

1 Magnus expansion for $A_t \in \mathbb{R}^{2,2}$, $B = 0$ deterministic

We will concern ourselves with the following SDE:

$$dX_t = A_t X_t dW_t$$

with

$$A_t = \begin{bmatrix} f^{11}(t) & f^{12}(t) \\ 0 & f^{22}(t) \end{bmatrix}$$

Here we chose:

$$f^{11}(t) \equiv 2, f^{12}(t) = t, f^{22}(t) \equiv -1,$$

1.1 Parameters

Parameter	value
t_0	0
T	1
N_{fine}	10000
N	1000
M_{fine}	1000
M	1000
d	2

t_0	0
T	1
N_{fine}	10000
N	1000
M_{fine}	1000
M	1000
d	2

1.2 Computational Times

Method	Log	Matrix Exp	Total
exact	0	0	0.732425
euler	0	0	4.68108
m1	0.0304975	4.79856	4.82906
m2	0.0430085	4.80114	4.84415
m3	0.0658781	4.77638	4.84226

1.3 Errors

- (i) Errors for $X(1, 1, :, :)$:
- (a) Reference method: exact

Error	euler	m1	m2	m3
(abs error) L2	0.0928766	21.7141	0	0
(rel error) min	0	0	0	0
(rel error) max	0.0219057	6.37577	0	0
(rel error) mean	0.0147946	2.19197	0	0

- (ii) Errors for $X(1, 2, :, :)$:

- (a) Reference method: exact

Error	euler	m1	m2	m3
(abs error) L2	0.0175783	3.40681	0.837159	0.346408
(rel error) min	0.00173942	0.334561	0.0791182	0.0154853
(rel error) max	0.0513247	3.04474	0.558977	0.2175
(rel error) mean	0.0270141	1.15961	0.27348	0.0944899

- (iii) Errors for $X(2, 2, :, :)$:

- (a) Reference method: exact

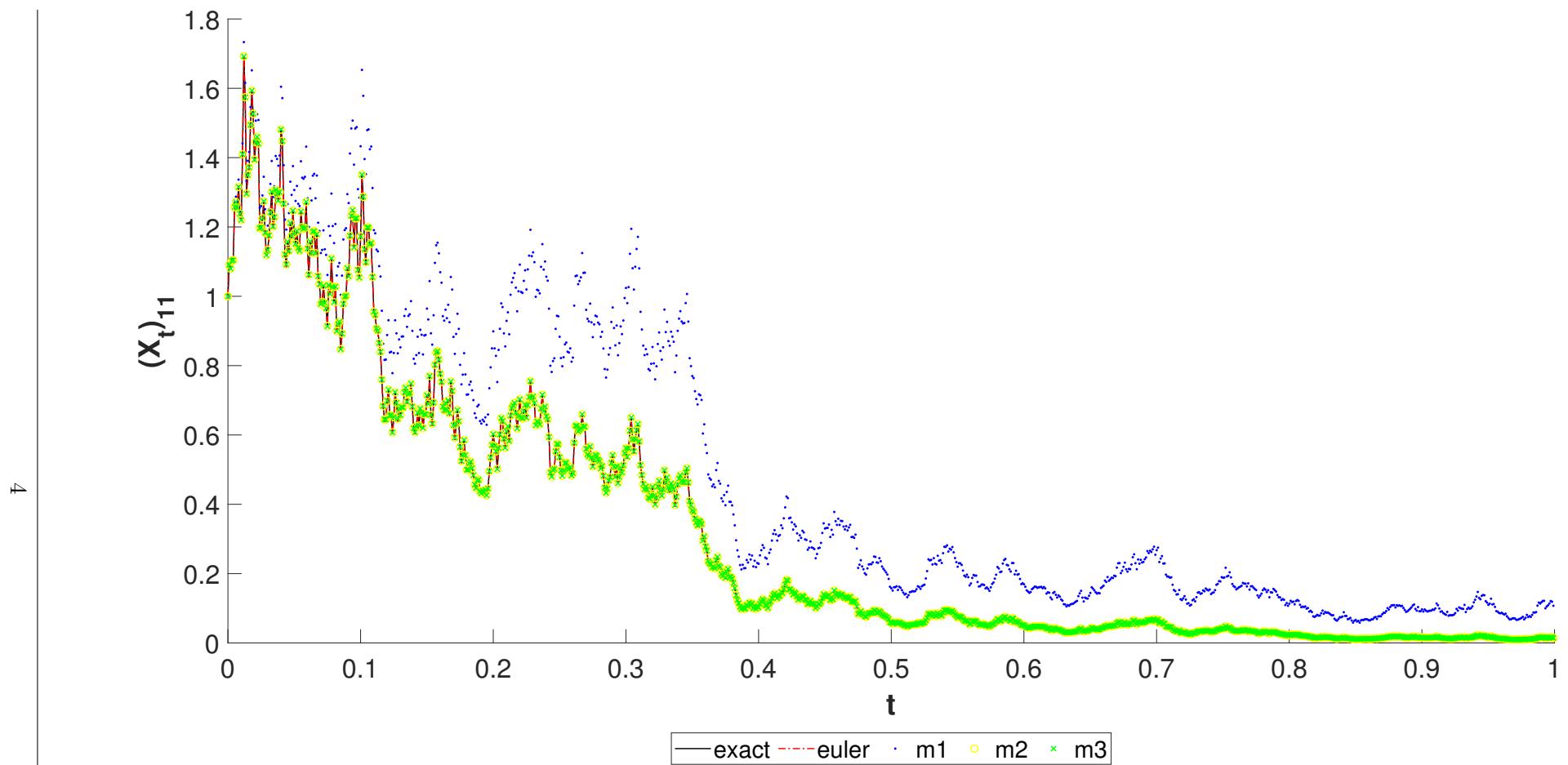
Error	euler	m1	m2	m3
(abs error) L2	0.0068628	0.538164	0	0
(rel error) min	0	0	0	0
(rel error) max	0.00552667	0.647979	0	0
(rel error) mean	0.00366393	0.297183	0	0

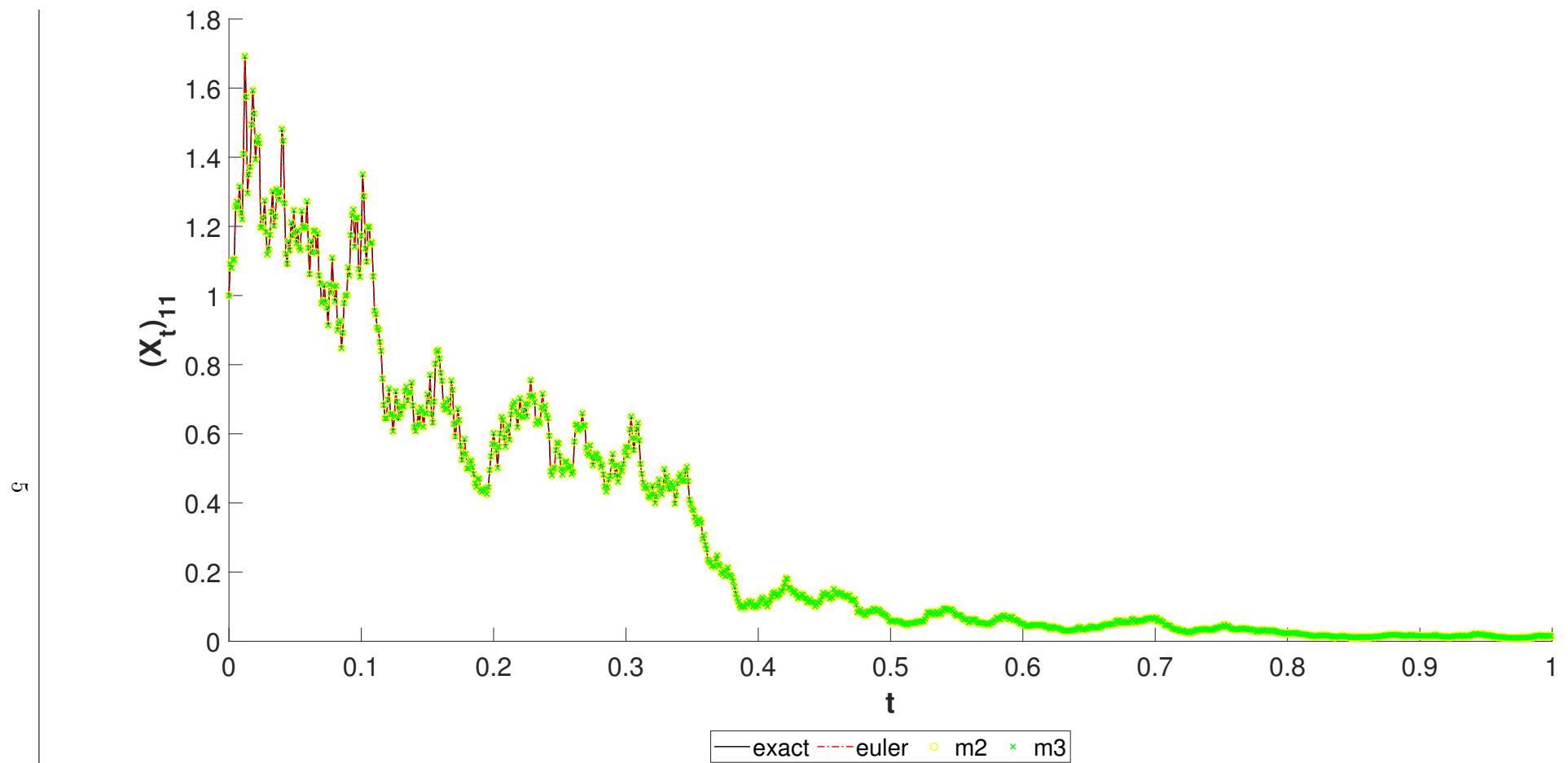
(iv) Total Errors:

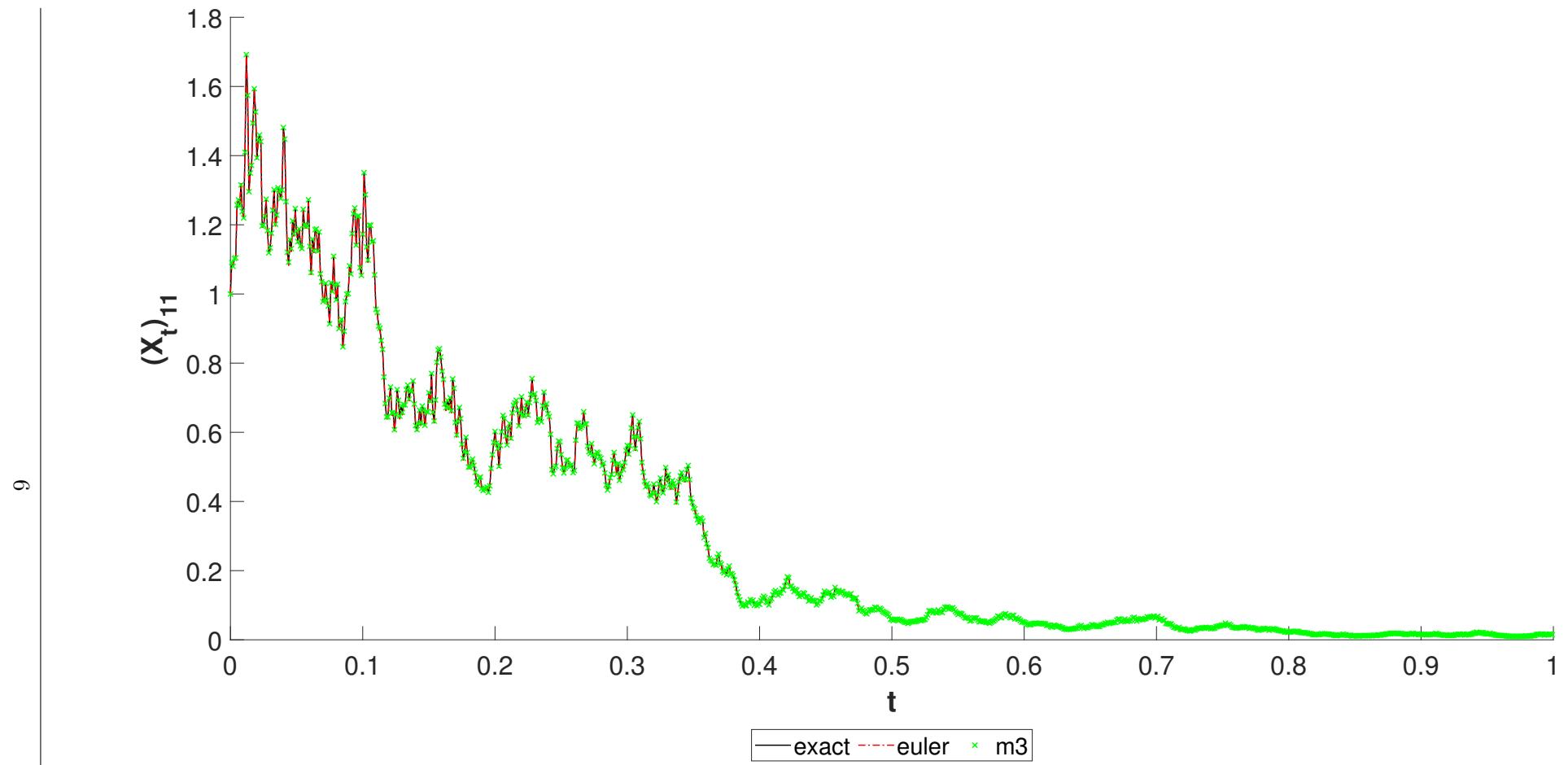
(a) Reference method: exact

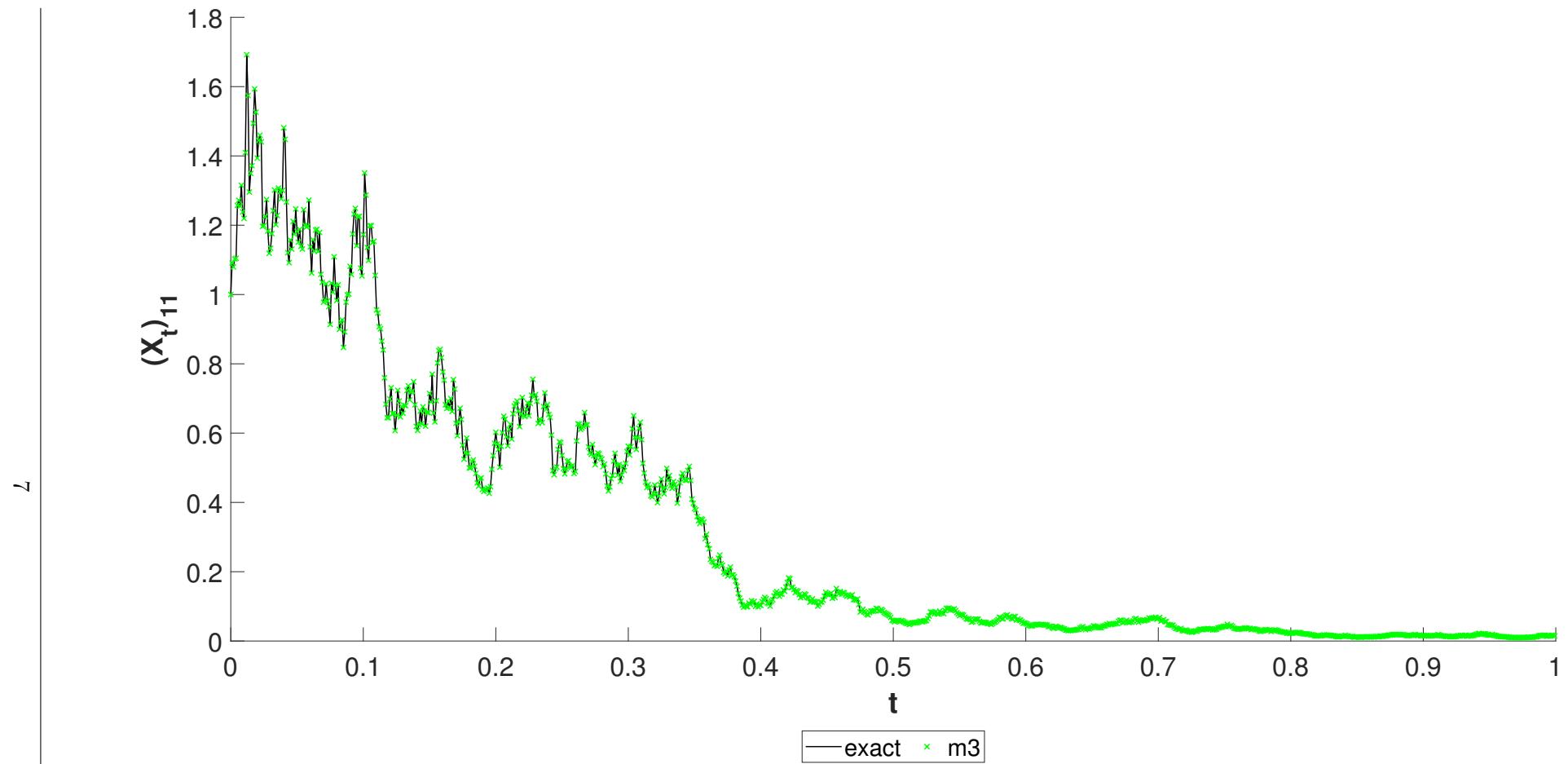
Method	$\mathbb{E}[Err_1]$
euler	1.06 %
m1	108 %
m2	3.06 %
m3	1.38 %

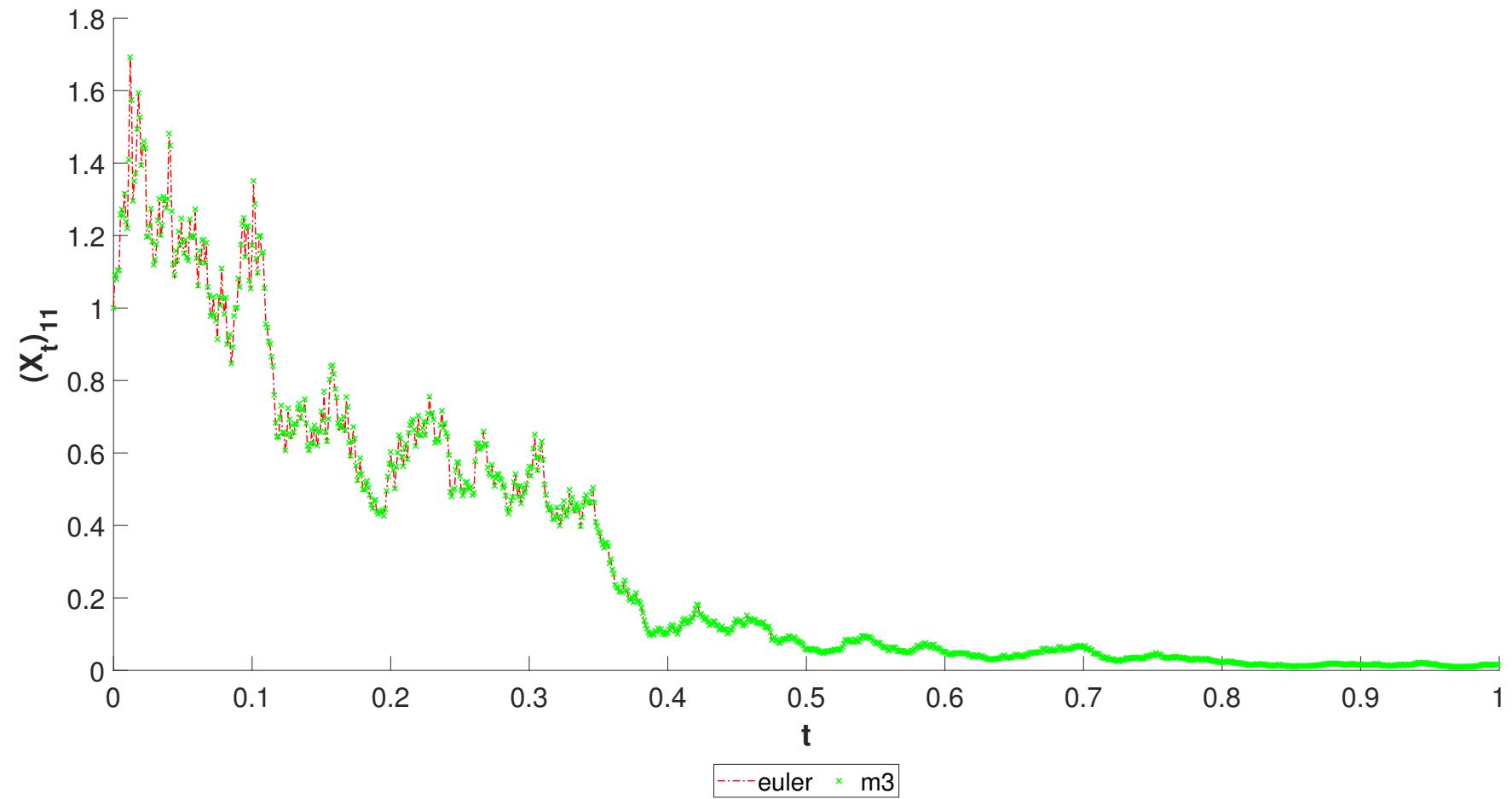
1.4 Plots

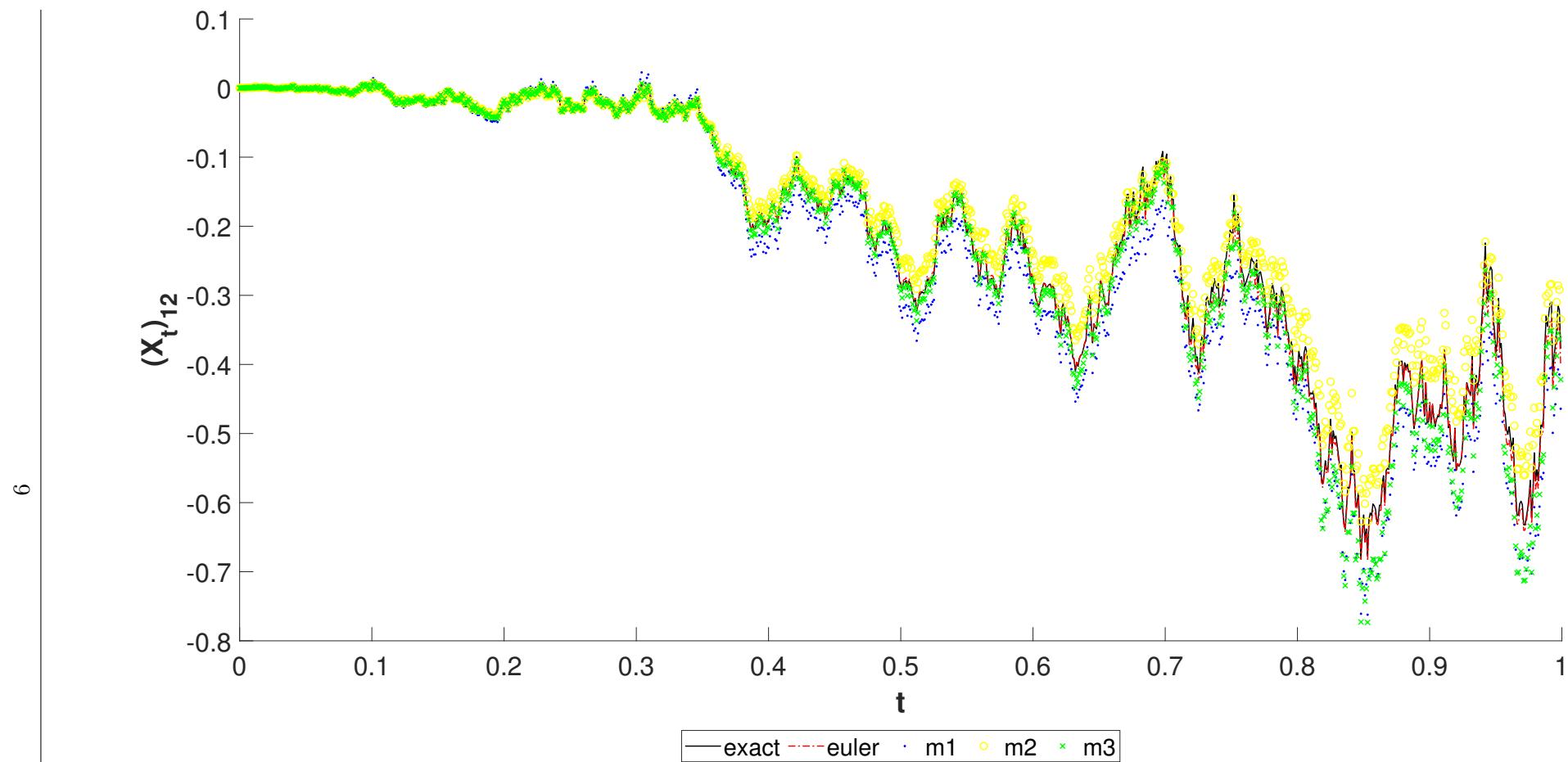


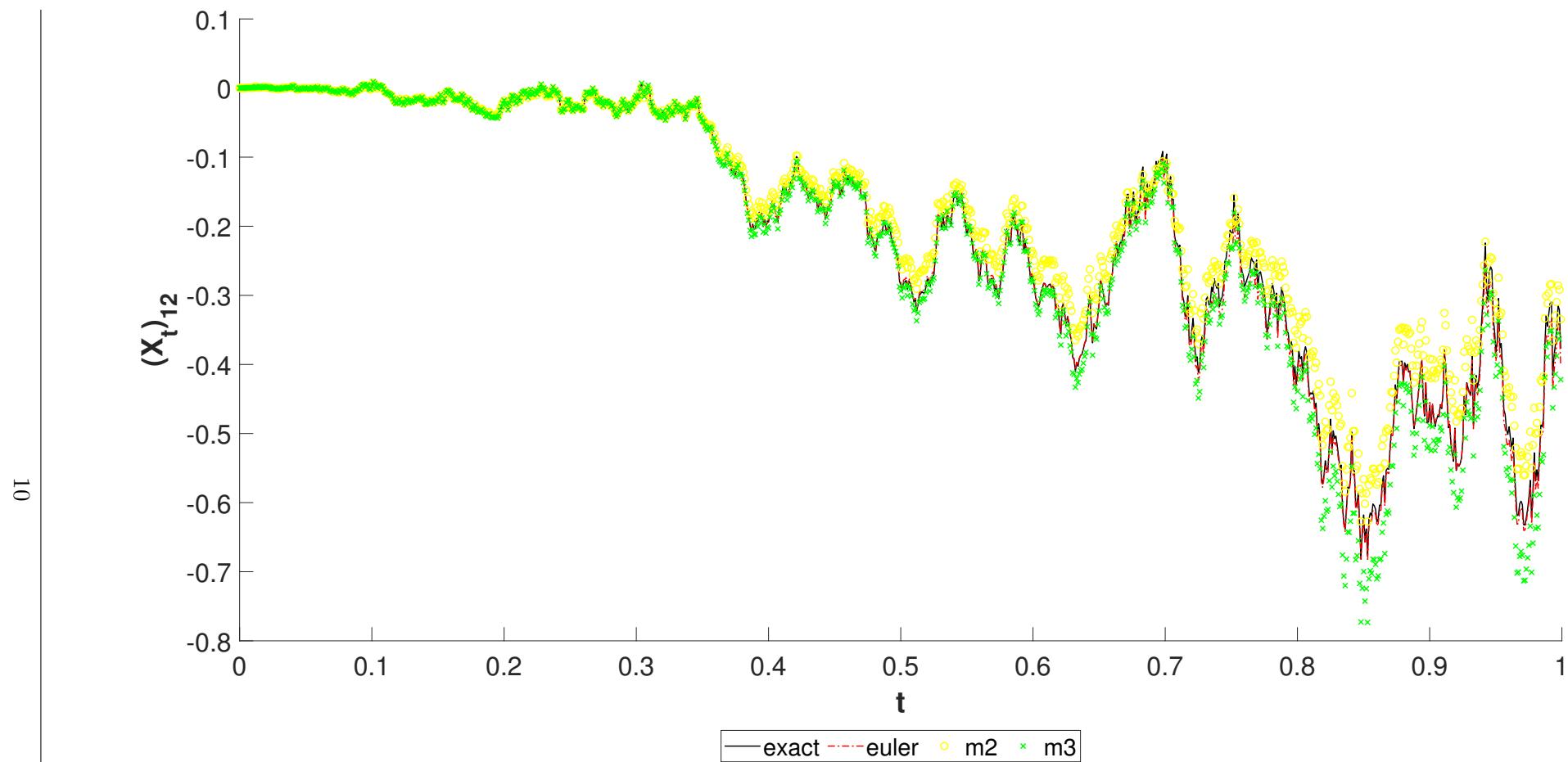


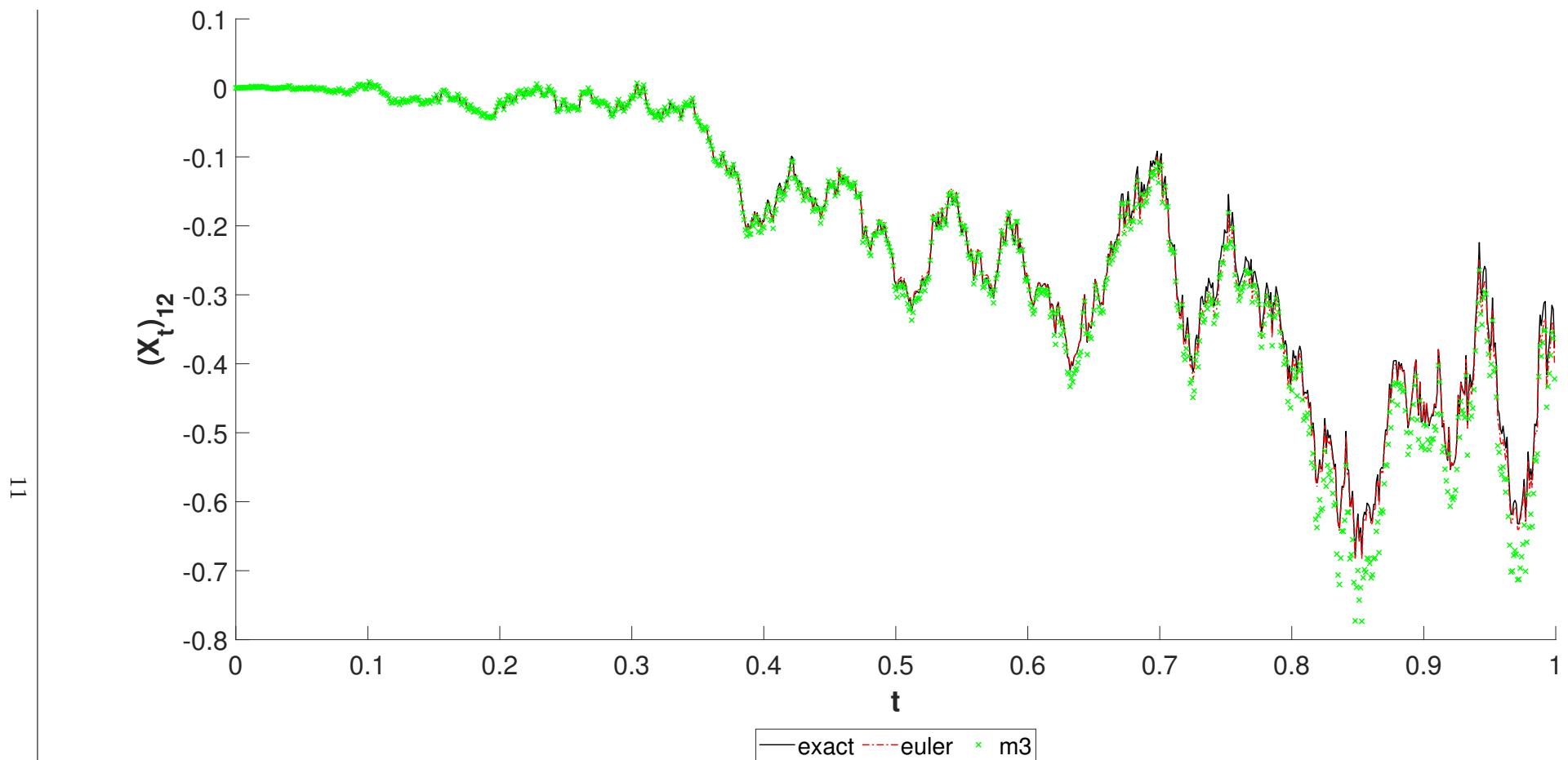


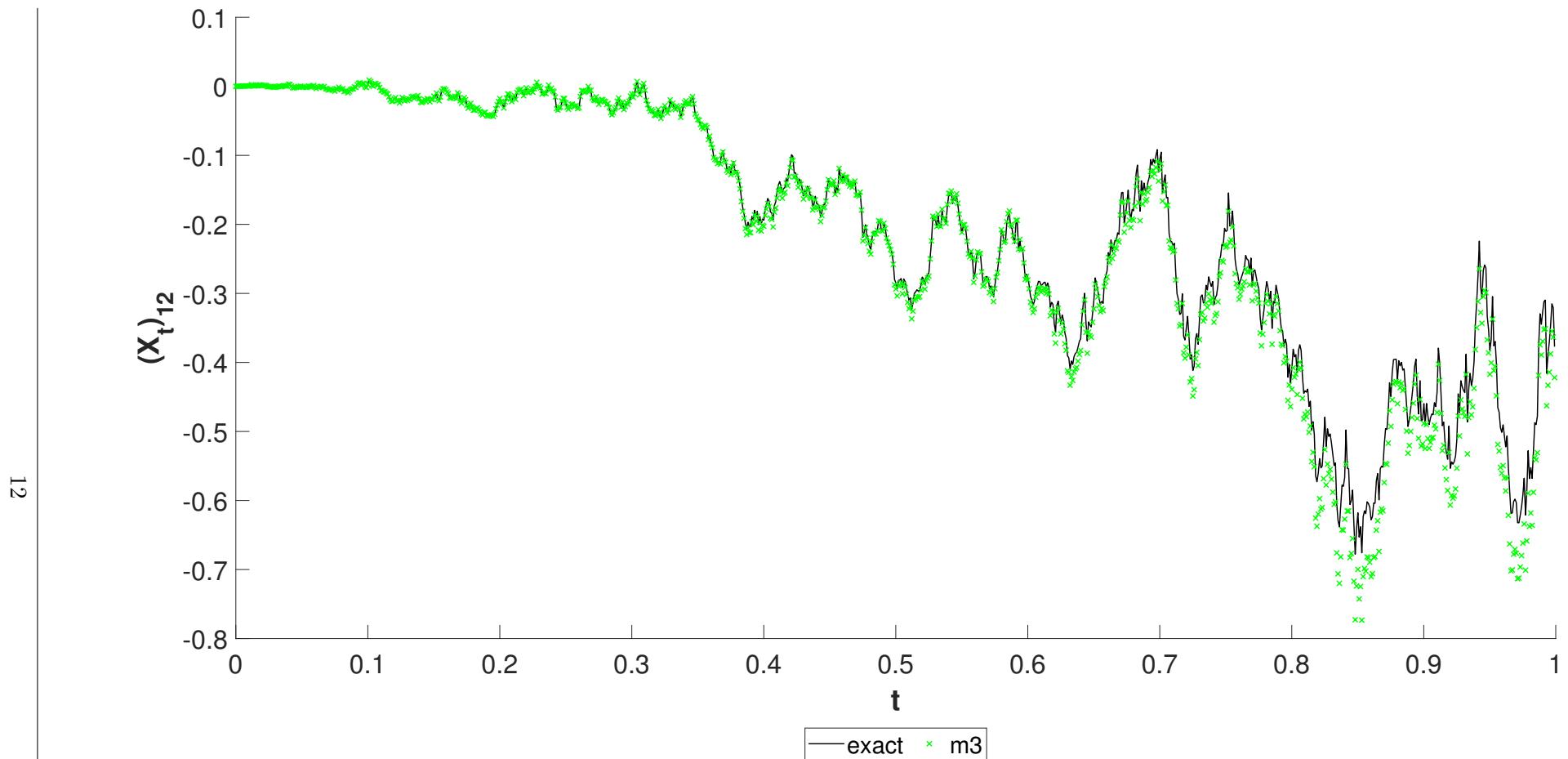


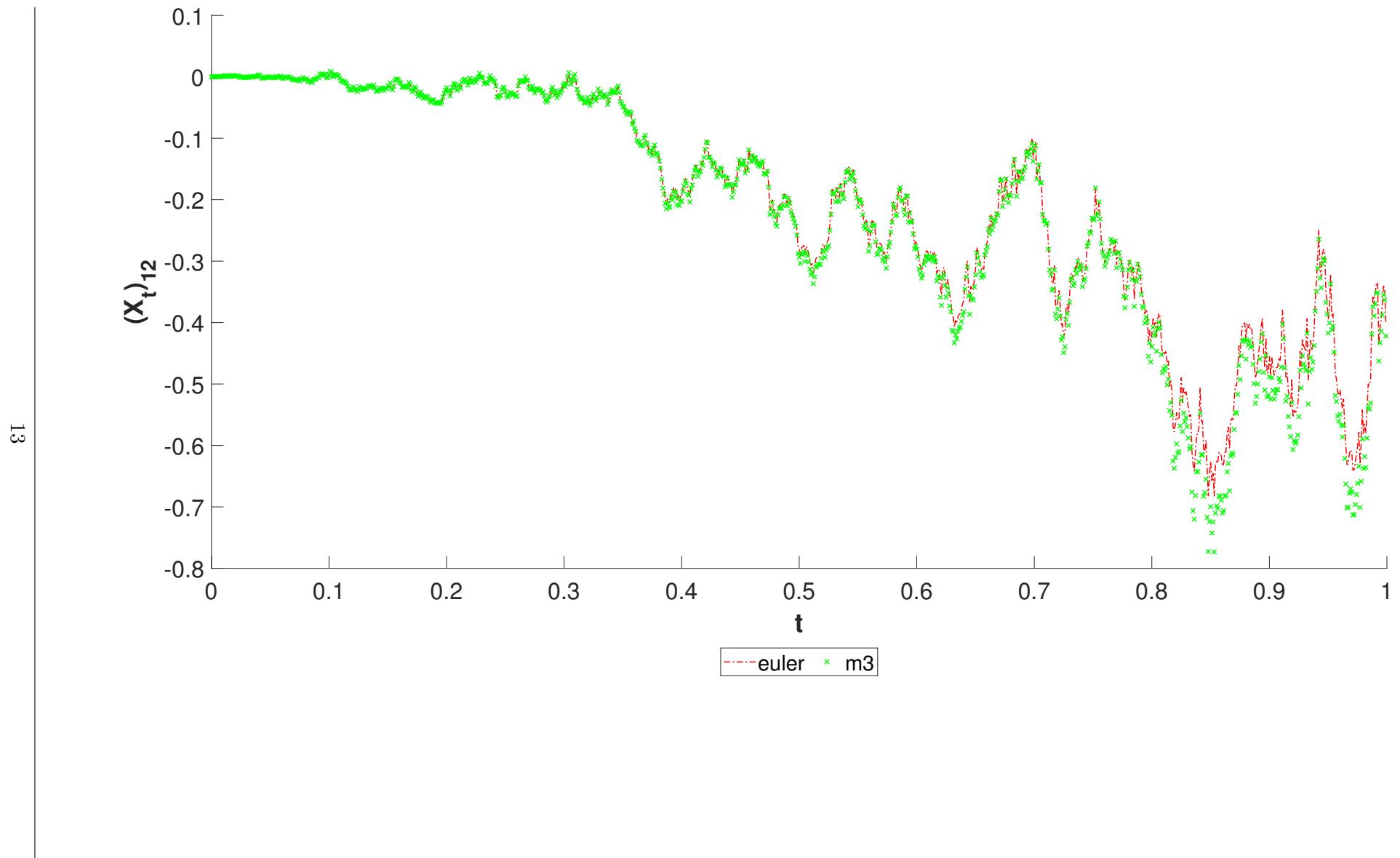


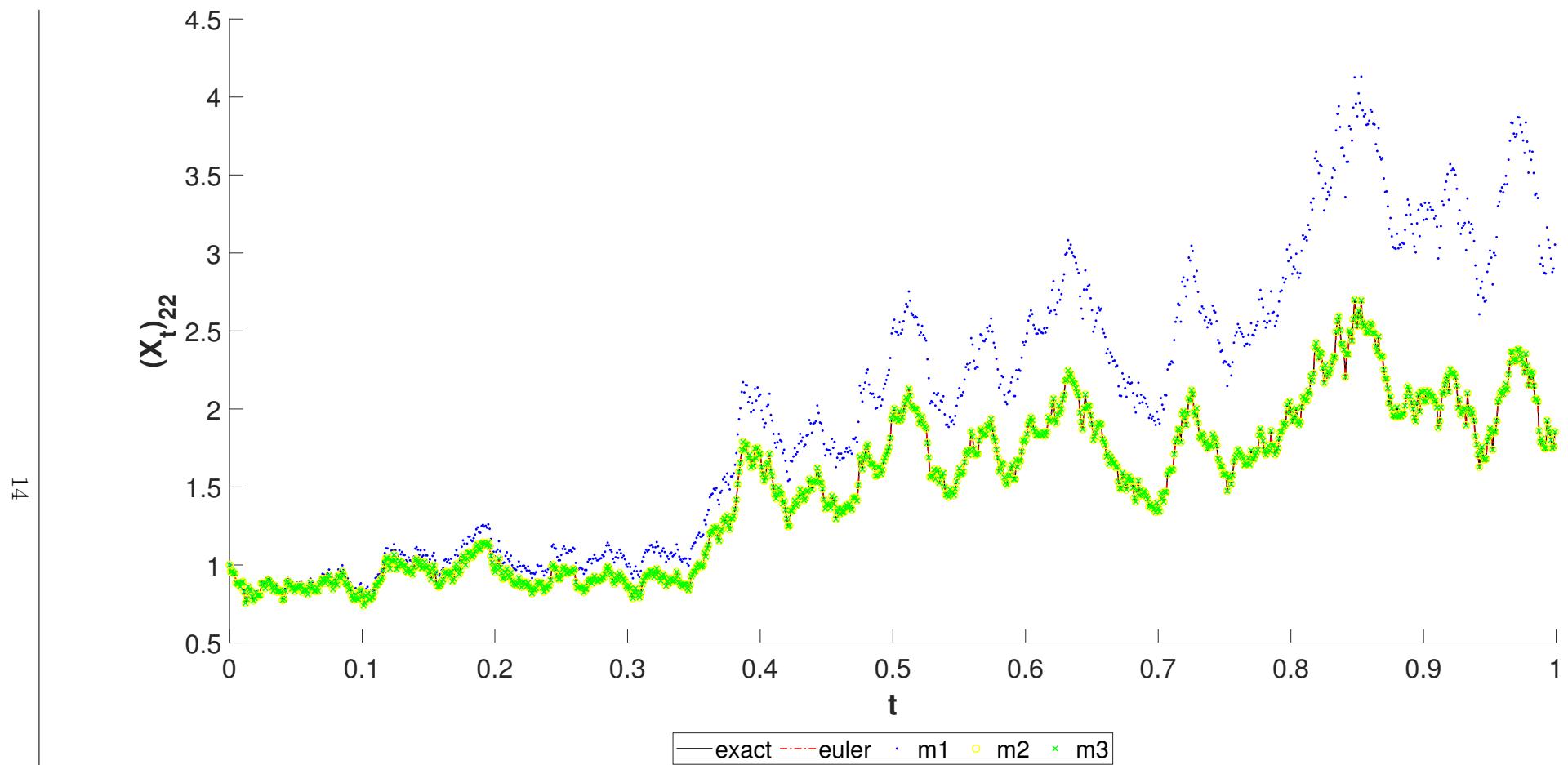


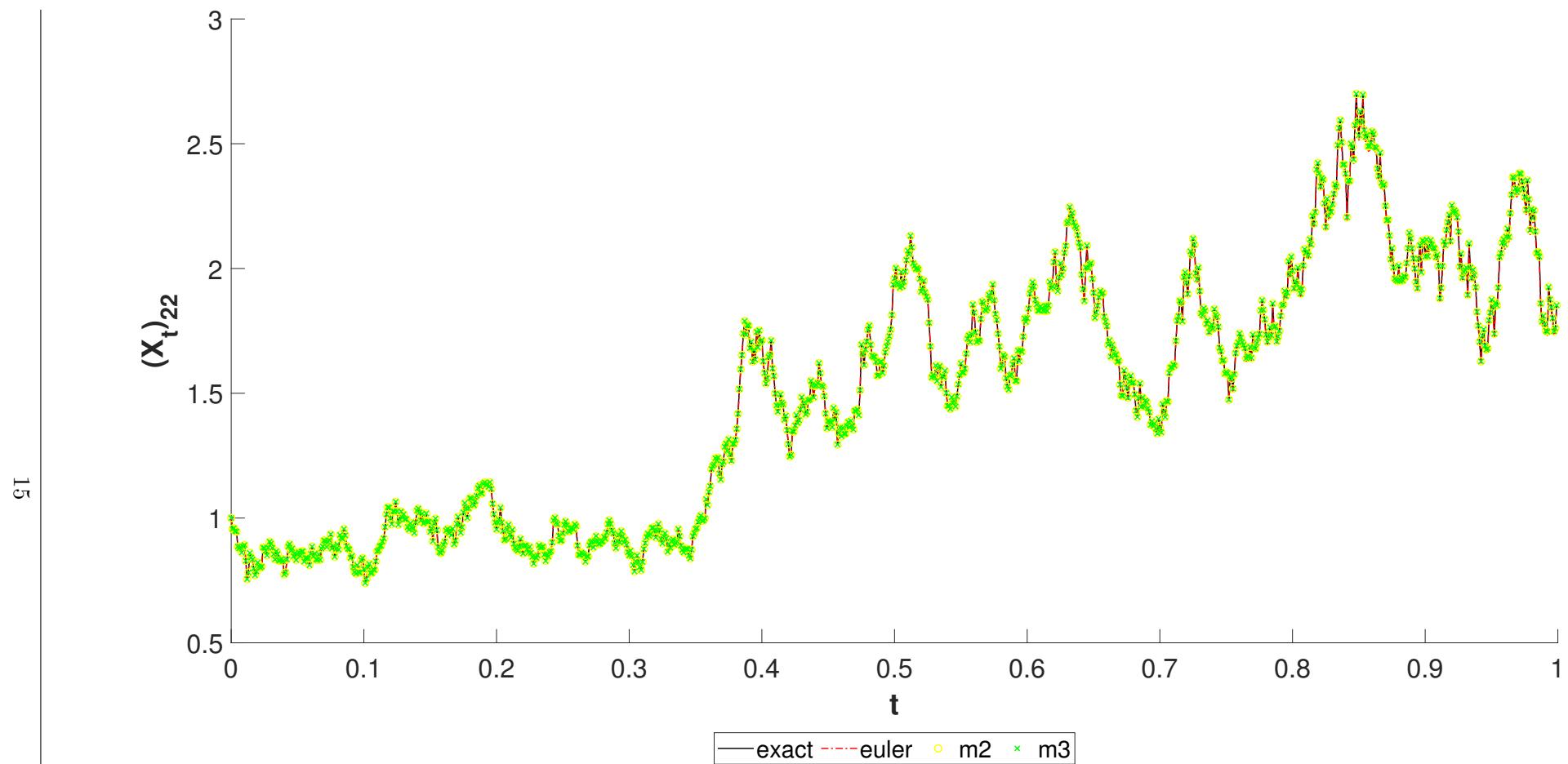


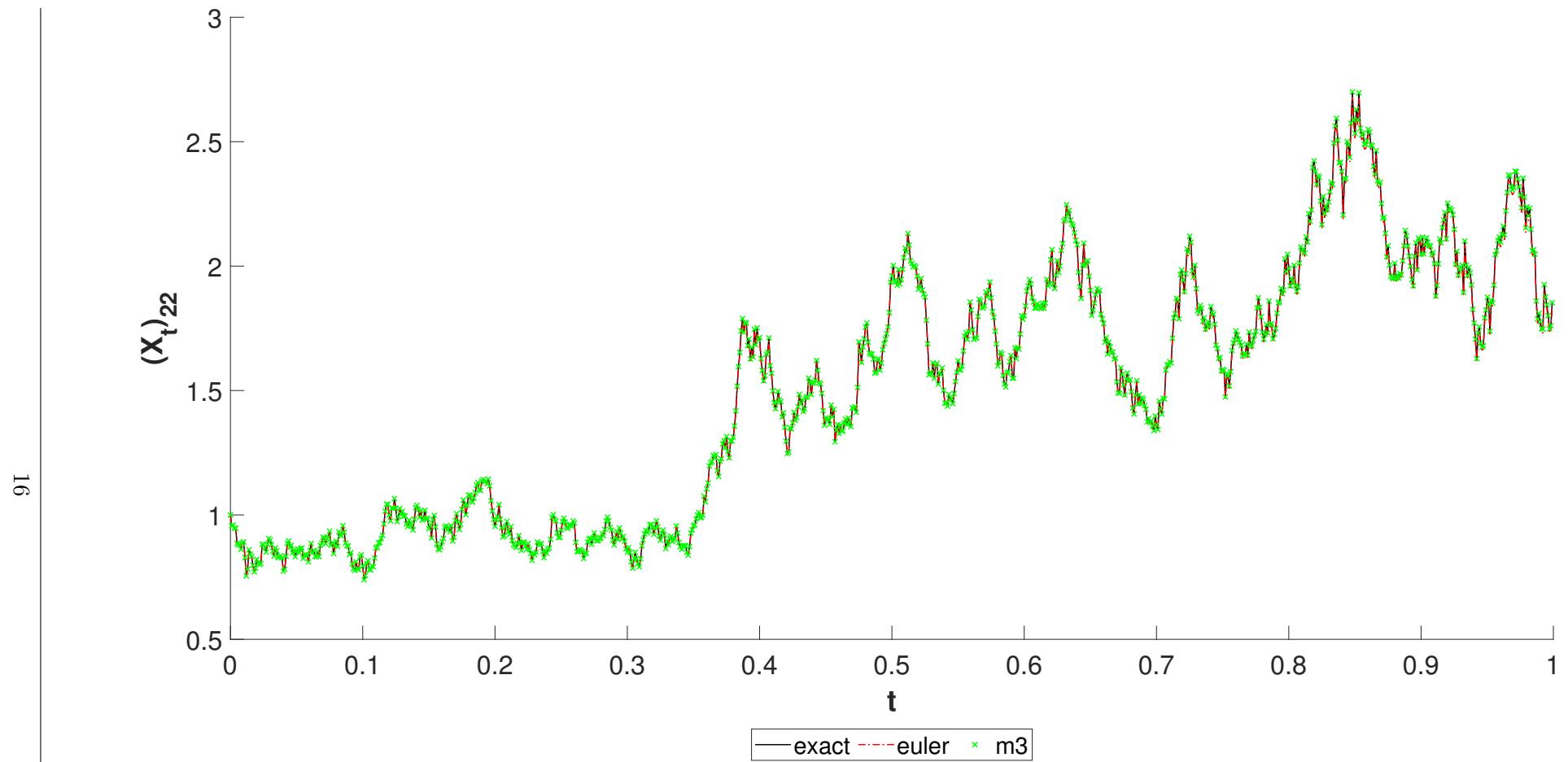


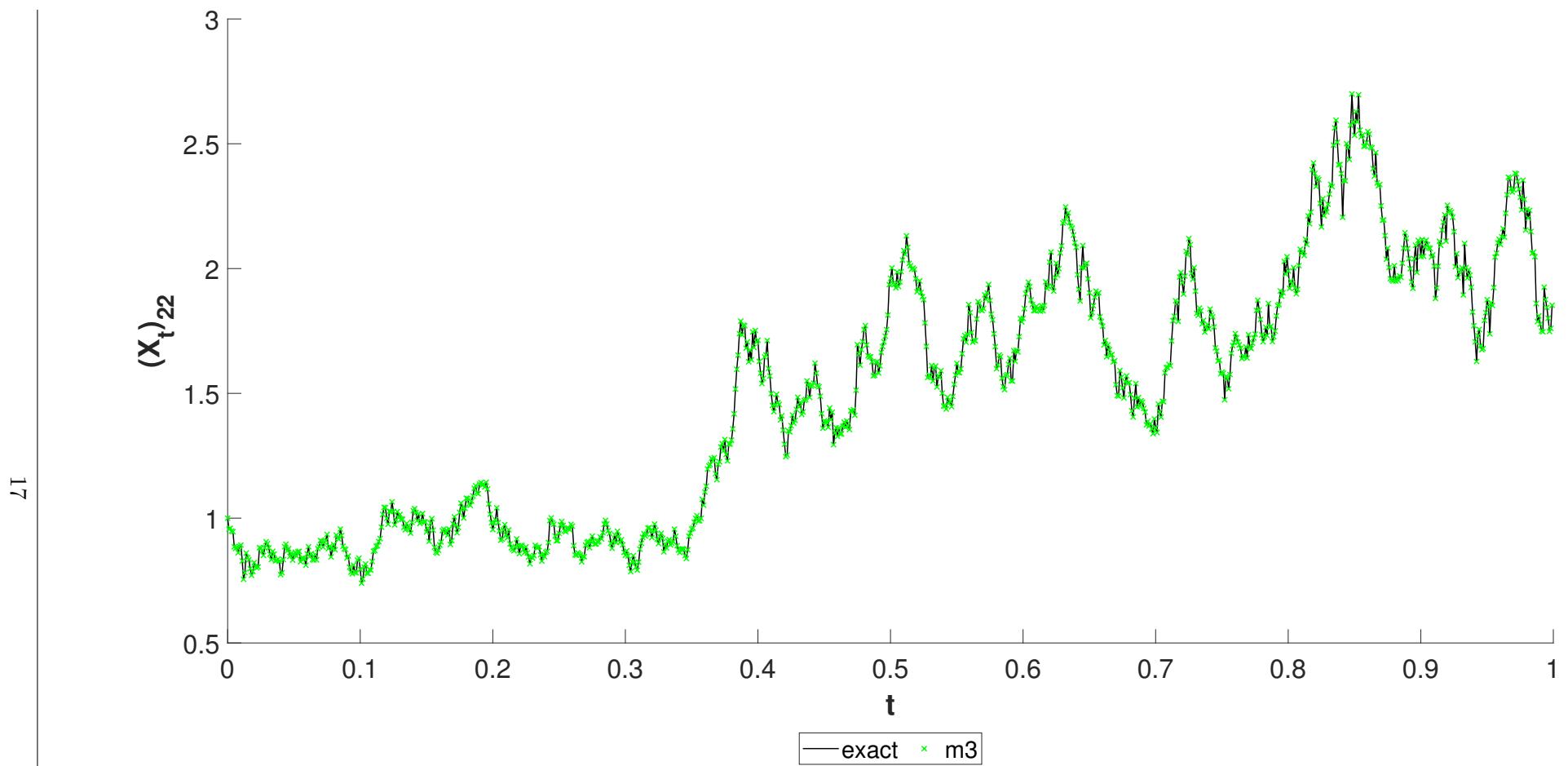


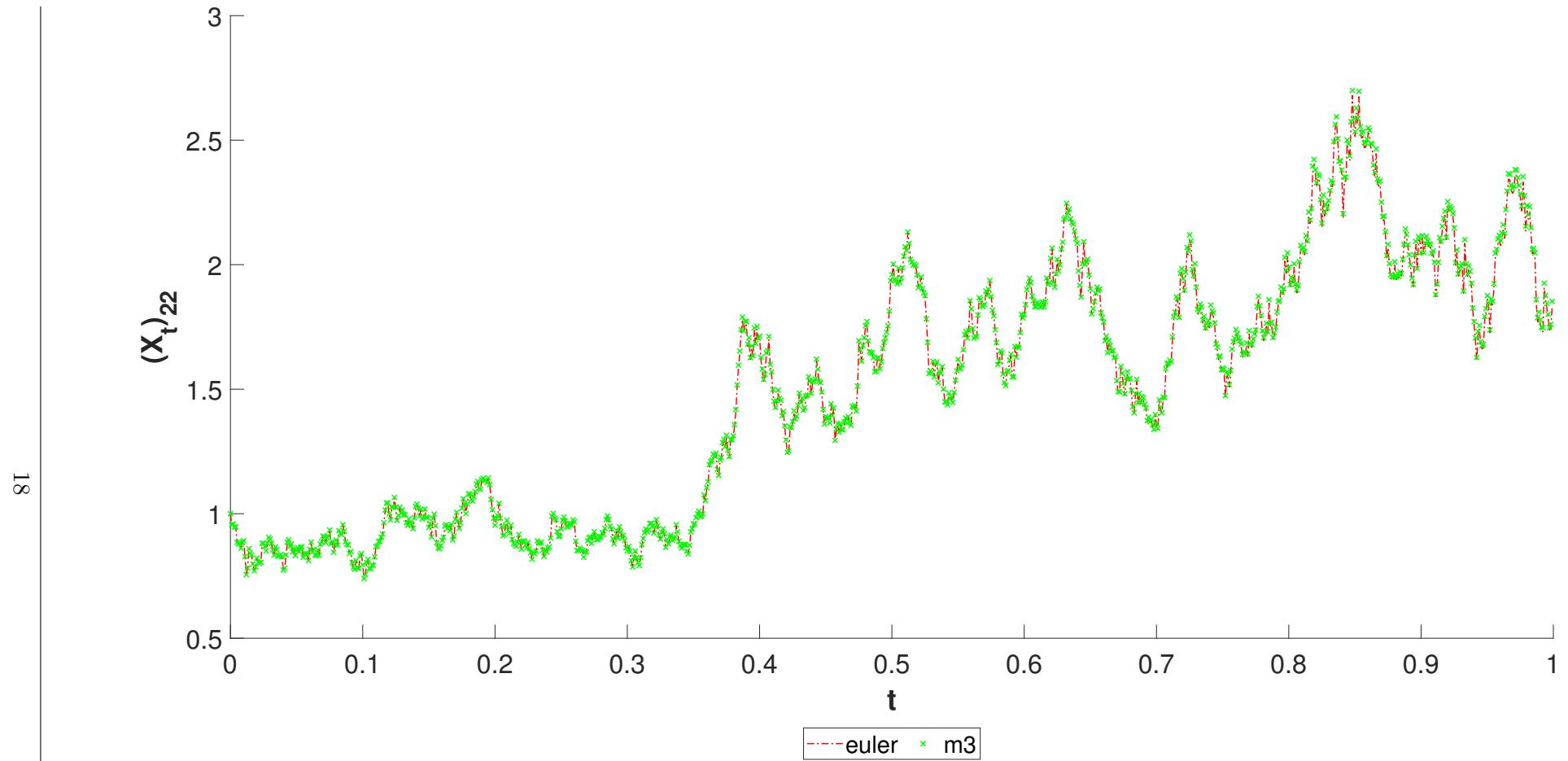


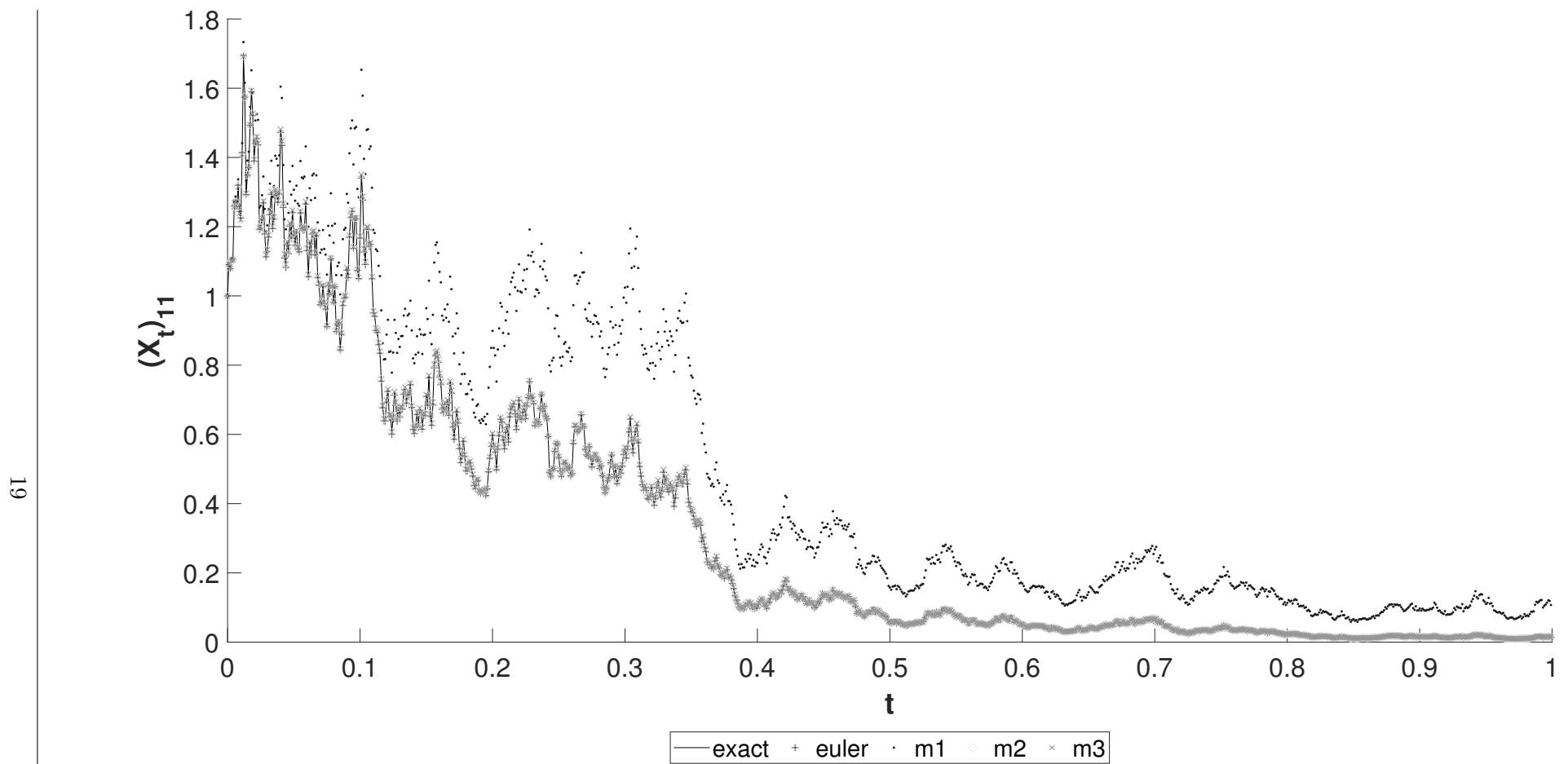


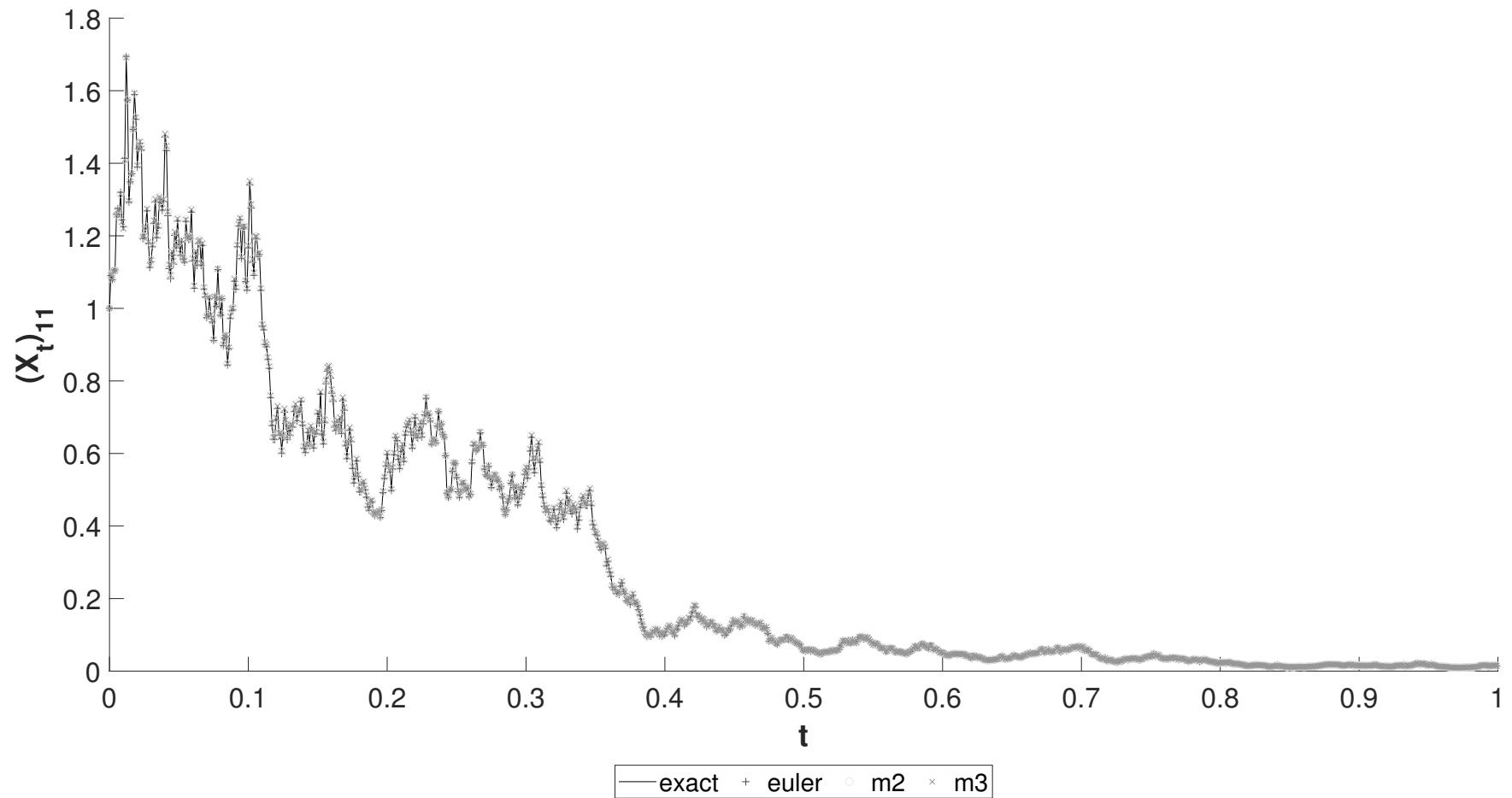


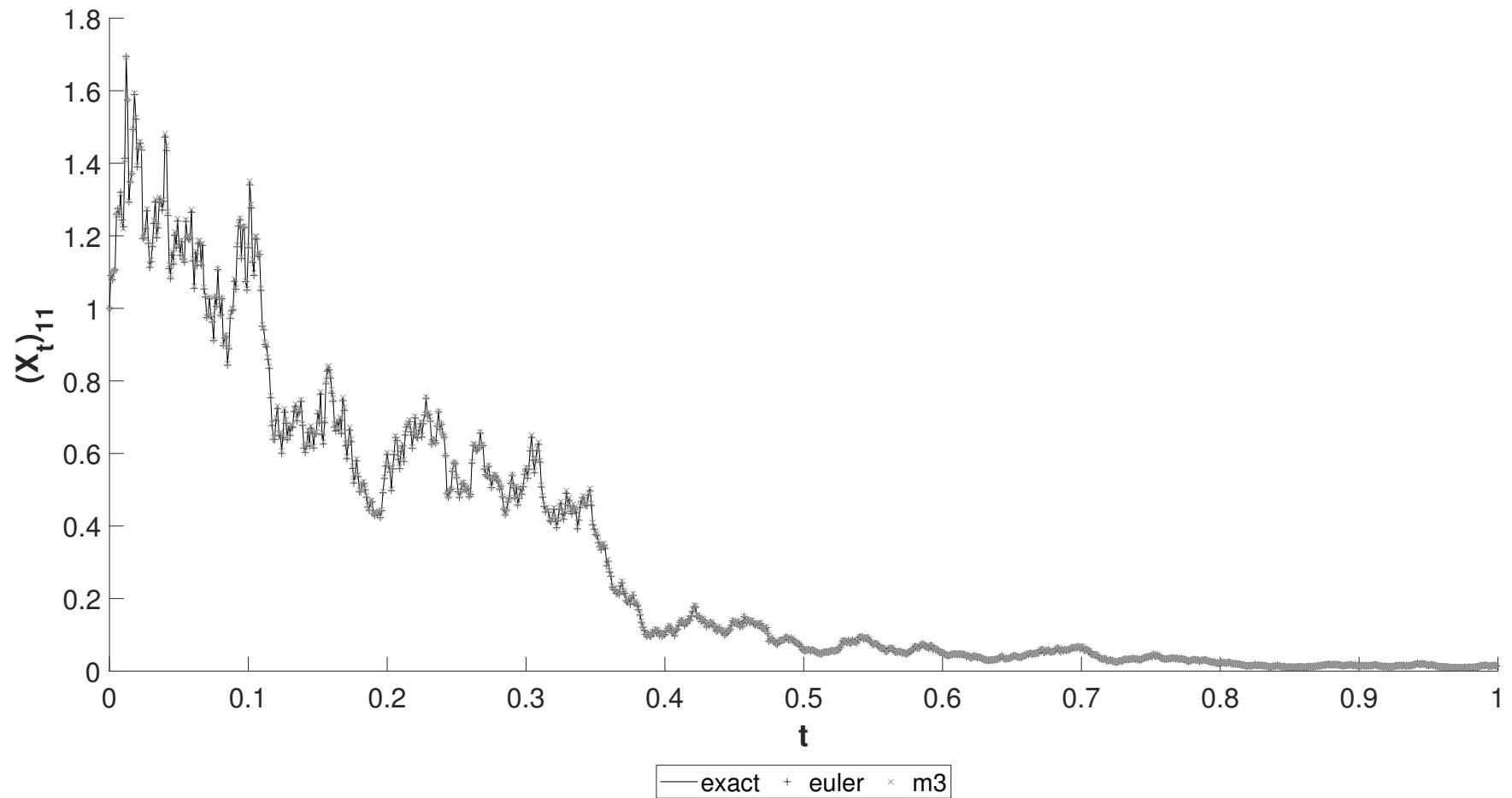


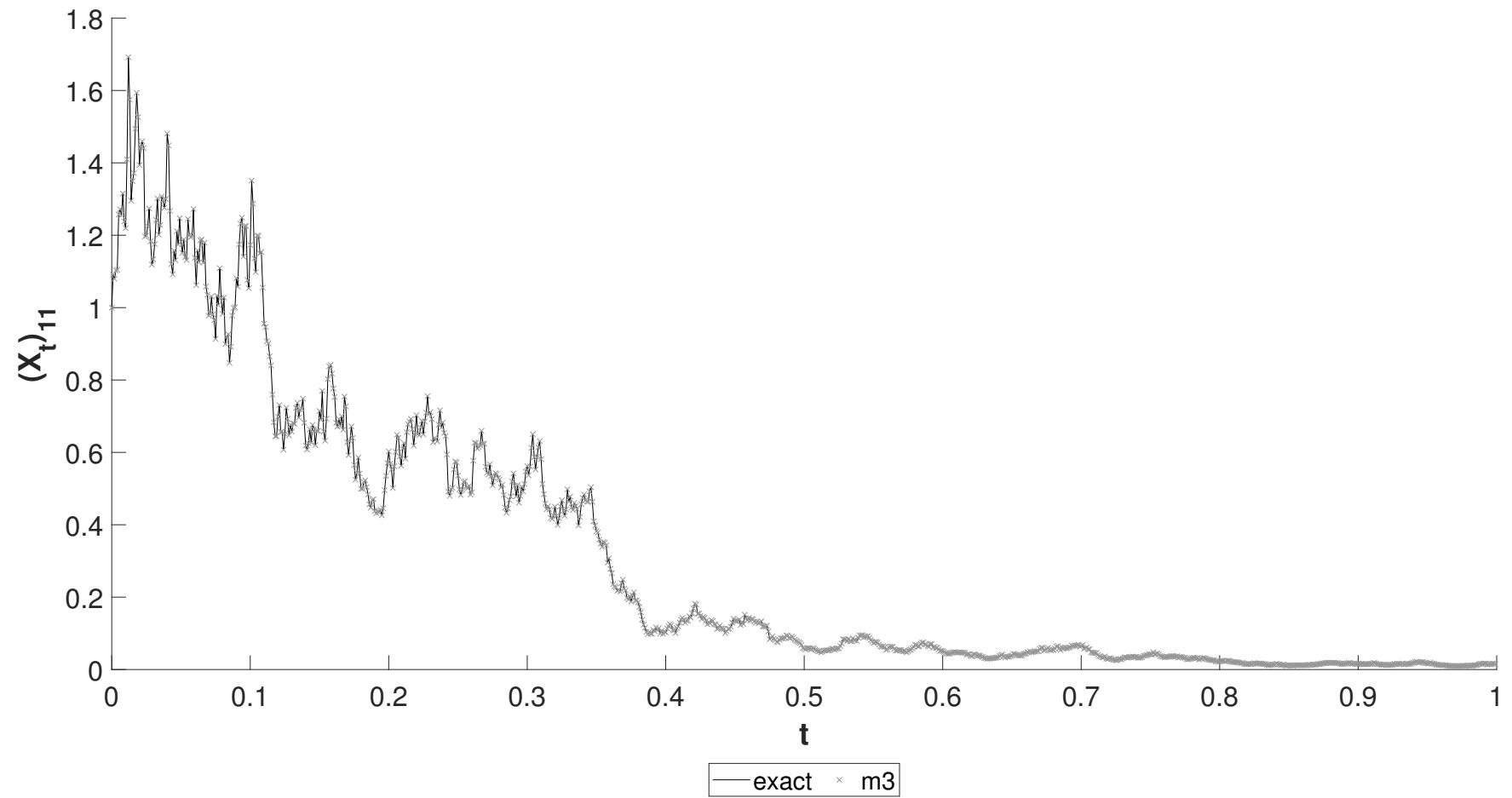


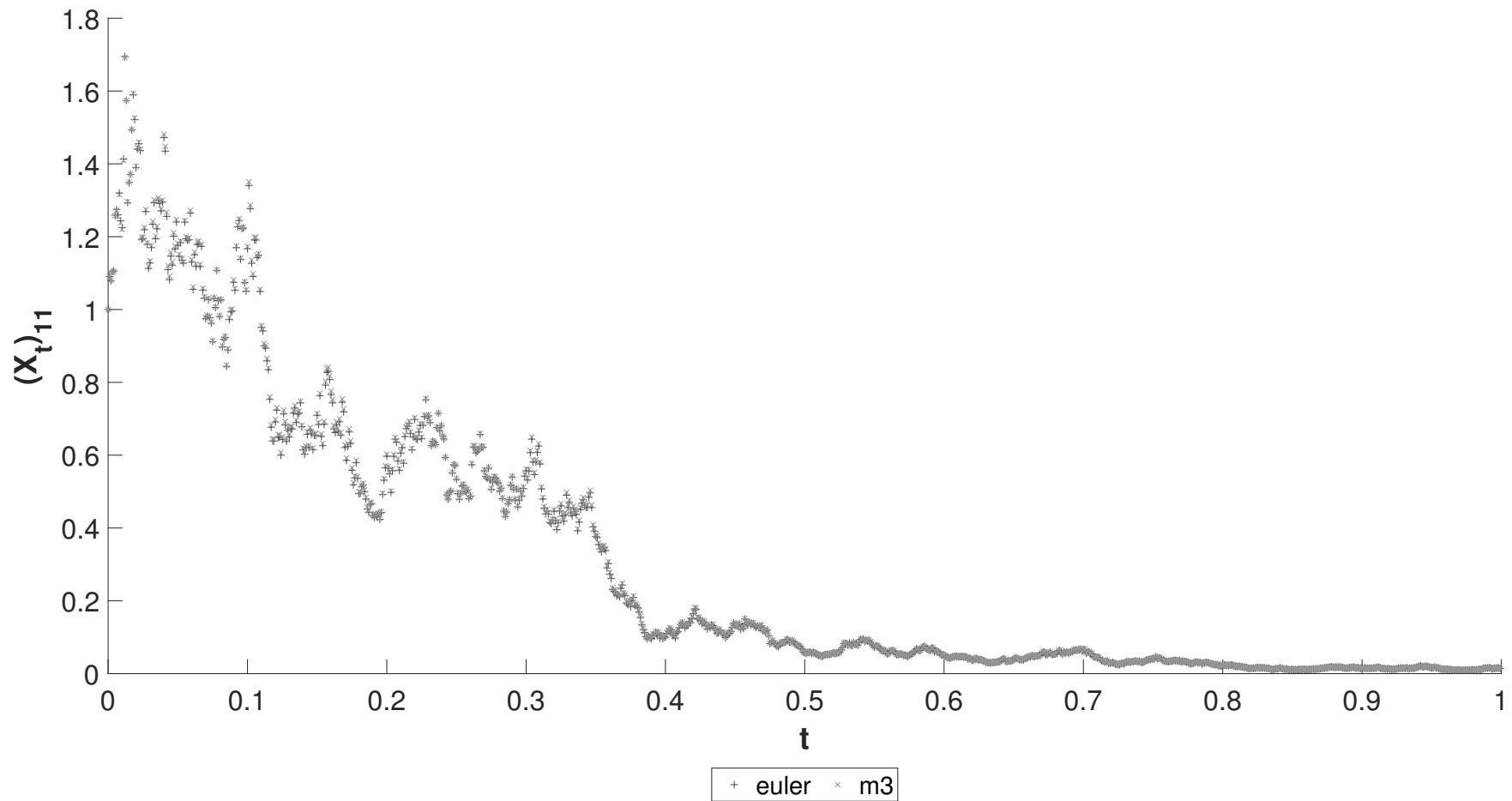


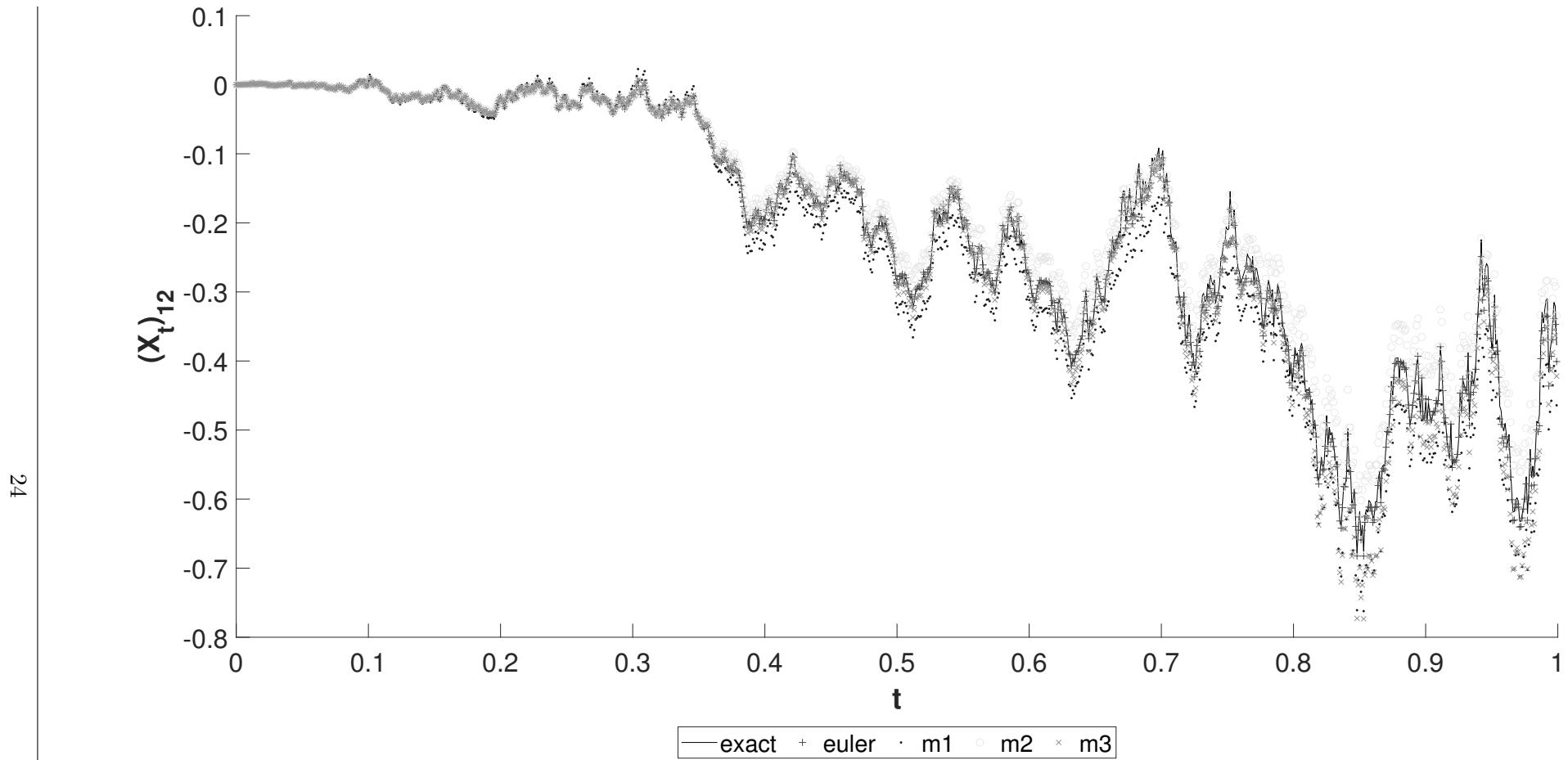


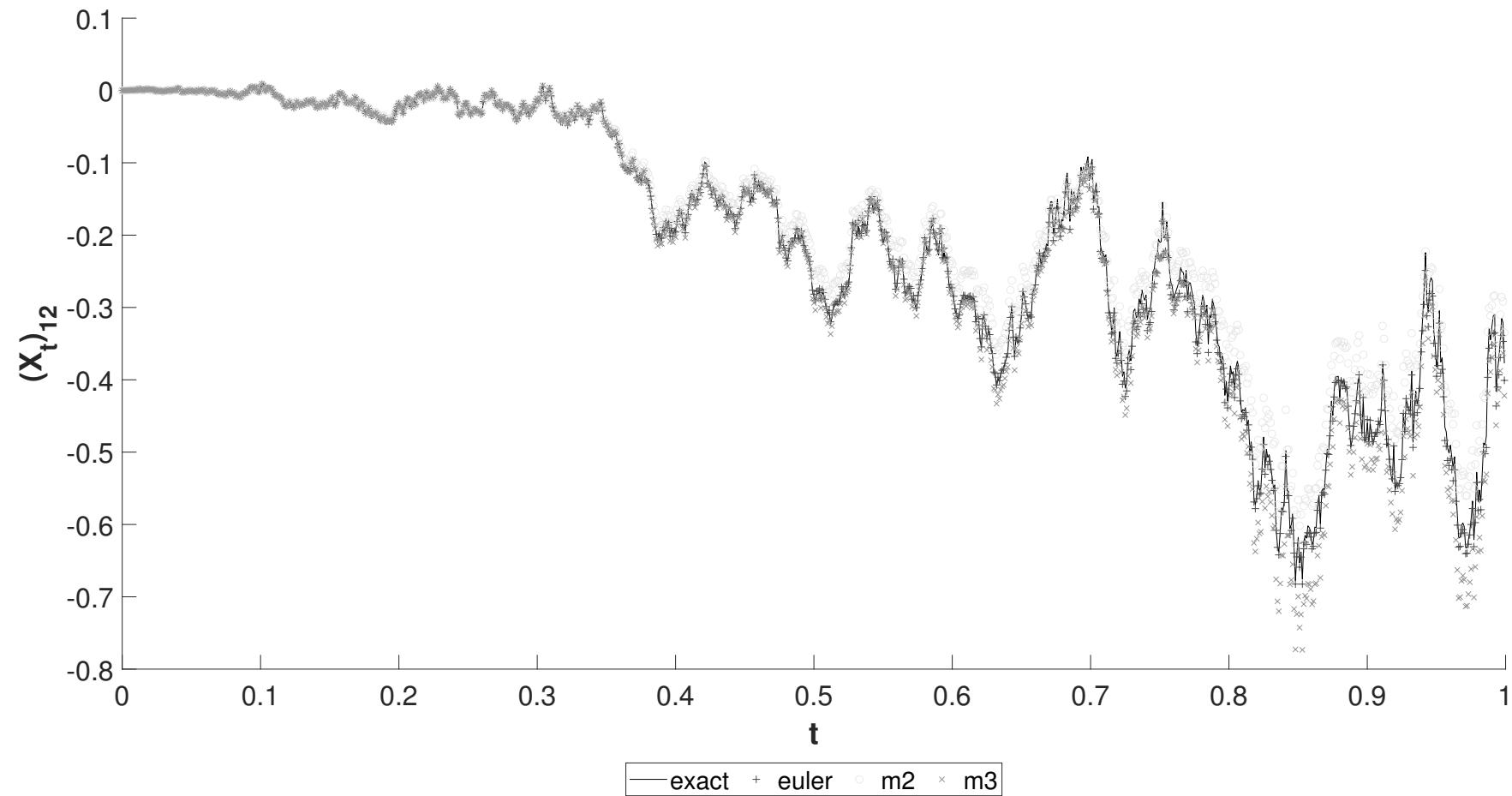


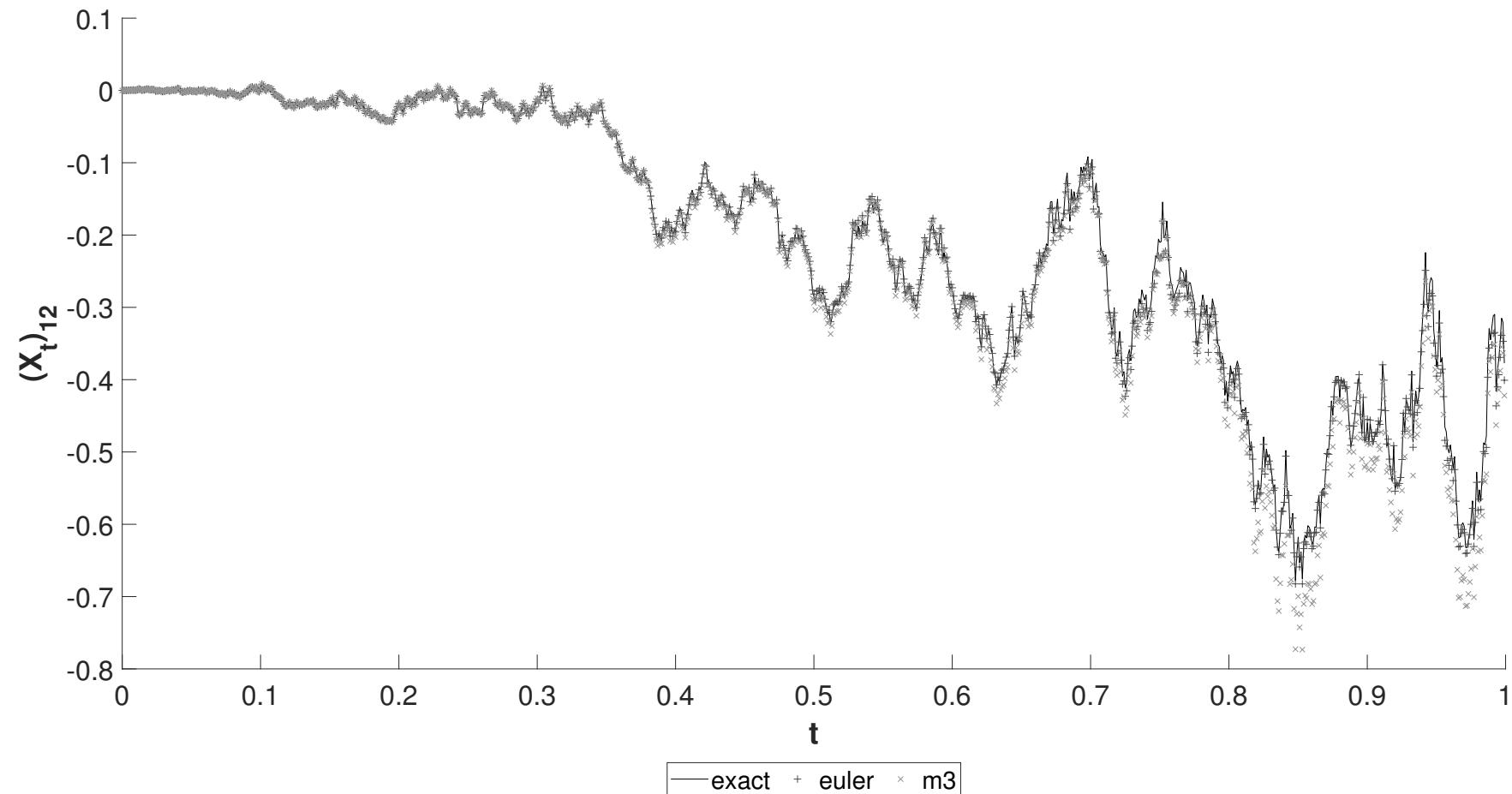


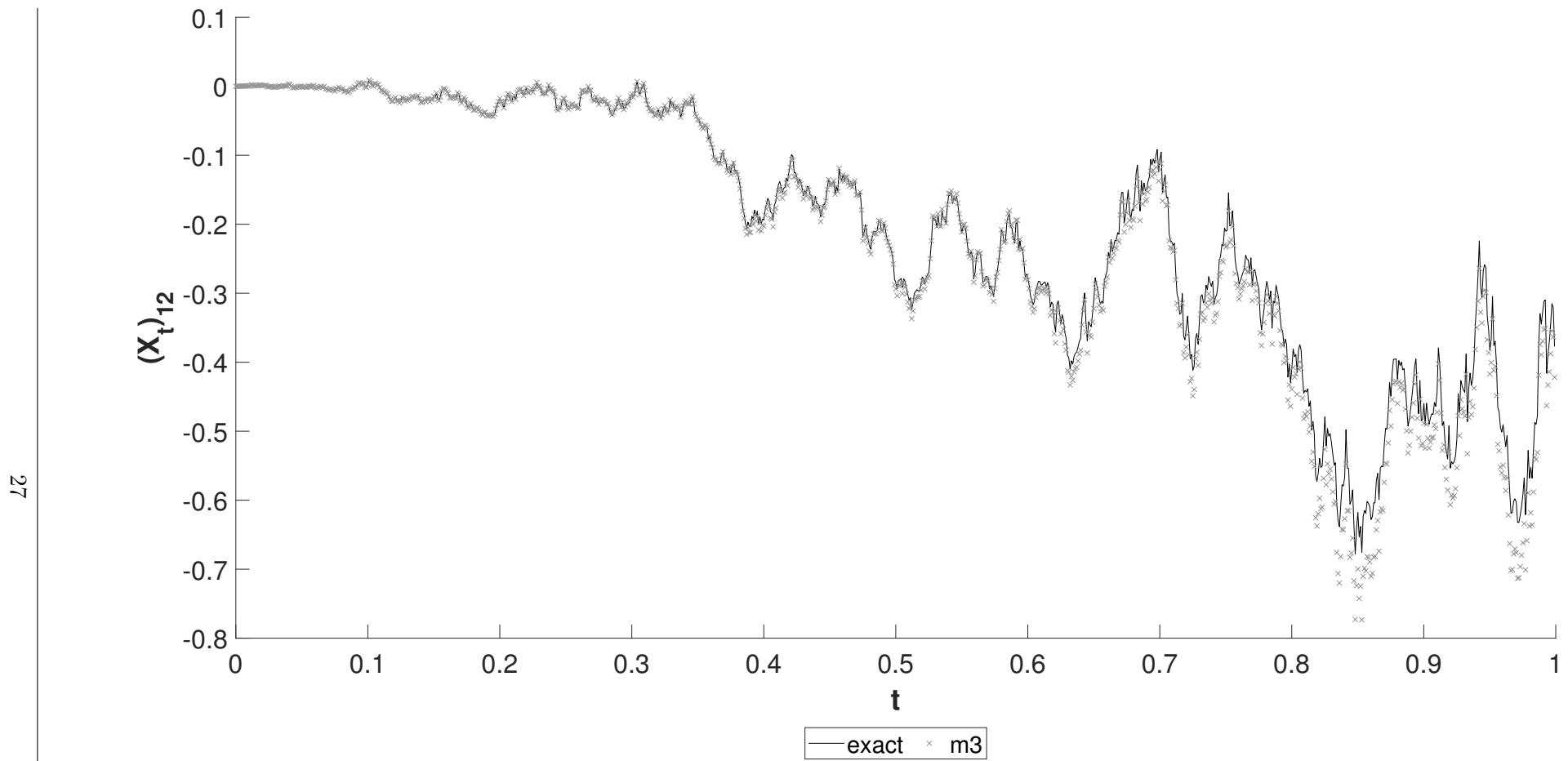


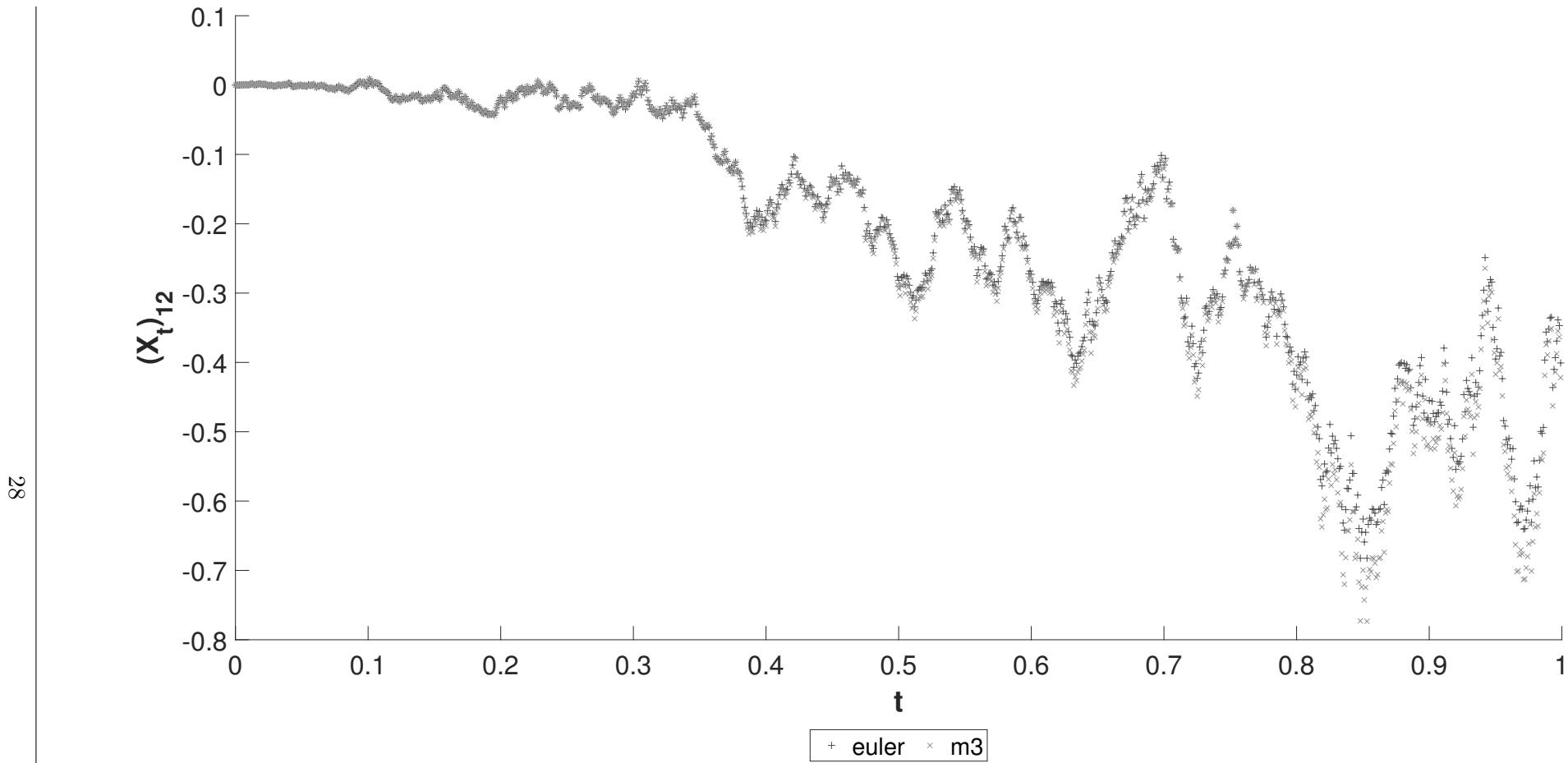


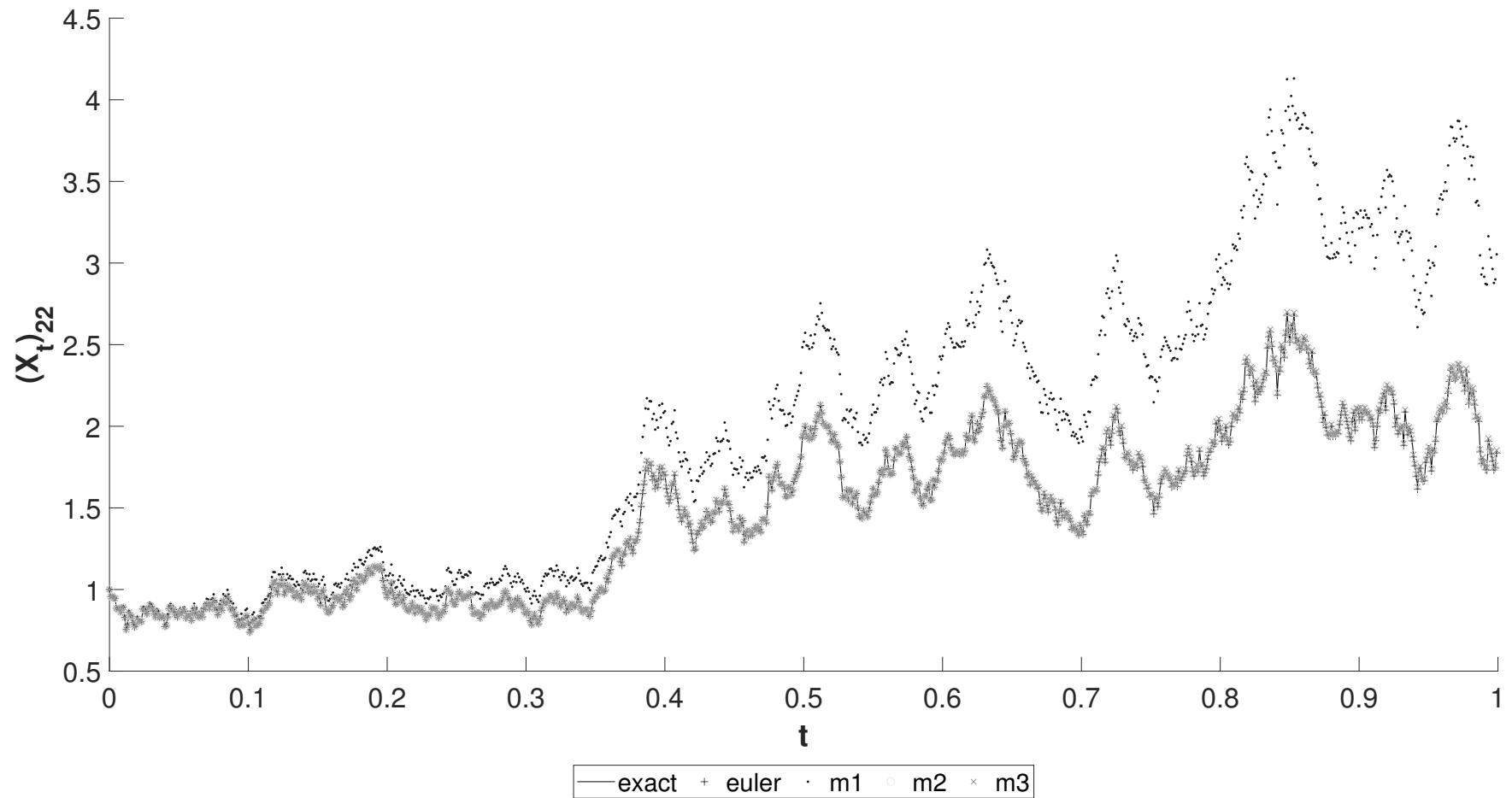


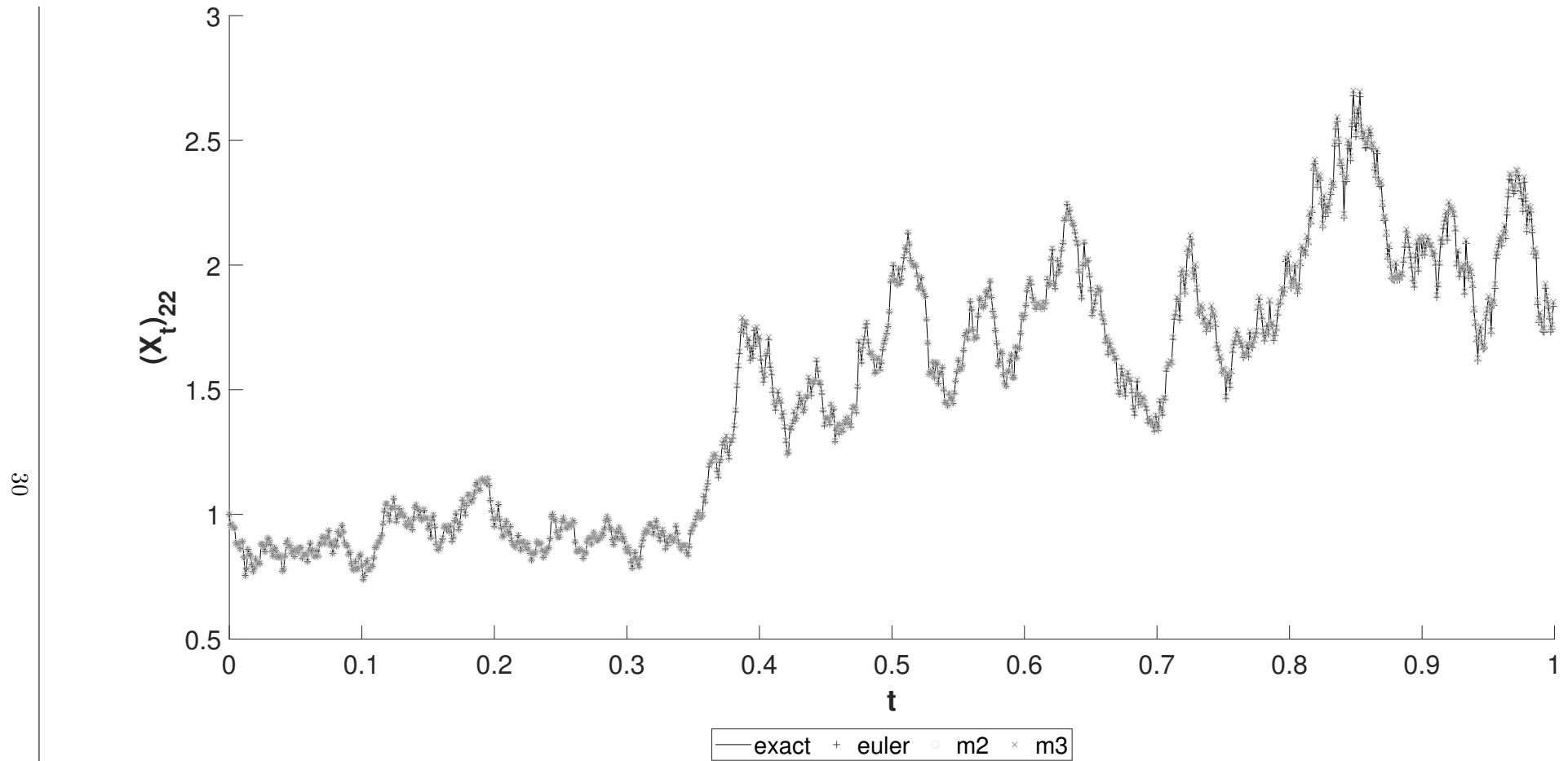


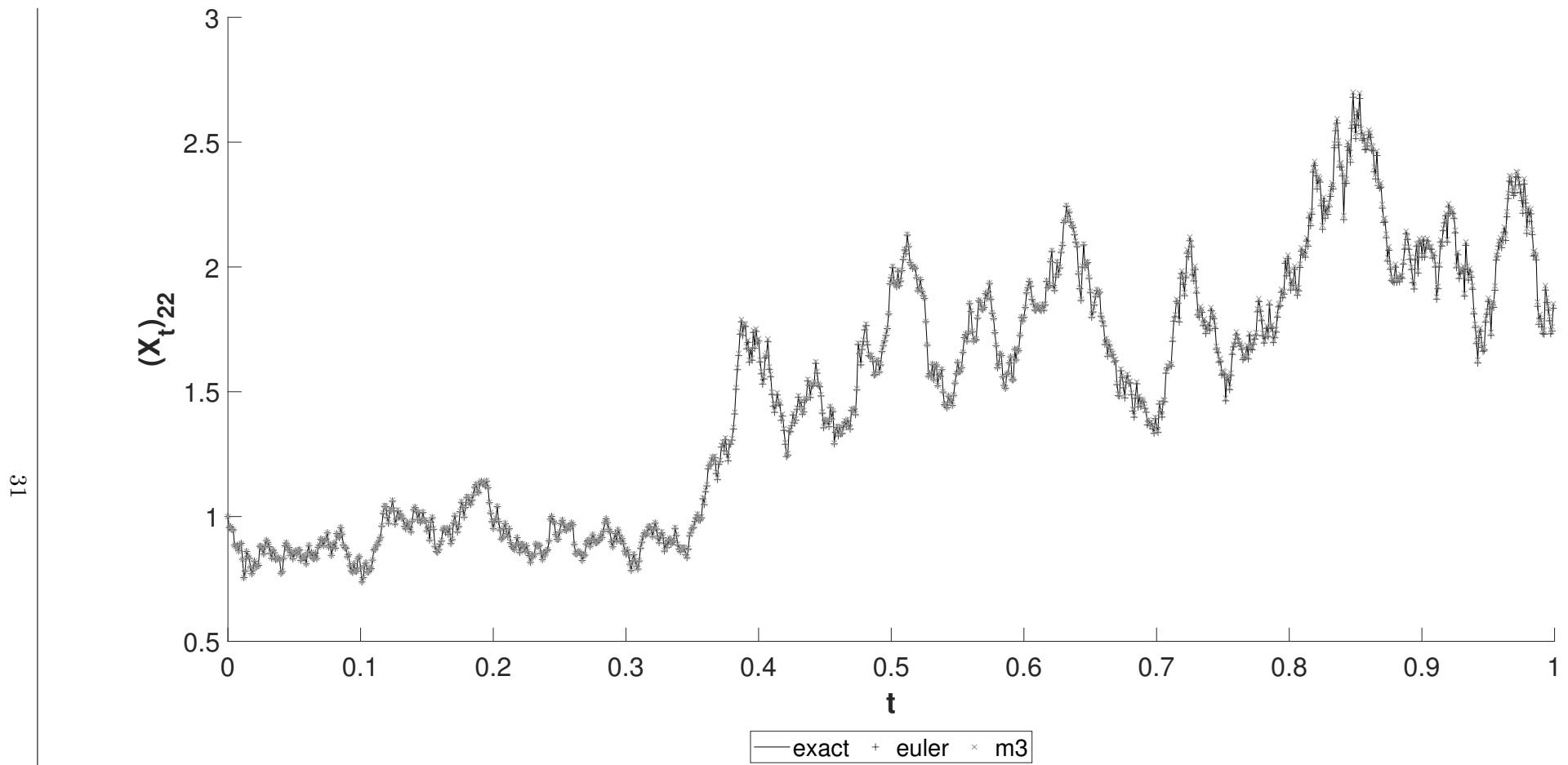


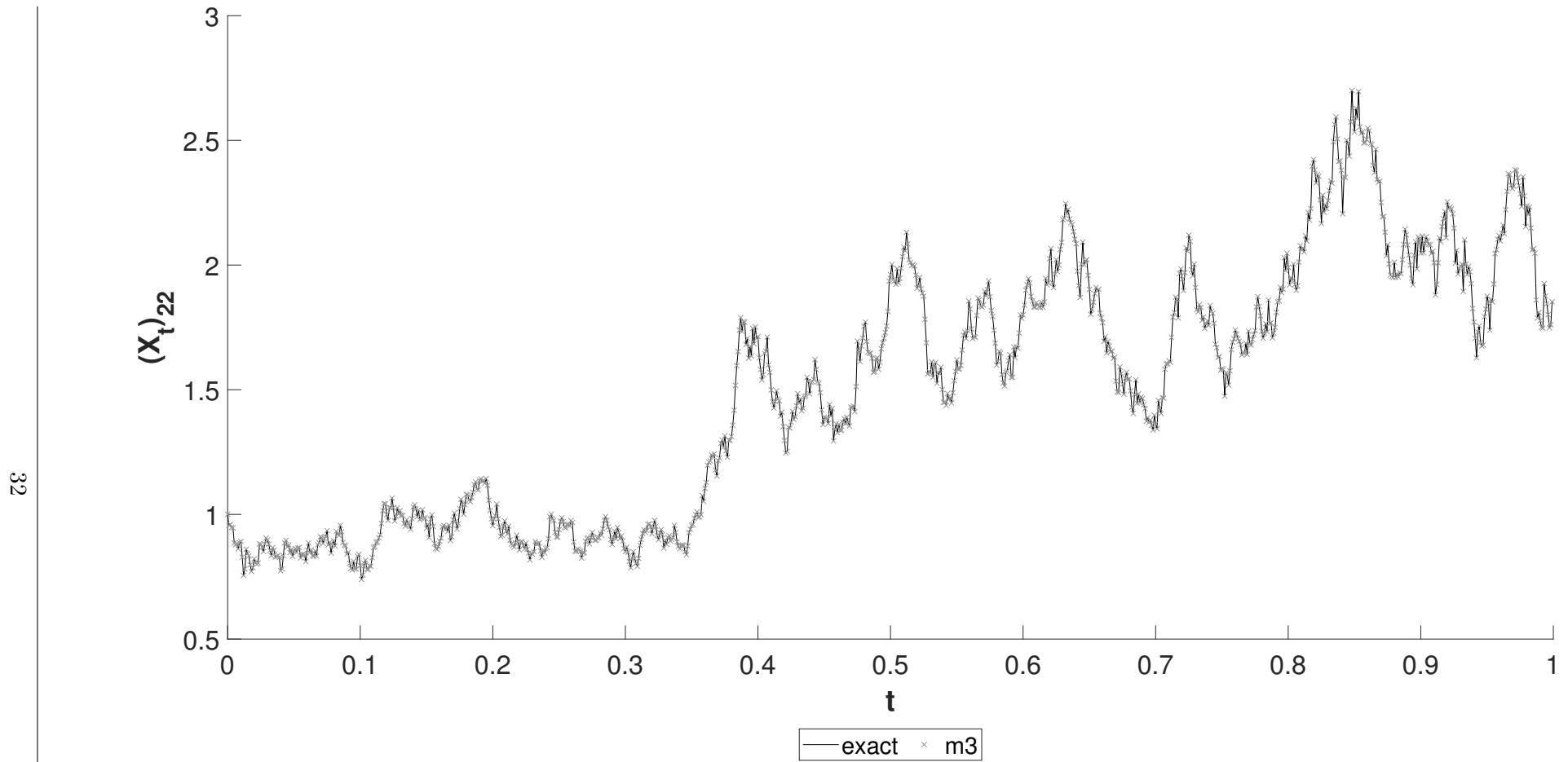


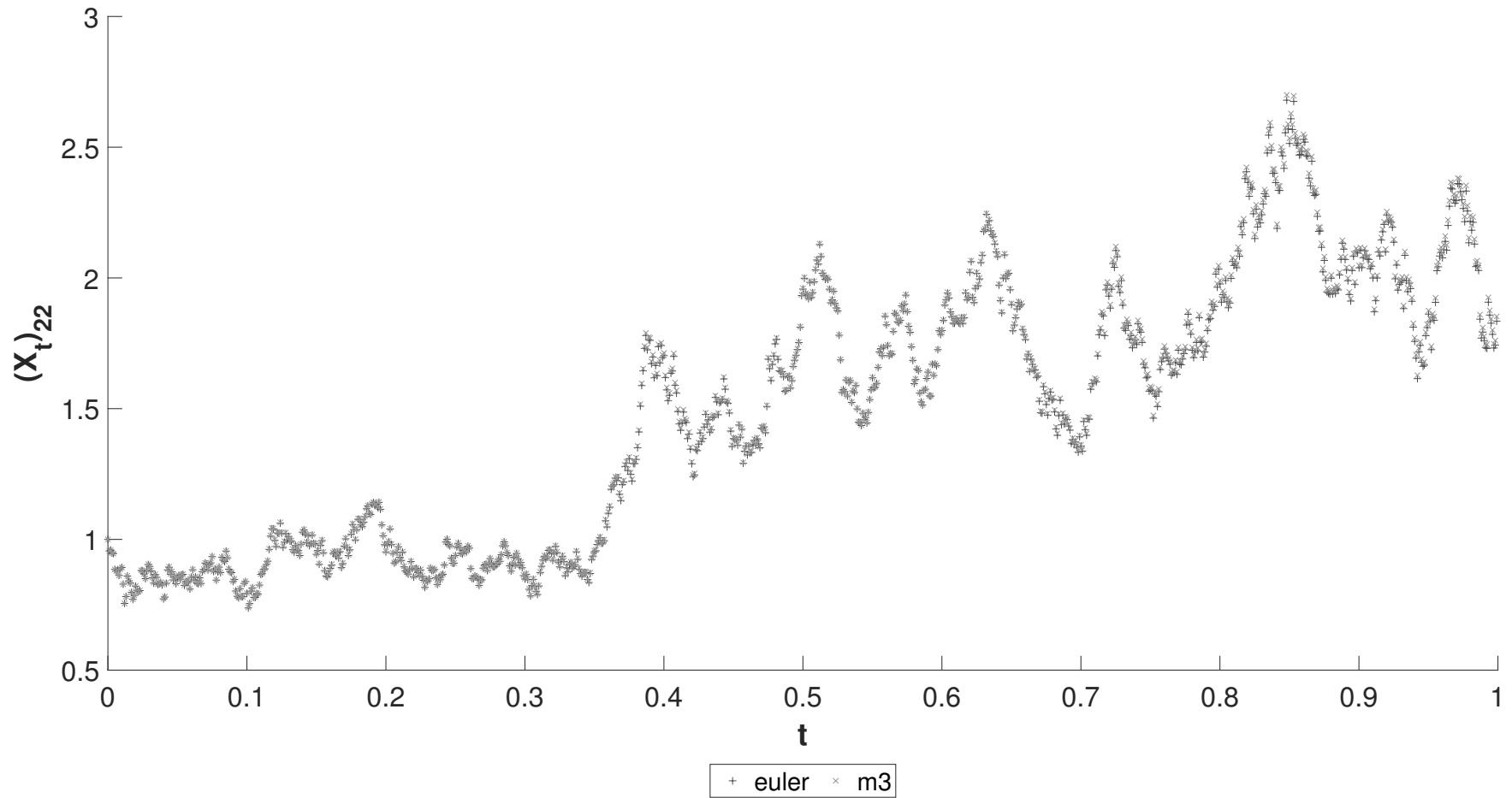












1.5 Error Plots

