

**Strategy 22:** Transposition can be used to set up constructive dilemma:

1.  $(A \supset B) \cdot (C \supset D)$
2.  $\sim B \vee \sim D$
3.  $(\sim B \supset \sim A) \cdot (C \supset D)$       1, Trans
4.  $(\sim B \supset \sim A) \cdot (\sim D \supset \sim C)$       3, Trans
5.  $\sim A \vee \sim C$       2, 4, CD

**Strategy 23:** Constructive dilemma can be used to set up tautology:

1.  $(A \supset C) \cdot (B \supset C)$
2.  $A \vee B$
3.  $C \vee C$       1, 2, CD
4.  $C$       3, Taut

**Strategy 24:** Material implication can be used to set up tautology:

1.  $A \supset \sim A$
2.  $\sim A \vee \sim A$       1, Impl
3.  $\sim A$       2, Taut

**Strategy 25:** Material implication can be used to set up distribution:

1.  $A \supset (B \cdot C)$
2.  $\sim A \vee (B \cdot C)$       1, Impl
3.  $(\sim A \vee B) \cdot (\sim A \vee C)$       2, Dist

## EXERCISE 7.4

I. For each of the following lists of premises, derive the indicated conclusion and complete the justification. For tautology, derive a conclusion that is simpler than the premise.

- ★(1) 1.  $H \vee F$   
 2.  $N \vee \sim S$   
 3.  $\sim G \vee Q$   
 4. \_\_\_\_\_, Impl
- (2) 1.  $R \supset (S \supset N)$   
 2.  $T \supset (U \vee M)$   
 3.  $K \cdot (L \supset W)$   
 4. \_\_\_\_\_, Exp
- (3) 1.  $G \equiv R$   
 2.  $H \supset P$   
 3.  $\sim F \vee T$   
 4. \_\_\_\_\_, Trans
- ★(4) 1.  $(B \supset N) \cdot (N \supset B)$   
 2.  $(R \vee F) \cdot (F \vee R)$   
 3.  $(K \supset C) \vee (C \supset K)$   
 4. \_\_\_\_\_, Equiv

- (5) 1.  $E \vee \sim E$   
 2.  $A \vee A$   
 3.  $G \bullet \sim G$   
 4. \_\_\_\_\_, Taut
- (6) 1.  $S \vee \sim M$   
 2.  $\sim N \bullet \sim T$   
 3.  $\sim L \supset Q$   
 4. \_\_\_\_\_, Trans
- ★(7) 1.  $\sim C \supset \sim F$   
 2.  $D \vee \sim P$   
 3.  $\sim R \bullet Q$   
 4. \_\_\_\_\_, Impl
- (8) 1.  $E \supset (R \bullet Q)$   
 2.  $(G \bullet N) \supset Z$   
 3.  $(S \supset M) \supset P$   
 4. \_\_\_\_\_, Exp
- (9) 1.  $(D \bullet H) \vee (\sim D \bullet \sim H)$   
 2.  $(F \supset J) \bullet (\sim F \supset \sim J)$   
 3.  $(N \vee T) \bullet (\sim N \vee \sim T)$   
 4. \_\_\_\_\_, Equiv
- ★(10) 1.  $L \supset (A \supset A)$   
 2.  $K \supset (R \vee \sim R)$   
 3.  $S \supset (G \bullet G)$   
 4. \_\_\_\_\_, Taut
- (11) 1.  $K \bullet (S \vee B)$   
 2.  $\sim F \supset \sim J$   
 3.  $\sim E \vee \sim M$   
 4. \_\_\_\_\_, Trans
- (12) 1.  $H \supset (K \bullet J)$   
 2.  $(N \vee E) \supset B$   
 3.  $C \supset (H \supset A)$   
 4. \_\_\_\_\_, Exp
- ★(13) 1.  $(A \supset \sim C) \bullet (C \supset \sim A)$   
 2.  $(W \supset \sim T) \bullet (\sim T \supset W)$   
 3.  $(M \supset \sim E) \bullet (\sim M \supset E)$   
 4. \_\_\_\_\_, Equiv
- (14) 1.  $(\sim K \vee M) \equiv S$   
 2.  $T \vee (F \bullet G)$   
 3.  $R \equiv (N \bullet \sim H)$   
 4. \_\_\_\_\_, Impl
- (15) 1.  $(S \vee S) \supset D$   
 2.  $K \supset (T \bullet \sim T)$   
 3.  $(Q \supset Q) \supset M$   
 4. \_\_\_\_\_, Taut

II. In the following symbolized arguments, derive the line needed to obtain the conclusion (last line), and supply the justification for both lines.

- ★(1) 1.  $\sim J \vee M$   
 2.  $M \supset B$   
 3. \_\_\_\_\_  
 4.  $J \supset B$  \_\_\_\_\_

- (2) 1.  $(J \cdot F) \supset N$   
 2.  $J$   
 3. \_\_\_\_\_  
 4.  $F \supset N$  \_\_\_\_\_

- (3) 1.  $C \supset A$   
 2.  $A \supset C$   
 3. \_\_\_\_\_  
 4.  $C \equiv A$  \_\_\_\_\_

- ★(4) 1.  $(G \supset K) \cdot (T \supset K)$   
 2.  $G \vee T$   
 3. \_\_\_\_\_  
 4.  $K$  \_\_\_\_\_

- (5) 1.  $(G \supset B) \cdot (\sim C \supset \sim H)$   
 2.  $G \vee H$   
 3. \_\_\_\_\_  
 4.  $B \vee C$  \_\_\_\_\_

- (6) 1.  $J \supset (M \supset Q)$   
 2.  $J \cdot M$   
 3. \_\_\_\_\_  
 4.  $Q$  \_\_\_\_\_

- ★(7) 1.  $H \supset (\sim C \vee R)$   
 2. \_\_\_\_\_  
 3.  $(H \cdot C) \supset R$  \_\_\_\_\_

- (8) 1.  $\sim G \supset \sim T$   
 2.  $G \supset N$   
 3. \_\_\_\_\_  
 4.  $T \supset N$  \_\_\_\_\_

- (9) 1.  $K \supset (A \supset F)$   
 2.  $\sim F$   
 3. \_\_\_\_\_  
 4.  $\sim(K \cdot A)$  \_\_\_\_\_

- ★(10) 1.  $H \supset \sim H$   
 2. \_\_\_\_\_  
 3.  $\sim H$  \_\_\_\_\_

- (11) 1.  $\sim S$   
 2. \_\_\_\_\_  
 3.  $S \supset K$  \_\_\_\_\_

- (12) 1.  $M \supset (M \supset D)$   
 2. \_\_\_\_\_  
 3.  $M \supset D$  \_\_\_\_\_

- ★(13) 1.  $(N \supset A) \cdot (\sim N \supset \sim A)$   
 2. \_\_\_\_\_  
 3.  $N \equiv A$  \_\_\_\_\_

- (14) 1.  $E \cdot R$   
 2. \_\_\_\_\_  
 3.  $E \equiv R$  \_\_\_\_\_

- (15) 1.  $Q \supset (\sim W \supset \sim G)$   
 2. \_\_\_\_\_  
 3.  $(Q \cdot G) \supset W$  \_\_\_\_\_

III. Use the eighteen rules of inference to derive the conclusions of the following symbolized arguments.

- ★(1) 1.  $(S \cdot K) \supset R$   
 2.  $K$  /  $S \supset R$

- (2) 1.  $T \supset (F \vee F)$   
 2.  $\sim(F \cdot F)$  /  $\sim T$

- (3) 1.  $G \supset E$   
 2.  $H \supset \sim E$  /  $G \supset \sim H$

- ★(4) 1.  $S \equiv Q$   
 2.  $\sim S$  /  $\sim Q$

- (5) 1.  $\sim N \vee P$   
 2.  $(N \supset P) \supset T$  /  $T$

- (6) 1.  $F \supset B$   
 2.  $B \supset (B \supset J)$  /  $F \supset J$
- ★(7) 1.  $(B \supset M) \cdot (D \supset M)$   
 2.  $B \vee D$  /  $M$
- (8) 1.  $Q \supset (F \supset A)$   
 2.  $R \supset (A \supset F)$   
 3.  $Q \cdot R$  /  $F \equiv A$
- (9) 1.  $T \supset (\sim T \vee G)$   
 2.  $\sim G$  /  $\sim T$
- ★(10) 1.  $(B \supset G) \cdot (F \supset N)$   
 2.  $\sim(G \cdot N)$  /  $\sim(B \cdot F)$
- (11) 1.  $(J \cdot R) \supset H$   
 2.  $(R \supset H) \supset M$   
 3.  $\sim(P \vee \sim J)$  /  $M \cdot \sim P$
- (12) 1.  $T$  /  $S \supset T$
- ★(13) 1.  $K \supset (B \supset \sim M)$   
 2.  $D \supset (K \cdot M)$  /  $D \supset \sim B$
- (14) 1.  $(O \supset C) \cdot (\sim S \supset \sim D)$   
 2.  $(E \supset D) \cdot (\sim E \supset \sim C)$  /  $O \supset S$
- (15) 1.  $\sim(U \cdot W) \supset X$   
 2.  $U \supset \sim U$  /  $\sim(U \vee \sim X)$
- ★(16) 1.  $T \supset R$   
 2.  $T \supset \sim R$  /  $\sim T$
- (17) 1.  $S \vee \sim N$   
 2.  $\sim S \vee Q$  /  $N \supset Q$
- (18) 1.  $M \supset (U \supset H)$   
 2.  $(H \vee \sim U) \supset F$  /  $M \supset F$
- ★(19) 1.  $\sim R \vee P$   
 2.  $R \vee \sim P$  /  $R \equiv P$
- (20) 1.  $\sim H \supset B$   
 2.  $\sim H \supset D$   
 3.  $\sim(B \cdot D)$  /  $H$
- (21) 1.  $J \supset (G \supset L)$  /  $G \supset (J \supset L)$
- ★(22) 1.  $S \supset (L \cdot M)$   
 2.  $M \supset (L \supset R)$  /  $S \supset R$
- (23) 1.  $F \supset (A \cdot K)$   
 2.  $G \supset (\sim A \cdot \sim K)$   
 3.  $F \vee G$  /  $A \equiv K$

- (24) 1.  $(I \supset E) \supset C$   
 2.  $C \supset \sim C$  /  $I$
- ★(25) 1.  $T \supset G$   
 2.  $S \supset G$  /  $(T \vee S) \supset G$
- (26) 1.  $H \supset U$  /  $H \supset (U \vee T)$
- (27) 1.  $Q \supset (W \cdot D)$  /  $Q \supset W$
- ★(28) 1.  $P \supset (\sim E \supset B)$   
 2.  $\sim(B \vee E)$  /  $\sim P$
- (29) 1.  $(G \supset J) \supset (H \supset Q)$   
 2.  $J \cdot \sim Q$  /  $\sim H$
- (30) 1.  $I \vee (N \cdot F)$   
 2.  $I \supset F$  /  $F$
- ★(31) 1.  $K \equiv R$   
 2.  $K \supset (R \supset P)$   
 3.  $\sim P$  /  $\sim R$
- (32) 1.  $C \supset (\sim L \supset Q)$   
 2.  $L \supset \sim C$   
 3.  $\sim Q$  /  $\sim C$
- (33) 1.  $(E \supset A) \cdot (F \supset A)$   
 2.  $E \vee G$   
 3.  $F \vee \sim G$  /  $A$
- ★(34) 1.  $(F \cdot H) \supset N$   
 2.  $F \vee S$   
 3.  $H$  /  $N \vee S$
- (35) 1.  $T \supset (H \cdot J)$   
 2.  $(H \vee N) \supset T$  /  $T \equiv H$
- (36) 1.  $T \supset \sim(A \supset N)$   
 2.  $T \vee N$  /  $T \equiv \sim N$
- ★(37) 1.  $(D \supset E) \supset (E \supset D)$   
 2.  $(D \equiv E) \supset \sim(G \cdot \sim H)$   
 3.  $E \cdot G$  /  $G \cdot H$
- (38) 1.  $(O \supset R) \supset S$   
 2.  $(P \supset R) \supset \sim S$  /  $\sim R$
- (39) 1.  $(L \vee P) \supset U$   
 2.  $(M \supset U) \supset I$   
 3.  $P$  /  $I$
- ★(40) 1.  $A \equiv W$   
 2.  $\sim A \vee \sim W$   
 3.  $R \supset A$  /  $\sim(W \vee R)$



- (41) 1.  $(S \vee T) \supset (S \supset \sim T)$   
 2.  $(S \supset \sim T) \supset (T \supset K)$   
 3.  $S \vee T$  /  $S \vee K$

- (42) 1.  $G \equiv M$   
 2.  $G \vee M$   
 3.  $G \supset (M \supset T)$  /  $T$

- ★(43) 1.  $O \supset (Q \cdot N)$   
 2.  $(N \vee E) \supset S$  /  $O \supset S$

- (44) 1.  $H \equiv I$   
 2.  $H \supset (I \supset F)$   
 3.  $\sim(H \vee I) \supset F$  /  $F$

- ★(45) 1.  $P \supset A$   
 2.  $Q \supset B$  /  $(P \vee Q) \supset (A \vee B)$

IV. Translate the following arguments into symbolic form and then use the eighteen rules of inference to derive the conclusion of each. Use the translation letters in the order in which they are listed.

- ★1. If sports-shoe manufacturers decline to use kangaroo hides in their products, then Australian hunters will cease killing millions of kangaroos yearly. It is not the case that both Australian hunters will cease killing millions of kangaroos yearly and the kangaroo will not be saved from extinction. Therefore, if sports-shoe manufacturers decline to use kangaroo hides in their products, then the kangaroo will be saved from extinction. ( $D, C, S$ )
2. If there is a direct correlation between what a nation spends for health care and the health of its citizens, then America has the lowest incidence of disease and the lowest mortality rates of any nation on Earth. But America does not have the lowest mortality rates of any nation on Earth. Therefore, there is not a direct correlation between what a nation spends for health care and the health of its citizens. ( $C, D, M$ )
3. It is not the case that strict controls exist on either the manufacture or the sale of handguns. Therefore, if strict controls exist on the sale of handguns, then the use of handguns in the commission of crimes has decreased. ( $M, S, U$ )
- ★4. If birth-control devices are made available in high school clinics, then the incidence of teenage pregnancy will decrease. Therefore, if both birth-control information and birth-control devices are made available in high school clinics, then the incidence of teenage pregnancy will decrease. ( $D, P, I$ )
5. If Congress enacts a law that either establishes a religion or prohibits the free exercise of religion, then that law is unconstitutional. Therefore, if Congress enacts a law that establishes a religion, then that law is unconstitutional. ( $E, P, U$ )
6. If cigarette smokers are warned of the hazards of smoking and they continue to smoke, then they cannot sue tobacco companies for any resulting lung cancer or emphysema. Cigarette smokers are warned of the hazards of smoking.