1 SPDE model

We will concern ourselves with the following SPDE:

$$du(t,x) = \frac{a}{2}(\partial_{xx}u)(t,x)dt + \sigma(\partial_x u)(t,x)dW_t,$$

with $a = 0.2$, $\sigma = 0.15$ We will cut off the spatial space via $[x_a, x_b] \subset \mathbb{R}$.

1.1 Parameters

Parameter	value
t_0	0
T	0.1
N	1000
M	50
d	201
x_a	-2
x_b	2

1.2 Computational Times

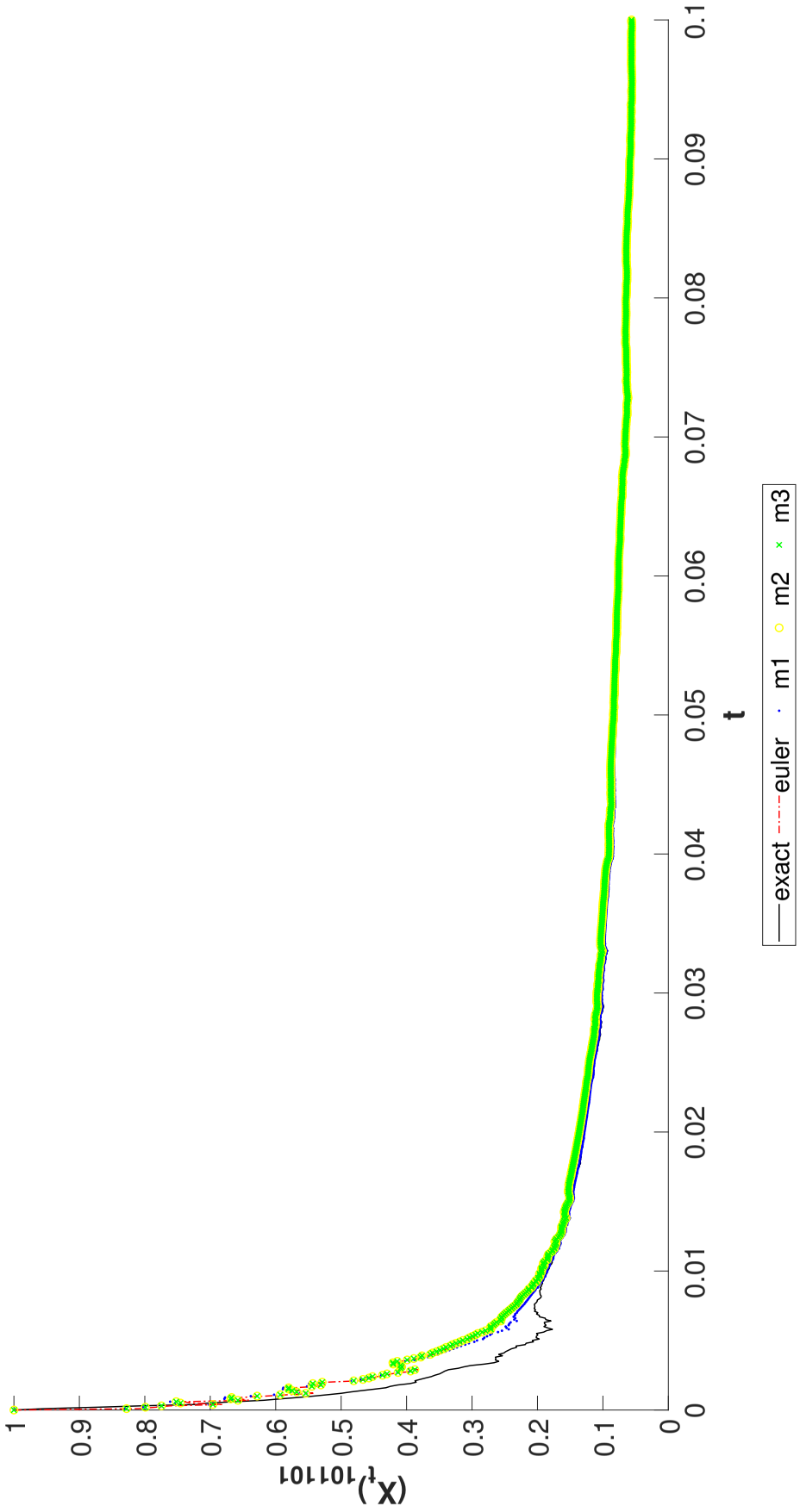
Method	Log	Matrix Exp	Total
exact	0	0	61.7775
euler	0	0	85.4141
m1	7.74737	144.738	152.486
m2	21.9373	141.264	163.201
m3	26.8905	144.387	171.278

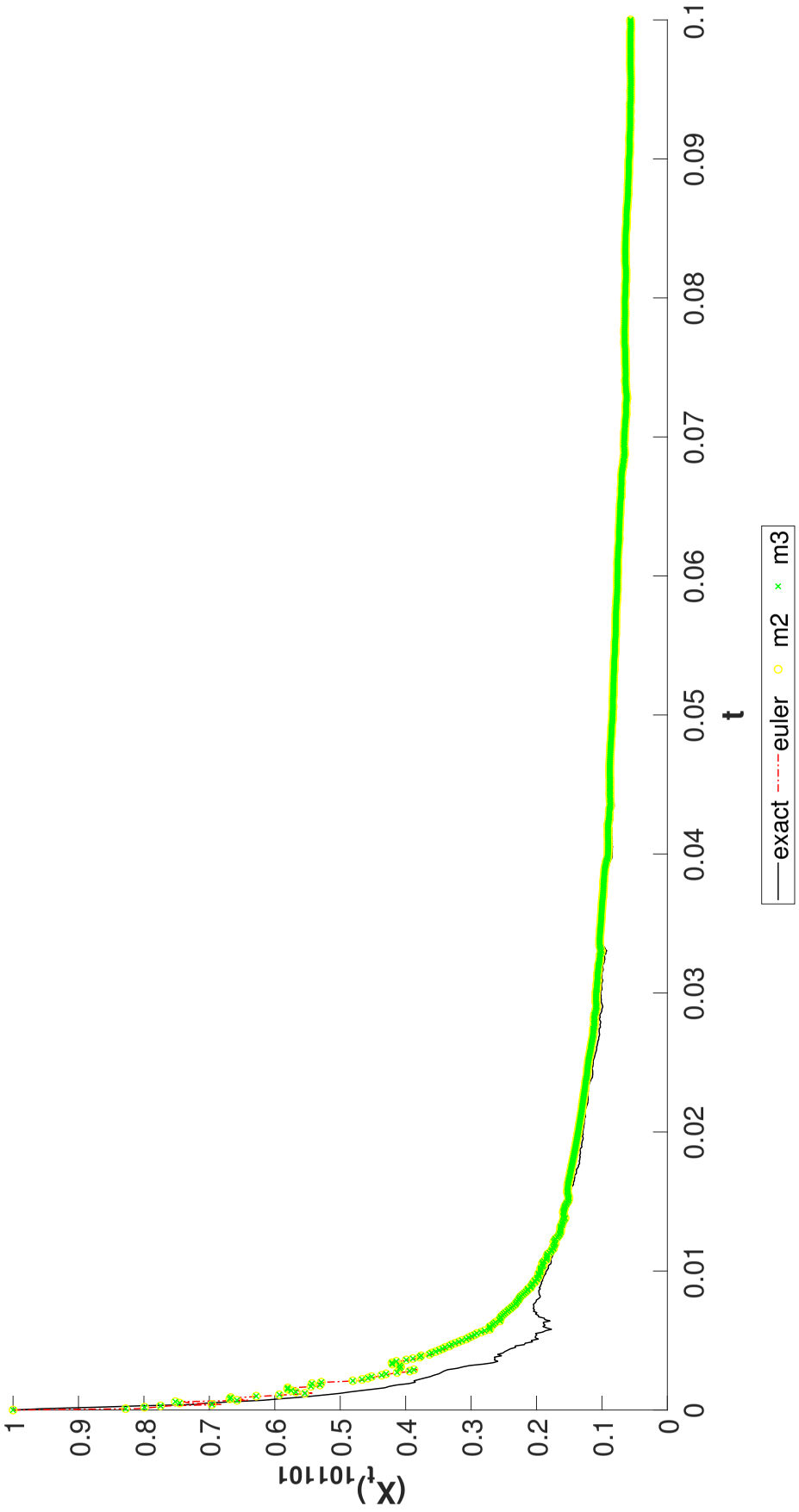
1.3 Errors

(i) Total Errors:
   (a) Reference method: exact

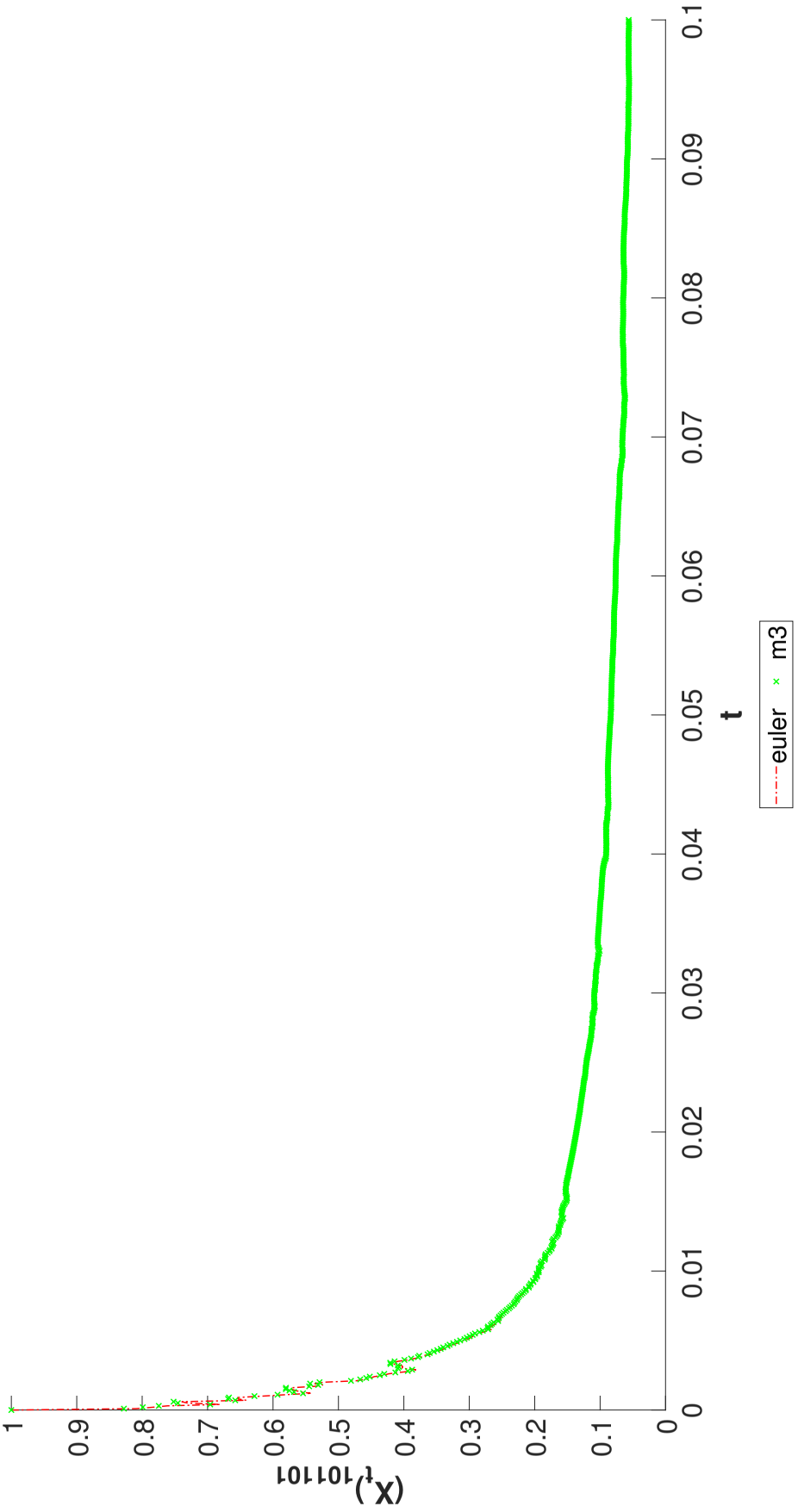
Method	$\mathbb{E}[Err_{0.1}]$
euler	2.52 %
m1	4.61 %
m2	2.53 %
m3	2.53 %

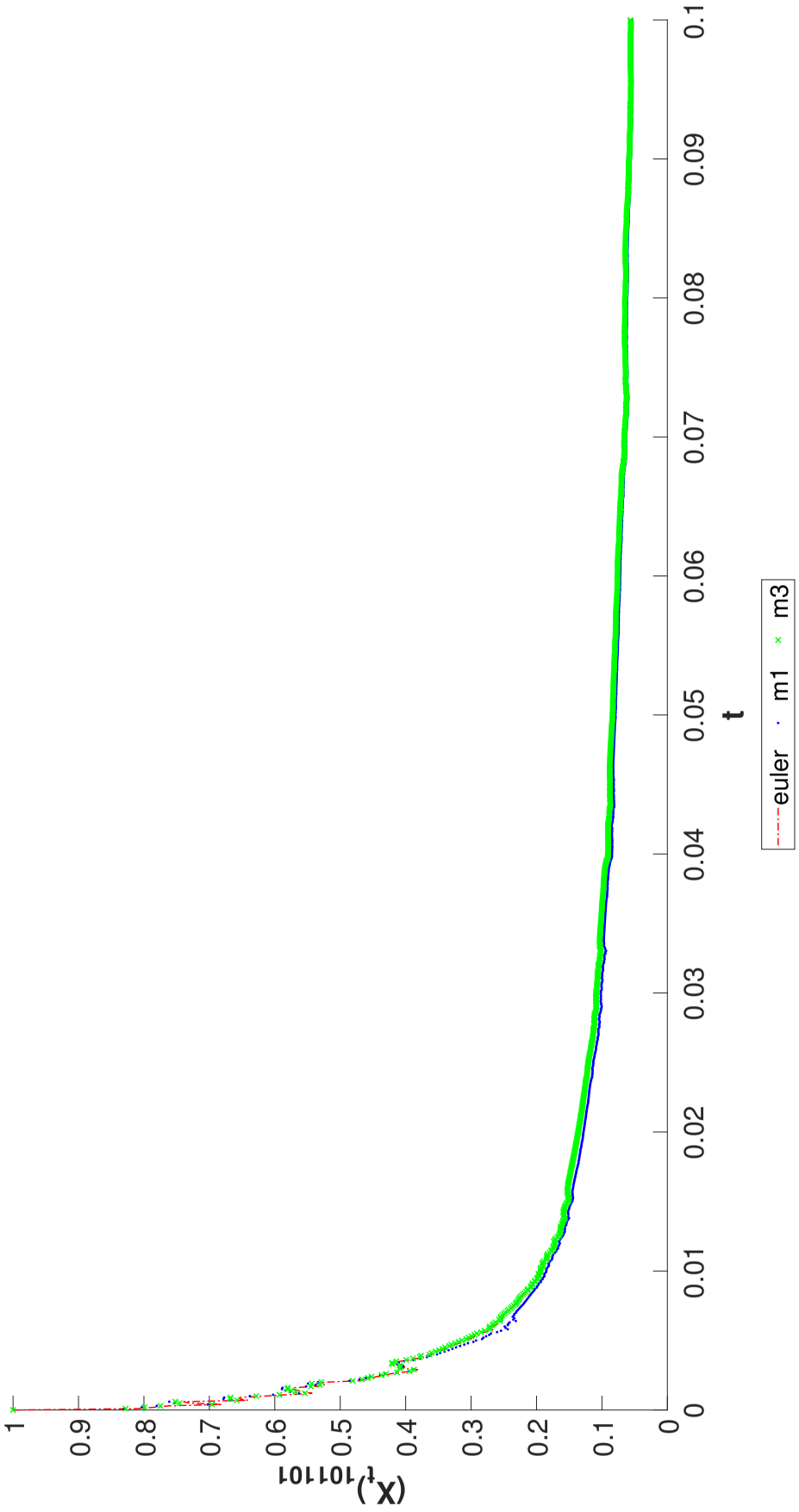
1.4 Plots


(X_t)_101101
t
exact
euler
m1
m2
m3
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

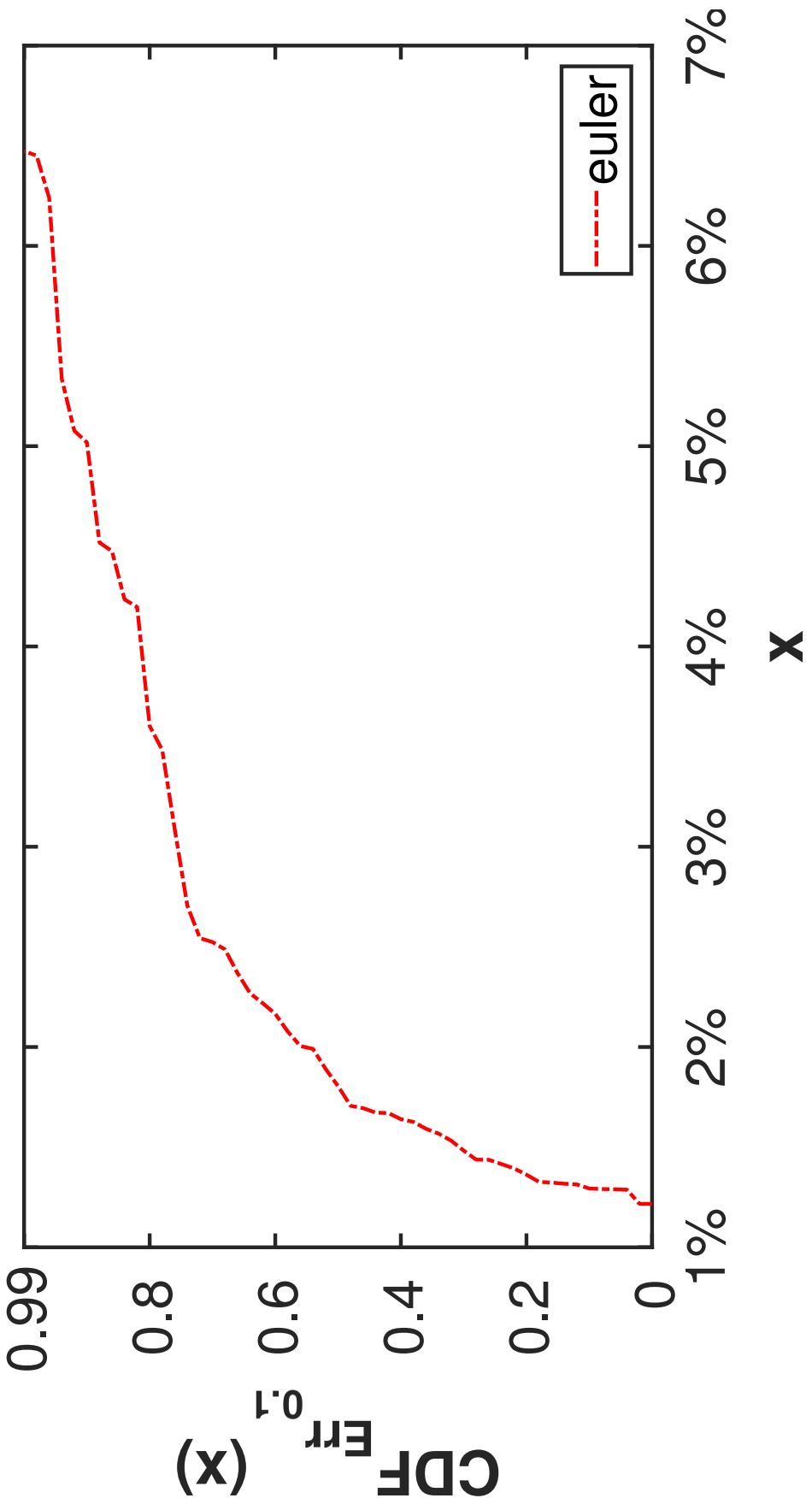

1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
t
exact
euler
m2
m3

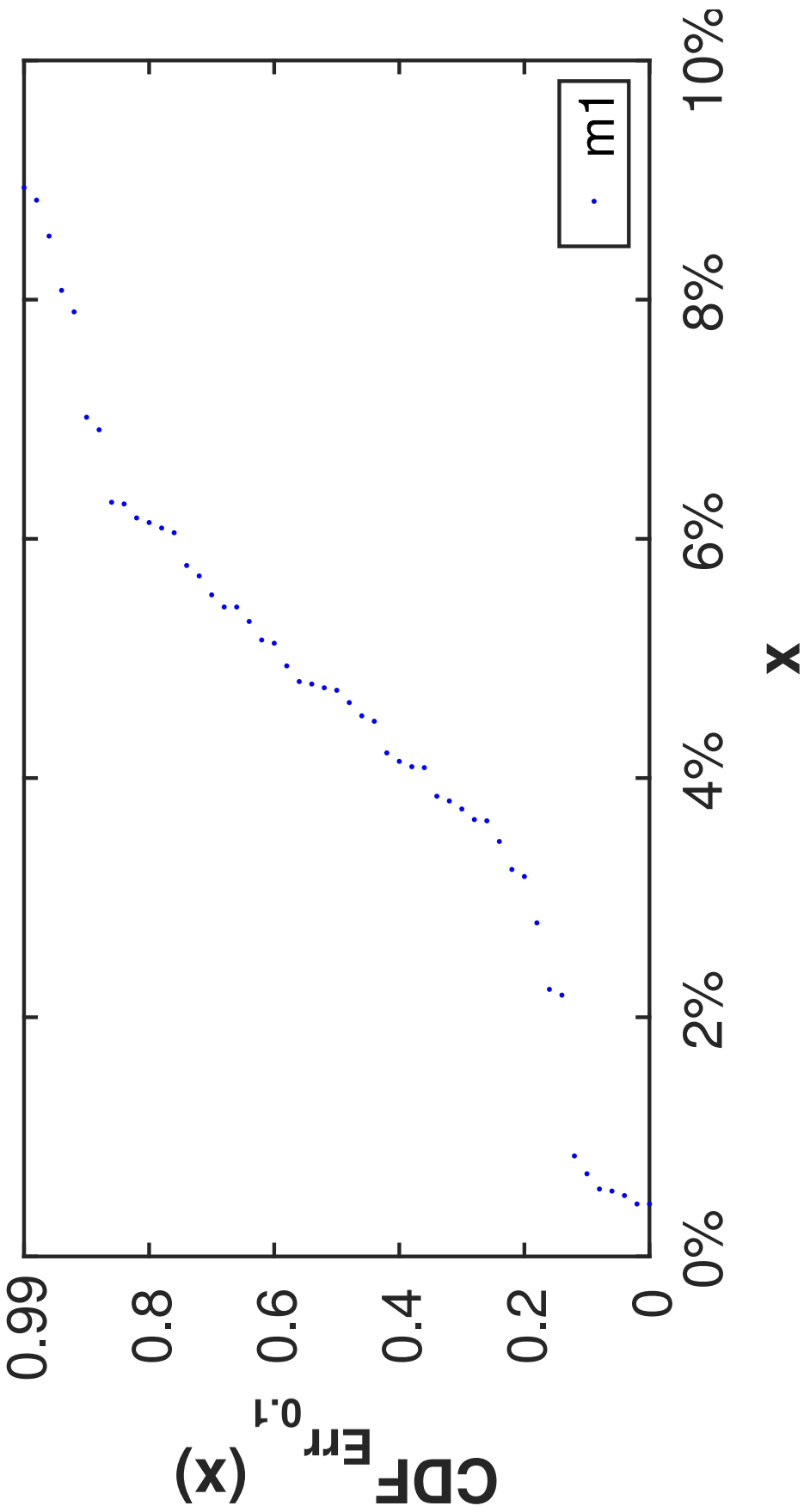

(X_t)_101101
t
exact
euler
m3
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

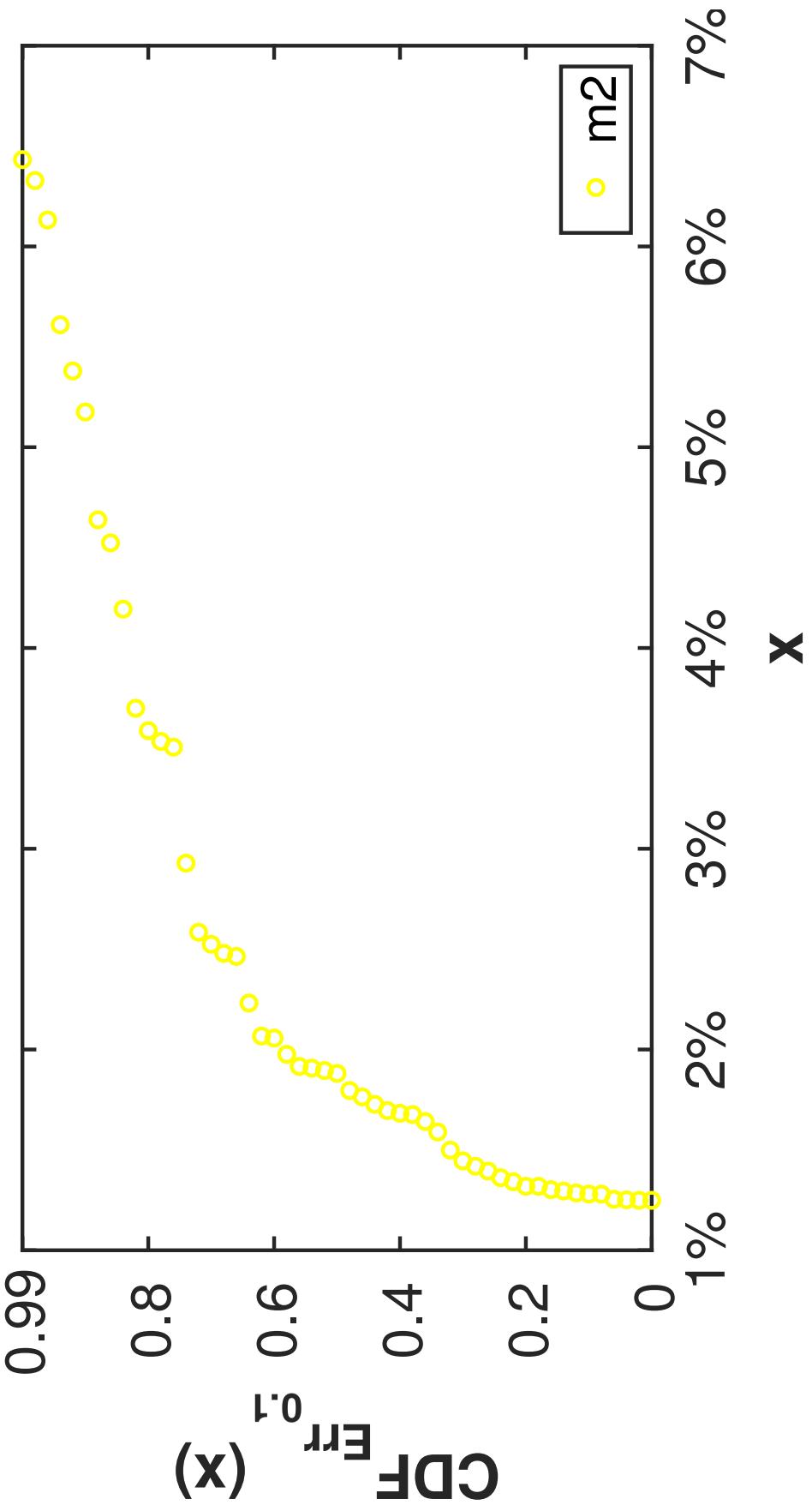

(x_t)_101101
t
euler
m3
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1

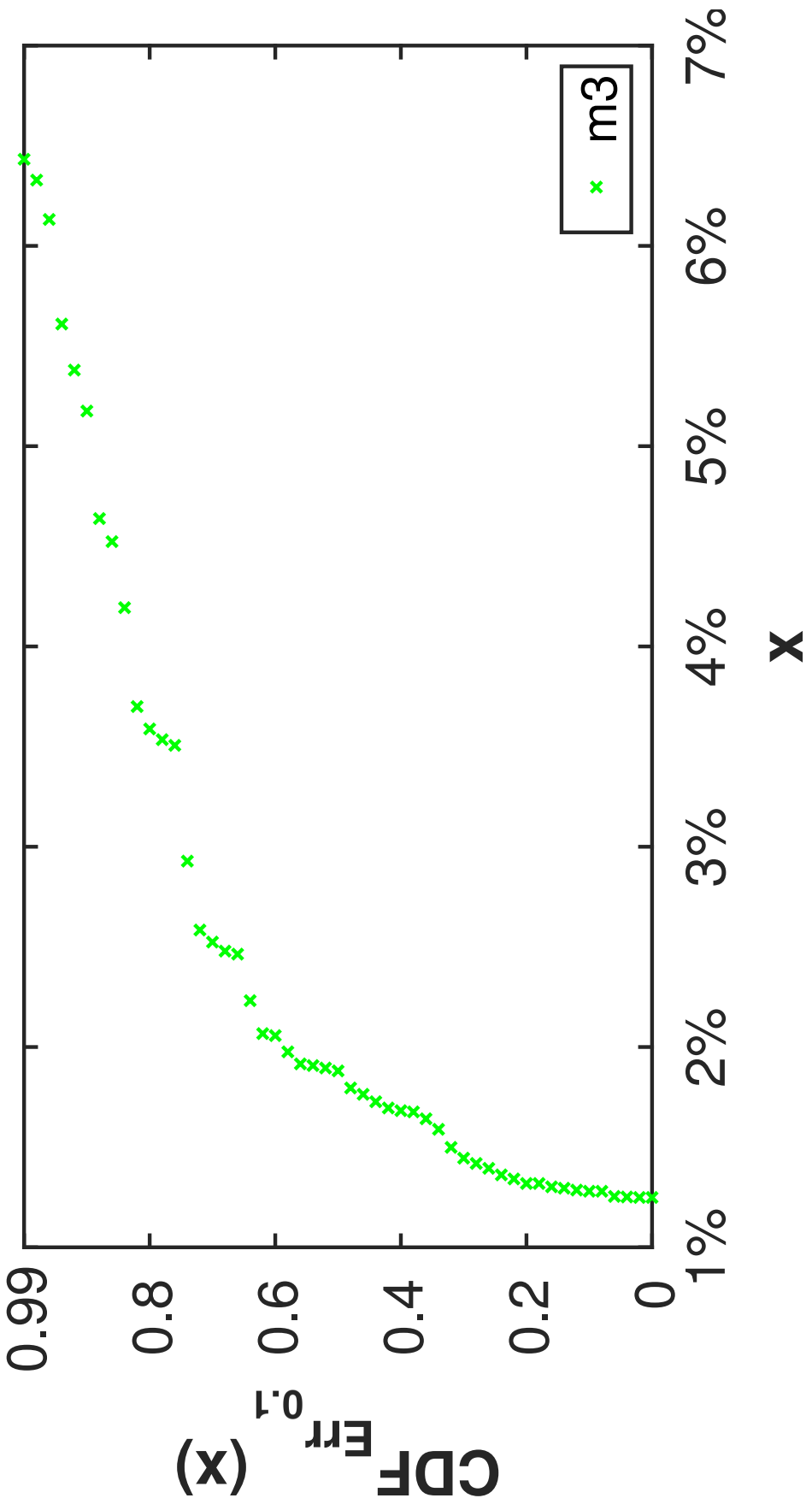

(X_t)_101101
t
euler
m1
m3
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

1.5 Error Plots


euler
0.99
0.8
0.6
0.4
0.2
0
1%
2%
3%
4%
5%
6%
7%
x
CDF err0.1 (x)


m1
0.99
0.8
0.6
0.4
0.2
0
0%
2%
4%
6%
8%
10%
x
CDF Err 0.1 (x)


m2
0.99
0.8
0.6
0.4
0.2
0
1%
2%
3%
4%
5%
6%
7%
x
CDF err 0.1 (x)


m3
0.99
0.8
0.6
0.4
0.2
0
1%
2%
3%
4%
5%
6%
7%
x
CDF err 0.1 (x)