

1 Magnus expansion for $A \in \mathbb{R}^{2,2}$, $B = 0$ deterministic

We will concern ourselves with the following SDE:

$$dX_t = A_t X_t dW_t$$

with

$$A_t = \begin{bmatrix} f^{11}(t) & f^{12}(t) \\ 0 & f^{22}(t) \end{bmatrix}$$

Here we chose:

$$A_t = \frac{\tilde{A}_t}{\sigma(\tilde{A}_t)},$$

where $\sigma(A_t)$ is the spectral radius and

$$\tilde{A}_t = \begin{bmatrix} 2 & t \\ 0 & -1 \end{bmatrix}$$

1.1 Parameters

Parameter	value
t_0	0
T	1
<code>N_fine</code>	10000
N	1000
<code>M_fine</code>	1000
M	1000
d	2

t_0	0
T	1
<code>N_fine</code>	10000
N	1000
<code>M_fine</code>	1000
M	1000
d	2

1.2 Computational Times

Method	Log	Matrix Exp	Total
exact	0	0	0.795271
euler	0	0	4.67155
m1	0.0267684	4.57533	4.6021
m2	0.089908	4.55504	4.64495
m3	0.245747	4.60379	4.84954

1.3 Errors

(i) Errors for $X(1, 1, :, :)$:

(a) Reference method: exact

Error	euler	m1	m2	m3
(abs error) L2	0.00612649	0.431044	0.000170138	0.000170138
(rel error) min	0	0	0	0
(rel error) max	0.00519639	0.576061	$9.72097e - 05$	$9.72097e - 05$
(rel error) mean	0.00373177	0.279633	$4.52479e - 05$	$4.52479e - 05$

(ii) Errors for $X(1, 2, :, :)$:

(a) Reference method: exact

Error	euler	m1	m2	m3
(abs error) L2	0.00213927	0.0771372	0.0217018	0.0084237
(rel error) min	0.000120492	0.0197843	0.0056921	0.00628867
(rel error) max	0.0140499	0.433351	0.138571	0.0323207
(rel error) mean	0.00771088	0.239473	0.065535	0.0158499

(iii) Errors for $X(2, 2, :, :)$:

(a) Reference method: exact

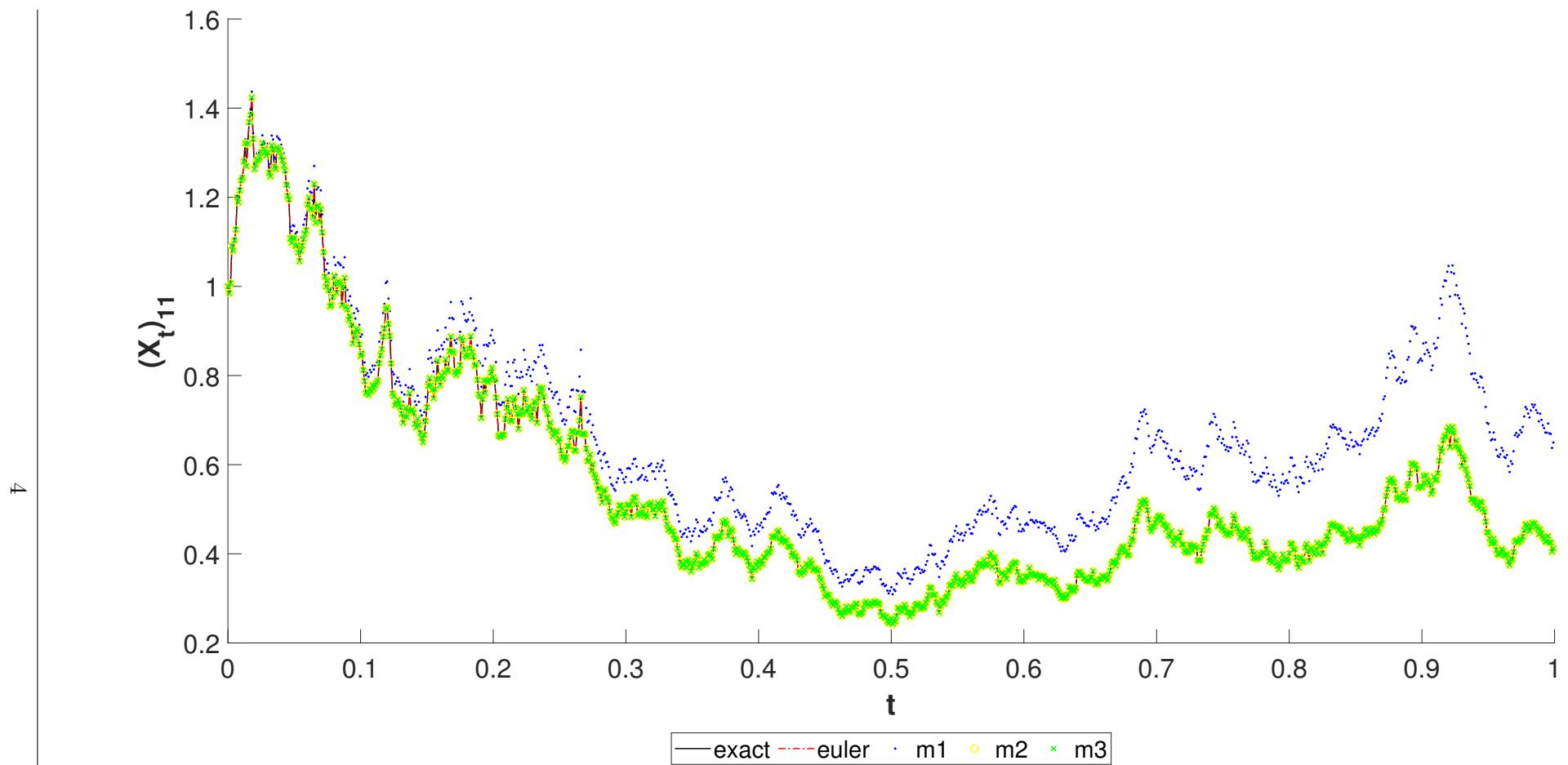
Error	euler	m1	m2	m3
(abs error) L2	0.00131998	0.0788425	$5.32062e - 05$	$5.32062e - 05$
(rel error) min	0	0	0	0
(rel error) max	0.0013011	0.120453	$4.61903e - 05$	$4.61903e - 05$
(rel error) mean	0.000933184	0.0618502	$2.20918e - 05$	$2.20918e - 05$

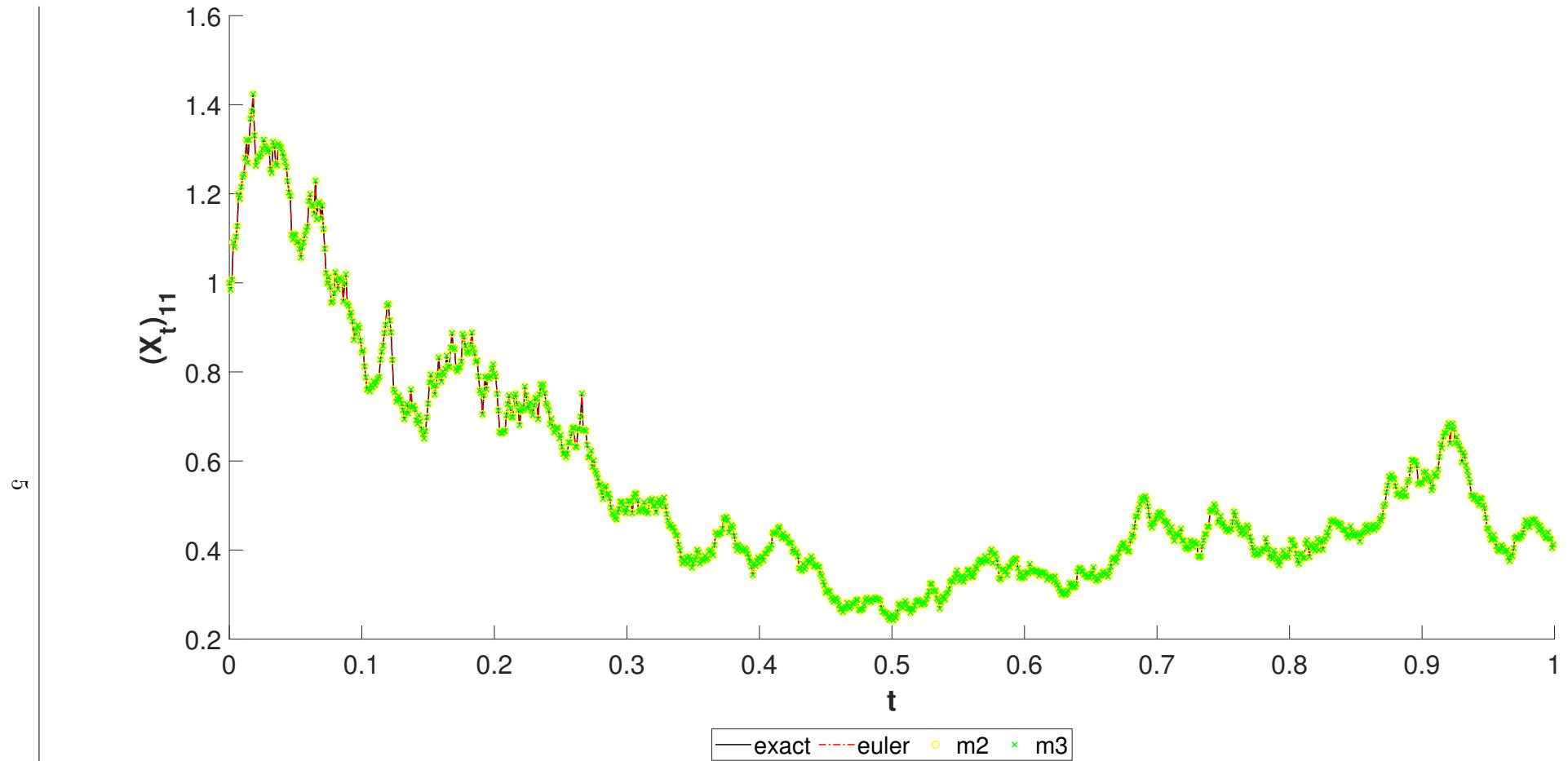
(iv) Total Errors:

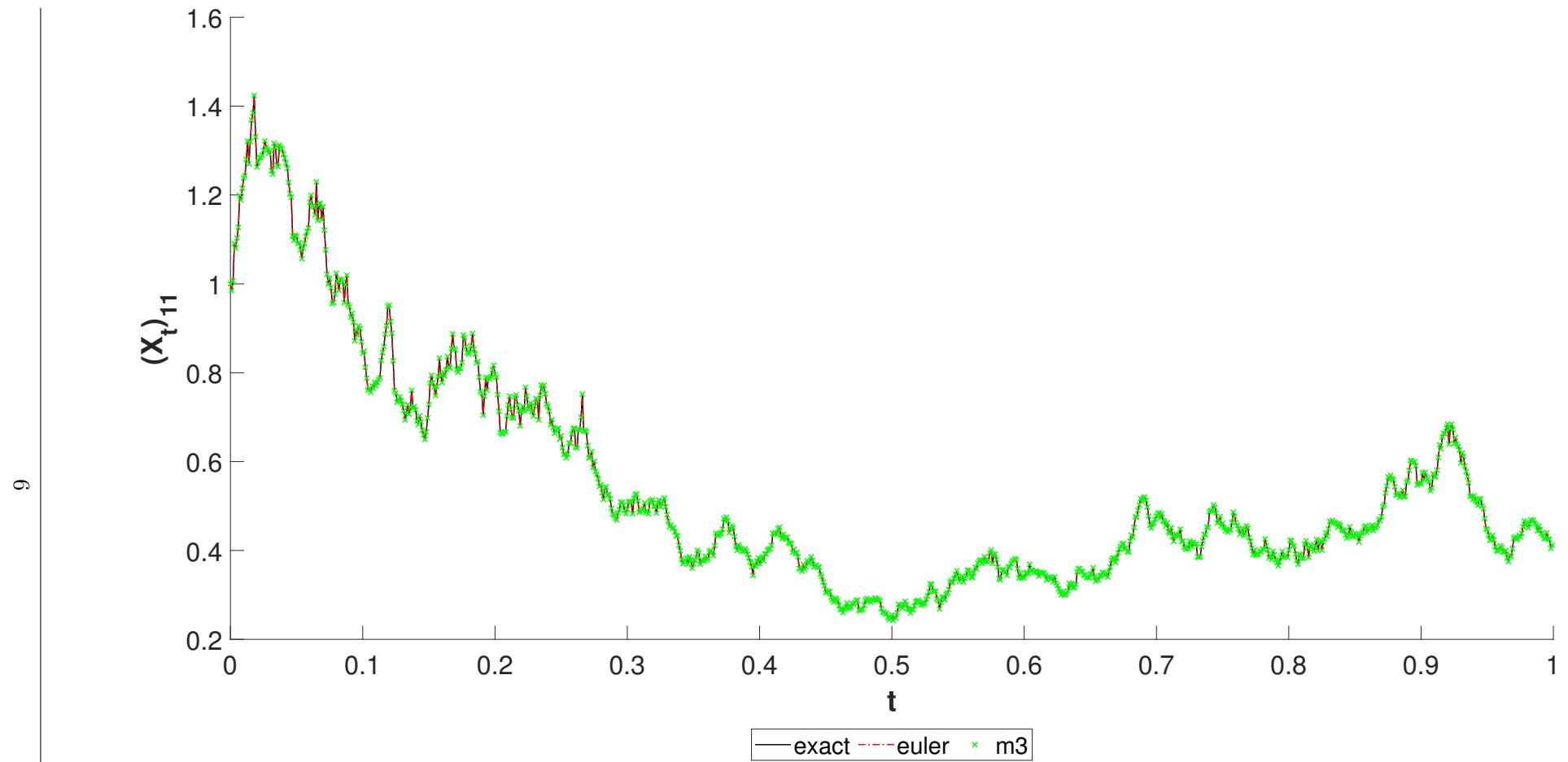
(a) Reference method: exact

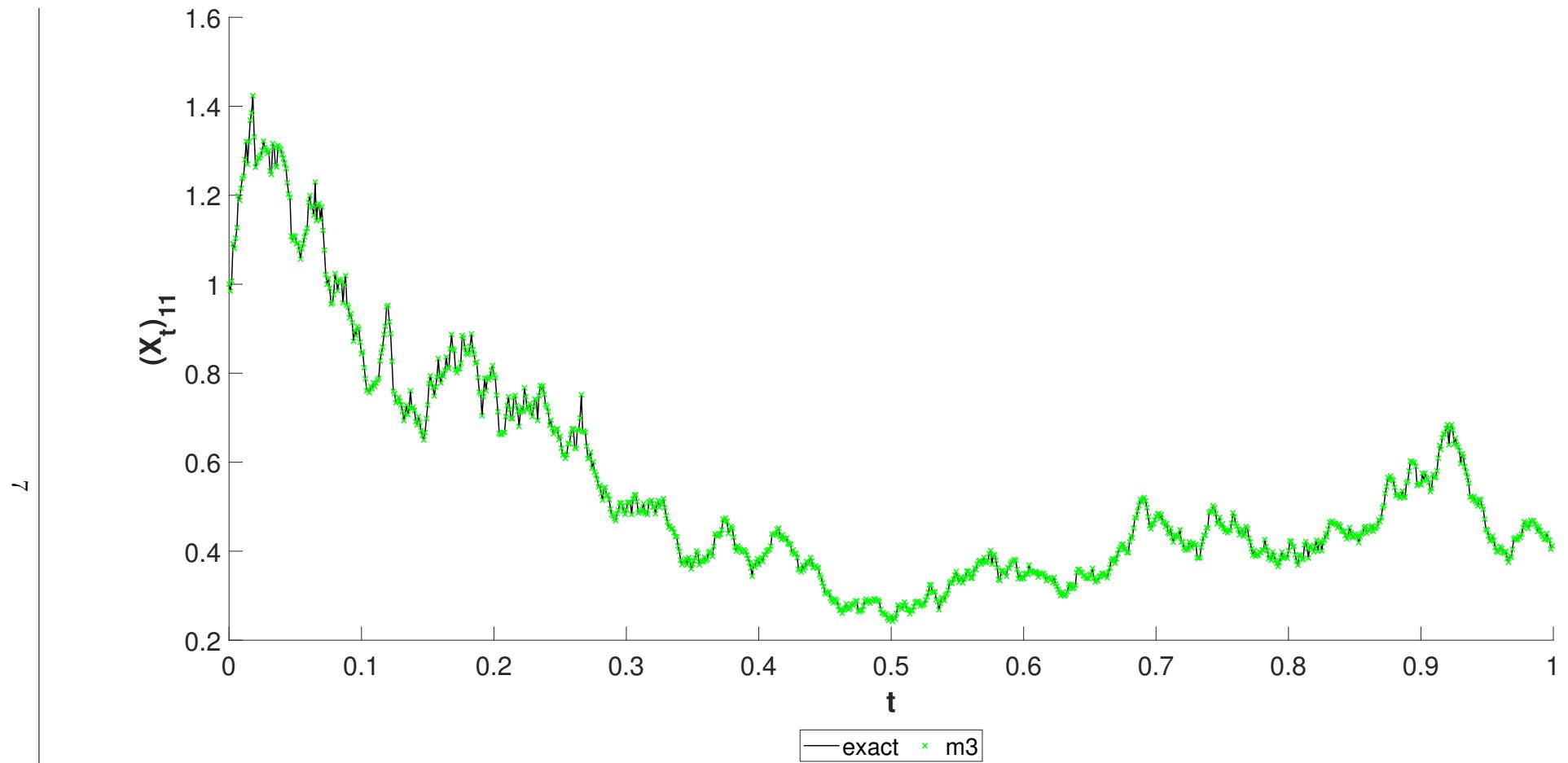
Method	$\mathbb{E}[Err_1]$
euler	0.27 %
m1	17.6 %
m2	0.523 %
m3	0.147 %

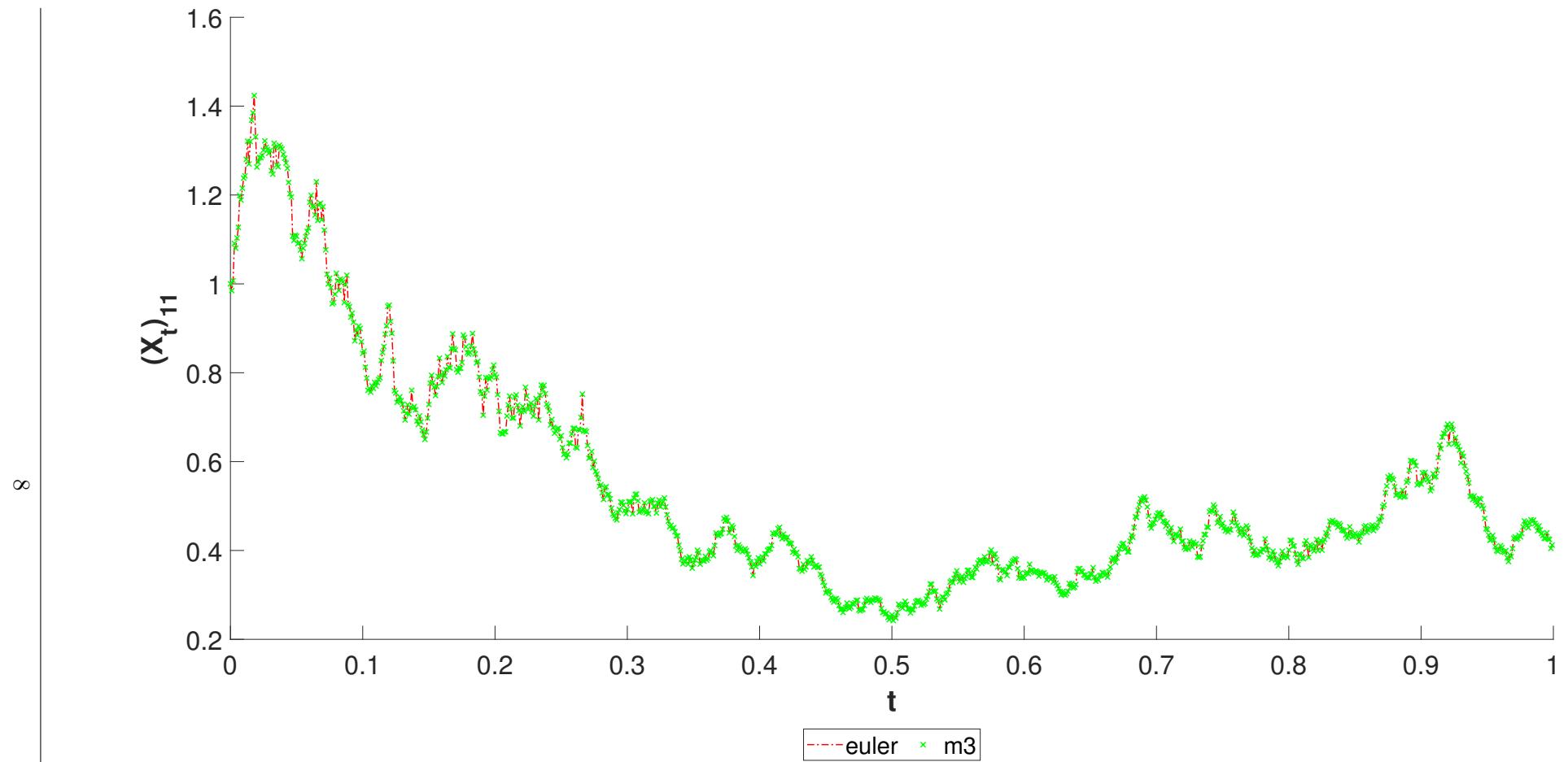
1.4 Plots

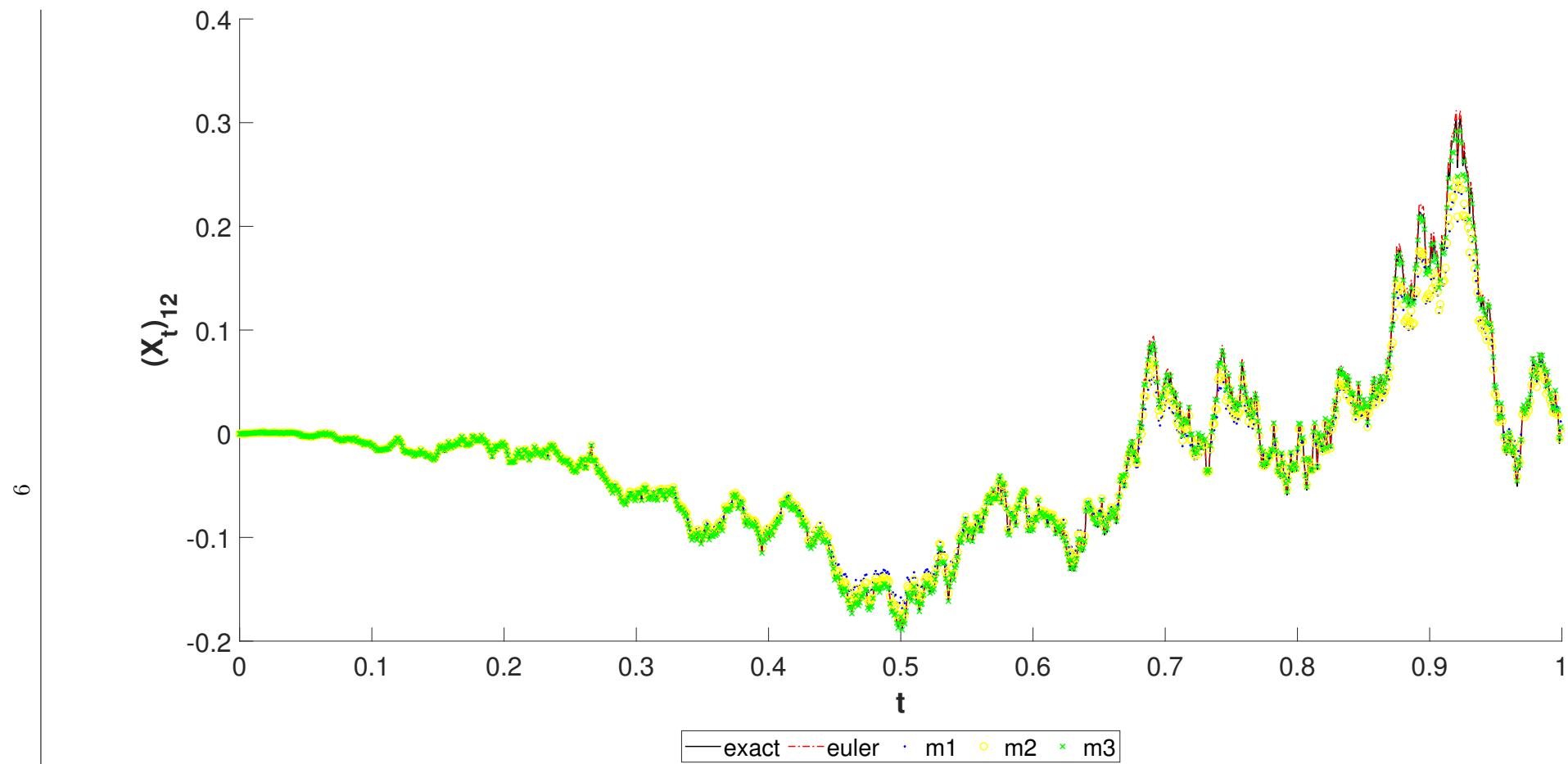


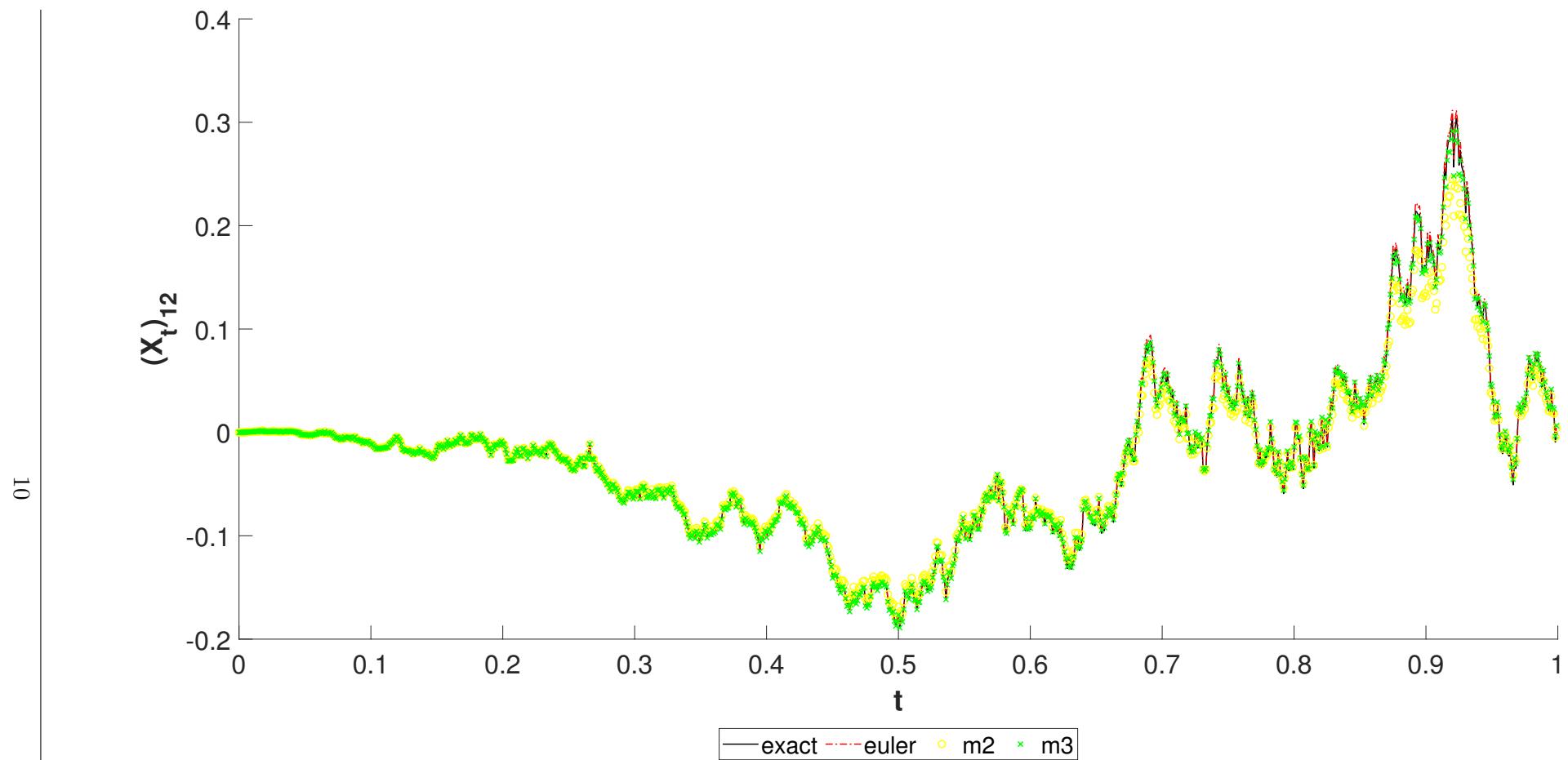


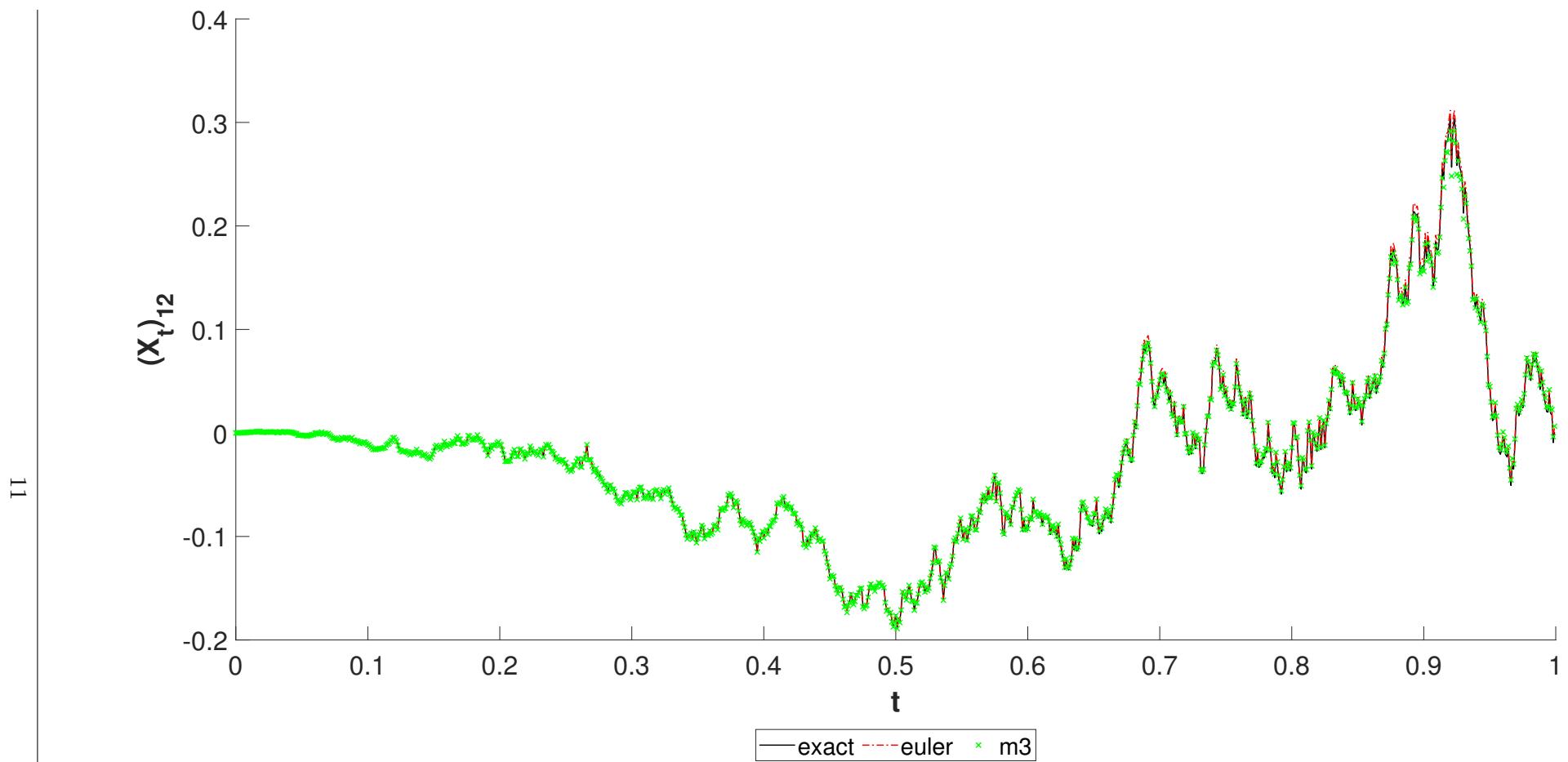


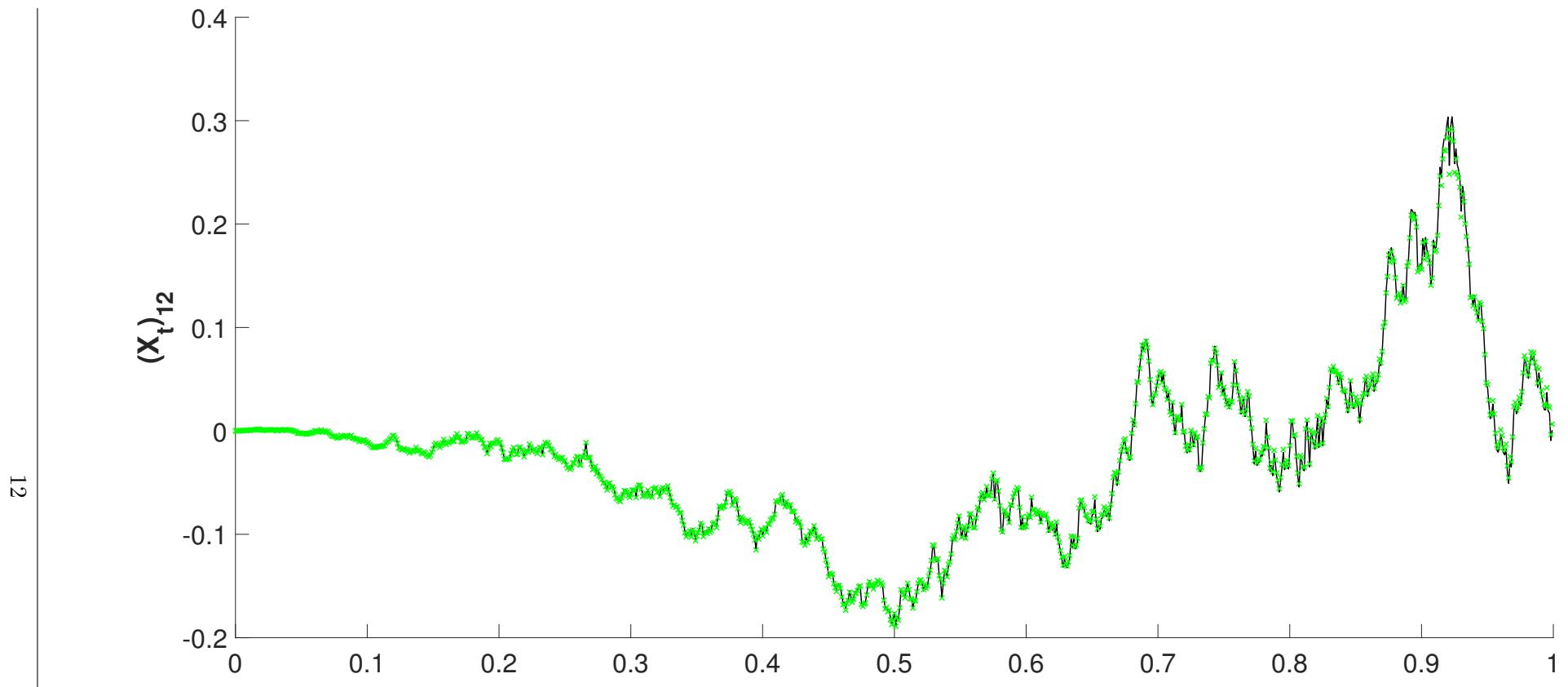


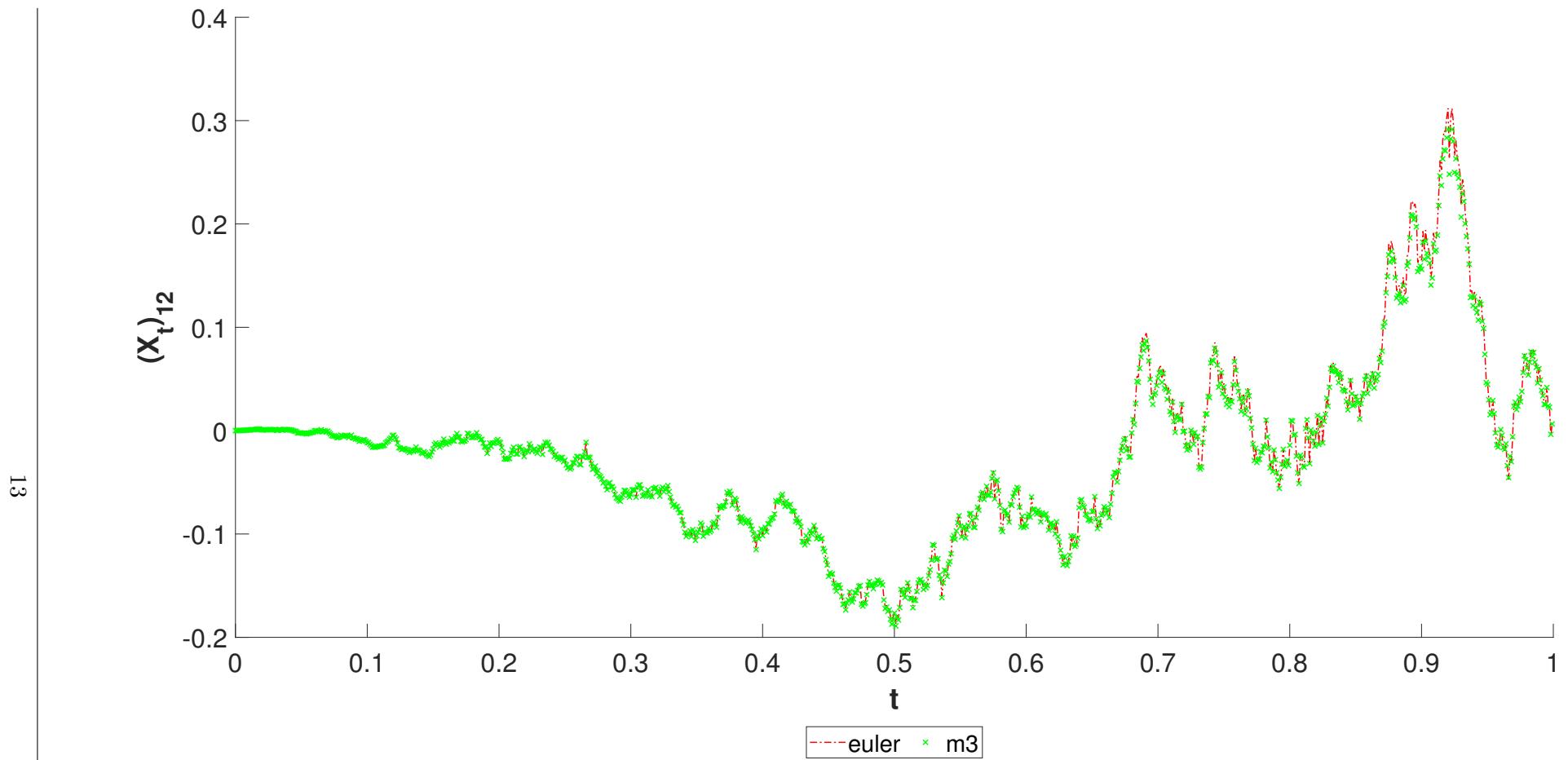


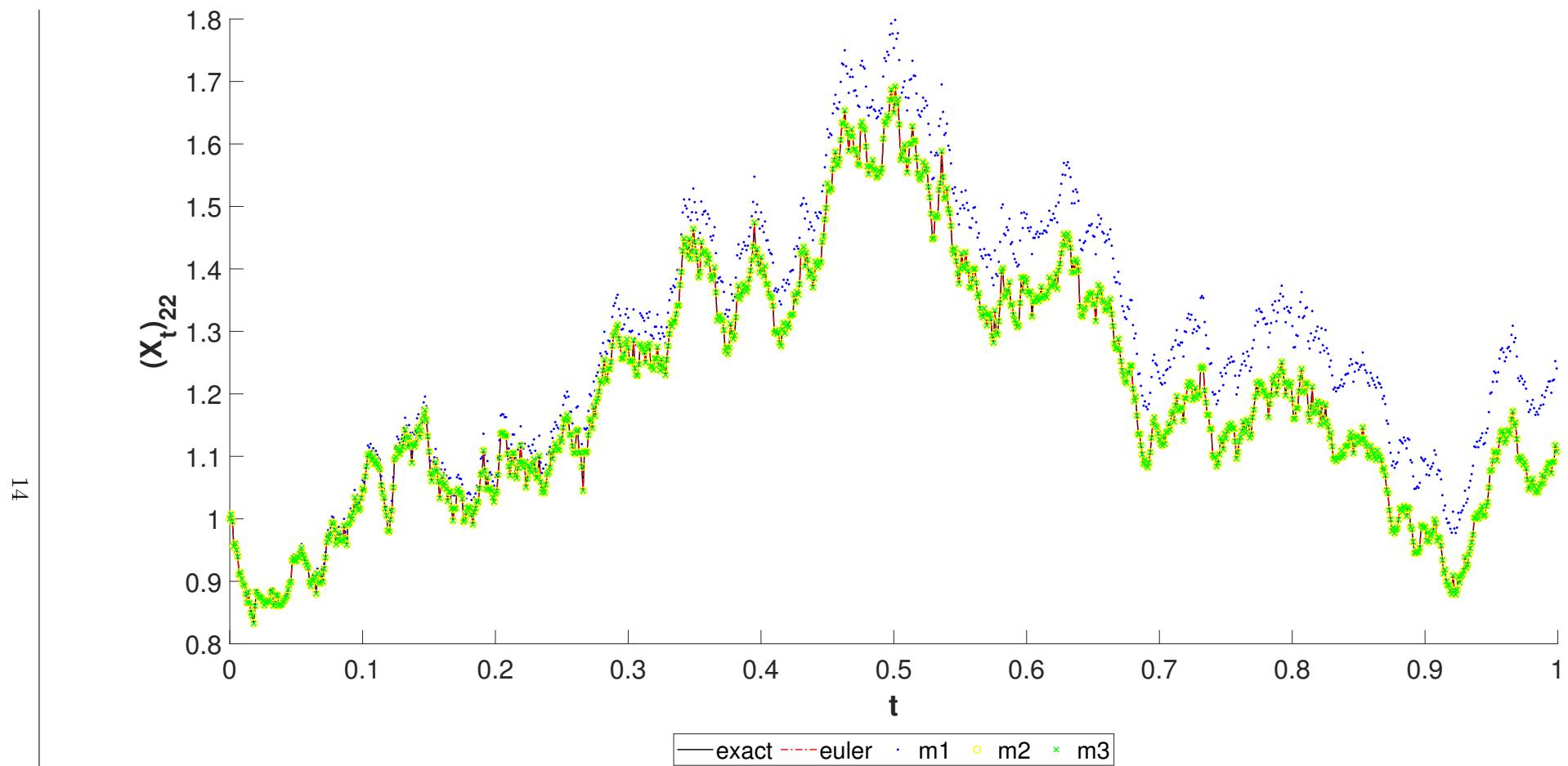


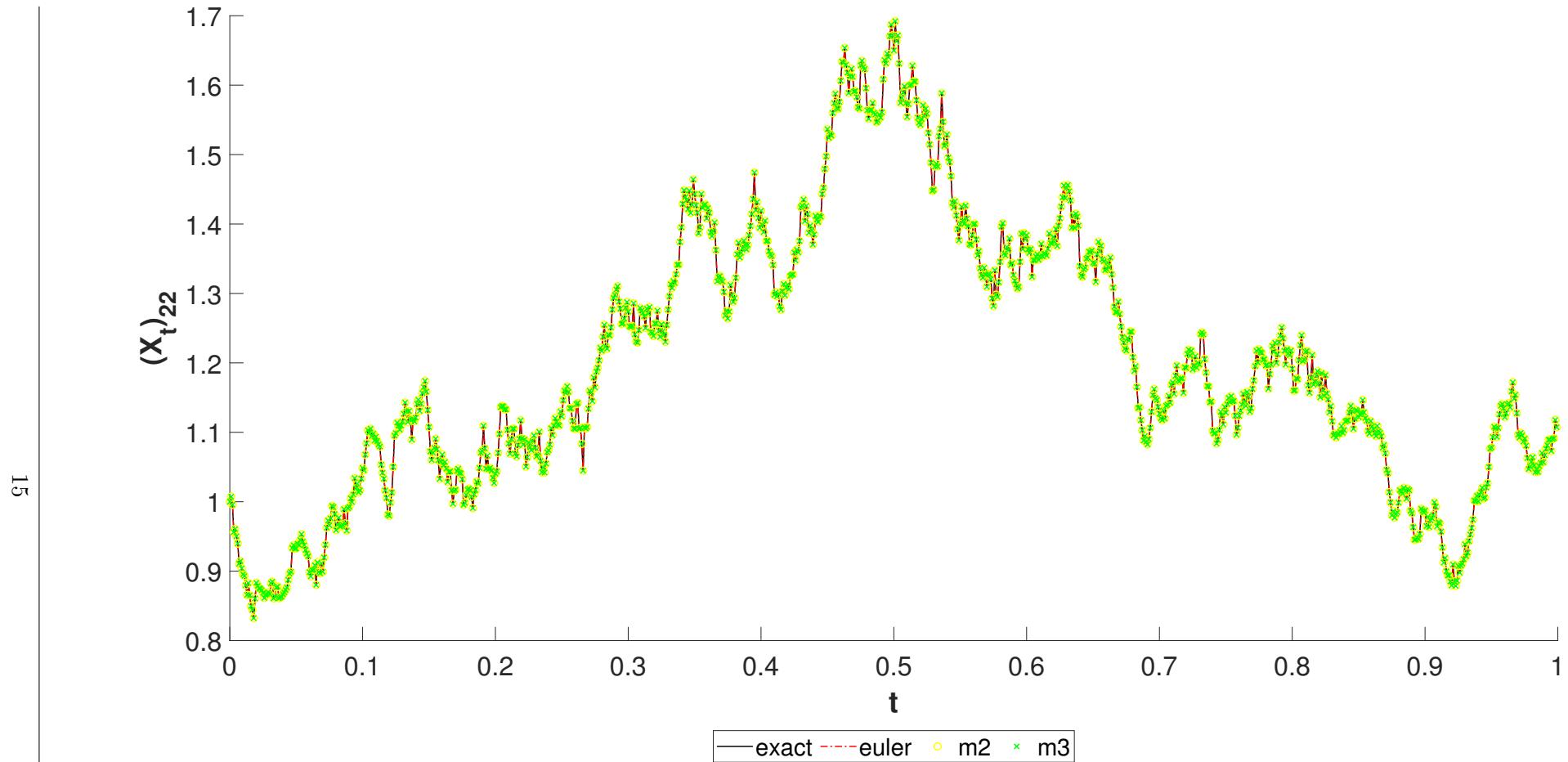


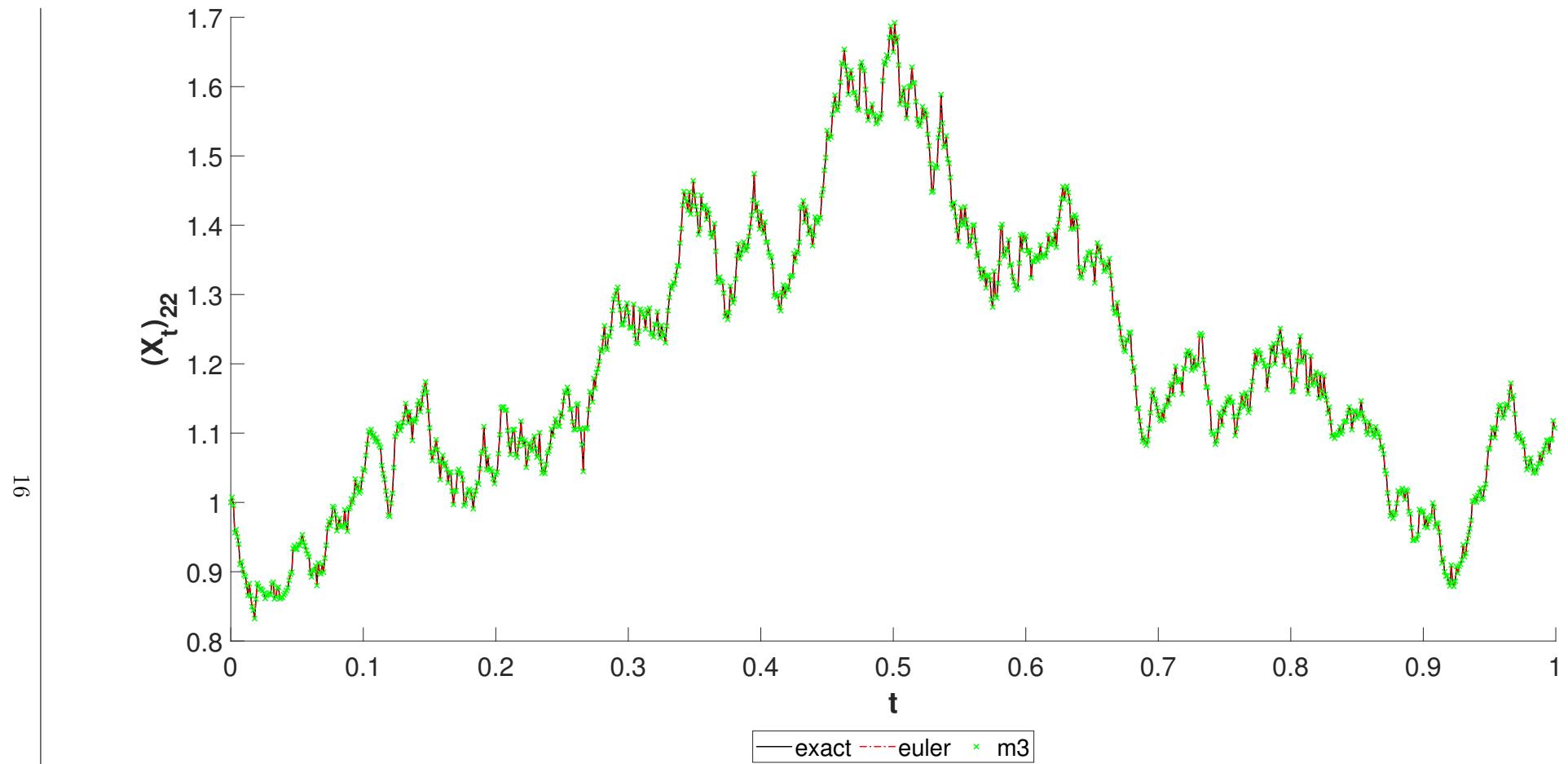


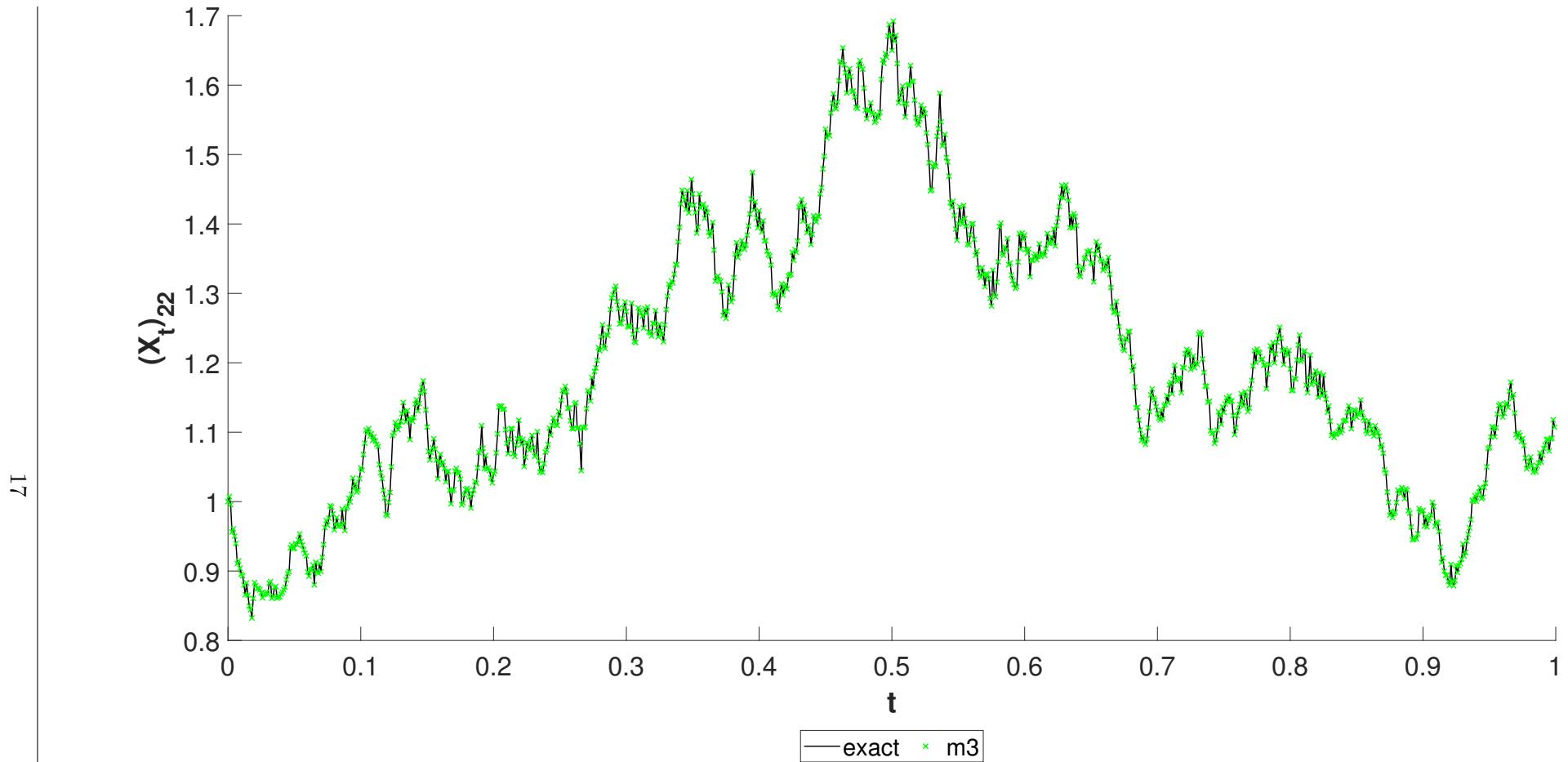


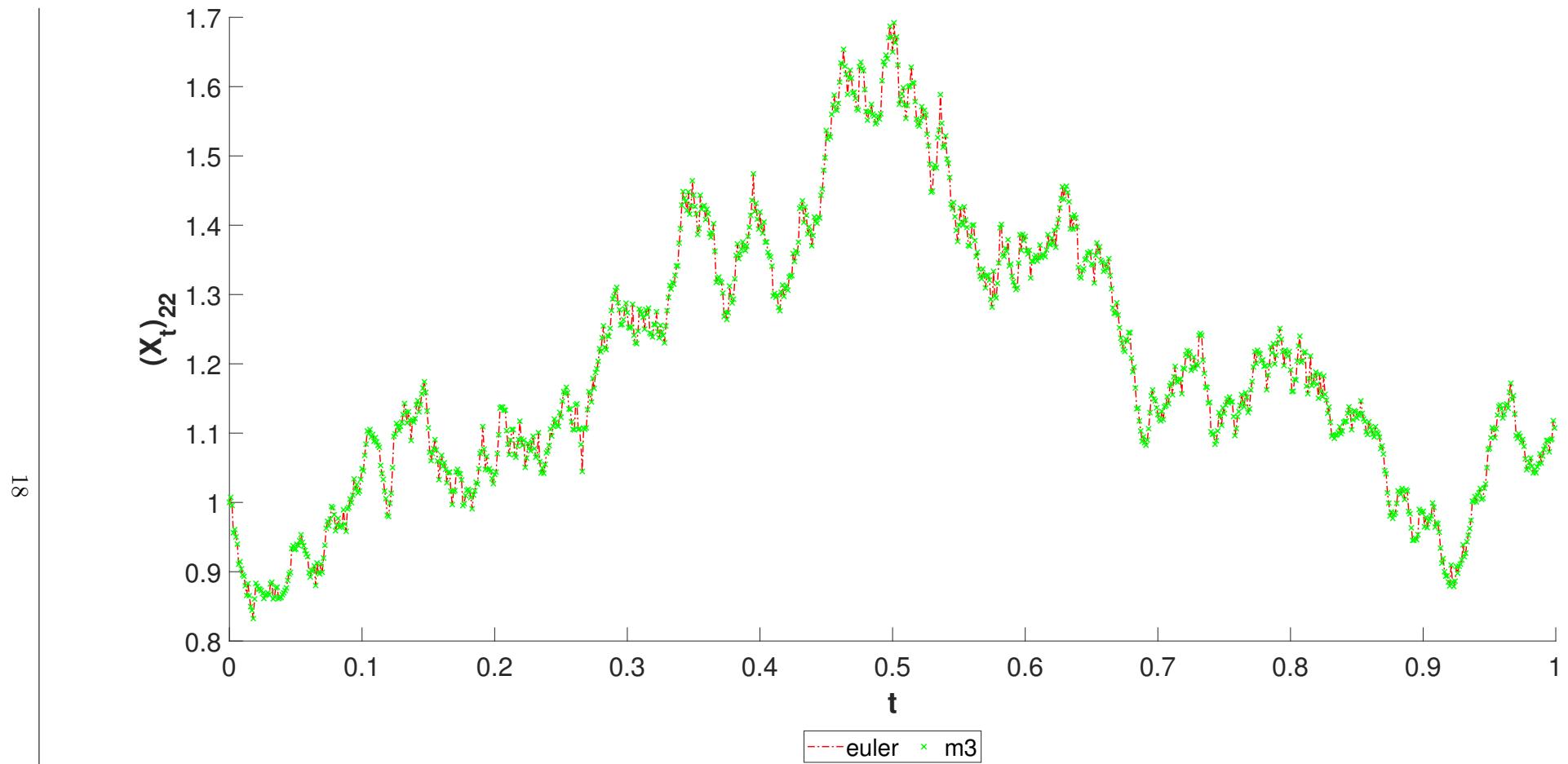


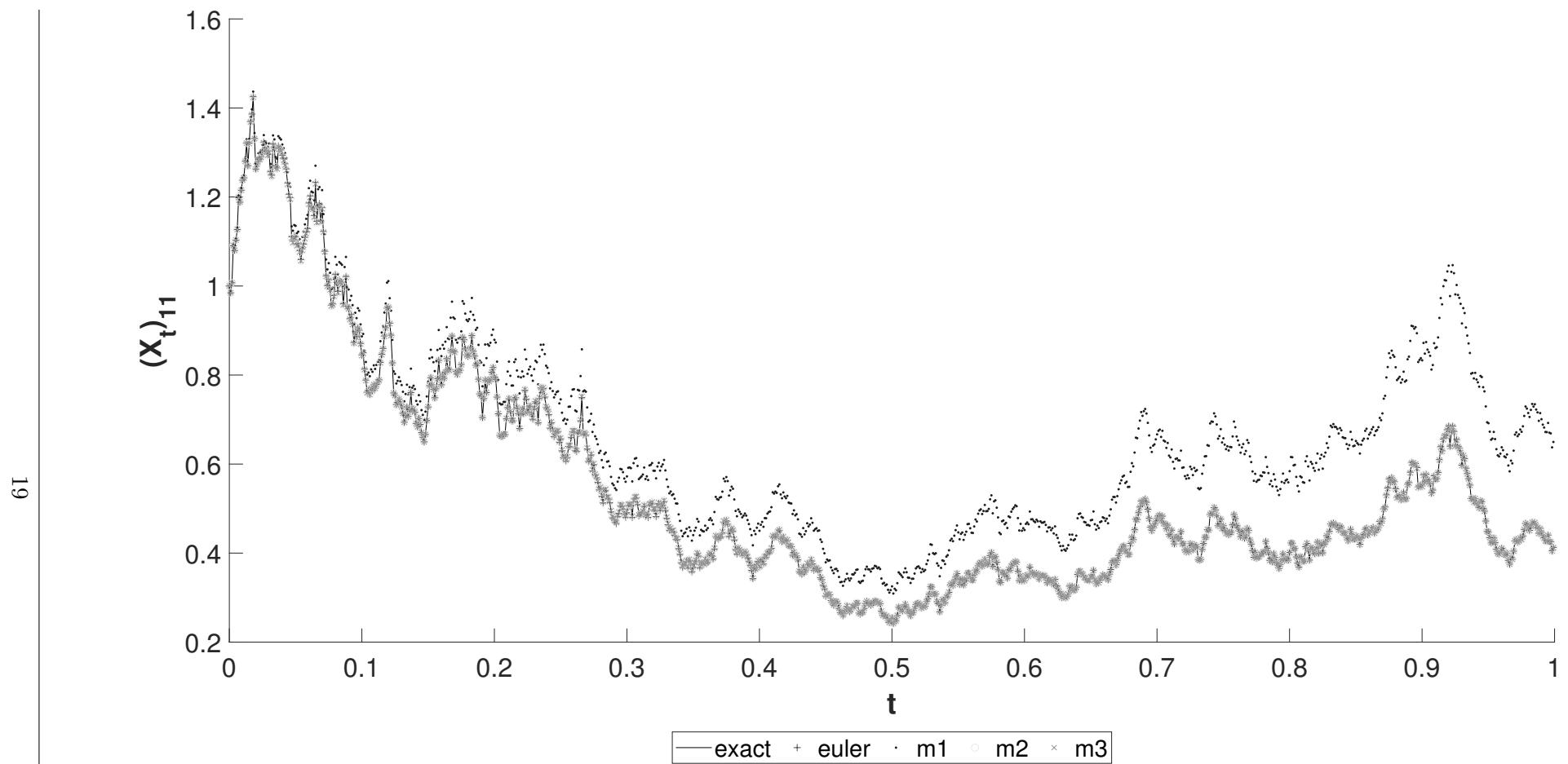


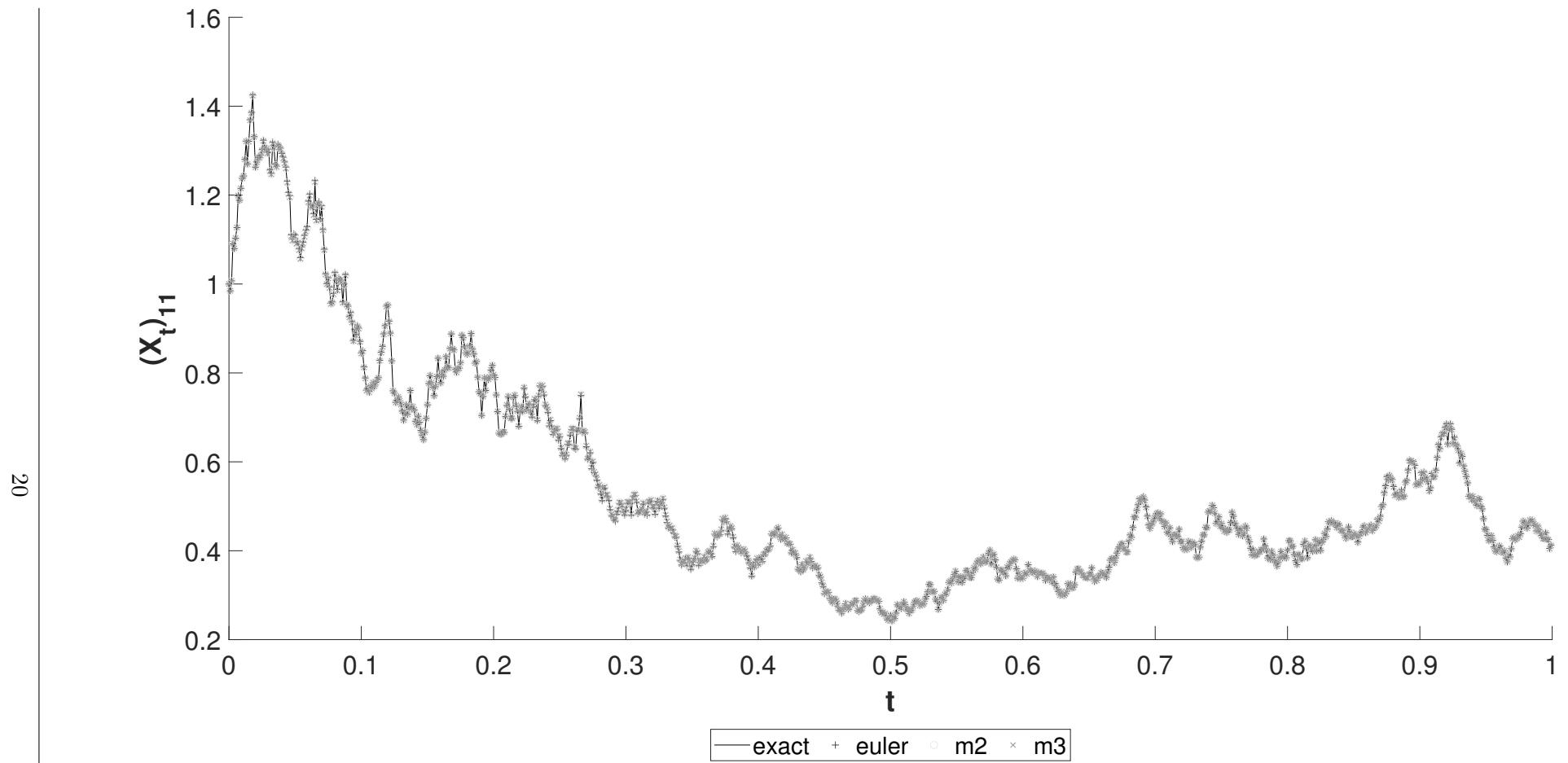


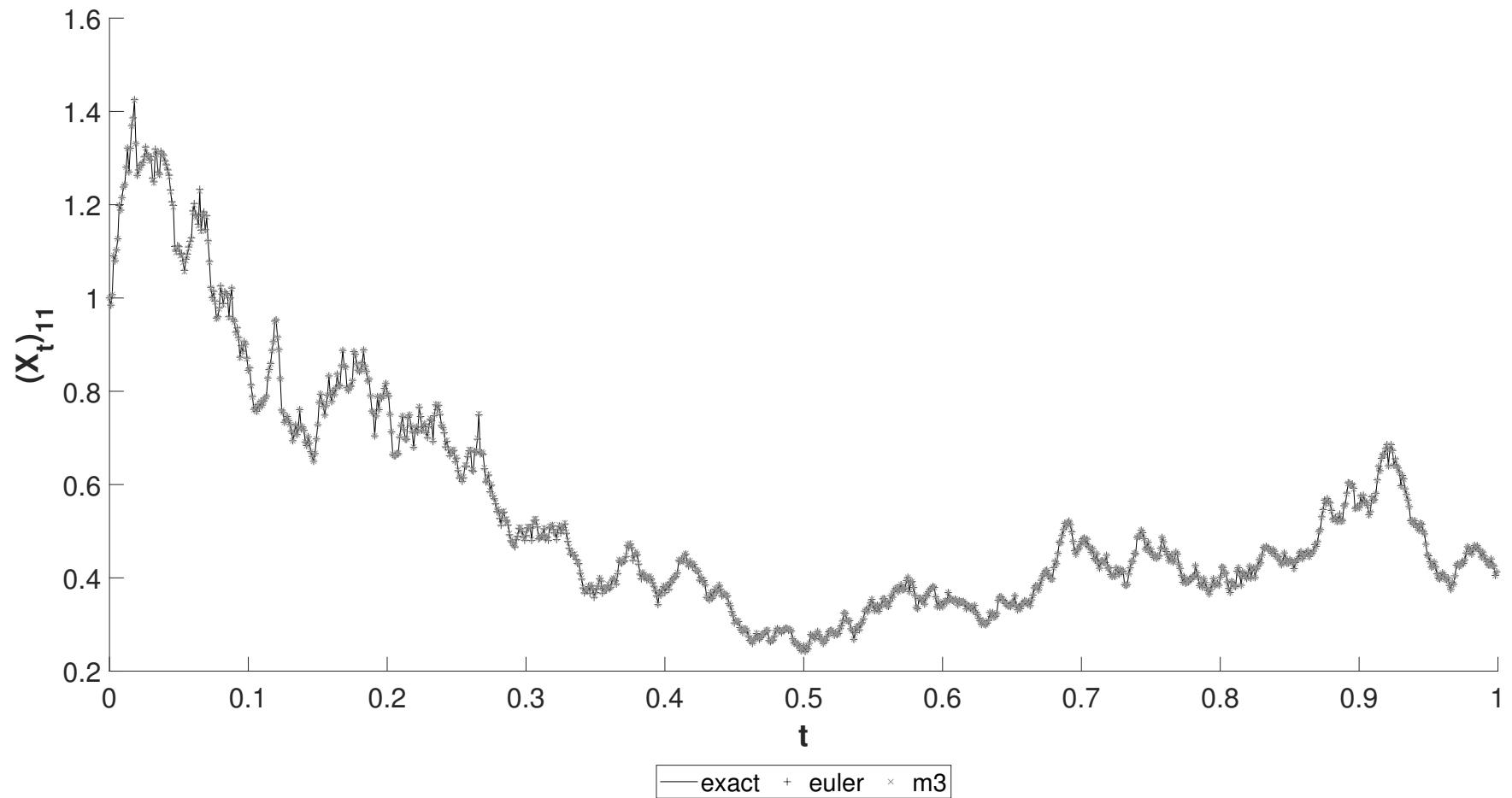


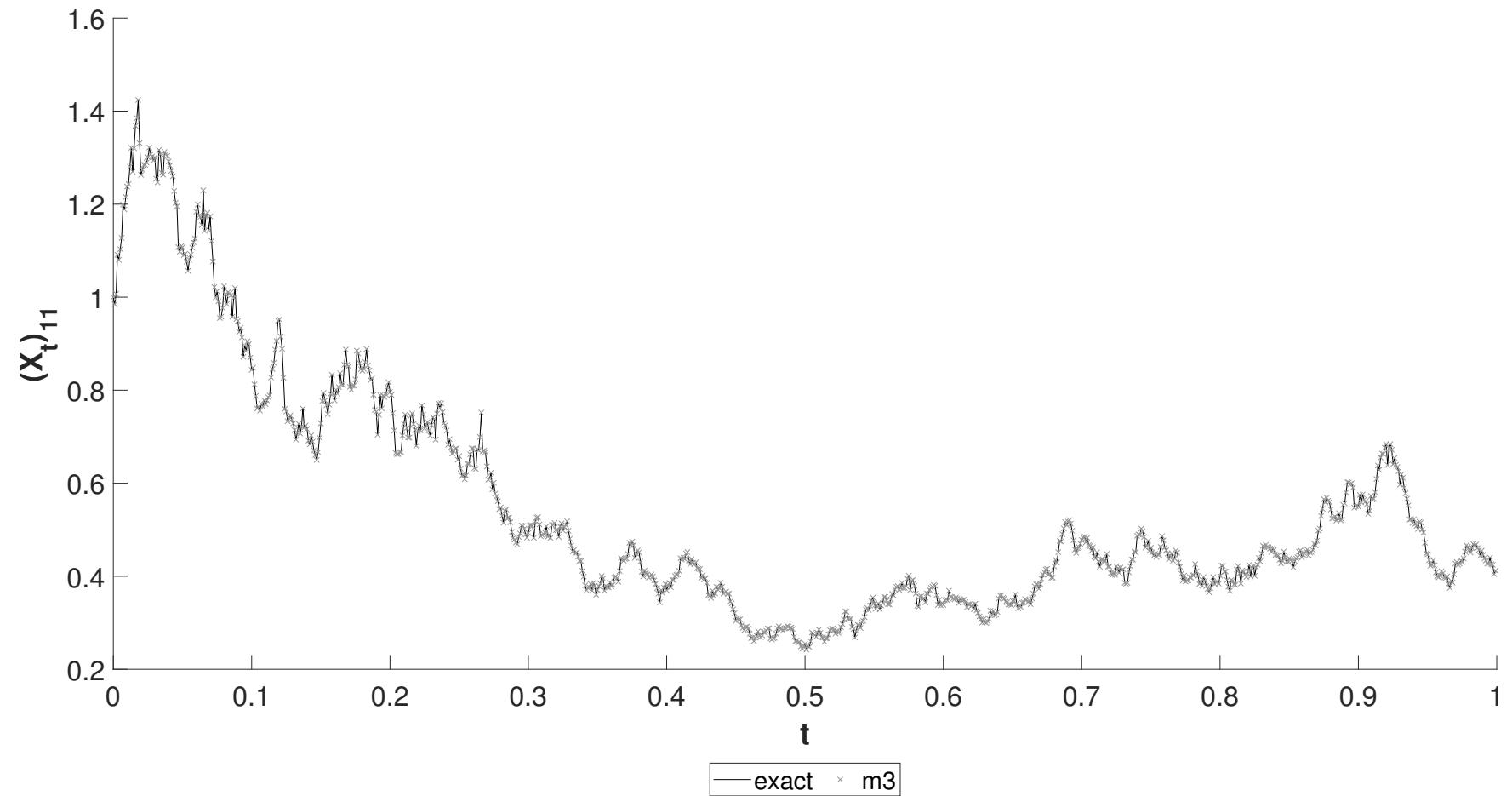


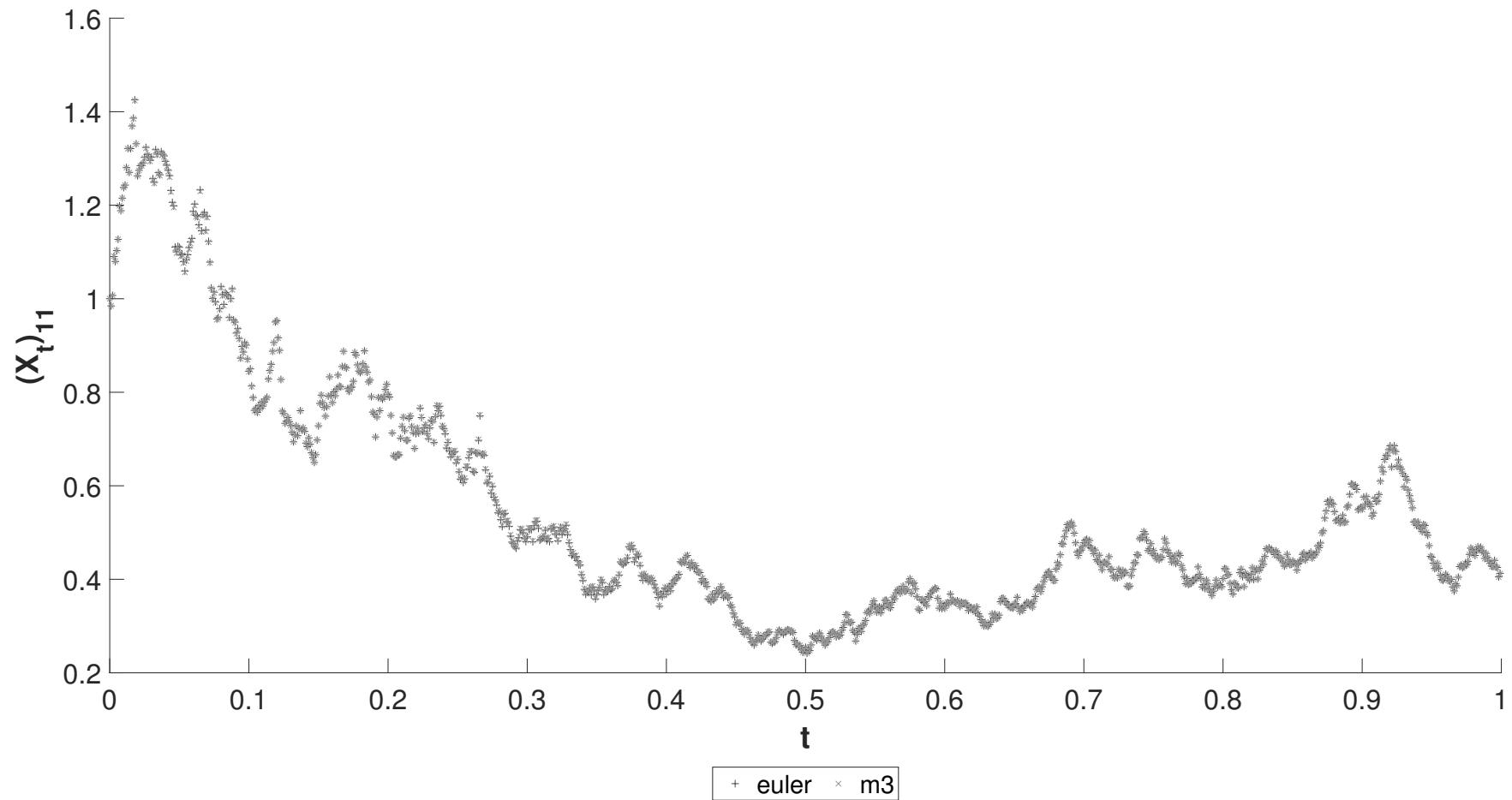


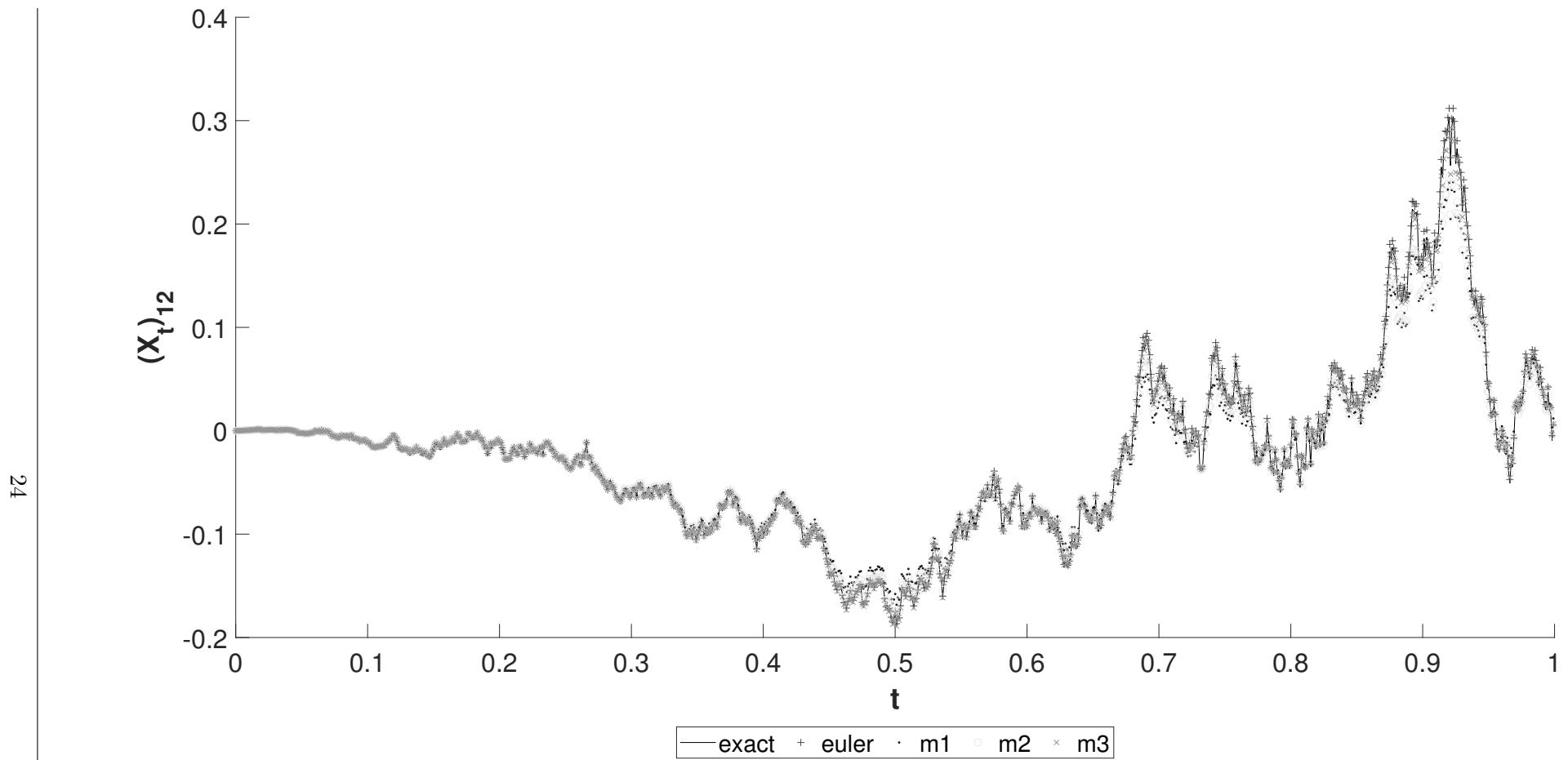


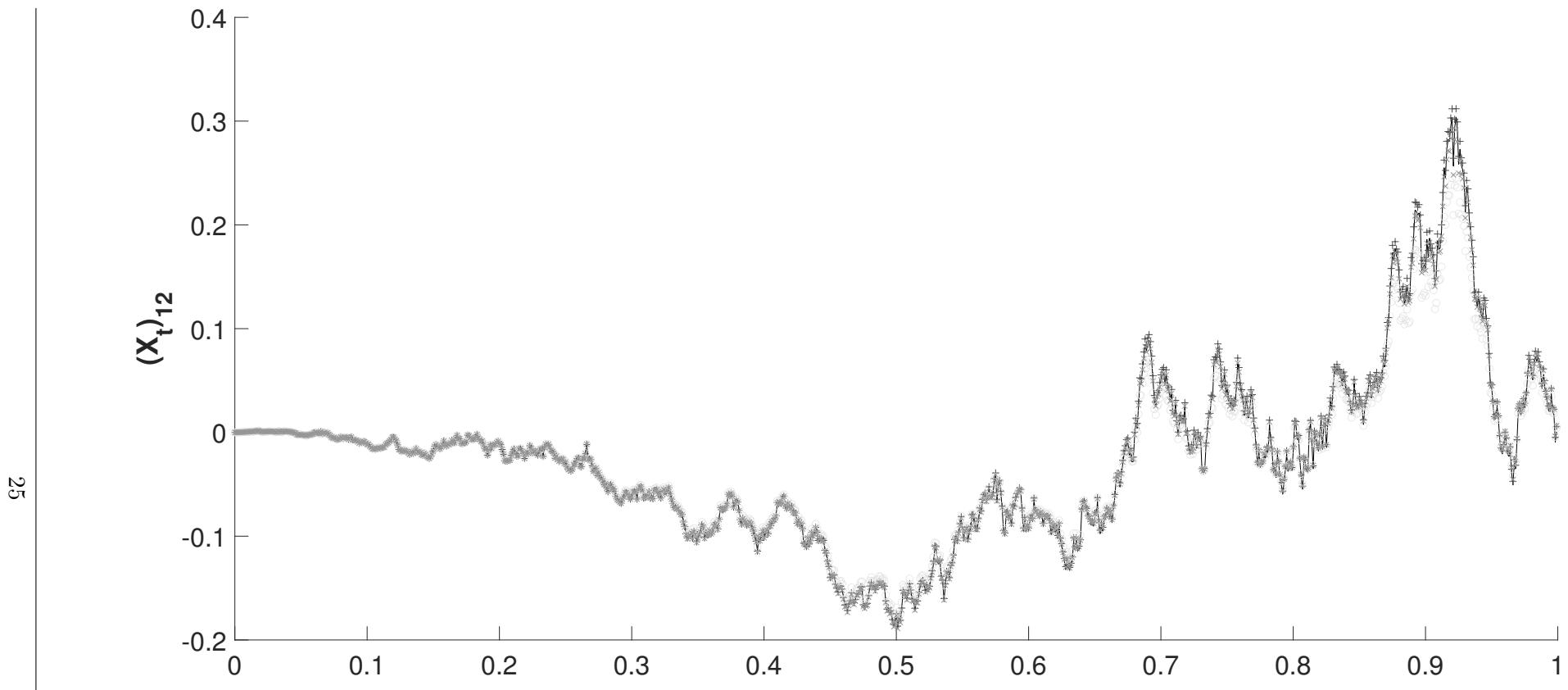


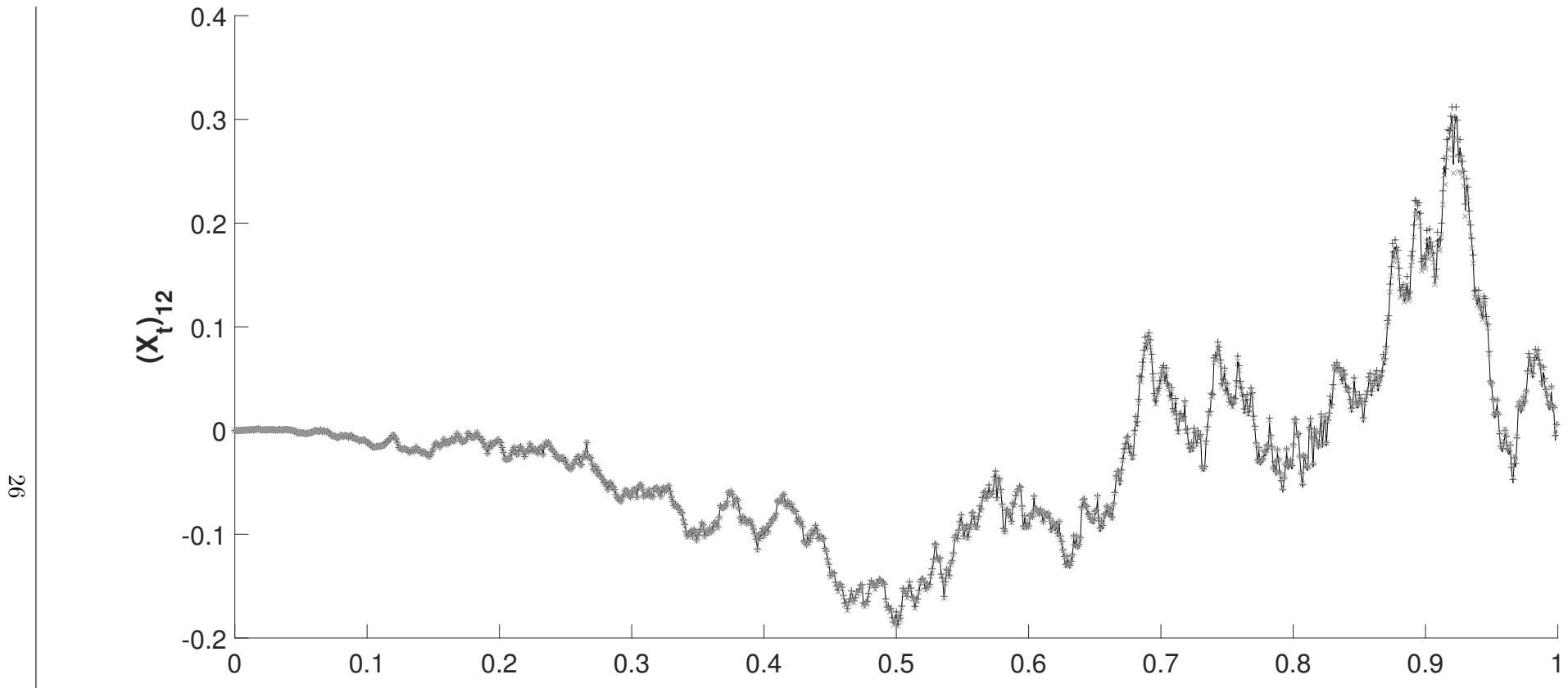


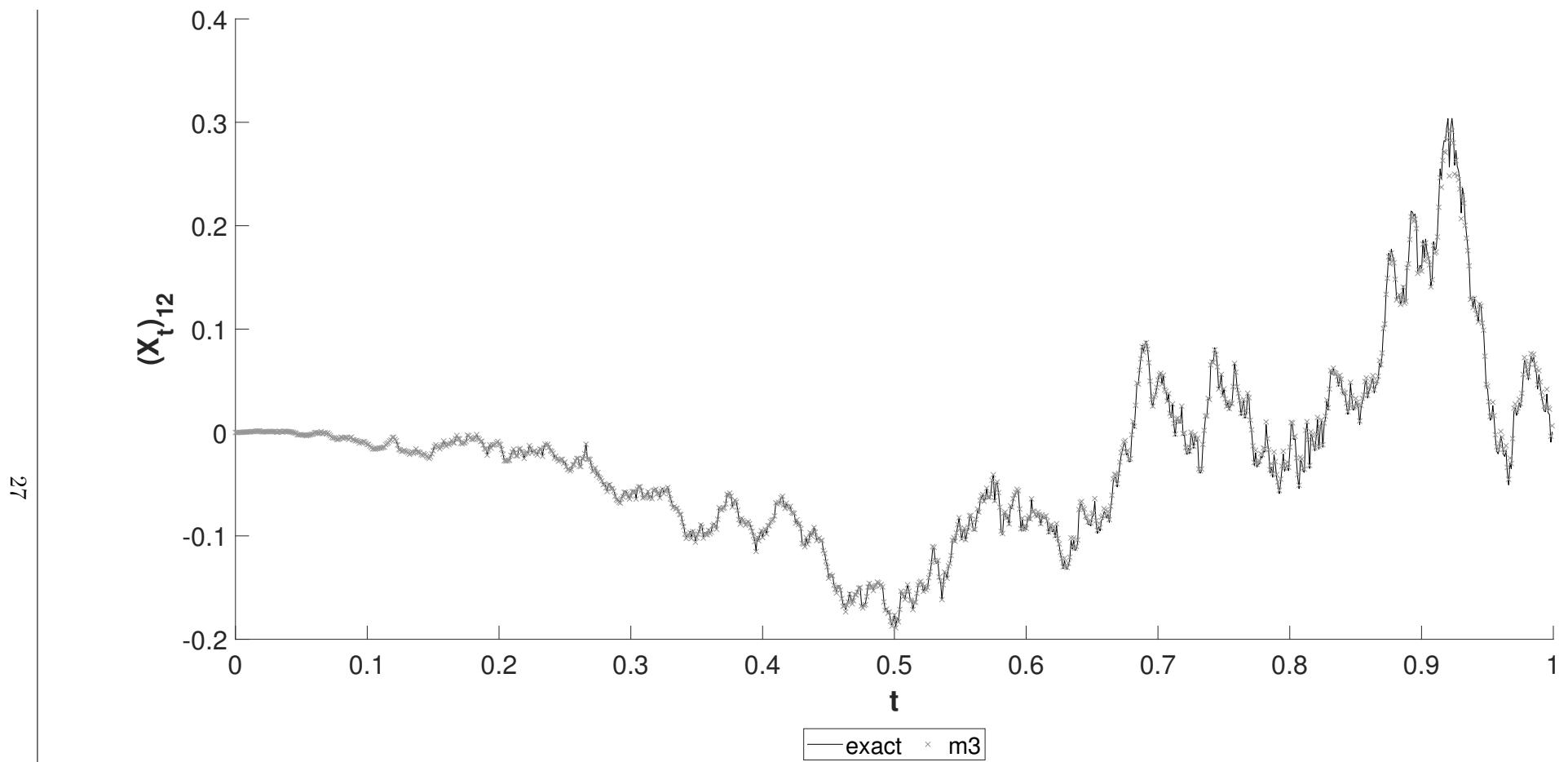


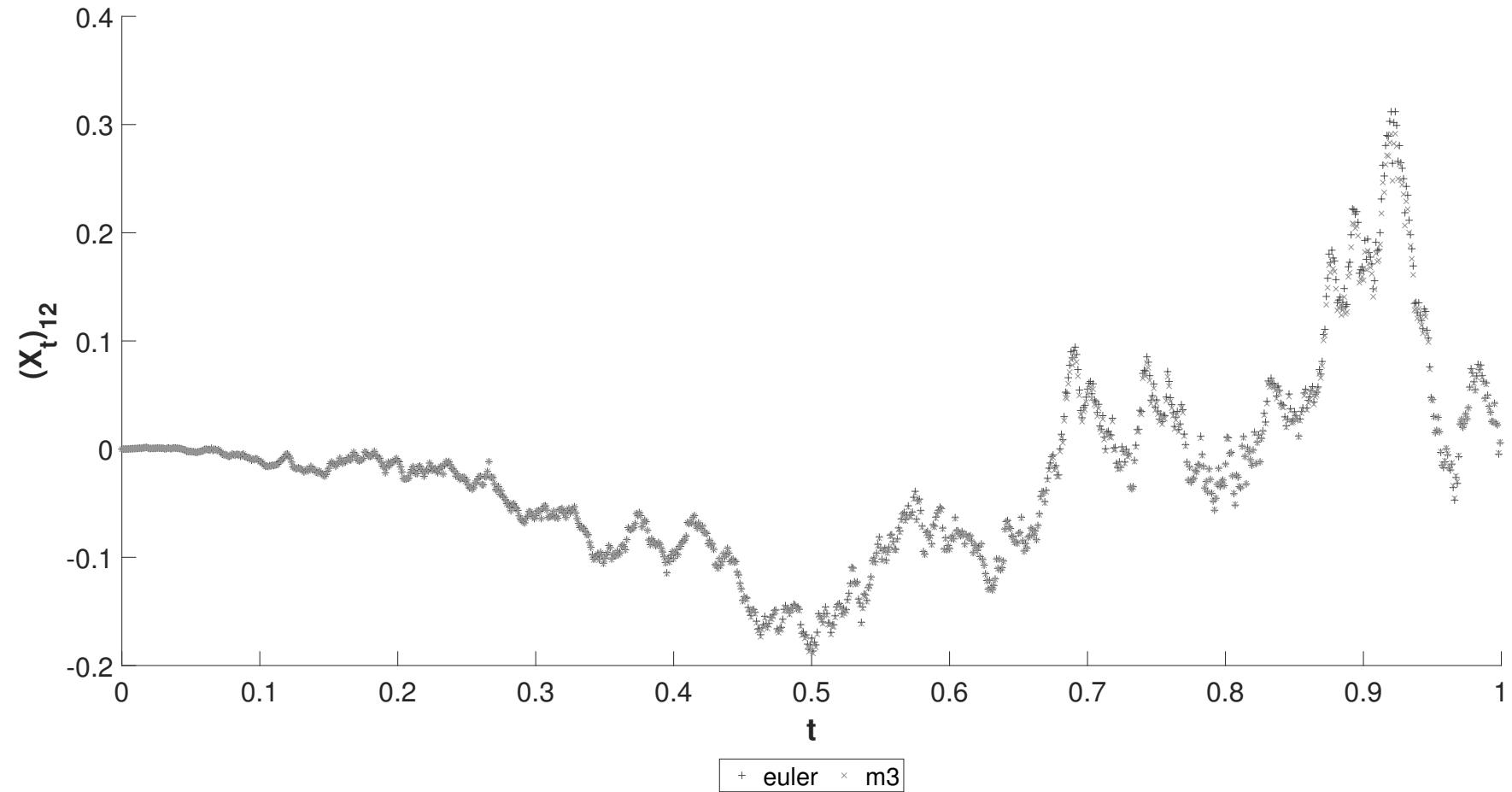


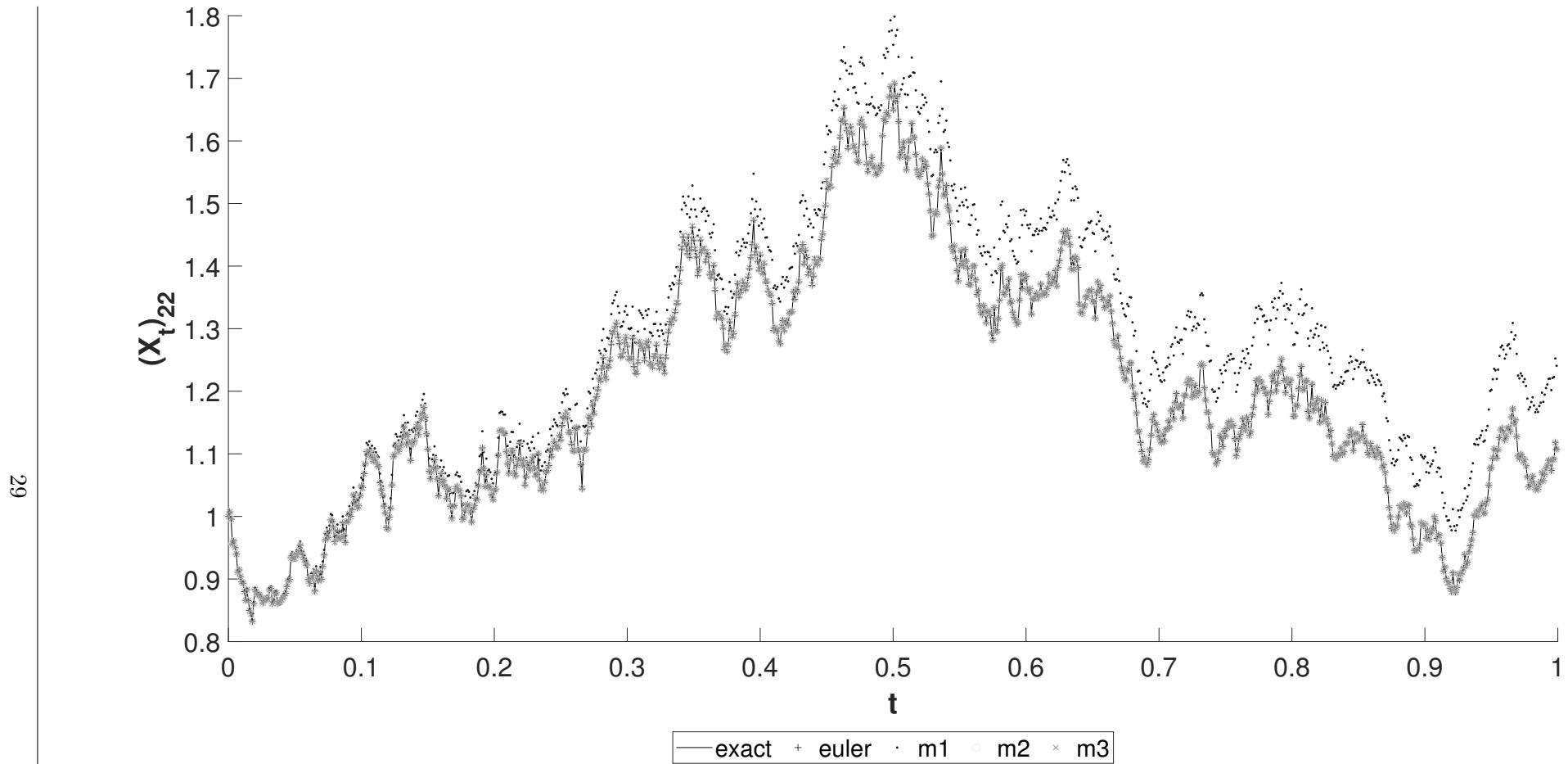


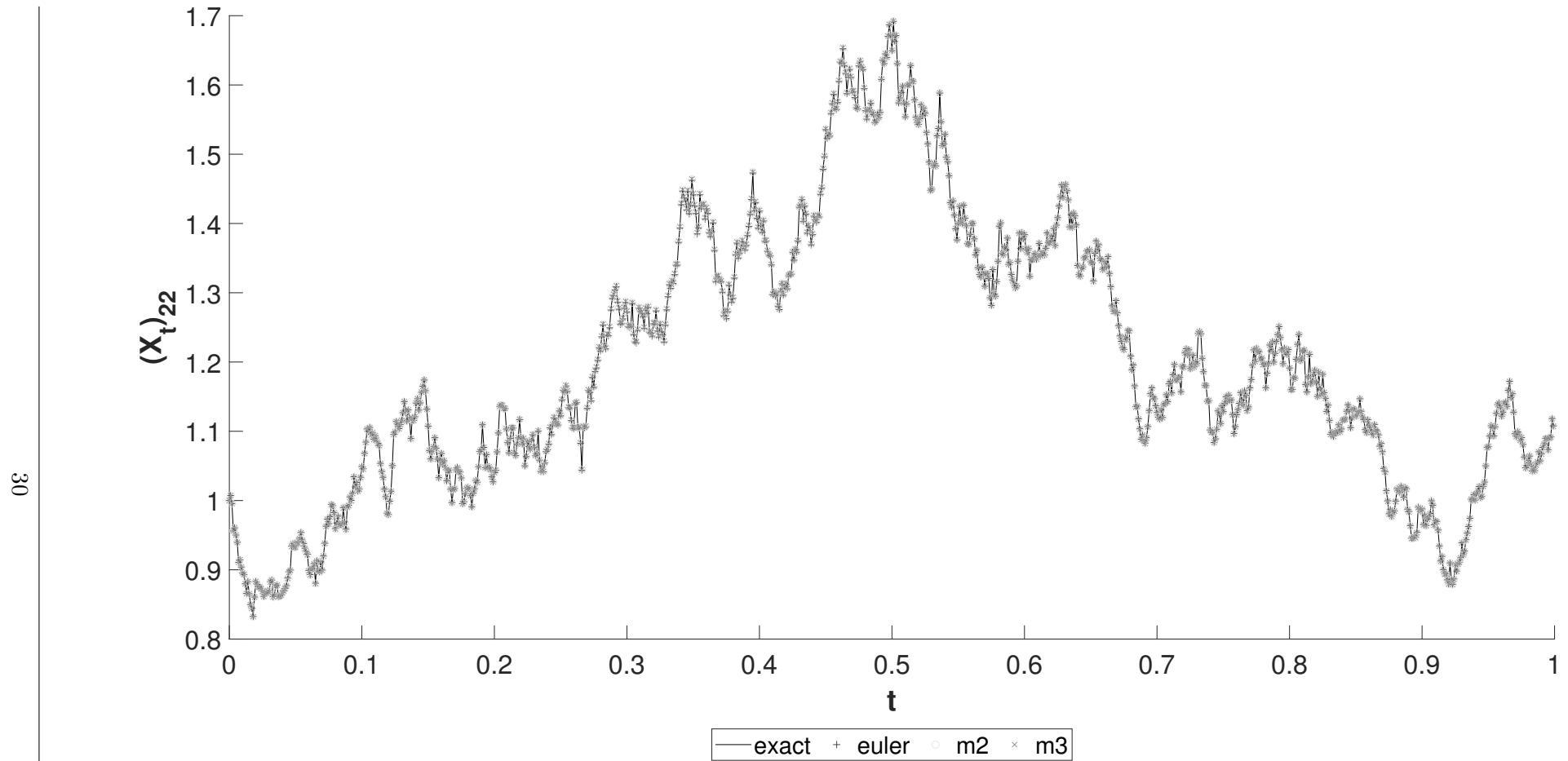


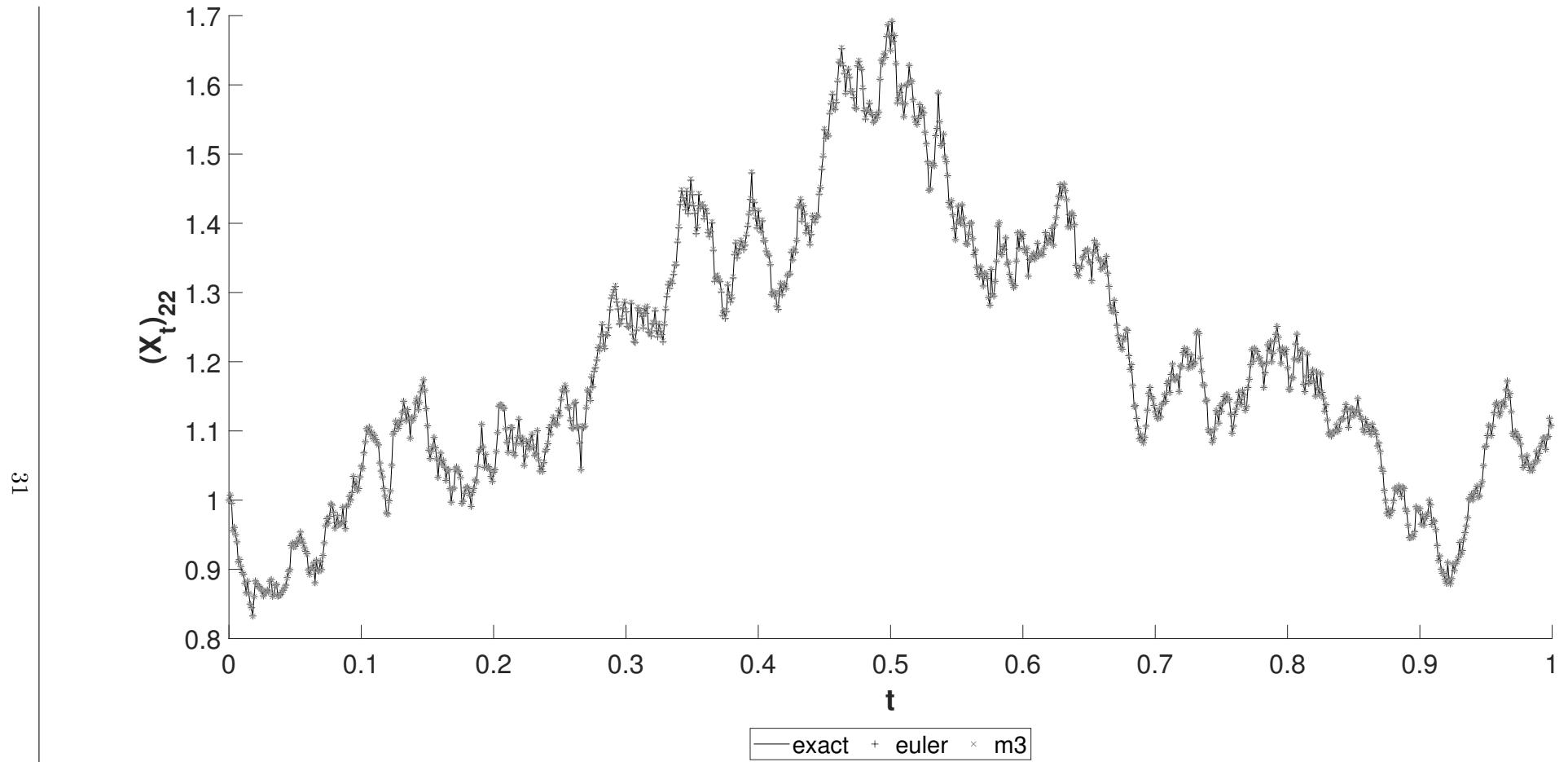


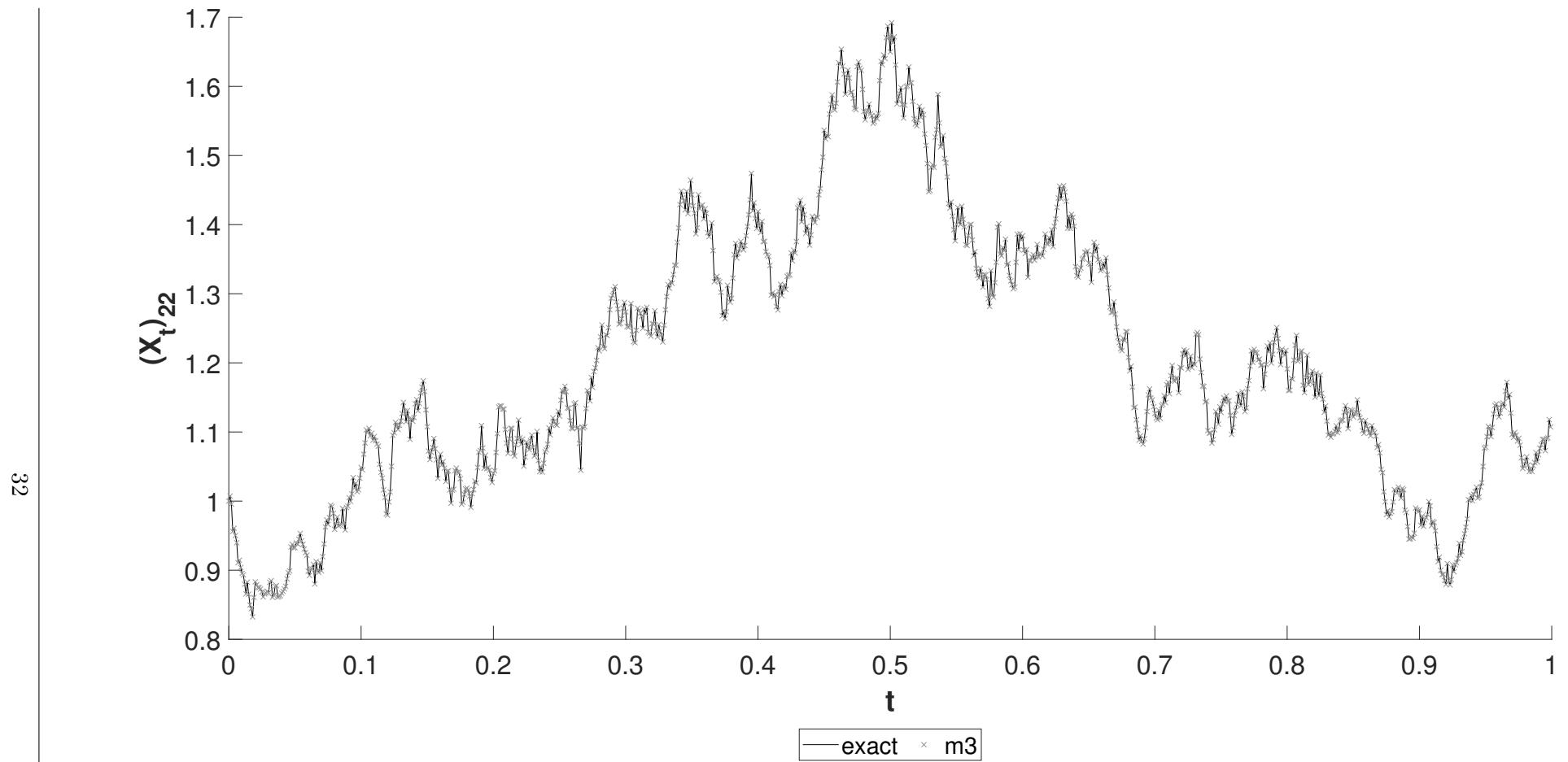


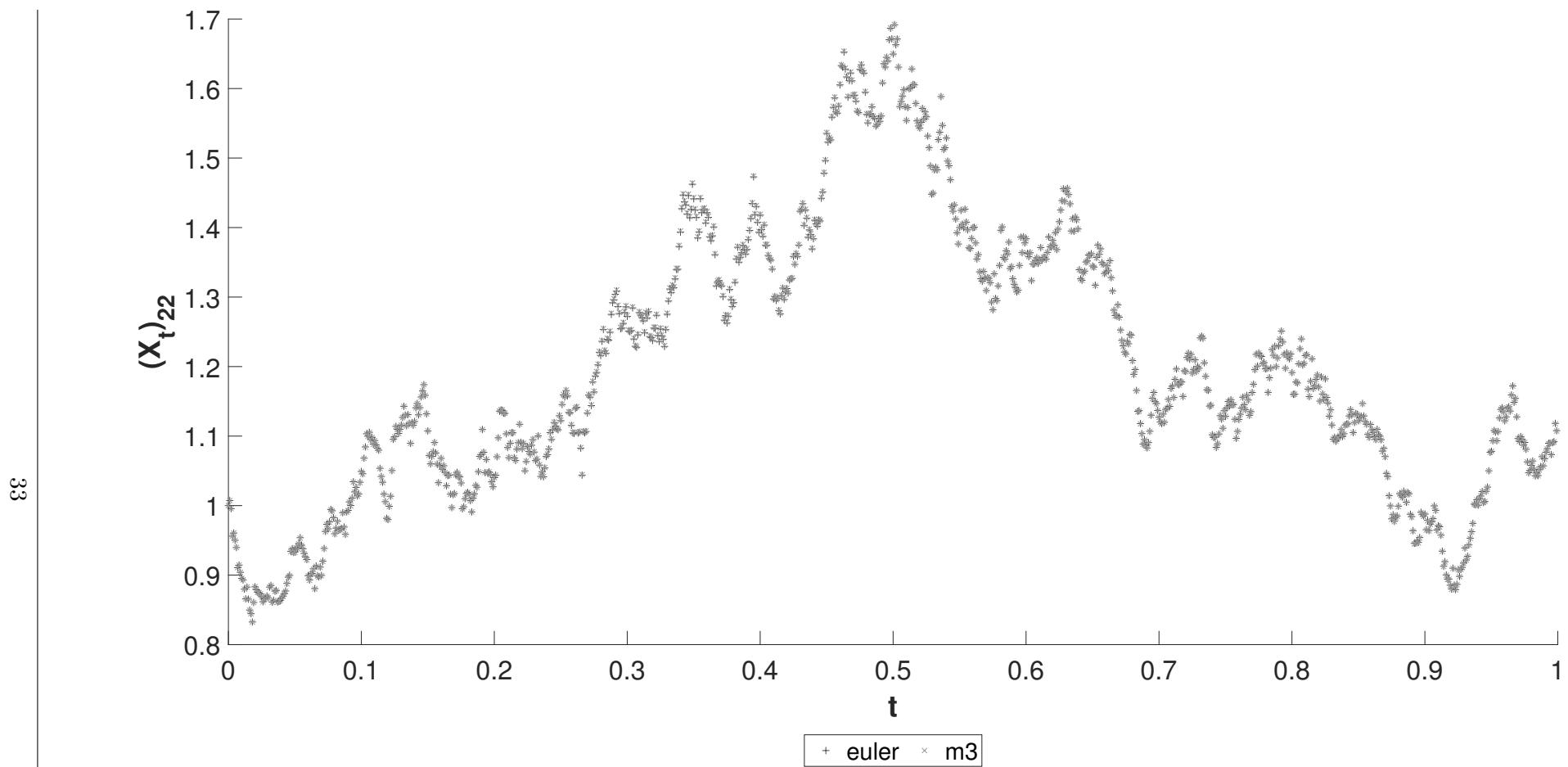












1.5 Error Plots

