

1 2D Magnus

$$\begin{cases} \left(-\partial_t u_t(x, v) + h(x, v)u_t(x, v) + f^x(x, v)\partial_x u_t(x, v) + f^v(x, v)\partial_v u_t(x, v) \right. \\ \quad \left. + \frac{1}{2}g^{xx}(x, v)\partial_{xx}u_t(x, v) + g^{xv}(x, v)\partial_{xv}u_t(x, v) + \frac{1}{2}g^{vv}(x, v)\partial_{vv}u_t(x, v) \right) dt \\ \quad + (\sigma^x(x, v)\partial_x u_t(x, v) + \sigma^v(x, v)\partial_v u_t(x, v)) dW_t = 0 \\ u_0(x, v) = \phi(x, v) \end{cases}$$

1.1 Model Coefficient Functions

$$\begin{aligned} h(x, v) &:= 0 \\ f^x(x, v) &:= -v \\ f^v(x, v) &:= 0 \\ g^{xx}(x, v) &:= 0 \\ g^{xv}(x, v) &:= 0 \\ g^{vv}(x, v) &:= \frac{1.1000}{x^2 + 1} + 1.1000 \\ \sigma^x(x, v) &:= 0 \\ \sigma^v(x, v) &:= 0.3162 \sqrt{\frac{1}{x^2 + 1} + 1} \\ \phi(x, v) &:= e^{-0.5000 v^2 - 0.5000 x^2} \end{aligned}$$

1.2 Numerical Parameters

t_0	T	time step $\Delta^{\text{euler ref}}$	time step Δ^{euler}	time step Δ^{magnus}	simulations M
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0	0.1	0.0001	0.001	0.025	100
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lower bound a_x upper bound b_x number of points n_x

-4	4	100
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lower bound a_v upper bound b_v number of points n_v

-4	4	100
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1.3 Computational Times

Method	Computational Time	Number of time evaluations
exact	0	1
euler ref	27.1117	1
m1	1.28799	1
m2	2.44808	1
m3	6.81088	1

1.4 Errors

1.4.1 Error regions

- (i) $\kappa = 1$: $[-1.941, 1.941] \times [-1.941, 1.941]$
- (ii) $\kappa = 2$: $[-0.990, 0.990] \times [-0.990, 0.990]$
- (iii) $\kappa = 4$: $[-0.515, 0.515] \times [-0.515, 0.515]$

1.4.2 Abs Errors

Average Absolute Error at time 0.100

Method	$\kappa = 1$	$\kappa = 2$	$\kappa = 4$
m1	1.873e-03	3.706e-03	5.914e-03
m2	7.992e-05	1.541e-04	2.445e-04
m3	7.982e-05	1.542e-04	2.444e-04

1.4.3 Error Distributions

Error distributions at time 0.100

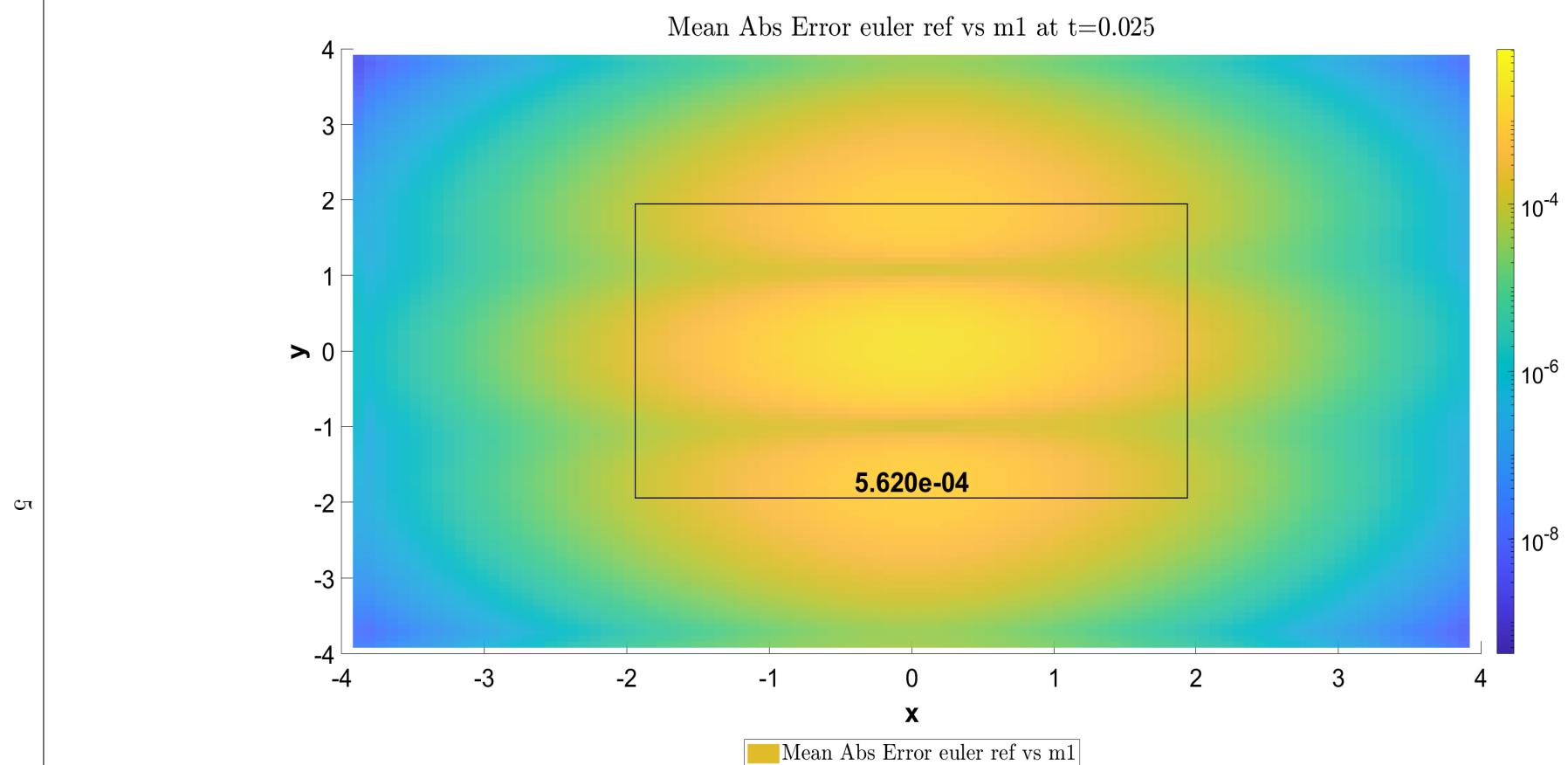
Method	$\kappa = 1$	$\kappa = 2$	$\kappa = 4$
m1	5.946e-03	6.122e-03	7.229e-03
m2	2.531e-04	2.569e-04	3.001e-04
m3	2.528e-04	2.568e-04	3.001e-04

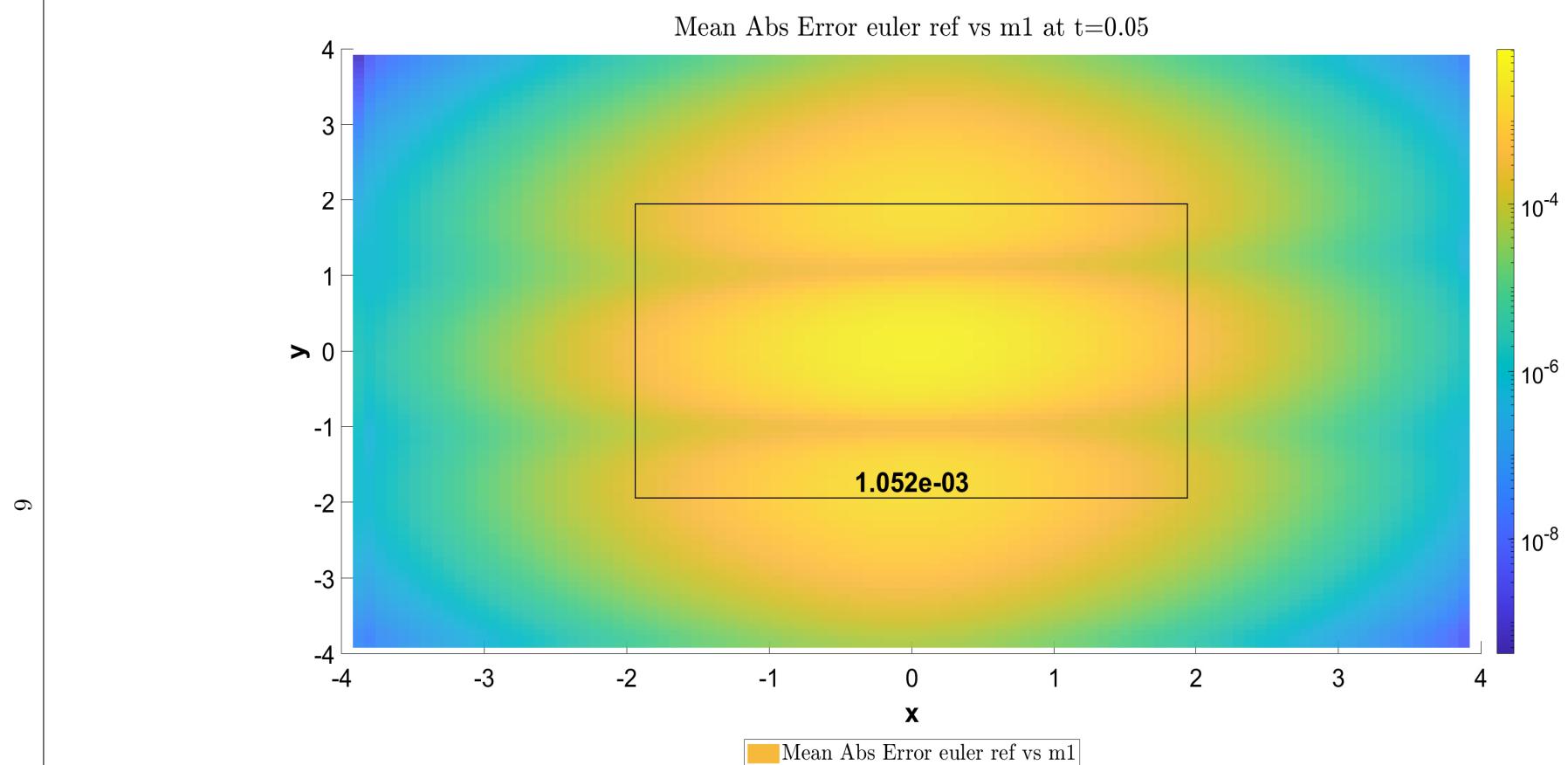
1.5 Plots of Mean Absolute Errors

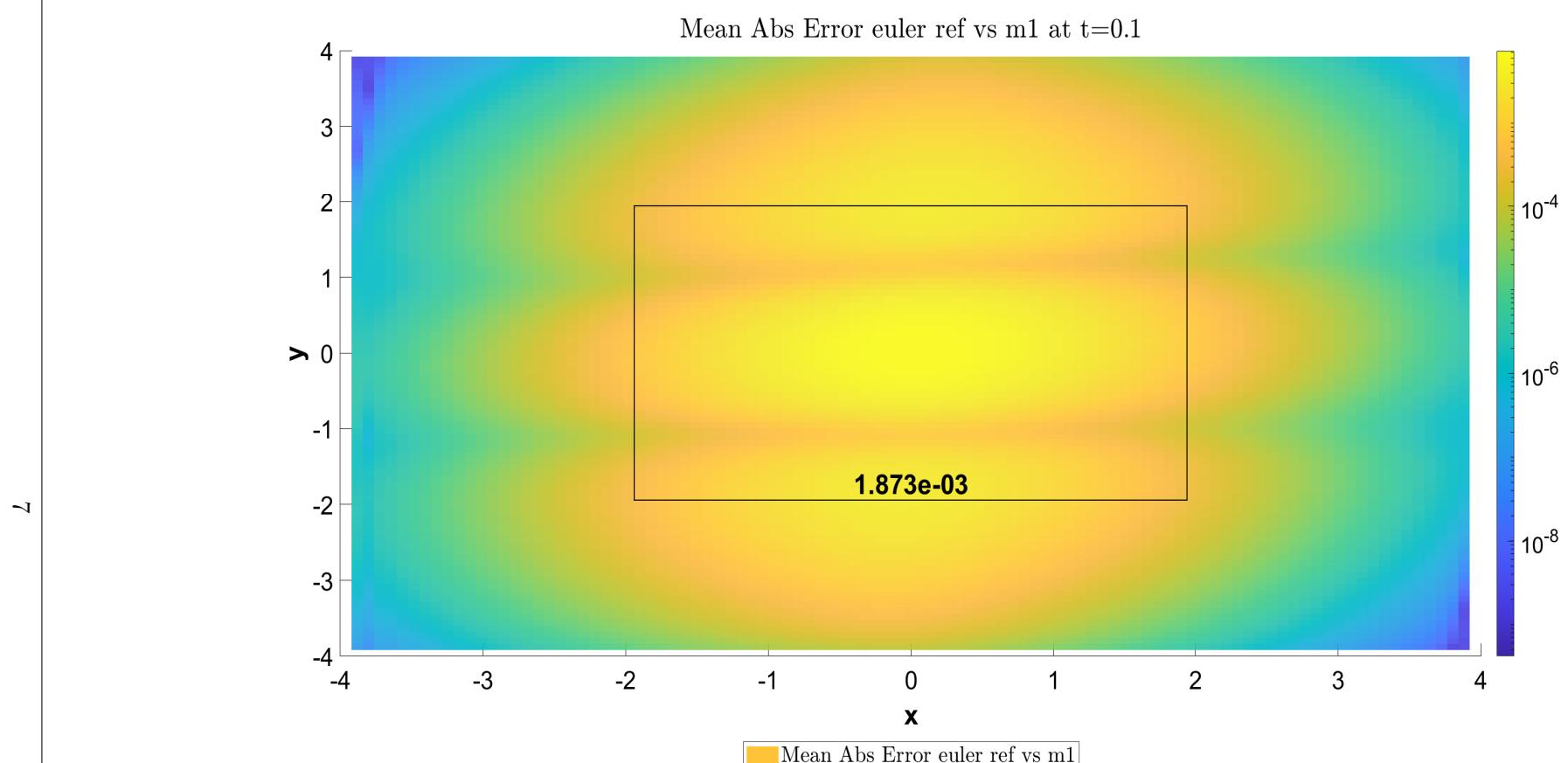
1.6 Reference method: Euler Ref

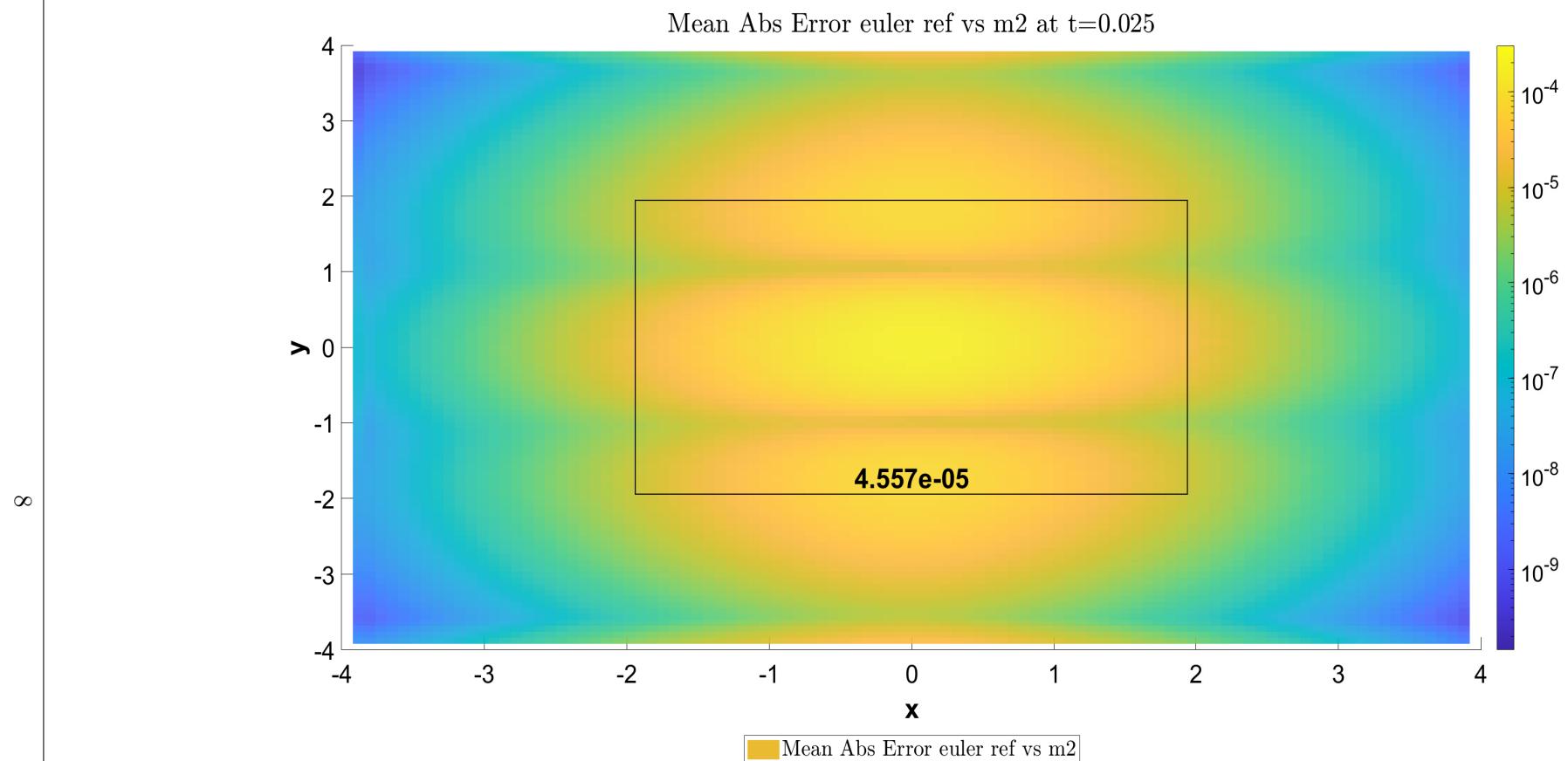
1.7 Plots of CDFs

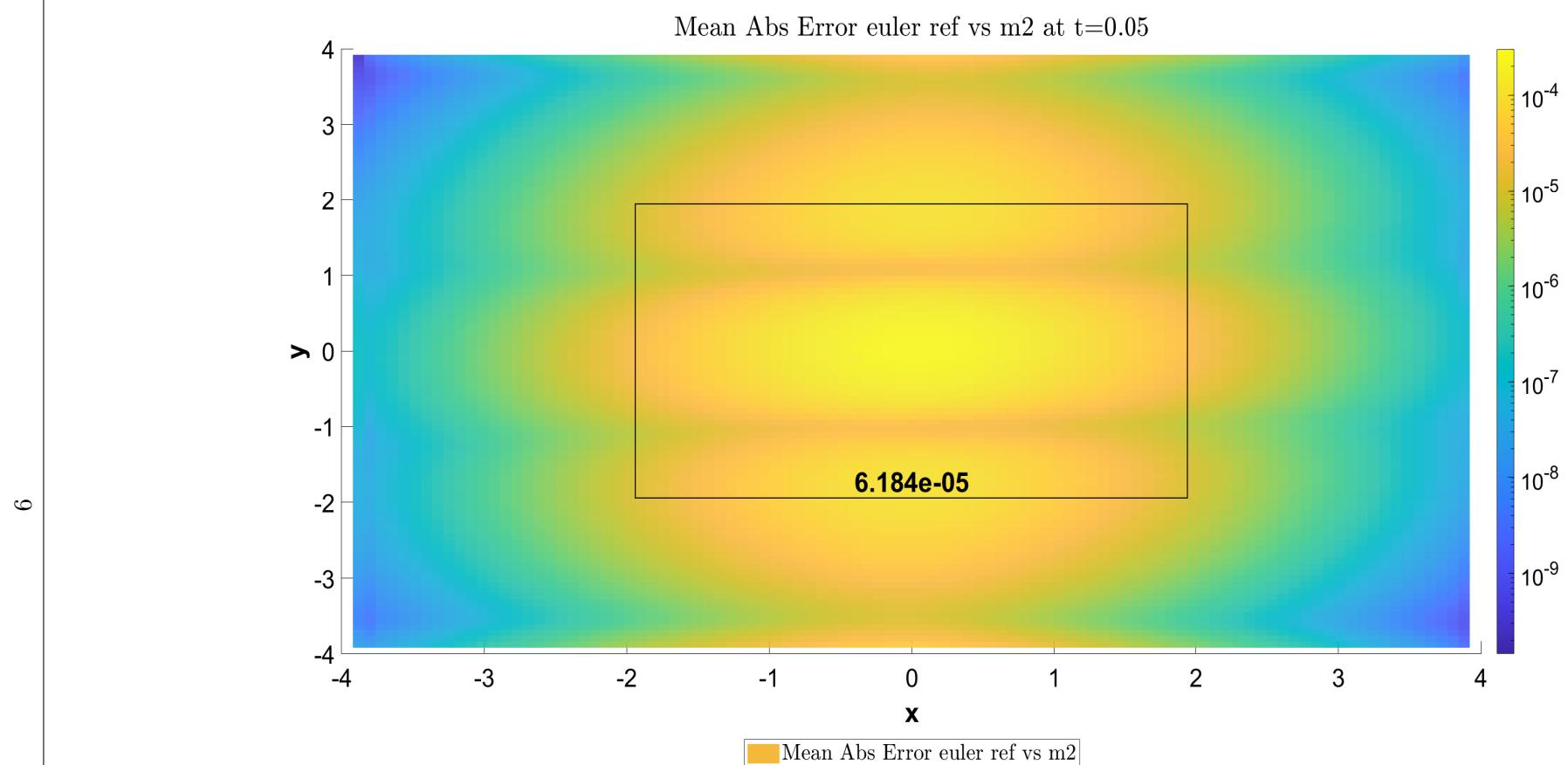
1.7.1 Euler Ref as reference

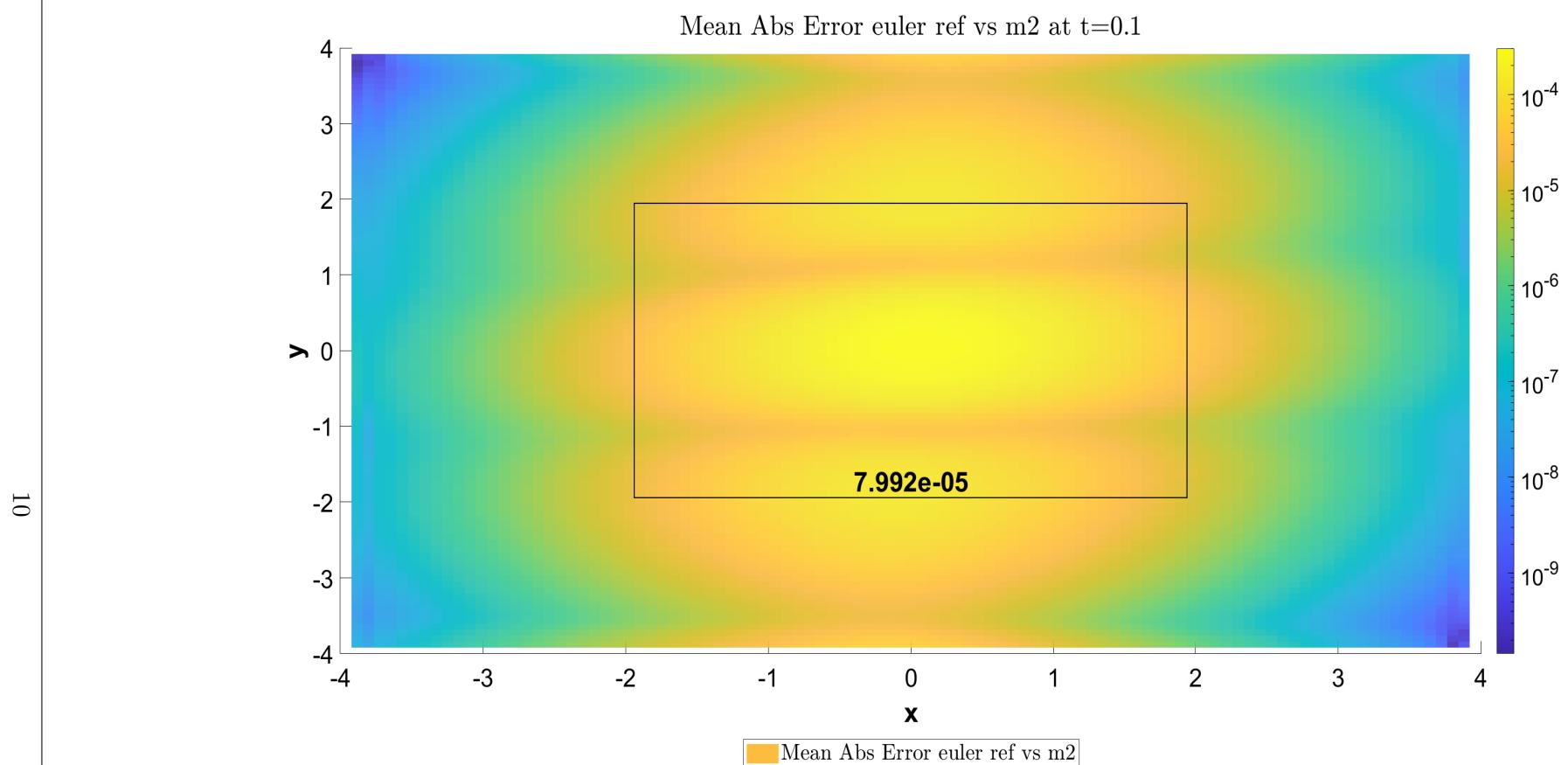


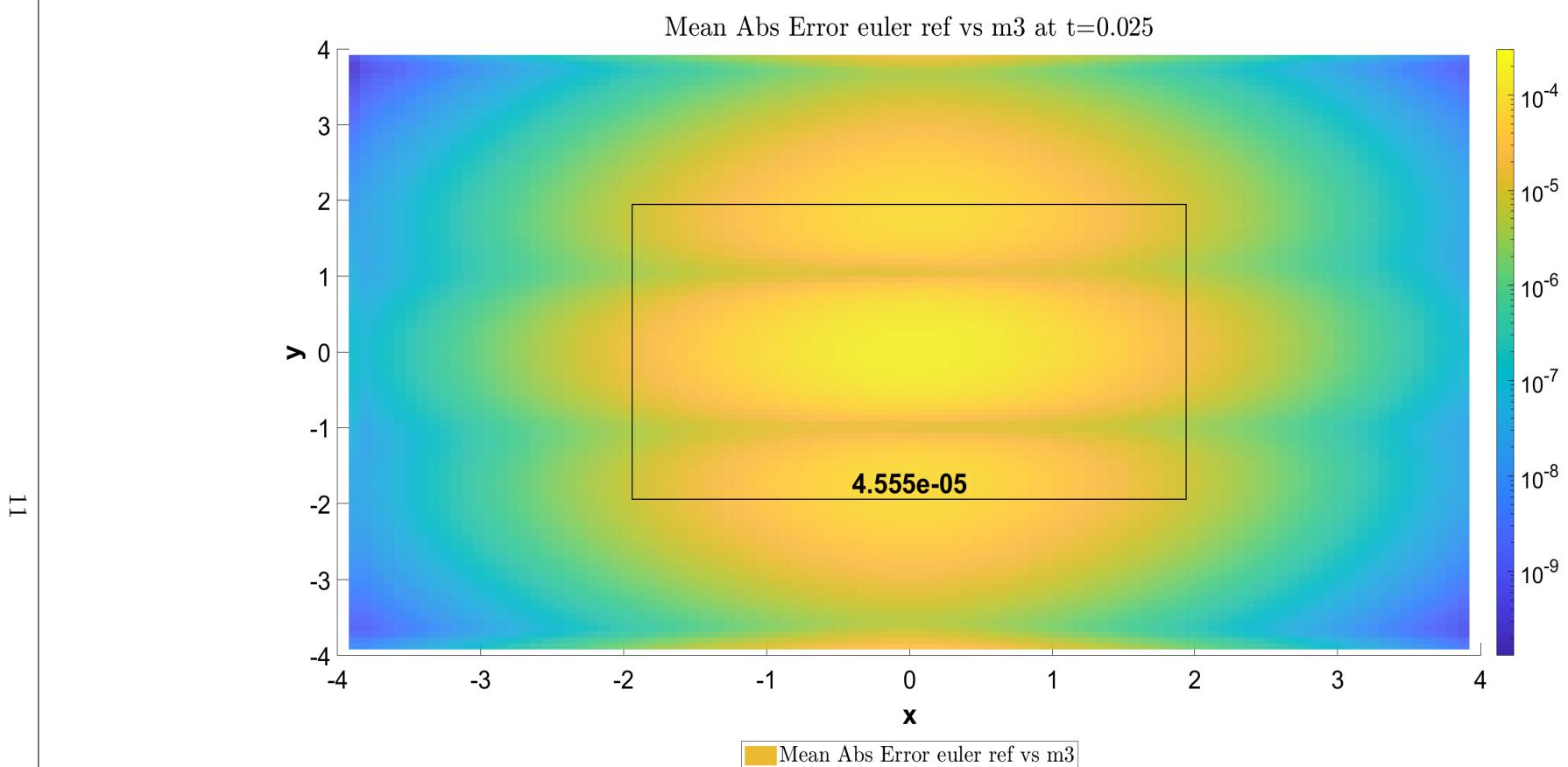


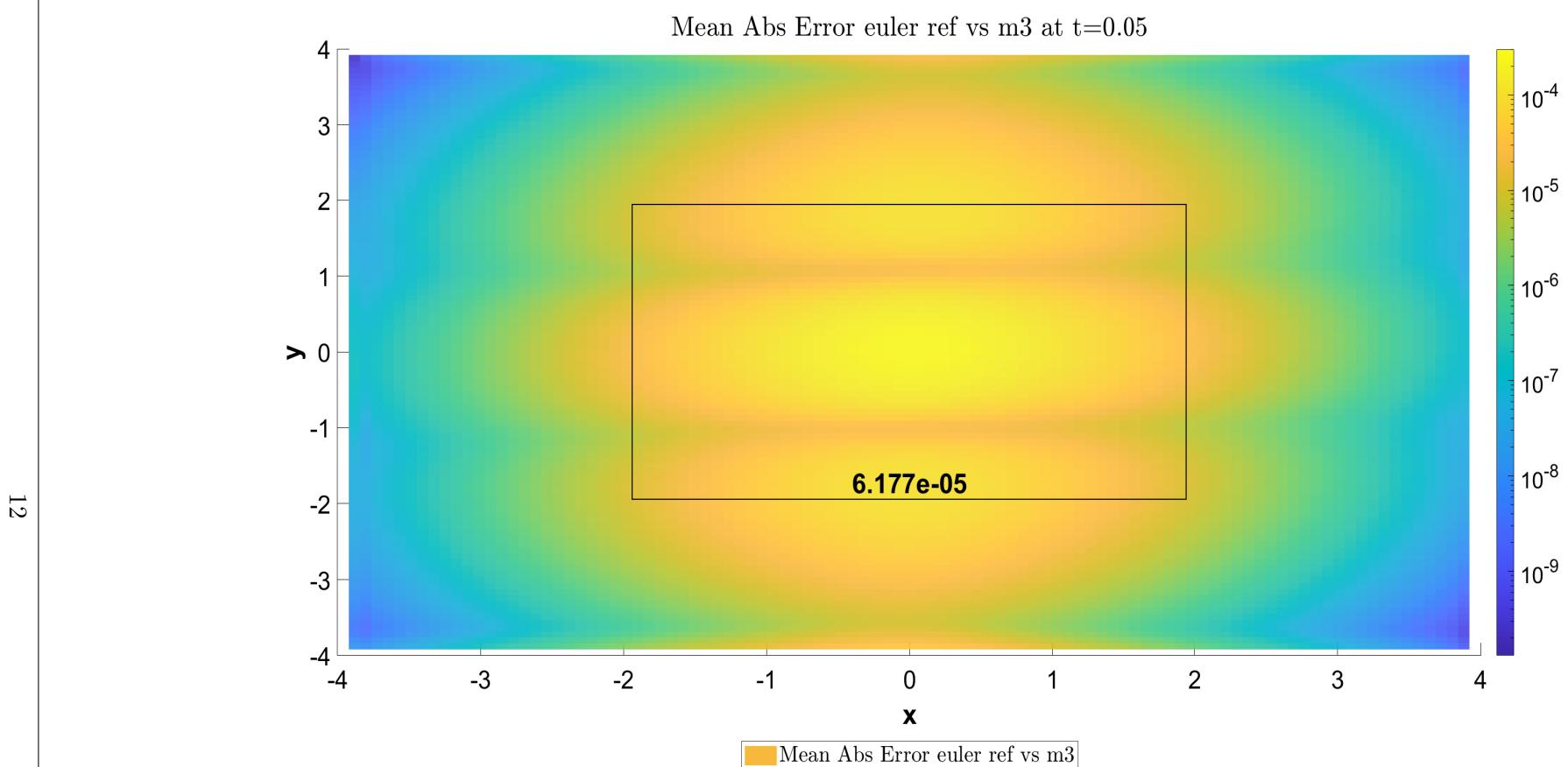


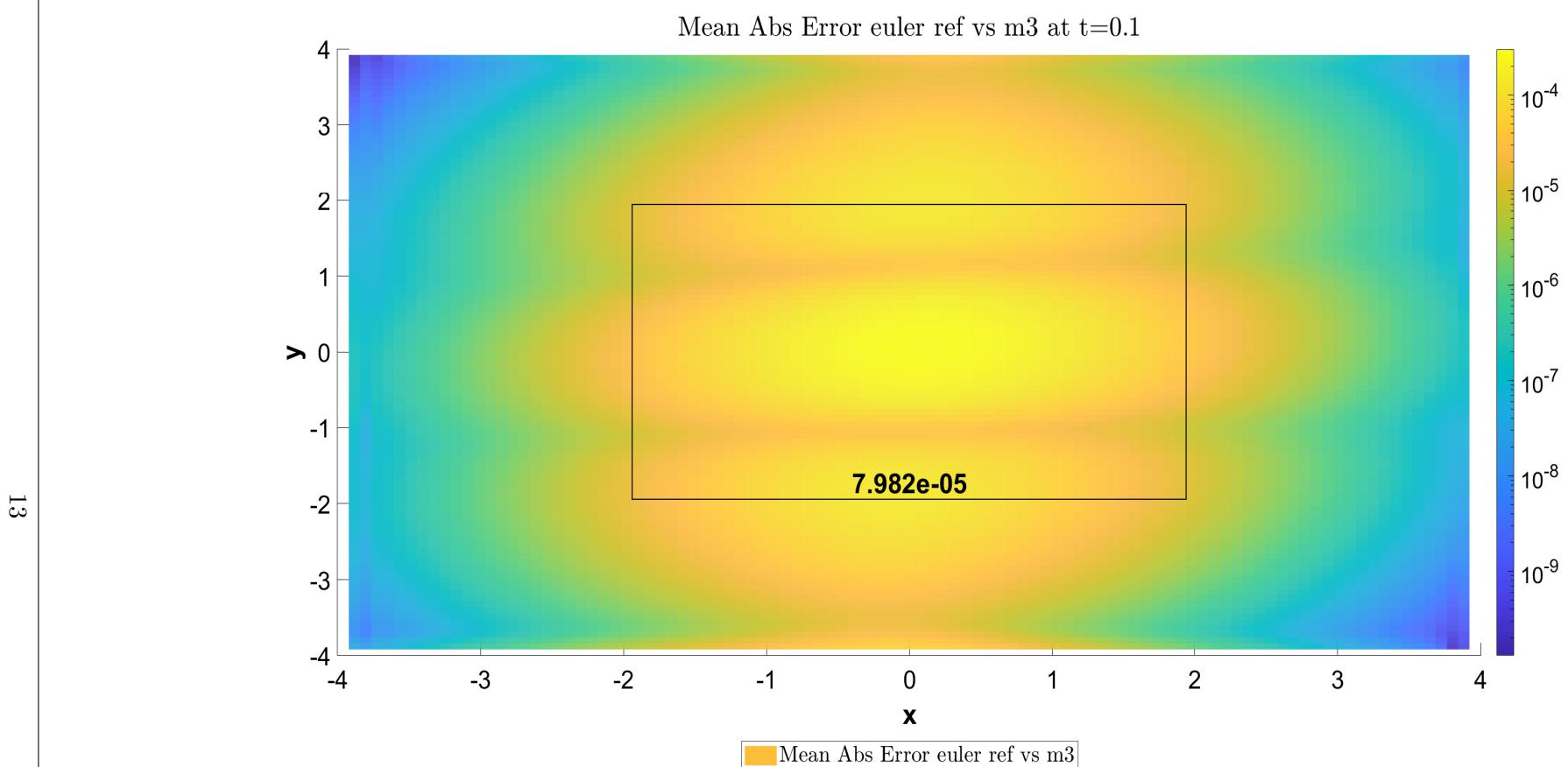


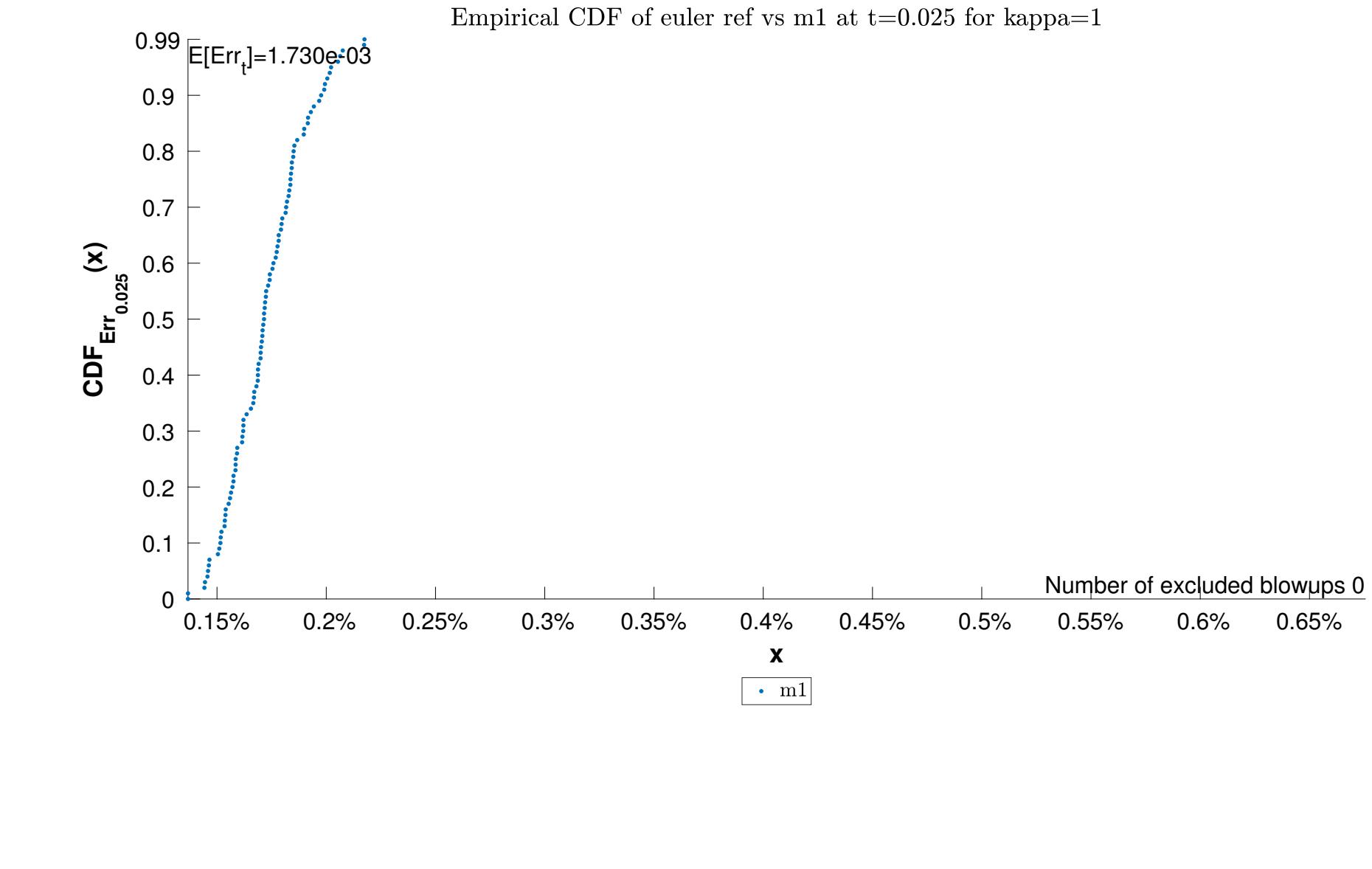


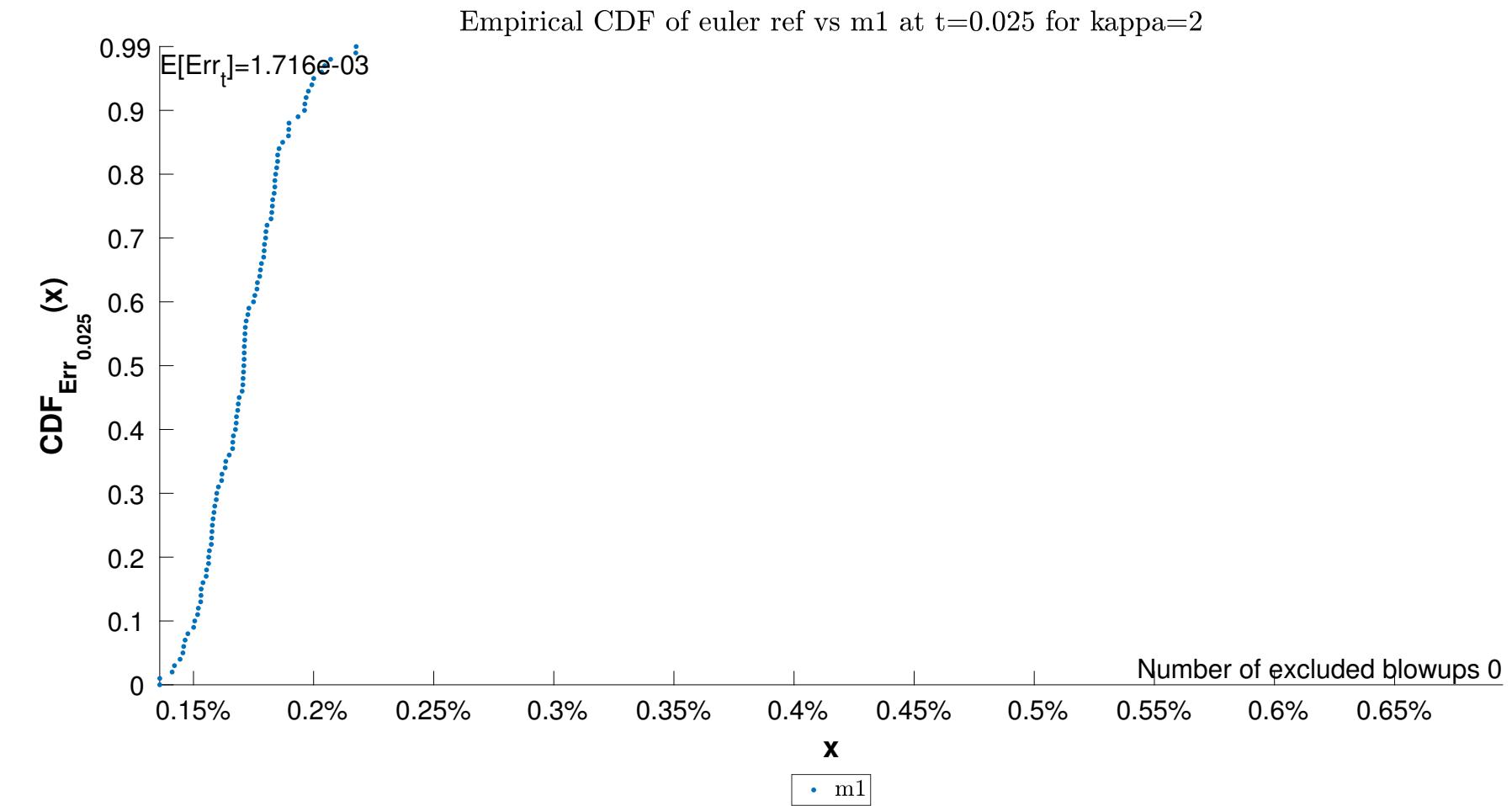




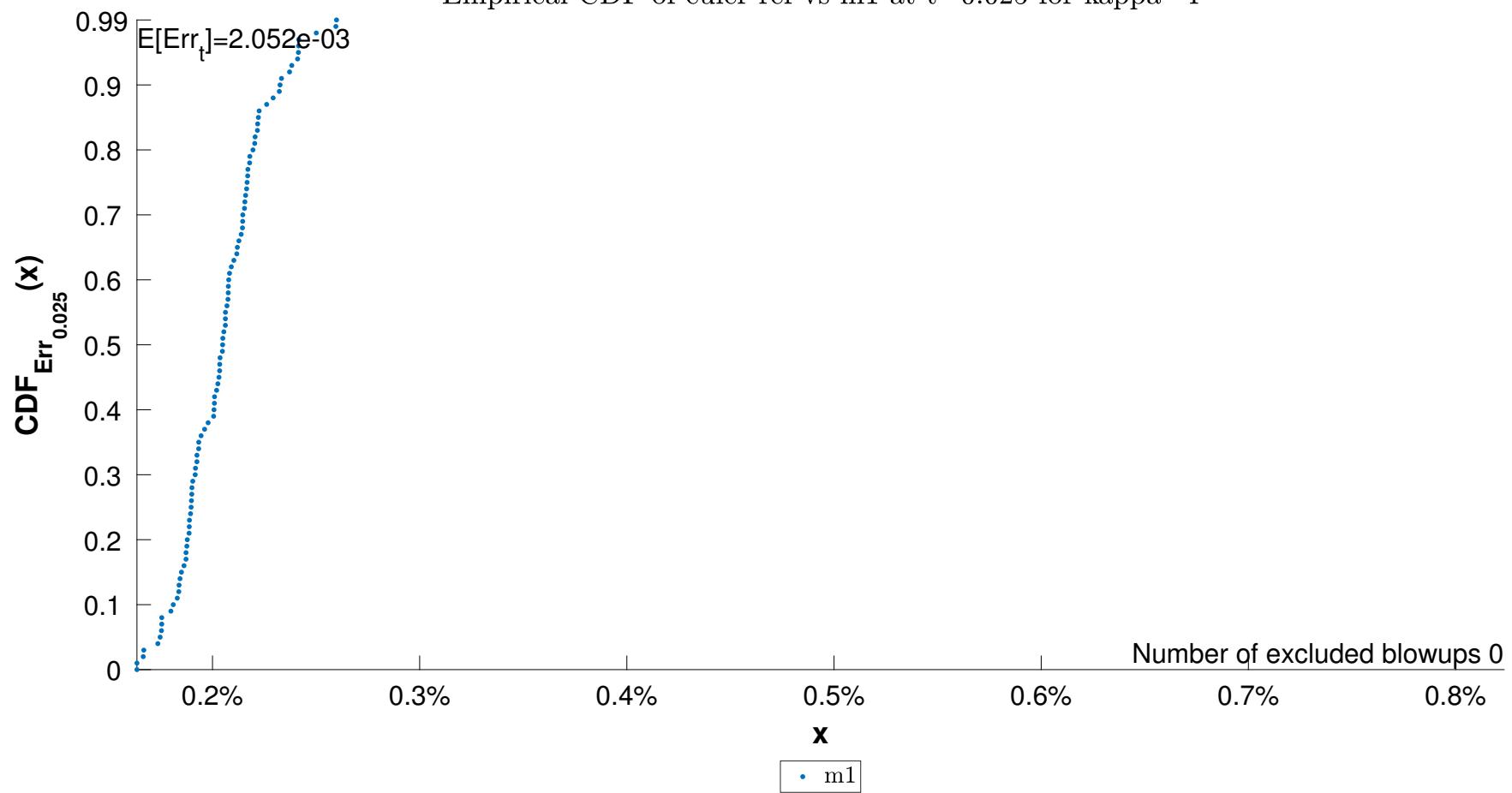


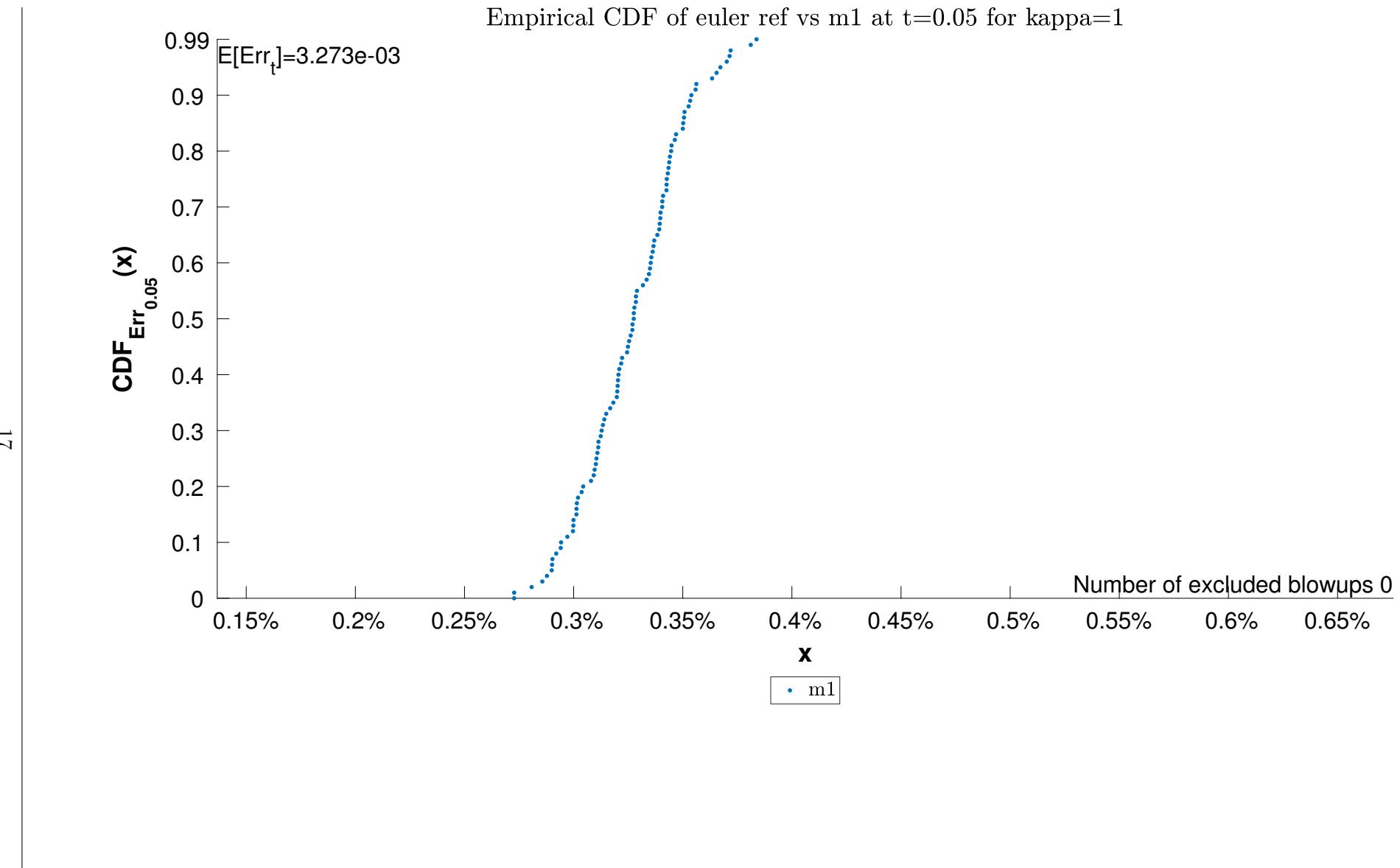


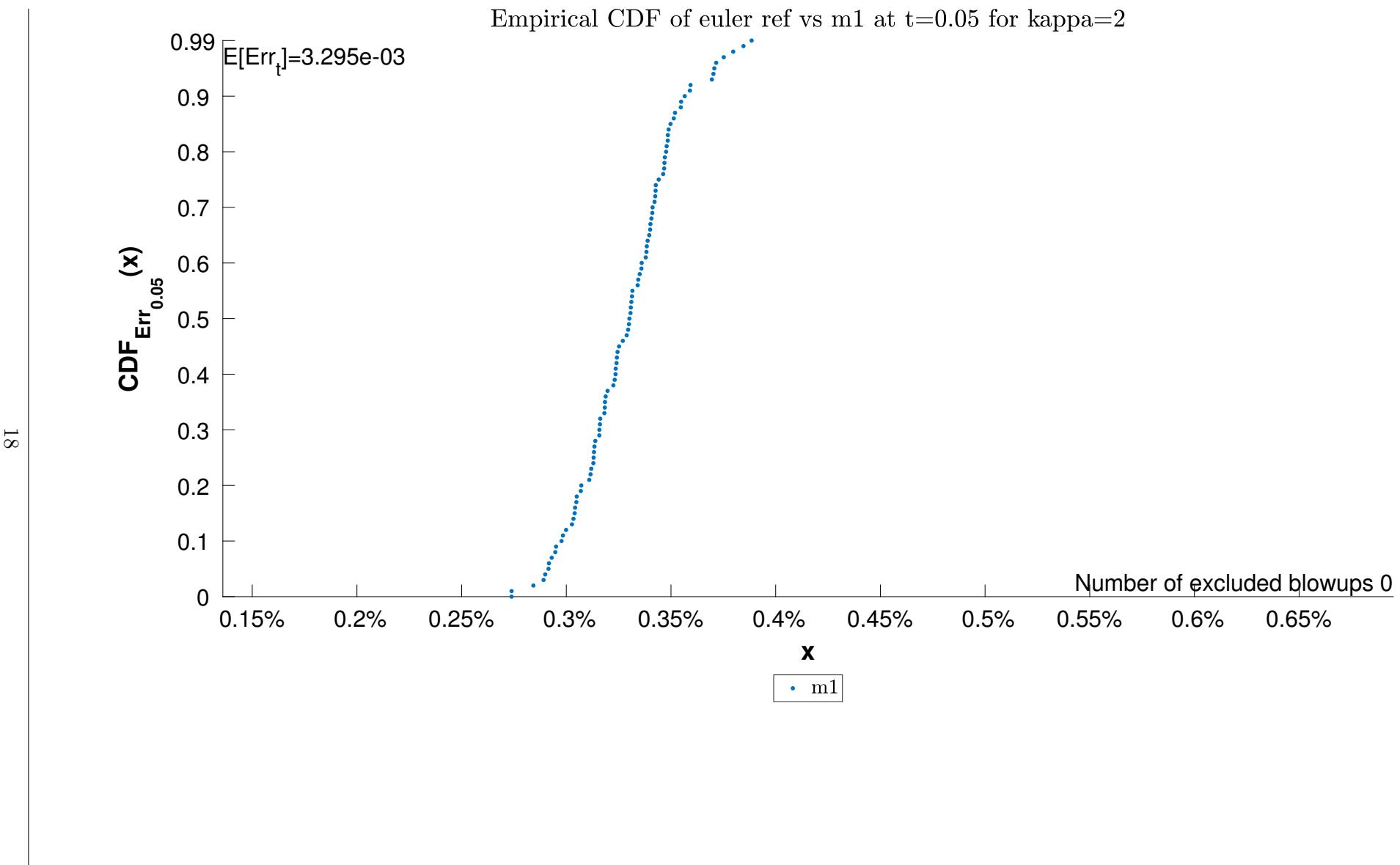


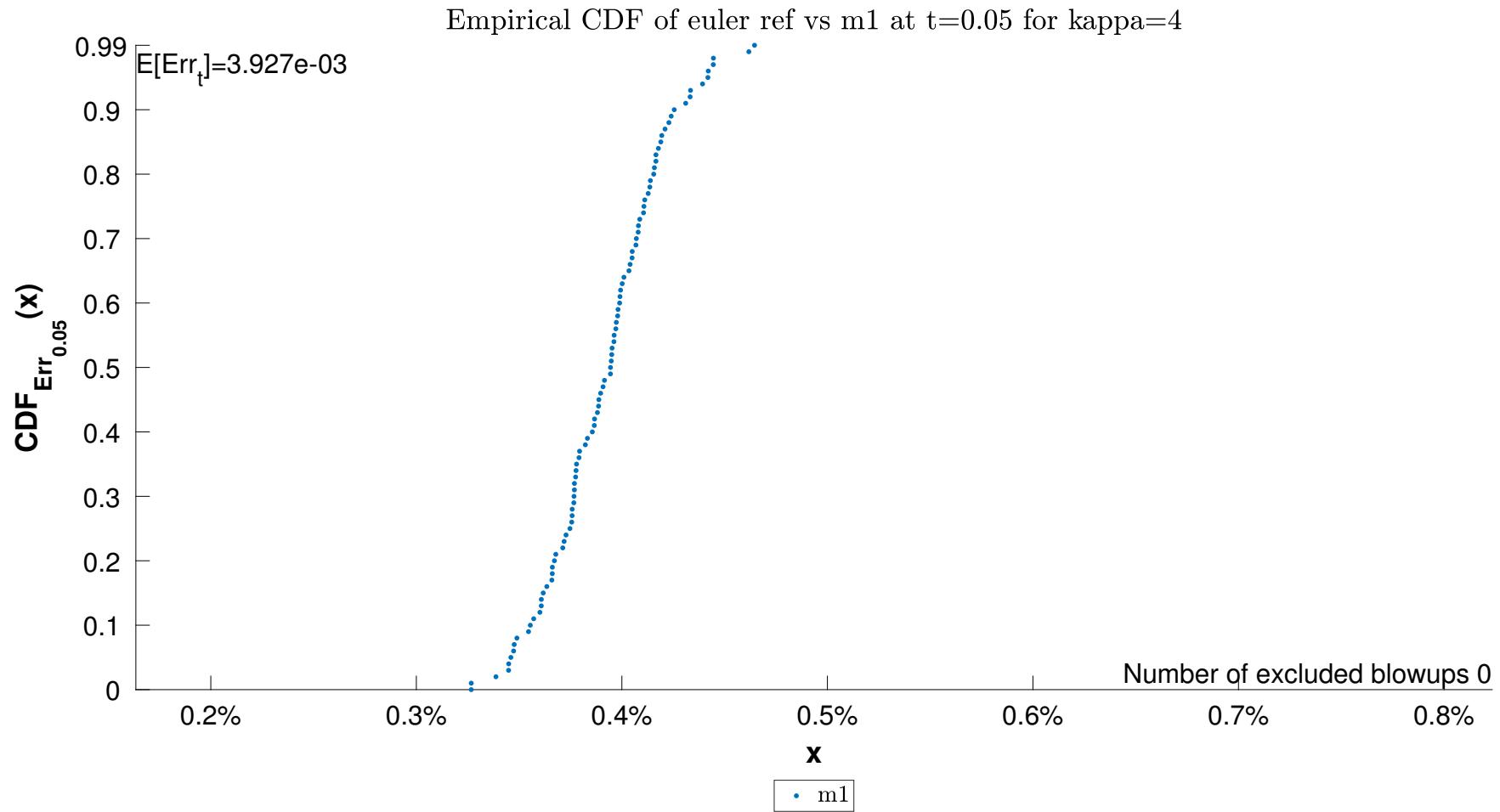


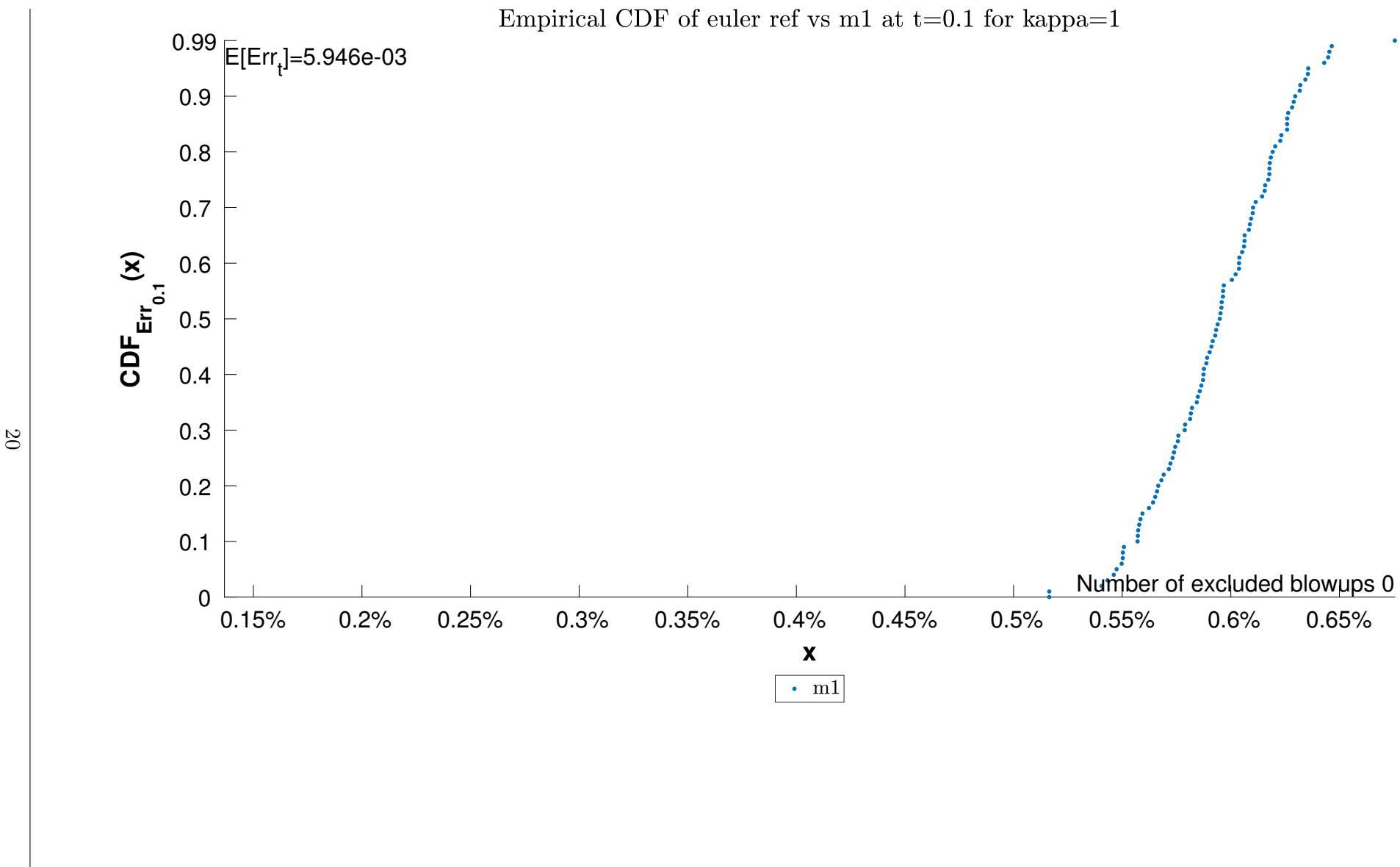
Empirical CDF of euler ref vs m1 at t=0.025 for kappa=4

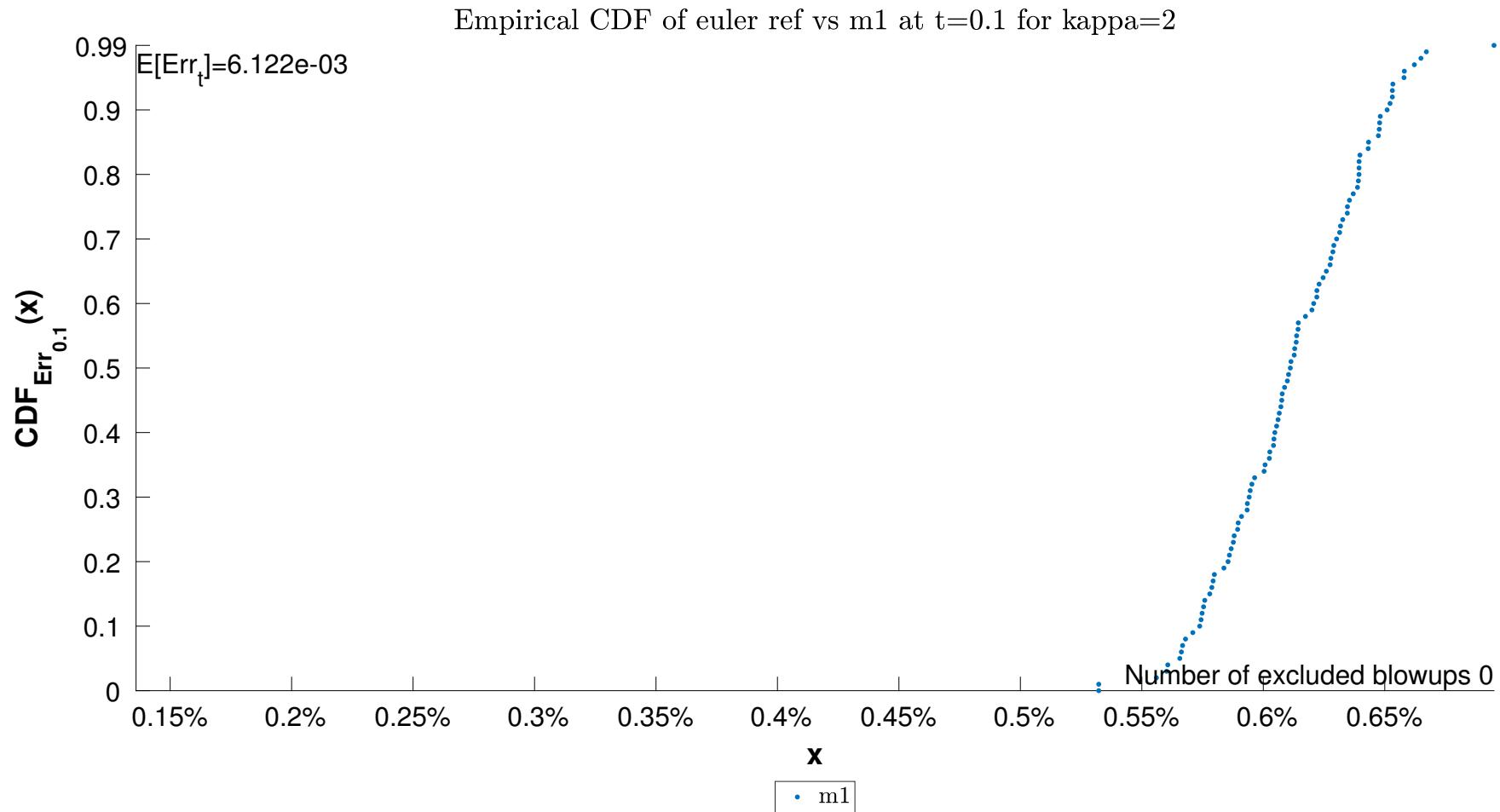


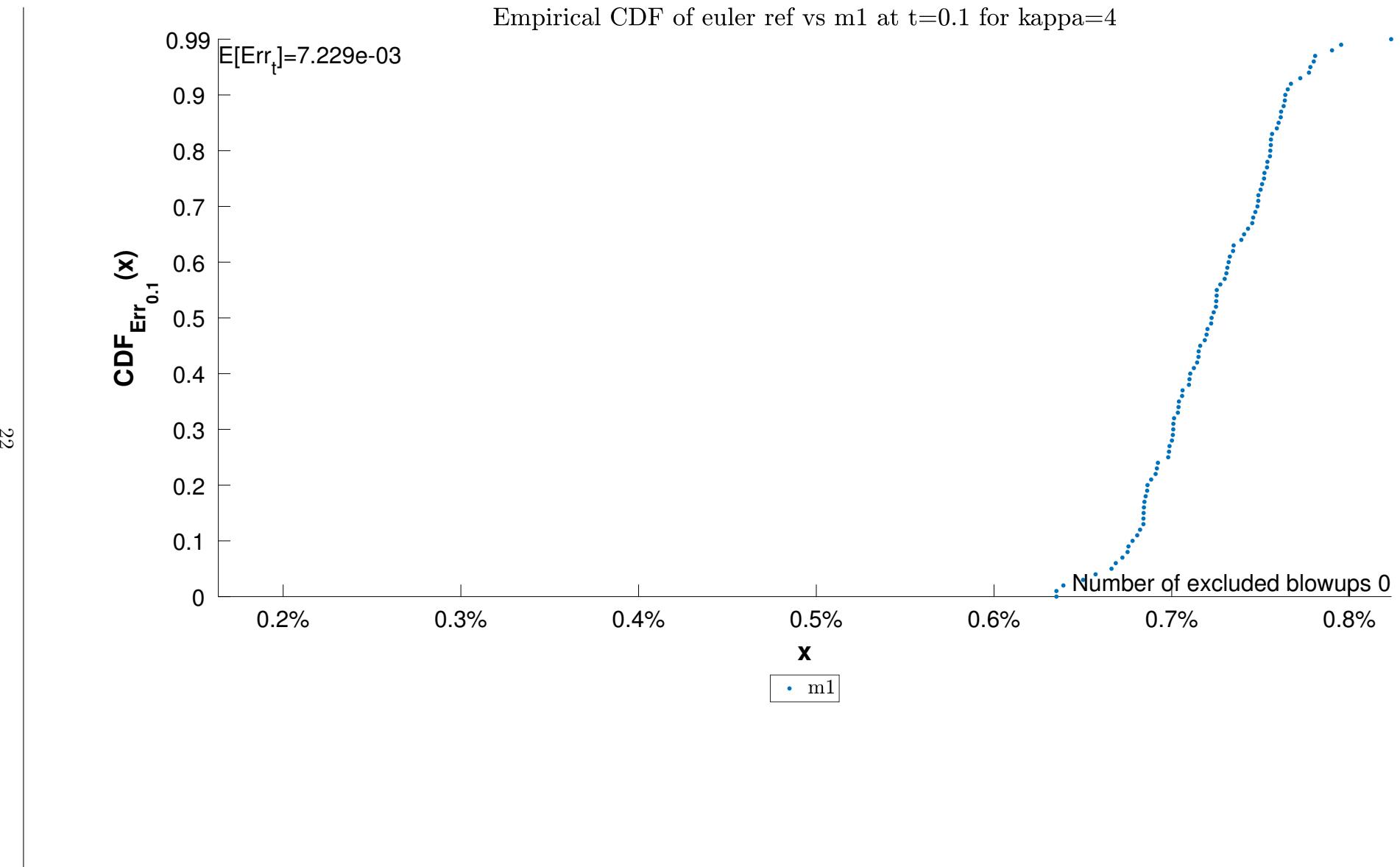


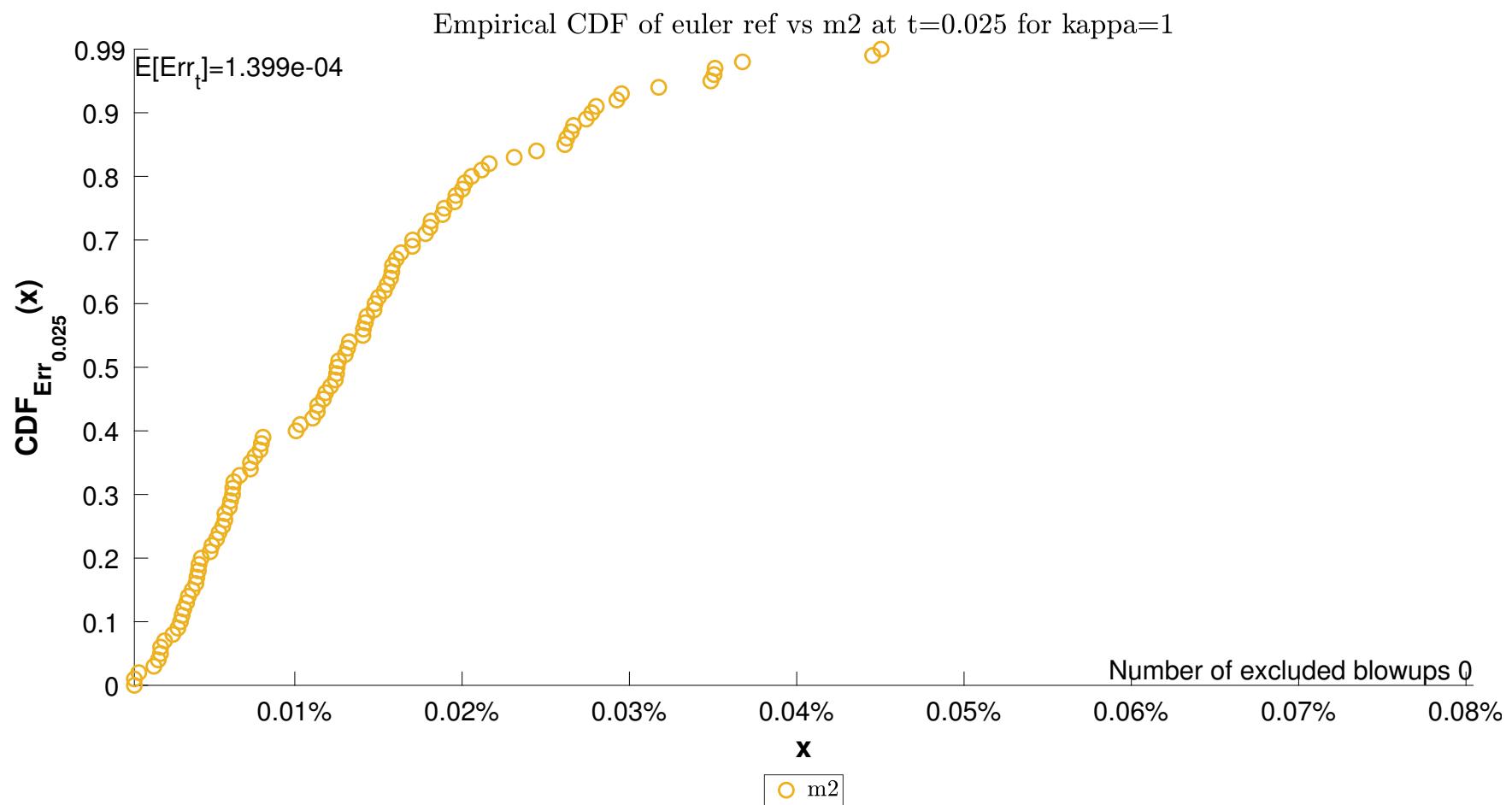


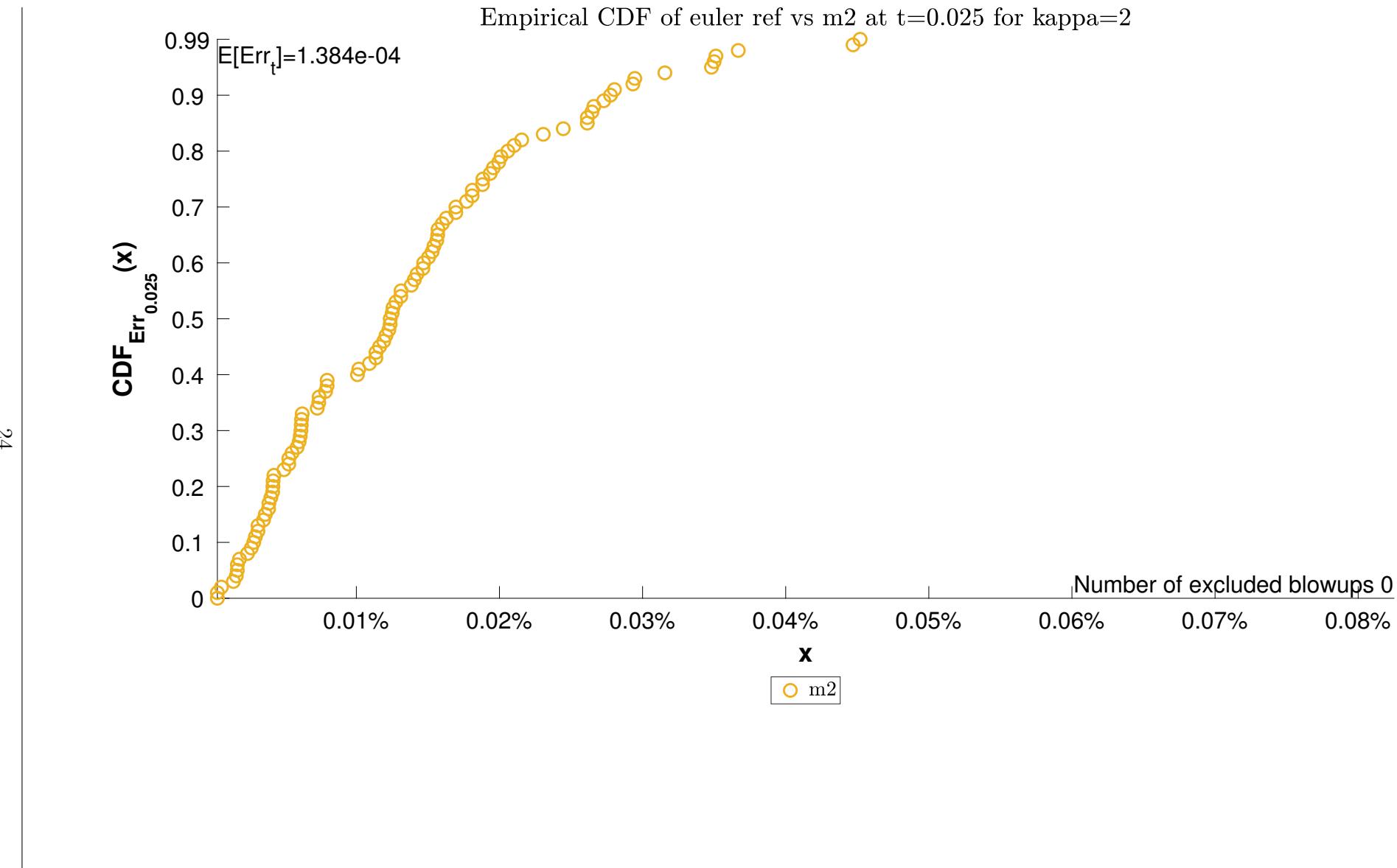


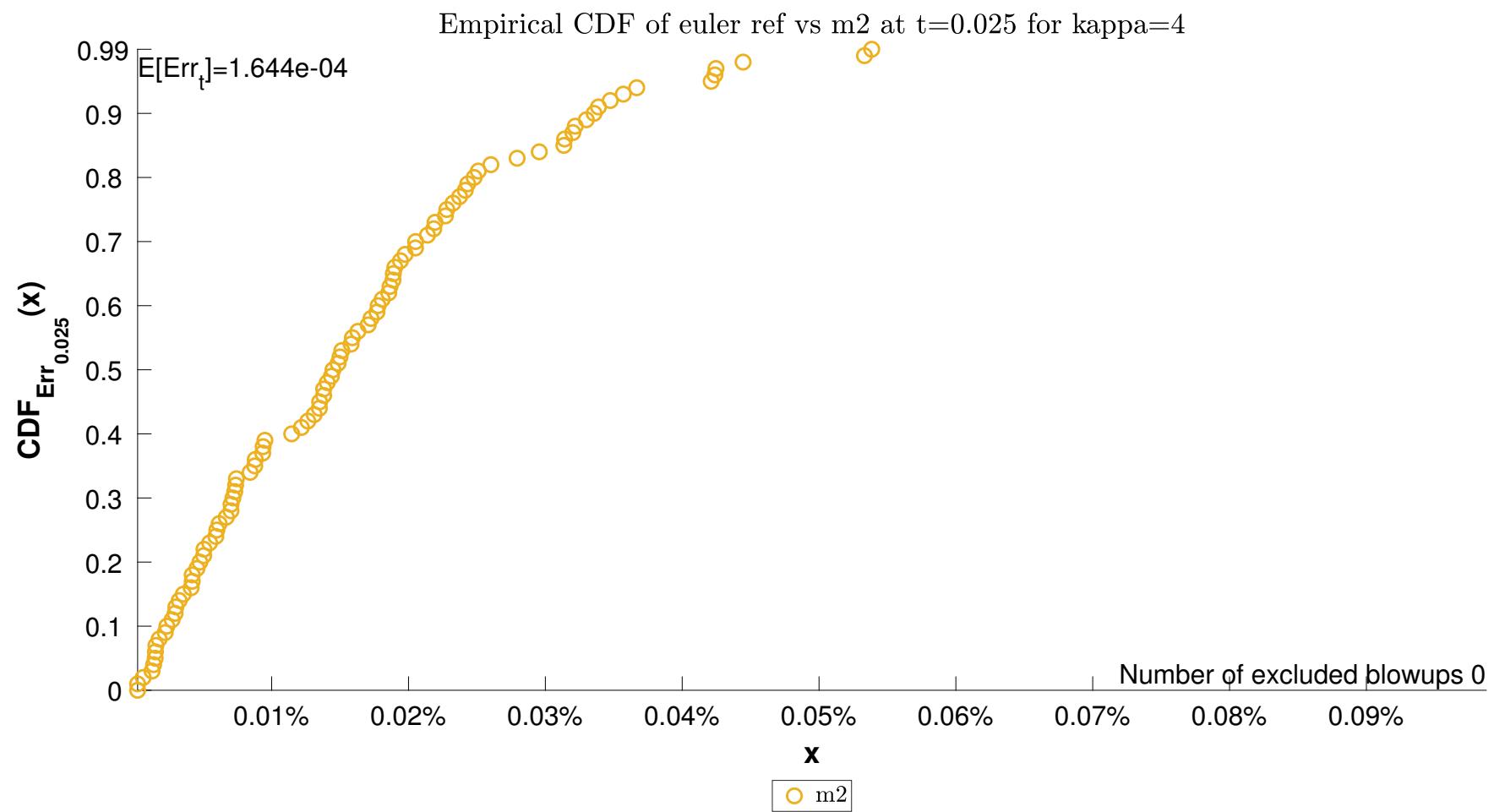


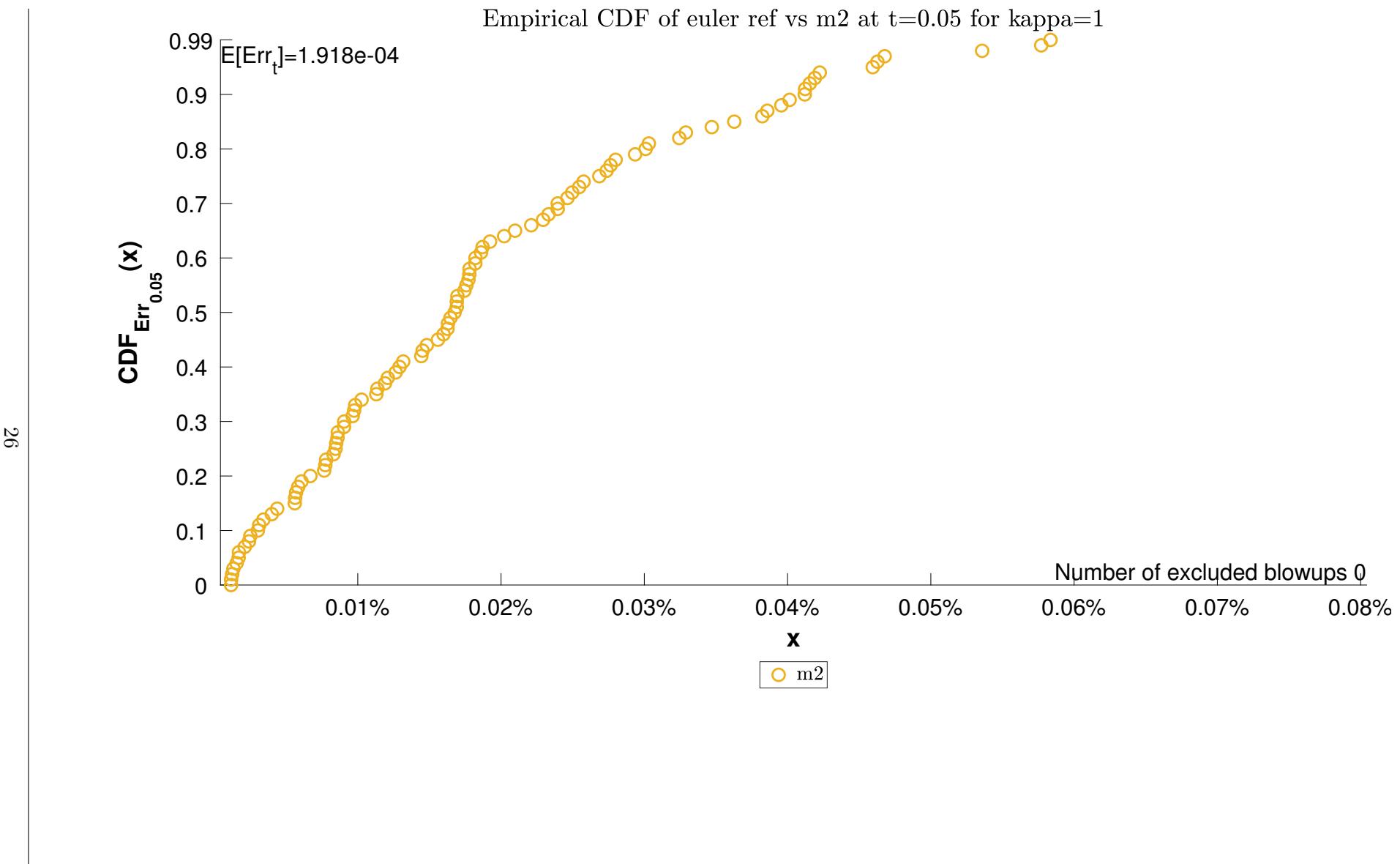


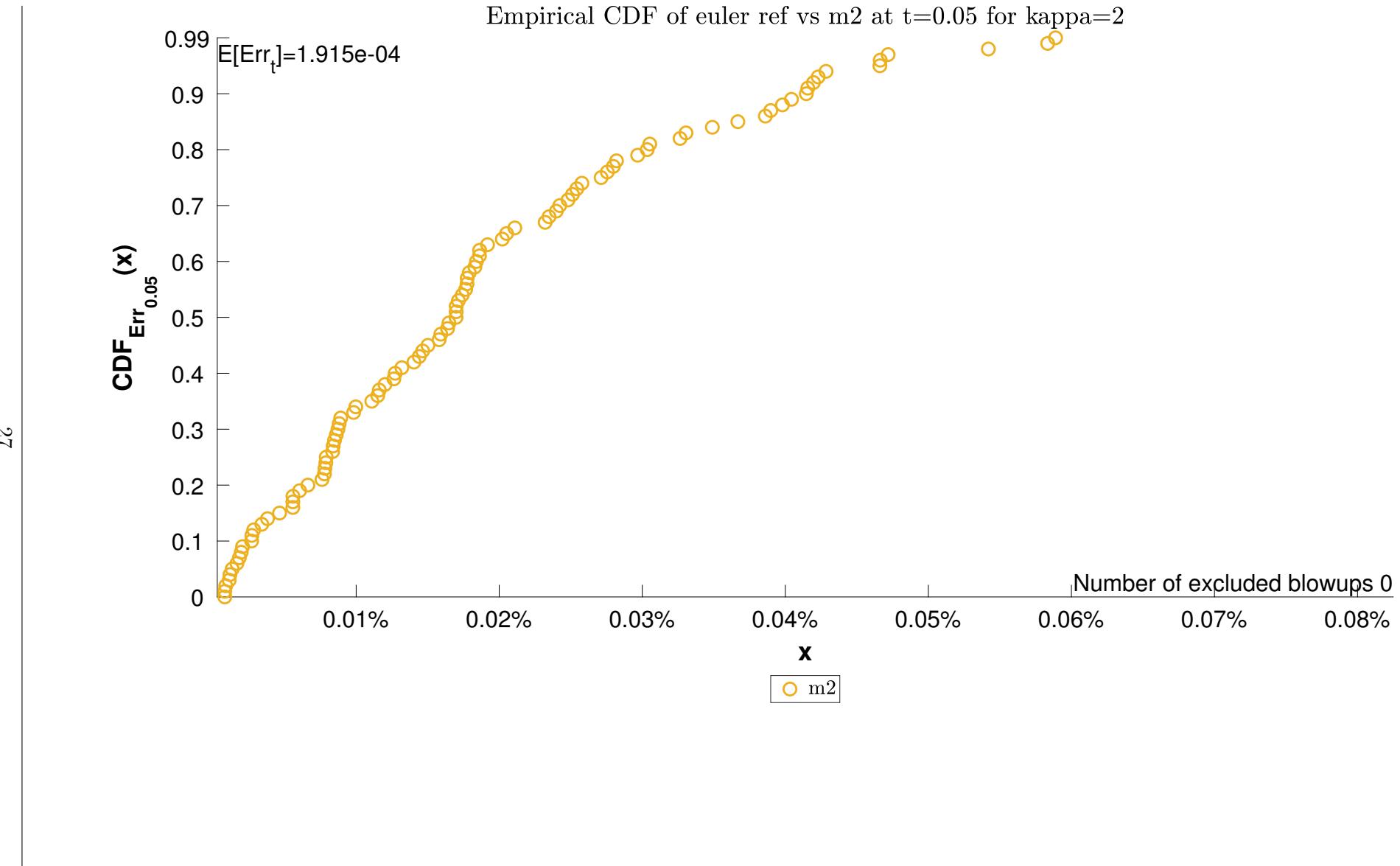


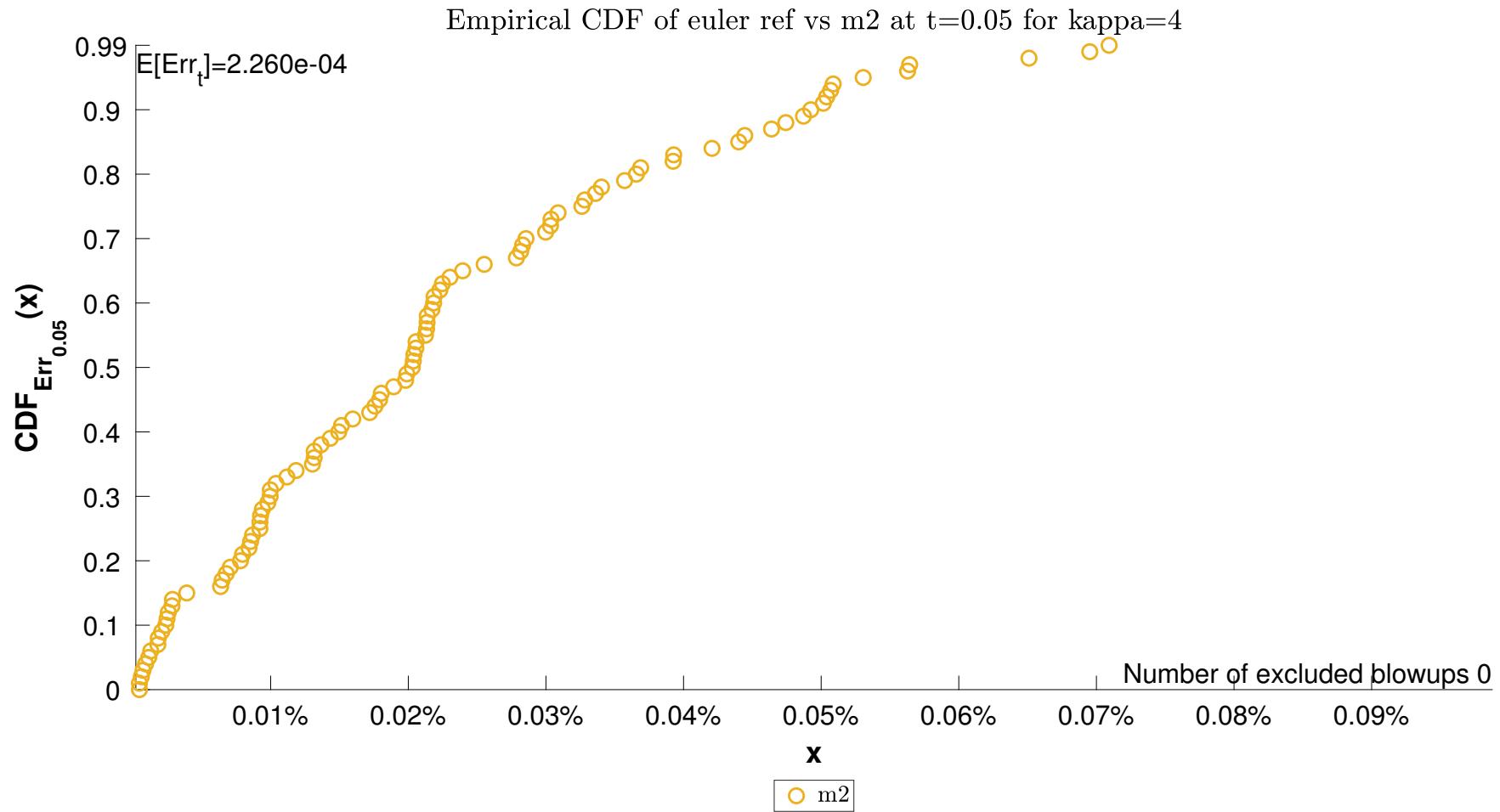


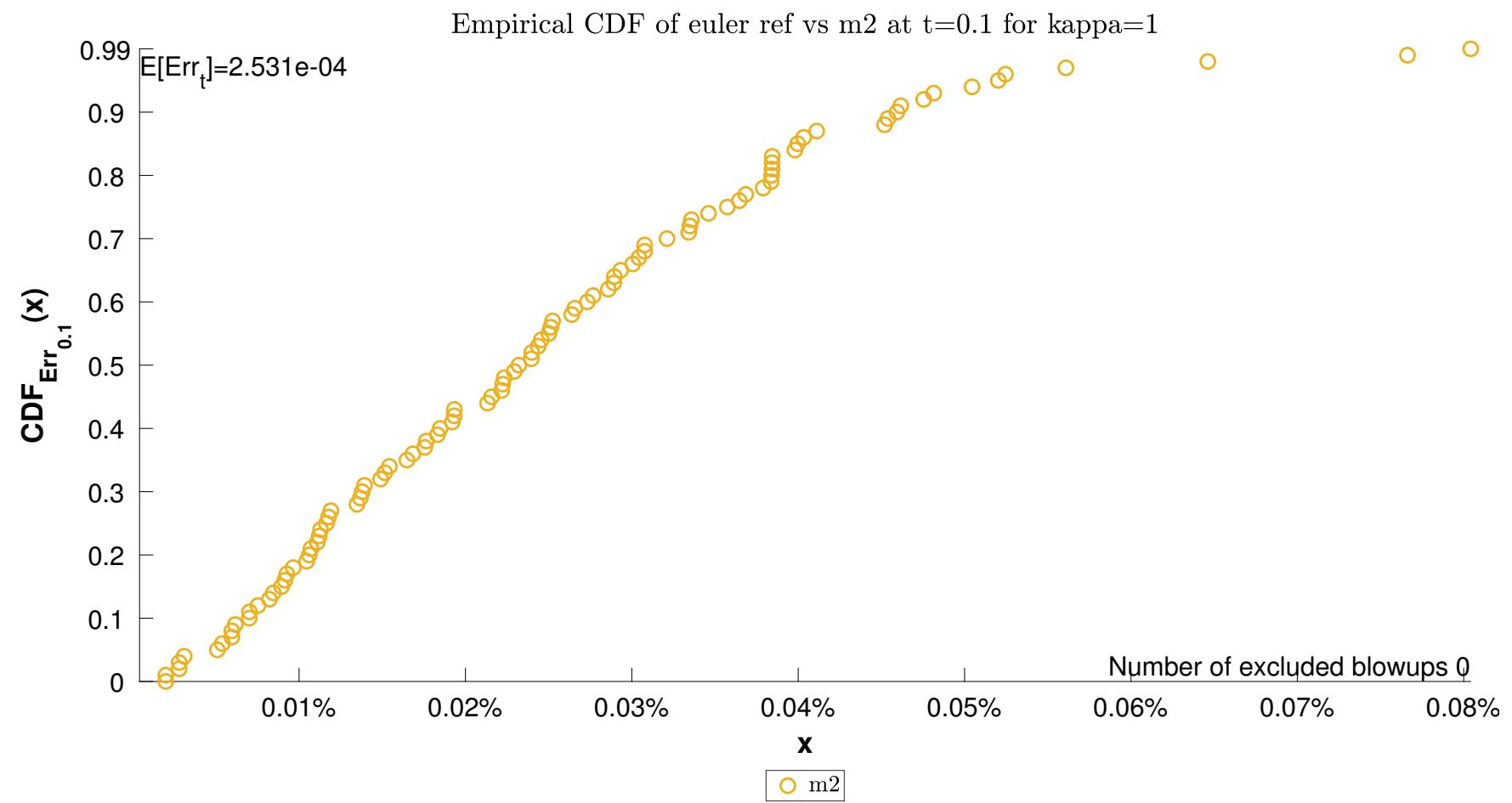


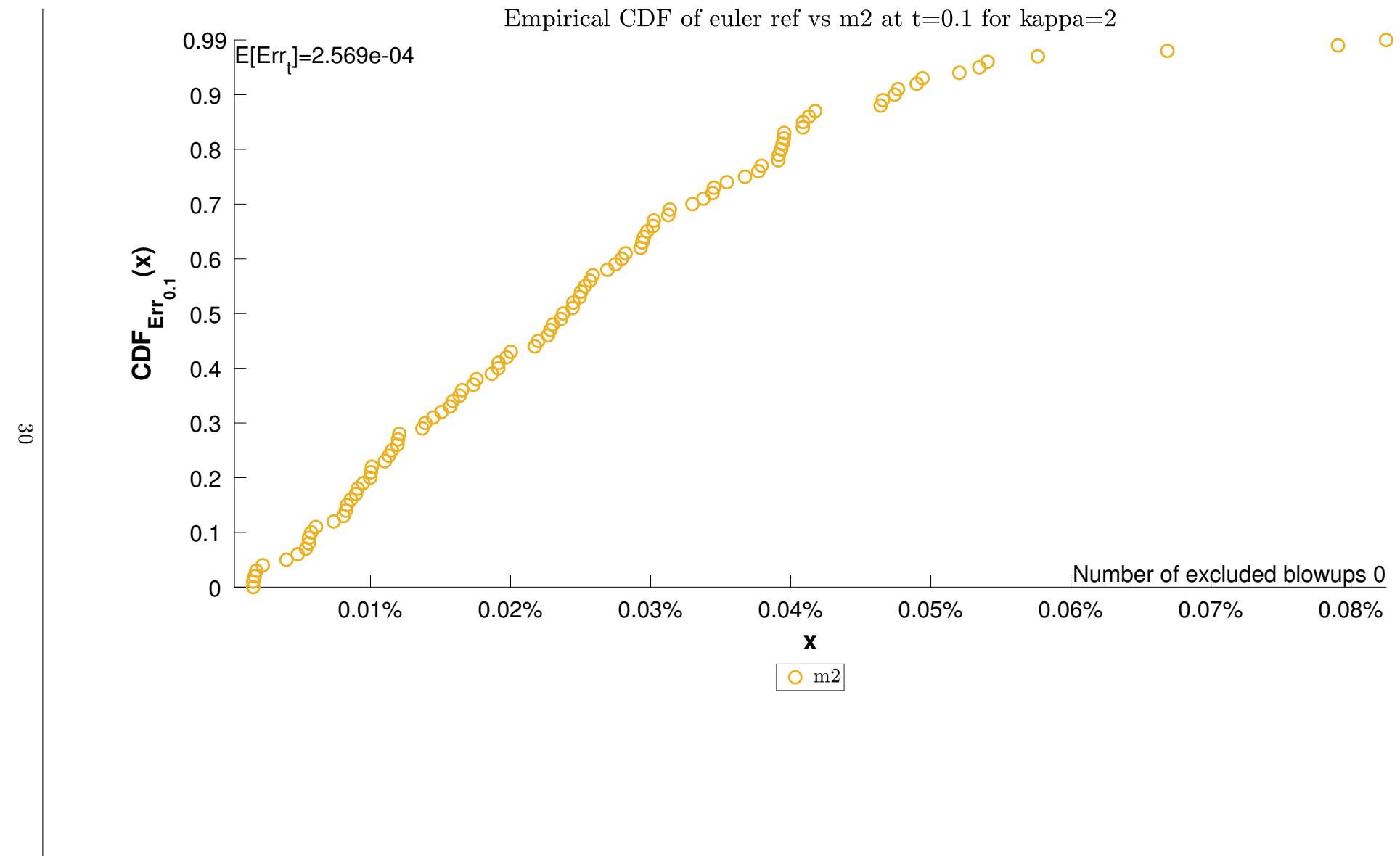


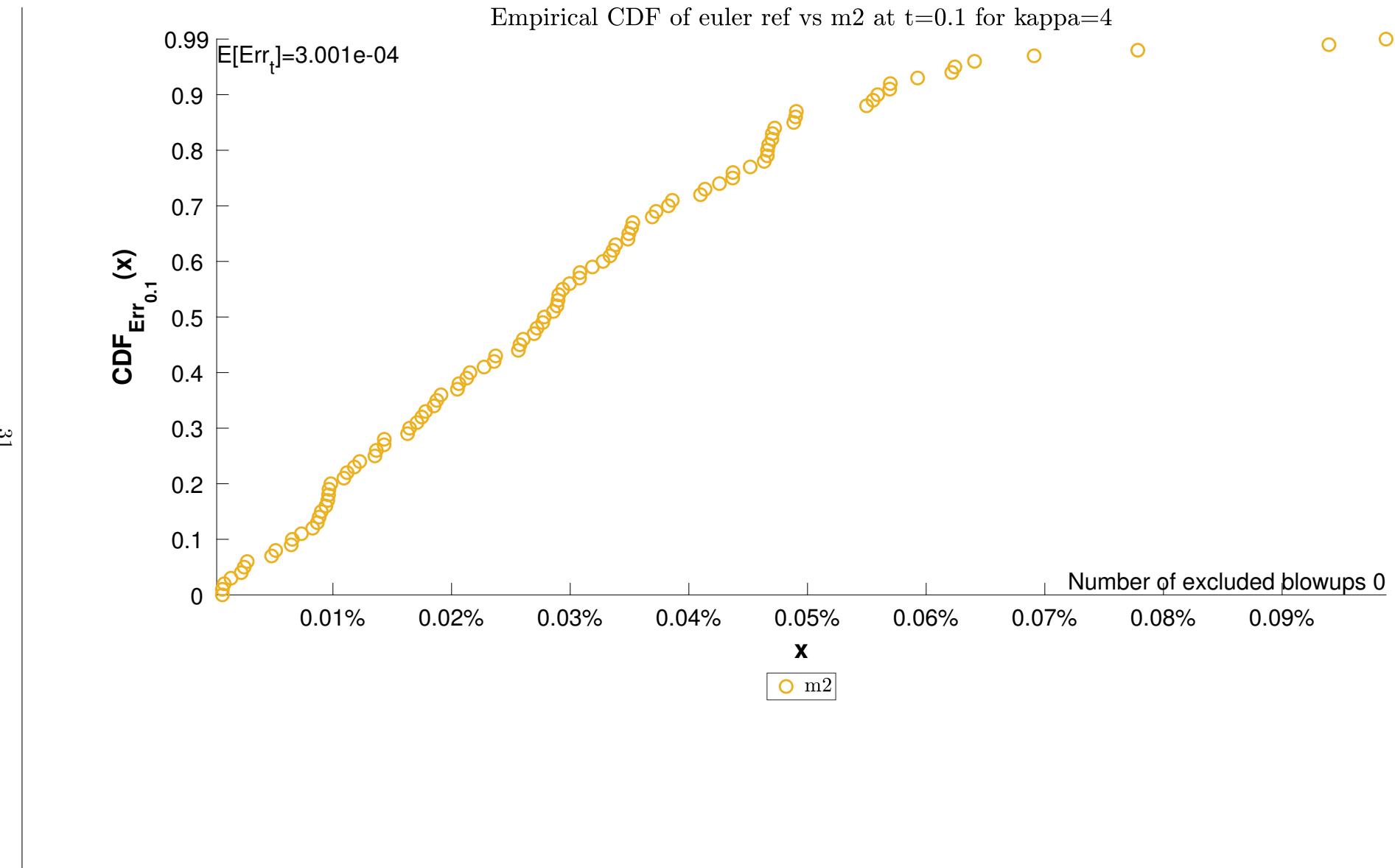


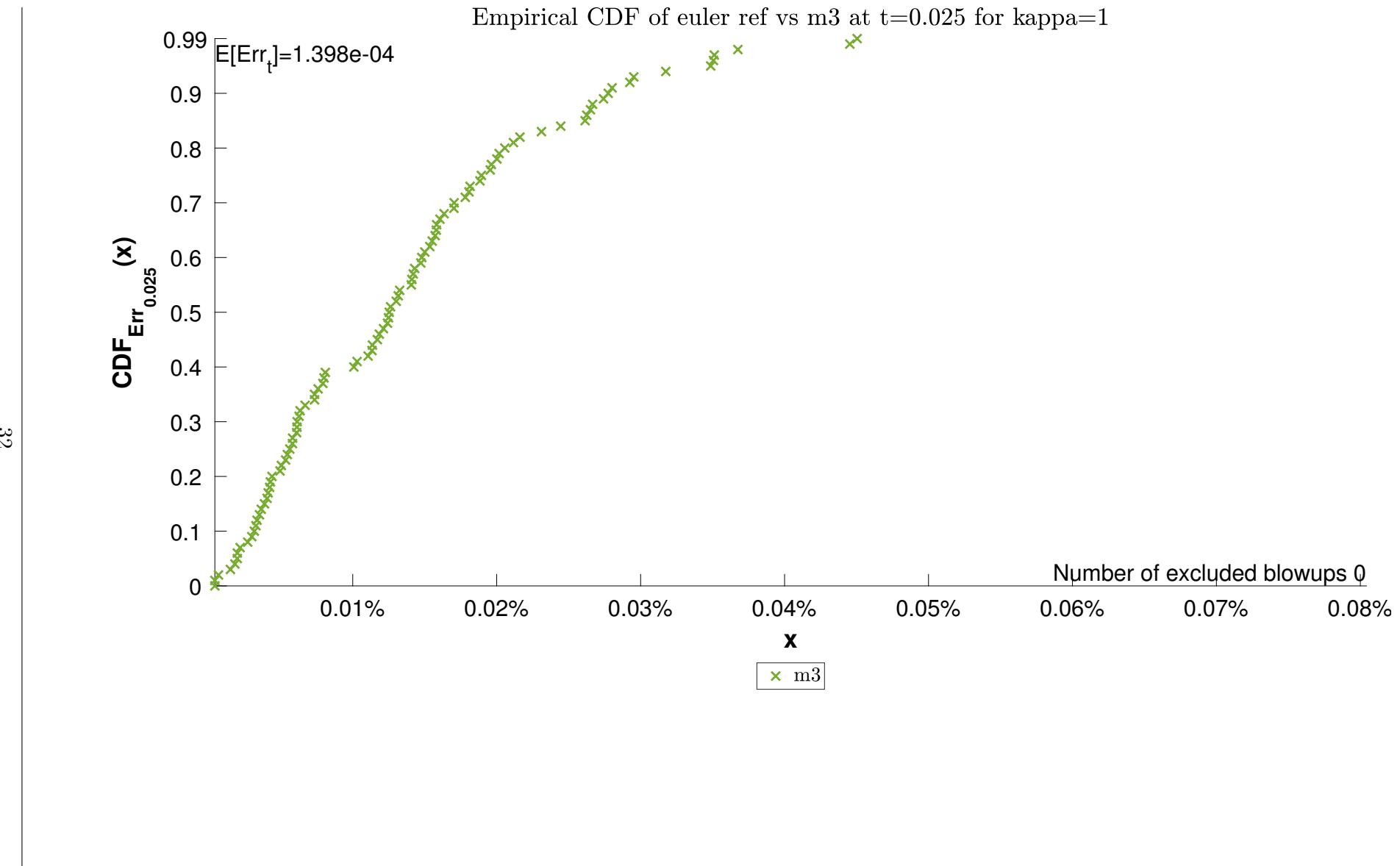


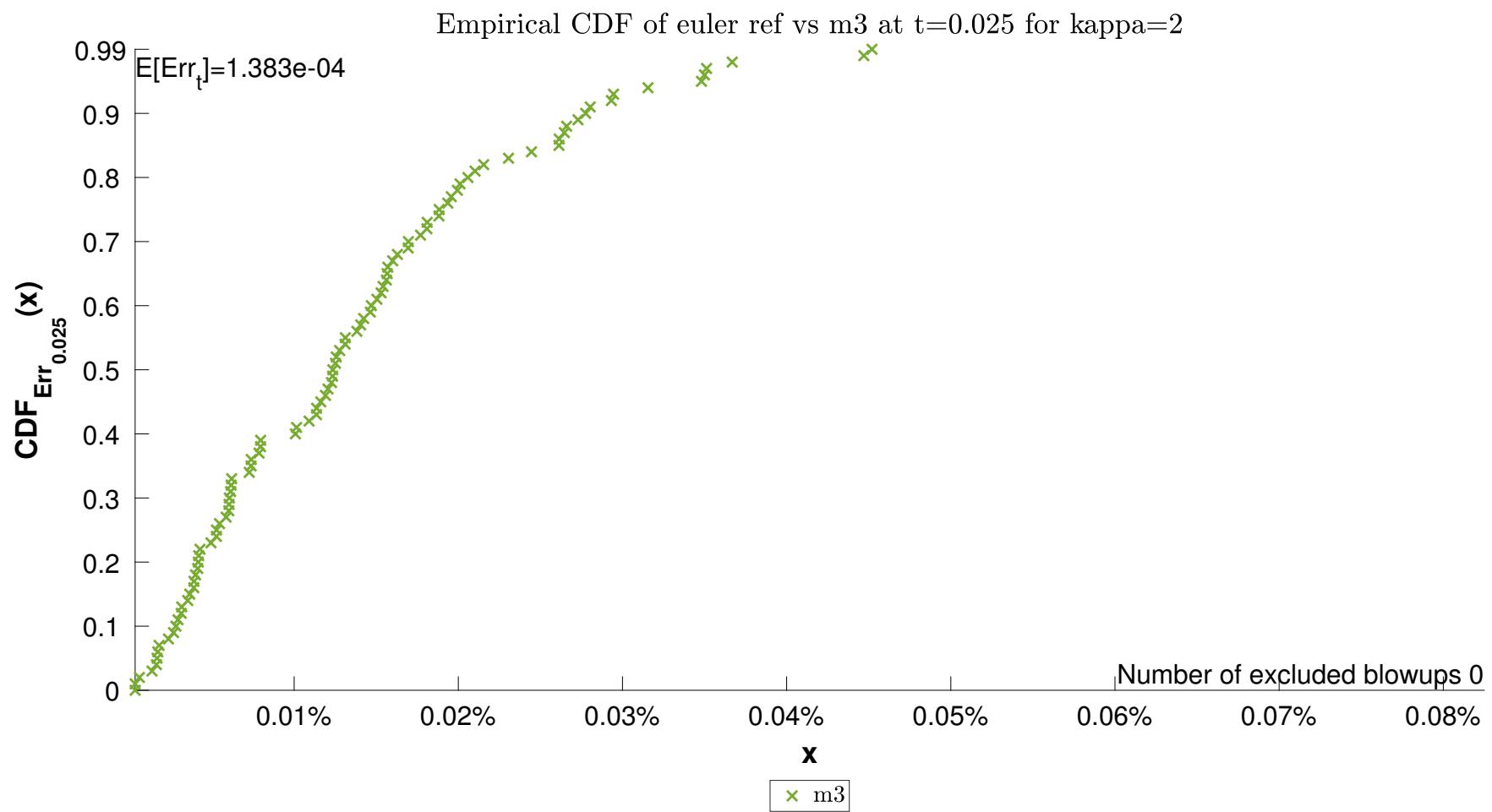


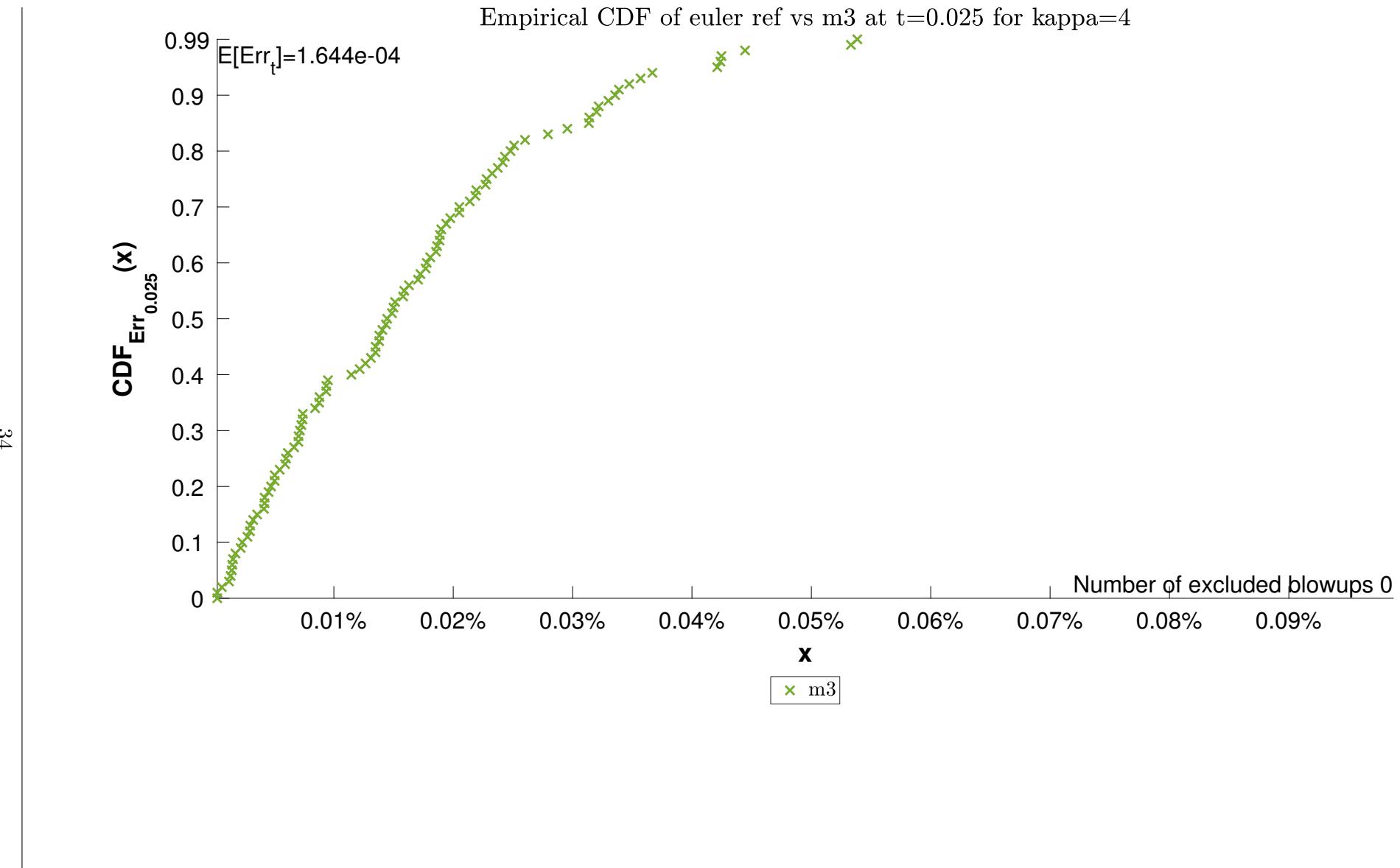


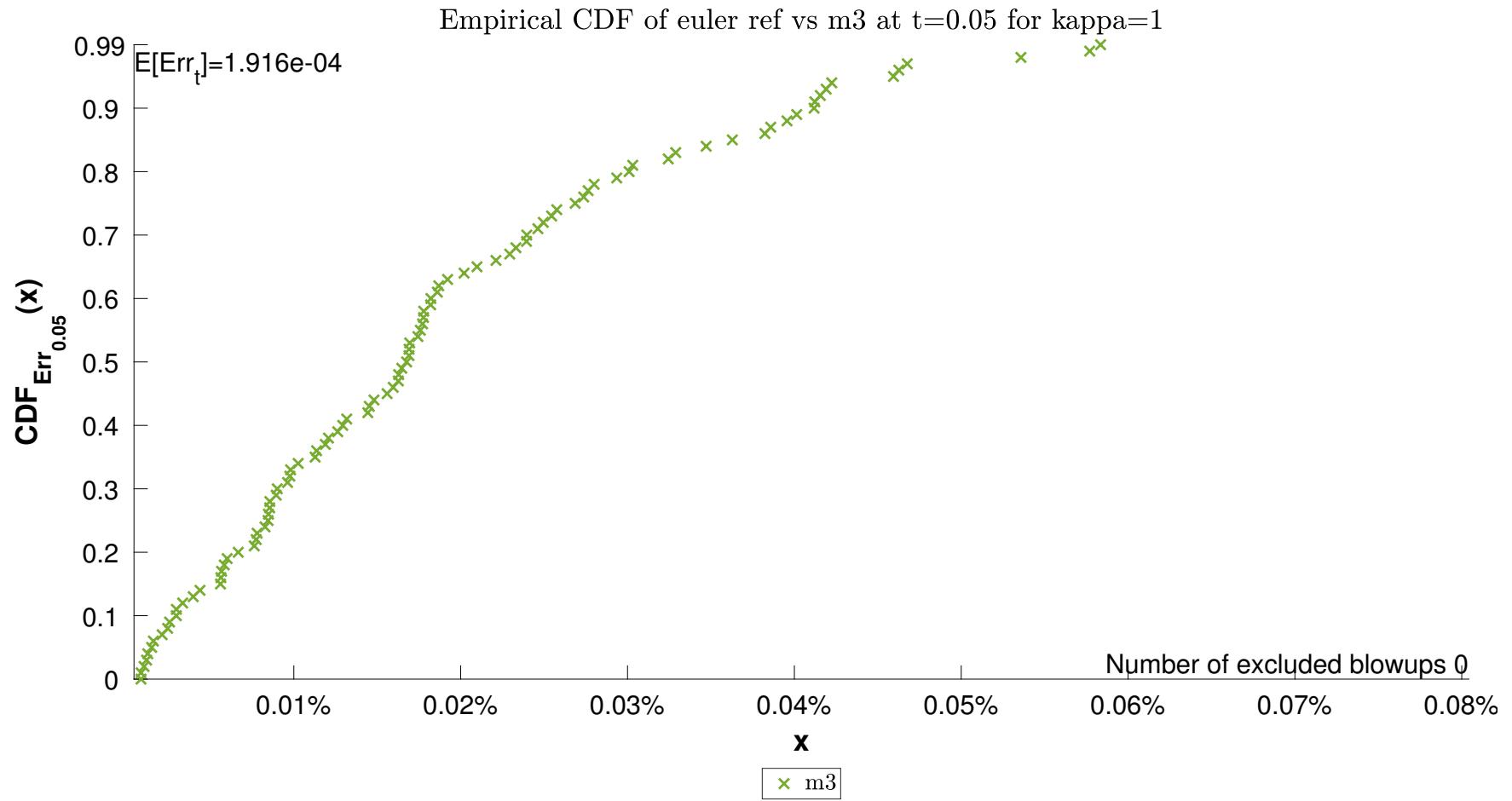


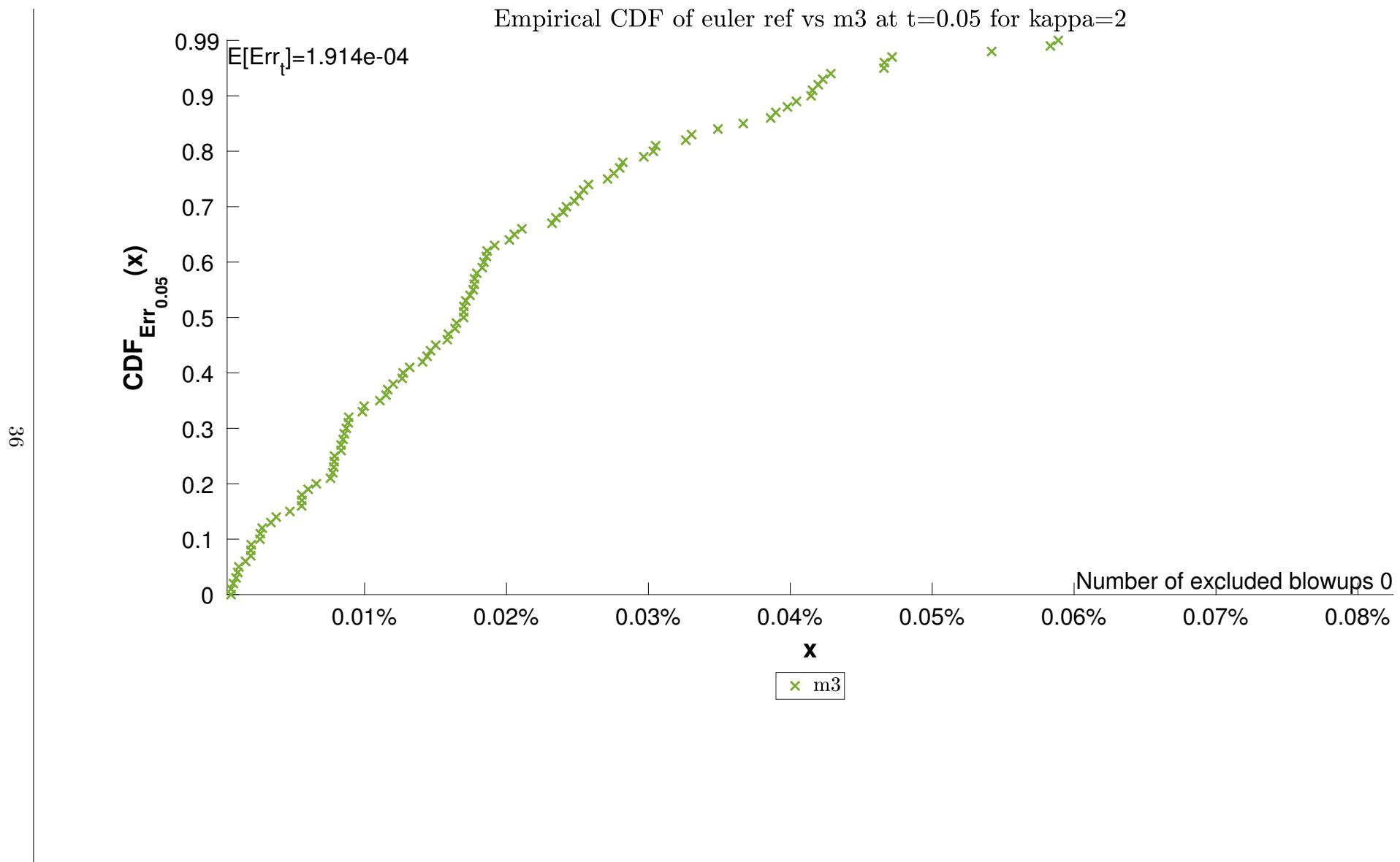


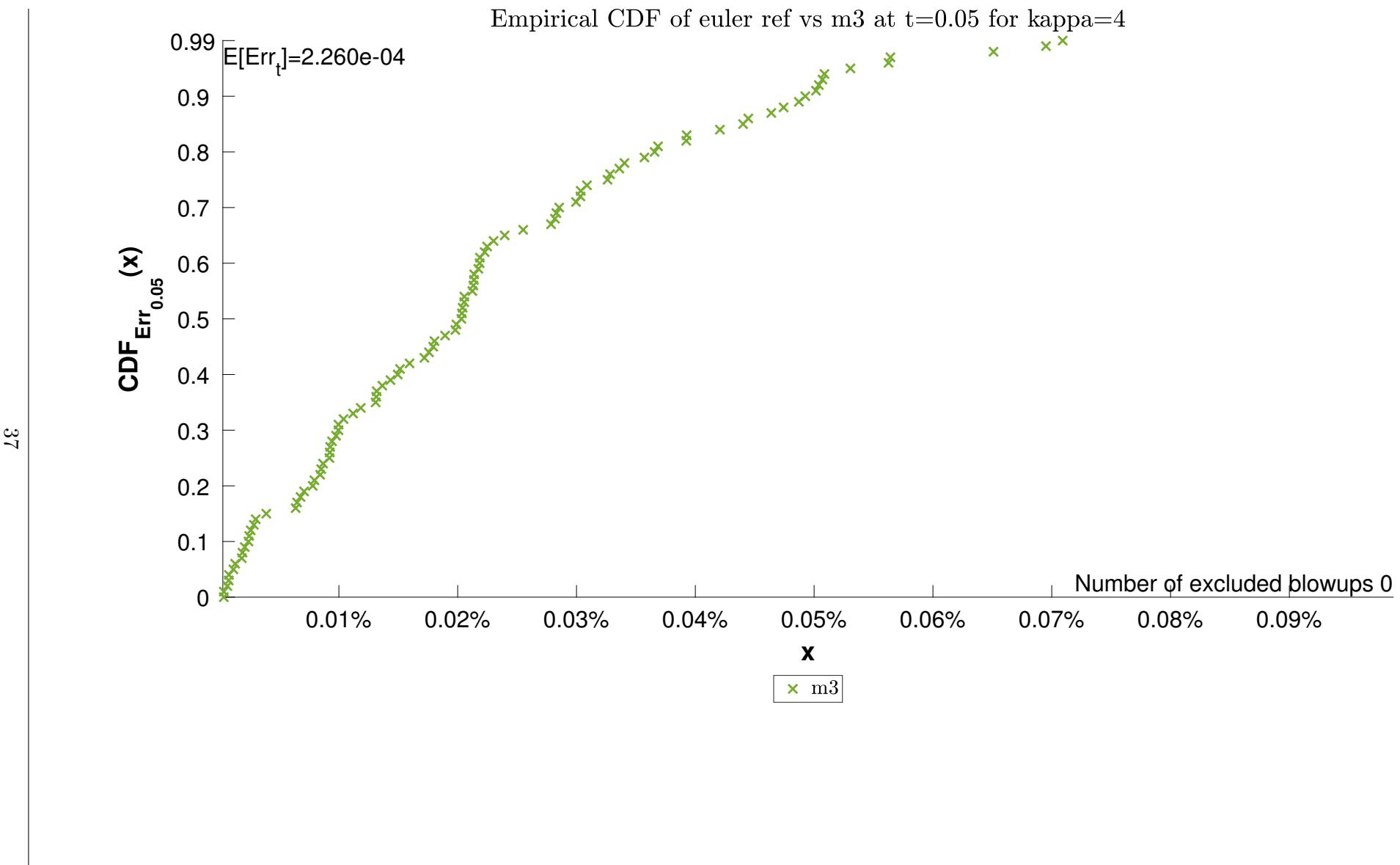


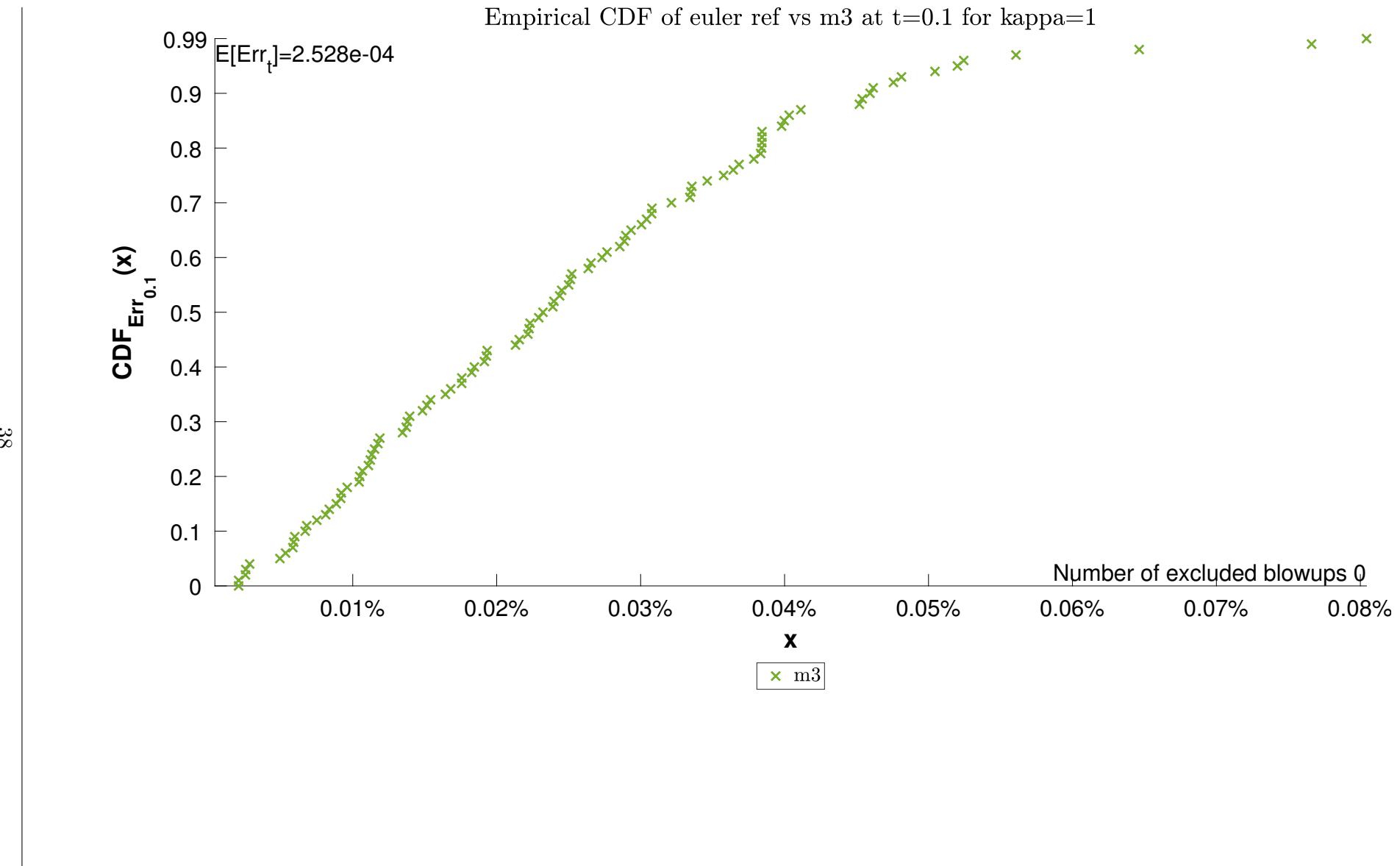


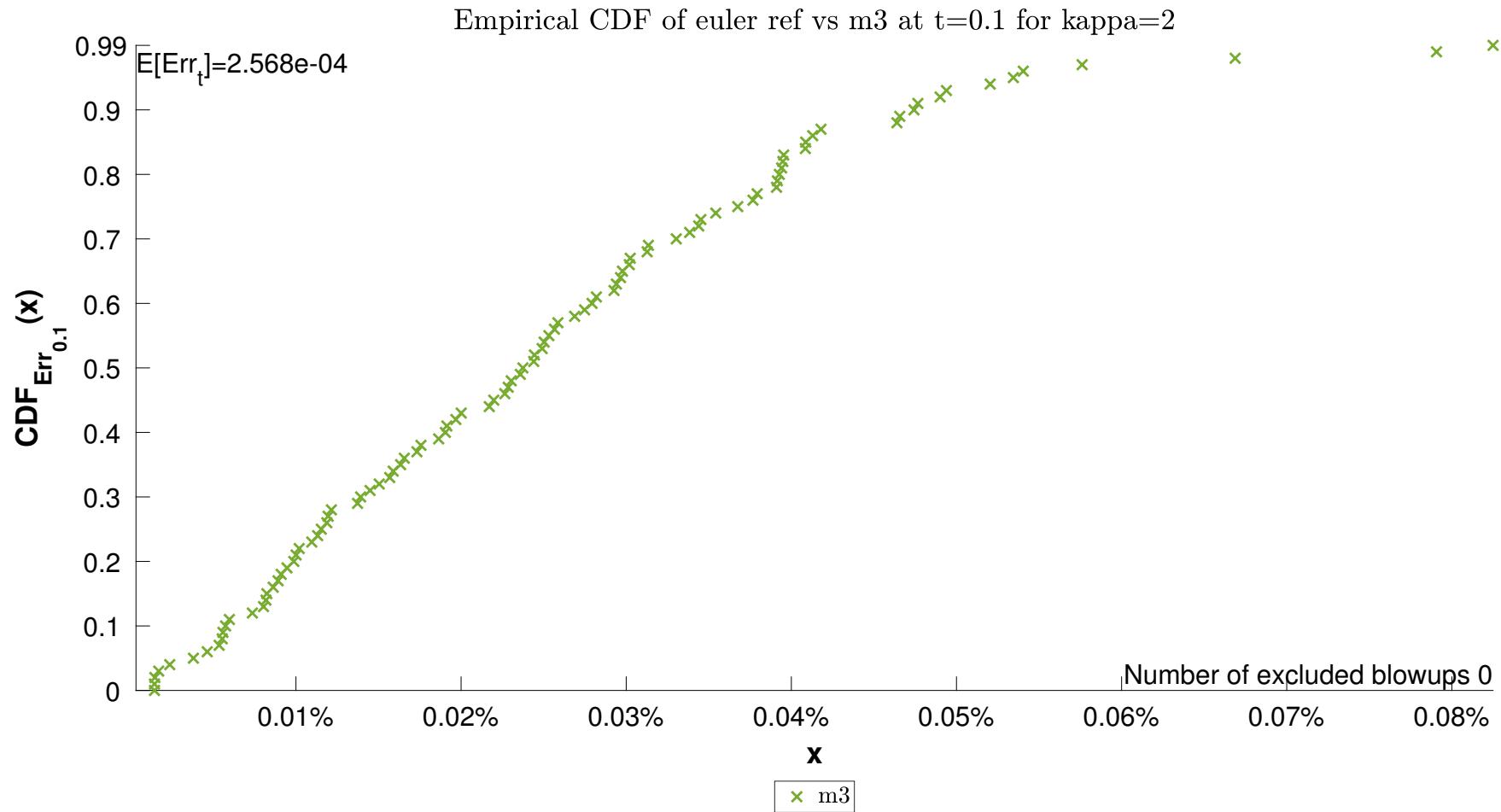


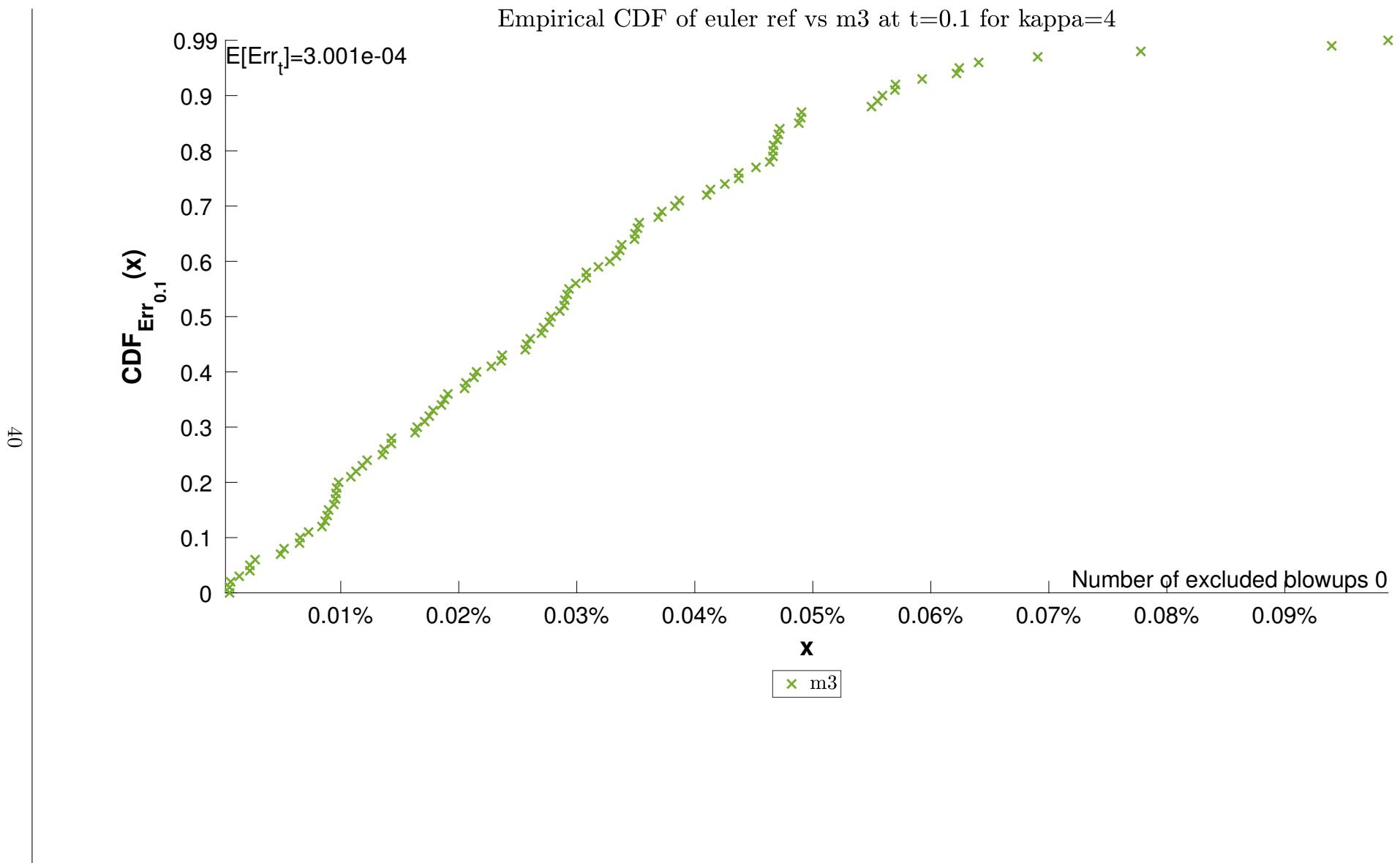








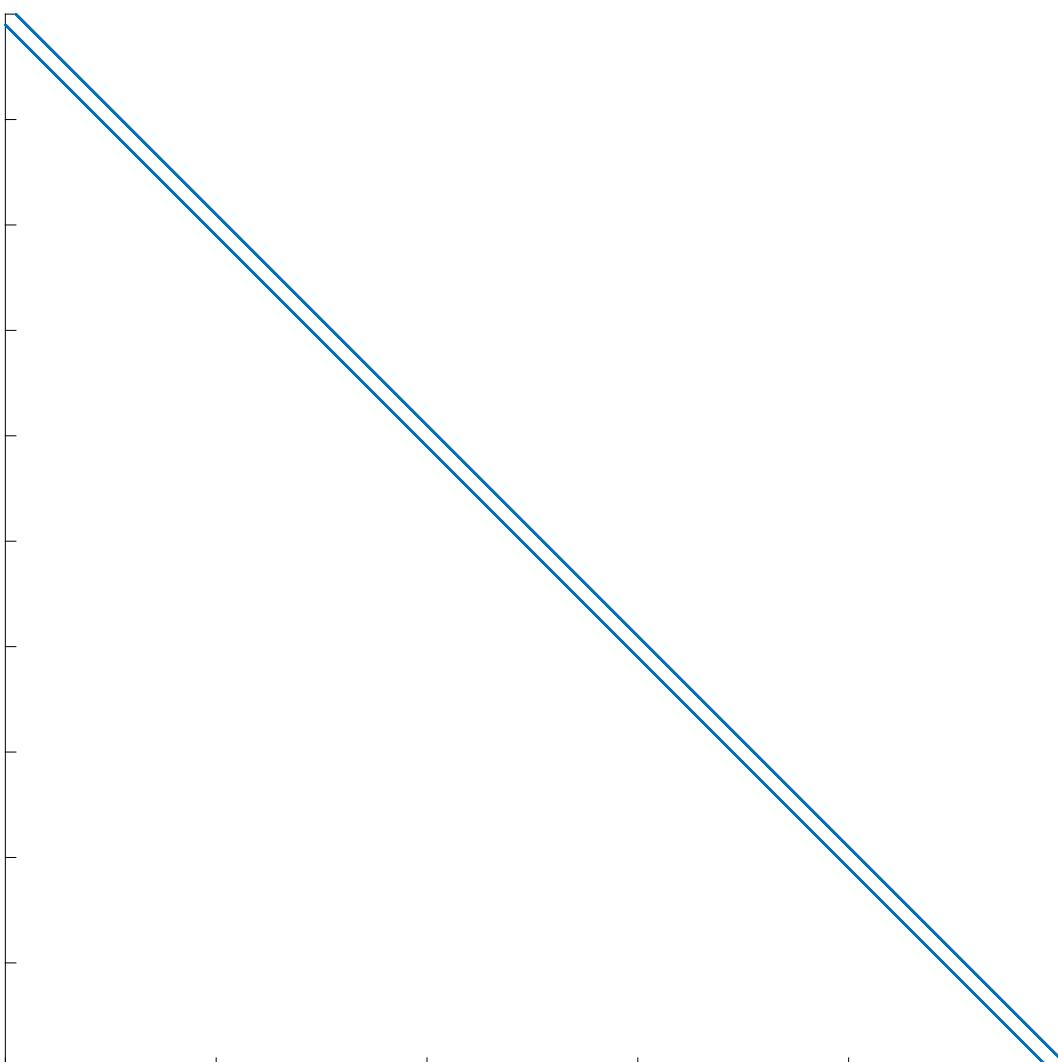




1.8 Sparsity Patterns

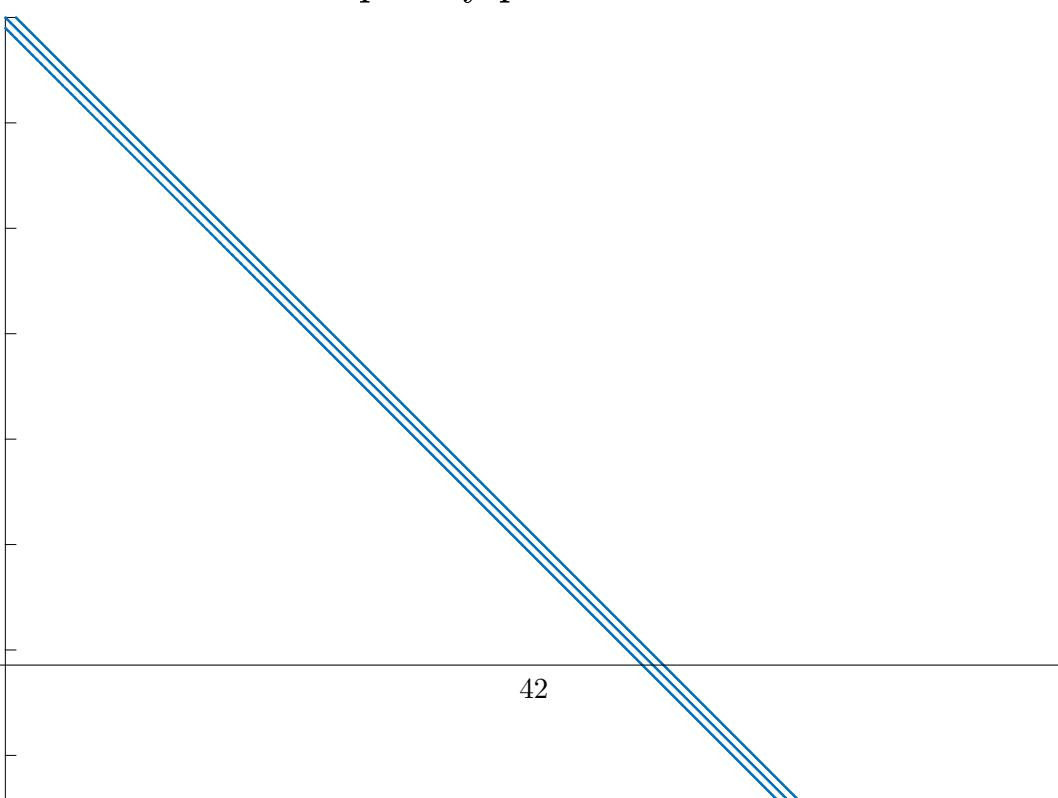
Sparsity pattern of A

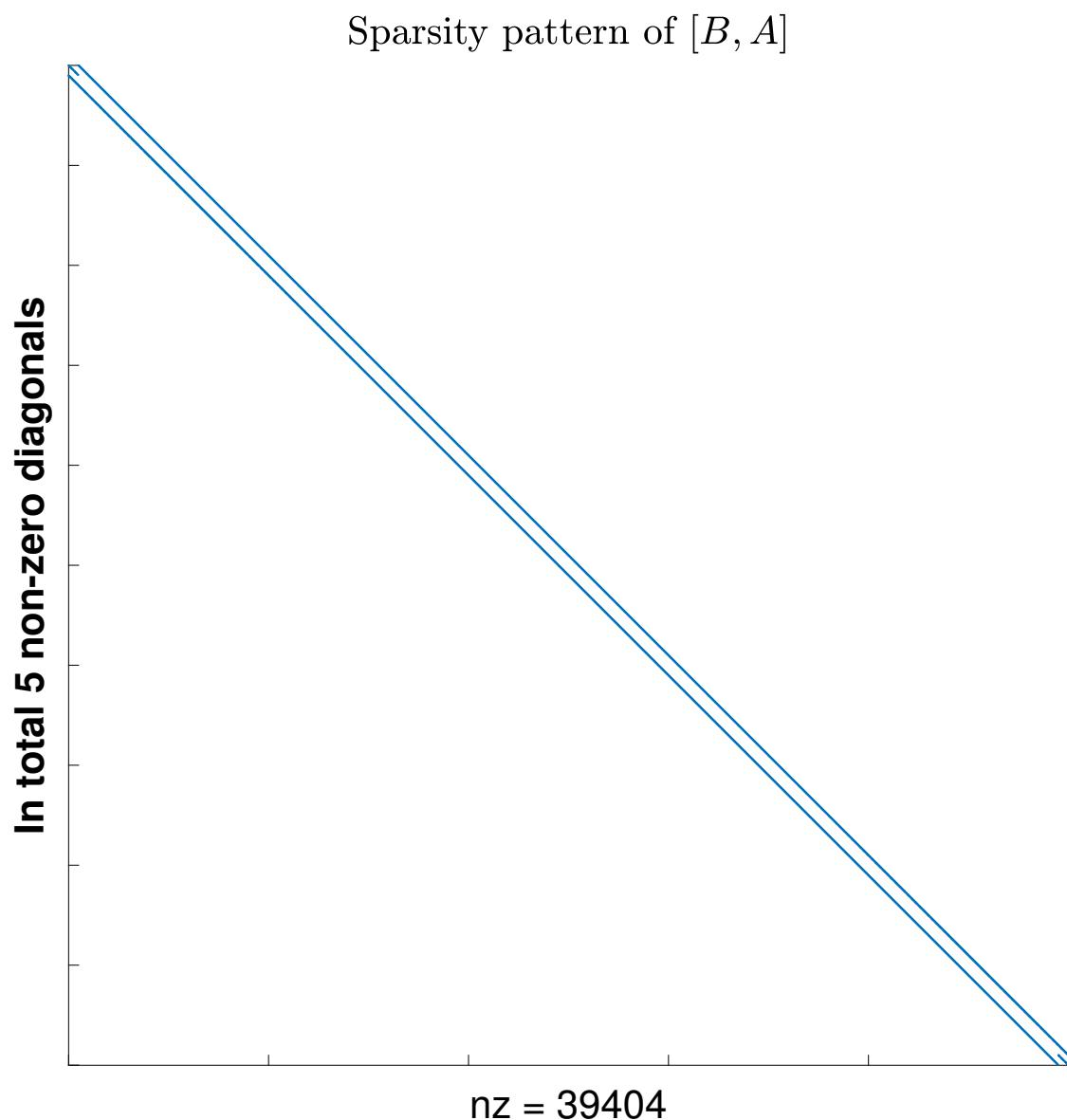
In total 2 non-zero diagonals

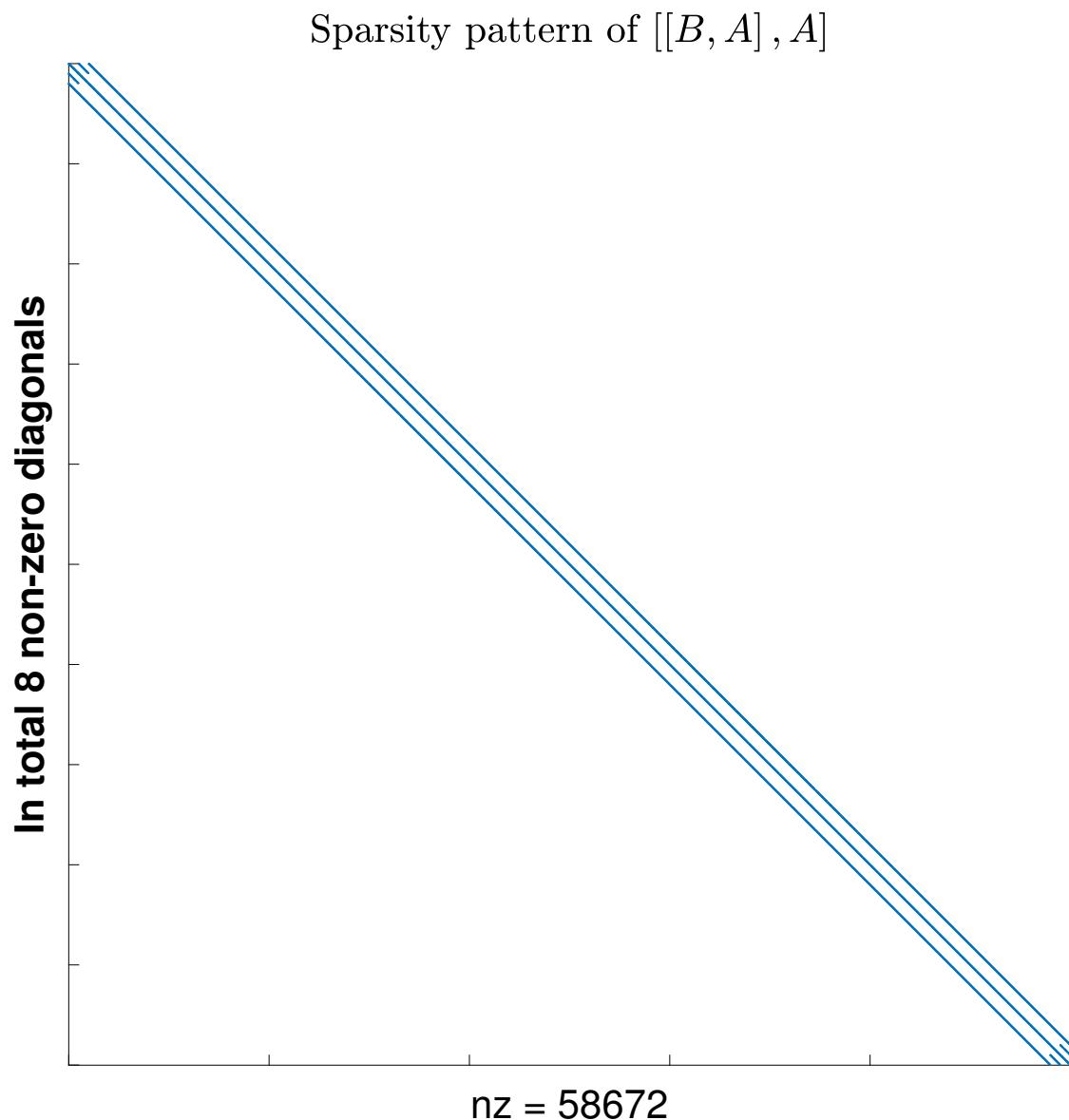


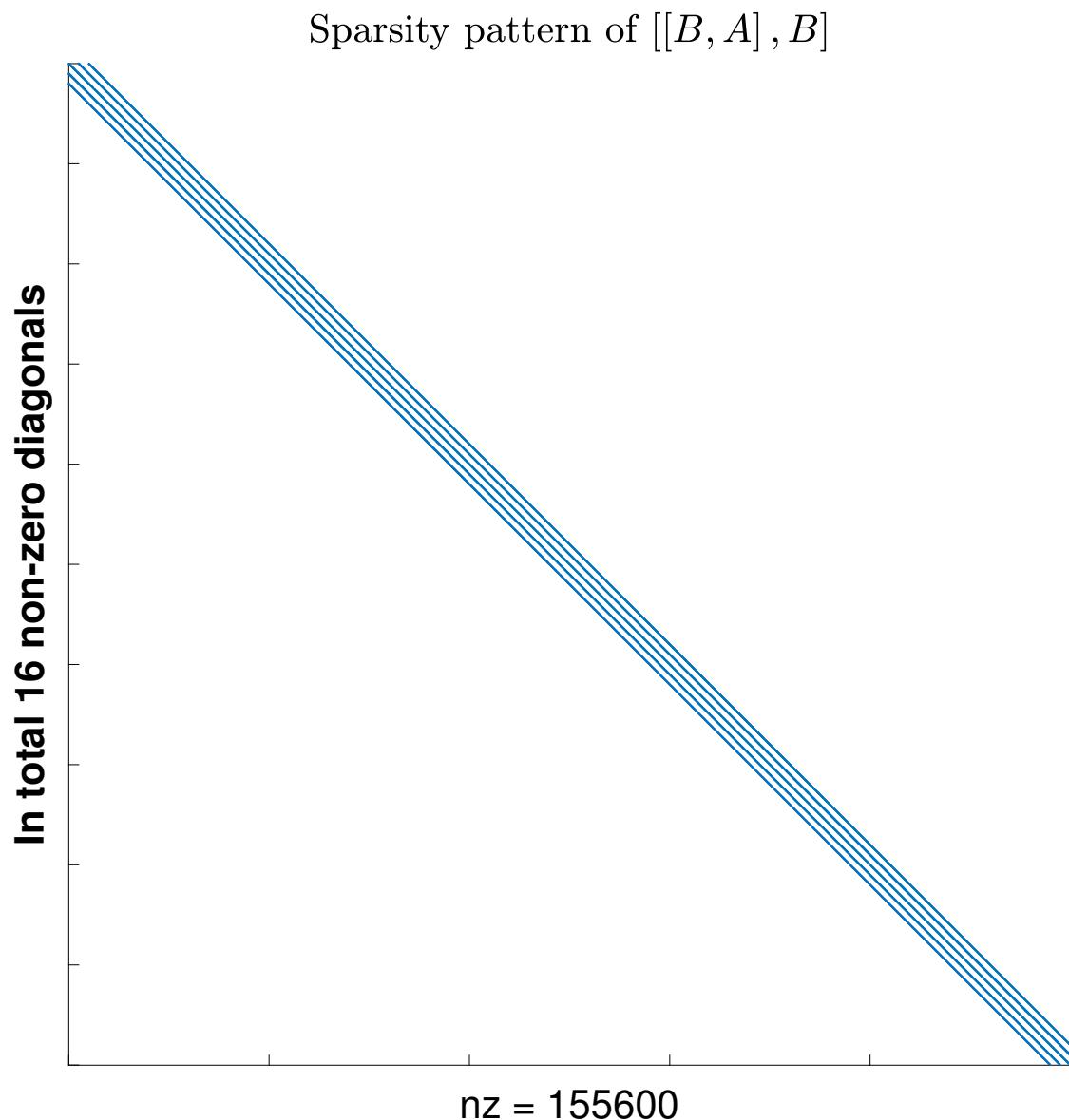
$\text{nz} = 19800$
Sparsity pattern of B

In total 5 non-zero diagonals









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