an A and an O or an E and an I.

With the AO pair, there are two possibilities that distribute the right terms: OAO-3 and AOO-II. These are the valid syllogisms Bocardo and Baroco. With the EI pair, there are four possibilities, which are all valid. They are the EIO sisters: Ferio, Festino, Ferison, and Fresison.

So there you have it. Those five rules completely characterize the possible valid Aristotelian syllogisms. Any other patterns you might notice among the valid syllogisms can be derived from these five rules. For instance, Problem (1) in exercise set E of Section 4.2 asked if you could have a valid Aristotelian syllogism with two particular premises. If you did that problem, hopefully you saw that the answer was "no." We could, in fact, make this one of our five rules above. But we don't need to. When we showed that these five rules were sufficient to characterize validity, we also showed that any other rule characterizing validity that we care to come up with can be derived from the rules we already set out. So, let's state the idea that a syllogism cannot have two particular premises as a rule, and show how it can be derived. This will be our statement of the rule:

Derived Rule 1: A valid Aristotelian syllogism cannot have two particular premises.

And let's call the associated fallacy the FALLACY OF PARTICULAR PREMISES. To show that this rule can be derived from the previous five, it is sufficient to show that any syllogism that violates this rule will also violate one of the previous five rules. Thus there will always be a reason, independent of this rule, that can explain why that syllogism is false.

So suppose we have a syllogism with two particular premises. If we want to avoid violating rule 1, we need to distribute the middle term, which means that both premises cannot be mood I, because mood-I statements don't distribute any term. We also know that both statements can't be mood O, because rule 3 says we can't have two negative premises. Therefore our syllogism has one premise that is I and one premise that is O. It thus has exactly one negative premise, and by rule 4, must have a negative conclusion, either an E or an O. But an argument with premises I and O can only have one term distributed: if the conclusion is mood O, then two terms are distributed; and if it is mood E then all three terms are distributed. Thus any syllogism that manages to avoid rules 1, 3, and 4 will fall victim to rule 2. Therefore any syllogism with two particular premises will violate one of the five basic rules.

Practice Exercises

Part A Determine whether the following arguments are valid by seeing if they violate any of the five basic rules. If they are invalid, list the rules they violate. If they are valid, name their form. For conditionally valid arguments, label them valid if the existential premise is given explicitly, and invalid if it is not.

Example 1: All M are P, and all S are M. Therefore no S are P.

Answer: Invalid. It violates rule 2, because P is distributed in the conclusion but not the premises, and rule 4, because it has a negative conclusion and two affirmative premises.

Example 2: No P are M, and all S are M. Therefore some S are not P.

Answer: Invalid. It violates rule 5 because it is missing the existential premise "Some S exist."

(1) Some M are P, and some M are S. Therefore, no S are P.

(2) Some P are M, and some M are not S. Therefore, all S are P.

(3) All P are M, and no M are S. Therefore, no S are P.

(4) Some P are not M, some M are S. Therefore, all S are P.

(5) No M are P, and all S are M. Also, some S exist. Therefore some S are not P.

(6) All P are M, and no S are M. Therefore some S are not P.

(7) Some M are P, and all M are S. Therefore some S are P.

(8) All M are P, and all S are M. Therefore some S are not P.

(9) Some M are not P, and all S are M. Therefore, some S are P.

(10) Some P are M, and some M are not S. Therefore some S are not P.

Part B Determine whether the following arguments are valid by seeing if they violate any of the five basic rules. If they are invalid, list the rules they violate. If they are valid, name their form. For conditionally valid arguments, label them valid if the existential premise is given explicitly, and invalid if it is not.

(1) Some M are not P, and no S are M. Therefore, all S are P.

(2) No M are P, and some S are M. Therefore, some S are not P.

(3) All P are M, and no S are M. Therefore no S are P.

(4) All P are M, and all M are S. Also, some S exist. Therefore some S are P.

(5) All P are M, and no S are M. Therefore some S are not P.

(6) All M are P, and no M are S. Therefore, some S are P.

(7) No P are M, and all M are S. Therefore, some S are not P.

(8) Some M are not P, and some M are S. Therefore, some S are not P.

(9) Some M are P, and all M are S. Therefore, some S are not P.

(10) No P are M, and no M are S. Therefore no S are P.

4.5 Validity and the Counterexample Method

Except for a brief discussion of logically structured English in section 4.1, so far, we have only been evaluating arguments that use variables for the subject, middle, and predicate terms. Now, we will be looking in detail at issues that come up when we try to evaluate categorical arguments that come up in ordinary English. In this section we will consider how your knowledge of the real world terms discussed in an argument can distract you from evaluating the form of the argument itself. In this section we will consider difficulties in putting ordinary language arguments into logically structured English, so they can be analyzed using the techniques we have learned.

Let's go back again to the definition of validity on page 34. (It is always good for beginning student to reinforce their understanding of validity). We did this in the last chapter in Section **??**, and we are doing it again now. Validity is a fundamental concept in logic that can be confusing. A valid argument is not necessarily one with true premises or a true conclusion. An argument is valid if the premises *would* make the conclusion true *if* the premises were true.

This means that, as we have seen before, there can be valid arguments with false conclusions. Consider this argument:

> No reptiles are chihuahuas. But all dogs are reptiles. Therefore, no dogs are chihuahuas.

This seems silly, if only because the conclusion is false. We know that some dogs are chihuahuas. But the argument is still valid. In fact, it shares a form with an argument that makes perfect sense:

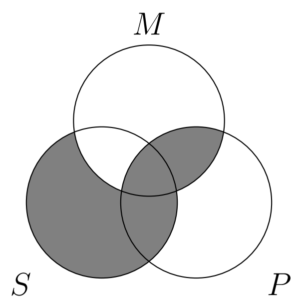
> No reptiles are dogs, but all chameleons are reptiles. Therefore, no dogs are chameleons.

Both of these arguments have the form Celarent:

P_1: No M are P.
P_2: All S are M.

C: No S are P.

This form is valid, whether the subject and predicate term are dogs and chameleons, or dogs and chihuahuas, which you can see from this Venn diagram.



This means you can't assume an argument is invalid because it has a false conclusion. The reverse is also true. You can't assume an argument is valid just because it has a true conclusion. Consider this

> All cats are animals, and some animals are dogs. Therefore no dogs are cats.

Makes sense, right? Everything is true. But the argument isn't valid. The premises aren't making the conclusion true. Other arguments with the same form have true premises and a false conclusion. Like this one.

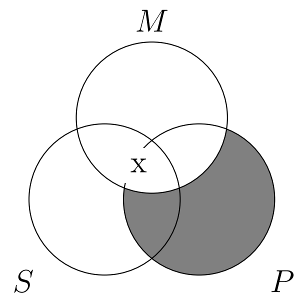
> All chihuahuas are animals, and some animals are dogs. Therefore, no dogs are chihuahuas.

Really, the arguments in both these passages have the same form: AEE-IV:

P_1: All P are M.
P_2: Some M are S.

C: No S are P.

This in an invalid form, and it remains invalid whether the P stands for cats or chihuahuas. You can see this in the Venn diagram:



All these examples bring out an important fact about the kind of logic we are doing in this chapter and the last one: this is *formal* logic. As we discussed on page 4 formal logic is a way of making our investigation content neutral. By using variables for terms instead of noun phrases in English we can show that certain ways of arguing are good or bad regardless of the topic being argued about. Parts This method will be extended in Part III, when we introduce the full formal language SL

The examples above also show us another way of proving that an argument given in English is invalid, called the counterexample method. As we have just seen, if you are given an argument in English with, say, false premises and a false conclusion, you cannot determine immediately whether the argument is valid. However, we can look at arguments that have the same form, and use them to see whether the argument is valid. If we can find an argument that has the exact same form as a given argument but has true premises and a false conclusion, then we know the argument is invalid. We just did that with the AEE-IV argument above. We were given an argument with true premises and a true conclusion involving cats, dogs, and animals. We were able to show this argument invalid by finding an argument with the same form that has true premises and a false conclusion, this time involving chihuahuas, dogs, and animals.

More precisely, we can define the COUNTEREXAMPLE METHOD as a method for determining if an argument with ordinary English words for terms is valid, where one consistently substitutes other English terms for the terms in the given argument to see if one can find an argument with the same form that has true premises and a false conclusion. Let's run through a couple more examples to see how this works.

First consider this argument in English:

> All tablet computers are computers. We know this because a computer is a kind of machine, and some machines are not tablet computers.

Every statement in this argument is true, but it doesn't seem right. The premises don't really relate to the conclusion. That means you can probably find an argument with the same form that has true premises and a false conclusion. Let's start by putting the argument in canonical form. Notice that the English passage had the conclusion first.

P_1: All computers are machines.
P_2: Some machines are not tablet computers.

C: All tablet computers are computers.

Let's find substitutes for "machines," "computers," and "tablet computers" that will give us true premises and a false conclusion. It is easiest to work with really common sense categories, like "dog" and "cat." It is also easiest to start with a false conclusion and then try to find a middle term that will give you true premises. "All dogs are cats" is a nice false conclusion to start with:

P_1: All cats are M.
P_2: Some M are not dogs.

C: All dogs are cats.

So what can we substitute for M (which used to be "machines") that will make P_1 and P_2 true? "Animals" works fine.

P_1: All cats are animals.
P_2: Some animals are not dogs.

C: All dogs are cats.

There you have it: a counterexample that shows the argument invalid. Let's try another one.

> Some diseases are viral, therefore some things caused by bacteria are not things that are caused by viruses, because all diseases are bacterial.

This will take a bit more unpacking. You can see from the indicator words that the conclusion is in the middle. We also have to fix "viral" and "things that are caused by viruses" so they match, and the same is true for "bacterial" and "things that are caused by bacteria." Once we have the sentences in canonical form, the argument will look like this:

P_1: Some diseases are things caused by viruses.
P_2: All diseases are things causes by bacteria.

C: Some things caused by bacteria are not things caused by viruses.

Once you manage to think through the complicated wording here, you can see that P_1 and the conclusion are true. Some diseases come from viruses, and not everything that comes from a bacteria comes from a virus. But P_2 is false. All diseases are not caused by bacteria. In fact, P_1 contradicts P_2. But none of this is enough to show the argument is invalid. To do that, we need to find an argument with the same form that has true premises and a false conclusion.

Let's go back to the simple categories: "dogs," "animals," etc. We need a false conclusion. Let's go with "Some dogs are not animals."

P_1: Some M are dogs.
P_2: All M are animals.

C: Some dogs are not animals.

We need a middle term that will make the premises true. It needs to be a class that is more general than "dogs" but more narrow than "animals." "Mammals" is a good standby here.

P_1: Some mammals are dogs.
P_2: All mammals are animals.

C: Some dogs are not animals.

And again we have it, a counterexample to the given syllogism.

Practice Exercises

Part A Each of the following arguments are invalid. Prove that they are invalid by providing a counterexample that has the same form, but true premises and a false conclusion.

Example: No pants are shirts, and no shirts are dresses, so some pants are not dresses.

Answer: No dogs are reptiles, and no reptiles are mammals, so some dogs are not mammals.

(1) Some mammals are animals, and some dogs are mammals. Therefore, all dogs are animals.

(2) Some evergreens are not spruces, and all spruces are trees. Therefore some trees are not evergreens.

(3) No wild animals are completely safe to be around, and some pigs are not completely safe to be around. Therefore some pigs are wild.

(4) Some sodas are clear, and some healthy drinks are clear. Therefore no sodas are healthy.

(5) No hamburgers are rocks, no igneous rocks are hamburgers. Therefore all igneous rocks are rocks.

(6) Some plants are not food, and some foods are vegetables. Therefore some plants are vegetables.

(7) All apartment buildings are buildings, and some buildings are residential buildings. Therefore some residential buildings are not apartment buildings.

(8) No foods are games, and some games are video games. Therefore no video games are food.

(9) Some trucks are rentals, and no cars are trucks. Therefore some cars are rentals.

(10) Some online things are not fun, and some fun things are not video games. Therefore, some video games are online.

Part B Each of the following arguments are invalid. Prove that they are invalid by providing a counterexample that has the same form, but true premises and a false conclusion.

(1) All cars are machines, and all vehicles are machines. Therefore all cars are vehicles.

(2) Some black animals are panthers. No panthers are sheep. Therefore some sheep are black.

(3) Some paper money is not counterfeit, and some counterfeit monies are quarters. Therefore no quarters are paper money.

(4) Some spacecraft are man-made, and no comets are man-made. Therefore some comets are not spacecraft.

(5) No planets are stars, and no planets are fusion reactions. Therefore all stars are fusion reactions.

(6) All straight lines are lines, and no straight lines are curved lines. Therefore, all curved lines are lines.

(7) Some physical objects are not writing implements, and all pencils are writing implements. Therefore, all pencils are physical objects.

(8) No traumatic memories are pleasant, but some memories are traumatic. Therefore, some memories are pleasant.

(9) Some farms are for-profit enterprises, and all munitions factories are for-profit enterprises. Therefore, no munitions factories are farms.

(10) No parasites are western brook lampreys, and all western brook lampreys are lampreys. Therefore some lampreys are parasitic.

4.6 Ordinary Language Arguments

In the last section we saw that arguments in ordinary language can throw us off because our knowledge of the truth or falsity of the statements in them can cloud our perception of the validity of the argument. Now we are going to look at ways we might need to transform ordinary language arguments in order to put them in standard form (see p. 105) and use our evaluative tools.

Basic Transformations into Logically Structured English

Arguments are composed of statements, so all of the guidelines we discussed for putting ordinary English statements into logically structured English in Section 3.5 apply here. Also, as you saw in the exercises in Section 4.1, we need to be aware that the conclusion of an ordinary language argument can occur anywhere in the passage. Consider this argument

> Every single penguin in the world is a bird. Therefore, at least one bird in the world is not a person, because not one person is a penguin.

The indicator words let us know the conclusion is the sentence in the middle of this passage, "At least one bird in the world is not a person." The quantifier in this sentence is nonstandard. As we saw on page 84 "at least one" should be changed to "some." That would give us "Some birds are not people" for the conclusion.

Similar changes need to be made to the two premises. Thus the argument and translation key look like this:

S**:** Birds
M**:** Penguins
P**:** People

$$
\begin{array}{rl}
\mathrm{P}_1\!: & \text{No } P \text{ are } M. \\
\mathrm{P}_2\!: & \underline{\text{All } M \text{ are } S.} \\
\mathrm{C}\!: & \text{Some } S \text{ are not } P.
\end{array}
$$

Now that we have it in this form, we can see that it is the argument Fesapo (EAO-4), which is valid if you add the premise that some penguins exist.

Another issue that can come up in ordinary language arguments is the use of synonyms. Consider another argument about penguins:

> No Adélie penguins can fly, because all members of *Pygoscelis adeliae* are members of the family Sphenisciformes, and no Sphenisciformes can fly.

You might think this argument can't be a proper syllogism, because it has four terms in it: Adélie penguins, *Pygoscelis adeliae*, Sphenisciformes, and things that can fly. However, a quick trip to Wikipedia can confirm that *Pygoscelis adeliae* is just the scientific name from the Adélie penguin, and for that matter "Sphenisciformes" are just penguins. So really, the argument just has three terms, and there is no problem representing the argument like this:

S**:** Adélie penguins
M**:** Penguins
P**:** Things that can fly

$$
\begin{array}{rl}
\mathrm{P}_1\!: & \text{No } M \text{ are } P. \\
\mathrm{P}_2\!: & \underline{\text{All } S \text{ are } M.} \\
\mathrm{C}\!: & \text{No } S \text{ are } P.
\end{array}
$$

And this is our good friend Celarent (EAE-1).

Generally speaking, arguments involving different names for the same person can work the same way as synonyms. Consider this argument:

> Bertrand Russell was a philosopher. We know this because Mr. Russell was a logician, and all logicians are philosophers.

"Bertrand Russell" and "Mr. Russell" are names for the same person, so we can represent them using the same variable. Back on page 84 we learned that names for individuals need to be transformed into classes using phrases like "objects identical with ..." or "people identical with ...." Thus the argument turns out to be Celarent's best friend, Barbara (AAA-I):

S: People identical with Bertrand Russell
M: Logicians
P: Philosophers

$$\begin{array}{rl} \text{P}_1\text{:} & \text{All } M \text{ are } P. \\ \text{P}_2\text{:} & \underline{\text{All } S \text{ are } M.} \\ \text{C:} & \text{All } S \text{ are } P. \end{array}$$

Merging proper nouns into one variable doesn't always work the way merging synonyms does, however. Sometimes exchanging one name with another can change the truth value of a sentence, even when the two names refer to the same person. Any comic book fan will tell you that the sentence "J. Jonah Jameson hates Spider-Man" is true. Also, "Peter Parker" and "Spider-Man" refer to the same person. However "J. Jonah Jameson hates Peter Parker" is false. These situations are called "intentional contexts." We won't be dealing with them in this textbook, but you should be aware that they exist.

Another thing to be aware of when you assign variables to terms is that often you will need to find the right general category that will allow your terms to hook up with each other. Consider this argument.

> Bertrand Russell did not believe in God. I know this because Mr. Russell was not a Christian, and all Christians believe in God

Here again we need to use one variable for both "Bertrand Russell" and "Mr. Russell." But we also have to change the verb phrase "believes in God" into a predicate term (see page 82). The main trick here is to transform the verb phrase "believes in God" and the singular terms referring to Bertrand Russell in a way that will match. The trick is to use the word "people" each time. "Believes in God" becomes "people who believe in God," and the two names for Bertrand Russell need to become "people identical with Bertrand Russell." By including the word "people" in each case, we ensure that the terms will be comparable.

S: People identical to Bertrand Russell
M: Christians
P: People who believe in God

P_1: All M are P.
P_2: No S are M.
C: No S are P.

The argument here is AEE-1, which is invalid. It violates rule 2, because P is distributed in the conclusion, but not the premises. More informally, we can say that just because Russell wasn't a Christian doesn't mean he was an atheist or an agnostic. (In fact, Russell said that he was an atheist in some ways, and an agnostic in others, but this argument doesn't show that.)

Complement Classes, Obversion, and Contraposition

In the last subsection, we saw that we can represent synonyms using the same variable, thus reducing the number of terms in an argument down to the three terms required for an Aristotelian syllogism. Something similar can be done when you have terms that are complements of other terms. Back on page 92 we said that the complement of a class was the class of things that are not in the original class. So the complement of the class "penguins" is everything that is not a penguin. Sometimes, if you have an argument with more than three terms, you can reduce the number of terms down to three, because some of the terms are complements of each other, like "penguins" and "non-penguins."

Consider the following argument, which talks about the stoics, an ancient philosophical school and pioneers in logical thinking.

> No stoics are non-logicians, and all non-philosophers are non-stoics. Therefore all logicians are philosophers.

This argument has six terms: stoics, logicians, philosophers, non-stoics, non-logicians, and non-philosophers. Since the second three are all complements of the first, we should be able to reduce the number of terms to three. Once we do this, we might find that the argument is valid—it looks like it might be Barbara. We can begin by assigning S, M, and P to the terms "stoics," "logicians," and "philosophers." We can then represent the other six terms as non-S, non-M, and non-P.

S: Logicians
M: Stoics
P: Philosophers

P$_1$: No M are non-P.
P$_2$: All non-S are non-M.
C: All S are P.

The argument still has six terms, so we can't evaluate it as is. However, we can get rid of three of these terms. The secret is to apply two of the transformation tools we learned about in Section 3.6, obversion and contraposition. In that section, we saw that there were different ways you could alter categorical statements that would sometimes yield a logically equivalent statement. Two of those methods—obversion and contraposition—involved taking the complements of existing terms. In the cases where these transformations yield logically equivalent terms, we can use them to change the premises of an argument into statements that lack the terms we want to get rid of.

In obversion, you change the quality of a sentence—switch it from affirmative to negative or vice versa—and then replace the predicate term with its complement. This process always yields a logically equivalent statement, so we can always use it. Applying obversion to the first premise of the argument above allows us to eliminate the term non-P.

P$_1$: All M are P.
P$_2$: All non-S are non-M.
C: All S are P.

We can also use contraposition to eliminate terms, but here we must be more careful. Contraposition only yields logically equivalent statements when it is applied to mood-A or mood-O statements. Fortunately, Premise 2 is a mood-A statement, so we can change the argument to look like this:

P$_1$: All M are P.
P$_2$: All M are S.
C: All S are P.

And now the argument only has three terms, and we can evaluate it. As you can see it is not Barbara at all, but one of her unnamed evil step-sisters, AAA-3. Again, remember not to use contraposition on mood-E or mood-I statements. You can use obversion on any statement, but contraposition only works on mood A or mood O.

Ordinary English has other prefixes besides “non-” to denote the complements of classes. “Un-” is the most common prefix, but English also uses “in-,” “dis-,” and “a-.” These can be handled the same way “non-” is. Consider,

> All dogs in the park must be leashed. Some unleashed pets in the park are cats. Therefore, some cats in the park are not dogs.

To make this argument work, we need to get the predicate of the major premise to match the subject of the minor premise. The first step is just turning the verb phrase “must be leashed” into a noun phrase that matches “unleashed pets.” We also should use the qualification “in the park” consistently across the terms.

S: Cats in the park
M: Leashed pets in the park
P: Dogs in the park

P_1: All P are M.
P_2: Some non-M are S.

C: Some S are not P.

Now the trick is to get the terms M and non-M to match. We could try to transform the second premise, but that won't do us any good. It is an E statement, so we can't use contraposition, and obversion won't change the subject term.

Instead of changing the “non-M” to “M” in the second premise, we need to change the “M” to “non-M” in the first premise. We can do this using obversion, which works on statements in any mood.

P_1: No P are non-M.

P_2: Some non-M are S.

C: Some S are not P.

And now we can see that the argument is Fresison (EIO-4), which is valid.

Practice Exercises

Part A The following arguments are given with variables for the terms. Use obversion and contraposition to reduce the number of terms to three. State which operations you are using on which statements, and give the resulting syllogism in canonical form. Finally, use Venn diagrams to evaluate the argument.

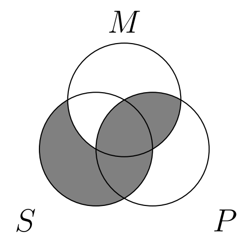
Example: No M are P, and all non-M are non-S. Therefore all S are non-P.

Answer: Use contraposition on the minor premise and obversion on the conclusion to get the following

P$_1$: No M are P.
P$_2$: All S are M.

C: No S are P.

Valid, Celartent (EAE-1)



(1) No P are M, and all non-S are non-M. Therefore some S are P.

(2) No P are M, and all S are M. Therefore all S are non-P.

(3) No M are P, and some S are M. Therefore some non-P are not non-S.

(4) All non-P are non-M, and some S are M. Therefore all S are P.

(5) All non-M are non-P, and no S are M. Therefore no S are P.

(6) All P are M, and all S are non-M. Also, some S exist. Therefore, some S are not P.

(7) Some P are not non-M. All non-M are non-S. Some S exist. Therefore some S are P.

(8) All non-M are non-P, and all S are non-M. Therefore no S are P.

(9) All P are M, and all M are non-S. Therefore all P are non-S

(10) All M are P, and some non-M are not S. Therefore some S are not non-P.

Part B The following arguments are given with variables for the terms. Use obversion and contraposition to reduce the number of terms to three. State which operations you are using on which statements, and give the resulting syllogism in canonical form. Finally, use Venn diagrams to evaluate the argument.

(1) All non-M are non-P, and no S are M. Therefore no S are P.

(2) All non-P are non-M, and all non-M are non-S. Therefore all S are P.

(3) All non-M are non-P, and no M are S. Also, some S exist. Therefore some S are not P.

(4) Some M are P and no M are non-S. Therefore some S are P.

(5) No P are M, and all M are non-S. Therefore no S are P.

(6) No P are M, and all S are M. Also some S exist. Therefore some non-P are not non-S.

(7) All P are M, and no S are M. Therefore all S are non-P.

(8) All M are P, and all M are S. Also, some M exist. Therefore some S are not non-P.

(9) All non-P are non-M, and some M are not S. Also some M exist. Therefore, some non-P are S.

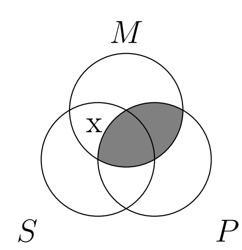
(10) All non-M are non-P, and all M are non-S. Therefore all S are non-M.

Part C For each inference make a translation key and put the argument in standard form. If you use obversion or contraposition to reduce the number of the terms, make a note of it. Then construct a Venn diagram for it, and determine whether the inference is valid.

Example: No one who studies logic is completely stupid, and some philosophers are not non-logicians. Therefore some philosophers are not completely stupid.

Answer: S: Philosophers
M: Logicians
P: People who are completely stupid
Obversion on the minor premise

P$_1$: No M are P.
P$_2$: Some S are M.
C: Some S are not P.



Ferio (EIO-1)
Unconditionally valid

(1) All of Lorain County is outside of Cuyahoga County, but at least some of

Cleveland is in Cuyahoga County. Therefore some of Lorain County is not in Cleveland.

(2) All Civics are vehicles. After all, any automobile is a vehicle, and a Civic is a kind of car.

(3) Cows are animals. After all, anything that is not an animal is also not a farm animal, and cows are farm animals.

(4) Some digits are fingers, and if something is a digit, then it is a body part. Therefore some body parts are non-fingers.

(5) Earth is a planet. Therefore Earth is not a moon, because no planets are moons.

(6) All trees are non-animals. Some trees are deciduous. Therefore some non-animals are not evergreen.

(7) Some relatives are not blood relatives, and some in-laws are mothers-in-law. Therefore, some non-relatives are not non-mothers-in-law.

(8) Ludwig Wittgenstein was not English. Therefore Ludwig Wittgenstein was a philosopher, because some English people are philosophers.

(9) Some liquids are non-alcoholic. This is because only liquids are drinks, and some non-alcoholic things are not non-drinks.

(10) No advertisements are misleading. Therefore all pandering things are non-misleading, because no advertisements are pandering.

Part D For each inference make a translation key and put the argument in standard form. Then construct a Venn diagram for it and determine whether the inference is valid.

(1) Some machines are likely to break, because some machines are elaborate, and nothing that is elaborate is unlikely to break.

(2) Only mammals are dogs, and no mammals are reptiles. Also, *Canis familiaris* really do exist. Therefore some dogs are not reptiles.

(3) All of Ohio is in the United States. Therefore none of Ohio is in Canada, because no part of Canada is in the United States.

(4) All snerps are snine. This is because all non-snine things are also non-sneeps, and all sneeps are snine.

(5) No non-forks are pointy. Therefore, all pointy things are forks, because no pointy things are non-forks.

(6) Some arguments are not invalid. After all, anything that is persuasive is valid, and anything that is persuasive is also an argument. Furthermore, we know that some arguments exist.

(7) Some things that sing don't sink in water. You can tell because some bricks do not sing, and all bricks float.

(8) Crayons are not precision tools. Therefore all toys are non-precision tools, because some toys are crayons.

(9) Some things that aren't beliefs are not objectionable. This is because all things that are not well founded are non-beliefs, and all things that are well founded are unobjectionable.

(10) Some Kaiju are not characters from Toho studios. Therefore Godzilla is not a Kaiju, because Godzilla is a character from Toho studios.

4.7 Enthymemes

In the real world, arguments are often presented with pieces missing. The missing piece could be an important premise or sometimes even the conclusion. Arguments with parts missing are called ENTHYMEMES. We discuss them extensively in the chapter on incomplete arguments in the full version of this text. Here we will be dealing with them specifically as they are used with Aristotelian syllogisms. Because we are now dealing with more real-world arguments, we will return to our practice of giving the context in italics before passages (see page 7). Remember, that this context is not a part of the argument. It is just there to help interpret the passage.

The simplest and most common reason one might leave out a part of an Aristotelian syllogism is brevity. Sometimes part of an argument is common knowledge or completely obvious from the context and simply doesn't need to be said. Consider this common inference that might pop into your head one afternoon:

> *Susan is working from home* Well, the dog is barking. I guess that means the mail is here.

Here the missing premise is that dogs bark at letter carriers. But this isn't something Susan is going to think of consciously. It is completely common knowledge, and probably something Susan has a lot of first-hand experience with as a dog owner. So the passage above basically represents Susan's inference as it occurs to her.

If in order to evaluate the argument, though, we need to make all of the reasoning explicit. This means putting the whole thing in standard form and then finding the missing premise. Once you do that, it becomes clear that this common, everyday inference is actually very complicated. The simplest way to put the argument in canonical form would simply be this:

P: The dog is barking.

C: The mail is here.

But if we are going to use the tools of Aristotelian logic to show that this is valid, we are going to need to convert these statements into categorical statements. The "now" functions here as a singular term, so we needed to use the awkward "times identical with" construction (see page 84). Also, we needed to find a general category to put things under, in this case "times," to be sure that all the terms matched. (This is similar to the "persons identical to Bertrand Russell example on page 151.)

P: All times identical with now are times when the dog is barking.

C: All times identical with now are times when the mail arrives.

Once we do this, we can see that the conclusion is a statement in mood A. The minor term is "times identical with now" and the major term is "times when the mail arrives." The premise is a minor premise, because it has the minor term in it, and the middle term is "times when the dog is barking." From this we can see that the missing premise is the major premise. When we add it, the full argument looks like this:

P_1: All times when the dog is barking are times when the mail arrives*
P_2: All times identical with now are times when the dog is barking.

C: All times identical with now are times when the mail arrives.

Remember that the asterisks means that an implicit premise has been made explicit. Once we have that in place, however, we can see that this argument is just a version of Barbara (AAA-1). The full argument is quite wordy, which shows how much sophisticated reasoning is going on in your brain when you make even the most everyday of inferences.

Enthymemes can also be used to hide premises that are needed for the argument, but that the audience might not accept if their attention were drawn to them. Consider this argument:

> *A used car salesperson is showing you a Porsche Cayenne Hybrid SUV*
> And it is a hybrid electric car, so you know it gets good gas mileage.

Here again we have an argument about a singular object—this car the salesperson is showing you. So when we represent the argument, we need to use the awkward "things identical with" construction.

P: All things identical with this vehicle are hybrid electric vehicles.

C: All things identical with this vehicle are vehicles that get good gas mileage.

Again, we have two mood-A statements. The minor term is "things identical with this vehicle," the middle term is "hybrid electric vehicles" and the major term is "vehicles that get good gas mileage." The missing premise must be the major premise "All hybrid electric vehicles get good gas mileage."

P_1: All hybrid electric vehicles are vehicles that get good gas mileage.*
P_2: All things identical with this vehicle are hybrid electric vehicles

C: All things identical with this vehicle are vehicles that get good gas mileage.

But wait, is this implicit premise even true? Actually it isn't—the Cayenne itself only gets 20 miles per gallon in the city and 24 on the highway—no wonder the salesperson kept this premise implicit! Compare the Cayenne's mileage to that of a regular gas-powered car like the Honda Fit, which gets 33 city and 41 highway. It is true that hybrids generally get better mileage than similarly sized cars that run on gasoline only, but that isn't the reasoning the salesperson is presenting here. Someone who was really all that concerned about mileage probably should not be in the market for a luxury SUV like the Cayenne.

When filling in missing parts of an enthymeme it is important to remember the principle of charity. The principle of charity says that we should try to interpret other people in a way that makes reasoning coherent and their beliefs reasonable. With the example of the salesperson above, one might argue that we were not being as charitable as we could have been. Perhaps the salesperson didn't expect us to believe that *all* hybrids get good gas mileage. Perhaps they only expected us to believe that *most* hybrids get good gas mileage.

If you interpret the argument this way, you are getting a more believable premise in exchange for a weaker inference. The original argument had the strongest possible inference—it was genuinely valid. If you change the major premise to something involving "most," you will have an argument that is at best strong, not valid. The premise is now believable, but the inference is fallible. Also note that if you do decide to use "most" when filling in the missing premise, the argument becomes inductive, and thus is something that you would evaluate using the tools from the parts of the complete version of this textbook on inductive and scientific reasoning, not the tools of Aristotelian logic. In this section we will only be looking at enthymemes where you fill in premises in a way that yields Aristotelian syllogisms.

Sometimes enthymemes are missing a conclusion, rather than a premise. Consider this example:

> *Annabeth, age 7, is pestering her mother Susan with strange questions.*
>
> **Annabeth**: Mommy, do any dogs have forked tongues?
>
> **Susan**: Well, dear, all dogs are mammals, and no mammals have forked tongues. So do you think any dogs have forked tongues?

Teachers and parents often replace the conclusion with a rhetorical question like this because it forces the audience to fill in the missing conclusion for themselves. The effect is going to be different for different audiences, however. A child might be pleased with herself for figuring out the answer with just a few hints. Often replacing the conclusion with a rhetorical question when talking to an adult can be demeaning, for instance, when someone glibly says "you do the math," even when there is no math involved in the issue.

In any case, the proper conclusion for the argument above is "No dogs have forked tongues." The argument is Celarent (EAE-1), and in canonical form it looks like this:

P_1: No mammals have forked tongues.
P_2: All dogs are mammals.

C: No dogs have forked tongues.

Evaluating an enthymeme requires four steps. First, you need to figure out if the missing statement is a premise or a conclusion. As you might expect, indicator phrases will be helpful here. Words like "because" or "therefore" will mark one of the sentences you are given as a premise or a conclusion, and you can use this information to determine what hasn't been given. In fact, if the enthymeme contains any indicator words at all, you can be pretty confident that the missing statement is a premise. If the enthymeme contains a conclusion indicator word like "therefore," you know that the statement after it is the conclusion. But even if the indicator word is a premise indicator word, you the missing statement is still likely to be a premise, because the premise indicator words are generally preceded by conclusions. Consider this following argument.

> *Susan is now wondering about what reptilian characteristics some mammals do have* Well, I know some mammals have scales, because armadillos have scales.

Here the "because" lets us know that "Armadillos have scales" is a premise, and "some mammals have scales" is the conclusion. Thus the missing statement is one of the premises.

Once you have figured out what part of the syllogism is missing, the second step is to write the parts of the syllogism you do have in standard form. For this example, that is pretty easy, but you do need to provide the missing quantifier for the premise.

P: All armadillos have scales.

C: Some mammals have scales.

Step three is to figure out what terms are in the missing statements. The two statements you have been given will contain a total of three terms, one of which will be repeated once in each statement. Since every term in an Aristotelian syllogism appears twice, we know that the two terms that only appear once must be the ones that appear in the missing statement. In the example above, the terms are "mammals," "armadillos," and "things with scales." The term "things with scales" is the one that appears twice. So the missing statement will use the terms "armadillos" and "mammals."

Step four is to determine the mood of the missing statement and the overall figure of the syllogism. This will let you fill in everything else you need to know to complete the syllogism. The rules for a valid syllogism listed in Section 4.4 can be a big help here.

In the example we have been working with we know that the conclusion is "Some mammals have scales." This means that the major term is "things with scales" and the minor term is "mammals." The premise that we are given, "All armadillos have scales" must then be the major premise. So we know this much:

P_1: All armadillos have scales.
P_2: [something with armadillos and mammals]

C: Some mammals have scales.

We also know that the middle term is in the subject position of the major premise, which means that the whole syllogism is either figure 1 or figure 3. Because the major premise and the conclusion are both affirmative, we know by Rule 4 (page 137) that the minor premise must be affirmative. That leaves us only mood-A and mood-I statements.

So there are four possibilities remaining: the argument is either AAI-1, AII-1, AAI-3 or AII-3. A quick glance at the Table 4.11 tells us that all of these are valid. (Although AAI-1 and AAI-3 are conditionally valid.) So any of these are correct answers. Let's go with AII-1 (Darii):

P_1: All armadillos have scales
P_2: Some mammals are armadillos.

C: Some mammals have scales.

Sometimes, when people give enthymemes, there is no way to fill in the missing premise that will make the argument valid. This can happen either because the person making the argument is confused or because they are being deceptive. Let's look at one example:

> *Now Annabeth wants a pet armadillo* If an animal makes a good pet, then it must be cute. So armadillos must make good pets.

There is no way to fill in Annabeth's reasoning here that can turn this into a valid syllogism. To see this, we need to first put it in standard form, which means converting the conditional "if...then" statement into a mood-A statement.

P: All animals that make good pets are animals that are cute.

C: All armadillos are animals that make good pets.

It seems like the missing premise here should be "All armadillos are cute," but adding that premise doesn't give us a valid argument.

P_1: All animals that make good pets are animals that are cute.
P_2: All armadillos are animals that are cute.

C: All armadillos are animals that make good pets.

The argument is AAA-2, which has an undistributed middle. In fact, there is no way to get from P_1 to the conclusion with an Aristotelian categorical statement. The major term has the premise in the subject position, which means that the argument can only be figure 2 or 4. But a quick look at Table 4.11 lets us know that the only valid argument with a mood-A major premise and a mood-A conclusion is Barbara, which is figure 1. Nice try, kid.

In other cases, the rules given in Section 4.4 can be used to determine whether an enthymeme with a missing conclusion can be valid. In the argument showing that some mammals have scales (p. 163), we used Rule 4 to show that the missing premise had to be affirmative. In an enthymeme with a missing conclusion, you might note that there are two negative premises, so that there actually is no valid conclusion to draw. In other situations you might be able to determine that the missing conclusion must be negative. The rules for valid syllogisms are your friends when working with enthymemes, because they allow you to consider broad categories of syllogisms, rather than having to work on a trial and error basis with the Venn diagram method.

Practice Exercises

Part A Write each enthymeme below in standard form and supply the premise or conclusion that makes the argument valid, marking it with an asterisk. If no statement can make the argument valid, write "invalid."

Example: Edinburgh is in Scotland, and no part of Scotland is sunny.

Answer:
P_1: All places in Edinburgh are places in Scotland.
P_2: No places in Scotland are places that are sunny.
C: No places in Edinburgh are sunny.*

(1) Dogs are mammals, which means that they aren't reptiles.

(2) Some pastas must be whole wheat, because some linguine is whole wheat.

(3) If you have ten dollars, you can see the movie. So therefore none of the kids will see the movie.

(4) Nothing divine is evil, so no gods are evil.

(5) No logicians are ignorant of Aristotle, and some people who are ignorant of Aristotle are foolish.

(6) No holidays are work days, and some work days are not Mondays.

(7) Some trees are not evergreens. Therefore some evergreens are not spruces.

(8) No jellyfish are vertebrates, but some animals that make good pets are vertebrates.

(9) Some chairs are not houses, and all tables are houses.

(10) No doodlebugs are octofish. Therefore no thing-havers are octofish.

Part B Write each enthymeme below in standard form and supply the premise or conclusion that makes the argument valid, marking it with an asterisk. If no statement can make the argument valid, write "invalid."

(1) No board games are online games, because all online games are video games.

(2) No coins are paper money. Therefore no coins are two dollar bills.

(3) Some vegetables are peppers. Therefore some foods are peppers.

(4) Everyone who fights for justice is a saint. Therefore no politicians are saints.

(5) Some children are not getting treats because no one who was naughty gets treats.

(6) On days when there is more than a foot of snow, school is canceled. Therefore some Mondays this winter school will be canceled.

(7) All churches are religious institutions. Therefore, some churches are not Christian.

(8) All rocks are food, and some vegetables are rocks.

(9) Some houses are not offices, and some residences are not offices.

(10) No snirt are hirt. Therefore some blorp are not snirt.

4.8 Sorites Categorical Arguments

The Aristotelian tradition mostly focused on syllogisms with two premises; however, there are fun things we can do with longer arguments. These were explored, among other places, in a 19th century textbook called *Symbolic Logic* (Dodgson, 1896), by the mathematician and logician Charles Lutwidge Dodgson. (Dodgson is actually better known by his pen name, Lewis Carroll, under which he wrote the children's classics *Alice's Adventures in Wonderland* and *Through the Looking Glass*.)

Categorical arguments with more than two premises are called SORITES CATEGORICAL ARGUMENTS, or just "Sorites" (pronounced "sore-EYE-tease") for short. The name comes from the Greek word "Soros" meaning "heap." This kind of sorites should not be confused with the sorites paradoxes we talk about in the chapter on real world evaluation in the complete version of this text, which were arguments that exploited vague premises.

Here is a simple example of a categorical sorites.

P_1: All carrots are vegetables.
P_2: No vegetables are houses.
P_3: All houses are buildings.

C: No carrots are buildings.

We have more premises and terms than we do in the classic Aristotelian syllogism, but they are arranged in a way that is a natural extension of the Aristotelian syllogism. Every term appears twice, except for the major and minor terms of the conclusion. Each premise shares one term with each of the other premises. Really, this is just like a regular Aristotelian syllogism with more middle terms.

Because the form of the argument above is an extension of the standard form of an Aristotelian syllogism, you can actually break it apart into two valid Aristotelian syllogisms if you supply the missing statements.

P$_1$: All carrots are vegetables.
P$_2$: No vegetables are houses.
C: No carrots are houses.*

P$_1$: No carrots are houses.*
P$_2$: All houses are buildings.
C: No carrots are buildings.

The argument on the left takes the first two premises from the sorites argument and derives a conclusion from them that was not stated in the original argument. We then slide this conclusion over and use it as the major premise for the next argument, where it gets combined with the final premise and the conclusion of the sorites argument. The result is two valid arguments, Calemes and Celarent, which have been chained together. The conclusion of the Calemes argument is an intermediate conclusion, and the conclusion of the Celarent is the final conclusion. You can think of the sorites argument as a sort of abbreviation for this chain of valid arguments.

The task for this section is to evaluate the validity of sorites arguments. To do this, we will need to put them in standard form, just as we did for ordinary Aristotelian arguments. We will define STANDARD FORM FOR A SORITES CATEGORICAL ARGUMENT as a sorites argument that has been put into logically structured English with the following criteria: (1) each statement in the argument is in standard form for a categorical statement in logically structured English, (2) each instance of a term is in the same format and is used in the same sense, (3) the major premise is first in the list of premises and the minor premise is last, and (4) the middle premises are arranged so that premises that share a term are adjacent to one another.

Some passages will take more work to get into standard form for sorites than others. We will just look at situations where you need to rearrange the order of the premises and fix the terms so that they match. We will also have an example using variables for terms. We will use the letters A, B, C, ... for the terms in sorites categorical arguments, rather than S, M, and P.

P_1: All D are E.
P_2: Some C are D.
P_3: All C are B.
P_4: All A are non-B.

C: Some E are not A.

In this argument, P_3 and P_4 don't match up, because one talks about B and the other non-B. We can just use obversion on P_4:

P_1: All D are E.
P_2: Some C are D.
P_3: All C are B.
P_4: No A are B.

C: Some E are not A.

Now we need to put the premises in the proper order. The predicate for the conclusion is A, so the statement "No A are B" needs to be the first premise. The statement containing E needs to be the last premise, and the middle two premises need to be swapped so each premise will share a term with the statements on either side of it.

P_1: No A are B.
P_2: All C are B.
P_3: Some C are D.
P_4: All D are E.

C: Some E are not A.

In this example, the letters wind up in ordinary alphabetical order. Not every example will work this way.

One we have the arguments in standard form, our job is to evaluate them. We will look at three methods for doing this. The first will require you to break down the sorites into its component syllogisms and evaluate them separately. The second two will let you evaluate the argument all at once.

Checking Each Inference with Venn Diagrams

The most thorough way to check a categorical sorites argument is to break it down into its component Aristotelian syllogisms and check each of those separately. Look at the argument on the previous page. We need to identify the implicit intermediate conclusions and write out each syllogism separately. The first two premises of this argument are "No A are B," and "All C are B." That's a mood-E statement and a mood-A statement, with both middle terms on the right. A glance at Table 4.11 lets us know that we can conclude a mood-E statement from this, "No C are A." So the first component argument is Cesare (EAE-II).

P_1: No A are B.
P_2: All C are B.

C: No C are A.*

The conclusion of this argument, "No C are A," is an implicit intermediate step, which means it is the major premise for the next component syllogism. The minor premise is P_3, "Some C are D." This gives us a mood-E and a mood-O statement. The new middle term is C. Again we can consult Table 4.11 to see that this sets up Ferison (EIO-3):

P_1: No C are A.*
P_2: Some C are D.

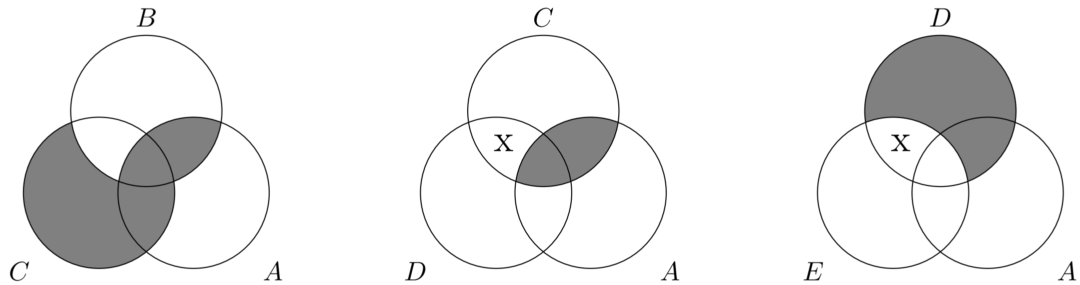
C: Some D are not A.*

The conclusion here becomes the major premise of the next component argument. The minor premise is P_4 of the original sorites. This means that the last step is Bocardo (OAO-3).

P_1: Some D are not A.*
P_2: All D are E.

C: Some E are not A.

To wrap things up, we can use three Venn diagrams to confirm that each step is valid. This step is needed to be sure that the terms appear in the correct position in each diagram. The three Venn diagrams are shown together with the



P$_1$: No A are B.
P$_2$: All C are B.

C: No C are A.

Cesare (EAE-II)

P$_1$: No C are A.
P$_2$: Some C are D.

C: Some D are not A.

Fresison (EIO-IV)

P$_1$: Some D are not A.
P$_2$: All D are E.

C: Some E are not A.

Bocardo (OAO-3)

Figure 4.4: Example of a valid sorites broken down into its component arguments

corresponding arguments in Figure 4.4. Notice that the major term is always represented by the lower right circle: it is the predicate of the conclusion of each component argument. The subject of the conclusion for each component argument then becomes the middle term for the next argument.

This example turned out to be valid, but not every sorites you will be asked to evaluate will work that way. Consider this example, given in standard form.

P$_1$: All B are A.

P$_2$: All C are B.

P$_3$: Some C are not D.

C: No D are A.

If you look at the first two premises, you can easily see that they entail "All C are A." The first component argument is a simple Barbara. But if you plug that conclusion into the next component argument, the result is invalid.

P_1: All B are A. P_2: All C are B. ――――― C: All C are A.*	P_1: All C are A.* P_2: Some C are not D. ――――― C: No D are A.
Barbara (AAA-I)	**AOE-III (Invalid)**

You will also encounter situations where two early premises in a sorites don't lead to any valid conclusion, so there is simply no way to fill in the missing steps. Consider this:

P_1: No D are B.
P_2: No D are C.
P_3: All A are C.
―――――
C: All A are B.

Here the first two premises are negative, so we know by Rule 3 that there is no conclusion we can validly draw here. Thus there is nothing to plug in as the major premise for the second component argument.

P_1: No D are B. P_2: No D are C. ――――― C: ???	P_1: ??? P_2: All A are C. ――――― C: All A are B.
Incomplete	**Incomplete**

Actually, once you note that the first two premises violate Rule 3 for Aristotelian syllogisms, there really isn't a need to break down the argument further. This brings us to the second method for evaluating sorites categorical arguments.

Checking the Whole Sorites Using Rules

Rather than filling in all the missing intermediate conclusions, we can see directly whether a sorites is valid by applying the rules we went over in Section 4.4. To spare you from flipping back and forth too much in the text, let's repeat the five basic rules here, modified so that they work with the longer arguments we are discussing.

Rule 1:	Each middle term in a valid categorical sorites argument must be distributed at least once.
Rule 2:	If a term is distributed in the conclusion of a valid categorical sorites argument, then it must also be distributed in one of the premises.
Rule 3:	A valid Aristotelian syllogism cannot have two or more negative premises.
Rule 4:	A valid Aristotelian syllogism can have a negative conclusion if and only if it has exactly one negative premise.
Rule 5:	A valid Aristotelian syllogism with only universal premises cannot have a particular conclusion.

Here "conclusion" means the final conclusion of the whole sorites, not any of the implicit intermediate conclusions. The middle terms are the ones that appear in two premises. Also, as was the case with Aristotelian syllogisms, if Rule 5 is broken, the argument can be fixed by adding an appropriate existential premise. We will look at how to do that shortly.

To evaluate an sorites argument using the rules, it helps not only to have the argument in standard form, but also to mark each term as distributed or undistributed. Remember that a term is distributed in a statement if the statement is making a claim about that whole class. (See our original discussion on page 59.) Table 4.20 is a reminder of which kinds of sentence distribute which terms. Basically, universal statements distribute the subject, and negative statements distribute the predicate. In the example below, distributed terms are marked with a superscript D.

P_1: No B^{D} are A^{D}.
P_2: All C^{D} are B.
P_3: Some C are D.
P_4: All E^{D} are D.

C: All E^{D} are A.

Mood	Form	Terms Distributed
A	All S are P	S
E	No S are P	S and P
I	Some S are P	None
O	Some S are not P	P

Table 4.20: Moods and distribution

To check the above argument using the rules we just run through the rules in order, and see if they are all followed. Rule 1 says that every middle term must be distributed in at least one premise. The middle terms here are B, C, and D. We can quickly check to see that B is distributed in premise 1, C is distributed in premise 2, but D is never distributed. You can check this by trying to break the argument down into component syllogisms, as we did in the first section.

Sometimes the argument you are given will satisfy the two distribution rules but fail to satisfy other rules. Consider this example.

P_1: No C^{D} are B^{D}.
P_2: Some A are B.
P_3: No A^{D} are D^{D}.
C: Some C are D.

Here the two middle terms are A and B, which are distributed in the third and first premises, respectively, so the first rule is satisfied. No terms are distributed in the conclusion, so neither need to be distributed in the premises, and the second rule is satisfied. This argument fails Rule 3, however, because it contains two negative premises. Again, you can confirm the results of this method using the results of the previous method.

The final case we need to look at is an argument that fails Rule 5, and thus needs an existential premise added.

P_1: No A^{D} are D^{D}.
P_2: All C^{D} are A.
P_3: All C^{D} are B.
C: Some B are not D^{D}.

Rules 1–4 are satisfied: the middle terms are A and C, which are both distributed; D is distributed in the conclusion and in P_1; there is exactly one negative premise and the conclusion is also negative. However, Rule 5 is not satisfied, because all the premises are universal, but the conclusion is particular. This means that for the argument to be valid, we will need to add an existential premise.

But what existential premise should we add? Remember that the term that must exist in order for a conditionally valid argument to be actually valid is called the critical term (see page 128. When looking at conditionally valid arguments in Section 4.3, we basically just found the critical term by trial and error. For instance, when we used Venn diagrams to show that Felapton was conditionally valid (p. 130), we saw that adding either the premise "Some S exist" or the premise "Some P exist" would not help, but adding "Some M exist" would help. On the other hand, when considering the argument EII-3 (p. 131), we saw that there were no existential premises that we could add that would help. However, as the number of terms we have to deal with increases, the trial and error method becomes less appealing. On top of that, our previous trial and error method was guided by the Venn diagrams for the syllogisms, and we haven't even looked yet at how to do Venn diagrams for sorites arguments.

The trick to finding the critical term in sorites arguments is called the superfluous distribution rule. The SUPERFLUOUS DISTRIBUTION RULE says that in a conditionally valid argument, the critical term will be the one that is distributed more times in the premises than is necessary to satisfy Rules 1 and 2. In the argument above, the middle term C is distributed in two premises, but it only needs to be distributed once to satisfy Rule 1. So we can make the argument above valid by adding the premise "Some C exist":

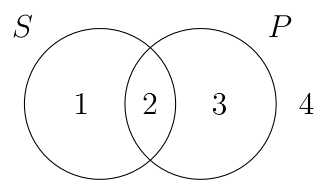
P_1: No A^{D} are D^{D}.
P_2: All C^{D} are A.
P_3: All C^{D} are B.
P_4: Some C exist.
C: Some B are not D^{D}.

You can confirm that this is a valid argument now using the method of filling in the missing intermediate conclusions described in the previous subsection. You will find that this argument actually consists of a Celarent (EAE-1) and a Felapton (EAO-3). The latter is conditionally valid and has the critical term as its middle term.

A Venn Diagram with More Than Three Terms

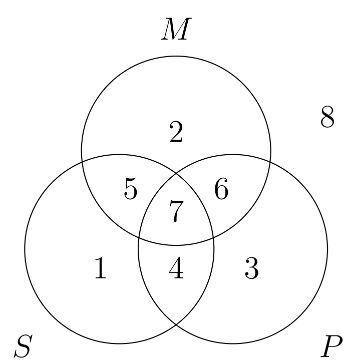
The last method we will consider is the most advanced but also the most fun. Your instructor may decide to skip this section, given that the prior two techniques are sufficient to evaluate sorites arguments. The Venn diagrams we have been making for this section have had three circles to represent three terms. The arguments we are dealing with have four or more terms, so if we want to represent them all at once, we will need a diagram with more shapes. To see how to set this up we need to remember the principles behind the original design of the diagram.

Recall that John Venn's original insight was that we should always begin by arranging the shapes so that every possible combination of the terms was represented (see pages 61 and 112). If we have two terms, we can use circles arranged like this,

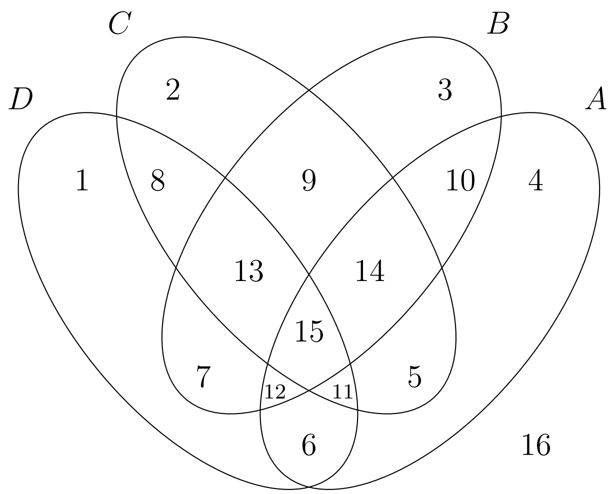


where area 1 represents things that are S but not P, area 2 represents things that are both S and P, area 3 represents things that are only P, and area 4 represents things that are neither S nor P.

Three terms give us eight possibilities to represent, so we draw the diagram like this:



A four-term sorites will have 16 possible combinations of terms. To represent all of these, we will need to stretch out our circles into ellipses.



The diagram is complex, and it takes some practice to learn to work with it. However, the basic methods for using it are the same as with the three-term diagram. Consider this argument, taken from Lewis Carroll's logic textbook (Dodgson, 1896).

P_1: Nobody who is despised can manage a crocodile.

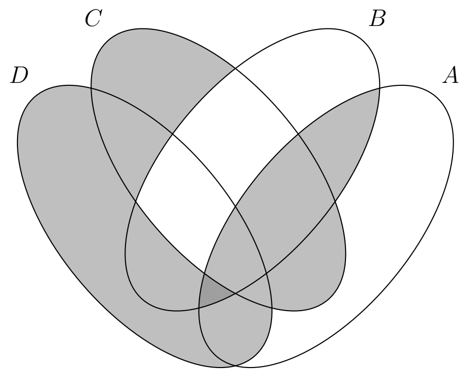
P_2: Illogical persons are despised.

P_3: Babies are illogical.

C: No babies can manage a crocodile.

If we set the major term "Persons who can manage a crocodile" as A, the minor term "babies" as D, and the two middle terms as B and C, we get this for the Venn diagram.

A**:** Persons who can manage a crocodile
B**:** Persons who are despised
C**:** Illogical persons
D**:** Babies



Practice Exercises

Part A Rewrite the following arguments in standard form, reducing the number of terms if necessary. Then supply the missing intermediate conclusions and evaluate the arguments with Venn diagrams.

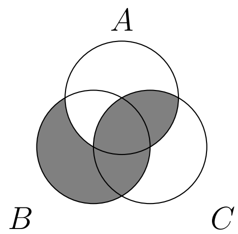
Example: P_1: No D are non-B.
P_2: All B are A.
P_3: No A are C.

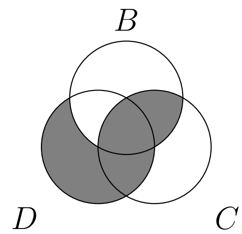
C: Some D are not C.

Answer: Standard form:

P_1: No A are C.
P_2: All B are A.
P_3: All D are B.

C: Some D are not C.





$$\begin{array}{rl} P_1: & \text{No } A \text{ are } C. \\ P_2: & \underline{\text{All } B \text{ are } A.} \\ C: & \text{No } B \text{ are } C. \end{array}$$

Celarent (EAE-I)

$$\begin{array}{rl} P_1: & \text{No } B \text{ are } C. \\ P_2: & \underline{\text{All } D \text{ are } B.} \\ C: & \text{Some } D \text{ are not } C. \end{array}$$

Celaront (EAO-1)

Conditionally valid. It also needs the premise “Some D exist.”

(1) P_1: All B are A.
P_2: Some C are D.
P_3: No D are A.
C: Some C are not B.

(2) P_1: Some B are C.
P_2: All D are A.
P_3: Some C are D.
C: All A are B.

(3) P_1: All B are C.
P_2: Some B are A.
P_3: All D are non-A.
C: Some C are not D.

(4) P_1: No A are B.
P_2: No C are B.
P_3: All D are A.
P_4: All E are C.
C: No D are E.

(5) P_1: All A are E.
P_2: All E are D.
P_3: All A are B.
P_4: All D are C.
C: All B are C.

(6) P_1: All A are E.
P_2: All E are D.
P_3: Some B are C.
P_4: All B are A.
C: Some C are D.

(7) P_1: All A are B.
P_2: No C are E.
P_3: No D are non-A.
P_4: Some E are D.
C: Some B are not C.

(8) P_1: All D are A.
P_2: All A are C.
P_3: All E are B.
P_4: Some D are not B.
C: Some C are not E.

(9) P_1: All E are D.
P_2: No E are F.
P_3: All B are A.
P_4: All C are B.
P_5: All D are C.
C: Some A are not F.

(10) P_1: All B are A.
P_2: Some C are E.
P_3: No A are D.
P_4: All C are B.
P_5: All E are F.
C: Some D are F.

Part B Rewrite the following arguments in standard form, reducing the number of terms if necessary. Then supply the missing intermediate conclusions and evaluate the arguments with Venn diagrams.

(1) P_1: No D are C.
P_2: No A are B.
P_3: Some D are A
C: Some C are not B.

(2) P_1: All C are B.
P_2: No B are D.
P_3: All A are D.
C: Some A are not C.

(3) P_1: Some M are not P.
P_2: No M are non-$M2$.
P_3: No $M2$ are non-S.
C: Some S are not P.

(4) P_1: All A are B.
P_2: All E are A.
P_3: All D are C.
P_4: All D are B.
C: Some E are C.

(5) P_1: Some E are C.
P_2: Some A are C.
P_3: All B are D.
P_4: All E are B.
C: Some A are D.

(6) P_1: No B are C.
P_2: Some B are not A.
P_3: All C are D.
P_4: All E are A.
C: Some D are not E.

(7) P_1: All D are A.
P_2: All C are D.
P_3: No E are B.
P_4: No B are A.
C: No C are E.

(8) P_1: No A are non-D.
P_2: No D are C.
P_3: All B are C.
P_4: All non-B are non-E
C: No E are A.

(9) P_1: No E are F.
P_2: All D are B.
P_3: All C are E.
P_4: Some D are not F.
P_5: All B are A.
C: Some C are not A.

(10) P_1: No B are D.
P_2: All E are A.
P_3: All F are C.
P_4: All C are E.
P_5: All D are F.
C: No B are A.

Part C Do the exercises in Part A, but skip writing out the missing intermediate conclusions, and evaluate them using the rules method instead of the Venn diagram method.

Example:
P$_1$: No D are non-B.
P$_2$: All B are A.
P$_3$: No A are C.

C: Some D are not C.

Answer:
P$_1$: No A^{D} are C^{D}.
P$_2$: All B^{D} are A.
P$_3$: All D^{D} are B.

C: Some D are not C^{D}.

Rule 1: Pass. Middle terms A and B are distributed in P$_1$ and P$_2$.
Rule 2: Pass. The conclusion distributes C, which is also distributed in P$_1$.
Rule 3: Pass. There is one negative premise, P$_1$.
Rule 4: Pass. The conclusion is negative, and there is exactly one negative premise.
Rule 5: Fail. The premises are all universal and the conclusion is particular.

Conditionally valid. It needs the premise "Some D exist."

Part D Do the exercises in Part B, but skip writing out the missing intermediate conclusions, and evaluate them using the rules method instead of the Venn diagram method.

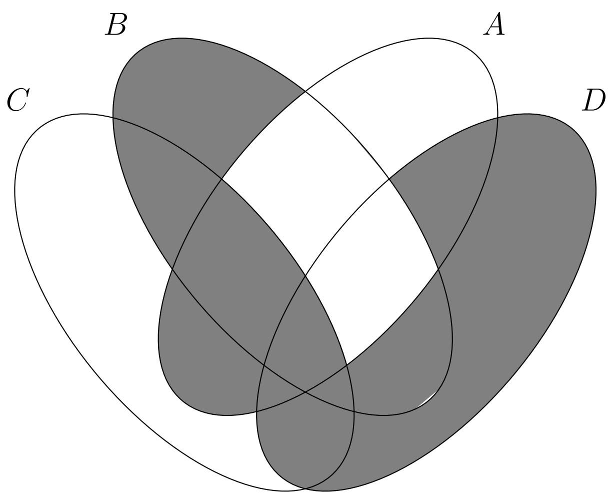
Part E Rewrite the following arguments in standard form, reducing terms if necessary, and then prove that they are valid using a single, four-term Venn diagram.

Example:
P$_1$: No D are non-B.
P$_2$: All B are A.
P$_3$: No A are C.

C: Some D are not C.

Answer:

P_1: No A are C.
P_2: All B are A.
P_3: All D are B.
C: Some D are not C.



Conditionally valid. If you had a premise that told you to write an x in the D ellipse, it would clearly fall outside of C.

(1) P_1: No D are B.
P_2: All A are C.
P_3: All C are B.
C: No D are A.

(2) P_1: All A are B.
P_2: No D are C.
P_3: All C are A.
C: No D are B.

(3) P_1: All B are D.
P_2: All B are C.
P_3: No A are D.
C: Some C are not A.

(4) P_1: No A are C.
P_2: All B are D.
P_3: All C are B.
C: No A are D.

(5) P_1: All D are A.
P_2: Some C are B.
P_3: Some B are not D.
C: Some non-C are not non-A.

Part F Rewrite the following arguments in standard form, reducing terms if necessary, and then prove that they are valid using a single, four-term Venn diagram.

(1) P_1: No A are C.
P_2: All B are D.
P_3: Some B are A.
C: Some S are not C.

(2) P_1: No P are M.
P_2: All M are $M2$.
P_3: No $M2$ are S.
C: No S are P.

(3) P_1: All A are C.
P_2: All B are C.
P_3: All A are D.
C: Some D are B.

(4) P_1: Some non-D are not non-C.
P_2: All D are B.
P_3: No B are A.
C: Some A are not C.

(5) P_1: No C are non-B.
P_2: Some D are A.
P_3: All D are C.
C: Some B are not A.

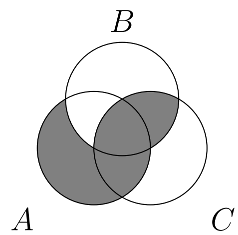
Part G Rewrite the following arguments in standard form using variables and a translation key. Then evaluate using any method you want. The example problem, along with exercises (2), (4), (7), and (10), come from Lewis Carroll's logic textbook (Dodgson, 1896). Other exercises are just Lewis Carroll themed.

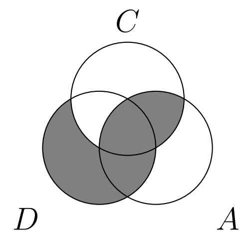
Example: My saucepans are the only things I have that are made of tin. I find all your presents very useful, but none of my saucepans are of the slightest use. Therefore, none of your presents are made of tin.

Answer:

A**:** Things of mine made from tin
B**:** Saucepans
C**:** Useful things
D**:** Presents from you

P_1: All A are B.
P_2: No B are C.
P_3: All D are C.
C: No D are A.





P_1: All A are B.
P_2: No B are C.

C: No A are C.*

Celarent (EAE-I)

P_1: No A are C.*
P_2: All D are C.

C: No D are A.

Cesare (EAE-II)

(1) All metal things are solid, and some of those metal things are also chairs. All chairs are furniture. Therefore some solid things are furniture.

(2) Every one who is sane can do Logic. No lunatics are fit to serve on a jury. None of *your* sons can do Logic. Therefore, none of your sons can serve on a jury.

(3) All hat-wearers are heaven-sent. This is because all platypuses are heaven-sent, all secret agents are platypuses, and some secret agents wear hats.

(4) No ducks waltz. No officers ever decline to waltz. All my poultry are ducks. Therefore, none of my poultry are officers.

(5) Some things are uffish, but nothing that is uffish is vorpal. Therefore some uffish things are not swords, because everything that is vorpal goes snicker-snack, and everything that goes snicker-snack is a sword.

(6) No animals are plants. Some animals are mammals. Some mammals are dogs. All dachshunds are dogs. Therefore, no dachshunds are plants.

(7) Things sold in the street are of no great value. Nothing but rubbish can be had for a song. Eggs of the Great Auk are very valuable. It is only what is sold in the streets that is really rubbish. Therefore the eggs of the Great Auk cannot be had for a song.

(8) All life forms are physical objects, and all bacteria are life forms. Also, all *E. coli* are bacteria, so all rod shaped things are physical objects, because all *E. coli* are rod-shaped.

(9) Some playing cards shout “Off with his head!” All playing cards play croquet. Everyone who plays croquet uses hedgehogs for balls. Everyone who uses

hedgehogs for balls uses flamingos for mallets. The queen of hearts uses flamingos for mallets. Therefore some cards identical to the queen of hearts shout "Off with his head!"

(10) I despise anything that cannot be used as a bridge. Everything, that is worth writing an ode to, would be a welcome gift to me. A rainbow will not bear the weight of a wheelbarrow. Whatever can be used as a bridge will bear the weight of a wheel-barrow. I would not take, as a gift, a thing that I despise. Therefore a rainbow is not worth writing an ode to.

Part H Rewrite the following arguments in standard form using variables and a translation key, and supply intermediate conclusions. Then evaluate using any method you want. Exercises (3), (5), (7), (9), and (10) come from Lewis Carroll's logic textbook (Dodgson, 1896). Other exercises are just Lewis Carroll themed.

(1) All buildings are habitable places, and all houses are residences. Therefore all houses are buildings because all residences are habitable places.

(2) Not all cephalopods are moralizing, but cuttlefish are. All pompous squid are cephalopods. Therefore some pompous squid are not cuttlefish.

(3) Showy talkers think too much of themselves; No really well-informed people are bad company; and people who think too much of themselves are not good company. Therefore no showy talkers are really well informed.

(4) All white animals are late, and some late animals are rabbits. Also, all animals taking a watch out of their waistcoat pocket are white. Therefore, some rabbits are taking a watch out of their waistcoat pocket.

(5) No one takes in the *Times* unless he is well-educated. But no hedge-hogs can read, and those who cannot read are not well-educated. Therefore no hedge-hog takes in the *Times*.

(6) The Red Queen is a chess piece. Among the things that have to run faster and faster just to stay in the same place is the Red Queen. All lost children have to run faster and faster just to stay in the same place. Alice is a lost child. Therefore, Alice is one of the chess pieces.

(7) None of the unnoticed things met with at sea are mermaids. Things entered in the log as met with at sea are sure to be worth remembering. I have never met with anything worth remembering, when on a voyage. Things met with at sea, that are noticed, are sure to be recorded in the log. Therefore, I have never come across a mermaid at sea.

(8) A plum-pudding that is not really solid is mere porridge. Every plum-pudding

served at my table has been boiled in a cloth. A plum-pudding that is mere porridge is indistinguishable from soup. No plum puddings are really solid except what are served at my table. Therefore no plum-pudding that has not been boiled in cloth can be distinguished from soup.

(9) The only books in this library that I do *not* recommend for reading are unhealthy in tone. All the bound books are well-written. All the romances are healthy in tone, and I do not recommend you read any of the unbound books. Therefore all the romances in this library are well-written.

(10) No interesting poems are unpopular among people of real taste. No modern poetry is free from affectation. All *your* poems are on the subject of soap bubbles. No affected poetry is popular among people of real taste. No ancient poem is on the subject of soap-bubbles. Therefore all of *your* poems are uninteresting.

Part I

(1) Suppose you wanted to represent five terms using a Venn diagram. How would you arrange the ellipses?

(2) Can you figure out a principle for continually adding shapes to a Venn diagram that will always allow you to represent every combination of terms?

Key Terms

Aristotelian syllogism

Categorical syllogism

Conditional validity

Counterexample method

Critical term

Enthymeme

Fallacy of exclusive premises

Fallacy of illicit process

Fallacy of particular premises

Fallacy of the undistributed middle

Major premise

Major term

Middle term

Minor premise

Minor term

Negative-affirmative fallacy

Sorites categorical arguments

Standard form for an Aristotelian syllogism

Standard form for a sorites categorical argument

Syllogism mood

Translation key

Unconditional validity

Part III

Sentential Logic

Chapter 5

Sentential Logic

This chapter introduces a logical language called SL. It is a version of *sentential logic*, because the basic units of the language will represent statements, and a statement is usually given by a complete sentence in English.

5.1 Sentence Letters

In SL, capital letters, called SENTENCE LETTERS are used to represent simple statements. Considered only as a symbol of SL, the letter A could mean any statement. So when translating from English into SL, it is important to provide a symbolization key, or dictionary. The SYMBOLIZATION KEY provides an English language sentence for each sentence letter used in the symbolization.

Consider this argument:

1. There is an apple on the desk.
2. If there is an apple on the desk, then Jenny made it to class.

∴ Jenny made it to class.

This is obviously a valid argument in English. In symbolizing it, we want to preserve the structure of the argument that makes it valid. What happens if we replace each sentence with a letter? Our symbolization key would look like this:

A: There is an apple on the desk.
B: If there is an apple on the desk, then Jenny made it to class.
C: Jenny made it to class.

We would then symbolize the argument in this way:

1. A
2. B

$\therefore C$

There is no necessary connection between some sentence A, which could be any statement, and some other sentences B and C, which could also be anything. The structure of the argument has been completely lost in this translation.

The important thing about the argument is that the second premise is not merely *any* statement, logically divorced from the other statement in the argument. The second premise contains the first premise and the conclusion *as parts*. Our symbolization key for the argument only needs to include meanings for A and C, and we can build the second premise from those pieces. So we symbolize the argument this way:

1. A
2. If A, then C.

$\therefore C$

This preserves the structure of the argument that makes it valid, but it still makes use of the English expression "If… then…." Although we ultimately want to replace all of the English expressions with logical notation, this is a good start.

The individual sentence letters in SL are called atomic sentences, because they are the basic building blocks out of which more complex sentences can be built. We can identify atomic sentences in English as well. An ATOMIC SENTENCE is one that cannot be broken into parts that are themselves sentences. "There is an apple on the desk" is an atomic sentence in English, because you can't find any proper part of it that forms a complete sentence. For instance "an apple on the desk" is a noun phrase, not a complete sentence. Similarly "on the desk" is a prepositional phrase, and not a sentence, and "is an" is not any kind of phrase at all. This is what you will find no matter how you divide "There is an apple on the desk." On the other hand you can find two proper parts of "If there is an apple on the desk, then Jenny made it to class" that are complete sentences: "There is an apple on the desk" and "Jenny made it to class." As a general rule, we will want to use atomic sentences in SL (that is, the sentence letters) to represent atomic sentences in English. Otherwise, we will lose some of the logical structure of the English sentence, as we have just seen.

There are only 26 letters of the alphabet, but there is no logical limit to the number of atomic sentences. We can use the same letter to symbolize different atomic sentences by adding a subscript, a small number written after the letter. We could have a symbolization key that looks like this:

$\mathbf{A_1}$**:** The apple is under the armoire.
$\mathbf{A_2}$**:** Arguments in SL always contain atomic sentences.
$\mathbf{A_3}$**:** Adam Ant is taking an airplane from Anchorage to Albany.
⋮
$\mathbf{A_{294}}$**:** Alliteration angers otherwise affable astronauts.

Keep in mind that each of these is a different sentence letter. When there are subscripts in the symbolization key, it is important to keep track of them.

5.2 Sentential Connectives

Logical connectives are used to build complex sentences from atomic components. In SL, our logical connectives are called SENTENTIAL CONNECTIVES because they connect sentence letters. There are five sentential connectives in SL. This table summarizes them, and they are explained below.

Symbol	What it is called	What it means
$\sim$	negation	"It is not the case that..."
$\bullet$	conjunction	"Both ... and ..."
$\vee$	disjunction	"Either ... or ..."
$\supset$	conditional	"If ... then ..."
$\equiv$	biconditional	"... if and only if ..."

Negation

Consider how we might symbolize these sentences:

1. Mary is in Barcelona.
2. Mary is not in Barcelona.
3. Mary is somewhere other than Barcelona.

In order to symbolize sentence 1, we will need one sentence letter. We can provide a symbolization key:

B: Mary is in Barcelona.

Note that here we are giving B a different interpretation than we did in the previous section. The symbolization key only specifies what B means *in a specific context*. It is vital that we continue to use this meaning of B so long as we are talking about Mary and Barcelona. Later, when we are symbolizing different sentences, we can write a new symbolization key and use B to mean something else.

Now, sentence 1 is simply B. Sentence 2 is obviously related to sentence 1: it is basically 1 with a "not" added. We could put the sentence partly our symbolic language by writing "Not B." This means we do not want to introduce a different sentence letter for 2. We just need a new symbol for the "not" part. Let's use the symbol '$\sim$,' which we will call NEGATION. Now we can translate 'Not B' to $\sim B$.

Sentence 3 is about whether or not Mary is in Barcelona, but it does not contain the word "not." Nevertheless, it is obviously logically equivalent to sentence 2. They both mean: It is not the case that Mary is in Barcelona. As such, we can translate both sentence 2 and sentence 3 as $\sim B$.

A sentence can be symbolized as $\sim\mathcal{A}$ if it can be paraphrased in English as "It is not the case that $\mathcal{A}$."

Consider these further examples:

4. The widget is replaceable.
5. The widget is irreplaceable.
6. The widget is not irreplaceable.

If we let R mean "The widget is replaceable", then sentence 4 can be translated as R.

What about sentence 5? Saying the widget is irreplaceable means that it is not the case that the widget is replaceable. So even though sentence 5 is not negative in English, we symbolize it using negation as $\sim R$.

Sentence 6 can be paraphrased as "It is not the case that the widget is irreplaceable." Using negation twice, we translate this as $\sim\sim R$. The two negations in a row each work as negations, so the sentence means "It is not the case that it is not the case that R." If you think about the sentence in English, it is logically equivalent to sentence 4. So when we define logical equivalence in SL, we will make sure that R and $\sim\sim R$ are logically equivalent.

More examples:

7. Elliott is happy.
8. Elliott is unhappy.

If we let H mean "Elliot is happy", then we can symbolize sentence 7 as H.

However, it would be a mistake to symbolize sentence 8 as $\sim H$. If Elliott is unhappy, then he is not happy—but sentence 8 does not mean the same thing as "It is not the case that Elliott is happy." It could be that he is not happy but that he is not unhappy either. Perhaps he is somewhere between the two. In order to symbolize sentence 8, we would need a new sentence letter.

For any sentence $\mathcal{A}$: If $\mathcal{A}$ is true, then $\sim\mathcal{A}$ is false. If $\sim\mathcal{A}$ is true, then $\mathcal{A}$ is false. Using T for true and F for false, we can summarize this in a *characteristic truth table* for negation:

$\mathcal{A}$	$\sim\mathcal{A}$
T	F
F	T

We will discuss truth tables at greater length in the next chapter.

Conjunction

Consider these sentences:

9. Adam is athletic.

10. Barbara is athletic.
11. Adam is athletic, and Barbara is also athletic.

We will need separate sentence letters for 9 and 10, so we define this symbolization key:

A: Adam is athletic.
B: Barbara is athletic.

Sentence 9 can be symbolized as A.

Sentence 10 can be symbolized as B.

Sentence 11 can be paraphrased as "A and B." In order to fully symbolize this sentence, we need another symbol. We will use $\bullet$. We translate "A and B" as $A \bullet B$. The logical connective $\bullet$ is called the CONJUNCTION, and A and B are each called CONJUNCTS.

Notice that we make no attempt to symbolize "also" in sentence 11. Words like "both" and "also" function to draw our attention to the fact that two things are being conjoined. They are not doing any further logical work, so we do not need to represent them in SL.

Some more examples:

12. Barbara is athletic and energetic.
13. Barbara and Adam are both athletic.
14. Although Barbara is energetic, she is not athletic.
15. Barbara is athletic, but Adam is more athletic than she is.

Sentence 12 is obviously a conjunction. The sentence says two things about Barbara, so in English it is permissible to refer to Barbara only once. It might be tempting to try this when translating the argument: Since B means "Barbara is athletic", one might paraphrase the sentences as "B and energetic." This would be a mistake. Once we translate part of a sentence as B, any further structure is lost. B is an atomic sentence; it is nothing more than true or false. Conversely, "energetic" is not a sentence; on its own it is neither true nor false. We should instead paraphrase the sentence as "B and Barbara is energetic." Now we need to add a sentence letter to the symbolization key. Let E mean "Barbara is energetic." Now the sentence can be translated as $B \bullet E$.

> A sentence can be symbolized as $\mathcal{A} \bullet \mathcal{B}$ if it can be paraphrased in English as 'Both $\mathcal{A}$, and $\mathcal{B}$.' Each of the conjuncts must be a sentence.

Sentence 13 says one thing about two different subjects. It says of both Barbara and Adam that they are athletic, and in English we use the word "athletic" only once. In translating to SL, it is important to realize that the sentence can be paraphrased as, "Barbara is athletic, and Adam is athletic." This translates as $B \bullet A$.

Sentence 14 is a bit more complicated. The word "although" sets up a contrast between the first part of the sentence and the second part. Nevertheless, the sentence says both that Barbara is energetic and that she is not athletic. In order to make each of the conjuncts an atomic sentence, we need to replace "she" with "Barbara."

So we can paraphrase sentence 14 as, "*Both* Barbara is energetic, *and* Barbara is not athletic." The second conjunct contains a negation, so we paraphrase further: "*Both* Barbara is energetic *and it is not the case that* Barbara is athletic." This translates as $E \bullet {\sim}B$.

Sentence 15 contains a similar contrastive structure. It is irrelevant for the purpose of translating to SL, so we can paraphrase the sentence as "*Both* Barbara is athletic, *and* Adam is more athletic than Barbara." (Notice that we once again replace the pronoun "she" with her name.) How should we translate the second conjunct? We already have the sentence letter A which is about Adam's being athletic and B which is about Barbara's being athletic, but neither is about one of them being more athletic than the other. We need a new sentence letter. Let R mean "Adam is more athletic than Barbara." Now the sentence translates as $B \bullet R$.

> Sentences that can be paraphrased "$\mathcal{A}$, but $\mathcal{B}$" or "Although $\mathcal{A}$, $\mathcal{B}$" are best symbolized using conjunction $\mathcal{A} \bullet \mathcal{B}$.

It is important to keep in mind that the sentence letters A, B, and R are atomic sentences. Considered as symbols of SL, they have no meaning beyond being true or false. We have used them to symbolize different English language sentences that are all about people being athletic, but this similarity is completely lost when we translate to SL. No formal language can capture all the structure of the English language, but as long as this structure is not important to the argument there is nothing lost by leaving it out.

For any sentences $\mathcal{A}$ and $\mathcal{B}$, $\mathcal{A} \bullet \mathcal{B}$ is true if and only if both $\mathcal{A}$ and $\mathcal{B}$ are true. We can summarize this in the characteristic truth table for conjunction:

$\mathcal{A}$	$\mathcal{B}$	$\mathcal{A} \bullet \mathcal{B}$
T	T	T
T	F	F
F	T	F
F	F	F

Conjunction is symmetrical because we can swap the conjuncts without changing the truth value of the sentence. Regardless of what $\mathcal{A}$ and $\mathcal{B}$ are, $\mathcal{A} \bullet \mathcal{B}$ is logically equivalent to $\mathcal{B} \bullet \mathcal{A}$.

Disjunction

Consider these sentences:

16. Either Denison will play golf with me, or he will watch movies.
17. Either Denison or Ellery will play golf with me.

For these sentences we can use this symbolization key:

D: Denison will play golf with me.
E: Ellery will play golf with me.
M: Denison will watch movies.

Sentence 16 is "Either D or M." To fully symbolize this, we introduce a new symbol. The sentence becomes $D \vee M$. The $\vee$connective is called DISJUNCTION, and D and M are called DISJUNCTS.

Sentence 17 is only slightly more complicated. There are two subjects, but the English sentence only gives the verb once. In translating, we can paraphrase it as "Either Denison will play golf with me, or Ellery will play golf with me." Now it obviously translates as $D \vee E$.

A sentence can be symbolized as $\mathcal{A} \vee \mathcal{B}$ if it can be paraphrased in English as "Either $\mathcal{A}$ or $\mathcal{B}$." Each of the disjuncts must be a sentence.

Sometimes in English, the word "or" excludes the possibility that both disjuncts are true. This is called an **exclusive or**. An *exclusive or* is clearly intended when a restaurant menu says, "Entrees come with either soup or salad." You may have soup; you may have salad; but, if you want *both* soup *and* salad, then you will have to pay extra.

At other times, the word "or" allows for the possibility that both disjuncts might be true. This is probably the case with sentence 17, above. I might play with Denison, with Ellery, or with both Denison and Ellery. Sentence 17 merely says that I will play with *at least* one of them. This is called an **inclusive or**.

The symbol $\vee$ represents an *inclusive or*. So $D \vee E$ is true if D is true, if E is true, or if both D and E are true. It is false only if both D and E are false. We can summarize this with the characteristic truth table for disjunction:

$\mathcal{A}$	$\mathcal{B}$	$\mathcal{A} \vee \mathcal{B}$
T	T	T
T	F	T
F	T	T
F	F	F

Like conjunction, disjunction is symmetrical. $\mathcal{A} \vee \mathcal{B}$ is logically equivalent to $\mathcal{B} \vee \mathcal{A}$.

These sentences are somewhat more complicated:

18. Either you will not have soup, or you will not have salad.
19. You will have neither soup nor salad.
20. You get either soup or salad, but not both.

We let S_1 mean that you get soup and S_2 mean that you get salad.

Sentence 18 can be paraphrased in this way: "Either *it is not the case that* you get soup, or *it is not the case that* you get salad." Translating this requires both disjunction and negation. It becomes $\sim S_1 \vee \sim S_2$.

Sentence 19 also requires negation. It can be paraphrased as, "*It is not the case that* either you get soup or you get salad." We need some way of indicating that the negation does not just negate the right or left disjunct, but rather negates the entire disjunction. In order to do this, we put parentheses around the disjunction: "It is not the case that $(S_1 \vee S_2)$." This becomes simply $\sim(S_1 \vee S_2)$.

Notice that the parentheses are doing important work here. The sentence $\sim S_1 \vee S_2$ would mean "Either you will not have soup, or you will have salad."

Sentence 20 is an *exclusive or*. Although $\vee$ is an inclusive or, we can symbolize an exclusive or in SL. We just need more than one connective to do it. We can break the sentence into two parts. The first part says that you get one or the other. We translate this as $(S_1 \vee S_2)$. The second part says that you do not get both. We can paraphrase this as "It is not the case both that you get soup and that you get salad." Using both negation and conjunction, we translate this as $\sim(S_1 \bullet S_2)$. Now we just need to put the two parts together. As we saw above, "but" can usually be translated as a conjunction. Sentence 20 can thus be translated as $(S_1 \vee S_2) \bullet \sim(S_1 \bullet S_2)$.

Conditional

For the following sentences, let R mean "You will cut the red wire" and B mean "The bomb will explode."

21. If you cut the red wire, then the bomb will explode.
22. The bomb will explode only if you cut the red wire.

Sentence 21 can be translated partially as "If R, then B." We will use the symbol $\supset$ to represent logical entailment. Sentence 21 then becomes $R \supset B$. The connective is called a CONDITIONAL. The sentence on the left-hand side of the conditional (R in this example) is called the ANTECEDENT. The sentence on the right-hand side (B) is called the CONSEQUENT.

Sentence 22 is also a conditional. Since the word "if" appears in the second half of the sentence, it might be tempting to symbolize this in the same way as sentence 21. That would be a mistake.

The conditional $R \supset B$ says that *if* R were true, *then* B would also be true. It does not say that you cutting the red wire is the *only* way that the bomb could explode. Someone else might cut the wire, or the bomb might be on a timer. The sentence $R \supset B$ does not say anything about what to expect if R is false. Sentence 22 is different. It says that the only conditions under which the bomb will explode involve you having cut the red wire; i.e., if the bomb explodes, then you must have cut the wire. As such, sentence 22 should be symbolized as $B \supset R$.

It is important to remember that the connective $\supset$ says only that, if the antecedent is true, then the consequent is true. It says nothing about the *causal* connection between the two events. Translating sentence 22 as $B \supset R$ does not mean that the bomb exploding would somehow have caused you cutting the wire. Both sentence 21 and 22 suggest that, if you cut the red wire, you cutting the red wire would be the cause of the bomb exploding. They differ on the *logical* connection. If sentence 22 were true, then an explosion would tell us—those of us safely away from the bomb—that you had cut the red wire. Without an explosion, sentence 22 tells us nothing.

The paraphrased sentence "$\mathcal{A}$ only if $\mathcal{B}$" is logically equivalent to "If $\mathcal{A}$, then $\mathcal{B}$."

"If $\mathcal{A}$, then $\mathcal{B}$" means that if $\mathcal{A}$ is true, then so is $\mathcal{B}$. So we know that if the antecedent $\mathcal{A}$ is true but the consequent $\mathcal{B}$ is false, then the conditional "If $\mathcal{A}$ then $\mathcal{B}$" is false. What is the truth value of "If $\mathcal{A}$, then $\mathcal{B}$" under other circumstances? Suppose, for instance, that the antecedent $\mathcal{A}$ happened to be false. "If $\mathcal{A}$, then $\mathcal{B}$" would then not tell us anything about the actual truth value of the consequent $\mathcal{B}$, and it is unclear what the truth value of "If $\mathcal{A}$, then $\mathcal{B}$" would be.

In English, the truth of conditionals often depends on what *would* be the case if the antecedent *were true*—even if, as a matter of fact, the antecedent is false. This poses a problem for translating conditionals into SL. Considered as sentences of SL, R and B in the above examples have nothing intrinsic to do with each other. In order to consider what the world would be like if R were true, we would need to analyze what R says about the world. Since R is an atomic symbol of SL, however, there is no further structure to be analyzed. When we replace a sentence with a sentence letter, we consider it merely as some atomic sentence that might be true or false.

In order to translate conditionals into SL, we will not try to capture all the subtleties of the English language "If…, then…." Instead, the symbol $\supset$ will be what logicians call a material conditional. This means that when $\mathcal{A}$ is false, the conditional $\mathcal{A} \supset \mathcal{B}$ is automatically true, regardless of the truth value of $\mathcal{B}$. If both $\mathcal{A}$ and $\mathcal{B}$ are true, then the conditional $\mathcal{A} \supset \mathcal{B}$ is true.

In short, $\mathcal{A} \supset \mathcal{B}$ is false if and only if $\mathcal{A}$ is true and $\mathcal{B}$ is false. We can summarize this with a characteristic truth table for the conditional.

$\mathcal{A}$	$\mathcal{B}$	$\mathcal{A} \supset \mathcal{B}$
T	T	T
T	F	F
F	T	T
F	F	T

The conditional is asymmetrical. You cannot swap the antecedent and consequent without changing the meaning of the sentence, because $\mathcal{A} \supset \mathcal{B}$ and $\mathcal{B} \supset \mathcal{A}$ are not logically equivalent.

Biconditional

Consider these sentences:

23. The figure on the board is a triangle only if it has exactly three sides.
24. The figure on the board is a triangle if it has exactly three sides.
25. The figure on the board is a triangle if and only if it has exactly three sides.

Let T mean "The figure is a triangle" and S mean "The figure has three sides."

Sentence 23, for reasons discussed above, can be translated as $T \supset S$.

Sentence 24 is importantly different. It can be paraphrased as "If the figure has three sides, then it is a triangle." So it can be translated as $S \supset T$.

Sentence 25 says that T is true *if and only if* S is true; we can infer S from T, and we can infer T from S. This is called a BICONDITIONAL, because it entails the two conditionals $S \supset T$ and $T \supset S$. We will use $\equiv$to represent the biconditional; sentence 25 can be translated as $S \equiv T$.

We could abide without a new symbol for the biconditional. Since sentence 25 means "$T \supset S$ and $S \supset T$," we could translate it as $(T \supset S) \bullet (S \supset T)$. We would need parentheses to indicate that $(T \supset S)$ and $(S \supset T)$ are separate conjuncts; the expression $T \supset S \bullet S \supset T$ would be ambiguous.

Because we could always write $(\mathcal{A} \supset \mathcal{B}) \bullet (\mathcal{B} \supset \mathcal{A})$ instead of $\mathcal{A} \equiv \mathcal{B}$, we do not strictly speaking *need* to introduce a new symbol for the biconditional. Nevertheless, logical languages usually have such a symbol. SL will have one, which makes it easier to translate phrases like "if and only if."

$\mathcal{A} \equiv \mathcal{B}$ is true if and only if $\mathcal{A}$ and $\mathcal{B}$ have the same truth value. This is the characteristic truth table for the biconditional:

$\mathcal{A}$	$\mathcal{B}$	$\mathcal{A} \equiv \mathcal{B}$
T	T	T
T	F	F
F	T	F
F	F	T

5.3 Other Symbolization

We have now introduced all of the connectives of SL. We can use them together to translate many kinds of sentences. Consider these examples of sentences that use the English-language connective "unless":

26. Unless you wear a jacket, you will catch cold.
27. You will catch cold unless you wear a jacket.

Let J mean "You will wear a jacket" and let D mean "You will catch a cold."

We can paraphrase sentence 26 as "Unless J, D." This means that if you do not wear a jacket, then you will catch cold; with this in mind, we might translate it as $\sim J \supset D$. It also means that if you do not catch a cold, then you must have worn a jacket; with this in mind, we might translate it as $\sim D \supset J$.

Which of these is the correct translation of sentence 26? Both translations are correct, because the two translations are logically equivalent in SL.

Sentence 27, in English, is logically equivalent to sentence 26. It can be translated as either $\sim J \supset D$ or $\sim D \supset J$.

When symbolizing sentences like sentence 26 and sentence 27, it is easy to get turned around. Since the conditional is not symmetric, it would be wrong to translate either sentence as $J \supset \sim D$. Fortunately, there are other logically equivalent expressions. Both sentences mean that you will wear a jacket or—if you do not wear a jacket—then you will catch a cold. So we can translate them as $J \vee D$. (You might worry that the "or" here should be an *exclusive or*. However, the sentences do not exclude the possibility that you might *both* wear a jacket *and* catch a cold; jackets do not protect you from all the possible ways that you might catch a cold.)

> If a sentence can be paraphrased as "Unless $\mathcal{A}$, $\mathcal{B}$," then it can be symbolized as $\mathcal{A} \vee \mathcal{B}$.

5.4 Recursive Syntax for SL

The previous two sections gave you a rough, informal sense of how to create sentences in SL. If I give you an English sentence like "Grass is either green or brown," you should be able to write a corresponding sentence in SL: "$A \vee B$." In this section we want to give a more precise definition of a sentence in SL. When we defined sentences in English, we did so using the concept of truth: Sentences were units of language that can be true or false. In SL, it is possible to define what counts as a sentence without talking about truth. Instead, we can just talk about the structure of the sentence. This is one respect in which a formal language like SL is more precise than a natural language like English.

The structure of a sentence in SL considered without reference to truth or falsity is called its syntax. More generally SYNTAX refers to the study of the properties of language that are there even when you don't consider meaning. Whether a sentence is true or false is considered part of its meaning. In this chapter, we will be giving a purely syntactical definition of a sentence in SL. The contrasting term is SEMANTICS the study of aspects of language that relate to meaning, including truth and falsity. (The word "semantics" comes from the Greek word for "mark")

If we are going to define a sentence in SL just using syntax, we will need to carefully distinguish SL from the language that we use to talk about SL. When you create an artificial language like SL, the language that you are creating is called the OBJECT LANGUAGE. The language that we use to talk about the object language is called the METALANGUAGE. Imagine building a house. The object language is like the house itself. It is the thing we are building. While you are building a house, you might put up scaffolding around it. The scaffolding isn't part of the the house. You just use it to build the house. The metalanguage is like the scaffolding.

The object language in this chapter is SL. For the most part, we can build this language just by talking about it in ordinary English. However we will also have to build some special scaffolding that is not a part of SL, but will help us build SL. Our metalanguage will thus be ordinary English plus this scaffolding.

An important part of the scaffolding are the METAVARIABLES These are the fancy script letters we have been using in the characteristic truth tables for the connectives: $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, etc. These are letters that can refer to any sentence in SL. They can represent sentences like P or Q, or they can represent longer sentences, like $(((A \vee B) \bullet G) \supset (P \equiv Q))$. Just as the sentence letters A, B, etc. are variables that range over any English sentence, the metavariables $\mathcal{A}$, $\mathcal{B}$, etc. are variables that range over any sentence in SL, including the sentence letters A, B, etc.

As we said, in this chapter we will give a syntactic definition for "sentence of SL." The definition itself will be given in mathematical English, the metalanguage.

sentence letters with subscripts, as needed	$A, B, C, \ldots, Z$ $A_1, B_1, Z_1, A_2, A_{25}, J_{375}, \ldots$
connectives	$\sim, \bullet, \vee, \supset, \equiv$
parentheses	(,)

Most random combinations of these symbols will not count as sentences in SL. Any random connection of these symbols will just be called a "string" or "expression" Random strings only become meaningful sentences when the are structured according to the rules of syntax. We saw from the earlier two sections that individual sentence letters, like A and G_{13} counted as sentences. We also saw that we can put these sentences together using connectives so that $\sim A$ and $\sim G_{13}$ is a sentence. The problem is, we can't simply list all the different sentences we can put together this way, because there are infinitely many of them. Instead, we will define a sentence in SL by specifying the process by which they are constructed.

Consider negation: Given any sentence $\mathcal{A}$ of SL, $\sim\mathcal{A}$ is a sentence of SL. It is important here that $\mathcal{A}$ is not the sentence letter A. Rather, it is a metavariable: part of the metalanguage, not the object language. Since $\mathcal{A}$ is not a symbol of SL, $\sim\mathcal{A}$ is not an expression of SL. Instead, it is an expression of the metalanguage that allows us to talk about infinitely many expressions of SL: all of the expressions that start with the negation symbol.

We can say similar things for each of the other connectives. For instance, if $\mathcal{A}$ and $\mathcal{B}$ are sentences of SL, then $(\mathcal{A} \bullet \mathcal{B})$ is a sentence of SL. Providing clauses like this for all of the connectives, we arrive at the following formal definition for a SENTENCE OF SL:

1. Every atomic sentence is a sentence.
2. If $\mathcal{A}$ is a sentence, then $\sim\mathcal{A}$ is a sentence of SL.
3. If $\mathcal{A}$ and $\mathcal{B}$ are sentences, then $(\mathcal{A} \bullet \mathcal{B})$ is a sentence.
4. If $\mathcal{A}$ and $\mathcal{B}$ are sentences, then $(\mathcal{A} \vee \mathcal{B})$ is a sentence.
5. If $\mathcal{A}$ and $\mathcal{B}$ are sentences, then $(\mathcal{A} \supset \mathcal{B})$ is a sentence.
6. If $\mathcal{A}$ and $\mathcal{B}$ are sentences, then $(\mathcal{A} \equiv \mathcal{B})$ is a sentence.
7. All and only sentences of SL can be generated by applications of these rules.

We can apply this definition to see whether an arbitrary string is a sentence. Suppose we want to know whether or not $\sim\sim\sim D$ is a sentence of SL. Looking at the second clause of the definition, we know that $\sim\sim\sim D$ is a sentence *if* $\sim\sim D$ is a sentence. So now we need to ask whether or not $\sim\sim D$ is a sentence. Again looking at the second clause of the definition, $\sim\sim D$ is a sentence *if* $\sim D$ is. Again, $\sim D$ is a sentence *if* D is a sentence. Now D is a sentence letter, an atomic sentence of SL, so we know that D is a sentence by the first clause of the definition. So for a compound formula like $\sim\sim\sim D$, we must apply the definition repeatedly. Eventually we arrive at the atomic sentences from which the sentence is built up.

Definitions like this are called recursive. RECURSIVE DEFINITIONS begin with some specifiable base elements and define ways to indefinitely compound the base elements. Just as the recursive definition allows complex sentences to be built up from simple parts, you can use it to decompose sentences into their simpler parts. To determine whether or not something meets the definition, you may have to refer back to the definition many times.

When you use a connective to build a longer sentence from shorter ones, the shorter sentences are said to be in the SCOPE of the connective. So in the sentence $(A \bullet B) \supset C$, the scope of the connective $\supset$ includes $(A \bullet B)$ and C. In the sentence $\sim(A \bullet B)$ the scope of the $\sim$ is $(A \bullet B)$. On the other hand, in the sentence $\sim A \bullet B$ the scope of the $\sim$ is just A.

The last connective that you add when you assemble a sentence using the recursive definition is the MAIN CONNECTIVE of that sentence. For example: The main logical operator of $\sim(E \vee (F \supset G))$ is negation, $\sim$. The main logical operator of $(\sim E \vee (F \supset G))$ is disjunction, $\vee$. The main connective of any sentence will have all the rest of the sentence in its scope.

Notational conventions

A sentence like $(Q \bullet R)$ must be surrounded by parentheses, because we might apply the definition again to use this as part of a more complicated sentence. If we negate $(Q \bullet R)$, we get $\sim(Q \bullet R)$. If we just had $Q \bullet R$ without the parentheses and put a negation in front of it, we would have $\sim Q \bullet R$. It is most natural to read this as meaning the same thing as $(\sim Q \bullet R)$, something very different than $\sim(Q \bullet R)$. The sentence $\sim(Q \bullet R)$ means that it is not the case that both Q and R are true; Q might be false or R might be false, but the sentence does not tell us which. The sentence $(\sim Q \bullet R)$ means specifically that Q is false and that R is true. As such, parentheses are crucial to the meaning of the sentence.

So, strictly speaking, $Q \bullet R$ without parentheses is *not* a sentence of SL. When using SL, however, we will often be able to relax the precise definition so as to make things easier for ourselves. We will do this in several ways.

First, we understand that $Q \bullet R$ means the same thing as $(Q \bullet R)$. As a matter of convention, we can leave off parentheses that occur *around the entire sentence*.

Second, it can sometimes be confusing to look at long sentences with many nested pairs of parentheses. We adopt the convention of using square brackets [and] in place of parentheses. There is no logical difference between $(P \vee Q)$ and $[P \vee Q]$, for example. The unwieldy sentence

$$(((H \supset I) \vee (I \supset H)) \bullet (J \vee K))$$

could be written in this way:

$$\big[(H \supset I) \vee (I \supset H)\big] \bullet (J \vee K)$$

Third, we will sometimes want to translate the conjunction of three or more sentences. For the sentence "Alice, Bob, and Candice all went to the party," suppose we let A mean "Alice went," B mean "Bob went," and C mean "Candice went." The definition only allows us to form a conjunction out of two sentences, so we can translate it as $(A \bullet B) \bullet C$ or as $A \bullet (B \bullet C)$. There is no reason to distinguish between these, since the two translations are logically equivalent. There is no logical difference between the first, in which $(A \bullet B)$ is conjoined with C, and the second, in which A is conjoined with $(B \bullet C)$. So we might as well just write $A \bullet B \bullet C$. As a matter of convention, we can leave out parentheses when we conjoin three or more sentences.

Fourth, a similar situation arises with multiple disjunctions. "Either Alice, Bob, or Candice went to the party" can be translated as $(A \vee B) \vee C$ or as $A \vee (B \vee C)$. Since these two translations are logically equivalent, we may write $A \vee B \vee C$.

These latter two conventions only apply to multiple conjunctions or multiple disjunctions. If a series of connectives includes both disjunctions and conjunctions, then the parentheses are essential; as with $(A \bullet B) \vee C$ and $A \bullet (B \vee C)$. The parentheses are also required if there is a series of conditionals or biconditionals; as with $(A \supset B) \supset C$ and $A \equiv (B \equiv C)$.

We have adopted these four rules as notational conventions, not as changes to the definition of a sentence. Strictly speaking, $A \vee B \vee C$ is still not a sentence. Instead, it is a kind of shorthand. We write it for the sake of convenience, but we really mean the sentence $(A \vee (B \vee C))$.

If we had given a different definition for a sentence, then these could count as sentences. We might have written rule 3 in this way: "If $\mathcal{A}$, $\mathcal{B}$, $\ldots$ $\mathcal{Z}$ are sentences, then $(\mathcal{A} \bullet \mathcal{B} \bullet \ldots \bullet \mathcal{Z})$, is a sentence ." This would make it easier to translate some English sentences, but would have the cost of making our formal language more complicated. We would have to keep the complex definition in mind when we develop truth tables and a proof system. We want a logical language that is expressively simple and allows us to translate easily from English, but we also want a formally simple language. Adopting notational conventions is a compromise between these two desires.

Practice Exercises

Part A Using the symbolization key given, translate each English-language sentence into SL.

M: Those creatures are men in suits.
C: Those creatures are chimpanzees.
G: Those creatures are gorillas.

1. Those creatures are not men in suits.

2. Those creatures are men in suits, or they are not.
3. Those creatures are either gorillas or chimpanzees.
4. Those creatures are not gorillas, but they are not chimpanzees either.
5. Those creatures cannot be both gorillas and men in suits.
6. If those creatures are not gorillas, then they are men in suits
7. Those creatures are men in suits only if they are not gorillas.
8. Those creatures are chimpanzees if and only if they are not gorillas.
9. Those creatures are neither gorillas nor chimpanzees.
10. Unless those creatures are men in suits, they are either chimpanzees or they are gorillas.

Part B Using the symbolization key given, translate each English-language sentence into SL.

A: Mister Ace was murdered.
B: The butler did it.
C: The cook did it.
D: The Duchess is lying.
E: Mister Edge was murdered.
F: The murder weapon was a frying pan.

1. Either Mister Ace or Mister Edge was murdered.
2. If Mister Ace was murdered, then the cook did it.
3. If Mister Edge was murdered, then the cook did not do it.
4. Either the butler did it, or the Duchess is lying.
5. The cook did it only if the Duchess is lying.
6. If the murder weapon was a frying pan, then the culprit must have been the cook.
7. If the murder weapon was not a frying pan, then the culprit was neither the cook nor the butler.
8. Mister Ace was murdered if and only if Mister Edge was not murdered.
9. The Duchess is lying, unless it was Mister Edge who was murdered.
10. Mister Ace was murdered, but not with a frying pan.
11. The butler and the cook did not both do it.
12. Of course the Duchess is lying!

Part C Using the symbolization key given, translate each English-language sentence into SL.

E_1: Ava is an electrician.

E_2: Harrison is an electrician.
F_1: Ava is a firefighter.
F_2: Harrison is a firefighter.
S_1: Ava is satisfied with her career.
S_2: Harrison is satisfied with his career.

1. Ava and Harrison are both electricians.
2. If Ava is a firefighter, then she is satisfied with her career.
3. Ava is a firefighter, unless she is an electrician.
4. Harrison is an unsatisfied electrician.
5. Neither Ava nor Harrison is an electrician.
6. Both Ava and Harrison are electricians, but neither of them find it satisfying.
7. Harrison is satisfied only if he is a firefighter.
8. If Ava is not an electrician, then neither is Harrison, but if she is, then he is too.
9. Ava is satisfied with her career if and only if Harrison is not satisfied with his.
10. If Harrison is both an electrician and a firefighter, then he must be satisfied with his work.
11. It cannot be that Harrison is both an electrician and a firefighter.
12. Harrison and Ava are both firefighters if and only if neither of them is an electrician.

Part D Using the symbolization key given, translate each English-language sentence into SL.

J_1: John Coltrane played tenor sax.
J_2: John Coltrane played soprano sax.
J_3: John Coltrane played tuba
M_1: Miles Davis played trumpet
M_2: Miles Davis played tuba

1. John Coltrane played tenor and soprano sax.
2. Neither Miles Davis nor John Coltrane played tuba.
3. John Coltrane did not play both tenor sax and tuba.
4. John Coltrane did not play tenor sax unless he also played soprano sax.
5. John Coltrane did not play tuba, but Miles Davis did.
6. Miles Davis played trumpet only if he also played tuba.
7. If Miles Davis played trumpet, then John Coltrane played at least one of these three instruments: tenor sax, soprano sax, or tuba.
8. If John Coltrane played tuba then Miles Davis played neither trumpet nor tuba.

9. Miles Davis and John Coltrane both played tuba if and only if Coltrane did not play tenor sax and Miles Davis did not play trumpet.

Part E Give a symbolization key and symbolize the following sentences in SL.

1. Alice and Bob are both spies.
2. If either Alice or Bob is a spy, then the code has been broken.
3. If neither Alice nor Bob is a spy, then the code remains unbroken.
4. The German embassy will be in an uproar, unless someone has broken the code.
5. Either the code has been broken or it has not, but the German embassy will be in an uproar regardless.
6. Either Alice or Bob is a spy, but not both.

Part F Give a symbolization key and symbolize the following sentences in SL.

1. If Gregor plays first base, then the team will lose.
2. The team will lose unless there is a miracle.
3. The team will either lose or it won't, but Gregor will play first base regardless.
4. Gregor's mom will bake cookies if and only if Gregor plays first base.
5. If there is a miracle, then Gregor's mom will not bake cookies.

Part G For each argument, write a symbolization key and translate the argument as well as possible into SL.

1. If Dorothy plays the piano in the morning, then Roger wakes up cranky. Dorothy plays piano in the morning unless she is distracted. So if Roger does not wake up cranky, then Dorothy must be distracted.
2. It will either rain or snow on Tuesday. If it rains, Neville will be sad. If it snows, Neville will be cold. Therefore, Neville will either be sad or cold on Tuesday.
3. If Zoog remembered to do his chores, then things are clean but not neat. If he forgot, then things are neat but not clean. Therefore, things are either neat or clean—but not both.

Part H For each argument, write a symbolization key and translate the argument as well as possible into SL. The part of the passage in italics is there to provide context for the argument, and doesn't need to be symbolized.

1. It is going to rain soon. I know because my leg is hurting, and my leg hurts if its going to rain.
2. *Spider-man tries to figure out the bad guys plan.* If Doctor Octopus gets the uranium, he will blackmail the city. I am certain of this because if Doctor Octopus gets the uranium, he can make a dirty bomb, and if he can make a dirty bomb, he will blackmail the city.
3. *A westerner tries to predict the policies of the Chinese government.* If the Chinese government cannot solve the water shortages in Beijing, they will have to move the capital. They dont want to move the capital. Therefore they must solve the water shortage. But the only way to solve the water shortage is to divert almost all the water from the Yangzi river northward. Therefore the Chinese government will go with the project to divert water from the south to the north.

Part I

1. Are there any sentences of SL that contain no sentence letters? Why or why not?
2. In the chapter, we symbolized an *exclusive or* using $\vee$, $\bullet$, and $\sim$. How could you translate an *exclusive or* using only two connectives? Is there any way to translate an *exclusive or* using only one kind of connective?

Key Terms

Antecedent

Atomic sentence

Biconditional

Conditional

Conjunct

Conjunction

Consequent

Disjunct

Disjunction

Main connective

Metalanguage

Metavariables

Negation

Object language

Recursive definition

Scope

Semantics

Sentence letter

Sentence of SL

Sentential connective

Symbolization key

Syntax

Chapter 6

Truth Tables

This chapter introduces a way of evaluating sentences and arguments of SL called the truth table method. As we shall see, the truth table method is *semantic* because it involves one aspect of the meaning of sentences, whether those sentences are true or false. As we saw on page 201, semantics is the study of aspects of language related to meaning, including truth and falsity. Although it can be laborious, the truth table method is a purely mechanical procedure that requires no intuition or special insight.

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6.1 Basic Concepts

A formal language is built from two kinds of elements: logical symbols and nonlogical symbols. LOGICAL SYMBOLS have their meaning fixed by the formal language. In SL, the logical symbols are the sentential connectives and the parentheses. When writing a symbolization key, you are not allowed to change the meaning of the logical symbols. You cannot say, for instance, that the $\sim$ symbol will mean "not" in one argument and "perhaps" in another. The $\sim$ symbol always means logical negation. It is used to translate the English language word "not", but it is a symbol of a formal language and is defined by its truth conditions.

The NONLOGICAL SYMBOLS are defined simply as all the symbols that aren't logical. The nonlogical symbols in SL are the sentence letters. When we translate an argument from English to SL, for example, the sentence letter M does not have its meaning fixed in advance; instead, we provide a symbolization key that says how M should be interpreted in that argument. When translating from English to a formal language, we provided symbolization keys which were interpretations of all the nonlogical symbols we used in the translation.

In logic, when we study artificial languages, we investigate their semantics by providing an interpretation of the nonlogical symbols. An INTERPRETATION is a way of setting up a correspondence between elements of the object language and elements of some other language or logical structure. The symbolization keys we defined in Chapter 5 (p. 188) are a sort of interpretation.

The truth table method will also involve giving an interpretation of sentences, but they will be much simpler than the translation schemes we used in Chapter 2. We will not be concerned with what the individual sentence letters mean. We will only care whether they are true or false. We can do this, because of the way that the meaning of larger sentences is generated by the meaning of their parts.

Any nonatomic sentence of SL is composed of atomic sentences with sentential connectives. The truth value of the compound sentence depends only on the truth value of the atomic sentences that it comprises. In order to know the truth value of $(D \equiv E)$, for instance, you only need to know the truth value of D and the truth value of E. Connectives that work in this way are called truth functional. More technically, we define a TRUTH-FUNCTIONAL CONNECTIVE as an operator that builds larger sentences out of smaller ones, and fixes the truth value of the resulting sentence based only on the truth value of the component sentences.

Because all of the logical symbols in SL are truth functional, the only aspect of meaning we need to worry about in studying the semantics of SL is truth and falsity. If we want to know about the truth of the sentence $A \bullet B$, the only thing we need to know is whether A and B are true. It doesn't actually matter what else they mean. So if A is false, then $A \bullet B$ is false no matter what false sentence A is used to represent. It could be "I am the Pope" or "Pi is equal to 3.19." The larger sentence $A \bullet B$ is still false. So to give an interpretation of sentences in SL, all we need to do is create a truth assignment. A TRUTH ASSIGNMENT is a function that maps the sentence letters in SL onto our two truth values. In other words, we just need to assign Ts and Fs to all our sentence letters.

It is worth knowing that most languages are not built only out of truth functional connectives. In English, it is possible to form a new sentence from any simpler sentence $\mathcal{X}$ by saying "It is possible that $\mathcal{X}$." The truth value of this new sentence

$\mathcal{A}$	$\sim\mathcal{A}$
T	F
F	T

$\mathcal{A}$	$\mathcal{B}$	$\mathcal{A}\bullet\mathcal{B}$	$\mathcal{A}\vee\mathcal{B}$	$\mathcal{A}\supset\mathcal{B}$	$\mathcal{A}\equiv\mathcal{B}$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	T	F
F	F	F	F	T	T

Table 6.1: The characteristic truth tables for the connectives of SL.

does not depend directly on the truth value of $\mathcal{X}$. Even if $\mathcal{X}$ is false, perhaps in some sense $\mathcal{X}$ *could* have been true—then the new sentence would be true. Some formal languages, called *modal logics*, have an operator for possibility. In a modal logic, we could translate "It is possible that $\mathcal{X}$" as $\diamond\mathcal{X}$. However, the ability to translate sentences like these comes at a cost: The $\diamond$ operator is not truth-functional, and so modal logics are not amenable to truth tables.

6.2 Complete Truth Tables

In the last chapter we introduced the characteristic truth tables for the different connectives. To put them all in one place, the truth tables for the connectives of SL are repeated in Table 6.1. Notice that when we did this, we listed all the possible combinations of truth and falsity for the sentence letters in these basic sentences. Each line of the truth table is thus a *truth assignment* for the sentence letters used in the sentence we are giving a truth table for. Thus one line of the truth table is all we need to give an *interpretation* of the sentence, and the full table gives all the possible interpretations of the sentence.

The truth value of sentences that contain only one connective is given by the characteristic truth table for that connective. The characteristic truth table for conjunction, for example, gives the truth conditions for any sentence of the form $(\mathcal{A}\bullet\mathcal{B})$. Even if the conjuncts $\mathcal{A}$ and $\mathcal{B}$ are long, complicated sentences, the conjunction is true if and only if both $\mathcal{A}$ and $\mathcal{B}$ are true. Consider the sentence $(H \bullet I) \supset H$. We consider all the possible combinations of true and false for H and I, which gives us four rows. We then copy the truth values for the sentence letters and write them underneath the letters in the sentence.

H	I	$(H$	$\bullet$	$I)$	$\supset$	H
T	T	**T**		**T**		**T**
T	F	**T**		**F**		**T**
F	T	**F**		**T**		**F**
F	F	**F**		**F**		**F**

Now consider the subsentence $H \bullet I$. This is a conjunction $\mathcal{A} \bullet \mathcal{B}$ with H as $\mathcal{A}$ and with I as $\mathcal{B}$. H and I are both true on the first row. Since a conjunction is true when both conjuncts are true, we write a T underneath the conjunction symbol. We continue for the other three rows and get this:

H	I	$(H$	$\bullet$	$I)$	$\supset$	H
		$\mathcal{A}$	$\bullet$	$\mathcal{B}$		
T	T	T	**T**	T		T
T	F	T	**F**	F		T
F	T	F	**F**	T		F
F	F	F	**F**	F		F

The entire sentence is a conditional $\mathcal{A} \supset \mathcal{B}$ with $(H \bullet I)$ as $\mathcal{A}$ and with H as $\mathcal{B}$. On the second row, for example, $(H \bullet I)$ is false and H is true. Since a conditional is true when the antecedent is false, we write a T in the second row underneath the conditional symbol. We continue for the other three rows and get this:

H	I	$(H$	$\bullet$	$I)$	$\supset$	H
			$\mathcal{A}$		$\supset$	$\mathcal{B}$
T	T		T		**T**	T
T	F		F		**T**	T
F	T		F		**T**	F
F	F		F		**T**	F

The column of Ts underneath the conditional tells us that the sentence $(H \bullet I) \supset H$ is true regardless of the truth values of H and I. They can be true or false in any combination, and the compound sentence still comes out true. It is crucial that we have considered all of the possible combinations. If we only had a two-line truth table, we could not be sure that the sentence was not false for some other combination of truth values.

In this example, we have not repeated all of the entries in every successive table. When actually writing truth tables on paper, however, it is impractical to erase whole columns or rewrite the whole table for every step. Although it is more crowded, the truth table can be written in this way:

H	I	$(H$	$\bullet$	$I)$	$\supset$	H
T	T	T	T	T	T	T
T	F	T	F	F	T	T
F	T	F	F	T	T	F
F	F	F	F	F	T	F

Most of the columns underneath the sentence are only there for bookkeeping purposes. When you become more adept with truth tables, you will probably no longer need to copy over the columns for each of the sentence letters. In any case, the truth value of the sentence on each row is just the column underneath the *main connective* (see p. 203) of the sentence, in this case, the column underneath the conditional.

A COMPLETE TRUTH TABLE is a table that gives all the possible interpretations for a sentence or set of sentences in SL. It has a row for all the possible combinations of T and F for all of the sentence letters. The size of the complete truth table depends on the number of different sentence letters in the table. A sentence that contains only one sentence letter requires only two rows, as in the characteristic truth table for negation. This is true even if the same letter is repeated many times, as in this sentence:

$$[(C \equiv C) \supset C] \bullet {\sim}(C \supset C).$$

The complete truth table requires only two lines because there are only two possibilities: C can be true, or it can be false. A single sentence letter can never be marked both T and F on the same row. The truth table for this sentence looks like this:

C	$[($	C	$\equiv$	C	$)$	$\supset$	C	$]$	$\bullet$	$\sim$	$($	C	$\supset$	C	$)$
T		T	T	T		T	T		**F**	F		T	T	T	
F		F	T	F		F	F		**F**	F		F	T	F	

Looking at the column underneath the main connective, we see that the sentence is false on both rows of the table; i.e., it is false regardless of whether C is true or false.

A sentence that contains two sentence letters requires four lines for a complete truth table, as in the characteristic truth tables and the table for $(H \bullet I) \supset I$.

A sentence that contains three sentence letters requires eight lines. For example:

M	N	P	M	$\bullet$	$(N$	$\vee$	$P)$
T	T	T	T	**T**	T	T	T
T	T	F	T	**T**	T	T	F
T	F	T	T	**T**	F	T	T
T	F	F	T	**F**	F	F	F
F	T	T	F	**F**	T	T	T
F	T	F	F	**F**	T	T	F
F	F	T	F	**F**	F	T	T
F	F	F	F	**F**	F	F	F

From this table, we know that the sentence $M \bullet (N \vee P)$ might be true or false, depending on the truth values of M, N, and P.

A complete truth table for a sentence that contains four different sentence letters requires 16 lines. For five letters, 32 lines are required. For six letters, 64 lines, and so on. To be perfectly general: If a complete truth table has n different sentence letters, then it must have 2^n rows.

In order to fill in the columns of a complete truth table, begin with the right-most sentence letter and alternate Ts and Fs. In the next column to the left, write two Ts, write two Fs, and repeat. For the third sentence letter, write four Ts followed by four Fs. This yields an eight line truth table like the one above. For a 16 line truth table, the next column of sentence letters should have eight Ts followed by eight Fs. For a 32 line table, the next column would have 16 Ts followed by 16 Fs. And so on.

Practice Exercises

Part A Identify the main connective in the each sentence.

1. $\sim(A \vee \sim B)$
2. $\sim(A \vee \sim B) \vee \sim(A \bullet D)$
3. $[\sim(A \vee \sim B) \vee \sim(A \bullet D)] \supset E$
4. $[(A \supset B) \bullet C] \equiv [A \vee (B \bullet C)]$
5. $\sim\sim\sim[A \vee (B \bullet (C \vee D))]$

Part B Identify the main connective in the each sentence.

1. $[(A \equiv B) \bullet C] \supset D$
2. $[(D \bullet (E \bullet F)) \vee G] \equiv {\sim}[A \supset (C \vee G)]$
3. ${\sim}({\sim}Z \vee {\sim}H)$
4. $({\sim}(P \bullet S) \equiv G) \bullet Y$
5. $(A \bullet (B \supset C)) \vee {\sim}D$

Part C Assume A, B, and C are true and X, Y, and Z are false and evaluate the truth of the each sentence.

1. ${\sim}((A \bullet B) \supset X)$
2. $(Y \vee Z) \equiv ({\sim}X \equiv B)$
3. $[(X \supset A) \vee (A \supset X)] \bullet Y$
4. $(X \supset A) \vee (A \supset X)$
5. $[A \bullet (Y \bullet Z)] \vee A$

Part D Assume A, B, and C are true and X, Y, and Z are false and evaluate the truth of the each sentence.

1. ${\sim}{\sim}({\sim}{\sim}{\sim}A \vee X)$
2. $(A \supset B) \supset X$
3. $((A \vee B) \bullet (C \equiv X)) \vee Y$
4. $(A \supset B) \vee (X \bullet (Y \bullet Z))$
5. $((A \vee X) \supset Y) \bullet B$

Part E Write complete truth tables for the following sentences and mark the column that represents the possible truth values for the whole sentence.

1. ${\sim}(S \equiv (P \supset S))$
2. ${\sim}[(X \bullet Y) \vee (X \vee Y)]$
3. $(A \supset B) \equiv ({\sim}B \equiv {\sim}A)$
4. $[C \equiv (D \vee E)] \bullet {\sim}C$
5. ${\sim}(G \bullet (B \bullet H)) \equiv (G \vee (B \vee H))$

Part F Write complete truth tables for the following sentences and mark the column that represents the possible truth values for the whole sentence.

1. $(D \bullet \sim D) \supset G$
2. $(\sim P \vee \sim M) \equiv M$
3. $\sim\sim(\sim A \bullet \sim B)$
4. $[(D \bullet R) \supset I] \supset \sim(D \vee R)$
5. $\sim[(D \equiv O) \equiv A] \supset (\sim D \bullet O)$

6.3 Using Truth Tables

A complete truth table shows us every possible combination of truth assignments on the sentence letters. It tells us every possible way sentences can relate to truth. We can use this to discover all sorts of logical properties of sentences and sets of sentences.

Tautologies, contradictions, and contingent sentences

We defined a tautology as a statement that must be true as a matter of logic, no matter how the world is (p. **??**). A statement like "Either it is raining or it is not raining" is always true, no matter what the weather is like outside. Something similar goes on in truth tables. With a complete truth table, we consider all of the ways that the world might be. Each line of the truth table corresponds to a way the world might be. This means that if the sentence is true on every line of a complete truth table, then it is true as a matter of logic, regardless of what the world is like.

We can use this fact to create a test for whether a sentence is a tautology: if the column under the main connective of a sentence is a T on every row, the sentence is a tautology. Not every tautology in English will correspond to a tautology in SL. The sentence "All bachelors are unmarried" is a tautology in English, but we cannot represent it as a tautology in SL, because it just translates as a single sentence letter, like B. On the other hand, if something is a tautology in SL, it will also be a tautology in English. No matter how you translate $A \vee \sim A$, if you translate the As consistently, the statement will be a tautology.

Rather than thinking of complete truth tables as an imperfect test for the English notion of a tautology, we can define a separate notion of a tautology in SL based on truth tables. A statement is a SEMANTIC TAUTOLOGY IN SL if and only if the column under the main connective in the complete truth table for the sentence contains only Ts. This is actually the semantic definition of a tautology in SL, because it uses truth tables. Later we will create a separate, syntactic definition and show that it is equivalent to the semantic definition. We will be doing the same thing for all the concepts defined in this section.

Conversely, we defined a contradiction as a sentence that is false no matter how the world is (p. **??**). This means we can define a SEMANTIC CONTRADICTION IN SL as a sentence that has only Fs in the column under them main connective of its complete truth table. Again, this is the semantic definition of a contradiction.

Finally, a sentence is contingent if it is sometimes true and sometimes false (p. **??**). Similarly, a sentence is SEMANTICALLY CONTINGENT IN SL if and only if its complete truth table for has both Ts and Fs under the main connective.

From the truth tables in the previous section, we know that $(H \bullet I) \supset H$ is a tautology (p. 215), that $[(C \equiv C) \supset C] \bullet {\sim}(C \supset C)$ is a contradiction (p. 215), and that $M \bullet (N \vee P)$ is contingent (p. 216).

Logical equivalence

Two sentences are logically equivalent in English if they have the same truth value as a matter of logic (p. **??**). Once again, we can use truth tables to define a similar property in SL: Two sentences are SEMANTICALLY LOGICALLY EQUIVALENT IN SL if they have the same truth value on every row of a complete truth table.

Consider the sentences ${\sim}(A \vee B)$ and ${\sim}A \bullet {\sim}B$. Are they logically equivalent? To find out, we construct a truth table.

A	B	$\sim$	$(A$	$\vee$	$B)$	$\sim$	A	$\bullet$	$\sim$	B
T	T	**F**	T	T	T	F	T	**F**	F	T
T	F	**F**	T	T	F	F	T	**F**	T	F
F	T	**F**	F	T	T	T	F	**F**	F	T
F	F	**T**	F	F	F	T	F	**T**	T	F

Look at the columns for the main connectives; negation for the first sentence, conjunction for the second. On the first three rows, both are F. On the final row, both are T. Since they match on every row, the two sentences are logically equivalent.

Consistency

A set of sentences in English is consistent if it is logically possible for them all to be true at once (p. ??). This means that a sentence is SEMANTICALLY CONSISTENT IN SL if and only if there is at least one line of a complete truth table on which all of the sentences are true. It is semantically inconsistent otherwise.

Validity

Logic is the study of argument, so the most important use of truth tables is to test the validity of arguments. An argument in English is valid if it is logically impossible for the premises to be true and for the conclusion to be false at the same time (p. 34). So we can define an argument as SEMANTICALLY VALID IN SL if there is no row of a complete truth table on which the premises are all marked "T" and the conclusion is marked "F." An argument is invalid if there is such a row.

Consider this argument:

1. $\sim L \supset (J \vee L)$
2. $\sim L$

$\therefore\ J$

Is it valid? To find out, we construct a truth table.

J	L	$\sim$	L	$\supset$	$(J$	$\vee$	$L)$	$\sim$	L	J
T	T	F	T	**T**	T	T	T	**F**	T	**T**
T	F	T	F	**T**	T	T	F	**T**	F	**T**
F	T	F	T	**T**	F	T	T	**F**	T	**F**
F	F	T	F	**F**	F	F	F	**T**	F	**F**

Yes, the argument is valid. The only row on which both the premises are T is the second row, and on that row the conclusion is also T.

In Chapters 1 and 2 we used the three dots $\therefore$ to represent an inference in English. We used this symbol to represent any kind of inference. The truth table method gives us a more specific notion of a valid inference. We will call this semantic entailment and represent it using a new symbol, $\models$, called the "double turnstile." The $\models$ is like the $\therefore$, except for arguments verified by truth tables. When you use the double turnstile, you write the premises as a set, using curly brackets, { and }, which mathematicians use in set theory. The argument above would be written $\{\sim L \supset (J \vee L), \sim L\} \models J$.

More formally, we can define the double turnstile this way: $\{\mathcal{A}_1 \ldots \mathcal{A}_n\} \models \mathcal{B}$ if and only if there is no truth value assignment for which $\mathcal{A}_1 \ldots \mathcal{A}_n$ are true and $\mathcal{B}$ is false. Put differently, it means that $\mathcal{B}$ is true for any and all truth value assignments for which $\mathcal{A}_1 \ldots \mathcal{A}_n$ are true.

We can also use the double turnstile to represent other logical notions. Since a tautology is always true, it is like the conclusion of a valid argument with no premises. The string $\models \mathcal{C}$ means that $\mathcal{C}$ is true for all truth value assignments. This is equivalent to saying that the sentence is entailed by anything. We can represent logical equivalence by writing the double turnstile in both directions: $\mathcal{A} \Dashv\models \mathcal{B}$ For instance, if we want to point out that the sentence $A \bullet B$ is equivalent to $B \bullet A$ we would write this: $A \bullet B \Dashv\models B \bullet A$.

Practice Exercises

If you want additional practice, you can construct truth tables for any of the sentences and arguments in the exercises for the previous chapter.

Part A Determine whether each sentence is a tautology, a contradiction, or a contingent sentence, using a complete truth table.

1. $A \supset A$
2. $C \supset \sim C$
3. $(A \equiv B) \equiv \sim(A \equiv \sim B)$
4. $(A \supset B) \vee (B \supset A)$
5. $(A \bullet B) \supset (B \vee A)$
6. $[(\sim A \vee A) \vee B] \supset B$
7. $[(A \vee B) \bullet \sim A] \bullet (B \supset A)$

Part B Determine whether each sentence is a tautology, a contradiction, or a contingent sentence, using a complete truth table.

1. $\sim B \bullet B$
2. $\sim D \vee D$
3. $(A \bullet B) \vee (B \bullet A)$
4. $\sim[A \supset (B \supset A)]$
5. $A \equiv [A \supset (B \bullet \sim B)]$
6. $[(A \bullet B) \equiv B] \supset (A \supset B)$

Part C Determine whether each the following statements are equivalent using complete truth tables. If the two sentences really are logically equivalent, write "Logically equivalent." Otherwise write, "Not logically equivalent."

1. $A \dashv\vDash \sim A$
2. $A \bullet \sim A \dashv\vDash \sim B \equiv B$
3. $[(A \vee B) \vee C] \dashv\vDash [A \vee (B \vee C)]$
4. $A \vee (B \bullet C) \dashv\vDash (A \vee B) \bullet (A \vee C)$
5. $[A \bullet (A \vee B)] \supset B \dashv\vDash A \supset B$

Part D Determine whether each the following statements of equivalence are true or false using complete truth tables. If the two sentences really are logically equivalent, write "Logically equivalent." Otherwise write, "Not logically equivalent."

1. $A \supset A \dashv\vDash A \equiv A$
2. $\sim(A \supset B) \dashv\vDash \sim A \supset \sim B$
3. $A \vee B \dashv\vDash \sim A \supset B$
4. $(A \supset B) \supset C \dashv\vDash A \supset (B \supset C)$
5. $A \equiv (B \equiv C) \dashv\vDash A \bullet (B \bullet C)$

Part E Determine whether each set of sentences is consistent or inconsistent using a complete truth table.

1. $\{A \bullet \sim B, \sim(A \supset B), B \supset A\}$
2. $\{A \vee B, A \supset \sim A, B \supset \sim B\}$
3. $\{\sim(\sim A \vee B), A \supset \sim C, A \supset (B \supset C)\}$

4. $\{A \supset B,\ A \bullet {\sim}B\}$
5. $\{A \supset (B \supset C),\ (A \supset B) \supset C,\ A \supset C\}$

Part F Determine whether each set of sentences is consistent or inconsistent, using a complete truth table.

1. $\{{\sim}B,\ A \supset B,\ A\}$
2. $\{{\sim}(A \vee B),\ A \equiv B,\ B \supset A\}$
3. $\{\ A \vee B,\ {\sim}B,\ {\sim}B \supset {\sim}A\}$
4. $\{A \equiv B,\ {\sim}B \vee {\sim}A,\ A \supset B\}$
5. $\{(A \vee B) \vee C,\ {\sim}A \vee {\sim}B,\ {\sim}C \vee {\sim}B\}$

Part G Determine whether each argument is valid or invalid, using a complete truth table.

1. $A \supset A \models A$
2. $A \supset B,\ B \models A$
3. $A \equiv B,\ B \equiv C \models A \equiv C$
4. $A \supset B,\ A \supset C \models B \supset C$
5. $A \supset B,\ B \supset A \models A \equiv B$

Part H Determine whether each argument is valid or invalid, using a complete truth table.

1. $A \vee \big[A \supset (A \equiv A)\big] \models A$
2. $A \vee B,\ B \vee C,\ {\sim}B \models A \bullet C$
3. $A \supset B,\ {\sim}A \models {\sim}B$
4. $A,\ B \models {\sim}(A \supset {\sim}B)$
5. ${\sim}(A \bullet B),\ A \vee B,\ A \equiv B \models C$

6.4 Partial Truth Tables

In order to show that a sentence is a tautology, we need to show that it is T on every row. So we need a complete truth table. To show that a sentence is *not* a tautology, however, we only need one line: a line on which the sentence is F. Therefore, in order to show that something is not a tautology, it is enough to provide a one-line *partial truth table*—regardless of how many sentence letters the sentence might have in it.

Consider, for example, the sentence $(U \bullet T) \supset (S \bullet W)$. We want to show that it is *not* a tautology by providing a partial truth table. We fill in F for the entire sentence. The main connective of the sentence is a conditional. In order for the conditional to be false, the antecedent must be true (T) and the consequent must be false (F). So we fill these in on the table:

S	T	U	W	$(U$	$\bullet$	$T)$	$\supset$	$(S$	$\bullet$	$W)$
					T		**F**		F	

In order for the $(U \bullet T)$ to be true, both U and T must be true.

S	T	U	W	$(U$	$\bullet$	$T)$	$\supset$	$(S$	$\bullet$	$W)$
	T	T		T	T	T	**F**		F	

Now we just need to make $(S \bullet W)$ false. To do this, we need to make at least one of S and W false. We can make both S and W false if we want. All that matters is that the whole sentence turns out false on this line. Making an arbitrary decision, we finish the table in this way:

S	T	U	W	$(U$	$\bullet$	$T)$	$\supset$	$(S$	$\bullet$	$W)$
F	T	T	F	T	T	T	**F**	F	F	F

Showing that something is a contradiction requires a complete truth table. Showing that something is *not* a contradiction requires only a one-line partial truth table, where the sentence is true on that one line.

A sentence is contingent if it is neither a tautology nor a contradiction. So showing that a sentence is contingent requires a *two-line* partial truth table: The sentence must be true on one line and false on the other. For example, we can show that the sentence above is contingent with this truth table:

S	T	U	W	$(U$	$\bullet$	$T)$	$\supset$	$(S$	$\bullet$	$W)$
F	T	T	F	T	T	T	**F**	F	F	F
F	T	F	F	F	F	T	**T**	F	F	F

Property	Present	Absent
tautology	complete truth table	one-line partial truth table
contradiction	complete truth table	one-line partial truth table
contingent	two-line partial truth table	complete truth table
equivalent	complete truth table	one-line partial truth table
consistent	one-line partial truth table	complete truth table
valid	complete truth table	one-line partial truth table

Table 6.2: Complete or partial truth tables to test for different properties

Note that there are many combinations of truth values that would have made the sentence true, so there are many ways we could have written the second line.

Showing that a sentence is *not* contingent requires providing a complete truth table, because it requires showing that the sentence is a tautology or that it is a contradiction. If you do not know whether a particular sentence is contingent, then you do not know whether you will need a complete or partial truth table. You can always start working on a complete truth table. If you complete rows that show the sentence is contingent, then you can stop. If not, then complete the truth table. Even though two carefully selected rows will show that a contingent sentence is contingent, there is nothing wrong with filling in more rows.

Showing that two sentences are logically equivalent requires providing a complete truth table. Showing that two sentences are *not* logically equivalent requires only a one-line partial truth table: Make the table so that one sentence is true and the other false.

Showing that a set of sentences is consistent requires providing one row of a truth table on which all of the sentences are true. The rest of the table is irrelevant, so a one-line partial truth table will do. Showing that a set of sentences is inconsistent, on the other hand, requires a complete truth table: You must show that on every row of the table at least one of the sentences is false.

Showing that an argument is valid requires a complete truth table. Showing that an argument is *invalid* only requires providing a one-line truth table: If you can produce a line on which the premises are all true and the conclusion is false, then the argument is invalid.

Table 6.2 summarizes when a complete truth table is required and when a partial truth table will do.

6.5 Expressive Completeness

We could leave the biconditional ($\equiv$) out of the language. If we did that, we could still write "$A \equiv B$" so as to make sentences easier to read, but that would be shorthand for $(A \supset B) \bullet (B \supset A)$. The resulting language would be formally equivalent to SL, since $A \equiv B$ and $(A \supset B) \bullet (B \supset A)$ are logically equivalent in SL. If we valued formal simplicity over expressive richness, we could replace more of the connectives with notational conventions and still have a language equivalent to SL.

There are a number of equivalent languages with only two connectives. You could do logic with only the negation and the material conditional. Alternately you could just have the negation and the disjunction. You will be asked to prove that these things are true in the last problem set. You could even have a language with only one connective, if you designed the connective right. The *Sheffer stroke* is a logical connective with the following characteristic truth table:

$\mathcal{A}$	$\mathcal{B}$	$\mathcal{A}\vert\mathcal{B}$
T	T	F
T	F	T
F	T	T
F	F	T

The Sheffer stroke has the unique property that it is the only connective you need to have a complete system of logic. You will be asked to prove that this is true in the last problem set also.

Practice Exercises

If you want additional practice, you can construct truth tables for any of the sentences and arguments in the exercises for the previous chapter.

Part A Determine whether each sentence is a tautology, a contradiction, or a contingent sentence. Justify your answer with a complete or partial truth table where appropriate.

1. $A \supset {\sim}A$
2. $A \supset (A \bullet (A \lor B))$
3. $(A \supset B) \equiv (B \supset A)$
4. $A \supset {\sim}(A \bullet (A \lor B))$
5. ${\sim}B \supset [({\sim}A \bullet A) \lor B]$
6. ${\sim}(A \lor B) \equiv ({\sim}A \bullet {\sim}B)$
7. $[(A \bullet B) \bullet C] \supset B$
8. ${\sim}\big[(C \lor A) \lor B\big]$
9. $\big[(A \bullet B) \bullet {\sim}(A \bullet B)\big] \bullet C$
10. $(A \bullet B)] \supset [(A \bullet C) \lor (B \bullet D)]$

Part B Determine whether each sentence is a tautology, a contradiction, or a contingent sentence. Justify your answer with a complete or partial truth table where appropriate.

1. ${\sim}(A \lor A)$
2. $(A \supset B) \lor (B \supset A)$
3. $[(A \supset B) \supset A] \supset A$
4. ${\sim}[(A \supset B) \lor (B \supset A)]$
5. $(A \bullet B) \lor (A \lor B)$
6. ${\sim}(A \bullet B) \equiv A$
7. $A \supset (B \lor C)$
8. $(A \bullet {\sim}A) \supset (B \lor C)$
9. $(B \bullet D) \equiv [A \equiv (A \lor C)]$
10. ${\sim}[(A \supset B) \lor (C \supset D)]$

Part C Determine whether each the following statements of equivalence are true or false using complete truth tables. If the two sentences really are logically equivalent, write "Logically equivalent." Otherwise write, "Not logically equivalent."

1. $A \mathrel{=\!\!\!|}\, \models {\sim}A$
2. $A \supset A \mathrel{=\!\!\!|}\, \models A \equiv A$
3. $A \bullet (B \bullet C) \mathrel{=\!\!\!|}\, \models A \bullet {\sim}A$
4. $A \bullet {\sim}A \mathrel{=\!\!\!|}\, \models {\sim}B \equiv B$

5. $\sim(A \supset B) \dashv\models \sim A \supset \sim B$
6. $A \equiv B \dashv\models \sim[(A \supset B) \supset \sim(B \supset A)]$
7. $(A \bullet B) \supset (\sim A \vee \sim B) \dashv\models \sim(A \bullet B)$
8. $[(A \vee B) \vee C] \dashv\models [A \vee (B \vee C)]$
9. $(Z \bullet (\sim R \supset O)) \dashv\models \sim(R \supset \sim O)$

Part D Determine whether each the following statements of equivalence are true or false using complete truth tables. If the two sentences really are logically equivalent, write "Logically equivalent." Otherwise write, "Not logically equivalent."

1. $A \dashv\models A \vee A$
2. $A \dashv\models A \bullet A$
3. $A \vee \sim B \dashv\models A \supset B$
4. $(A \supset B) \dashv\models (\sim B \supset \sim A)$
5. $\sim(A \bullet B) \dashv\models \sim A \vee \sim B$
6. $((U \supset (X \vee X)) \vee U) \dashv\models \sim(X \bullet (X \bullet U))$
7. $((C \bullet (N \equiv C)) \equiv C) \dashv\models (\sim\sim\sim N \supset C)$
8. $[(A \vee B) \bullet C] \dashv\models [A \vee (B \bullet C)]$
9. $((L \bullet C) \bullet I) \dashv\models L \vee C$

Part E Determine whether each set of sentences is consistent or inconsistent. Justify your answer with a complete or partial truth table where appropriate.

1. $\{A \supset A, \sim A \supset \sim A, A \bullet A, A \vee A\}$
2. $\{A \supset \sim A, \sim A \supset A\}$
3. $\{A \vee B, A \supset C, B \supset C\}$
4. $\{A \vee B, A \supset C, B \supset C, \sim C\}$
5. $\{B \bullet (C \vee A), A \supset B, \sim(B \vee C)\}$
6. $\{(A \equiv B) \supset B, B \supset \sim(A \equiv B), A \vee B\}$
7. $\{A \equiv (B \vee C), C \supset \sim A, A \supset \sim B\}$
8. $\{A \equiv B, \sim B \vee \sim A, A \supset B\}$
9. $\{A \equiv B, A \supset C, B \supset D, \sim(C \vee D)\}$

10. $\{{\sim}(A \bullet {\sim}B),\ B \supset {\sim}A,\ {\sim}B\ \}$

Part F Determine whether each set of sentences is consistent or inconsistent. Justify your answer with a complete or partial truth table where appropriate.

1. $\{A \bullet B,\ C \supset {\sim}B,\ C\}$
2. $\{A \supset B,\ B \supset C,\ A,\ {\sim}C\}$
3. $\{A \vee B,\ B \vee C,\ C \supset {\sim}A\}$
4. $\{A,\ B,\ C,\ {\sim}D,\ {\sim}E,\ F\}$
5. $\{A \bullet (B \vee C),\ {\sim}(A \bullet C),\ {\sim}(B \bullet C)\}$
6. $\{A \supset B,\ B \supset C,\ {\sim}(A \supset C)\}$

Part G Determine whether each argument is valid or invalid. Justify your answer with a complete or partial truth table where appropriate.

1. $A \supset (A \bullet {\sim}A) \models {\sim}A$
2. $A \vee B,\ A \supset B,\ B \supset A \models A \equiv B$
3. $A \vee (B \supset A) \models {\sim}A \supset {\sim}B$
4. $A \vee B,\ A \supset B,\ B \supset A \models A \bullet B$
5. $(B \bullet A) \supset C,\ (C \bullet A) \supset B \models (C \bullet B) \supset A$
6. ${\sim}({\sim}A \vee {\sim}B),\ A \supset {\sim}C \models A \supset (B \supset C)$
7. $A \bullet (B \supset C),\ {\sim}C \bullet ({\sim}B \supset {\sim}A) \models C \bullet {\sim}C$
8. $A \bullet B,\ {\sim}A \supset {\sim}C,\ B \supset {\sim}D \models A \vee B$
9. $A \supset B \models (A \bullet B) \vee ({\sim}A \bullet {\sim}B)$
10. ${\sim}A \supset B,{\sim}B \supset C,{\sim}C \supset A \models {\sim}A \supset ({\sim}B \vee {\sim}C)$

Part H Determine whether each argument is valid or invalid. Justify your answer with a complete or partial truth table where appropriate.

1. $A \equiv {\sim}(B \equiv A) \models A$

2. $A \vee B$, $B \vee C$, $\sim A \models B \bullet C$
3. $A \supset C$, $E \supset (D \vee B)$, $B \supset \sim D \models (A \vee C) \vee (B \supset (E \bullet D))$
4. $A \vee B$, $C \supset A$, $C \supset B \models A \supset (B \supset C)$
5. $A \supset B$, $\sim B \vee A \models A \equiv B$

Part I Answer each of the questions below and justify your answer.

1. Suppose that $\mathcal{A}$ and $\mathcal{B}$ are logically equivalent. What can you say about $\mathcal{A} \equiv \mathcal{B}$?
2. Suppose that $(\mathcal{A} \bullet \mathcal{B}) \supset \mathcal{C}$ is contingent. What can you say about the argument "$\mathcal{A}$, $\mathcal{B}$, $\therefore$ $\mathcal{C}$"?
3. Suppose that $\{\mathcal{A}, \mathcal{B}, \mathcal{C}\}$ is inconsistent. What can you say about $(\mathcal{A} \bullet \mathcal{B} \bullet \mathcal{C})$?
4. Suppose that $\mathcal{A}$ is a contradiction. What can you say about the argument $\{\mathcal{A}$, $\mathcal{B}\} \models \mathcal{C}$?
5. Suppose that $\mathcal{C}$ is a tautology. What can you say about the argument $\{\mathcal{A}$, $\mathcal{B}\}$ $\models \mathcal{C}$"?
6. Suppose that $\mathcal{A}$ and $\mathcal{B}$ are *not* logically equivalent. What can you say about $(\mathcal{A} \vee \mathcal{B})$?

Part J

1. In section 3.5, we said that you could have a language that only used the negation and the material conditional. Prove that this is true by writing sentences that are logically equivalent to each of the following using only parentheses, sentence letters, negation ($\sim$), and the material conditional ($\supset$).
   (a) $A \vee B$
   (b) $A \bullet B$
   (c) $A \equiv B$
2. We also said in section 3.5 that you could have a language which used only the negation and the disjunction. Show this: Using only parentheses, sentence letters, negation ($\sim$), and disjunction ($\vee$), write sentences that are logically equivalent to each of the following.

(a) $A \bullet B$
(b) $A \supset B$
(c) $A \equiv B$

3. Write a sentence using the connectives of SL that is logically equivalent to $(A|B)$.
4. Every sentence written using a connective of SL can be rewritten as a logically equivalent sentence using one or more Sheffer strokes. Using only the Sheffer stroke, write sentences that are equivalent to each of the following.

   (a) $\sim A$
   (b) $(A \bullet B)$
   (c) $(A \vee B)$
   (d) $(A \supset B)$
   (e) $(A \equiv B)$

Key Terms

Complete truth table

Interpretation

Logical symbol

Nonlogical symbol

Semantically consistent in SL

Semantically contingent in SL

Semantically logically equivalent in SL

Semantically valid in SL

Semantic contradiction in SL

Semantic tautology in SL

Truth-functional connective

Truth assignment

Chapter 7

Proofs in Sentential Logic

7.1 Semantics and Syntax

In the last chapter, we introduced the truth table method, which allowed us to check to see if various logical properties were present, such as whether a statement is a tautology or whether an argument is valid. The method in that chapter was semantic, because it relied on the meaning of symbols, specifically, whether they were interpreted as true or false. The nice thing about that method was that it was completely mechanical. If you just followed the rules like a robot, you would eventually get the right answer. You didn't need any special insight and there were no tough decisions to make. The downside to this method was that the tables quickly became way too long. It just isn't practical to make a 32 line table every time you have to deal with five different sentence letters.

In this chapter, we are going to introduce a new method for checking for validity and other logical properties. This time our method is going to be purely syntactic. We won't be at all concerned with what our symbols mean. We are just going to look at the way they are arranged. Our method here will be called a system of natural deduction. When you use a system of natural deduction, you won't do it mechanically. You will need to understand the logical structure of the argument and employ your insight. This is actually one of the reasons people like systems of natural deduction. They let us represent the logical structure of arguments in a way we can understand. Learning to represent and manipulate arguments this way is a core mental skill, used in fields like mathematics and computer programming.

7.2 Substitution Instances and Proofs

Consider two arguments in SL:

Argument A

P_1: $P \vee Q$
P_2: ${\sim}P$
C: Q

Argument B

P_1: $P \supset Q$
P_2: P
C: Q

These are both valid arguments. Go ahead and prove that for yourself by constructing the four-line truth tables. These particular valid arguments are examples of important kinds of arguments that are given special names. Argument A is an example of a kind of argument traditionally called *disjunctive syllogism*. Given a disjunction and the negation of one of the disjuncts, the other disjunct follows as a valid consequence. Argument B makes use of a different valid form: Given a conditional and its antecedent, the consequent follows as a valid consequence. This is traditionally called *modus ponens*.

Both of the arguments above remain valid even if we substitute different sentence letters. You don't even need to run the truth tables again to see that these arguments are valid:

Argument A*

P_1: $A \vee B$
P_2: ${\sim}A$
C: B

Argument B*

P_1: $A \supset B$
P_2: A
C: B

Replacing P with A and Q with B changes nothing (so long as we are sure to replace *every* P with an A and every Q with a B). What's more interesting is that we can replace the individual sentence letters in Argument A and Argument B with longer sentences in SL and the arguments will still be valid, as long as we do the substitutions consistently. Here are two more perfectly valid instances of disjunctive syllogism and *modus ponens*.

Argument A**

P$_1$: $(C \bullet D) \vee (E \vee F)$
P$_2$: $\sim(C \bullet D)$

C: $E \vee F$

Argument B**

P$_1$: $(G \supset H) \supset (I \vee J)$
P$_2$: $(G \supset H)$

C: $I \vee J$

Again, you can check these using truth tables, although the 16 line truth tables begin to get tiresome. All of these arguments are what we call *substitution instances* of the same two logical forms. We call them that because you get them by replacing the sentence letters with other sentences, either sentence letters or longer sentences in SL. A substitution instance cannot change the sentential connectives of a sentence, however. The sentential connectives are what make the *logical form* of the sentence. We can write these logical forms using fancy script letters.

Disjunctive Syllogism
(DS)

P$_1$: $\mathcal{A} \vee \mathcal{B}$
P$_2$: $\sim\mathcal{A}$

C: $\mathcal{B}$

Modus Ponens
(MP)

P$_1$: $\mathcal{A} \supset \mathcal{B}$
P$_2$: $\mathcal{A}$

C: $\mathcal{B}$

As we explained in Chapter 5, the fancy script letters are *metavariables.* They are a part of our metalanguage and can refer to single sentence letters like P or longer sentences like $A \equiv (B \bullet (C \vee D))$.

Formally, we can define a **sentence form** as a sentence in SL that contains one or more metavariables in place of sentence letters. A **substitution instance** of that sentence form is then a sentence created by consistently substituting sentences for one or more of the metavariables in the sentence form. Here "consistently substituting" means replacing all instances of the metavariable with the same sentence. You cannot replace instances of the same metavariable with different sentences, or leave a metavariable as it is, if you have replaced other metavariables of that same type. An **argument form** is an argument that includes one or more sentence forms, and a **substitution instance** of the argument form is the argument obtained by consistently replacing the sentence forms in the argument form with their substitution instances.

Once we start identifying valid argument forms like this, we have a new way of showing that longer arguments are valid. Truth tables are fun, but doing the 1024 line truth table for an argument with 10 sentence letters would be tedious. Worse, we would never be sure we hadn't made a little mistake in all those Ts and Fs. Part of the problem is that we have no way of knowing *why* the argument is valid. The table gives you very little insight into how the premises work together.

The aim of a *proof system* is to show that particular arguments are valid in a way that allows us to understand the reasoning involved in the argument. Instead of representing all the premises and the conclusion in one table, we break the argument up into steps. Each step is a basic argument form of the sort we saw above, like disjunctive syllogism or *modus ponens*. Suppose we are given the premises $\sim L \supset (J \vee L)$ and $\sim L$ and wanted to show J. We can break this up into two smaller arguments, each of which is a substitution instance of a form we know is correct.

Argument 1	**Argument 2**
P_1: $\sim L \supset (J \vee L)$	P_1: $J \vee L$
P_2: $\sim L$	P_2: $\sim L$
C: $J \vee L$	C: J

The first argument is a substitution instance of *modus ponens* and the second is a substitution instance of disjunctive syllogism, so we know they are both valid. Notice also that the conclusion of the first argument is the first premise of the second, and the second premise is the same in both arguments. Together, these arguments are enough to get us from $\sim L \supset (J \vee L)$ and $\sim L$ to J.

These two arguments take up a lot of space, though. To complete our proof system, we need a system for showing clearly how simple steps can combine to get us from premises to conclusions. The system we will use in this book was devised by the American logician Irving Marmer Copi (1917–2002). We begin by writing our premises on numbered lines with a bar on the left and a little bar underneath to represent the end of the premises. Then we write "Want" on the side followed by the conclusion we are trying to reach. If we wanted to write out arguments 1 and 2 above, we would begin like this.

1	$\sim L \supset (J \vee L)$	
2	$\sim L$	Want: J

We then add the steps leading to the conclusion below the horizontal line, each time explaining off to the right why we are allowed to write the new line. This explanation consists of citing a rule and the prior lines the rule is applied to. In the example we have been working with we would begin like this

1	$\sim L \supset (J \vee L)$	
2	$\sim L$	Want: J
3	$J \vee L$	MP 1, 2

and then go like this

1	$\sim L \supset (J \vee L)$	
2	$\sim L$	Want: J
3	$J \vee L$	MP 1, 2
4	J	DS 2, 3

The little chart above is a *proof* that J follows from $\sim L \supset (J \vee L)$ and $\sim L$. We will also call proofs like this *derivations*. Formally, a **proof** is a sequence of sentences. The first sentences of the sequence are assumptions; these are the premises of the argument. Every sentence later in the sequence follows from earlier sentences by one of the **rules of inference**. A rule of inference is a valid argument form like disjunctive syllogism or *modus ponens*, which lets us add new sentences to a proof. The final sentence of the sequence is the conclusion of the argument.

Practice Exercises

Part A For each problem, a sentence form is given in metavariables. Identify which of the sentences after it are legitimate substitution instances of that form.

(1) $\mathcal{A} \bullet \mathcal{B}$:

a. $P \vee Q$

b. $(A \supset B) \bullet C$

c. $[(A \bullet B) \supset (B \bullet A)] \bullet (\sim A \bullet \sim B)$

d. $[((A \bullet B) \bullet C) \bullet D] \bullet F$

e. $(A \bullet B) \supset C$

(2) $\sim(\mathcal{P} \bullet \mathcal{Q})$

a. $\sim(A \bullet B)$

b. $\sim(A \bullet A)$

c. $\sim A \bullet B$

d. $\sim((\sim A \bullet B) \bullet (B \bullet \sim A))$

e. $\sim(A \supset B)$

(3) $\sim\mathcal{A}$

a. $\sim A \supset B$

b. $\sim(A \supset B)$

c. $\sim[(G \supset (H \vee I)) \supset G]$

d. $\sim G \bullet (\sim B \bullet \sim H)$

e. $\sim(G \bullet (B \bullet H))$

(4) $\sim\mathcal{A} \supset \mathcal{B}$

a. $\sim A \bullet B$

b. $\sim B \supset A$

c. $\sim(X \bullet Y) \supset (Z \vee B)$

d. $\sim(A \supset B)$

e. $A \supset \sim B$

(5) $\sim\mathcal{A} \equiv \sim\mathcal{Z}$

a. $\sim(P \equiv Q)$

b. $\sim(P \equiv Q) \equiv \sim(Q \equiv P)$

c. $\sim H \supset \sim G$

d. $\sim(A \bullet B) \equiv C$

e. $\sim[\sim(P \equiv Q) \equiv R] \equiv \sim S$

(6) $(\mathcal{A} \bullet \mathcal{B}) \vee \mathcal{C}$

a. $(P \vee Q) \bullet R$

b. $(\sim M \bullet \sim D) \vee C$

c. $(D \bullet R) \bullet (I \vee D)$

d. $[(D \supset O) \vee A] \bullet D$

e. $[(A \bullet B) \bullet C] \vee (D \vee A)$

(7) $(\mathcal{A} \bullet \mathcal{B}) \vee \mathcal{A}$

a. $((C \supset D) \bullet E) \vee A$

b. $(A \bullet A) \vee A$

c. $((C \supset D) \bullet E) \vee (C \supset D)$

d. $((G \bullet B) \bullet (Q \vee R)) \vee (G \bullet B)$

e. $(P \vee Q) \bullet P$

(8) $\mathcal{P} \supset (\mathcal{P} \supset \mathcal{Q})$

a. $A \supset (B \supset C)$

b. $(A \bullet B) \supset [(A \bullet B) \supset C]$

c. $(G \supset B) \supset [(G \supset B) \supset (G \supset B)]$

d. $M \supset [M \supset (D \bullet (C \bullet M))]$

e. $(S \vee O) \supset [(O \vee S) \supset A]$

(9) $\sim\mathcal{A} \vee (\mathcal{B} \bullet \sim\mathcal{B})$

a. $\sim P \vee (Q \bullet \sim P)$
b. $\sim A \vee (A \bullet \sim A)$
c. $(P \supset Q) \vee [(P \supset Q) \bullet \sim R]$
d. $\sim E \bullet (F \bullet \sim F)$
e. $\sim G \vee [(H \supset G) \bullet \sim(H \supset G)]$

(10) $(\mathcal{P} \vee \mathcal{Q}) \supset \sim(\mathcal{P} \bullet \mathcal{Q})$

a. $A \supset \sim B$
b. $(A \vee B) \supset \sim(A \bullet B)$
c. $(A \vee A) \supset \sim(A \bullet A)$
d. $[(A \bullet B) \vee (D \supset E)] \supset$
$\sim[(A \bullet B) \bullet (D \supset E)]$
e. $(A \bullet B) \supset \sim(A \vee B)$

Part B For each problem, a sentence form is given in sentence variables. Identify which of the sentences after it are legitimate substitution instances of that form.

(1) $\mathcal{P} \bullet \mathcal{P}$

a. $A \bullet B$
b. $D \vee D$
c. $Z \bullet Z$
d. $(Z \vee B) \bullet (Z \bullet B)$
e. $(Z \vee B) \bullet (Z \vee B)$

(2) $\mathcal{O} \bullet (\mathcal{N} \bullet \mathcal{N})$

a. $A \bullet (B \bullet C)$
b. $A \bullet (A \bullet B)$
c. $(A \bullet B) \bullet B$
d. $A \bullet (B \bullet B)$
e. $(C \supset D) \bullet (Q \bullet Q)$

(3) $\mathcal{H} \supset \mathcal{Z}$

a. $E \supset E$
b. $G \supset H$
c. $G \supset (I \supset K)$
d. $[(I \supset K) \supset G] \supset A$
e. $G \bullet (I \supset K)$

(4) $\sim\mathcal{H} \bullet \mathcal{C}$

a. $H \bullet C$
b. $\sim(H \bullet C)$
c. $\sim Q \bullet R$
d. $R \bullet \sim Q$
e. $\sim(X \equiv Y) \bullet (Y \supset Z)$

(5) $\sim(\mathcal{G} \equiv \mathcal{M})$

a. $\sim(K \equiv K)$
b. $\sim K \equiv K$
c. $\sim((I \equiv K) \equiv (S \bullet S))$
d. $\sim(H \supset (I \vee J)$
e. $\sim((H \vee F) \equiv (Z \supset D))$

(6) $(I \supset \mathcal{W}) \vee \mathcal{W}$

a. $(D \vee E) \supset E$
b. $(D \supset E) \vee E$
c. $D \supset (E \vee E)$
d. $((W \bullet L) \supset L) \vee W$
e. $((W \bullet L) \supset J) \vee J$

(7) $\mathcal{M} \vee (\mathcal{A} \vee \mathcal{A})$

a. $A \vee (A \vee A)$
b. $(A \vee A) \vee A$
c. $C \vee (C \vee D)$
d. $(R \supset K) \vee ((D \bullet G) \vee (D \bullet G))$
e. $(P \bullet P) \vee ((\sim H \bullet C) \vee (\sim H \bullet C))$

(8) $\mathcal{A} \supset \sim(\mathcal{G} \bullet \mathcal{G})$

a. $B \equiv \sim(G \bullet G)$
b. $O \supset \sim(R \bullet D)$
c. $(H \supset Z) \supset (\sim D \bullet D)$
d. $(O \bullet (N \bullet N)) \supset \sim(F \bullet F)$
e. $\sim D \bullet \sim((J \supset J) \bullet (O \equiv O)$

(9) $\sim((\mathcal{K} \supset \mathcal{K}) \vee \mathcal{K}) \bullet \mathcal{G}$

a. $\sim(D \supset D)(\vee D \bullet L)$
b. $\sim(D \supset (D \vee (D \bullet L))$
c. $\sim((D \supset D) \vee D) \bullet L$
d. $((\sim K \supset \sim K) \vee K) \bullet L$
e. $\sim((D \supset D) \vee D) \bullet ((D \supset D) \vee D)$

(10) $(\mathcal{B} \equiv (\mathcal{N} \equiv \mathcal{N})) \vee \mathcal{N}$

a. $(B \equiv (N \equiv (N \bullet N))) \vee N$
b. $((E \bullet T) \equiv (V \equiv V)) \vee V$
c. $(B \equiv (N \bullet N)) \vee B$
d. $A \equiv (N \equiv (N \vee N)))$
e. $((X \equiv N) \equiv N) \vee N$

Part C Decide whether the following are examples of MP (*modus ponens*).

(1) P_1: $A \supset B$
P_2: $B \supset C$
C: $A \supset C$

(2) P_1: $P \bullet Q$
P_2: P
C: Q

(3) P_1: $P \supset Q$
C: Q

(4) P_1: $D \supset E$
P_2: E
C: D

(5) P_1: $(P \bullet Q) \supset (Q \bullet V)$
P_2: $P \bullet Q$
C: $Q \bullet V$

Part D Decide whether the following are examples of DS (disjunctive syllogism).

(1) P_1: $(A \supset B) \vee (X \supset Y)$
P_2: $\sim A$
C: $X \supset Y$

(2) P_1: $[(S \vee T) \vee U] \vee V$
P_2: $\sim[(S \vee T) \vee U]$
C: V

(3) P_1: $P \vee Q$
P_2: P
C: $\sim Q$

(4) P_1: $\sim(A \vee B)$
P_2: $\sim A$
C: B

(5) P_1: $(P \vee Q) \vee R$
P_2: $\sim(P \vee Q)$
C: R

7.3 Rules of Implication I

In designing a proof system, we could just start with disjunctive syllogism and *modus ponens*. Whenever we discovered a valid argument that could not be proved with rules we already had, we could introduce new rules. Proceeding in this way, we would have an unsystematic grab bag of rules. We might accidentally add some strange rules, and we would surely end up with more rules than we need.

Instead, we will use a list of some of the most common valid argument forms as our rules. This will not give us the sparsest set of rules, but it will give us ones that are easy to use, thereby making our proof system more manageable. You may ask yourself: What happens if we find a valid argument that cannot be proved with our rules? In a later section, we will discuss this question in more depth. For now, just take it as true that our set of rules will be sufficient to prove every valid argument.

In this section, you will learn the first 4 rules. In total, the system will have 20. The first 8 rules of inference you will learn are called **rules of implication**. They consist of valid argument forms like disjunctive syllogism and *modus ponens*. We will get into the differences between rules of implication and the other rules in a later section. All of the rules introduced in this chapter are summarized starting on p.292.

Conjunction

Think for a moment: What would you need to show in order to prove $E \bullet F$?

Of course, you could show $E \bullet F$ by proving E and separately proving F. This holds even if the two conjuncts are not atomic sentences. If you can prove $[(A \vee J) \supset V]$ and $[(V \supset L) \equiv (F \vee N)]$, then you have effectively proved $[(A \vee J) \supset V] \bullet [(V \supset L) \equiv (F \vee N)]$. The following is the argument form for the rule conjunction, which will be abbreviated to (conj):

m	$\mathcal{A}$	
n	$\mathcal{B}$	
	$\mathcal{A} \bullet \mathcal{B}$	Conj m, n

m	$\mathcal{A}$	
n	$\mathcal{B}$	
	$\mathcal{B} \bullet \mathcal{A}$	Conj m, n

A line of proof must be justified by some rule, and here we have “Conj m, n” This means: Conjunction applied to line m and line n. Again, these are variables, not real line numbers; m is some line and n is some other line. If you have K on line 8 and L on line 15, you can prove $(K \bullet L)$ at some later point in the proof with the justification “Conj 8, 15.”

We have written two versions of the rule to indicate that you can write the conjuncts in any order. Even though K occurs before L in the proof, you can derive $(L \bullet K)$ from them using the right-hand version of conjunction. You do not need to mark this in any special way in the proof.

Simplification

What are you entitled to conclude from a sentence like $E \bullet F$? Surely, you are entitled to conclude E; if $E \bullet F$ is true, then E would be true. Similarly, you are entitled to conclude F. This rule is known as simplification, which we abbreviate as 'Simp':

m	$\mathcal{A} \bullet \mathcal{B}$		m	$\mathcal{A} \bullet \mathcal{B}$	
	$\mathcal{A}$	Simp m		$\mathcal{B}$	Simp m

When you have a conjunction on some line of a proof, you can use simplification to derive either of the conjuncts. Again, we have written two versions of the rule to indicate that it can be applied to either side of the conjunction. The simplification rule requires only one sentence, so we write one line number as the justification for applying it. For example, both of these moves are acceptable in derivations.

4	$A \bullet (B \vee C)$		10	$A \bullet (B \vee C)$	
5	A	Simp 4	...	...	
			15	$(B \vee C)$	Simp 10

Some textbooks will only let you use simplification on one side of a conjunction. They then make you *prove* that it works for the other side. We won't do this, because it is a pain in the neck.

Even with just these two rules, we can provide some proofs. Consider this argument.

$$\begin{array}{rl} & [(A \vee B) \supset (C \vee D)] \bullet [(E \vee F) \supset (G \vee H)] \\ \therefore & [(E \vee F) \supset (G \vee H)] \bullet [(A \vee B) \supset (C \vee D)] \end{array}$$

The main logical operator in both the premise and conclusion is a conjunction. Since the conjunction is symmetric, the argument is obviously valid. In order to provide a proof, we begin by writing down the premise. After the premises, we draw a horizontal line—everything below this line must be justified by a rule of proof. So the beginning of the proof looks like this:

$$\begin{array}{r|l}
1 & [(A \vee B) \supset (C \vee D)] \bullet [(E \vee F) \supset (G \vee H)] \\ \hline
\end{array}$$

From the premise, we can get each of the conjuncts by simplification. The proof now looks like this:

$$\begin{array}{r|ll}
1 & [(A \vee B) \supset (C \vee D)] \bullet [(E \vee F) \supset (G \vee H)] & \\ \hline
2 & [(A \vee B) \supset (C \vee D)] & \text{Simp } 1 \\
3 & [(E \vee F) \supset (G \vee H)] & \text{Simp } 1
\end{array}$$

The rule conjunction requires that we have each of the conjuncts available somewhere in the proof. They can be separated from one another, and they can appear in any order. So by applying the conjunction rule to lines 3 and 2, we arrive at the desired conclusion. The finished proof looks like this:

$$\begin{array}{r|ll}
1 & [(A \vee B) \supset (C \vee D)] \bullet [(E \vee F) \supset (G \vee H)] & \\ \hline
2 & [(A \vee B) \supset (C \vee D)] & \text{Simp } 1 \\
3 & [(E \vee F) \supset (G \vee H)] & \text{Simp } 1 \\
4 & [(E \vee F) \supset (G \vee H)] \bullet [(A \vee B) \supset (C \vee D)] & \text{Conj } 3, 2
\end{array}$$

This proof is trivial, but it shows how we can use rules of inference together to demonstrate the validity of an argument form. Also: Using a truth table to show that this argument is valid would have required a staggering 256 lines, since there are eight sentence letters in the argument.

Modus Ponens

In section 8.1, you were introduced to the rule *modus ponens*, which is one of the most commonly used deductive argument forms. Here it is again:

$$\begin{array}{r|ll}
m & \mathcal{A} \supset \mathcal{B} & \\
n & \mathcal{A} & \\
 & \mathcal{B} & \text{MP } m, n
\end{array}$$

$(A \supset B)$ means that if A is true, then so is B. So, if we have $(A \supset B)$ and A on separate lines of our proof, then we can conclude B. Since *modus ponens* requires two different lines, the justification will include the abbreviation "MP" followed by the numbers for the two lines.

Disjunctive Syllogism

Here we have another rule that you have already seen. What can you conclude from $M \vee N$? You cannot conclude M. Remembering the truth table for $\vee$, all that is needed for a disjunction to be true is for at least one of the disjuncts to be true. M could be true or N could be true, but since the disjunction could be true while one of the disjuncts is false, we cannot conclude anything about either M or N specifically. If you also knew that N was false, however, then you would be able to conclude M, since at least one of them must be true for $M \vee N$ to be true.

m	$\mathcal{A} \vee \mathcal{B}$		m	$\mathcal{A} \vee \mathcal{B}$	
n	$\sim\mathcal{B}$		n	$\sim\mathcal{A}$	
	$\mathcal{A}$	DS m, n		$\mathcal{B}$	DS m, n

Once again, the rule works on both sides of the sentential connective. For the justification, we will write "DS" followed by the two lines to which it is being applied.

Notation

The rules we have learned in this chapter give us enough to start doing some basic derivations in SL. This will allow us to prove things syntactically which would have been too cumbersome to prove using the semantic method of truth tables. We now need to introduce a few more symbols to be clear about what methods of proof we are using.

In Chapter 1, we used the three dots $\therefore$ to indicate generally that one thing followed from another. In chapter 3 we introduced the double turnstile, $\models$, to indicate that one statement could be proven from some others using truth tables. Now we are going to use a single turnstile, $\vdash$, to indicate that we can derive a statement from a bunch of premises, using the system of natural deduction we have begun to introduce in this section. Thus we will write $\{\mathcal{A}, \mathcal{B}, \mathcal{C}\} \vdash \mathcal{D}$, to indicate that there is a derivation going from the premises $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ to the conclusion $\mathcal{D}$. Note that these are metavariables, so I could be talking about any sentences in SL.

Practice Exercises

Part A For each problem, provide a proof starting with the premises and ending with the conclusion, using the 4 rules from this section.

(1) $\{P \supset Q, P \bullet R\} \vdash Q$

(2) $\{(P \bullet Q) \supset R, P, Q\} \vdash R$

(3) $\{\sim P \supset (Q \vee P), \sim P\} \vdash Q$

(4) $\{A \supset B, A \supset C, A\} \vdash B \bullet C$

(5) $\{(P \vee Q) \bullet (Q \supset R), \sim P\} \vdash R$

(6) $\{P \bullet Q, R \bullet S\} \vdash Q \bullet S$

(7) $\{\sim P, P \vee Q, Q \supset R\} \vdash R$

(8) $\{P, Q \vee R, ((Q \vee R) \bullet P) \supset R\} \vdash R$

7.4 Rules of Implication II

Addition

If M were true, then $M \vee N$ would also be true. The addition rule allows us to derive a disjunction if we have one of the two disjuncts:

m	$\mathcal{A}$			m	$\mathcal{A}$	
	$\mathcal{A} \vee \mathcal{B}$	Add m			$\mathcal{B} \vee \mathcal{A}$	Add m

Like conjunction and disjunctive syllogism, this rule can be applied two ways. Also notice that $\mathcal{B}$ can be *any* sentence whatsoever. So the following is a legitimate proof:

1	M	
2	$M \vee ([(A \equiv B) \supset (C \bullet D)] \equiv [E \bullet F])$	Add 1

This might seem odd. How can we prove a sentence that includes A, B, and the rest, from the simple sentence M—which has nothing to do with the other letters? The secret here is to remember that all the new letters are on just one side of a disjunction, and nothing on that side of the disjunction has to be true. As long as M is true, we can add whatever we want after a disjunction and the whole thing will continue to be true.

Modus Tollens

You may recognize the similarity between the names "*modus ponens*" and "*modus tollens*." *Modus ponens* goes from a conditional and its antecedent to the conditional's consequent. "*Modus tollens*," on the other hand, goes from a conditional and the negation of its consequent to the negation of its antecedent.

m	$\mathcal{A} \supset \mathcal{B}$	
n	$\sim\mathcal{B}$	
	$\sim\mathcal{A}$	MT m, n

If $(A \supset B)$ and $\sim B$ are true, then it must be the case that $\sim A$, since if A were true and B were false, then $(A \supset B)$ would be false.

Hypothetical Syllogism

Consider the following English argument: If Spot is a Golden Retriever, then he is a dog. If Spot is a dog, then he is an animal. Therefore, if Spot is a Golden Retriever, then he is an animal. This argument is an example of hypothetical syllogism.

m	$\mathcal{A} \supset \mathcal{B}$	
n	$\mathcal{B} \supset \mathcal{C}$	
	$\mathcal{A} \supset \mathcal{C}$	HS m, n

Suppose that $(A \supset B)$ and $(B \supset C)$ are both true. Now, imagine that A is true. By *modus ponens* B would also be true. Then, by *modus ponens* again, C must be true. If A is true, then C must be true, which amounts to saying that $(A \supset C)$ is true. This little bit of reasoning is meant to give you an intuitive understanding of hypothetical syllogism, but it is actually an example of a proof method that you will learn in a later section.

Constructive Dilemma

Constructive dilemma may look more complicated than any of the rules you've learned so far, but don't be intimidated by the addition of a third line. Constructive dilemma is a common form of argument in English. Consider the following argument: If a US company gains a monopoly on space travel, then most missions will go to mars. If a Chinese company gains a monopoly on space travel, then most missions will go to the moon. Either a US company will gain a monopoly on space travel or a Chinese company will. Therefore, either most missions will go to mars or they will go to the moon.

m	$\mathcal{A} \supset \mathcal{B}$	
n	$\mathcal{C} \supset \mathcal{D}$	
o	$\mathcal{A} \vee \mathcal{C}$	
	$\mathcal{B} \vee \mathcal{D}$	HS m, n, o

Note that to apply constructive dilemma you need two conditionals and one disjunction. The disjunction must have the antecedents of the conditionals as its disjuncts. Since you need to use three different lines, your justification will cite all three lines.

Practice Exercises

Part A For each problem, provide a proof starting with the premises and ending with the conclusion, using the 8 rules from this section.

(1) $\{P \supset {\sim}Q, R \supset Q, P\} \vdash {\sim}R$

(2) $\{P \supset Q, Q \supset R, {\sim}R\} \vdash {\sim}P$

(3) $\{P \supset (Q \supset R), P, {\sim}R\} \vdash {\sim}Q$

(4) $\{(P \supset Q) \bullet (P \vee S), S \supset T\} \vdash Q \vee T$

(5) $\{P \bullet {\sim}P\} \vdash {\sim}R$

(6) $\{{\sim}{\sim}P \supset {\sim}Q, {\sim}P \supset S, T \supset Q, {\sim}S\} \vdash {\sim}T$

(7) $\{T \supset P, T \supset S\} \vdash (P \vee Q) \bullet (R \vee S)$

(8) $\{P \supset Q, Q \supset R, P \supset S, P \vee P\} \vdash R \vee S$

7.5 Rules of Replacement I

So far we have only been dealing with one kind of rule—rules of implication. We will now begin learning the second kind—rules of replacement. There are some important differences between these two kinds of rules. You may have noticed that rules of implication must be applied to entire lines.

$$\begin{array}{r|ll} 1 & (M \supset N) \supset O & \\ 2 & M & \\ \hline 3 & N & \text{MP 1, 2} \end{array}$$

The above sequence of propositions does not represent a valid argument. Yes, line 2 is the antecedent of the conditional $(M \supset N)$, but it is not the antecedent of $((M \supset N) \supset O)$. *Modus ponens* must be applied to entire lines, so the antecedent we are using must be the antecedent for the main operator of the line containing the conditional. Below is a correct application of *modus ponens*.

$$\begin{array}{r|ll} 1 & (M \supset N) \supset O & \\ 2 & M \supset N & \\ \hline 3 & O & \text{MP 1, 2} \end{array}$$

Modus ponens is an example of a rule of implication. It must be applied to entire lines. In addition to only applying to entire lines, rules of implication only go in one direction. Take any of the rules you have learned so far and swap the conclusion for one of the premises. The resulting argument will be invalid. This can be easily checked with truth tables.

Rules of replacement differ on both of these points. They can be applied to parts of lines or whole lines. They also go in both directions. This is because rules of replacement represent logical equivalences. If two propositions are logically equivalent, then we can substitute one for the other, even inside of a sentence, and the truth value won't change. For an example in another domain, consider the following two statements in arithmetic:

$5+7=12$ $\quad\quad$ $(2+3)+7=12$

Since $(2+3)$ is equal to 5 we can swap them in any situation without changing the value of the resulting proposition. The same idea applies to logically equivalent sentences in SL.

Before moving on to the rules, a point of notation is required. Since rules of replacement are bidirectional, they will be presented as logical equivalences instead of proofs. "$\mathcal{A} :: \mathcal{B}$" will mean that $\mathcal{A}$ is logically equivalent to $\mathcal{B}$, and therefore they can be swapped where ever they appear.

Commutation

You may remember that while introducing rules like simplification and disjunctive syllogism that we made a point about how we were avoiding unnecessary trouble by admitting two versions of the rules. The trouble we were referring to is the requirement of applying the rule commutation in order to apply simplification or disjunctive syllogism in the way we wanted. However, commutation is a useful rule for other purposes and it demonstrates an important property of conjunctions and disjunctions.

$\mathcal{A} \vee \mathcal{B} :: \mathcal{B} \vee \mathcal{A}$ $\quad\quad$ $\mathcal{A} \bullet \mathcal{B} :: \mathcal{B} \bullet \mathcal{A}$

m	$\mathcal{A} \vee \mathcal{B}$	
	$\mathcal{B} \vee \mathcal{A}$	Comm m

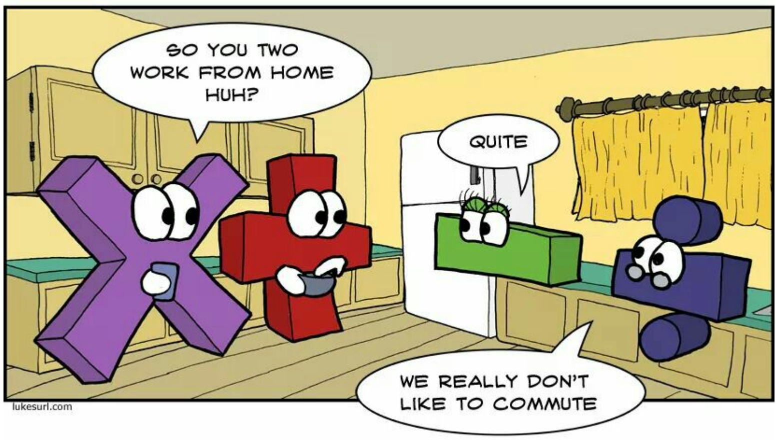
m	$\mathcal{B} \vee \mathcal{A}$	
	$\mathcal{A} \vee \mathcal{B}$	Comm m

m	$\mathcal{A} \bullet \mathcal{B}$	
	$\mathcal{B} \bullet \mathcal{A}$	Comm m

m	$\mathcal{B} \bullet \mathcal{A}$	
	$\mathcal{A} \bullet \mathcal{B}$	Comm m

Note that there are two different versions of commutation. This is because commutation can be applied to both disjunctions and conjunctions. If you remember the truth tables for disjunctions and conjunctions, then you will remember that, unlike conditionals, they do not have a direction. As such, we can swap the sides of the disjuncts without affecting the truth value of the disjunction.

You have probably heard of the commutative property of addition and multiplication in math classes before. $2+3=3+2$ and $2*3=3*2$. The same property is in play here. In fact, the semantics for the connectives is often called Boolean Algebra because we can treat it like algebra by giving true propositions the value 1 and false ones the value 0 and treating the connectives like addition and multiplication.



Association

Another rule that you may already have some understanding of is association. Just like commutation, association also applies in arithmetic. $2 + (3 + 5) = (2 + 3) + 5$ and $2 * (3 * 5) = (2 * 3) * 5$.

$\mathcal{A} \vee (\mathcal{B} \vee \mathcal{C}) :: (\mathcal{A} \vee \mathcal{B}) \vee \mathcal{C}$ $\qquad$ $\mathcal{A} \bullet (\mathcal{B} \bullet \mathcal{C}) :: (\mathcal{A} \bullet \mathcal{B}) \bullet \mathcal{C}$

Association allows us to move parentheses. There are two conditions. First, it only works for conjunctions and disjunctions. Second, the main connective of the sentence and the main connective of one of the disjuncts/conjuncts are the same.

Consider what would happen if we tried to apply association to $E \supset (F \supset G)$. $(E \supset F) \supset G$ is not logically equivalent. You can construct a truth table to check. In particular, consider the case where E is false and G is false—F can be either true or false. Now, consider what would happen if we tried to apply association when the two connectives do not match. $A \vee (B \bullet C)$ is not logically equivalent to $(A \vee B) \bullet C$. Consider the situation where A is true and C is false.

Double Negation

You probably learned in an English class that you should not use double negations in English sentences. In logic double negatives are perfectly okay. We even have a rule for both introducing and eliminating them.

$\mathcal{A} :: \sim\sim\mathcal{A}$

Whenever there are two negations, you can apply double negation to remove them. In addition, at any time you can add two negations to the front of a sentence.

Conditional Exchange

During some of the proofs you have already done, you may have wanted a conditional when you had a disjunction or conversely, you may have wanted a disjunction but you had a conditional. Well, lucky for you, there is a rule that allows us to substitute conditionals for disjunctions and vice versa.

$\sim\mathcal{A}\vee\mathcal{B} :: \mathcal{A}\supset\mathcal{B}$

Of course, you can use a truth table to check to see that the two sentence forms above are logically equivalent. Consider the two ways that a conditional can be true. Either its antecedent is false (the bottom two lines of the truth table) or its consequent is true (the first and third lines of the truth table). These are the two ways that a conditional can be true. They are represented by $(\sim\mathcal{A} \vee \mathcal{B})$

De Morgan's

De Morgan's law is one of the most useful rules in our proof system. While doing symbolization you may have struggled with sentences like "Neither A nor B," "Not both A and B," "Not A and not B," and "Not A or not B." These sentences test your understanding of the scope of the negations. De Morgan's allows us to move between some of these sentence.

$\sim(\mathcal{A} \bullet \mathcal{B}) :: \sim\mathcal{A} \vee \sim\mathcal{B}$ $\qquad\qquad$ $\sim(\mathcal{A} \vee \mathcal{B}) :: \sim\mathcal{A} \bullet \sim\mathcal{B}$

On the left, we have "not both A and B" is logically equivalent to "not A or not B." If "not both A and B" is true, then at least one of them must be false. On the right, we have "neither A nor B" is logically equivalent to "not A and not B." If neither of them is true, then both of them are false.

Practice Exercises

Part A For each problem, provide a proof starting with the premises and ending with the conclusion, using the 13 you've learned so far.

(1) $\{\sim(A \vee \sim B), C \supset \sim B\} \vdash \sim C$

(2) $\{\sim B\} \vdash \sim(A \bullet B)$

(3) $\{\sim A\} \vdash A \supset B$

(4) $\{\sim(A \bullet B), \sim\sim A\} \vdash \sim B$

(5) $\{A \supset B\} \vdash \sim B \supset \sim A$

(6) $\{A \supset (B \supset C)\} \vdash (A \bullet B) \supset C$

(7) $\{A \vee \sim B, \sim B \supset C\} \vdash A \vee C$

(8) $\{C \vee (A \vee B), A \supset D\} \vdash (B \vee C) \supset D$

7.6 Rules of Replacement II

Distribution

WARNING: This rule is one of the most difficult for students to remember and to apply in a proof. It is very important that you spend time memorizing and practicing this rule.

$\mathcal{A} \vee (\mathcal{B} \bullet \mathcal{C}) :: (\mathcal{A} \vee \mathcal{B}) \bullet (\mathcal{A} \vee \mathcal{C})$ $\qquad$ $\mathcal{A} \bullet (\mathcal{B} \vee \mathcal{C}) :: (\mathcal{A} \bullet \mathcal{B}) \vee (\mathcal{A} \bullet \mathcal{C})$

Perhaps the best way to demonstrate the reasoning behind distribution is to provide English arguments.

Left: Either Spot is a dog or Spot is both a cat and lazy. So, Spot is either a dog or lazy and Spot is either a dog or a cat.

Right: Spot is a dog and Spot is either a golden retriever or an Australian shepherd. So, either Spot is both a dog and a golden retriever or he is both a dog and an Australian shepherd.

Exportation

Like distribution, many students struggle with exportation, so it may be a good idea to spend some extra time learning this rule. The value exportation adds to our toolbox is the ability to change the antecedent of a line in a proof.

$\mathcal{A} \supset (\mathcal{B} \supset \mathcal{C}) :: (\mathcal{A} \bullet \mathcal{B}) \supset \mathcal{C}$

English Example: If you are a senior, then if you have completed your general education requirements, then you can write your thesis. So, if you are a senior who has completed their general education requirements, then you can write your thesis.

Contraposition

If you studied the categorical logic chapters of this book, then the next rule should look familiar to you. It even shares a name with the rule you learned in categorical logic.

$\mathcal{A}\supset\mathcal{B} :: \sim\mathcal{B}\supset\sim\mathcal{A}$

In the previous section you completed proofs that required you to swap the antecedent and consequent of a conditional and to negate both. In so doing you have already derived contraposition using conditional exchange, double negation, and commutation.

Biconditional Exchange

A biconditional is just two conditionals going in different directions. A biconditional like $(A \equiv B)$ just says "if A, then B, and if B, then A."

$\mathcal{A}\equiv\mathcal{B} :: (\mathcal{A}\supset\mathcal{B})\bullet(\mathcal{B}\supset\mathcal{A})$

Biconditional exchange is the only rule we have for breaking open biconditionals and for constructing them. So, if you need to do either of these tasks, then it should be clear what rule you need to use.

Duplication

You may have noticed in your practicing that sometimes you end up with a sentence like $(A \vee A)$, but that you had no way of getting just A from it. After all, we have no way of getting one disjunct from a disjunction unless we have the negation of the other side. But, you may have also noticed that A is logically equivalent to $(A \vee A)$. This rule will let you substitute $(A \vee A)$ for A and vice versa.

$\mathcal{A}\vee\mathcal{A} :: \mathcal{A}$ $\qquad\qquad$ $\mathcal{A}\bullet\mathcal{A} :: \mathcal{A}$

There are two versions of duplication, one for disjunction and one for conjunction. Of course, you can always apply simplification to $(A \bullet A)$ to get A, but without duplication you cannot go directly from A to $(A \bullet A)$.

Practice Exercises

Part A For each problem, provide a proof starting with the premises and ending with the conclusion, using the 18 rules/.

(1) $\{\sim(B \equiv A), A \supset B\} \vdash \sim A$

(2) $\{A \supset \sim B, A \supset B\} \vdash \sim A$

(3) $\{\sim C \supset B, A \vee C\} \vdash C \vee (B \bullet A)$

(4) $\{T \supset (H \bullet J), (H \vee N) \supset T\} \vdash T \equiv H$

(5) $\{\sim H \supset B, \sim H \supset D, \sim(B \bullet D)\} \vdash H$

(6) $\{I \vee (N \bullet F), I \supset F\} \vdash F$

(7) $\{(E \supset A) \bullet (F \supset A), E \vee G, F \vee \sim G\} \vdash A$

(8) $\{C \equiv B, (C \supset \sim B) \vee \sim B\} \vdash \sim C$

7.7 Conditional Proof

You will now learn another way of doing proofs. So far, you have learned the rules of implication and the rules of replacement. These rules only make use of the premises and whatever steps you got from the premises. Now, you will learn a method of proof that lets you assume something in order to prove your conclusion. For example, suppose that you want to prove that a conditional like $\mathcal{A}\supset\mathcal{B}$ is true. One way we could do this is assume $\mathcal{A}$ and show that from $\mathcal{A}$ and our other premises that we can derive $\mathcal{B}$. This would mean that if $\mathcal{A}$ is true, then, given our premises, $\mathcal{B}$ must also be true, and therefory $\mathcal{A}\supset\mathcal{B}$.

The following is a very simple example of a conditional proof. Note that when we assume $\sim P$ we start a proof within our proof. This interior proof is called a subproof.

$$
\begin{array}{r|lll}
1 & P \vee Q & & \\
\hline
2 & & \sim P & \text{Assp. CP Target: } Q \\
3 & & Q & \text{DS 1, 2} \\
4 & \sim P \supset Q & & \text{CP 2–3}
\end{array}
$$

Starting with the premise $(P \vee Q)$ we assume that $\sim P$ is true. From $(P \vee Q)$ and $\sim P$ we can conclude Q. Since Q is true under the assumption that $\sim P$ is true, it follows that if $\sim P$, then Q. You may be thinking that conditional proof is useless, since you could have just applied double negation and conditional exchange to the premise to get the conclusion without worrying about this subproof business. You would be right about this proof. Sometimes conditional proofs will be more complicated than proofs using just the 18 rules you've learned so far. However, most proofs will actually become easier with conditional proof. Consider the following example:

1	$P \supset Q$	
2	$R \supset S$	
3	$\quad (P \bullet R)$	Assp. CP Target: $(Q \bullet S)$
4	$\quad P$	Simp 3
5	$\quad R$	Simp 3
6	$\quad Q$	MP 1, 4
7	$\quad S$	MP 2, 5
8	$\quad (Q \bullet S)$	Conj 6, 7
9	$(P \bullet R) \supset (Q \bullet S)$	CP 3–8

This proof is much easier to do with a conditional proof than it is with just the 18 rules you've learned. If you're daring, then go ahead and give it a try without conditional proof. After giving it a good try, you can check the next page for a completed proof.

1	$P \supset Q$	
2	$R \supset S$	
3	$\sim P \vee Q$	CE 1
4	$\sim R \vee S$	CE 2
5	$(\sim P \vee Q) \vee \sim R$	Add 3
6	$(\sim R \vee S) \vee \sim P$	Add 4
7	$\sim R \vee (\sim P \vee Q)$	Comm 5
8	$(\sim R \vee \sim P) \vee Q$	Assoc 7
9	$(\sim P \vee \sim R) \vee Q$	Comm 8
10	$\sim P \vee (\sim R \vee S)$	Comm 6
11	$(\sim P \vee \sim R) \vee S$	Assoc 10
12	$((\sim P \vee \sim R) \vee Q) \bullet ((\sim P \vee \sim R) \vee S)$	Conj 9, 11
13	$(\sim P \vee \sim R) \vee (Q \bullet S)$	Dist 12
14	$\sim(P \bullet R) \vee (Q \bullet S)$	DeM 13
15	$(P \bullet R) \supset (Q \bullet S)$	CE 14

Did you get it? If not, don't worry. This proof is far from easy. Plus, now that you know how to do a conditional proof, you can do this proof in a much easier way.

Notes on Subproofs

Everything within a subproof is said under an assumption. If we draw some conclusion within a subproof, we only know that that conclusion follows from the premises plus the assumption. We cannot, therefore, use that line outside of the subproof. If we want to use a line from inside of a subproof outside of that subproof, then we must prove it without using the assumption that started the subproof.

Suppose I am arguing that if Winston committed murder, then he broke the law. Suppose also that I argue for this by assuming that Winston did murder someone.

In my hypothetical scenario where Winston murders someone I conclude that Winston used a gun. I cannot now conclude that Winston used a gun to kill someone. All I can conclude is that if Winston committed murder, then he used a gun for it.

7.8 Indirect Proof

Congratulations on making it this far. This is the final tool we will add to our natural deduction toolbox. Indirect proof, like conditional proof, requires the use of an assumption and a subproof. You may have seen this kind of argument before, since it is one of the most used methods of proof in mathematics and philosophy. First we assume the negation of what we want to prove. We then derive a contradiction. Since the assumption leads to a contradiction it must be false. So, the negation of our goal is false, which means that our goal is true. You may know this by its other name *reductio ad absurdum*.

1	A	
2	$\quad \sim(P \supset P)$	Assp. IP
3	$\quad \sim(\sim P \vee P)$	CE 2
4	$\quad \sim\sim P \bullet \sim P$	DeM 3
5	$\quad P \bullet \sim P$	DN 4
6	$\sim\sim(P \supset P)$	IP 2–5
7	$P \supset P$	DN 6

In this proof we prove $(P \supset P)$ from A. You may be thinking, "Isn't A completely unrelated to $(P \supset P)$?" You would be right. We can do this because $(P \supset P)$ is a tautology, and so the negation is a contradiction.

When doing an indirect proof we make an assumption, and then using the premises and the assumption we derive something of the form $\mathcal{A} \bullet \sim\mathcal{A}$. Once we have reached this conclusion within our subproof, we can discharge our assumption and conclude the negation of our assumption.

Notation

The single turnstile will work the same way the double turnstile did. So, in addition to the uses of the single turnstile above we can write $\vdash \mathcal{A}$ to indicate that $\mathcal{A}$ can be proven a tautology using syntactic methods. We can write $\mathcal{A} \dashv\vdash \mathcal{B}$ to say that $\mathcal{A}$ and $\mathcal{B}$ can be proven logically equivalent using these derivations. You will learn how to do these later things at the end of the chapter. In the meantime, we need to practice our basic rules of derivation.

Practice Exercises

Part A Derive the following.

(1) $\{A \supset B, A\} \vdash A \bullet B$

(2) $\{A \equiv D, C, [(A \equiv D) \bullet C] \supset (C \equiv B)\} \vdash B$

(3) $\{A \equiv B, B \equiv C, C \supset D, A\} \vdash D$

(4) $\{(A \supset {\sim}B) \bullet A, B \vee C\} \vdash C$

(5) $\{(A \supset B) \vee (C \supset (D \bullet E)), {\sim}(A \supset B), C\} \vdash D$

(6) $\{C \vee (B \bullet A), {\sim}C\} \vdash A \vee A$

(7) $\{A \vee B, {\sim}A, {\sim}B\} \vdash C$

Part B Derive the following.

(1) $\{A \bullet B, B \supset C\} \vdash A \bullet (B \bullet C)$

(2) $\{(P \vee R) \bullet (S \vee R), {\sim}R \bullet Q\} \vdash P \bullet (Q \vee R)$

(3) $\{(X \bullet Y) \supset Z, X \bullet W, W \supset Y\} \vdash Z$

(4) $\{A \vee (B \vee G), A \vee (B \vee H), {\sim}A \bullet {\sim}B\} \vdash G \bullet H$

(5) $\{P \bullet (Q \bullet {\sim}R), R \vee T\} \vdash T \vee S$

(6) $\{((A \supset D) \vee B) \vee C, \sim C, \sim B, A\} \vdash D$

(7) $\{A \vee \sim\sim B, \sim B \vee \sim C, C \vee A, \sim A\} \vdash D$

Part C Derive the following.

(1) $H \bullet A \vdash A \bullet H$

(2) $\{P \vee Q, D \supset E, \sim P \bullet D\} \vdash E \bullet Q$

(3) $\{\sim A \supset (A \vee \sim C), \sim A, \sim C \equiv D\} \vdash D$

(4) $\{\sim A \bullet C, A \vee B, (B \bullet C) \supset (D \bullet E)\} \vdash D$

(5) $\{A \supset (B \supset (C \supset D)), A \bullet (B \bullet C)\} \vdash D$

(6) $\{E \vee F, F \vee G, \sim F\} \vdash E \bullet G$

(7) $\{X \bullet (Z \vee Y), \sim Z, Y \supset \sim X\} \vdash A$

Part D Derive the following using indirect or conditional proof.

(1) $\sim\sim A \vdash A$

(2) $\{A \supset B, \sim B\} \vdash \sim A$

(3) $A \supset (\sim B \vee \sim C) \vdash A \supset \sim(B \bullet C)$

(4) $\sim(A \bullet B) \vdash \sim A \vee \sim B$

(5) $\{\sim F \supset G, F \supset H\} \vdash G \vee H$

(6) $\{(T \bullet K) \vee (C \bullet E), E \supset \sim C\} \vdash T \bullet K$

Part E Derive the following using indirect or conditional proof.

(1) $\{P \supset Q, P \supset \sim Q\} \vdash \sim P$

(2) $(C \bullet D) \vee E \vdash E \vee D$

(3) $M \vee (N \supset M) \vdash \sim M \supset \sim N$

(4) $\{A \vee B, A \supset C, B \supset C\} \vdash C$

(5) $A \supset (B \vee (C \vee D)) \vdash \sim[A \bullet (\sim B \bullet (\sim C \bullet \sim D))]$

7.9 Tautologies and Equivalences

So far all we've looked at is whether conclusions follow validly from sets of premises. However, as we saw in the chapter on truth tables, there are other logical properties we want to investigate: whether a statement is a tautology, a contradiction or a contingent statement, whether two statements are equivalent, and whether sets of sentences are consistent. In this section, we will look at using derivations to test for two properties which will be important in later sections, logical equivalence and being a tautology.

We can say that two statements are SYNTACTICALLY LOGICALLY EQUIVALENT IN SL if you can derive each of them from the other. We can symbolize this the same way we symbolized semantic equivalence. When we introduced the double turnstile (p. 221), we said we would write the symbol facing both directions to indicate that two sentences were semantically equivalent, like this: $A \bullet B \mathrel{=\!\!|}\models B \bullet A$. We can do the same thing with the single turnstile for syntactic equivalence, like this: $A \bullet B \dashv\vdash B \bullet A$.

For an example of how we can show two sentences to be syntactically equivalent, consider the sentences $P \supset (Q \supset R)$ and $(P \supset Q) \supset (P \supset R)$. To prove these logically equivalent using derivations, we simply use derivations to prove the equivalence one way, from P ⊃(Q ⊃R) to (P ⊃Q) ⊃(P ⊃R). And then we prove it going the other way, from (P ⊃Q) ⊃(P ⊃R) to P ⊃(Q ⊃R). We set up the proof going left to right like this:

$$
\begin{array}{lll}
1 & \quad P \supset (Q \supset R) & \qquad \text{Want: (P ⊃Q) ⊃(P ⊃R)}
\end{array}
$$

Since our want line is a conditional, we can set this up as a conditional proof. Once we set up the conditional proof, we also have a conditional in next want line, which means that we can put a conditional proof inside a conditional proof, like this.

$$
\begin{array}{lll}
1 & \quad P \supset (Q \supset R) & \qquad \text{Want: (P ⊃Q) ⊃(P ⊃R)} \\
2 & \qquad P \supset Q & \qquad \text{Assp. CP Target: P ⊃R} \\
3 & \qquad\quad P & \qquad \text{Assp. CP Target: R}
\end{array}
$$

The completed proof for the equivalence going in one direction will look like this.

1	$P \supset (Q \supset R)$	Want: (P ⊃Q) ⊃(P ⊃R)
2	$\quad P \supset Q$	Assp. CP Target: P ⊃R
3	$\quad\quad P$	Assp. CP Target: R
4	$\quad\quad Q \supset R$	MP 1, 3
5	$\quad\quad Q$	MP 2, 3
6	$\quad\quad R$	MP 4, 5
7	$\quad P \supset R$	CP 3–6
8	$(P \supset Q) \supset (P \supset R)$	CP 2–7

This shows that $P \supset (Q \supset R) \vdash (P \supset Q) \supset (P \supset R)$. In order to show $P \supset (Q \supset R) \dashv\vdash (P \supset Q) \supset (P \supset R)$, we need to prove the equivalence going the other direction. That proof will look like this:

1	$(P \supset Q) \supset (P \supset R)$	Want: P ⊃(Q ⊃R)
2	$\quad P$	Assp. CP Target: Q ⊃R
3	$\quad\quad Q$	Assp. CP Target: R
4	$\quad\quad {\sim}P \vee Q$	Add 3
5	$\quad\quad P \supset Q$	CE 4
6	$\quad\quad P \supset R$	MP 1, 5
7	$\quad\quad R$	MP 2, 6
8	$\quad Q \supset R$	CP 3–7
9	$P \supset (Q \supset R)$	CP 2–8

These two proofs show that $P \supset (Q \supset R)$ and $(P \supset Q) \supset (P \supset R)$ are equivalent, so we can write $P \supset (Q \supset R) \dashv\vdash (P \supset Q) \supset (P \supset R)$.

We can also prove that a sentence is a tautology using a derivation. A tautology is something that must be true as a matter of logic. If we want to put this in syntactic terms, we would say that a **syntactic tautology** in SL is a statement that can be derived without any premises, because its truth doesn't depend on anything else. Now that we have all of our rules for starting and ending subproofs, we can actually do this. Rather than listing any premises, we simply start a subproof at the beginning of the derivation. The rest of the proof can work only using premises assumed for the purposes of subproofs. By the end of the proof, you have discharged all these assumptions, and are left knowing a tautological statement without relying on any leftover premises. Consider the proof of the tautology: $(P \supset (Q \supset P))$.

1	$\quad P$	Assp. CP Target: Q⊃P
2	$\quad {\sim}Q \vee P$	Add 1
3	$\quad Q \supset P$	CE 2
4	$(P \supset (Q \supset P))$	CP 1–3

In the previous chapter, we expressed the fact that something could be proven a tautology using truth tables by writing the double turnstile in front of it. The tautology above could have been proven using truth tables, so we could write: $\models (P \supset (Q \supset P))$ In this chapter, we will use the single turnstile the same way, to indicate that a sentence can be proven to be a tautology using a derivation. Thus the above proof entitles us to write $\vdash (P \supset (Q \supset P))$.

We could have also proven it using indirect proof in the following way.

1	$\sim(P \supset (Q \supset P))$	Assp. IP
2	$\sim(\sim P \vee (Q \supset P))$	CE 1
3	$\sim\sim P \bullet \sim(Q \supset P)$	DeM 2
4	$\sim\sim P$	Simp 3
5	$\sim(Q \supset P)$	Simp 3
6	$\sim(\sim Q \vee P)$	CE 5
7	$\sim\sim Q \bullet \sim P$	DeM 6
8	$\sim P$	Simp 7
9	$\sim P \bullet \sim\sim P$	Conj 8, 4
10	$\sim\sim(P \supset (Q \supset P))$	IP 1–9
11	$P \supset (Q \supset P)$	DN 10

Some proofs of tautologies will be easier with conditional proof, like the one above, and others will be either easier with indirect proof or only doable with indirect proof. The general rule of thumb, is to do conditional proof if your goal is a conditional or biconditional and indirect proof otherwise.

Practice Exercises

Part A Prove each of the following equivalences

(1) $J \dashv\vdash J \vee (L \bullet \sim L)$

(2) $P \supset (Q \supset R) \dashv\vdash Q \supset (P \supset R)$

(3) $P \supset \sim P \dashv\vdash \sim P$

(4) $\sim(P \equiv Q) \dashv\vdash (P \equiv \sim Q)$

Part B Prove each of the following equivalences

(1) $(P \supset R) \bullet (Q \supset R) \dashv\vdash (P \vee Q) \supset R$

(2) $(P \supset (Q \vee R)) \dashv\vdash (P \supset Q) \vee (P \supset R)$

(3) $(P \equiv Q) \dashv\vdash {\sim}P \equiv {\sim}Q$

Part C Prove each of the following tautologies

(1) $\vdash O \supset O$

(2) $\vdash N \vee {\sim}N$

(3) $\vdash {\sim}(A \supset {\sim}C) \supset (A \supset C)$

(4) $\vdash P \equiv (P \vee (Q \bullet P))$

Part D Prove each of the following tautologies

(1) $\vdash (B \supset {\sim}B) \equiv {\sim}B$

(2) $\vdash (P \supset [P \supset Q]) \supset (P \supset Q)$

(3) $\vdash (P \vee {\sim}P) \bullet (Q \equiv Q)$

(4) $\vdash (P \bullet {\sim}P) \vee (Q \equiv Q)$

7.10 Soundness and completeness

In section 4.6, we saw that we could use derivations to test for the same concepts we used truth tables to test for. Not only could we use derivations to prove that an argument is valid, we could also use them to test if a statement is a tautology or a pair of statements are equivalent. We also started using the single turnstile the same way we used the double turnstile. If we could prove that $\mathcal{A}$ was a tautology with a truth table, we wrote $\models \mathcal{A}$, and if we could prove it using a derivation, we wrote $\vdash \mathcal{A}$.

You may have wondered at that point if the two kinds of turnstiles always worked the same way. If you can show that $\mathcal{A}$ is a tautology using truth tables, can you also always show that it is true using a derivation? Is the reverse true? Are these things also true for tautologies and pairs of equivalent sentences? As it turns out, the answer to all these questions and many more like them is yes. We can show this by defining all these concepts separately and then proving them equivalent. That is, we imagine that we actually have two notions of validity, $valid_{\models}$ and $valid_{\vdash}$ and then show that the two concepts always work the same way.

To begin with, we need to define all of our logical concepts separately for truth tables and derivations. A lot of this work has already been done. We handled all of the truth table definitions in Chapter 6. We have also already given syntactic definitions for a tautologies and pairs of logically equivalent sentences. The other definitions follow naturally. For most logical properties we can devise a test using derivations, and those that we cannot test for directly can be defined in terms of the concepts that we can define.

For instance, we defined a syntactic tautology as a statement that can be derived without any premises (p. **??**). Since the negation of a contradiction is a tautology, we can define a SYNTACTIC CONTRADICTION IN SL as a sentence whose negation can be derived without any premises. The syntactic definition of a contingent sentence is a little different. We don't have any practical, finite method for proving that a sentence is contingent using derivations, the way we did using truth tables. So we have to content ourselves with defining "contingent sentence" negatively. A sentence is SYNTACTICALLY CONTINGENT IN SL if it is not a syntactic tautology or contradiction.

A set of sentences is SYNTACTICALLY INCONSISTENT IN SL if and only if one can derive a contradiction from them. Consistency, on the other hand, is like contingency, in that we do not have a practical finite method to test for it directly. So again, we have to define a term negatively. A set of sentences is SYNTACTICALLY CONSISTENT IN SL if and only if they are not syntactically inconsistent.

Finally, an argument is SYNTACTICALLY VALID IN SL if and only if there is a derivation of it. All of these definitions are given in Table 7.5.

All of our concepts have now been defined both semantically and syntactically. How can we prove that these definitions always work the same way? A full proof here goes well beyond the scope of this book. However, we can sketch what it would be like. We will focus on showing the two notions of validity to be equivalent. From that the other concepts will follow quickly. The proof will have to go in two directions. First we will have to show that things which are syntactically valid will also be semantically valid. In other words, everything that we can prove using derivations could also be proven using truth tables. Put symbolically, we want to show that $valid_{\vdash}$ implies $valid_{\models}$. Afterwards, we will need to show things

Concept	**Truth table (semantic) definition**	**Derivation (syntactic) definition**
Tautology	A statement whose truth table only has Ts under the main connective	A statement that can be derived without any premises.
Contradiction	A statement whose truth table only has Fs under the main connective	A statement whose negation can be derived without any premises
Contingent sentence	A statement whose truth table contains both Ts and Fs under the main connective	A statement that is not a syntactic tautology or contradiction
Equivalent sentences	The columns under the main connectives are identical.	The statements can be derived from each other
Inconsistent sentences	Sentences which do not have a single line in their truth table where they are all true.	Sentences which one can derive a contradiction from
Consistent sentences	Sentences which have at least one line in their truth table where they are all true.	Sentences which are no inconsistent
Valid argument	An argument whose truth table has no lines where there are all Ts under main connectives for the premises and an F under the main connective for the conclusion.	An argument where can derive the conclusion from the premises

Table 7.5: Two ways to define logical concepts.

in the other directions, $valid_{\models}$ implies $valid_{\vdash}$.

This argument from $\vdash$ to $\models$ is the problem of SOUNDNESS. A proof system is **sound** if there are no derivations of arguments that can be shown invalid by truth tables. Demonstrating that the proof system is sound would require showing that *any* possible proof is the proof of a valid argument. It would not be enough simply to succeed when trying to prove many valid arguments and to fail when trying to prove invalid ones.

The proof that we will sketch depends on the fact that we initially defined a sentence of SL using a recursive definition (see p. 203). We could have also used recursive definitions to define a proper proof in SL and a proper truth table. (Although we didn't.) If we had these definitions, we could then use a *recursive proof* to show the soundness of SL. A recursive proof works the same way a recursive definition does.With the recursive definition, we identified a group of base elements that were stipulated to be examples of the thing we were trying to define. In the case of a well formed formula, the base class was the set of sentence letters A, B, C . . . . We just announced that these were sentences. The second step of a recursive definition is to say that anything that is built up from your base class using certain rules also counts as an example of the thing you are defining. In the case of a definition of a sentence, the rules corresponded to the five sentential connectives (see p. 203). Once you have established a recursive definition, you can use that definition to show that all the members of the class you have defined have a certain property. You simply prove that the property is true of the members of the base class, and then you prove that the rules for extending the base class don't change the property. This is what it means to give a recursive proof.

Even though we don't have a recursive definition of a proof in SL, we can sketch how a recursive proof of the soundness of SL would go. Imagine a base class of one-line proofs, one for each of our rules of inference. The members of this class would look like this $\{\mathcal{A}, \mathcal{B}\} \vdash \mathcal{A} \bullet \mathcal{B}$; $\mathcal{A} \bullet \mathcal{B} \vdash \mathcal{A}$; $\{\mathcal{A} \vee \mathcal{B}, {\sim}\mathcal{A}\} \vdash \mathcal{B}$ . . . etc. Since some rules have a couple different forms, we would have to have add some members to this base class, for instance $\mathcal{A} \bullet \mathcal{B} \vdash \mathcal{B}$ Notice that these are all statements in the metalanguage. The proof that SL is sound is not a part of SL, because SL does not have the power to talk about itself.

You can use truth tables to prove to yourself that each of these one-line proofs in this base class is $valid_{\models}$. For instance the proof $\{\mathcal{A}, \mathcal{B}\} \vdash \mathcal{A} \bullet \mathcal{B}$ corresponds to a truth table that shows $\{\mathcal{A}, \mathcal{B}\} \models \mathcal{A} \bullet \mathcal{B}$ This establishes the first part of our recursive proof.

The next step is to show that adding lines to any proof will never change a $valid_{\models}$ proof into an $invalid_{\models}$ one. We would need to show this for each of our rules of inference. So, for instance, for conjunction we need to show that for any proof $\mathcal{A}_1 \ldots \mathcal{A}_n \vdash \mathcal{B}$ adding a line where we use $\bullet$I to infer $\mathcal{C} \bullet \mathcal{D}$, where $\mathcal{C} \bullet \mathcal{D}$ can be legitimately inferred from $\{\mathcal{A}_1 \ldots \mathcal{A}_n, \mathcal{B}\}$, would not change a valid proof into an invalid proof. But wait, if we can legitimately derive $\mathcal{C} \bullet \mathcal{D}$ from these premises, then $\mathcal{C} and \mathcal{D}$ must be already available in the proof. They are either members of $\{\mathcal{A}_1 \ldots \mathcal{A}_n, \mathcal{B}\}$ or can be legitimately derived from them. As such, any truth table line in which the premises are true must be a truth table line in which $\mathcal{C}$ and $\mathcal{D}$ are true. According to the characteristic truth table for $\bullet$, this means that $\mathcal{C} \bullet \mathcal{D}$ is also true on that line. Therefore, $\mathcal{C} \bullet \mathcal{D}$ validly follows from the premises. This means that using the conjunction rule to extend a valid proof produces another valid proof.

In order to show that the proof system is sound, we would need to show this for the other inference rules. This tedious exercise falls beyond the scope of this book.

So we have shown that $\mathcal{A} \vdash \mathcal{B}$ implies $\mathcal{A} \models \mathcal{B}$. What about the other direction, that is why think that *every* argument that can be shown valid using truth tables can also be proven using a derivation.

This is the problem of completeness. A proof system has the property of COMPLETENESS if and only if there is a derivation of every semantically valid argument. Proving that a system is complete is generally harder than proving that it is sound. Proving that a system is sound amounts to showing that all of the rules of your proof system work the way they are supposed to. Showing that a system is complete means showing that you have included *all* the rules you need, that you haven't left any out. Showing this is beyond the scope of this book. The important point is that, happily, the proof system for SL is both sound and complete. This is not the case for all proof systems and all formal languages. Because it is true of SL, we can choose to give proofs or give truth tables—whichever is easier for the task at hand.

Now that we know that the truth table method is interchangeable with the method of derivation, you can chose which method you want to use for any given problem. Students often prefer to use truth tables, because a person can produce them purely mechanically, and that seems 'easier'. However, we have already seen that truth tables become impossibly large after just a few sentence letters. On the other hand, there are a couple situations where using derivations simply isn't possible. We syntactically defined a contingent sentence as a sentence that couldn't be proven to be a tautology or a contradiction. There is no practical way to prove

Logical property	To prove it present	To prove it absent
Being a tautology	Derive the statement	Find the false line in the truth table for the sentence
Being a contradiction	Derive the negation of the statement	Find the true line in the truth table for the sentence
Contingency	Find a false line and a true line in the truth table for the statement	Prove the statement or its negation
Equivalence	Derive each statement from the other	Find a line in the truth tables for the statements where they have different values
Consistency	Find a line in truth table for the sentence where they all are true	Derive a contradiction from the sentences
Validity	Derive the conclusion form the premises	Find a line in the truth table where the premises are true and the conclusion false.

Table 7.6: When to provide a truth table and when to provide a proof.

this kind of negative statement. We will never know if there isn't some proof out there that a statement is a contradiction and we just haven't found it yet. We have nothing to do in this situation but resort to truth tables. Similarly, we can use derivations to prove two sentences equivalent, but what if we want to prove that they are *not* equivalent? We have no way of proving that we will never find the relevant proof. So we have to fall back on truth tables again.

Table 7.6 summarizes when it is best to give proofs and when it is best to give truth tables.

Practice Exercises

Part A Use either a derivation or a truth table for each of the following.

(1) Show that $A \supset [((B \bullet C) \vee D) \supset A]$ is a tautology.

(2) Show that $A \supset (A \supset B)$ is not a tautology

(3) Show that the sentence $A \supset {\sim}A$ is not a contradiction.

(4) Show that the sentence $A \equiv \sim A$ is a contradiction.

(5) Show that the sentence $\sim(W \supset (J \vee J))$ is contingent

(6) Show that the sentence $\sim(X \vee (Y \vee Z)) \vee (X \vee (Y \vee Z))$ is not contingent

(7) Show that the sentence $B \supset \sim S$ is equivalent to the sentence $\sim\sim B \supset \sim S$

(8) Show that the sentence $\sim(X \vee O)$ is not equivalent to the sentence $X \bullet O$

(9) Show that the set $\{\sim(A \vee B), C, C \supset A\}$ is inconsistent.

(10) Show that the set $\{\sim(\mathrm{A} \vee \mathrm{B}), \sim\mathrm{B}, \mathrm{B} \supset \mathrm{A}\}$ is consistent

(11) Show that $\sim(A \vee (B \vee C))$ $\therefore \sim C$ is valid.

(12) Show that $\sim(A \bullet (B \vee C))$ $\therefore \sim C$ is invalid.

Part B Use either a derivation or a truth table for each of the following.

(1) Show that $A \supset (B \supset A)$ is a tautology

(2) Show that $\sim(((N \equiv Q) \vee Q) \vee N)$ is not a tautology

(3) Show that $Z \vee (\sim Z \equiv Z)$ is contingent

(4) show that $(L \equiv ((N \supset N) \supset L)) \vee H$ is not contingent

(5) Show that $(A \equiv A) \bullet (B \bullet \sim B)$ is a contradiction

(6) Show that $(B \equiv (C \vee B))$ is not a contradiction.

(7) Show that $((\sim X \equiv X) \vee X)$ is equivalent to X

(8) Show that $F \bullet (K \bullet R)$ is not equivalent to $(F \equiv (K \equiv R))$

(9) Show that the set $\{\sim(W \supset W), (W \equiv W) \bullet W, E \vee (W \supset \sim(E \bullet W))\}$ is inconsistent.

(10) Show that the set $\{\sim R \vee C, (C \bullet R) \supset \not R, (\sim(R \vee R) \supset R)\}$ is consistent.

(11) Show that $\sim\sim(C \equiv \sim C), ((G \vee C) \vee G) \therefore ((G \supset C) \bullet G)$ is valid.

(12) Show that $\sim\sim L, (C \supset \sim L) \supset C) \therefore \sim C$ is invalid.

Key Terms

Completeness

Soundness

Syntactically consistent in SL

Syntactically contingent in SL

Syntactically inconsistent in SL

Syntactically logically equivalent in SL

Syntactically valid in SL

Syntactic contradiction in SL

Syntactic tautology in SL

Appendix A

Other Symbolic Notation

In the history of formal logic, different symbols have been used at different times and by different authors. Often, authors were forced to use notation that their printers could typeset.

In one sense, the symbols used for various logical constants is arbitrary. There is nothing written in heaven that says that '$\sim$' must be the symbol for truth-functional negation. We might have specified a different symbol to play that part. Once we have given definitions for well-formed formulae (wff) and for truth in our logic languages, however, using '$\sim$' is no longer arbitrary. That is the symbol for negation in this textbook, and so it is the symbol for negation when writing sentences in our languages SL or QL.

This appendix presents some common symbols, so that you can recognize them if you encounter them in an article or in another book.

mmary of symbols	
negation	$\neg$, $\sim$
njunction	&, $\wedge$, $\bullet$
isjunction	$\vee$
onditional	$\rightarrow$, $\supset$
onditional	$\leftrightarrow$, $\equiv$

Negation Two commonly used symbols are the *hoe*, '$\neg$', and the *swung dash*, '$\sim$.' In some more advanced formal systems it is necessary to distinguish between two kinds of negation; the distinction is sometimes represented by using both '$\neg$' and '$\sim$.'

Disjunction The symbol '$\vee$' is typically used to symbolize inclusive disjunction.

Conjunction Conjunction is often symbolized with the *ampersand*, '&.' The ampersand is actually a decorative form of the Latin word 'et' which means 'and'; it is commonly used in English writing. As a symbol in a formal system, the ampersand is not the word 'and'; its meaning is given by the formal semantics for the language. Perhaps to avoid this confusion, some systems use a different symbol for conjunction. For example, '$\wedge$' is a counterpart to the symbol used for disjunction. Sometimes a single dot, '$\bullet$', is used. In some older texts, there is no symbol for conjunction at all; 'A and B' is simply written 'AB.'

Material Conditional There are two common symbols for the material conditional: the *arrow*, '$\rightarrow$', and the *hook*, '$\supset$.'

Material Biconditional The *double-headed arrow*, '$\leftrightarrow$', is used in systems that use the arrow to represent the material conditional. Systems that use the hook for the conditional typically use the *triple bar*, '$\equiv$', for the biconditional.

Quantifiers The universal quantifier is typically symbolized as an upside-down A, '$\forall$', and the existential quantifier as a backwards E, '$\exists$.' In some texts, there is no separate symbol for the universal quantifier. Instead, the variable is just written in parentheses in front of the formula that it binds. For example, 'all x are P' is written $(x)Px$.

In some systems, the quantifiers are symbolized with larger versions of the symbols used for conjunction and disjunction. Although quantified expressions cannot be translated into expressions without quantifiers, there is a conceptual connection between the universal quantifier and conjunction and between the existential quantifier and disjunction. Consider the sentence $\exists x Px$, for example. It means that *either* the first member of the UD is a P, *or* the second one is, *or* the third one is, ... Such a system uses the symbol '$\bigvee$' instead of '$\exists$.'

Polish notation

This section briefly discusses sentential logic in Polish notation, a system of notation introduced in the late 1920s by the Polish logician Jan Łukasiewicz.

ation SL	Polish notation
$\sim$	N
$\bullet$	K
$\vee$	A
$\supset$	C
$\equiv$	E

Lower case letters are used as sentence letters. The capital letter N is used for negation. A is used for disjunction, K for conjunction, C for the conditional, E for the biconditional. ('A' is for alternation, another name for logical disjunction. 'E' is for equivalence.)

In Polish notation, a binary connective is written *before* the two sentences that it connects. For example, the sentence $A \bullet B$ of SL would be written Kab in Polish notation.

The sentences $\sim A \supset B$ and $\sim(A \supset B)$ are very different; the main logical operator of the first is the conditional, but the main connective of the second is negation. In SL, we show this by putting parentheses around the conditional in the second sentence. In Polish notation, parentheses are never required. The left-most connective is always the main connective. The first sentence would simply be written $CNab$ and the second $NCab$.

This feature of Polish notation means that it is possible to evaluate sentences simply by working through the symbols from right to left. If you were constructing a truth table for $NKab$, for example, you would first consider the truth-values assigned to b and a, then consider their conjunction, and then negate the result. The general rule for what to evaluate next in SL is not nearly so simple. In SL, the truth table for $\sim(A \bullet B)$ requires looking at A and B, then looking in the middle of the sentence at the conjunction, and then at the beginning of the sentence at the negation. Because the order of operations can be specified more mechanically in Polish notation, variants of Polish notation are used as the internal structure for many computer programming languages.

Appendix B

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Appendix C

Glossary

Affirmative The quality of a statement without a “not” or a “no.” 58

Antecedent The sentence to the left of a conditional.. 197

Argument a connected series of statements designed to convince an audience of another statement. 9

Aristotelian syllogism A categorical syllogism where each statement is in one of the moods A, E, I, or O, and which has exactly three terms, arranged so that any two pairs of statements will share one term. 104

Atomic sentence A sentence that does not have any sentences as proper parts. 189

Biconditional The symbol $\equiv$, used to represent words and phrases that function like the English phrase “if and only if.” 199

Canonical form a method for representing arguments where each premise is written on a separate, numbered, line, followed by a horizontal bar and then the conclusion. Statements in the argument might be paraphrased for brevity and indicator words are removed. 10

Categorical syllogism An argument with two premises composed of categorical statements. 104

Cogent A property of arguments that holds when the argument is strong and the premises are true. 42

Complement The class of everything that is not in a given class. 92

Complete truth table A table that gives all the possible interpretations for a sentence or set of sentences in SL. 215

Completeness A property held by logical systems if and only if $\models$ implies $\vdash$ 270

Conclusion the statement that an argument is trying to convince an audience of. 9

Conclusion indicator a word or phrase such as "therefore" used to indicate that what follows is the conclusion of an argument. 10

Conditional The symbol $\supset$, used to represent words and phrases that function like the English phrase "if . . . then." 197

Conditional validity A kind if validity that Aristotelian syllogisms have if they are valid only given the assumption that the objects named by its terms actually exist. 122

Conjunct A sentences joined to another by a conjunction. 193

Conjunction The symbol $\bullet$, used to represent words and phrases that function like the English word "and." 193

Consequent The sentence to the right of a conditional. 197

Content neutrality the feature of the study of logic that makes it indifferent to the topic being argued about. If a method of argument is considered rational in one domain, it should be considered rational in any other domain, all other things being equal. 4

Contradictories Two statements that must have opposite truth values, so that one must be true and the other false. 66

Contraposition The process of transforming a categorical statement by reversing subject and predicate and replacing them with their complements. 95

Contraries Two statements that can't both be true, but can both be false. A set two inconsistent sentences. 66

Conversion The process of changing a sentence by reversing the subject and predicate. 90

Copula The form of the verb "to be" that links subject and predicate. 56

Counterexample method A method for determining whether an argument with ordinary English words for terms is valid. One consistently substitutes other English terms for the terms in the given argument to see whether one can find an argument with the same form that has true premises and a false conclusion. 145

Critical term the term that names things that must exist in order for a conditionally valid argument to be actually valid. 128

Critical thinking The use of metareasoning to improve our reasoning in practical situations. 5

Deductive A style of arguing where one attempts to use valid arguments. 43

Disjunct A sentences joined to another by a disjunction. 195

Disjunction The symbol $\vee$, used to represent words and phrases that function like the English word "or" in its inclusive sense. 195

Distribution A property of the terms of a categorical statement that is present when the statement makes a claim about the whole term. 59

Enthymeme An argument where a premise or conclusion has been left unstated. 158

Existential fallacy A fallacy committed in an Aristotelian syllogism where the conclusion is particular but both premises are universal. 139

Existential import An aspect of the meaning of a statement that which is present if the statement can only be true when the objects it describes exist. 76

Explainee The part of an explanation that one gains a greater understanding of as a result of the explainer. 23

Explainer The part of an explanation that provides greater understanding of the explainee. 23

Explanation A kind of reasoning where reasons are used to provide a greater understanding of something that is already known. 23

Expository passage A nonargumentative passage that organizes statements around a central theme or topic statement. 18

Fallacy of exclusive premises A fallacy committed in an Aristotelian syllogism where both premises are negative. 136

Fallacy of illicit process A fallacy committed in an Aristotelian syllogism when a term is distributed in the conclusion but is not distributed in the corresponding premise. This fallacy is called “illicit major” or “illicit minor” depending on which term is not properly distributed. 136

Fallacy of particular premises A fallacy committed in an Aristotelian syllogism where both premises are particular. 141

Fallacy of the undistributed middle A fallacy committed in an Aristotelian syllogism where the middle term is not distributed in either premise. 134

Formal logic A way of studying logic that achieves content neutrality by replacing parts of the arguments being studied with abstract symbols. Often this will involve the construction of full formal languages. 4

Inductive A style of arguing where one attempts to use strong arguments. 43

Inference the act of coming to believe a conclusion on the basis of some set of premises. 13

Informal logic The study of arguments given in ordinary language. 5

Interpretation A correspondence between nonlogical symbols of the object language and elements of some other language or logical structure. 212

Invalid A property of arguments that holds when the premises do not force the truth of the conclusion. The opposite of valid. 36

Logical equivalence A property held by a pair of sentences that must always have the same truth value. 70

Logical symbol A symbol that has its meaning fixed by the formal language. 211

Logically structured English English that has been regimented into a standard form to make its logical structure clear and to remove ambiguity. A stepping stone to full-fledged formal languages. 81

Main connective The last connective that you add when you assemble a sentence using the recursive definition. 203

Major premise The one premise in an Aristotelian syllogism that names the major term. 105

Major term The term that is used as the predicate of the conclusion of an Aristotelian syllogism. 105

Metalanguage The language logicians use to talk about the object language. In this textbook, the metalanguage is English, supplemented by certain symbols like metavariables and technical terms like "valid." 201

Metavariables A variable in the metalanguage that can represent any sentence in the object language. 202

Middle term The one term in an Aristotelian syllogism that does not appear in the conclusion. 105

Minor premise The one premise in an Aristotelian syllogism that names the minor term. 105

Minor term The term that is used as the subject of the conclusion of an Aristotelian syllogism. 105

Mood-A statement A quantified categorical statement of the form "All S are P." 59

Mood-E statement A quantified categorical statement of the form "No S are P." 59

Mood-I statement A quantified categorical statement of the form "Some S are P." 59

Mood-O statement A quantified categorical statement of the form "Some S are not P." 59

Narrative A nonargumentative passage that describes a sequence of events or actions. 20

Negation The symbol $\sim$, used to represent words and phrases that function like the English word "not". 191

Negative The quality of a statement containing a "not" or "no." 58

Negative-affirmative fallacy A fallacy committed in an Aristotelian syllogism where the conclusion is negative but both of the premises are positive or the conclusion is affirmative but one or more of the premises is negative. 138

Nonlogical symbol A symbol that is not logical, that is, that does not have its meaning fixed by the formal language. 211

Object language A language that is constructed and studied by logicians. In this textbook, the object languages are SL and QL. 201

Obversion The process of transforming a categorical statement by changing its quality and replacing the predicate with its complement. 92

Particular The quantity of a statement that uses the quantifier “some.” 58

Predicate class The second class named in a quantified categorical statement. 56

Premise a statement in an argument that provides evidence for the conclusion 9

Premise indicator a word or phrase such as “because” used to indicate that what follows is the premise of an argument. 10

Quality The status of a categorical statement as affirmative or negative. 58

Quantified categorical statement A statement that makes a claim about a certain quantity of the members of a class or group. 56

Quantifier The part of a categorical sentence that specifies a portion of a class. 56

Quantity The portion of the subject class described by a categorical statement. Generally “some” or “none.” 58

Reason The premise of an argument or the explainer in an explanation; the part of reasoning that provides logical support for the target proposition. 23

Recursive definition A definition that defines a term by identifying base class and rules for extending that class. 203

Rhetoric The study of effective persuasion. 6

Scope The sentences that are joined by a connective. These are the sentences the connective was applied to when the sentence was assembled using a recursive definition. 203

Semantic contradiction in SL A statement that has only Fs in the column under the main connective of its complete truth table. 219

Semantic tautology in SL A statement that has only Ts in the column under the main connective of its complete truth table. 218

Semantically consistent in SL A property held by sets of sentences if and only if the complete truth table for that set contains one line on which all the sentences are true 220

Semantically contingent in SL A property held by a sentence in SL if and only if the complete truth table for that sentence has both Ts and Fs under its main connective. 219

Semantically logically equivalent in SL A property held by pairs of sentences if and only if the complete truth table for those sentences has identical columns under the two main connectives. 219

Semantically valid in SL A property held by arguments if and only if the complete truth table for the argument contains no rows where the premises are all true and the conclusion false. 220

Semantics The meaning of a bit of language is its meaning, including truth and falsity. 201

Sentence letter A single capital letter, used in SL to represent a basic sentence. 188

Sentence of SL A string of symbols in SL that can be built up using according to the recursive rules given on page 203

Sentential connective A logical operator in SL used to combine sentence letters into larger sentences. 190

Simple statement of belief A kind of nonargumentative passage where the speaker simply asserts what they believe without giving reasons. 17

Sorites categorical argument A categorical argument with more than two premises. 166

Sound A property of arguments that holds if the argument is valid and has all true premises. 37

Soundness A property held by logical systems if and only if $\vdash$ implies $\models$ 269

Square of opposition A way of representing the four basic propositions and the ways they relate to one another. 66

Standard form for a categorical statement A categorical statement that has been put into logically structured English, with the following elements in the following order: (1) The quantifiers "all," "some," or "no"; (2) the subject term; (3) the copula "are" or "are not"; and (4) the predicate term. 82

Standard form for a sorites categorical argument A sorites argument that has been put into logically structured English with the following criteria: (1)

each statement in the argument is in standard form for a categorical statement in logically structured English, (2) each instance of a term is in the same format and is used in the same sense, (3) the major premise is first in the list of premises and the minor premise is last, and (4) the middle premises are arranged so that premises that share a term are adjacent to one another. 167

Standard form for an Aristotelian syllogism An Aristotelian syllogism that has been put into logically structured English with the following criteria: (1) all of the individual statements are in standard form, (2) each instance of a term is in the same format and is used in the same sense, and (3) the major premise appears first, followed by the minor premise, and then the conclusion. 105

Statement mood The classification of a categorical statement based on its quantity and quality. 58

Strong A property of arguments which holds when the premises, if true, mean the conclusion must be likely to be true. 42

Subalternation The relationship between a universal categorical statement and the particular statement with the same quality. 68

Subcontraries Two categorical statements that cannot both be false, but might both be true. 68

Subject class The first class named in a quantified categorical statement. 56

Superfluous distribution rule A rule that says that in a conditionally valid argument, the critical term will be the one that is distributed more times in the premises than is necessary to satisfy Rules 1 and 2. 174

Syllogism mood The classification of an Aristotelian syllogism based on the moods of statements it contains. The mood is designated simply by listing the three letters for the moods of the statements in the argument, such as AAA, EAE, AII, etc. 105

Symbolization key A list that shows which English sentences are represented by which sentence letters in SL. It is also called a dictionary. 188

Syntactic contradiction in SL A statement in SL whose negation can be derived without any premises. 267

Syntactically consistent in SL A property held by sets of sentences in SL if and only if they are not syntactically inconsistent. 267

Syntactically contingent in SL A property held by a statement in SL if and only if it is not a syntactic tautology or a syntactic contradiction. 267

Syntactically inconsistent in SL A property held by sets of sentences in SL if and only if one can derive a contradiction from them. 267

Syntactically logically equivalent in SL A property held by pairs of statements in SL if and only if there is a derivation which takes you from each one to the other one. 262

Syntactically valid in SL A property held by arguments in SL if and only if there is a derivation that goes from the premises to the conclusion. 267

Syntax The structure of a bit of language, considered without reference to truth, falsity, or meaning. 201

Target proposition The conclusion of an argument or the explainee in an explanation; the part of reasoning that is logically supported by the reasons. 23

Translation key A list that assigns English phrases or sentences to variable names. 107

Truth assignment A function that maps the sentence letters in SL onto truth values. 212

Truth-functional connective an operator that builds larger sentences out of smaller ones and fixes the truth value of the resulting sentence based only on the truth value of the component sentences. 212

Unconditional validity A kind of validity that an Aristotelian syllogism has regardless of whether the objects named by its terms actually exist. 122

Universal The quantity of a statement that uses the quantifier "all." 58

Vacuous truth The kind of truth possessed by statements that do not have existential import and refer to objects that do not exist. 76

Valid A property of arguments where it is impossible for the premises to be true and the conclusion false. 34

Venn diagram A diagram that represents categorical statements using circles that stand for classes. 60

Weak A property of arguments that are neither valid nor strong. In a weak argument, the premises would not even make the conclusion likely, even if they were true. 42

Appendix D

Quick Reference

Characteristic Truth Tables

$\mathcal{A}$	$\sim\mathcal{A}$
T	F
F	T

$\mathcal{A}$	$\mathcal{B}$	$\mathcal{A}\bullet\mathcal{B}$	$\mathcal{A}\lor\mathcal{B}$	$\mathcal{A}\supset\mathcal{B}$	$\mathcal{A}\equiv\mathcal{B}$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	T	F
F	F	F	F	T	T

Symbolization

Sentential Connectives (chapter 5)

It is not the case that P.	$\sim P$
Either P, or Q.	$(P \lor Q)$
Neither P, nor Q.	$\sim(P \lor Q)$ or $(\sim P \bullet \sim Q)$
Both P, and Q.	$(P \bullet Q)$
If P, then Q.	$(P \supset Q)$
P only if Q.	$(P \supset Q)$
P if and only if Q.	$(P \equiv Q)$

Unless P, Q. P unless Q. $(P \vee Q)$

Predicates (chapter ??)

All Fs are Gs.	$\forall x(Fx \supset Gx)$
Some Fs are Gs.	$\exists x(Fx \bullet Gx)$
Not all Fs are Gs.	$\sim\forall x(Fx \supset Gx)$ or $\exists x(Fx \bullet \sim Gx)$
No Fs are Gs.	$\forall x(Fx \supset \sim Gx)$ or $\sim\exists x(Fx \bullet Gx)$

Identity (section ??)

Only j is G.	$\forall x(Gx \equiv x = j)$
Everything besides j is G.	$\forall x(x \neq j \supset Gx)$
j is more R than anyone else.	$\forall x(x \neq j \supset Rjx)$
The F is G.	$\exists x(Fx \bullet \forall y(Fy \supset x = y) \bullet Gx)$

'The F is not G' can be translated two ways:

It is not the case that the F is G. (wide)	$\sim\exists x(Fx \bullet \forall y(Fy \supset x = y) \bullet Gx)$
The F is non-G. (narrow)	$\exists x(Fx \bullet \forall y(Fy \supset x = y) \bullet \sim Gx)$

Using identity to symbolize quantities

There are at least _____ Fs.

one	$\exists x Fx$
two	$\exists x_1 \exists x_2(Fx_1 \bullet Fx_2 \bullet x_1 \neq x_2)$
three	$\exists x_1 \exists x_2 \exists x_3(Fx_1 \bullet Fx_2 \bullet Fx_3 \bullet x_1 \neq x_2 \bullet x_1 \neq x_3 \bullet x_2 \neq x_3)$

four $\exists x_1 \exists x_2 \exists x_3 \exists x_4 (Fx_1 \bullet Fx_2 \bullet Fx_3 \bullet Fx_4 \bullet x_1 \neq x_2 \bullet x_1 \neq x_3 \bullet x_1 \neq x_4 \bullet x_2 \neq x_3 \bullet x_2 \neq x_4 \bullet x_3 \neq x_4)$

n $\exists x_1 \cdots \exists x_n (Fx_1 \bullet \cdots \bullet Fx_n \bullet x_1 \neq x_2 \bullet \cdots \bullet x_{n-1} \neq x_n)$

There are at most ______ Fs.

One way to say 'at most n things are F' is to put a negation sign in front of one of the symbolizations above and say $\sim$ 'at least $n+1$ things are F.' Equivalently:

one $\forall x_1 \forall x_2 \big[(Fx_1 \bullet Fx_2) \supset x_1 = x_2\big]$

two $\forall x_1 \forall x_2 \forall x_3 \big[(Fx_1 \bullet Fx_2 \bullet Fx_3) \supset (x_1 = x_2 \vee x_1 = x_3 \vee x_2 = x_3)\big]$

three $\forall x_1 \forall x_2 \forall x_3 \forall x_4 \big[(Fx_1 \bullet Fx_2 \bullet Fx_3 \bullet Fx_4) \supset (x_1 = x_2 \vee x_1 = x_3 \vee x_1 = x_4 \vee x_2 = x_3 \vee x_2 = x_4 \vee x_3 = x_4)\big]$

n $\forall x_1 \cdots \forall x_{n+1} \big[(Fx_1 \bullet \cdots \bullet Fx_{n+1}) \supset (x_1 = x_2 \vee \cdots \vee x_n = x_{n+1})\big]$

There are exactly ______ Fs.

One way to say 'exactly n things are F' is to conjoin two of the symbolizations above and say 'at least n things are F' $\bullet$ 'at most n things are F.' The following equivalent formulae are shorter:

zero $\forall x {\sim} Fx$

one $\exists x \big[Fx \bullet {\sim}\exists y (Fy \bullet x \neq y)\big]$

two $\exists x_1 \exists x_2 \big[Fx_1 \bullet Fx_2 \bullet x_1 \neq x_2 \bullet {\sim}\exists y \big(Fy \bullet y \neq x_1 \bullet y \neq x_2\big)\big]$

three $\exists x_1 \exists x_2 \exists x_3 \big[Fx_1 \bullet Fx_2 \bullet Fx_3 \bullet x_1 \neq x_2 \bullet x_1 \neq x_3 \bullet x_2 \neq x_3 \bullet {\sim}\exists y (Fy \bullet y \neq x_1 \bullet y \neq x_2 \bullet y \neq x_3)\big]$

n $\exists x_1 \cdots \exists x_n \big[Fx_1 \bullet \cdots \bullet Fx_n \bullet x_1 \neq x_2 \bullet \cdots \bullet x_{n-1} \neq x_n \bullet {\sim}\exists y (Fy \bullet y \neq x_1 \bullet \cdots \bullet y \neq x_n)\big]$

Specifying the size of the UD

Removing F from the symbolizations above produces sentences that talk about the size of the UD. For instance, 'there are at least 2 things (in the UD)' may be symbolized as $\exists x \exists y(x \neq y)$.

Basic Rules of Proof

Reiteration

$$
\begin{array}{r|ll}
m & \mathcal{A} & \\
 & \mathcal{A} & \text{R } m
\end{array}
$$

Conjunction Introduction

$$
\begin{array}{r|ll}
m & \mathcal{A} & \\
n & \mathcal{B} & \\
 & \mathcal{A} \bullet \mathcal{B} & \bullet\text{I } m, n
\end{array}
\qquad
\begin{array}{r|ll}
m & \mathcal{A} & \\
n & \mathcal{B} & \\
 & \mathcal{B} \bullet \mathcal{A} & \bullet\text{I } m, n
\end{array}
$$

Conjunction Elimination

$$
\begin{array}{r|ll}
m & \mathcal{A} \bullet \mathcal{B} & \\
 & \mathcal{A} & \bullet\text{E } m
\end{array}
\qquad
\begin{array}{r|ll}
m & \mathcal{A} \bullet \mathcal{B} & \\
 & \mathcal{B} & \bullet\text{E } m
\end{array}
$$

Disjunction Introduction

$$
\begin{array}{r|ll}
m & \mathcal{A} & \\
 & \mathcal{A} \vee \mathcal{B} & \vee\text{I } m
\end{array}
\qquad
\begin{array}{r|ll}
m & \mathcal{A} & \\
 & \mathcal{B} \vee \mathcal{A} & \vee\text{I } m
\end{array}
$$

Disjunction Elimination

$$\begin{array}{r|ll} m & \mathcal{A} \lor \mathcal{B} & \\ n & {\sim}\mathcal{B} & \\ & \mathcal{A} & \lor\text{E } m, n \end{array} \qquad \begin{array}{r|ll} m & \mathcal{A} \lor \mathcal{B} & \\ n & {\sim}\mathcal{A} & \\ & \mathcal{B} & \lor\text{E } m, n \end{array}$$

Conditional Introduction

$$\begin{array}{r|ll} m & \quad \begin{array}{|l} \mathcal{A} \\ \hline \mathcal{B} \end{array} & \begin{array}{l} \text{want } \mathcal{B} \\ \\ \end{array} \\ n & & \\ & \mathcal{A} \supset \mathcal{B} & \supset\text{I } m\text{–}n \end{array}$$

Conditional Elimination

$$\begin{array}{r|ll} m & \mathcal{A} \supset \mathcal{B} & \\ n & \mathcal{A} & \\ & \mathcal{B} & \supset\text{E } m, n \end{array}$$

Biconditional Introduction

$$\begin{array}{r|ll} m & \quad \begin{array}{|l} \mathcal{A} \\ \hline \mathcal{B} \end{array} & \begin{array}{l} \text{want } \mathcal{B} \\ \\ \end{array} \\ n & & \\ p & \quad \begin{array}{|l} \mathcal{B} \\ \hline \mathcal{A} \end{array} & \begin{array}{l} \text{want } \mathcal{A} \\ \\ \end{array} \\ q & & \\ & \mathcal{A} \equiv \mathcal{B} & \equiv\text{I } m\text{–}n,\ p\text{–}q \end{array}$$

Biconditional Elimination

$$
\begin{array}{l|ll}
m & \mathcal{A} \equiv \mathcal{B} & \\
n & \mathcal{B} & \\
 & \mathcal{A} & \equiv\text{E } m, n
\end{array}
\qquad
\begin{array}{l|ll}
m & \mathcal{A} \equiv \mathcal{B} & \\
n & \mathcal{A} & \\
 & \mathcal{B} & \equiv\text{E } m, n
\end{array}
$$

Negation Introduction

$$
\begin{array}{l|l|ll}
m & & \underline{\mathcal{A}} & \text{for reductio} \\
n-1 & & \mathcal{B} & \\
n & & {\sim}\mathcal{B} & \\
 & {\sim}\mathcal{A} & & {\sim}\text{I } m\text{–}n
\end{array}
$$

Negation Elimination

$$
\begin{array}{l|l|ll}
m & & \underline{{\sim}\mathcal{A}} & \text{for reductio} \\
n-1 & & \mathcal{B} & \\
n & & {\sim}\mathcal{B} & \\
 & \mathcal{A} & & {\sim}\text{E } m\text{–}n
\end{array}
$$

[4]

Quantifier Rules

Existential Introduction

$$
\begin{array}{r|ll}
m & \mathcal{A} & \\
 & \exists \chi \mathcal{A}[\chi || c] & \exists\mathrm{I}\ m
\end{array}
$$

χ may replace some or all occurrences of c in $\mathcal{A}$.

Existential Elimination

$$
\begin{array}{r|lll}
m & \exists \chi \mathcal{A} & & \\
n & & \begin{array}{|l} \mathcal{A}[c|\chi] \\ \hline \end{array} & \\
p & & \begin{array}{|l} \mathcal{B} \end{array} & \\
 & \mathcal{B} & & \exists\mathrm{E}\ m,\ n\text{–}p
\end{array}
$$

The constant c must not appear in $\exists \chi \mathcal{A}$, in $\mathcal{B}$, or in any undischarged assumption.

Universal Introduction

$$
\begin{array}{r|ll}
m & \mathcal{A} & \\
 & \forall \chi \mathcal{A}[\chi | c] & \forall\mathrm{I}\ m
\end{array}
$$

c must not occur in any undischarged assumptions.

Universal Elimination

$$
\begin{array}{r|ll}
m & \forall \chi \mathcal{A} & \\
 & \mathcal{A}[c|\chi] & \forall\mathrm{E}\ m
\end{array}
$$

Identity Rules

Identity Introduction

$$\begin{array}{l|ll} & \mathcal{c} = \mathcal{c} & =\text{I} \end{array}$$

Identity Elimination

$$\begin{array}{l|ll} m & \mathcal{c} = \mathcal{d} & \\ n & \mathcal{A} & \\ & \mathcal{A}[\mathcal{c}||\mathcal{d}] & =\text{E } m, n \\ & \mathcal{A}[\mathcal{d}||\mathcal{c}] & =\text{E } m, n \end{array}$$

One constant may replace some or all occurrences of the other.

Derived Rules

Constructive Dilemma (CD)

$$\begin{array}{l|ll} m & \mathcal{A} \vee \mathcal{B} & \\ n & \mathcal{A} \supset \mathcal{C} & \\ p & \mathcal{B} \supset \mathcal{C} & \\ & \mathcal{C} & \vee{*}\ m, n, p \end{array}$$

Modus Tollens (MT)

$$\begin{array}{l|ll} m & \mathcal{A} \supset \mathcal{B} & \\ n & {\sim}\mathcal{B} & \\ & {\sim}\mathcal{A} & \text{MT } m, n \end{array}$$

Hypothetical Syllogism (HS)

$$\begin{array}{l|ll} m & \mathcal{A} \supset \mathcal{B} & \\ n & \mathcal{B} \supset \mathcal{C} & \\ & \mathcal{A} \supset \mathcal{C} & \text{HS } m, n \end{array}$$

Replacement Rules

Commutivity (Comm)

$$(\mathcal{A} \bullet \mathcal{B}) \iff (\mathcal{B} \bullet \mathcal{A})$$
$$(\mathcal{A} \vee \mathcal{B}) \iff (\mathcal{B} \vee \mathcal{A})$$
$$(\mathcal{A} \equiv \mathcal{B}) \iff (\mathcal{B} \equiv \mathcal{A})$$

DeMorgan (DeM)

$$\sim(\mathcal{A} \vee \mathcal{B}) \iff (\sim\mathcal{A} \bullet \sim\mathcal{B})$$
$$\sim(\mathcal{A} \bullet \mathcal{B}) \iff (\sim\mathcal{A} \vee \sim\mathcal{B})$$

Double Negation (DN)

$$\sim\sim\mathcal{A} \iff \mathcal{A}$$

Material Conditional (MC)

$$(\mathcal{A} \supset \mathcal{B}) \iff (\sim\mathcal{A} \vee \mathcal{B})$$
$$(\mathcal{A} \vee \mathcal{B}) \iff (\sim\mathcal{A} \supset \mathcal{B})$$

Biconditional Exchange (≡ex)

$$[(\mathcal{A} \supset \mathcal{B}) \bullet (\mathcal{B} \supset \mathcal{A})] \iff (\mathcal{A} \equiv \mathcal{B})$$

Quantifier Negation (QN)

$$\sim\forall \chi \mathcal{A} \iff \exists \chi \sim\mathcal{A}$$
$$\sim\exists \chi \mathcal{A} \iff \forall \chi \sim\mathcal{A}$$

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