# Indirect Truth Tables and Four Rules

Indirect truth tables have been the focus of a number of questions (specifically as related to the homework problems #14 and #15 from page 366). As such, I've decided to take another stab at explaining them here.

Let's look at #15 from page 366.

| N ∨ ~O | P ∨ O | P ⊃ Q | (N ∨ Q) ⊃ (R • S) | S ⊃ (R ⊃ T) | O ⊃ (T ⊃ U) | // U |
|---|---|---|---|---|---|---|
| | | | | | | |

The directions state to use the indirect method for determining whether the argument is valid or invalid. The quickest way to do this is to set the conclusion to false (in this case, just the sentence letter U).

| N ∨ ~O | P ∨ O | P ⊃ Q | (N ∨ Q) ⊃ (R • S) | S ⊃ (R ⊃ T) | O ⊃ (T ⊃ U) | // U |
|---|---|---|---|---|---|---|
| | | | | | F | F |

Now we try to figure out a way for each of the premises to receive a T under their dominant operator. This, of course, has been the most confusing part since we have *very little* information about the remaining premises. Let's begin with the premise we have the most info about. We know that since U is false that any truth table line which has O and T as true will result in the sixth premise being false, so those truth values are no options. We also note that O also plays an important role in the first and second premise - hence, we can get a lot done by focusing on O's truth value. Let's try having O be false, but everything else (except U) be true. Then, the first premise is true, the second premise is true, the third premise is true, the fourth premise is true, the fifth premise is true, *and* the sixth premise is true - but the conclusion is false! That is EXACTLY what we wanted.

| N ∨ ~O | P ∨ O | P ⊃ Q | (N ∨ Q) ⊃ (R • S) | S ⊃ (R ⊃ T) | O ⊃ (T ⊃ U) | // U |
|---|---|---|---|---|---|---|
| T **T** T F | T **T** F | T **T** T | T T T **T** T T T | T **T** T T T | F **T** T F F | F |

So, the process for creating an indirect truth table turns out to be very simple.

1. Determine what truth values are needed to make the conclusion false. Those will be the lines we will focus on for the truth tables.
2. Fill in the values for the conclusion everywhere else the sentence letters occur in the relevant premises.

3. Then, see if applying either T or all F to the remaining sentence letters makes the dominant operator for the premises come out as true.
4. If step three works, then you are done and the argument is invalid.
5. If step three does not work, ask yourself if there is a minor adjustment to one or more of the sentence letters that will make the premises all true.
6. If step 5 works, then you are done and the argument is invalid.

What should be somewhat clear now is that indirect truth tables work well for proving invalidity when you have complex premises and simple conclusions. As you can see from the exercises on page 365-367, all of Hurley's problems involve very simple conclusions. As such, our method should suffice.

If this is still unclear, then I would encourage your to come see Zach or I during office hours (or schedule a time with us). Moreover, you can always do a less-indirect-truth-table. Do steps 1 and 2. But, instead of doing 3, create all the lines on truth table by computing all the T / F combinations following our $2^n$ formula. Then, get rid of the lines which do not match step 1. Use the remaining lines to compute all the truth values for the premises. If there are any lines on which the premises get Ts but the conclusion is an F, then the argument is invalid. Depending on the number of sentence letters in the premises, this could be quite big, but, if the indirect way is not clear, then this is your only option short of a direct truth table.

## Chapter 7

All together we will have 18 rules which we can use in our proofs. Section 7.1 introduces us to the first four rules.

1. *Modus ponens* (MP)

| | |
|---|---|
| $p \supset q$ | If Su Lin is a panda, then Su Lin is cute. |
| $p$ | Su Lin is a panda. |
| $q$ | Su Lin is cute. |

2. *Modus tollens* (MT)

| | |
|---|---|
| $p \supset q$ | If Koko is a koala, then Koko is cuddly. |
| $\sim q$ | Koko is not cuddly. |
| $\sim p$ | Koko is not a koala. |

3. Pure hypothetical syllogism (HS)

| | |
|---|---|
| $p \supset q$ | If Leo is a lion, then Leo roars. |
| $q \supset r$ | If Leo roars, then Leo is fierce. |
| $p \supset r$ | If Leo is a lion, then Leo is fierce. |

4. Disjunctive syllogism (DS)

| | |
|---|---|
| $p \vee q$ | Scooter is either a mouse or a rat. |
| $\sim p$ | Scooter is not a mouse. |
| $q$ | Scooter is a rat. |

Using these rules is more straightforward than the truth tables were, but will also require your ‘seeing’ the connections between the stated premises and the needed conclusion. One important thing to note is that the P’s and Q’s in the above characterization of each rule are **not** meant to imply that only sentence letters can stand in for the P’s and Q’s. Complex wffs can be in the P and Q positions. For example:

These arguments are all instances of ***modus ponens*** (MP):

| | | |
|---|---|---|
| $\sim F \supset (G \equiv H)$ | $(A \vee B) \supset \sim(C \bullet D)$ | $K \bullet L$ |
| $\sim F$ | $A \vee B$ | $(K \bullet L) \supset [(R \supset S) \bullet (T \supset U)]$ |
| $G \equiv H$ | $\sim(C \bullet D)$ | $(R \supset S) \bullet (T \supset U)$ |

These arguments are all instances of ***modus tollens*** (MT):

| | | |
|---|---|---|
| $(D \vee F) \supset K$ | $\sim G \supset \sim(M \vee N)$ | $\sim T$ |
| $\sim K$ | $\sim\sim(M \vee N)$ | $[(H \vee K) \bullet (L \vee N)] \supset T$ |
| $\sim(D \vee F)$ | $\sim\sim G$ | $\sim[(H \vee K) \bullet (L \vee N)]$ |

These arguments are all instances of **pure hypothetical syllogism** (HS):

| | | |
|---|---|---|
| $A \supset (D \bullet F)$ | $\sim M \supset (R \supset S)$ | $(L \supset N) \supset [(S \vee T) \bullet K]$ |
| $(D \bullet F) \supset \sim H$ | $(C \vee K) \supset \sim M$ | $(C \equiv F) \supset (L \supset N)$ |
| $A \supset \sim H$ | $(C \vee K) \supset (R \supset S)$ | $(C \equiv F) \supset [(S \vee T) \bullet K]$ |

These arguments are all instances of **disjunctive syllogism** (DS):

| | | |
|---|---|---|
| $U \vee \sim(W \bullet X)$ | $\sim(E \vee F)$ | $\sim B \vee [(H \supset M) \bullet (S \supset T)]$ |
| $\sim U$ | $(E \vee F) \vee (N \supset K)$ | $\sim\sim B$ |
| $\sim(W \bullet X)$ | $N \supset K$ | $(H \supset M) \bullet (S \supset T)$ |

The structure of our proofs will also be rigorous so that there is no confusion on our part as to what your reasoning is in the proof. Hurley uses the convention of listing the premises in numbered order, and then stating the conclusion you must prove by first using a slashed line, /, followed by the desired wff. The rules you use and the lines you are using the rules on will be listed to the right of the new line you are creating:

1. $A \supset \sim B$
2. $C \supset B$
3. $A$      / $\sim C$
4. $\sim B$      1, 3, MP

So, in this example, line 4 is the line that was derived from lines 1 and 3 via the rule MP (modus ponens). As usual, practice makes perfect and so you will gain comfort with the rules the more that you use them. A more complicated problem using multiple rules to derive the conclusion is the following problem (which we did in class):

| | |
|---|---|
| 1. $F \supset G$ | |
| 2. $F \vee H$ | |
| 3. $\sim G$ | |
| 4. $H \supset (G \supset I)$ | / $F \supset I$ |

And the proof is as follows:

| | |
|---|---|
| 1. $F \supset G$ | |
| 2. $F \vee H$ | |
| 3. $\sim G$ | |
| 4. $H \supset (G \supset I)$ | / $F \supset I$ |
| 5. $\sim F$ | 1, 3, MT |
| 6. $H$ | 2, 5, DS |
| 7. $G \supset I$ | 4, 6, MP |
| 8. $F \supset I$ | 1, 7, HS |