

1. *Modus ponens* (MP)
$$\begin{array}{l} p \supset q \\ p \\ \hline q \end{array}$$

If Su Lin is a panda, then Su Lin is cute.
 Su Lin is a panda.

 Su Lin is cute.

2. *Modus tollens* (MT)
$$\begin{array}{l} p \supset q \\ \sim q \\ \hline \sim p \end{array}$$

If Koko is a koala, then Koko is cuddly.
 Koko is not cuddly.

 Koko is not a koala.

3. Pure hypothetical syllogism (HS)

$$\begin{array}{l} p \supset q \\ q \supset r \\ \hline p \supset r \end{array}$$

If Leo is a lion, then Leo roars.
 If Leo roars, then Leo is fierce.

 If Leo is a lion, then Leo is fierce.

4. Disjunctive syllogism (DS)

$$\begin{array}{l} p \vee q \\ \sim p \\ \hline q \end{array}$$

Scooter is either a mouse or a rat.
 Scooter is not a mouse.

 Scooter is a rat.

Constructive dilemma (CD)

$$(p \supset q) \cdot (r \supset s)$$

$$\begin{array}{l} p \vee r \\ \hline q \vee s \end{array}$$

If Oscar is a dog, then you'll have fleas, and
 if Oscar is a cat, then you'll have fur balls.
 Oscar is either a dog or a cat.

 You'll have either fleas or fur balls.

Simplification (Simp)

$$\begin{array}{l} p \cdot q \\ \hline p \end{array}$$

Eliza has long legs and runs fast.

 Eliza has long legs.

Conjunction (Conj)

$$\begin{array}{l} p \\ q \\ \hline p \cdot q \end{array}$$

Roxy has big eyes.
 Roxy has a tail.

 Roxy has big eyes and a tail.

Addition (Add)

$$\begin{array}{l} p \\ \hline p \vee q \end{array}$$

Theo has spots.

 Theo has either spots or stripes.

9. De Morgan's rule (DM):

$$\sim(p \cdot q) :: (\sim p \vee \sim q)$$

$$\sim(p \vee q) :: (\sim p \cdot \sim q)$$

10. Commutativity (Com):

$$(p \vee q) :: (q \vee p)$$

$$(p \cdot q) :: (q \cdot p)$$

11. Associativity (Assoc):

$$[p \vee (q \vee r)] :: [(p \vee q) \vee r]$$

$$[p \cdot (q \cdot r)] :: [(p \cdot q) \cdot r]$$

12. Distribution (Dist):

$$[p \cdot (q \vee r)] :: [(p \cdot q) \vee (p \cdot r)]$$

$$[p \vee (q \cdot r)] :: [(p \vee q) \cdot (p \vee r)]$$

13. Double negation (DN):

$$p :: \sim\sim p$$

14. Transposition (Trans):

$$(p \supset q) :: (\sim q \supset \sim p)$$

17. Exportation (Exp):

$$[(p \cdot q) \supset r] :: [p \supset (q \supset r)]$$

15. Material implication (Impl):

$$(p \supset q) :: (\sim p \vee q)$$

18. Tautology (Taut):

$$p :: (p \vee p)$$

$$p :: (p \cdot p)$$

16. Material equivalence (Equiv):

$$(p \equiv q) :: [(p \supset q) \cdot (q \supset p)]$$

$$(p \equiv q) :: [(p \cdot q) \vee (\sim p \cdot \sim q)]$$