1 Magnus expansion for A, B constant and deterministic

We will concern ourselves with the following SDE:

$$dX_t = BX_t dt + AX_t dW_t$$

with

$$A = \begin{bmatrix} 0.335302 & -0.645492 \\ -0.264419 & 0.634641 \end{bmatrix}$$

and

$$B = \begin{bmatrix} -0.0572262 & 0.0493763 \\ -0.665366 & 0.742744 \end{bmatrix}$$

The spectral norm of A is 1. and the spectral norm of B is 1.

1.1 Parameters

Parameter	value
t_0	0
T	1
`N_fine`	10001
N	101
`M_fine`	10000
M	10000
d	2

1.2 Computational Times

Method	Log	Matrix Exp	Total
euler	0	0	18.5065
m1	0.0293016	4.90805	4.93735
m2	0.0499197	4.87559	4.92551
m3	0.0849477	4.82618	4.91113

1.3 Errors

(i) Errors for $X(1, 1, :, :)$:

(a) Reference method: euler

Error	m1	m2	m3
(abs error) L2	0.272075	0.0279567	0.0104659
(rel error) min	0	0	0
(rel error) max	0.410348	0.0547169	0.0176534
(rel error) mean	0.140555	0.0131195	0.00492488

(ii) Errors for $X(1, 2, :, :)$:

(a) Reference method: euler

Error	m1	m2	m3
(abs error) L2	0.510175	0.0664473	0.0162412
(rel error) min	0.0263207	0.00458545	0.00457875
(rel error) max	1.17028	0.1652	0.0397644
(rel error) mean	0.624329	0.065942	0.0169359

(iii) Errors for $X(2, 1, :, :)$:

(a) Reference method: euler

Error	m1	m2	m3
(abs error) L2	0.315418	0.0241787	0.00605357
(rel error) min	0.016521	0.00150533	0.00164692
(rel error) max	0.567606	0.043233	0.00828375
(rel error) mean	0.342371	0.0205964	0.00587691

(iv) Errors for $X(2, 2, :, :)$:

(a) Reference method: euler

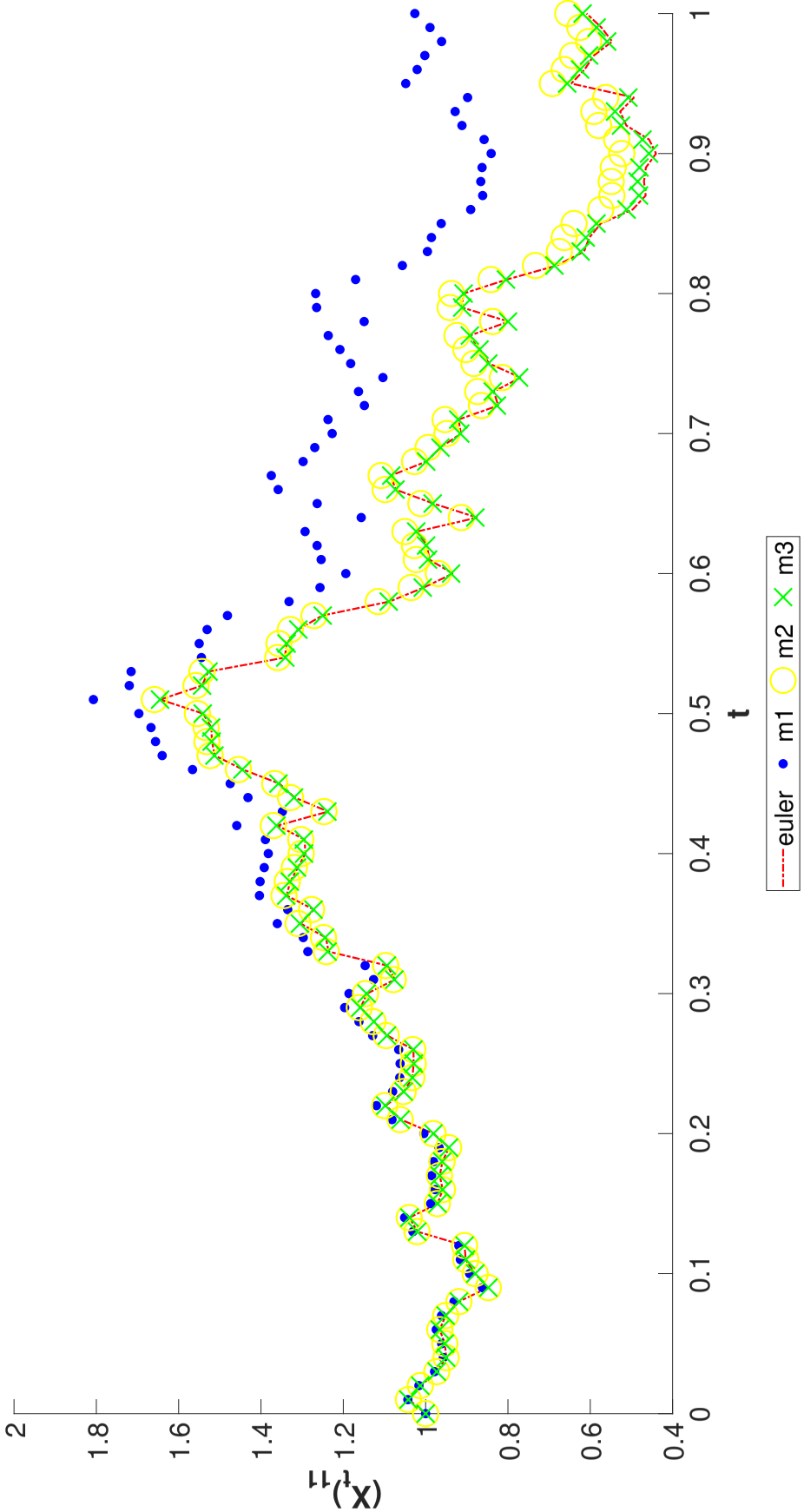
Error	m1	m2	m3
(abs error) L2	0.625094	0.0167095	0.0133713
(rel error) min	0	0	0
(rel error) max	0.422431	0.0108088	0.00794262
(rel error) mean	0.187248	0.00412377	0.0035569

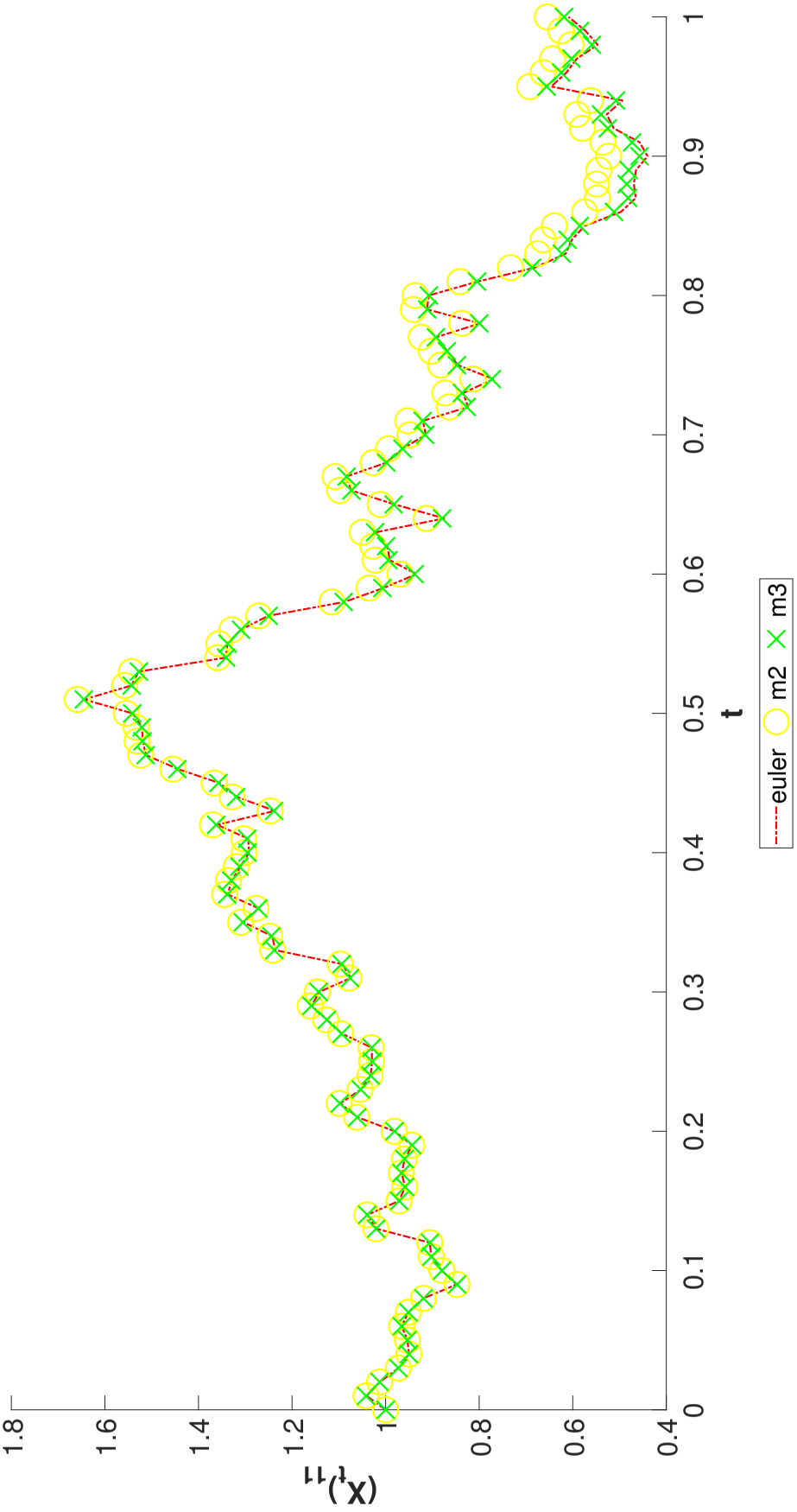
(v) Total Errors:

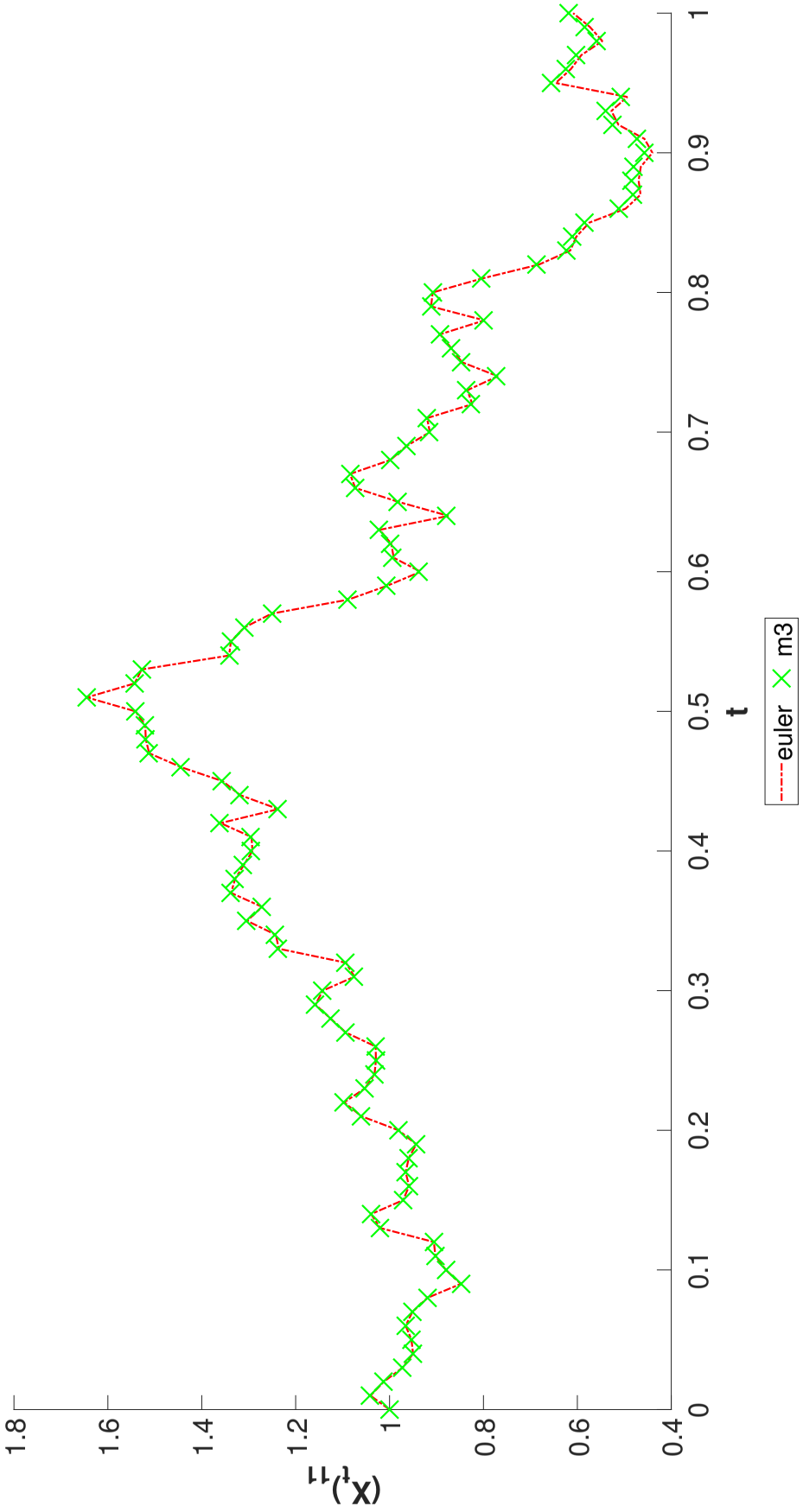
(a) Reference method: euler

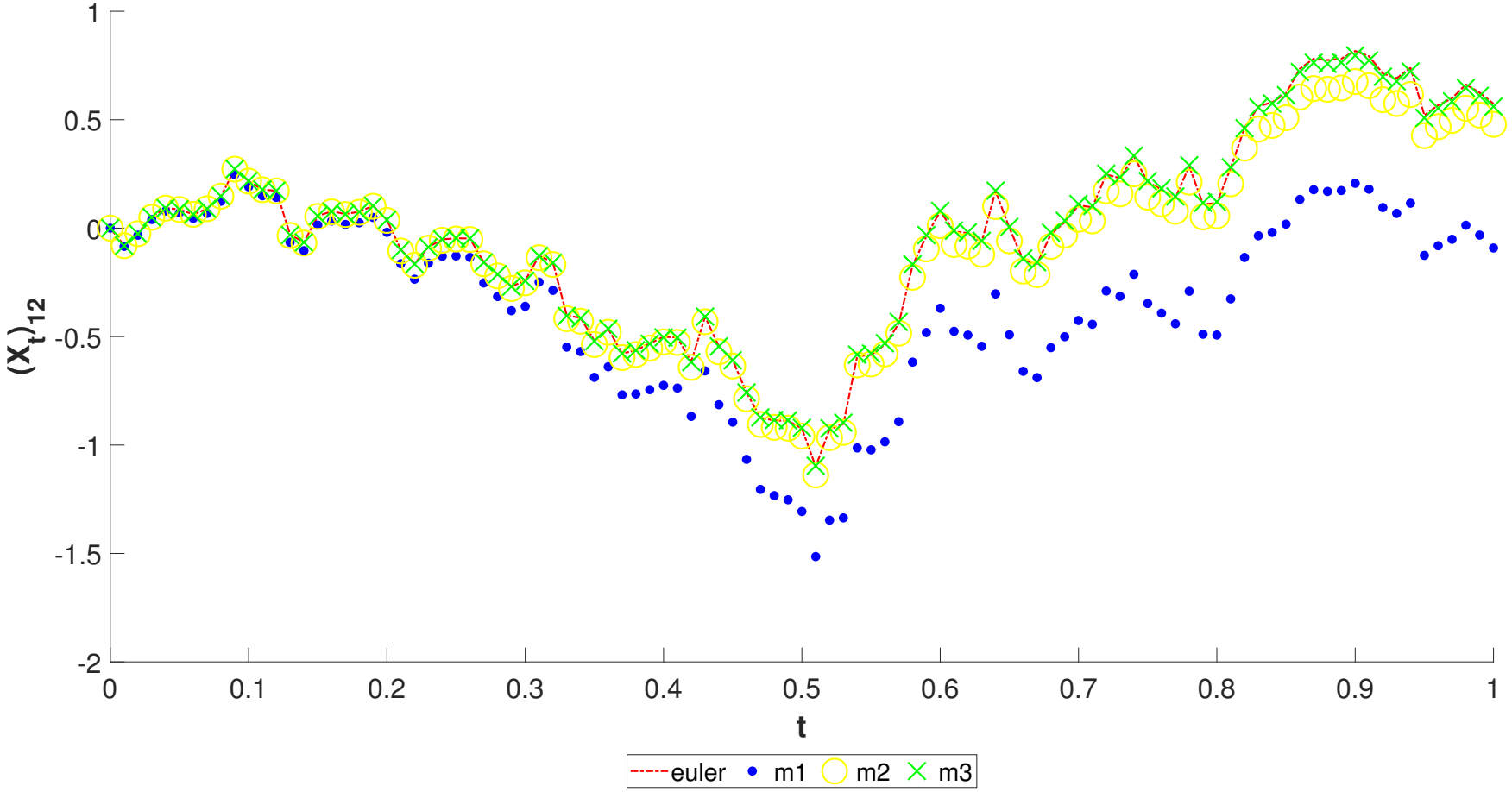
Method	$\mathbb{E}[Err_1]$
m1	20.9 %
m2	1.56 %
m3	0.543 %

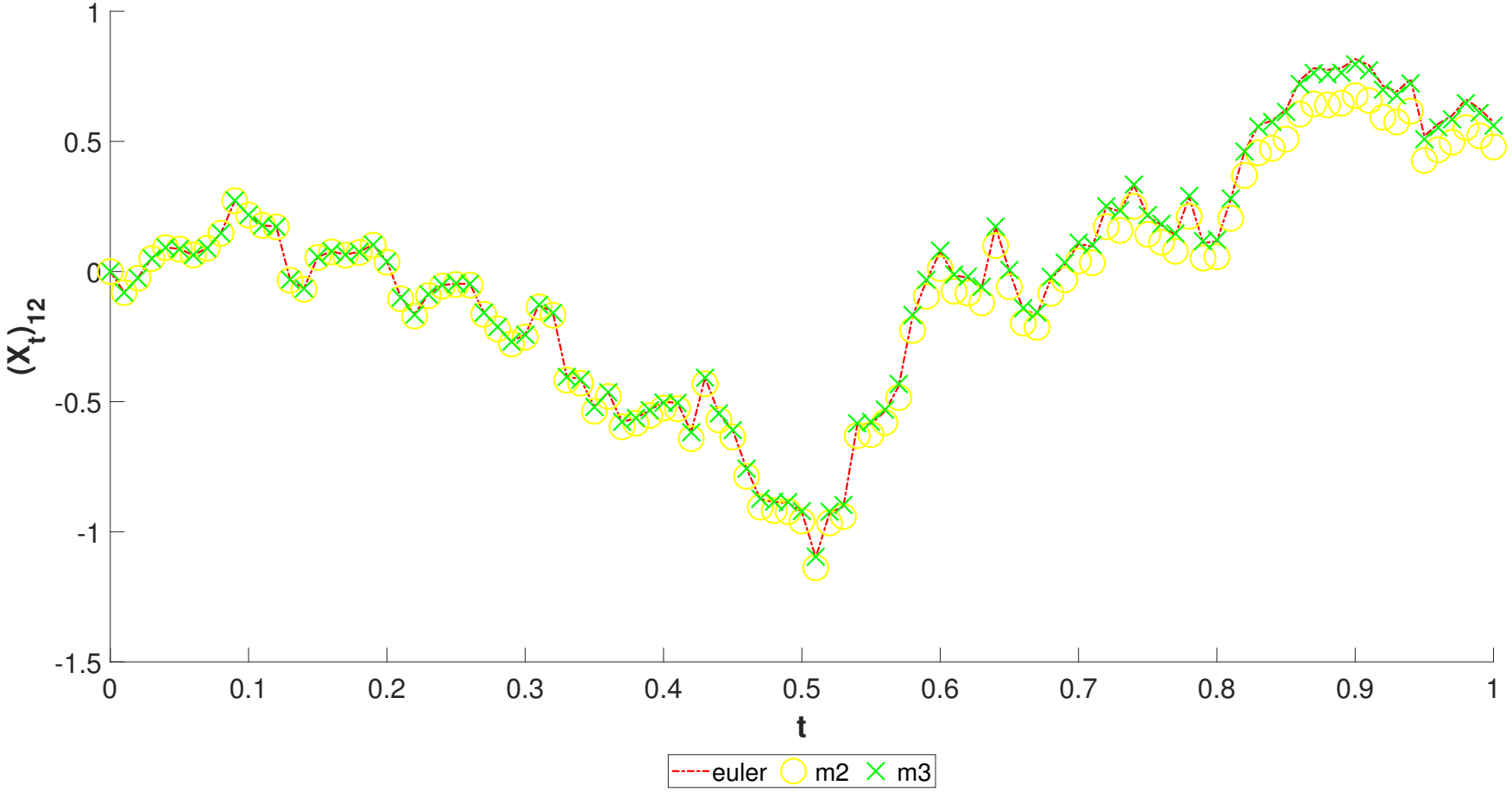
1.4 Plots

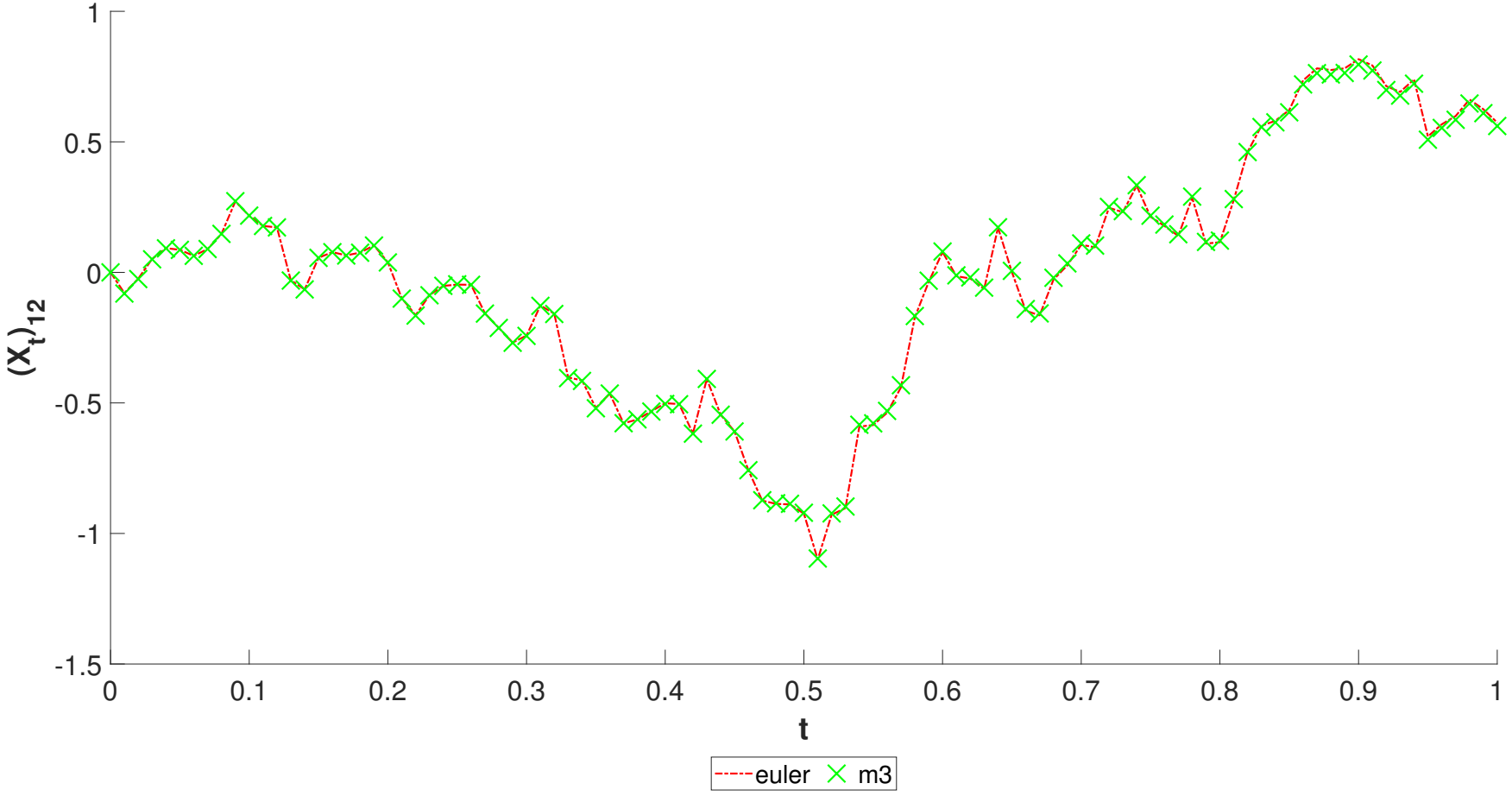

2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t
euler
m1
m2
m3

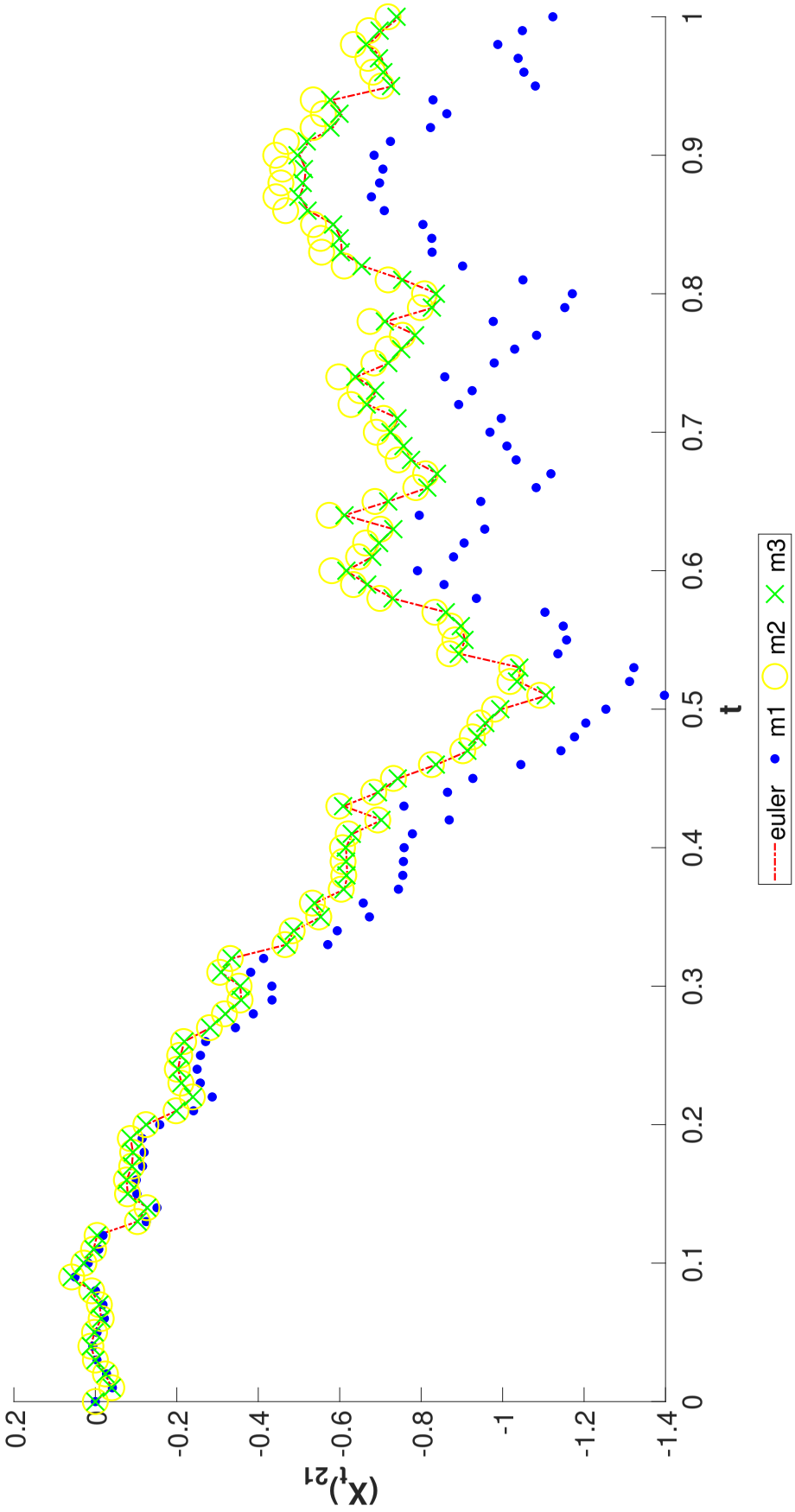

1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
$(X_t)_{11}$
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t
euler
m2
m3

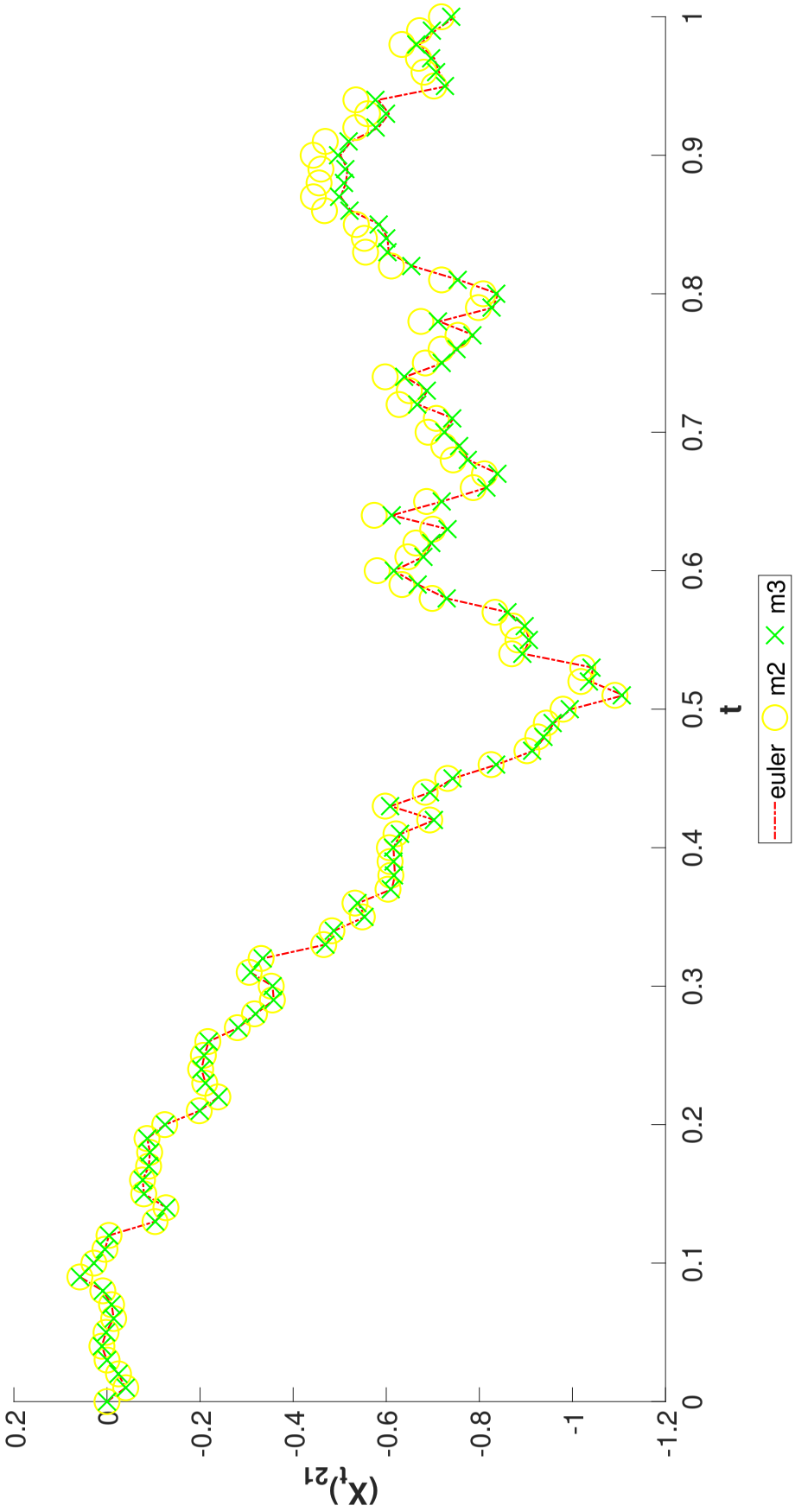

1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t
euler
m3

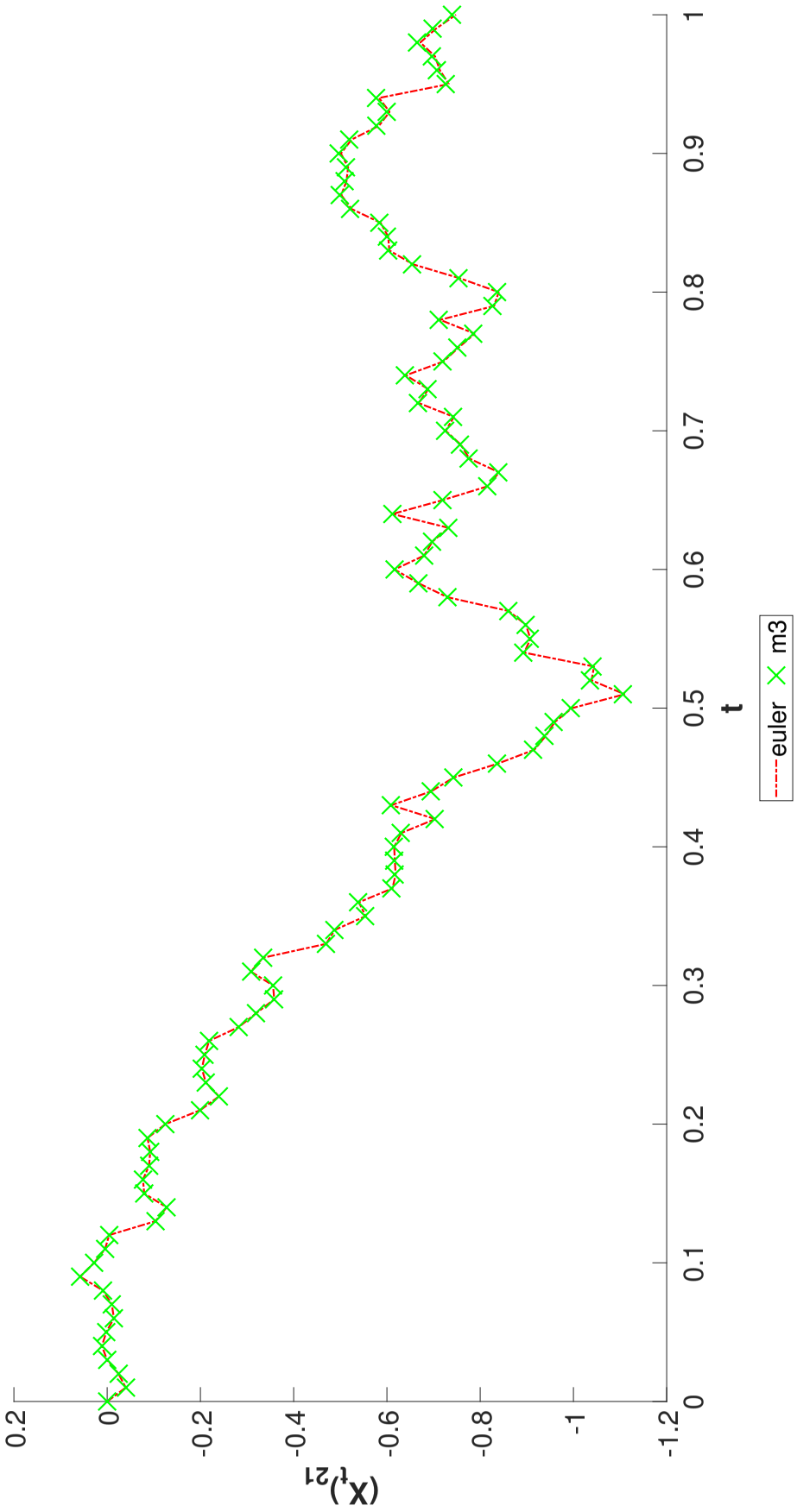

1
0.5
0
-0.5
-1
-1.5
-2
$(X_t)_{12}$
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t
euler
m1
m2
m3

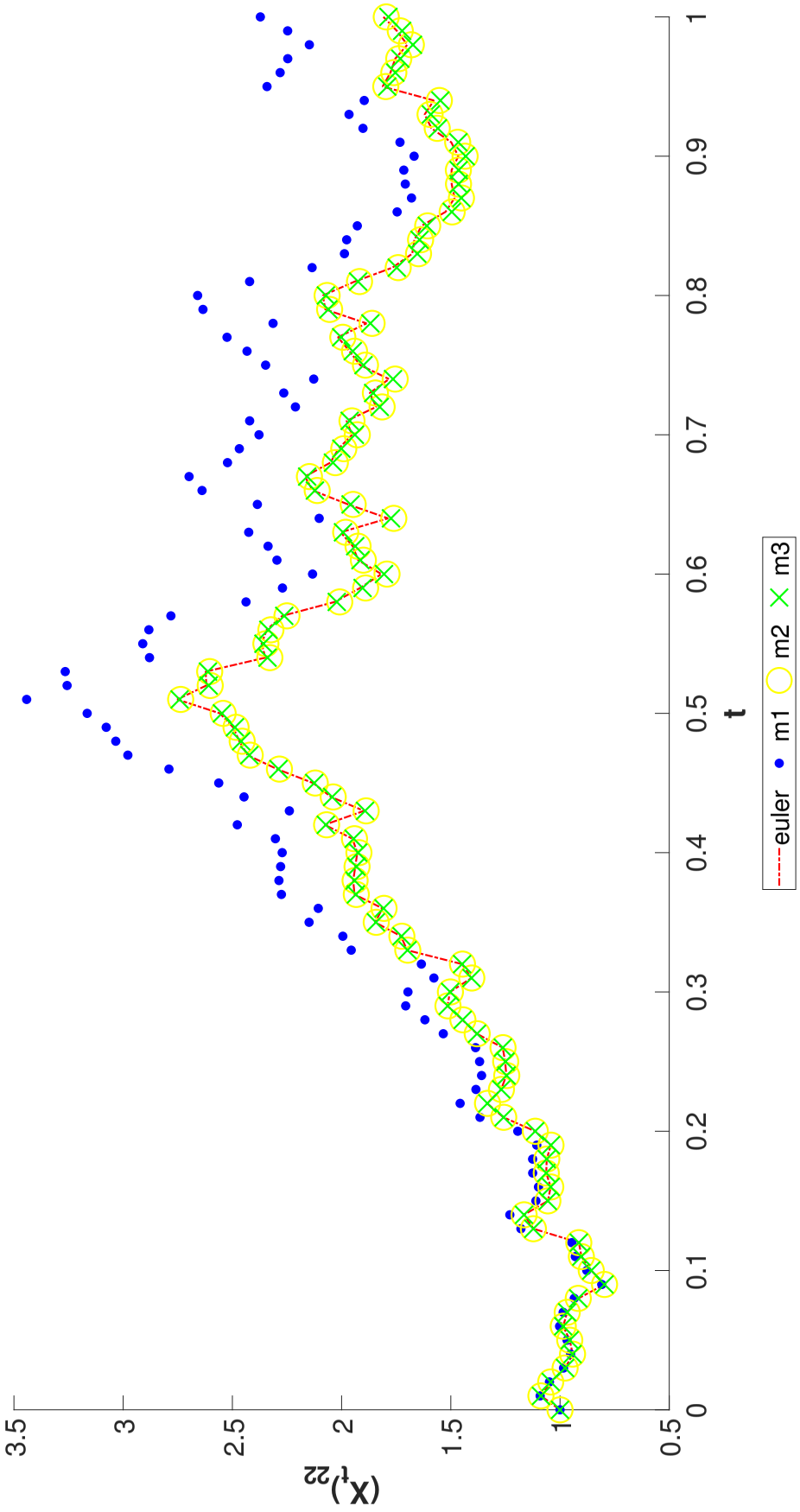

1
0.5
0
-0.5
-1
-1.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t
$(X_t)_{12}$
euler
m2
m3

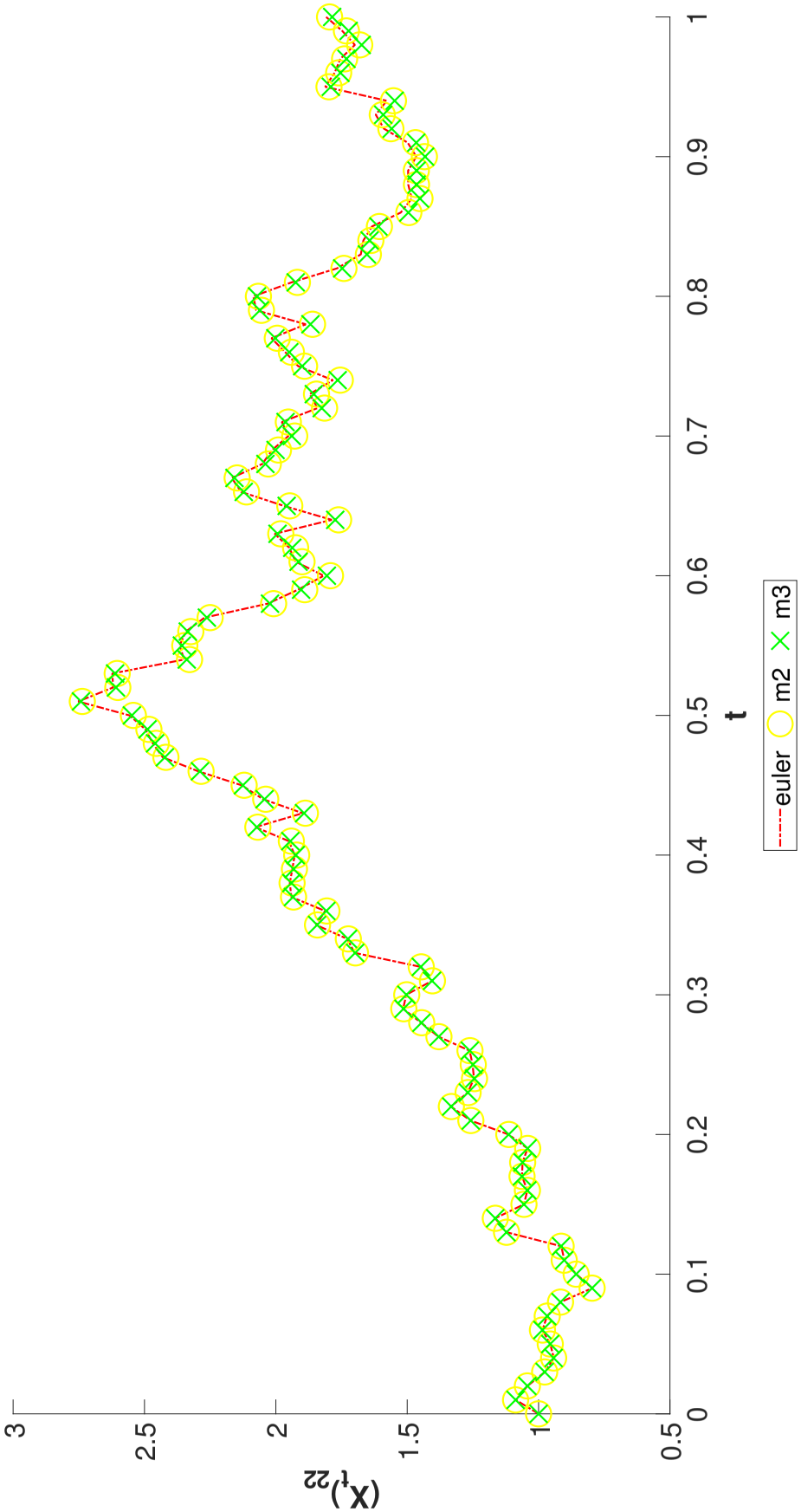

1
0.5
0
-0.5
-1
-1.5
$(X_t)_{12}$
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t
euler
m3

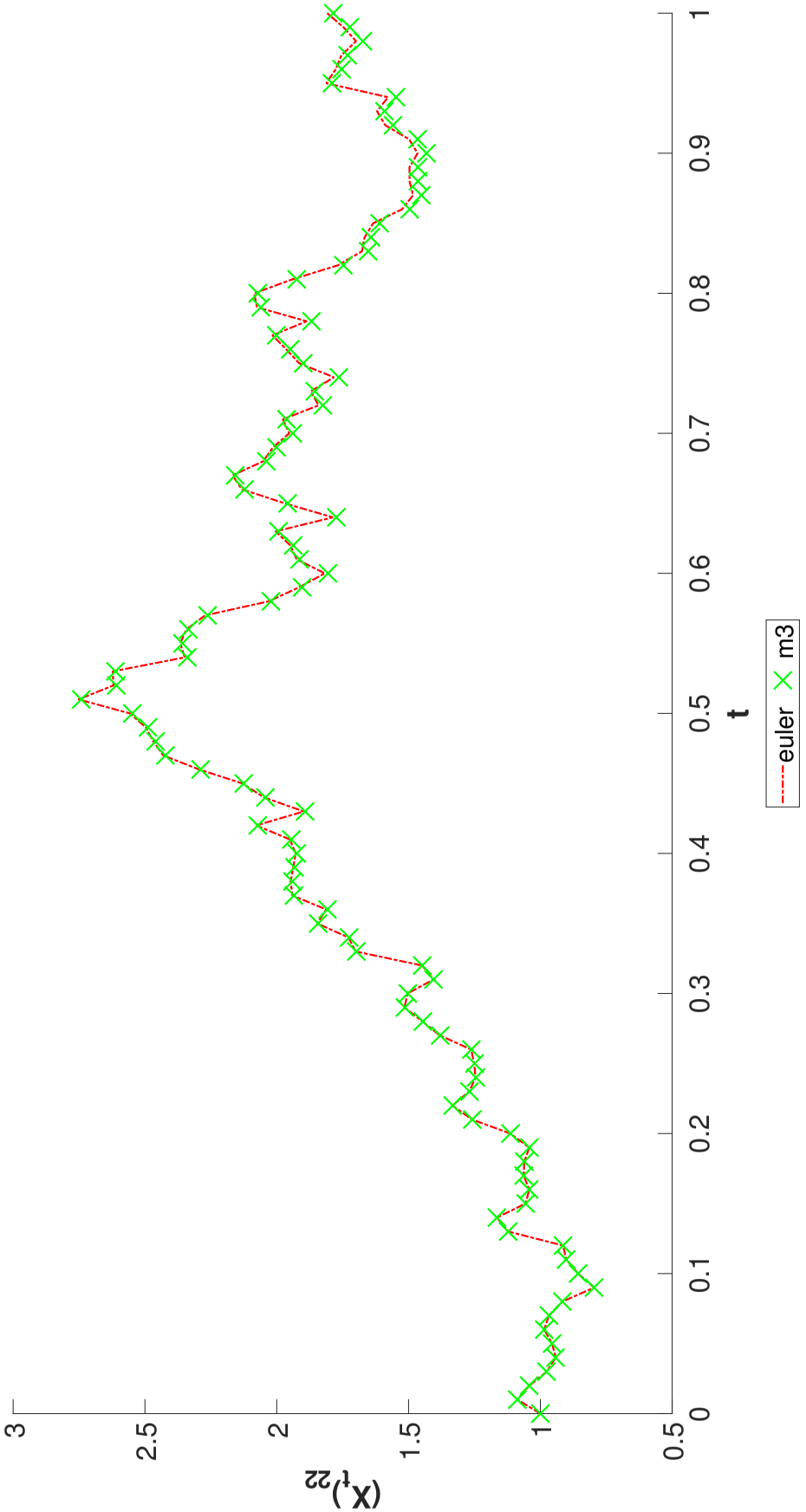

0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
(X_t)_21
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t
euler
m1
m2
m3


0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
(X_t)_21
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t
euler
m2
m3

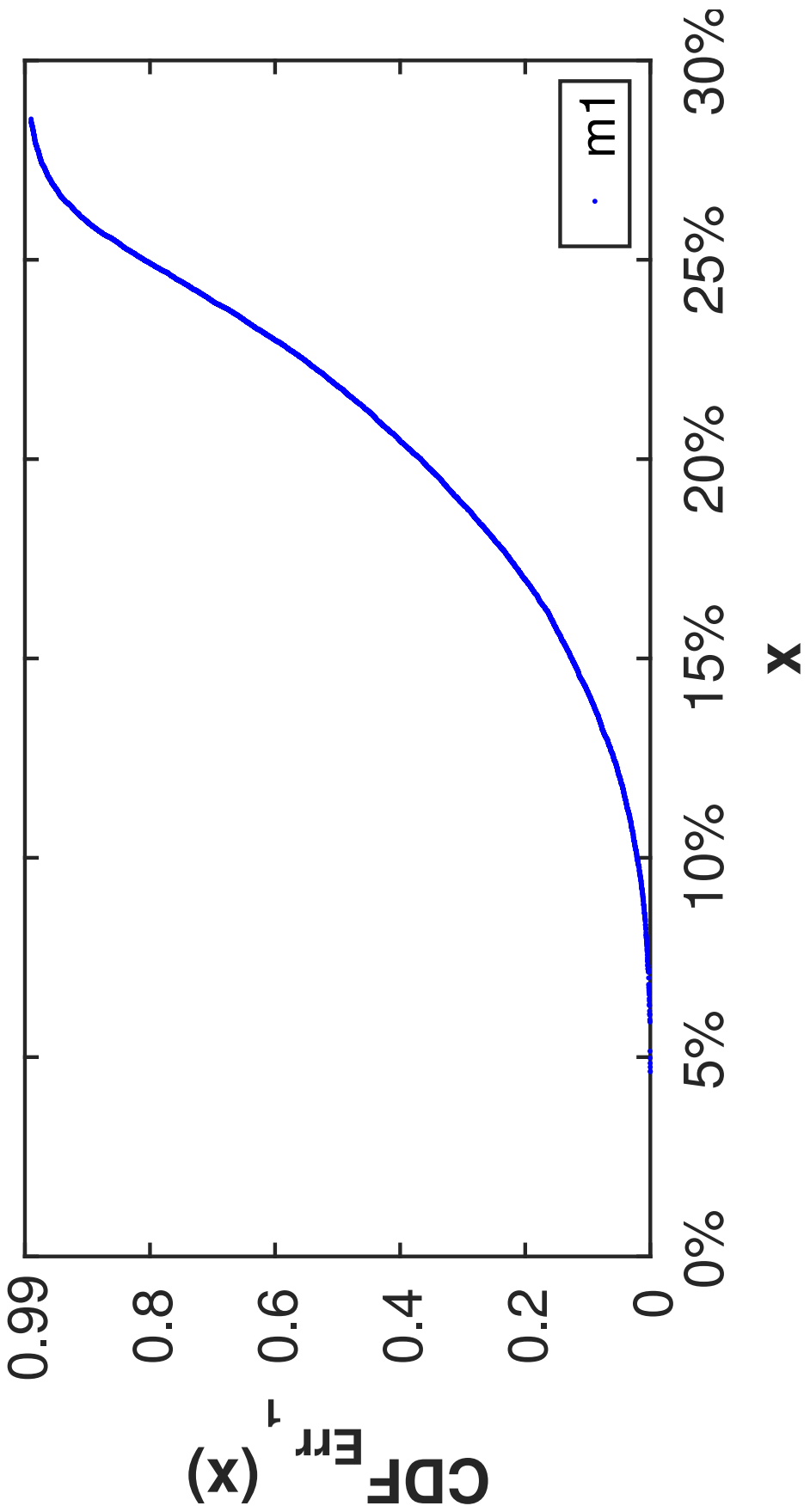

0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
(X_t)_21
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t
euler
m3

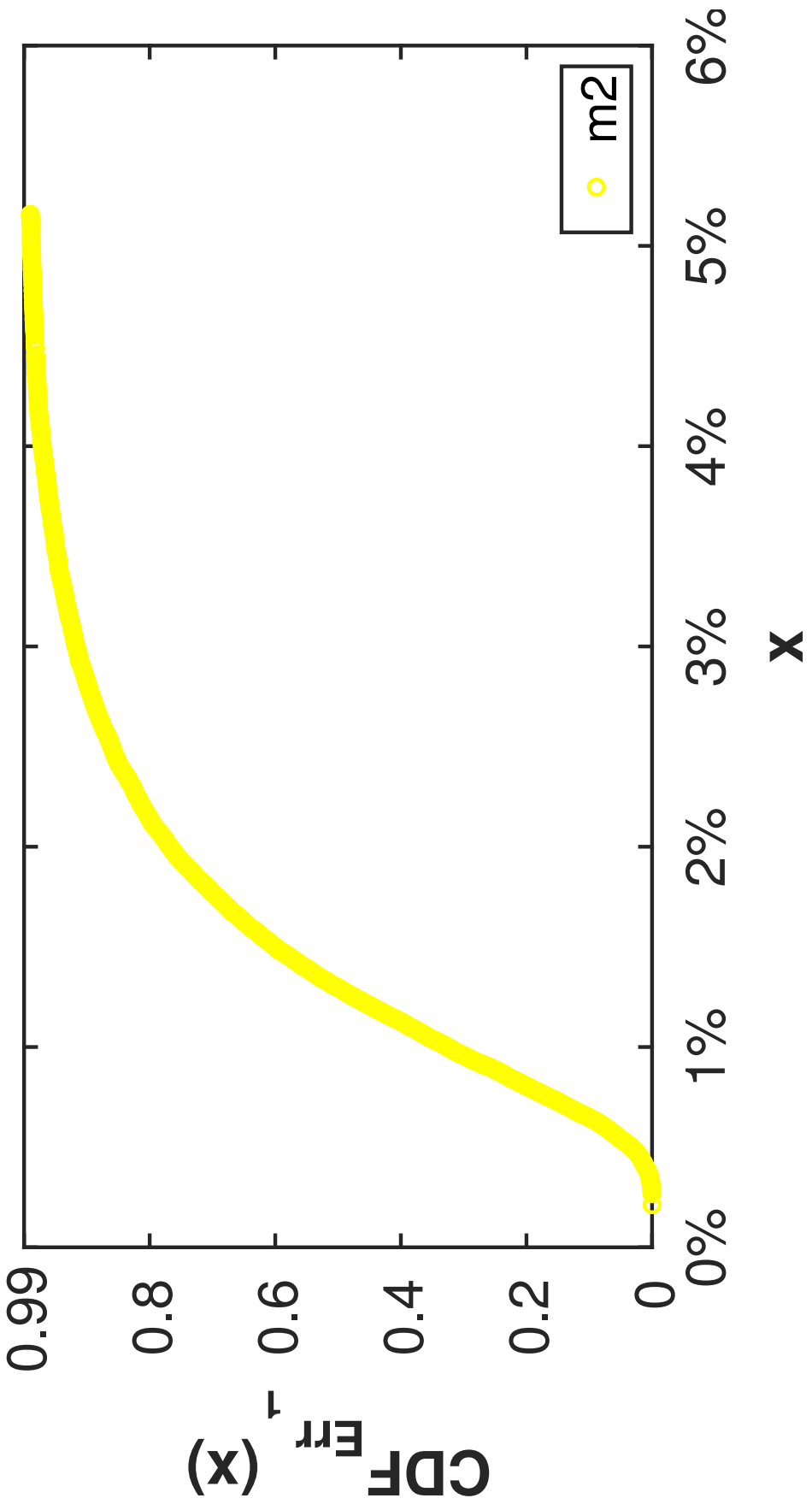

(X_t)_22
t
euler
m1
m2
m3
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0.5
1
1.5
2
2.5
3
3.5

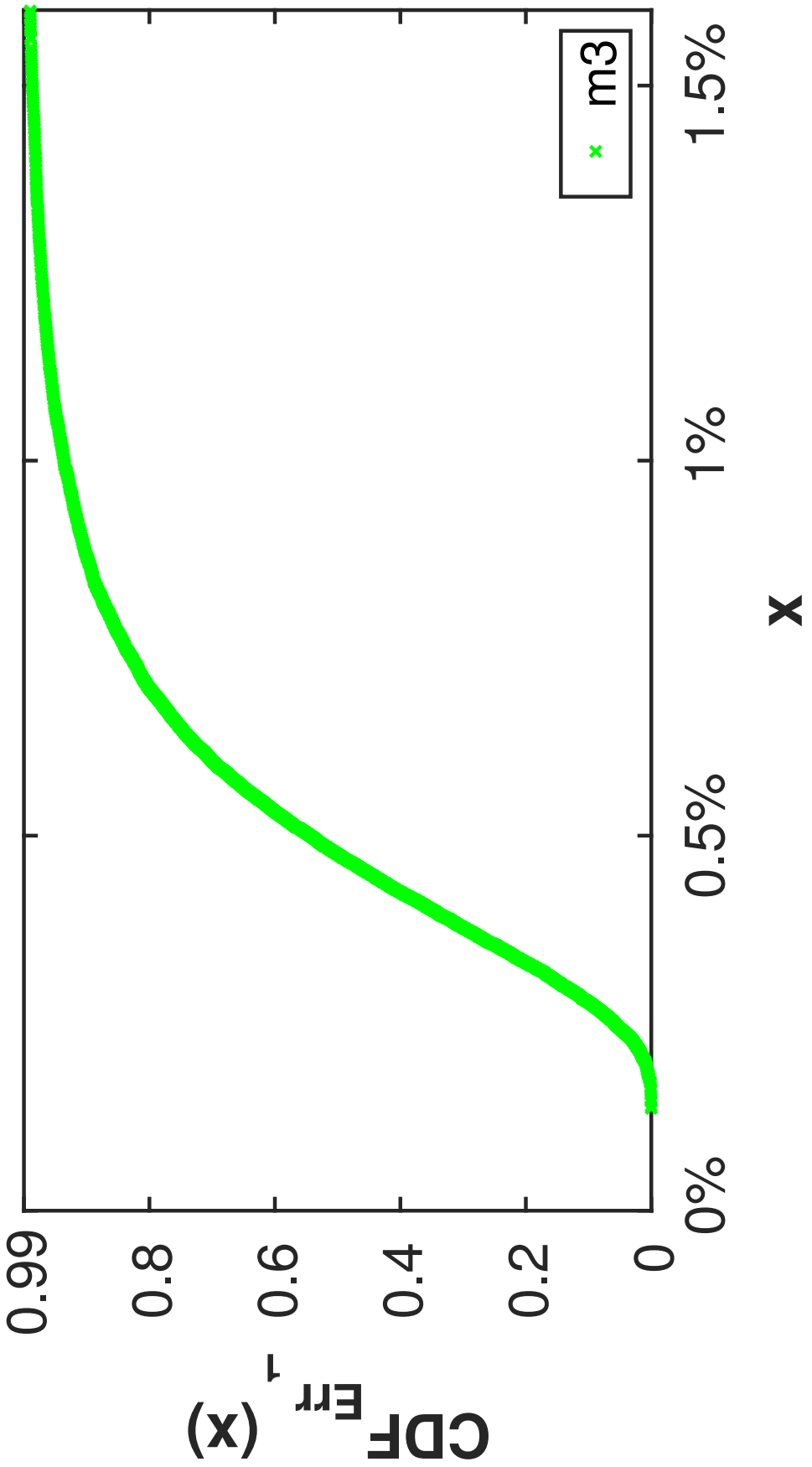

(X_t)_22
t
euler
m2
m3
0.5
1
1.5
2
2.5
3
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

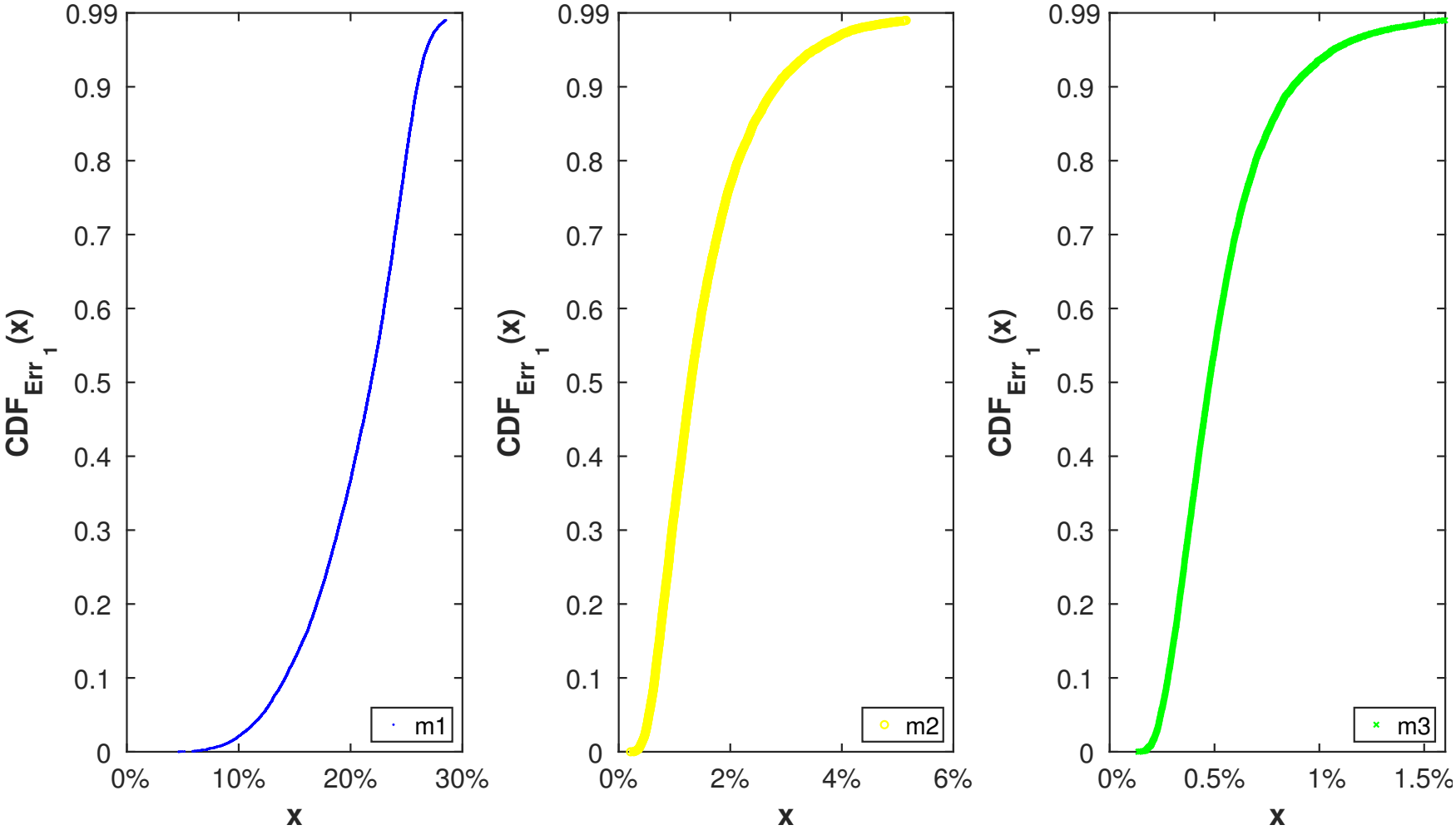

(X_t)_22
t
euler
m3
0.5
1
1.5
2
2.5
3
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

1.5 Error Plots

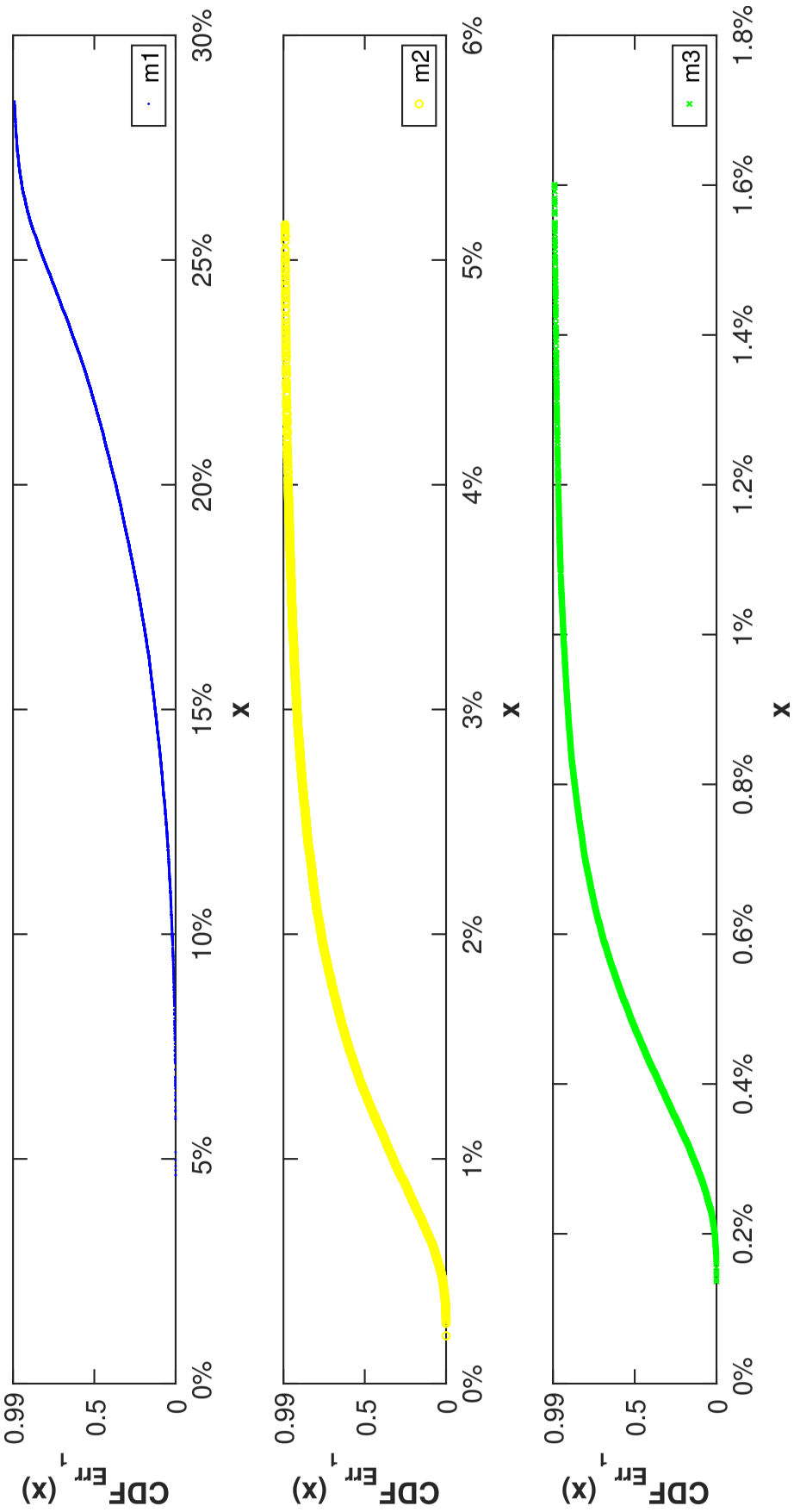

m1
0.99
0.8
0.6
0.4
0.2
0
0%
5%
10%
15%
20%
25%
30%
x
CDF_err_1(x)


m2
0.99
0.8
0.6
0.4
0.2
0
0%
1%
2%
3%
4%
5%
6%
x
CDF_Err_1(x)


m3
0.99
0.8
0.6
0.4
0.2
0
0%
0.5%
1%
1.5%
x
CDF_Err_1(x)



CDF_Err_1(x)
0.99
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0%
10%
20%
30%
x
m1
CDF_Err_1(x)
0%
2%
4%
6%
x
m2
CDF_Err_1(x)
0%
0.5%
1%
1.5%
x
m3


m1
m2
m3
x
$CDF_{Err_1}(x)$
0.99
0.5
0
0%
5%
10%
15%
20%
25%
30%
1%
2%
3%
4%
5%
6%
0.2%
0.4%
0.6%
0.8%
1%
1.2%
1.4%
1.6%
1.8%