Conditional Proof

Conditional proof is a powerful proof technique whereby we assume something and then see what can be derived from that assumption. For example, if an economist wants to assess what would happen if a particular economic law was passed, they pretend the law is passed, and then examine what the likely effects would be. Similarly, for conditional proof in propositional logic, we will assume some wff to be true, and then will use the 18 rules to derive a particular conclusion. Thus, what we have proved is: if <assumption>, then <conclusion derived>. This process, then, is how we *create* conditionals. **Note**: This does *not* entail that the assumption is in fact true; what it proves is that if it is true, then the particular conclusion follows - and that is just what the horseshoe is meant to symbolize.

In order to maintain a clear and explicit proof, we are going to introduce conditional proof by a using an indented vertical line which begins with our assumption, and ends with our desired conclusion. Then, we discharge the assumption and conclusion with the horseshoe connecting them. Note the first part of the conditional proof is cited as ACP (assumed for conditional proof) and the discharged line cites every line up and including the ACP line and is the rule CP (conditional proof).

1. $A \supset (B \cdot C)$	
2. $(B \vee D) \supset E$	/ $A \supset E$
	3. A	ACP
	4. $B \cdot C$	1, 3, MP
	5. B	4, Simp
	6. $B \vee D$	5, Add
	7. E	2, 6, MP
8. $A \supset E$	3–7, CP

How do you know what to assume? Well, in the proof above, A is assumed. Why? Well, you could use the 18 rules on premise 1 and 2 and try to derive some new lines. At some point, however, you'll hit a wall and won't be able to get close to the desired conclusion. This is a good indication that some kind of conditional proof will be called for. Additionally, the desired conclusion itself is a conditional which itself is not easily found in the premises. Hence, it would be worth a shot to try and create it.

There is no hard-and-fast rule about what to assume and when. It is, at least when you first start using conditional proof, a trial and error period where you try and fail until you succeed. You *must* work with the rules enough so that you are comfortable with their application. Long story short, you must work through a lot of proofs in order to get good at proofs. Some example problems are below:

Example 1:

1.	$G \supset (H \cdot I)$	
2.	$J \supset (K \cdot L)$	
3.	$G \vee J$	/ $H \vee K$
	4. G	ACP
	5. $H \cdot I$	1, 4, MP
	6. H	5, Simp
7.	$G \supset H$	4–6, CP
	8. J	ACP
	9. $K \cdot L$	2, 8, MP
	10. K	9, Simp
11.	$J \supset K$	8–10, CP
12.	$(G \supset H) \cdot (J \supset K)$	7, 11, Conj
13.	$H \vee K$	3, 12, CD

Example 2:

1.	$L \supset [M \supset (N \vee O)]$	
2.	$M \supset \sim N$	/ $L \supset (\sim M \vee O)$
	3. L	ACP
	4. $M \supset (N \vee O)$	1, 3, MP
	5. M	ACP
	6. $N \vee O$	4, 5, MP
	7. $\sim N$	2, 5, MP
	8. O	6, 7, DS
	9. $M \supset O$	5–8, CP
	10. $\sim M \vee O$	9, Impl
11.	$L \supset (\sim M \vee O)$	3–10, CP