

1 Magnus expansion for A, B constant and deterministic

We will concern ourselves with the following SDE:

$$dX_t = BX_t dt + AX_t dW_t$$

with

$$A = \begin{bmatrix} 0.335302 & -0.645492 \\ -0.264419 & 0.634641 \end{bmatrix}$$

and

$$B = \begin{bmatrix} -0.0572262 & 0.0493763 \\ -0.665366 & 0.742744 \end{bmatrix}$$

The spectral norm of A is 1. and the spectral norm of B is 1.

1.1 Parameters

Parameter	value
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t_0	0
T	1
<code>N_fine</code>	10001
N	101
<code>M_fine</code>	1000
M	1000
d	2

1.2 Computational Times

Method	Log	Matrix Exp	Total
euler	0	0	6.90796
m1	0.0109259	0.600221	0.611147
m2	0.0205114	0.55821	0.578721
m3	0.0525858	0.694051	0.746637

1.3 Errors

(i) Errors for $X(1, 1, :, :)$:

(a) Reference method: euler

Error	m1	m2	m3
(abs error) L2	0.256408	0.025446	0.00935141
(rel error) min	0	0	0
(rel error) max	0.3819	0.0474896	0.0152656
(rel error) mean	0.135906	0.0127001	0.00475491

(ii) Errors for $X(1, 2, :, :)$:

(a) Reference method: euler

Error	m1	m2	m3
(abs error) L2	0.480924	0.0605773	0.0155642
(rel error) min	0.0255849	0.00424483	0.00424515
(rel error) max	1.22312	0.167935	0.0407477
(rel error) mean	0.620758	0.0644437	0.0165662

(iii) Errors for $X(2, 1, :, :)$:

(a) Reference method: euler

Error	m1	m2	m3
(abs error) L2	0.297623	0.0223517	0.00492149
(rel error) min	0.0146654	0.00191757	0.00171391
(rel error) max	0.568568	0.0429021	0.00798183
(rel error) mean	0.343587	0.0205081	0.00587289

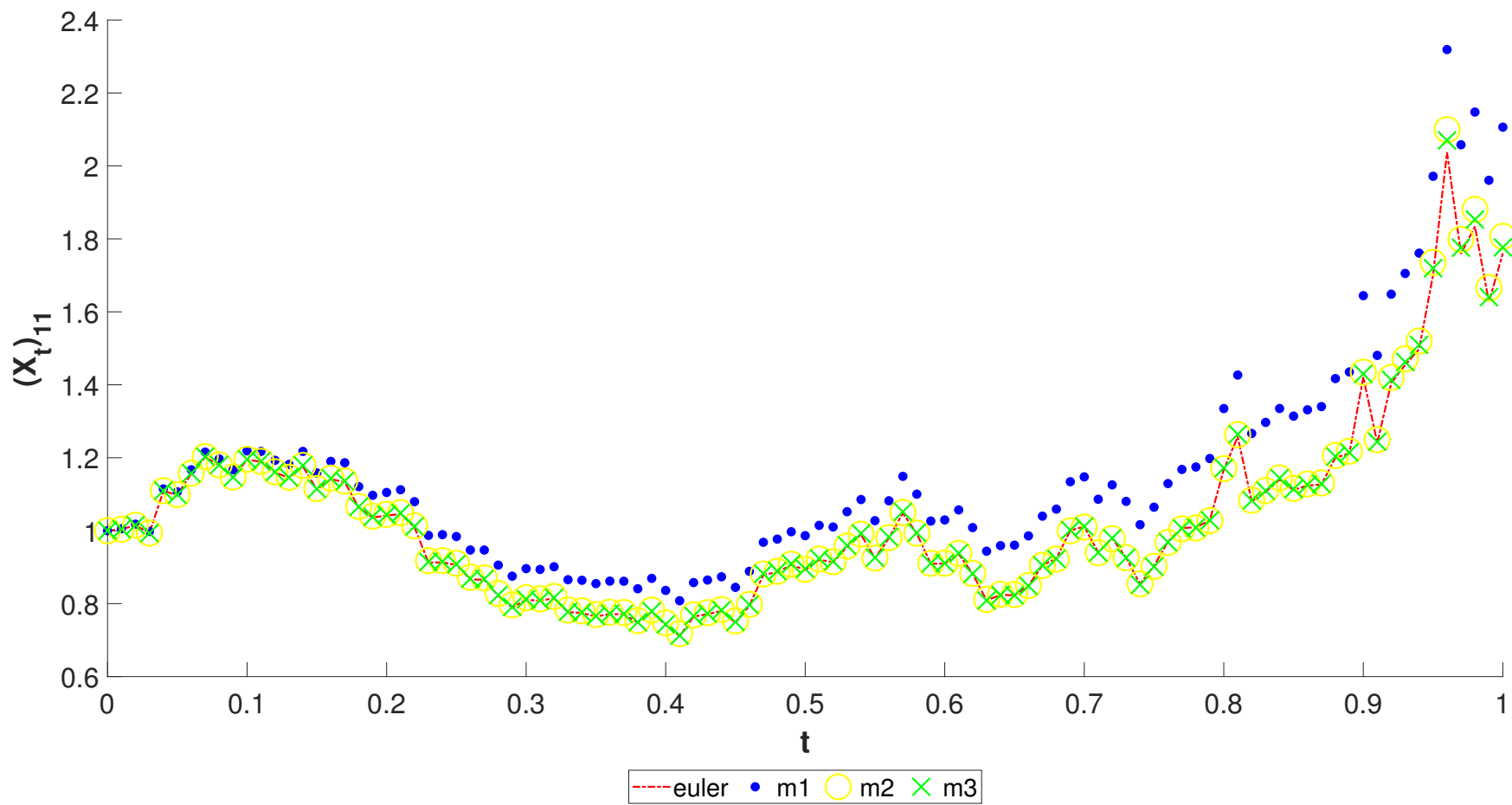
(iv) Errors for $X(2, 2, :, :)$:

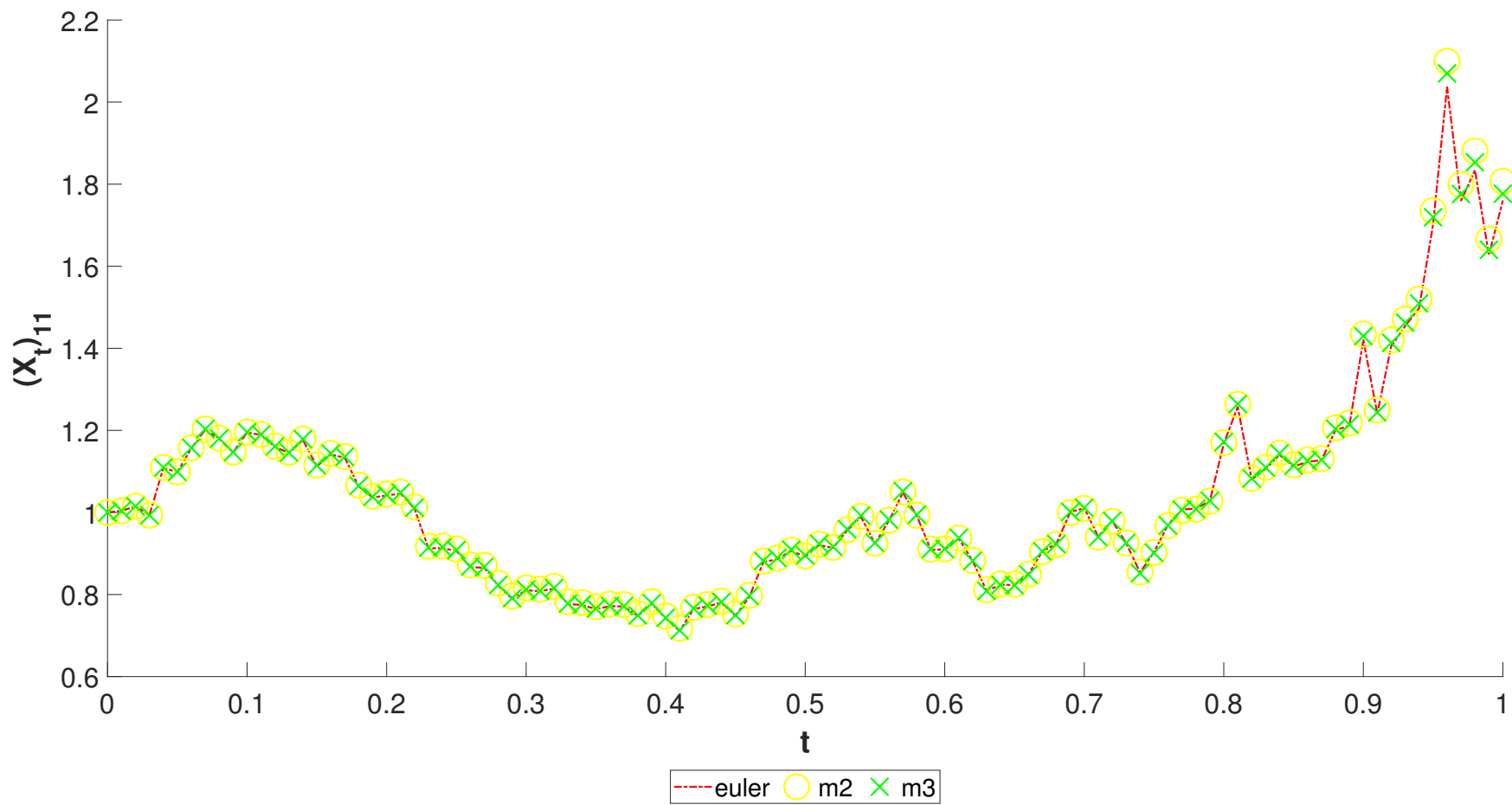
(a) Reference method: euler

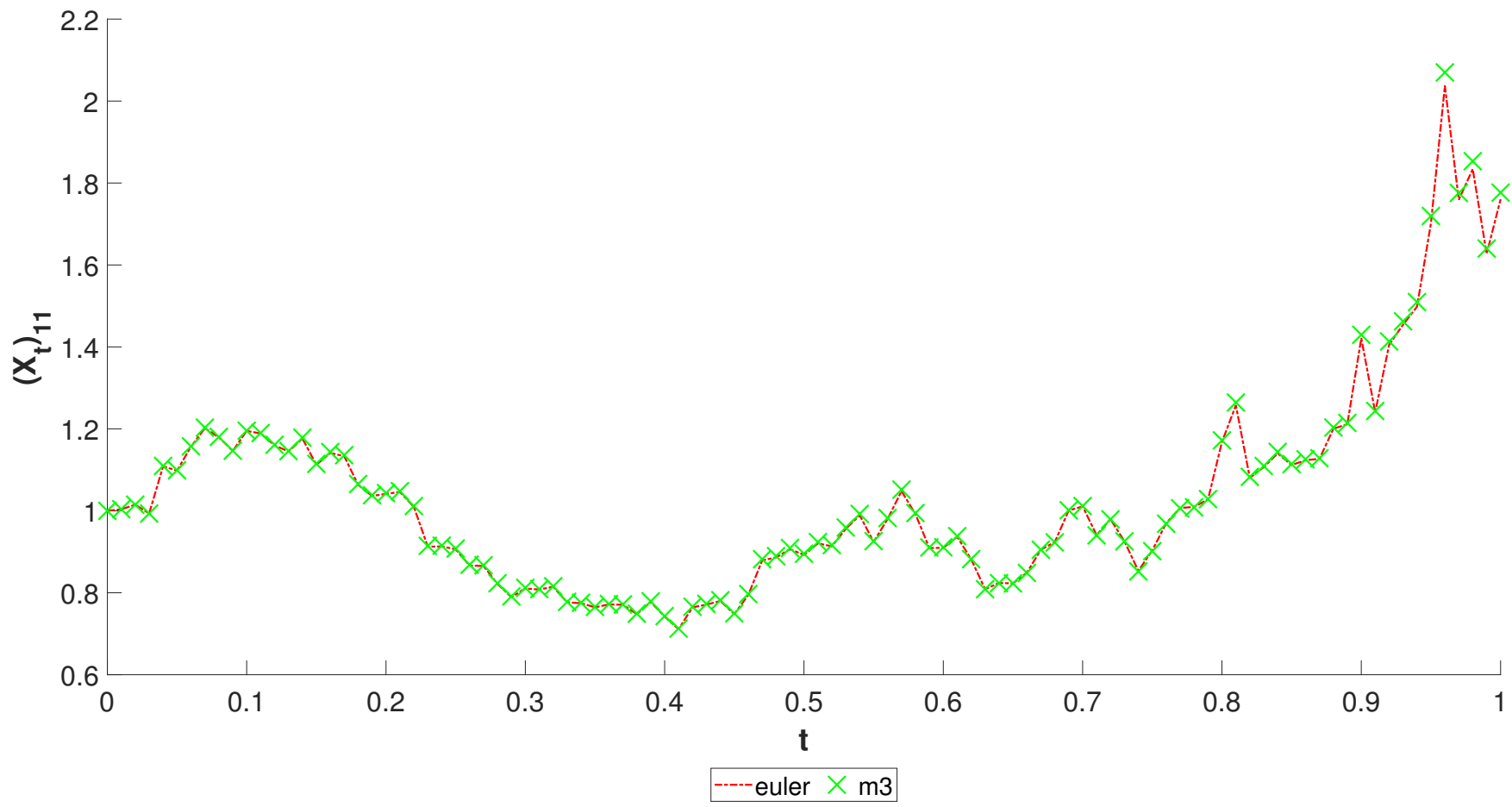
	Error	m1	m2	m3
	(abs error) L2	0.591488	0.0161652	0.0116628
	(rel error) min	0	0	0
	(rel error) max	0.424588	0.0106245	0.00748974
	(rel error) mean	0.188398	0.00411253	0.00353543
(v)	Total Errors:			
(a)	Reference method: euler			

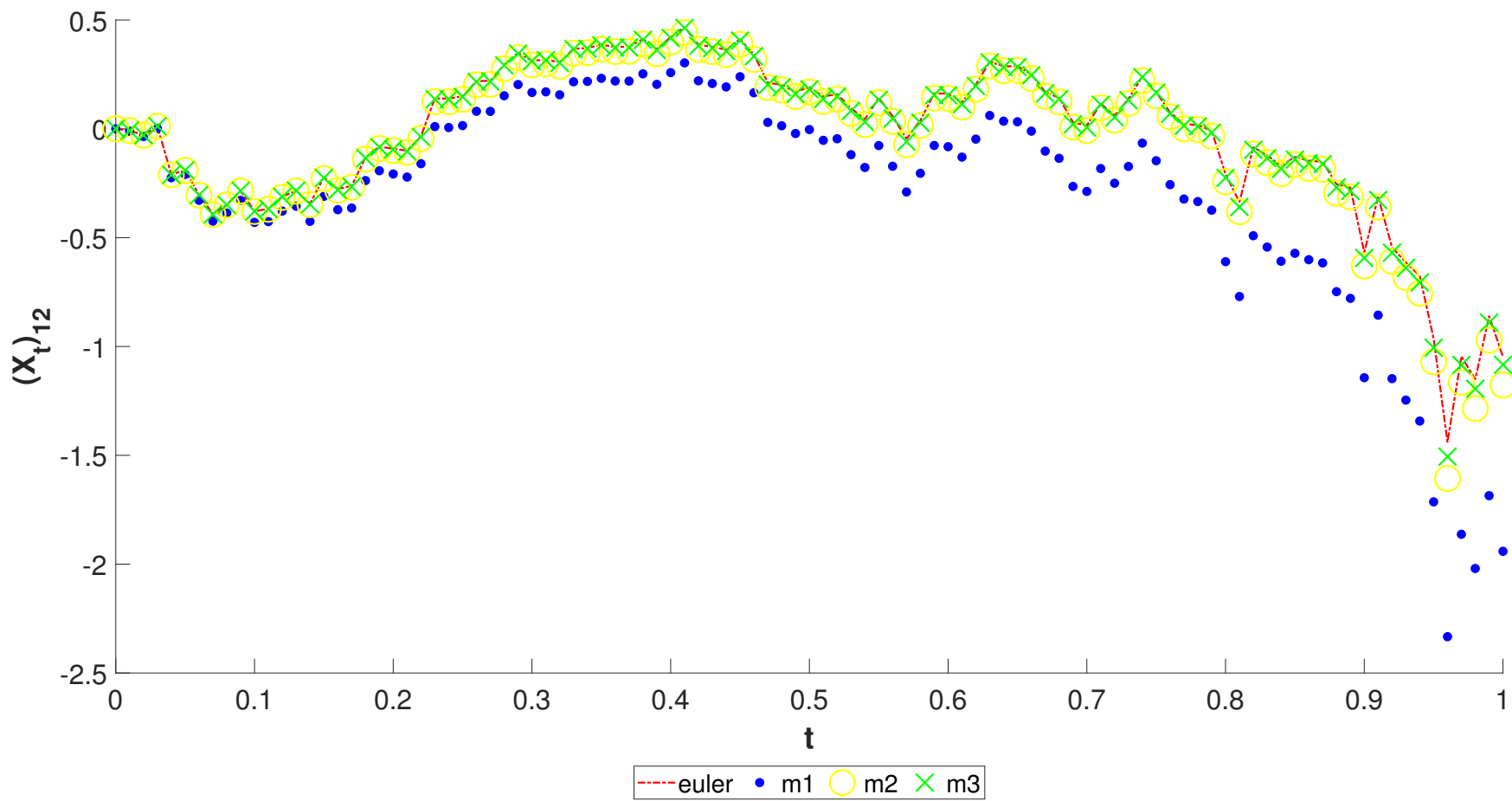
Method	$\mathbb{E}[Err_1]$
m1	20.8 %
m2	1.53 %
m3	0.532 %

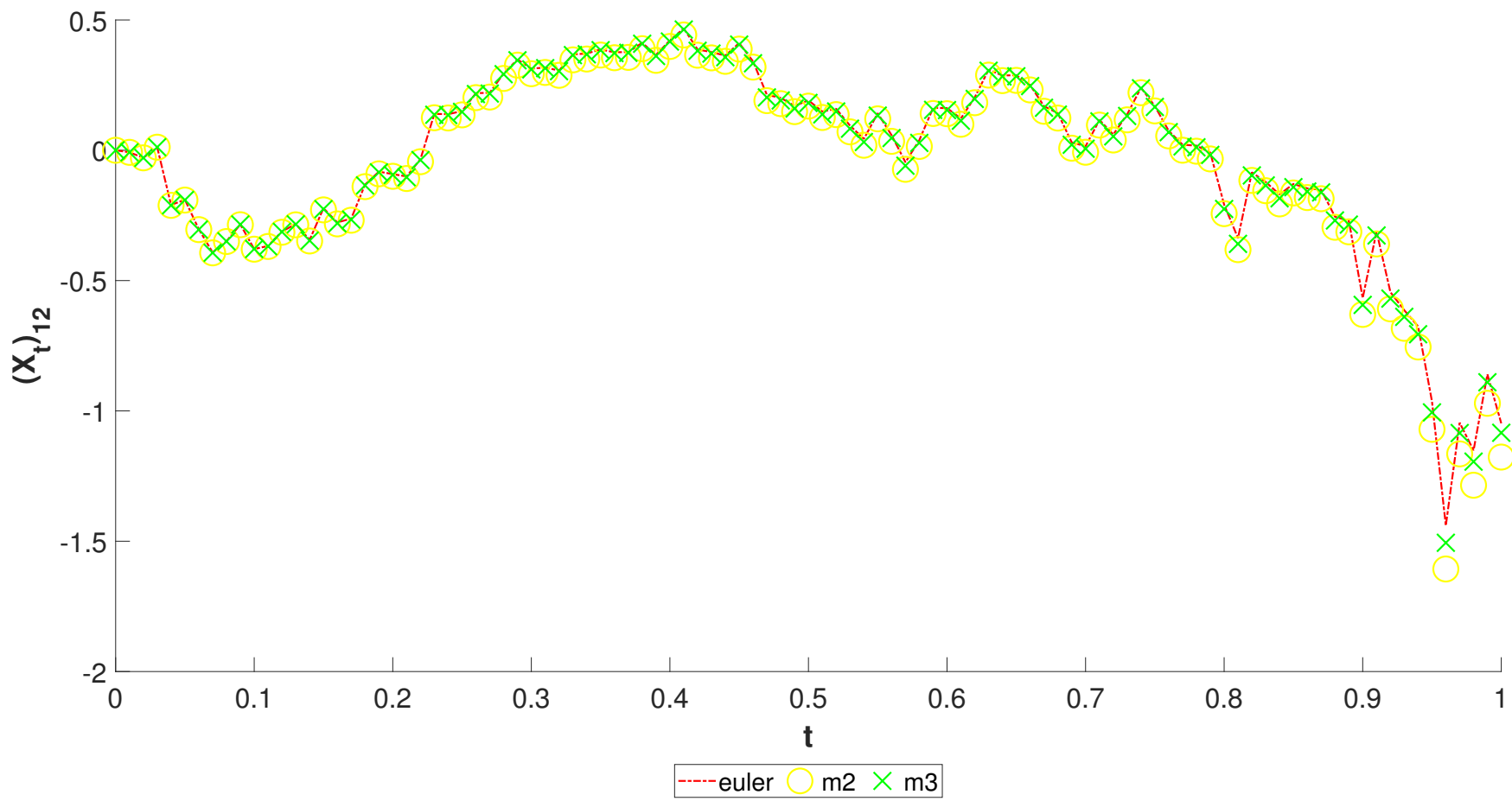
1.4 Plots

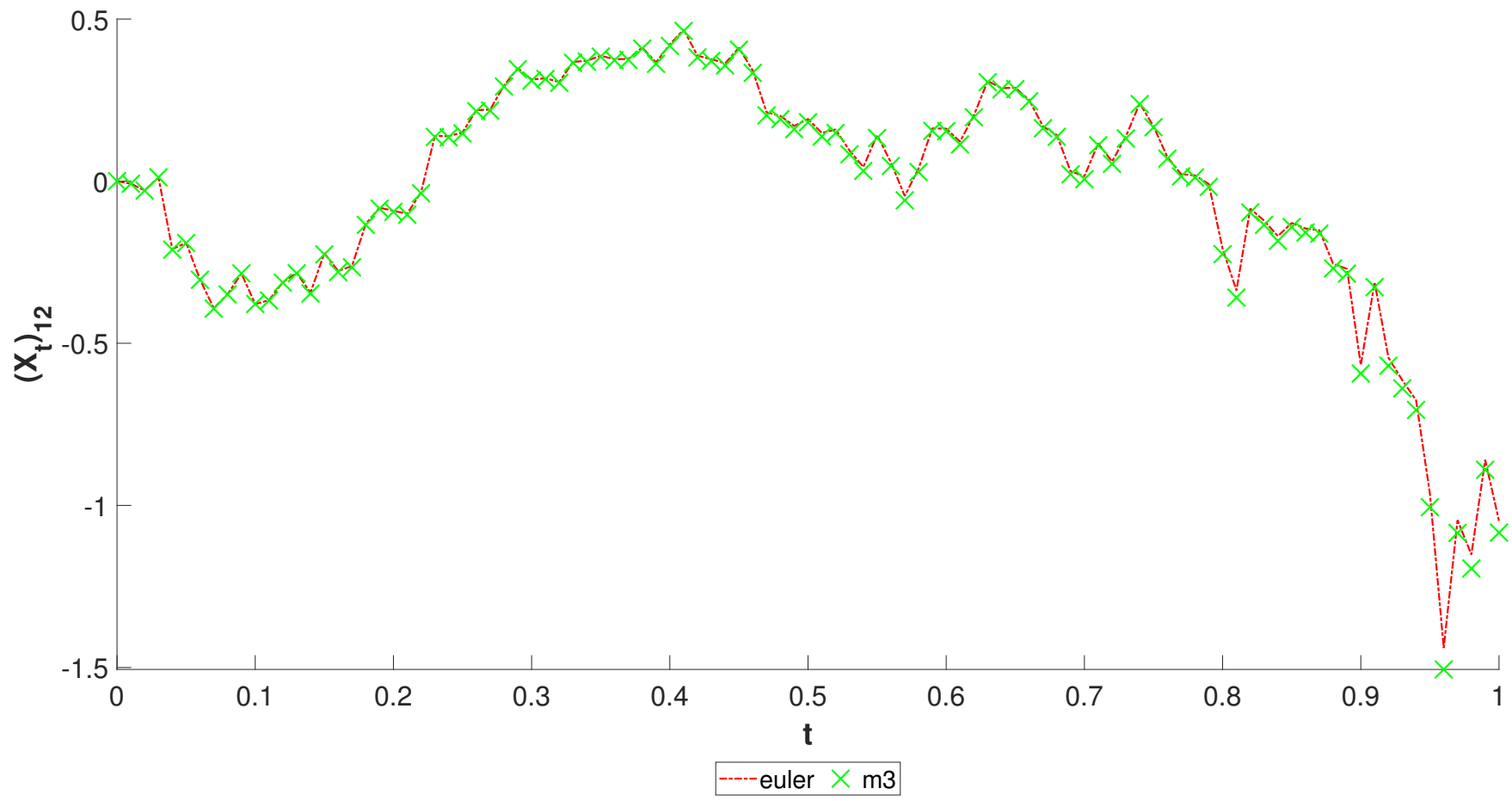


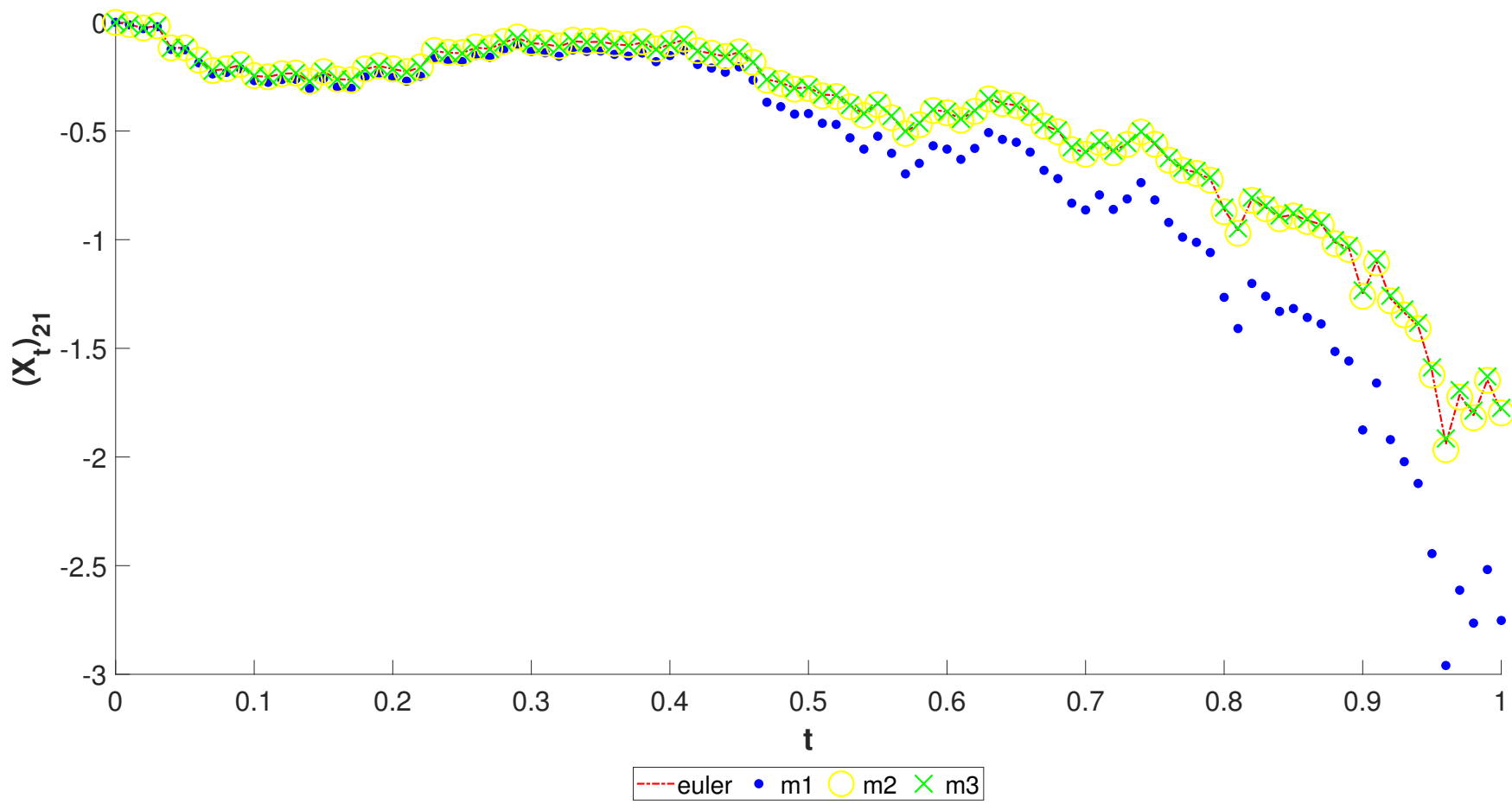


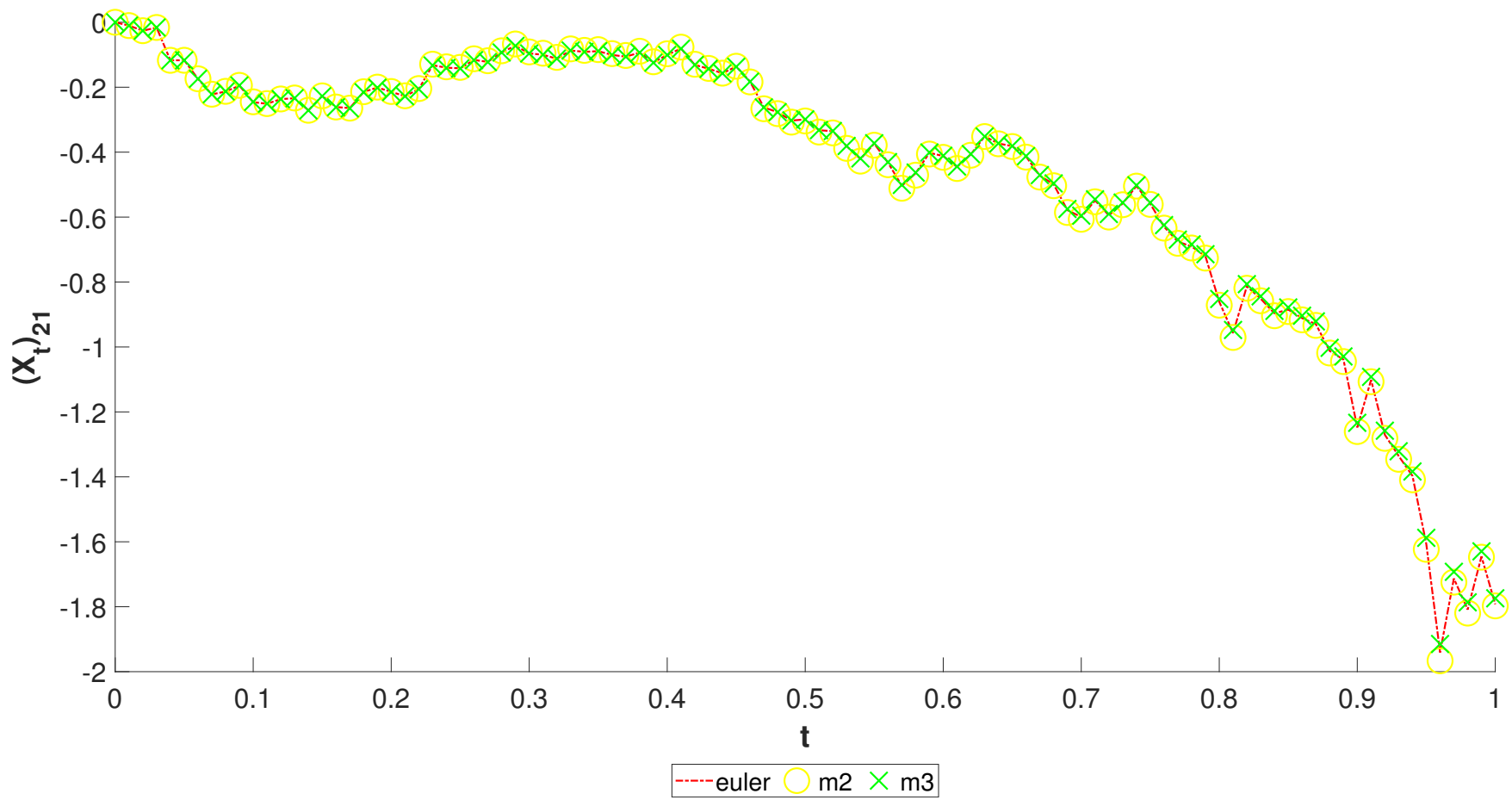


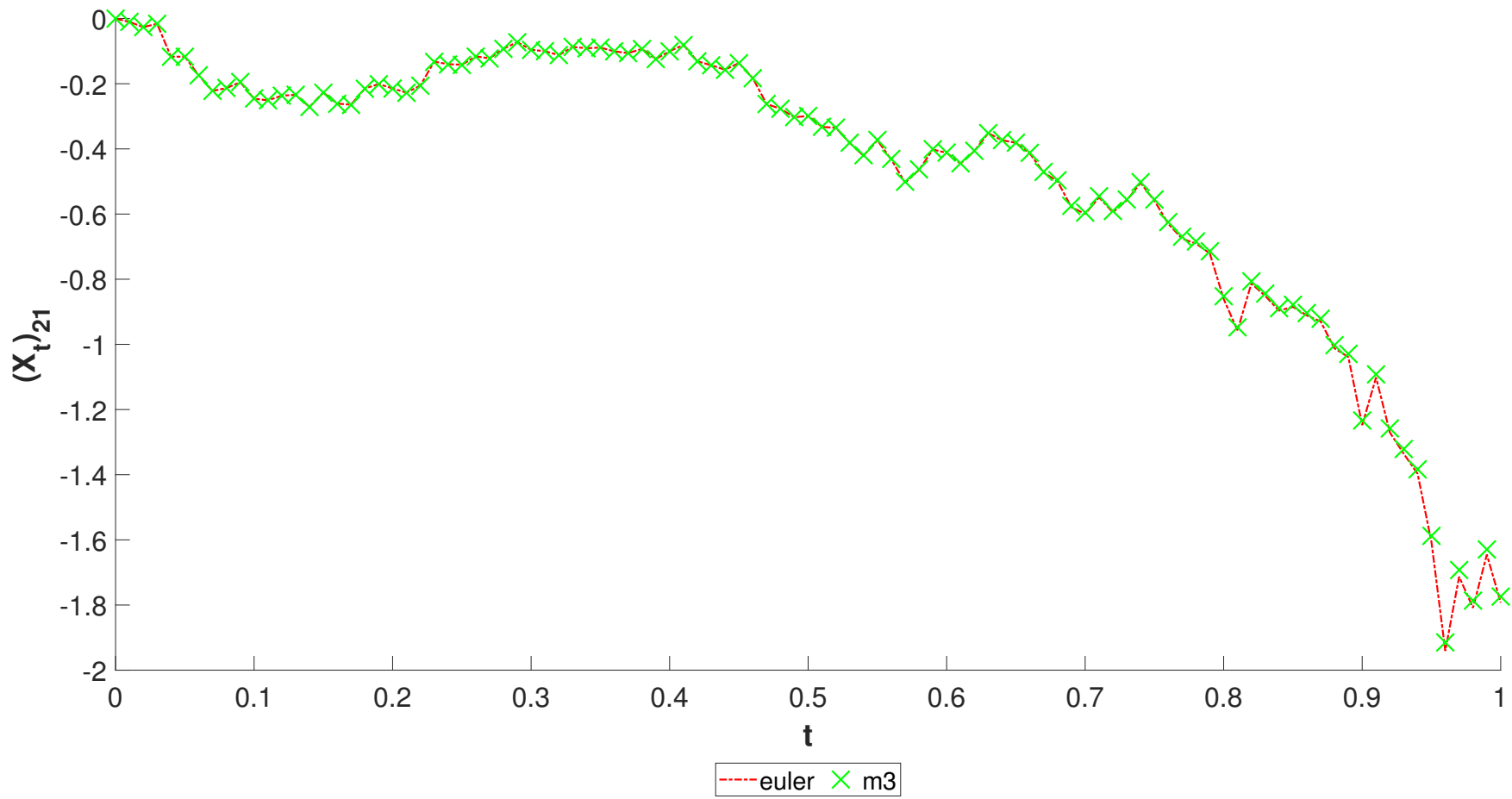


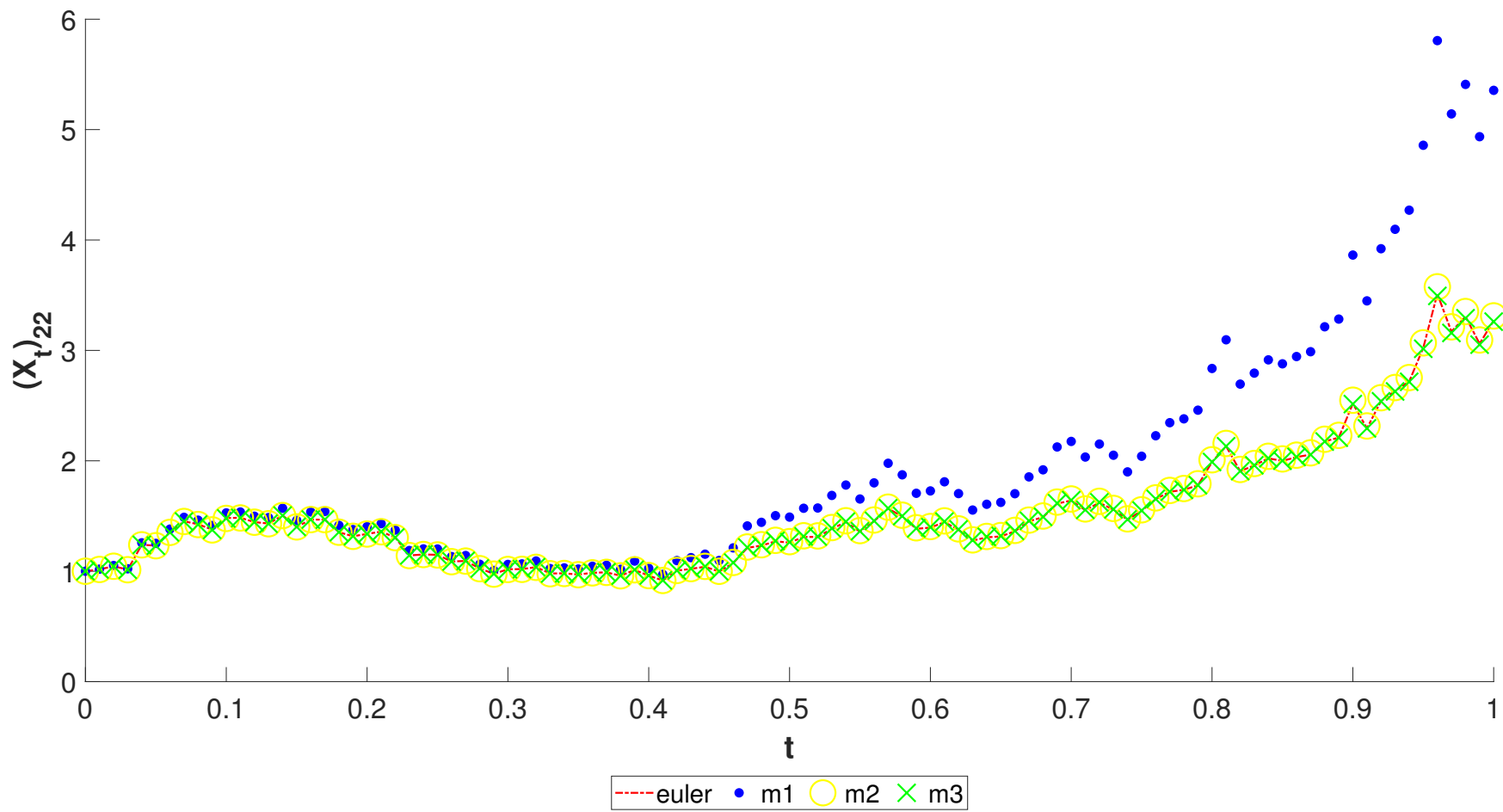


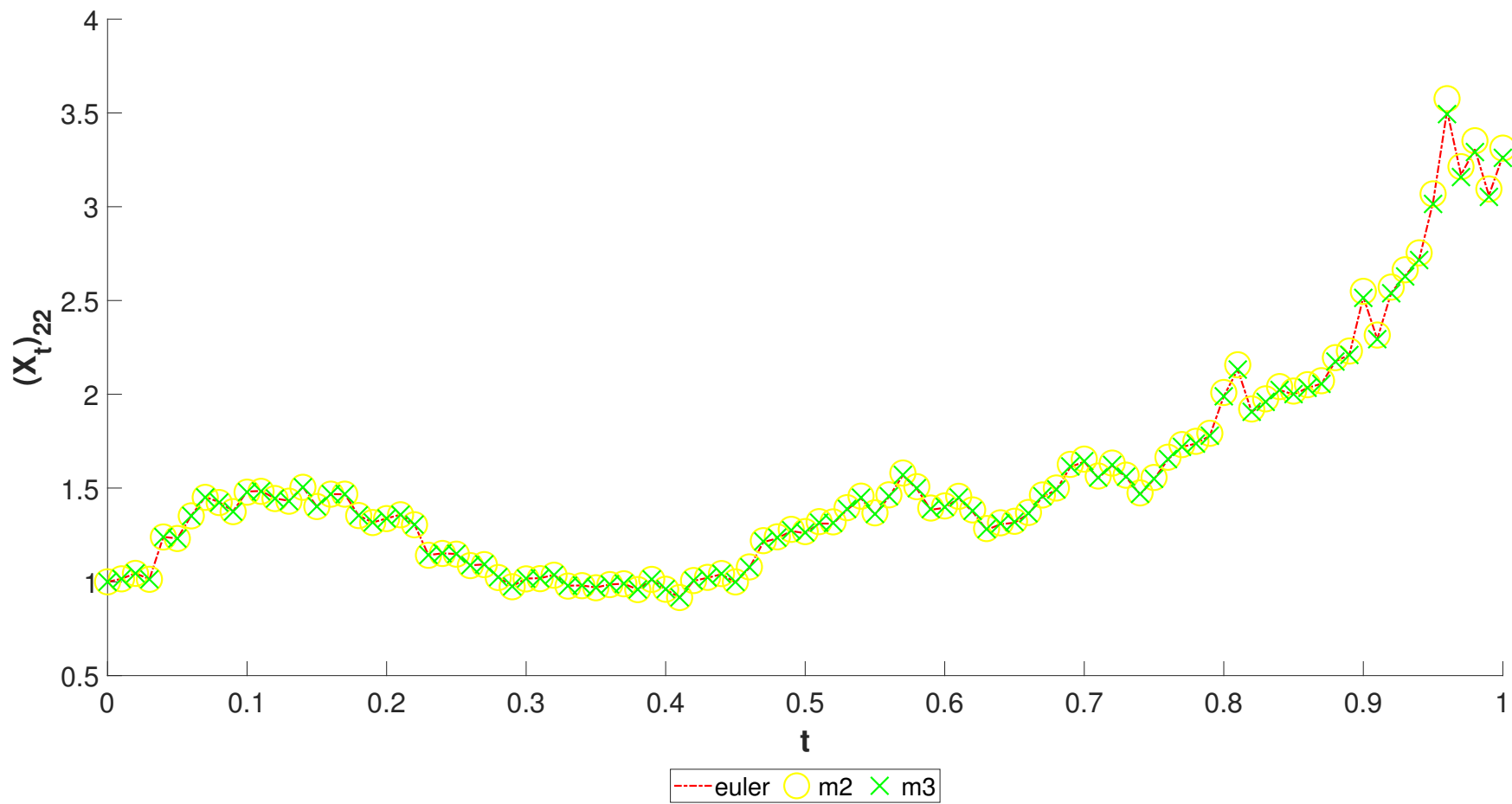


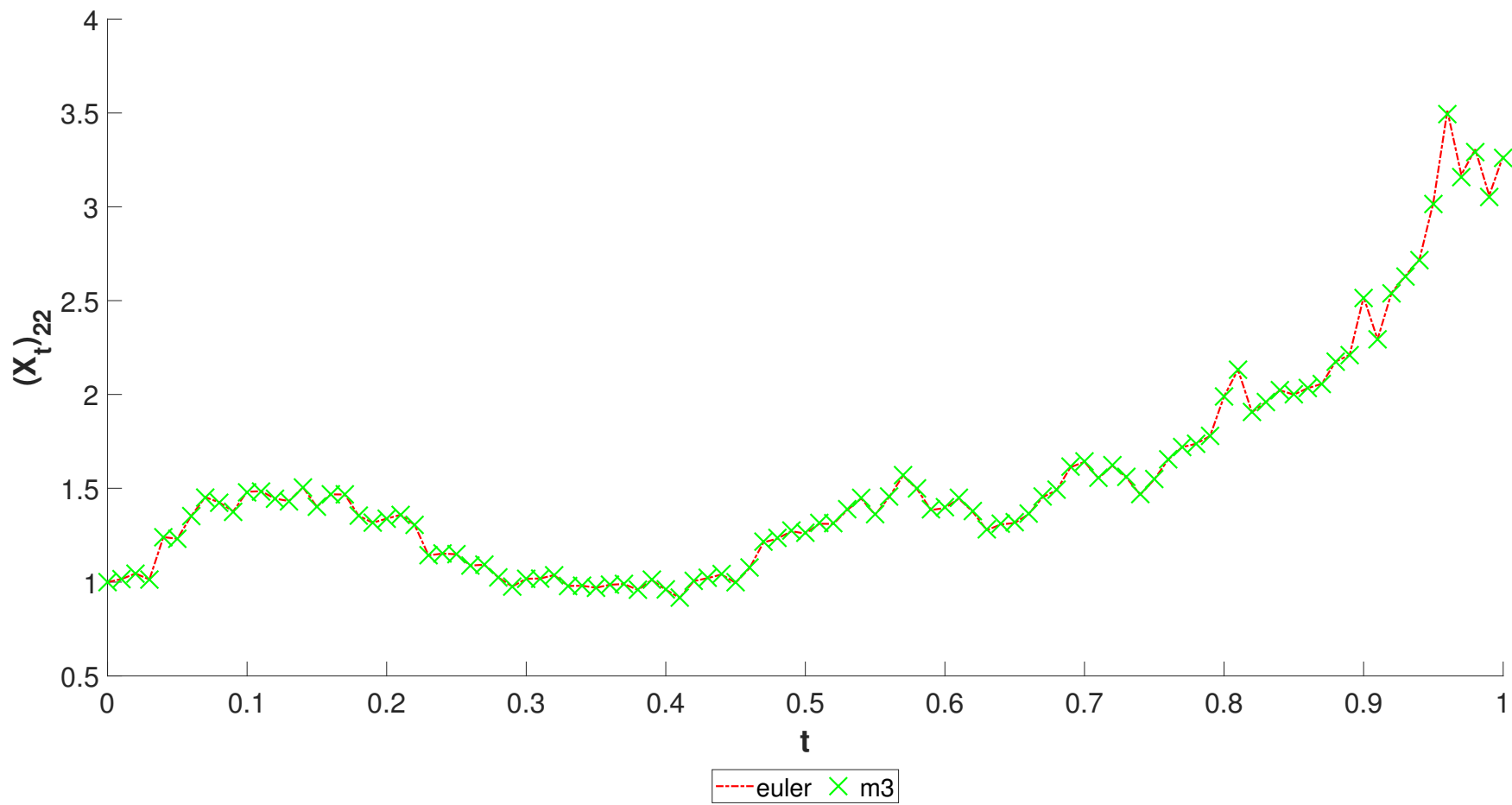












1.5 Error Plots

