

1 Magnus expansion for A , B constant and deterministic

We will concern ourselves with the following SDE:

$$dX_t = BX_t dt + AX_t dW_t$$

with

$$A = \begin{bmatrix} 0.335302 & -0.645492 \\ -0.264419 & 0.634641 \end{bmatrix}$$

and

$$B = \begin{bmatrix} -0.0572262 & 0.0493763 \\ -0.665366 & 0.742744 \end{bmatrix}$$

The spectral norm of A is 1. and the spectral norm of B is 1.

1.1 Parameters

Parameter	value
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t_0	0
T	1
N_{fine}	10000
N	1000
M_{fine}	1000
M	1000
d	2

1.2 Computational Times

Method	Log	Matrix Exp	Total
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euler	0	0	7.02831
m1	0.0260201	4.7858	4.81182
m2	0.0494365	4.86597	4.91541
m3	0.0827815	4.75041	4.8332

1.3 Errors

(i) Errors for $X(1, 1, :, :)$:

(a) Reference method: euler

Error	m1	m2	m3
(abs error) L2	0.239192	0.0248999	0.0100847
(rel error) min	0	0	0
(rel error) max	0.408967	0.0540652	0.0169514
(rel error) mean	0.13977	0.0131549	0.00439208

(ii) Errors for $X(1, 2, :, :)$:

(a) Reference method: euler

Error	m1	m2	m3
(abs error) L2	0.442901	0.0609911	0.0153019
(rel error) min	0.0116113	0.00219708	0.0021864
(rel error) max	1.17432	0.166052	0.0425755
(rel error) mean	0.632279	0.0670159	0.0165127

(iii) Errors for $X(2, 1, :, :)$:

(a) Reference method: euler

Error	m1	m2	m3
(abs error) L2	0.275927	0.0223561	0.00477048
(rel error) min	0.0145007	0.00191422	0.00190031
(rel error) max	0.56626	0.0432767	0.00831428
(rel error) mean	0.338108	0.0208735	0.0060625

(iv) Errors for $X(2, 2, :, :)$:

(a) Reference method: euler

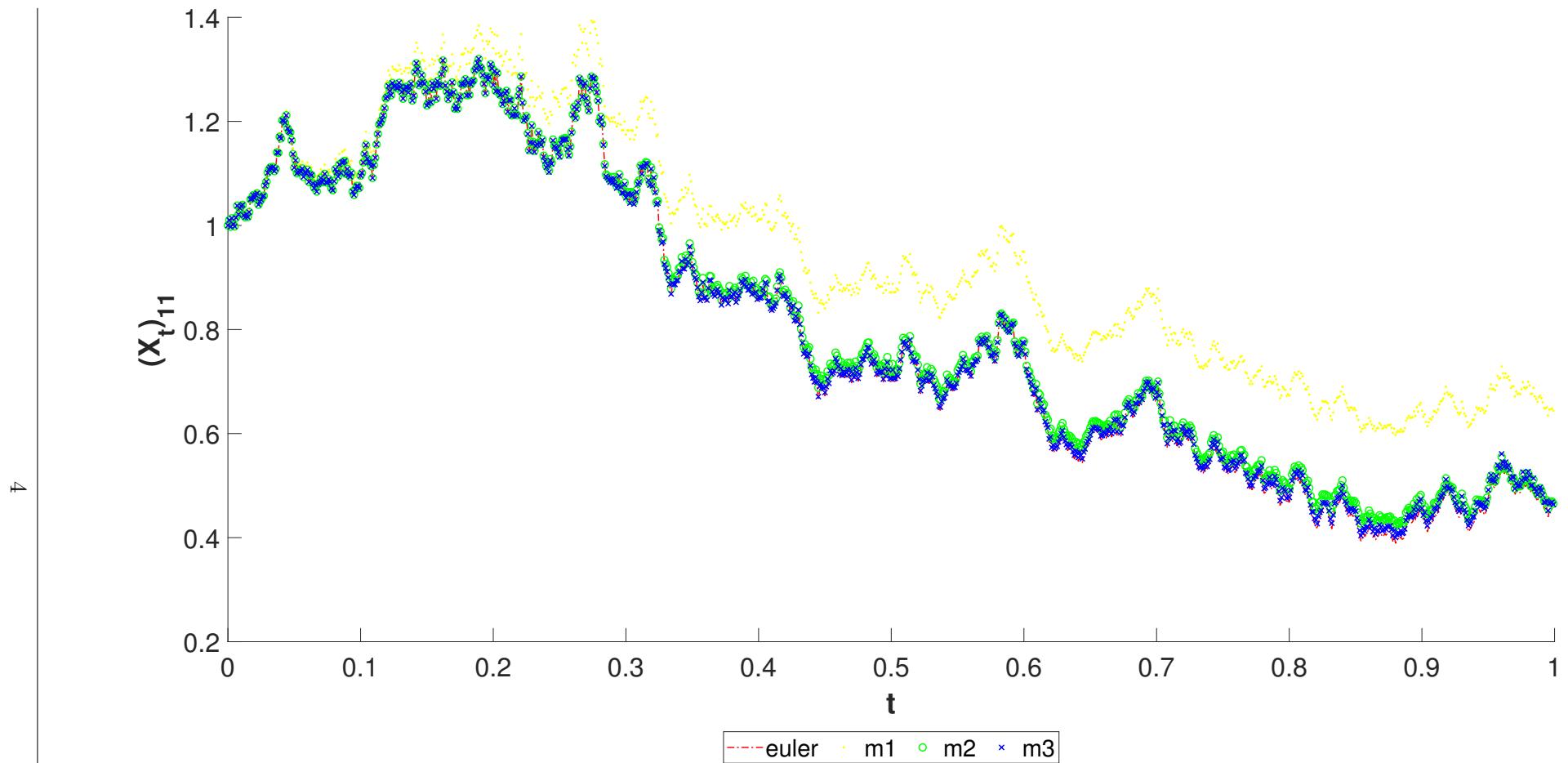
Error	m1	m2	m3
(abs error) L2	0.546056	0.0141749	0.0107696
(rel error) min	0	0	0
(rel error) max	0.413116	0.00981888	0.00737206
(rel error) mean	0.183406	0.00394856	0.00338999

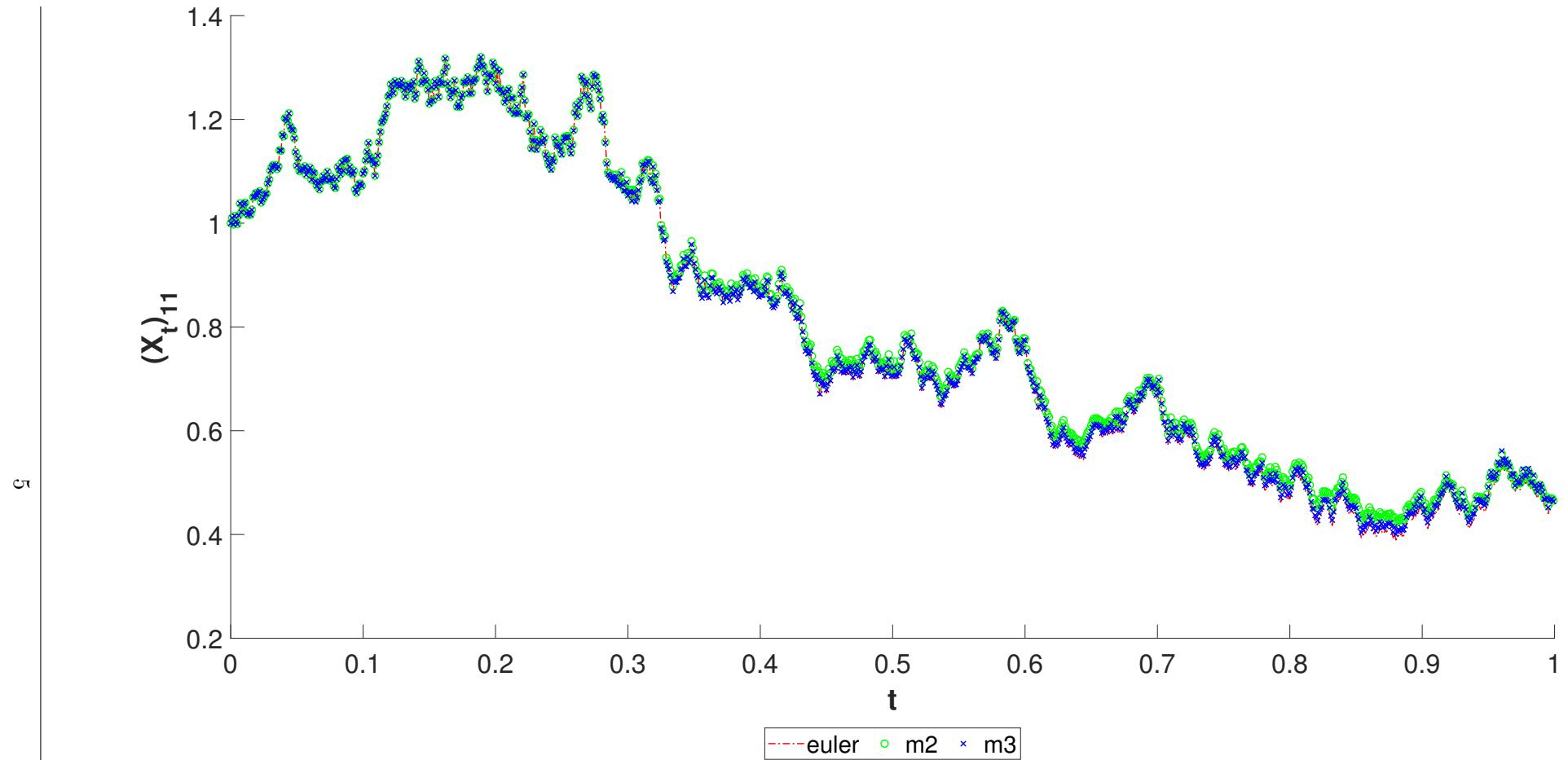
(v) Total Errors:

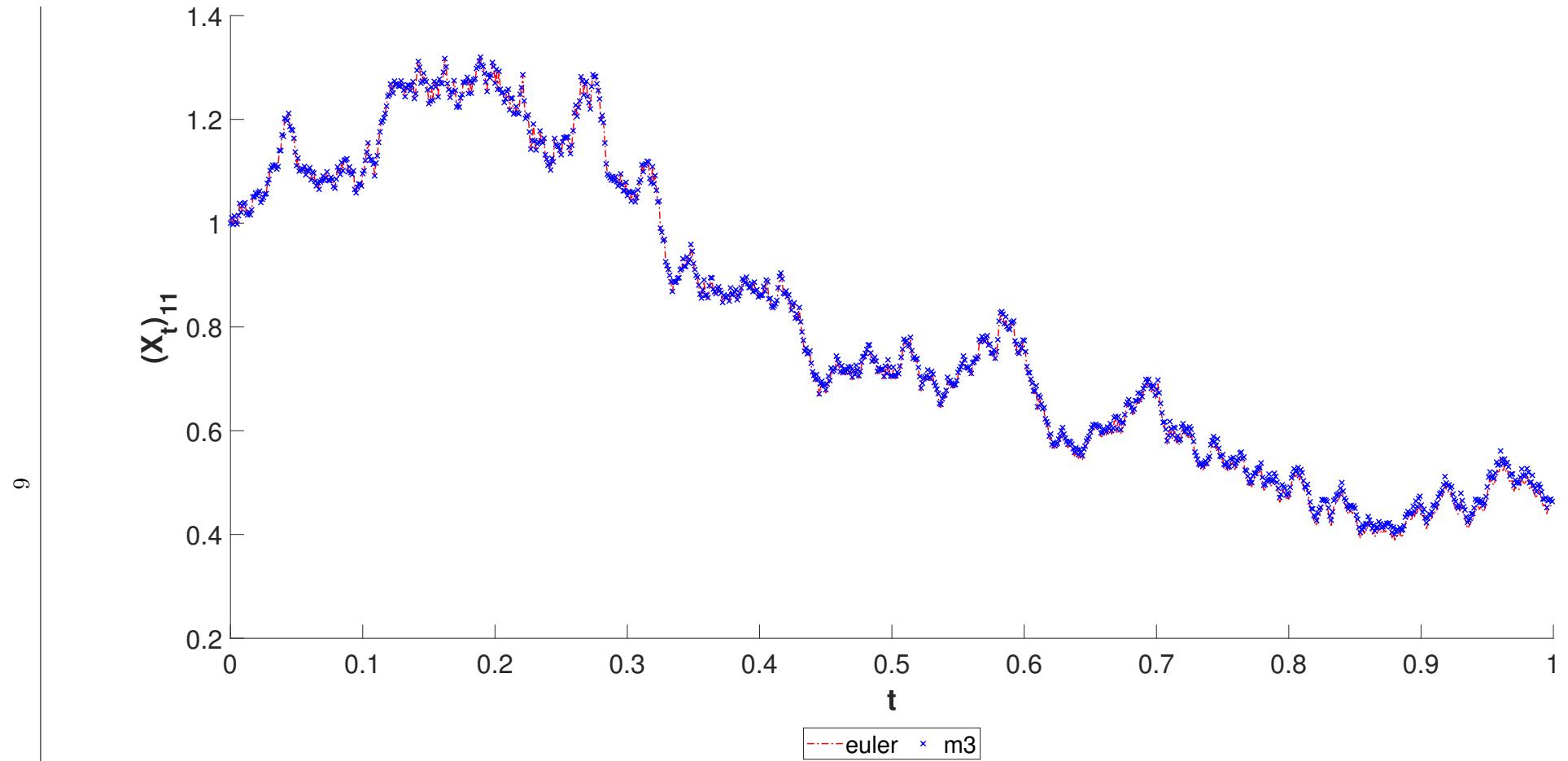
(a) Reference method: euler

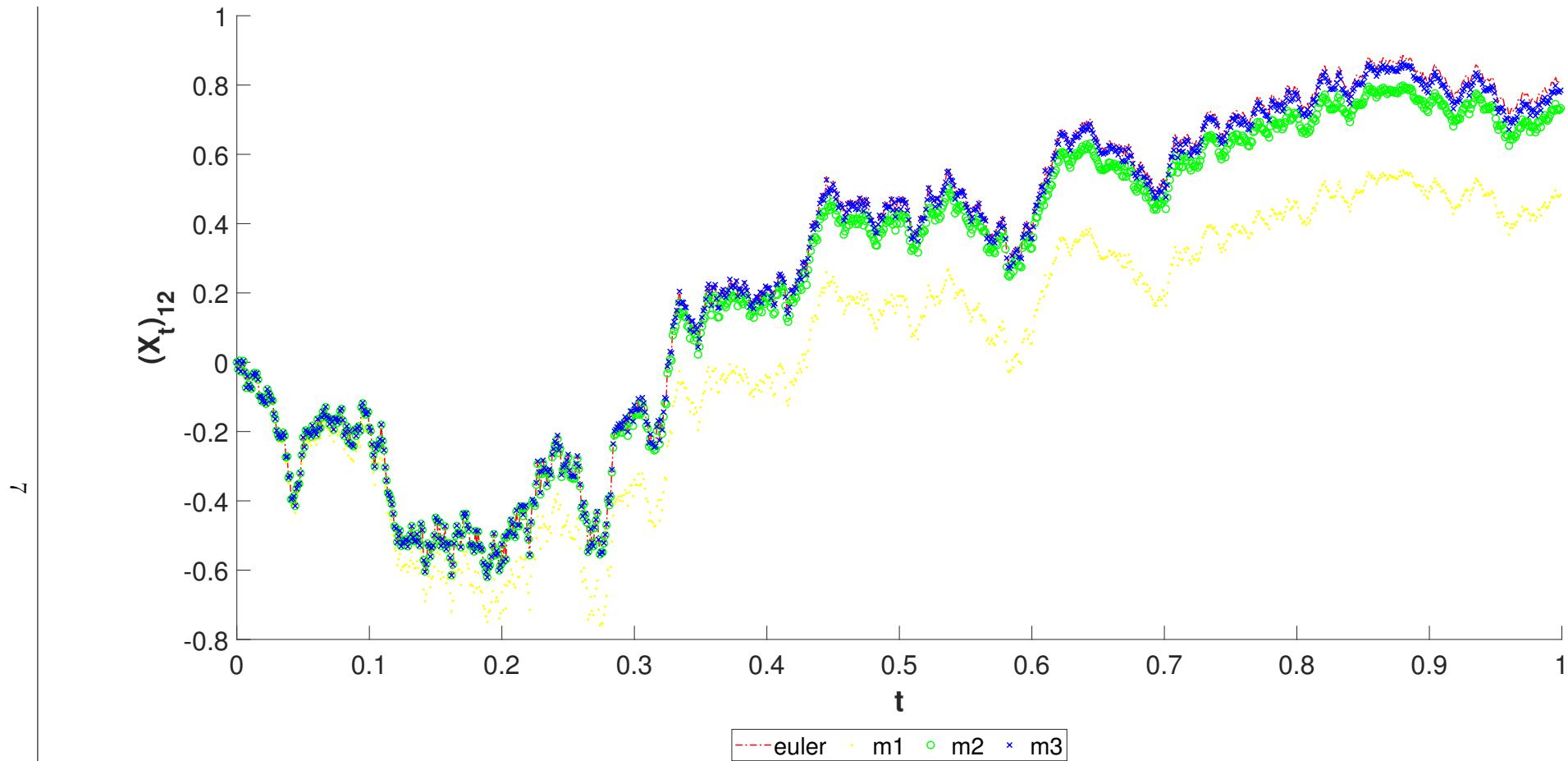
Method	$\mathbb{E}[Err_1]$
m1	20.7 %
m2	1.58 %
m3	0.507 %

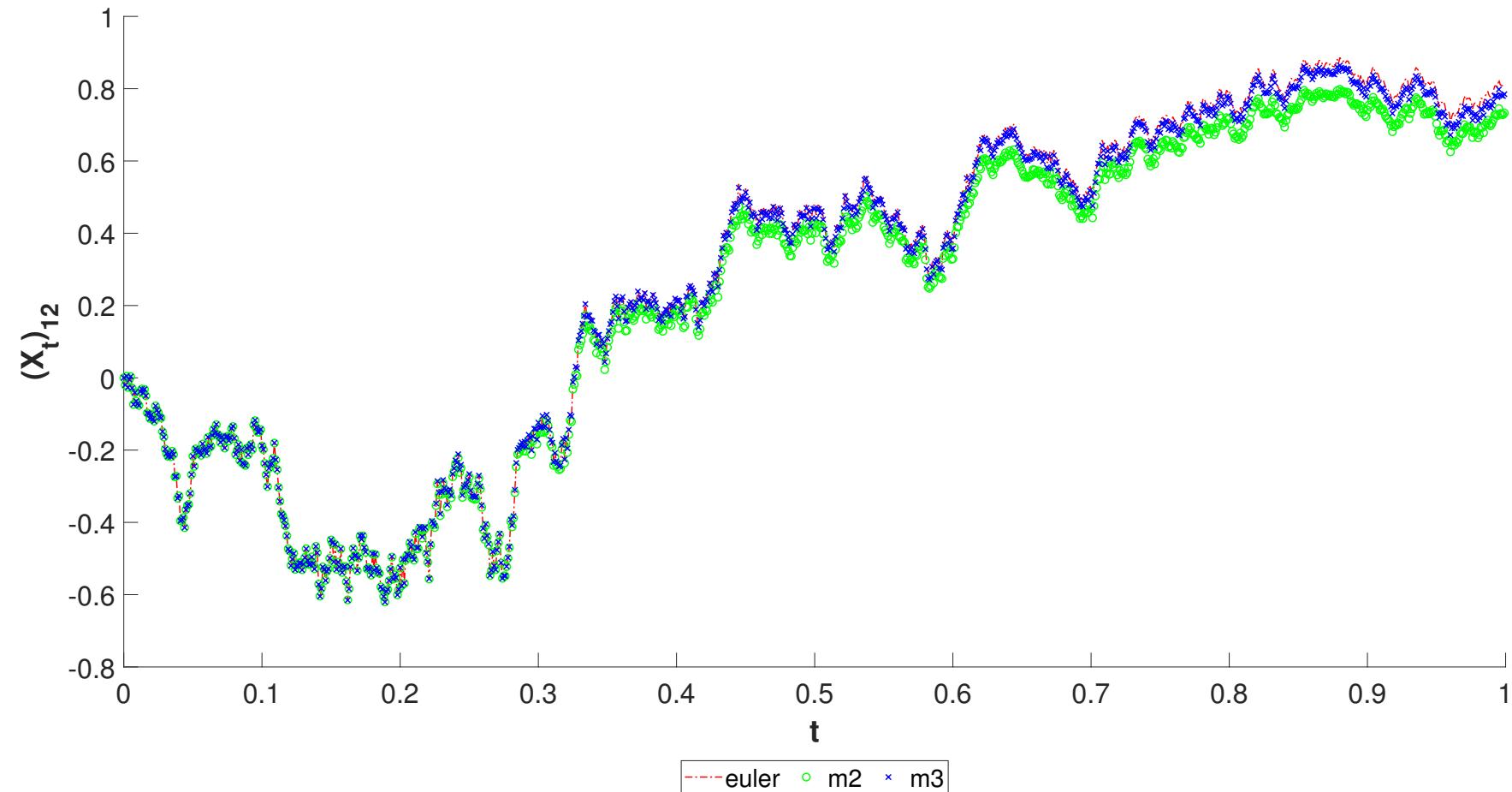
1.4 Plots

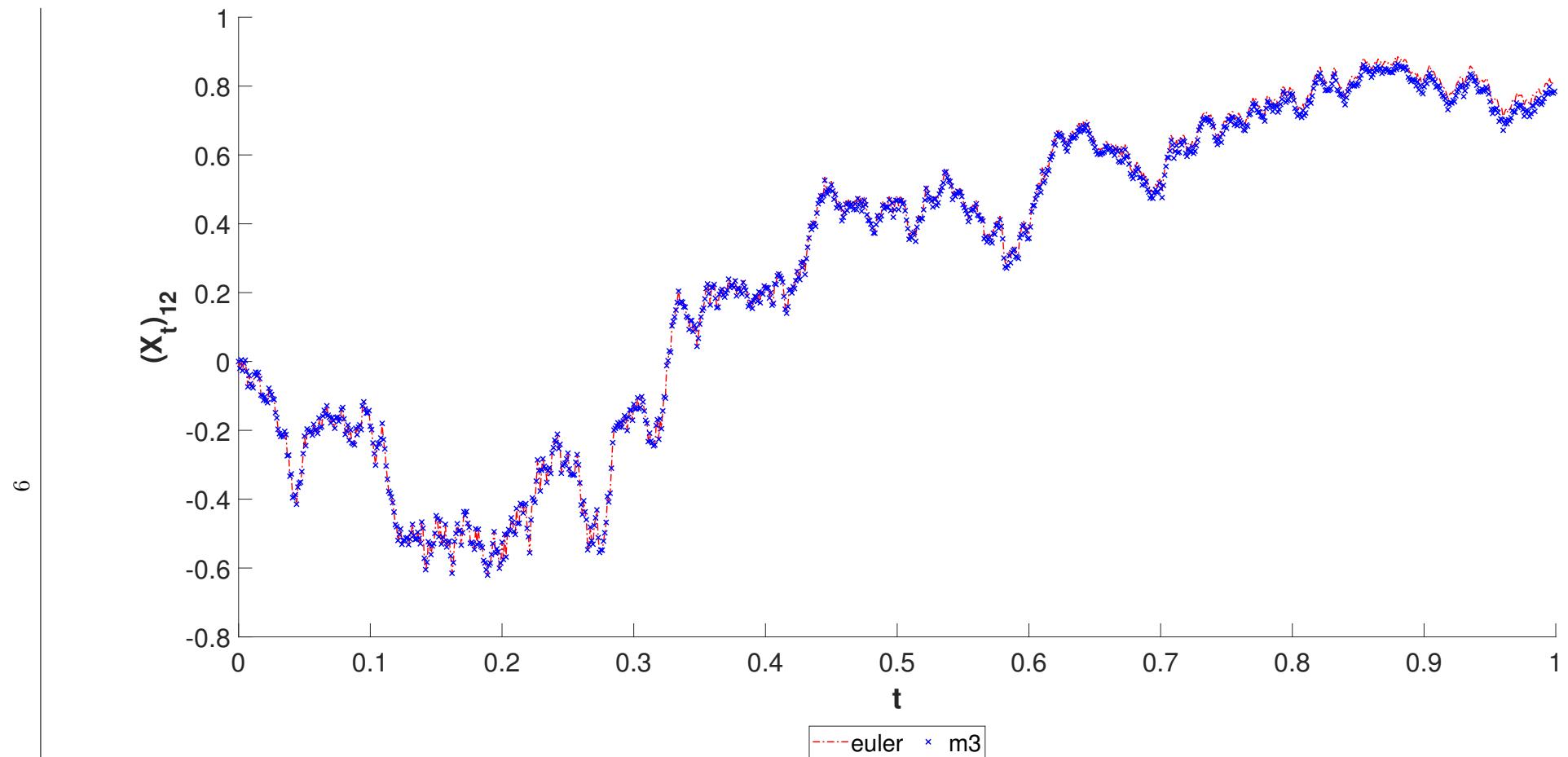


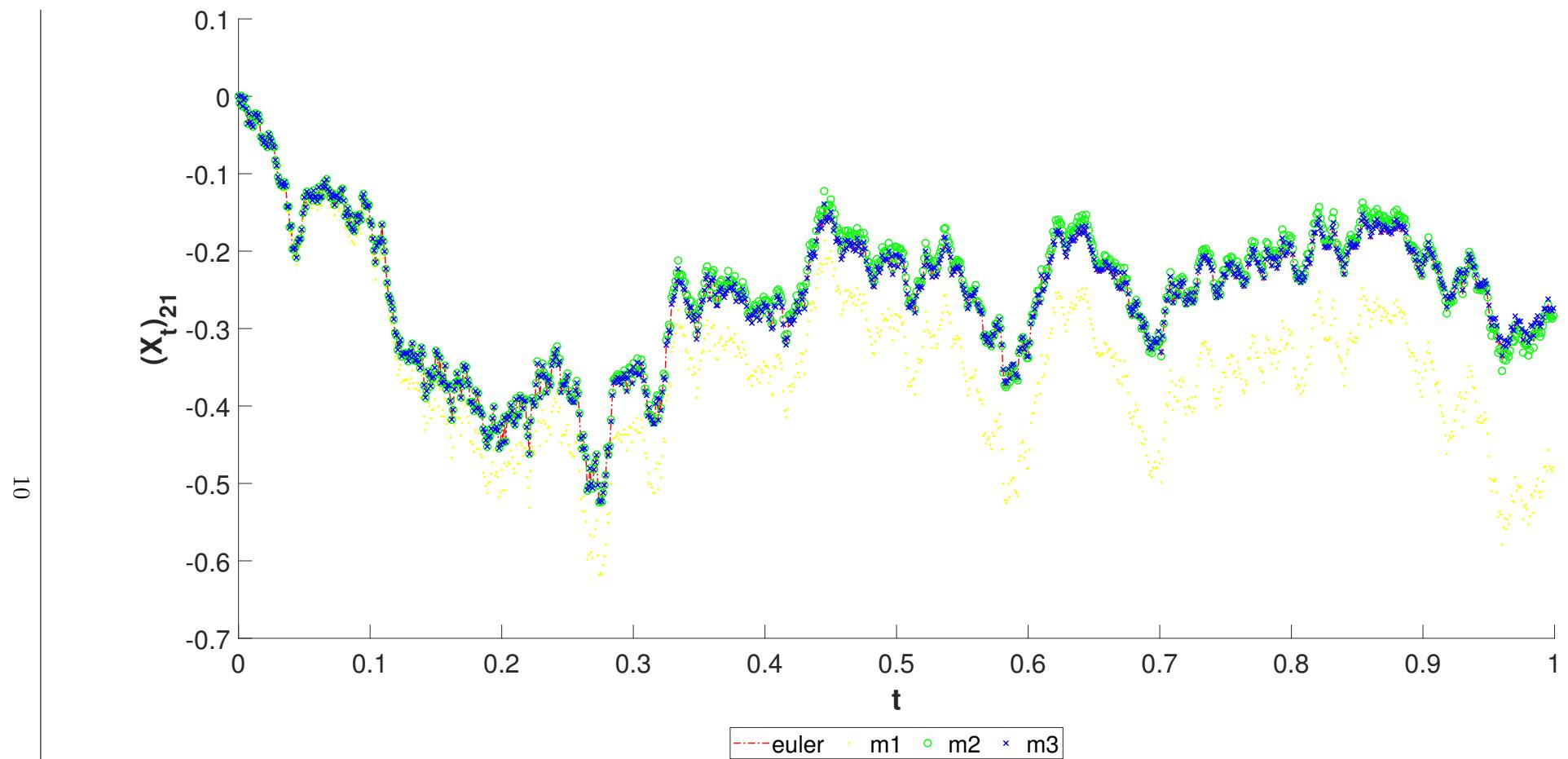


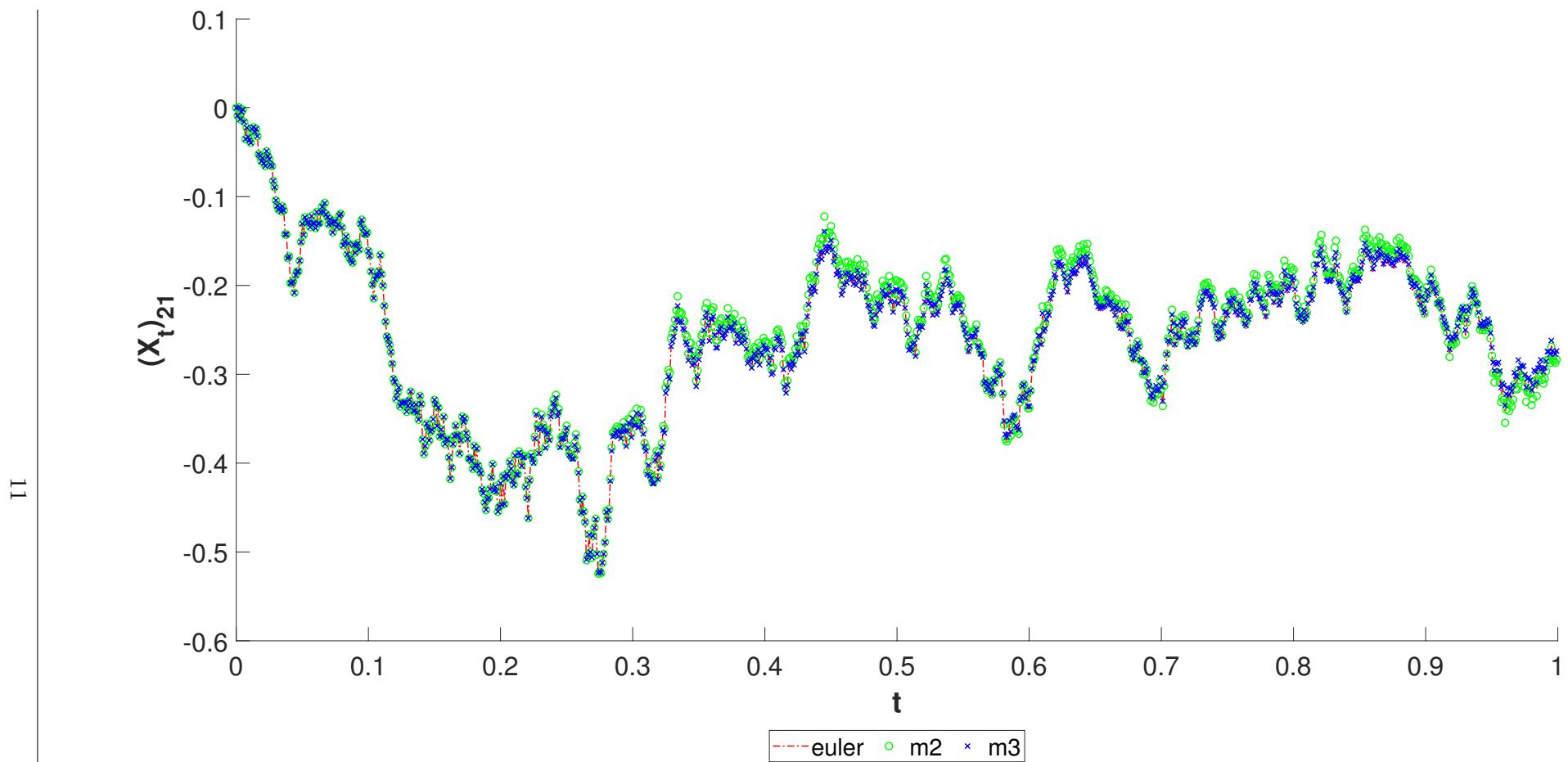


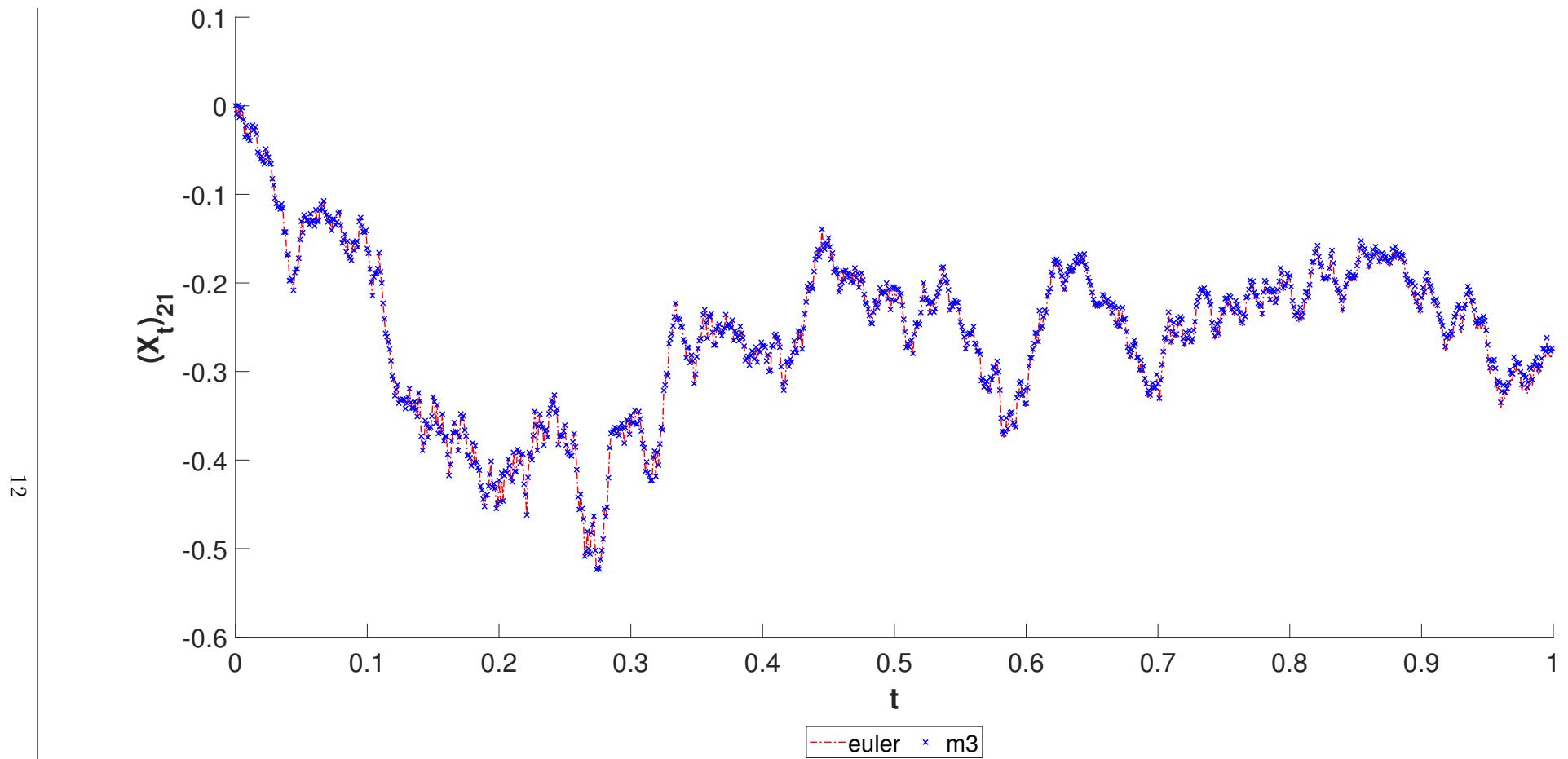


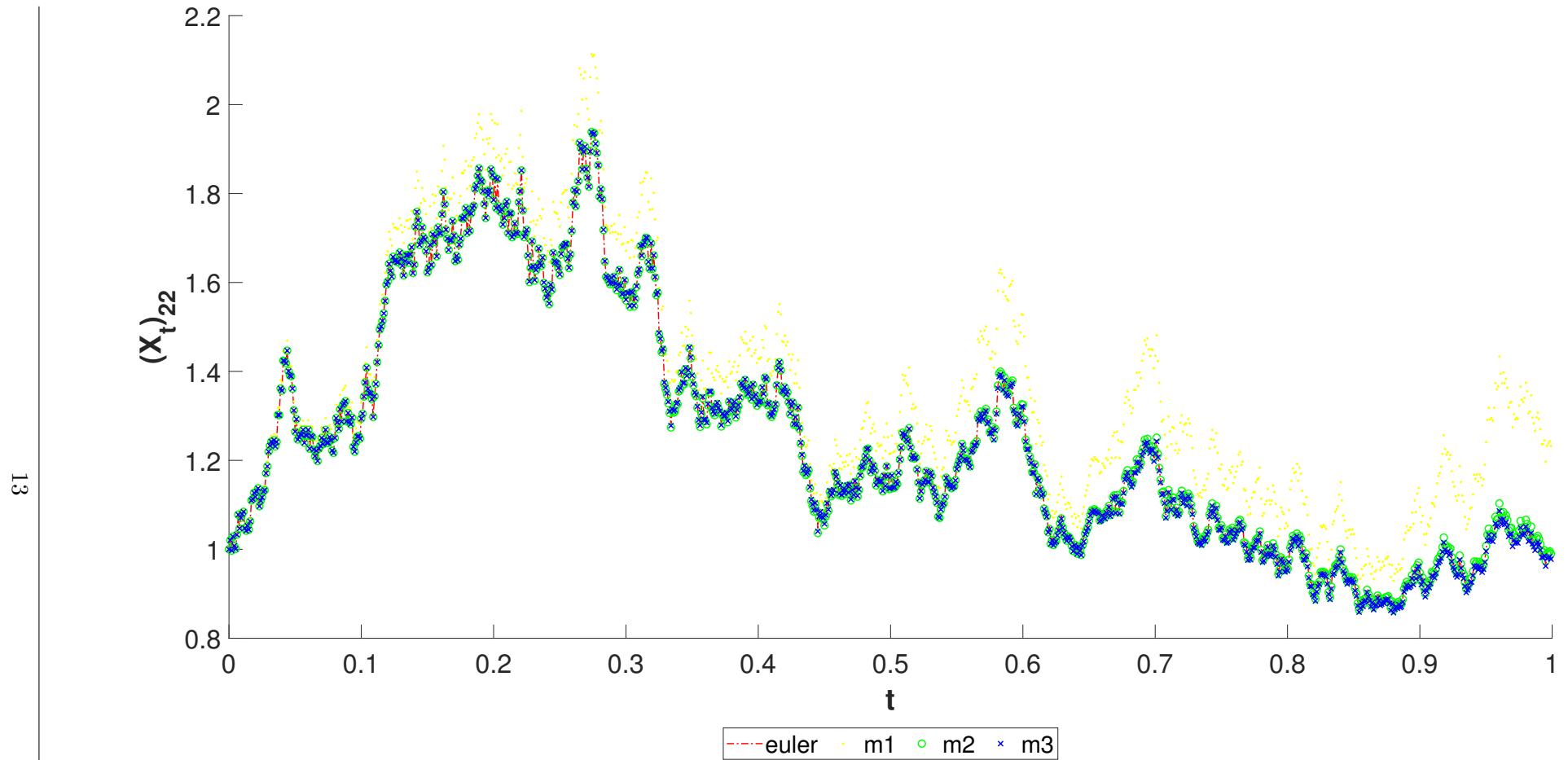


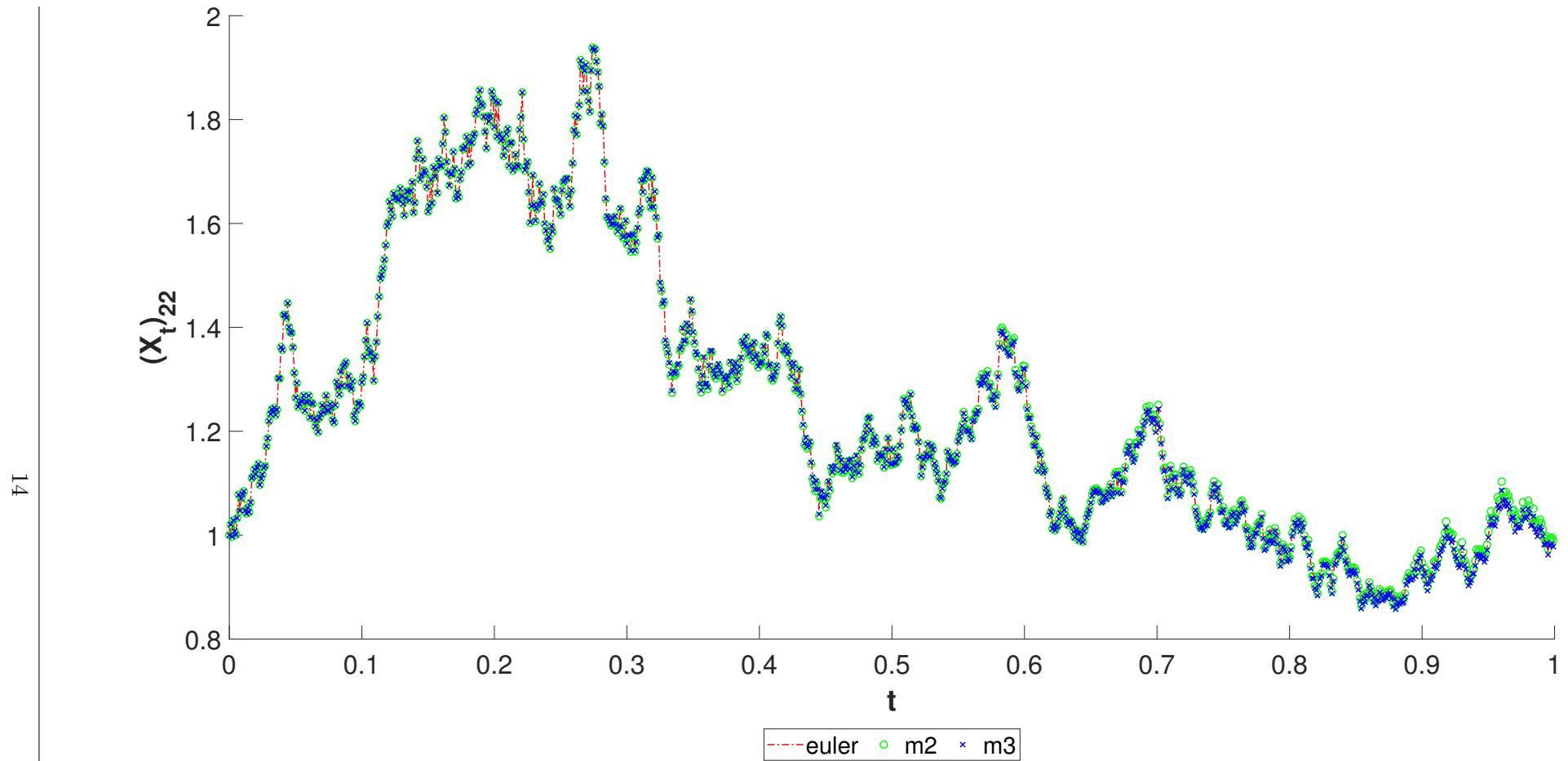


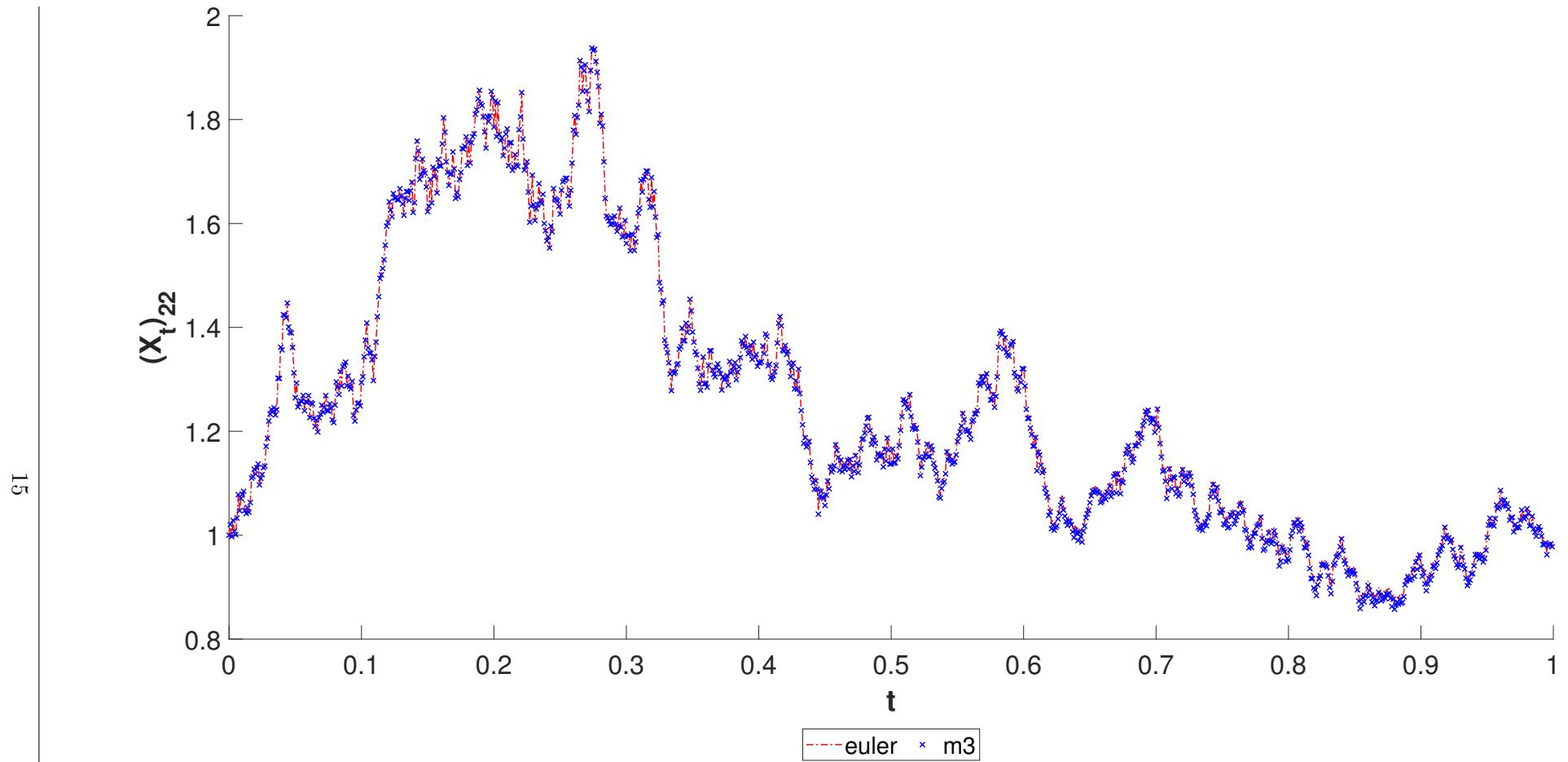


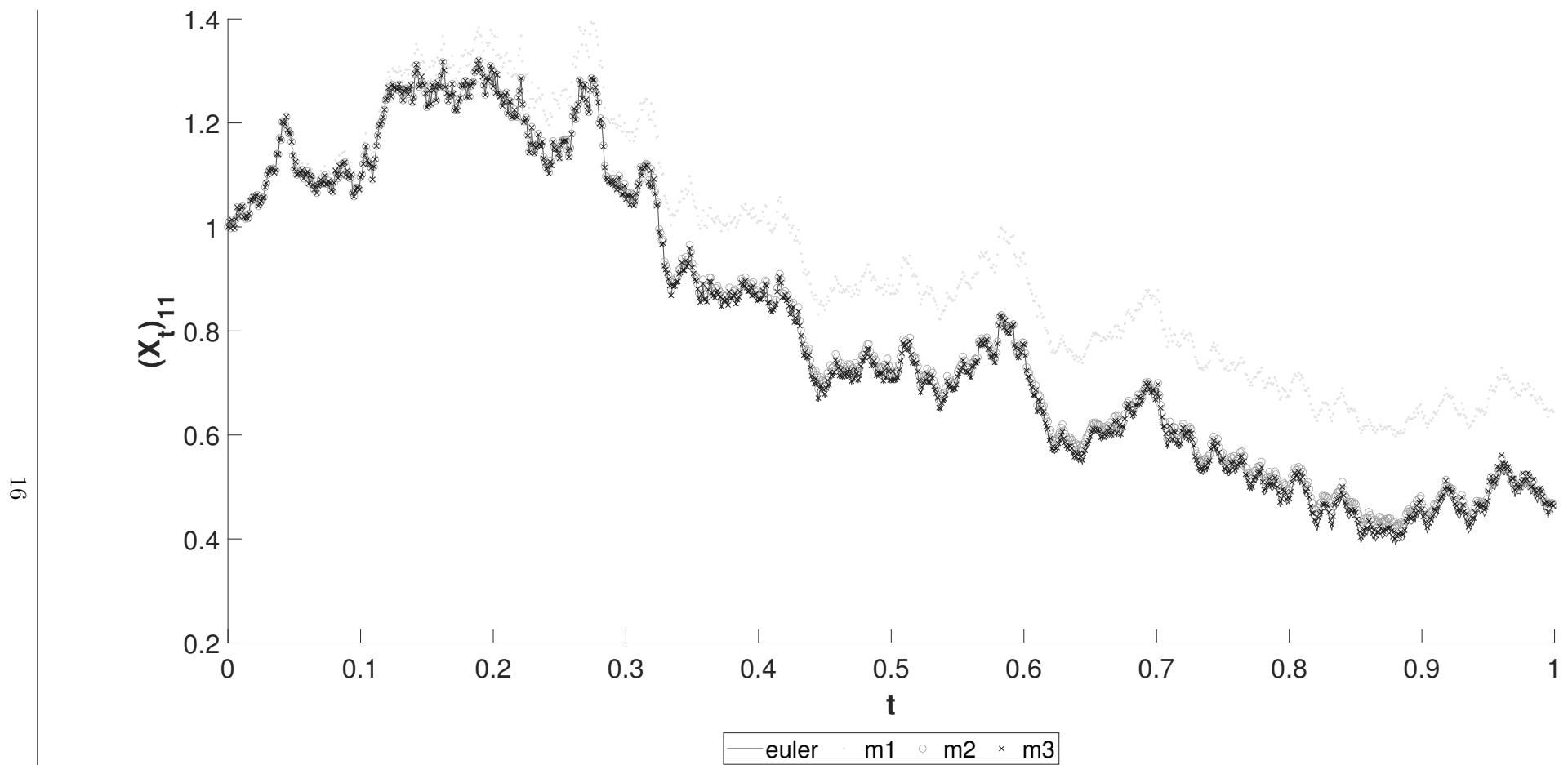


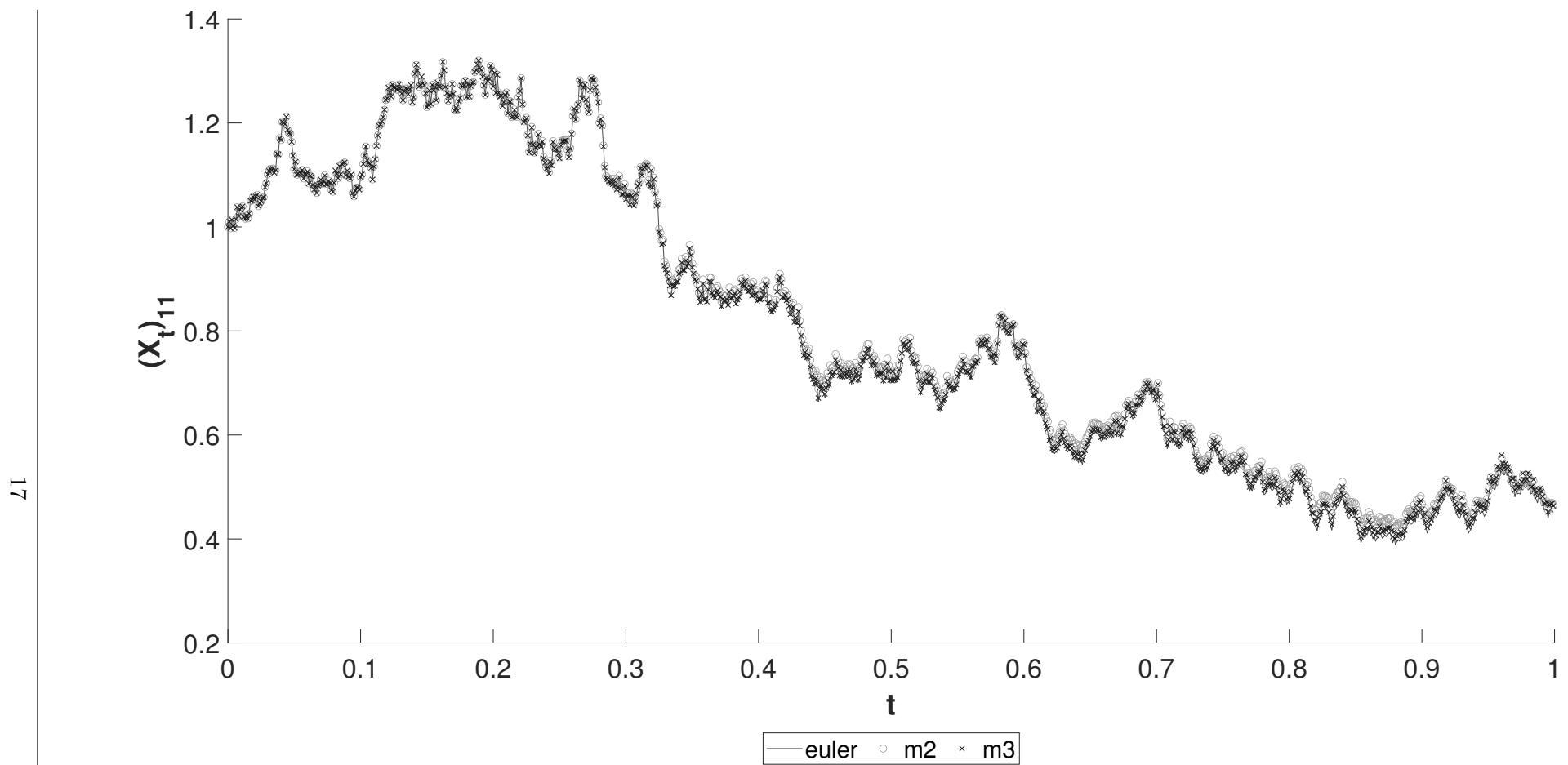


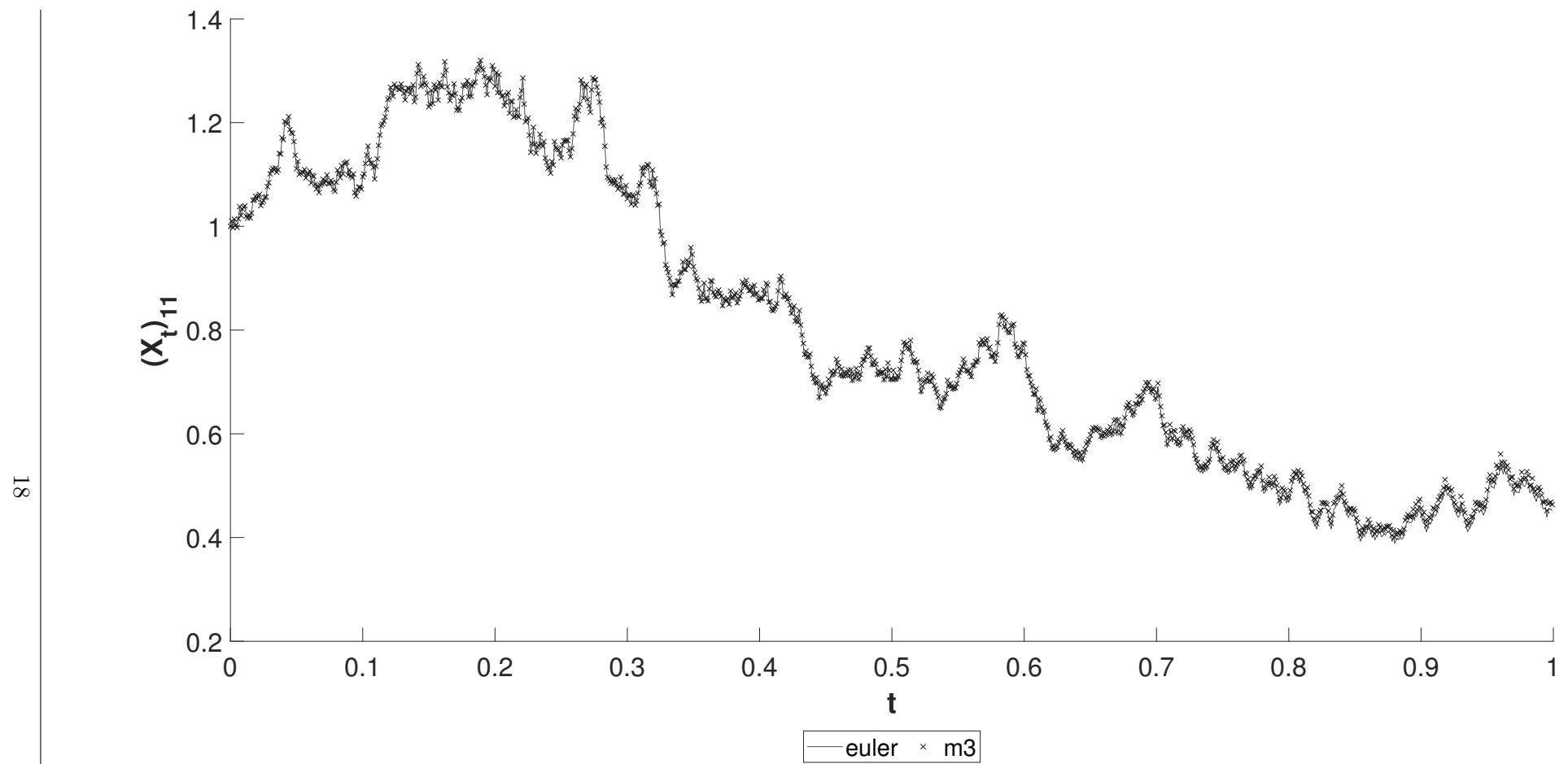


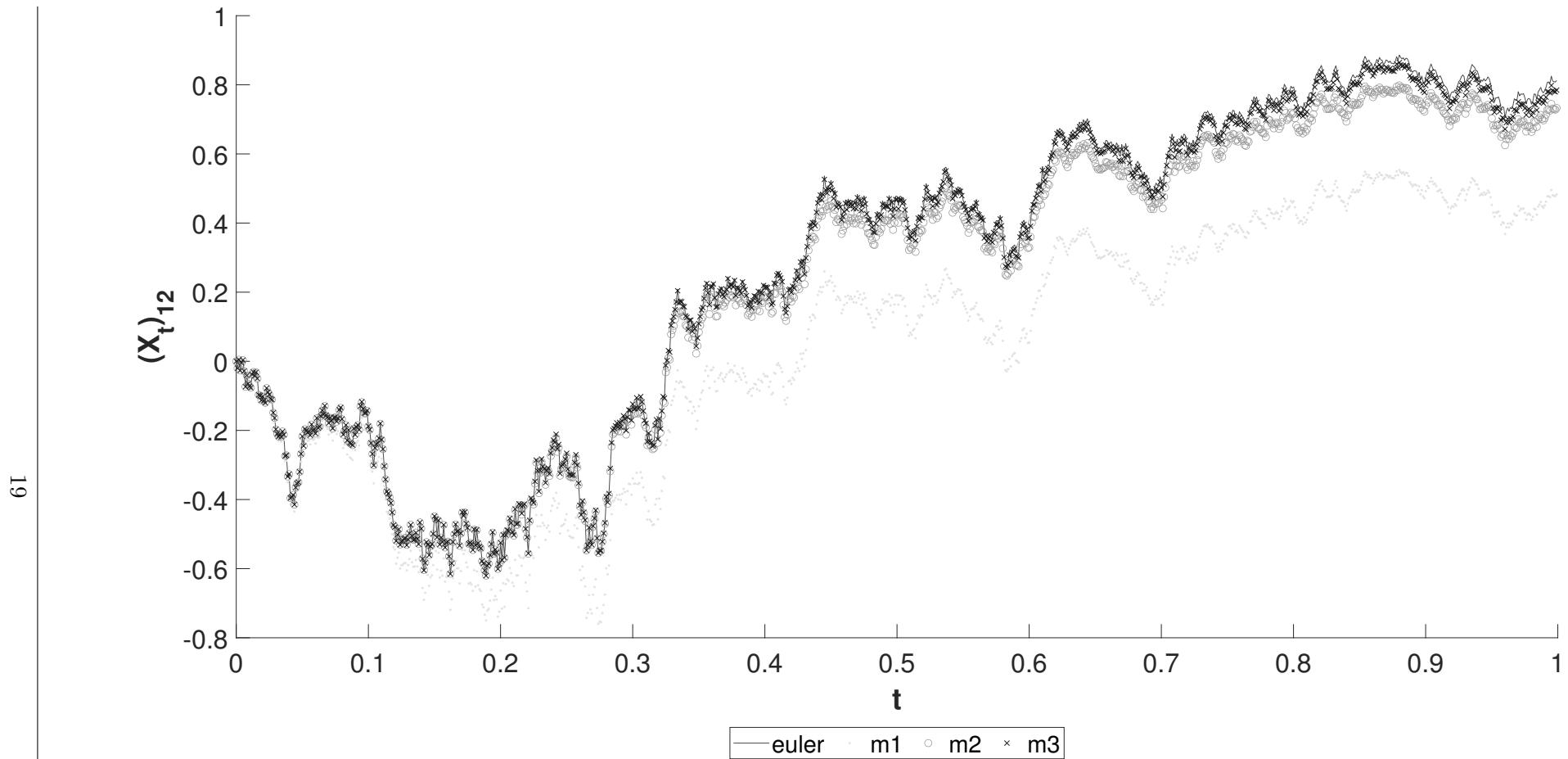


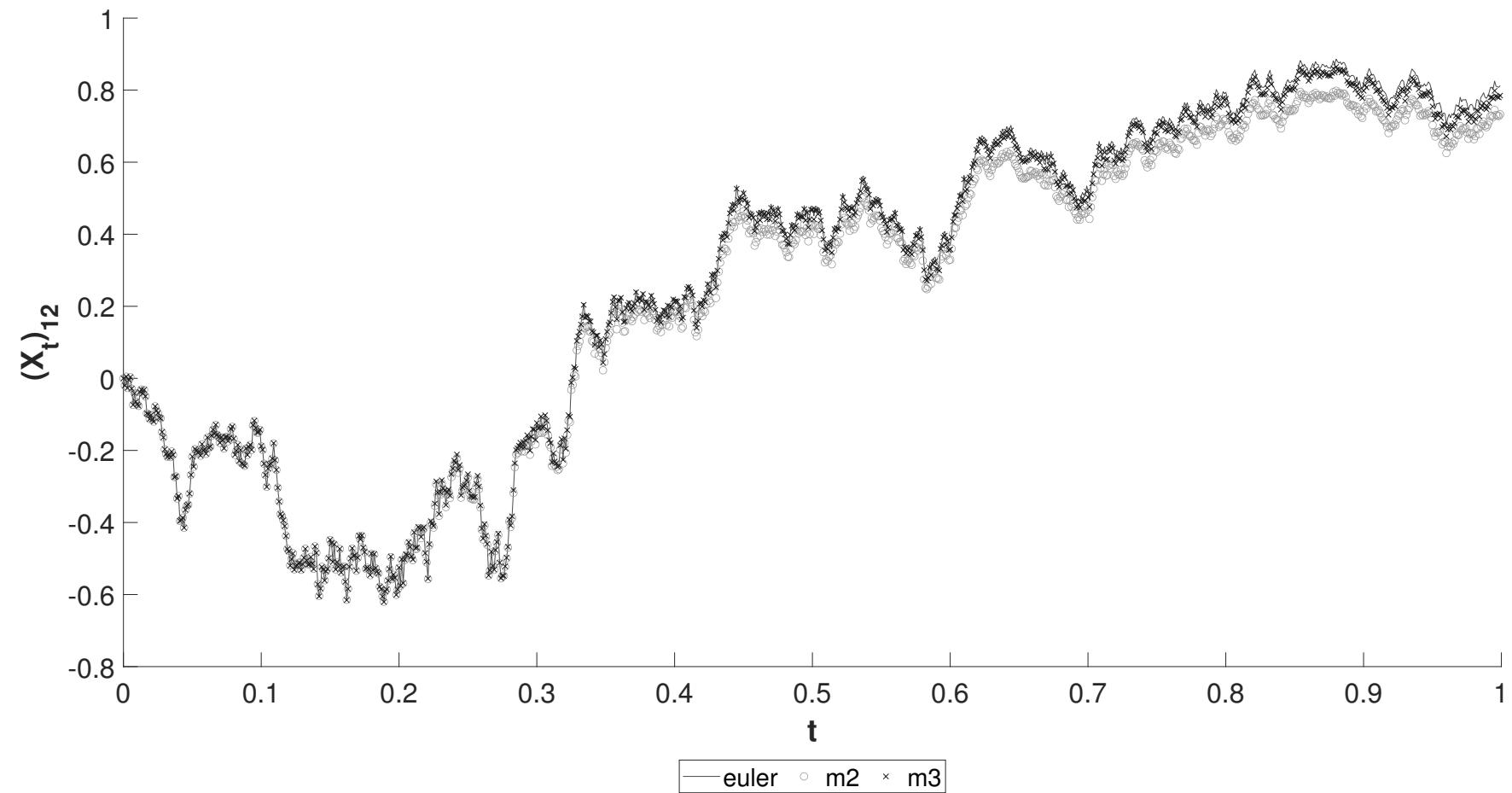


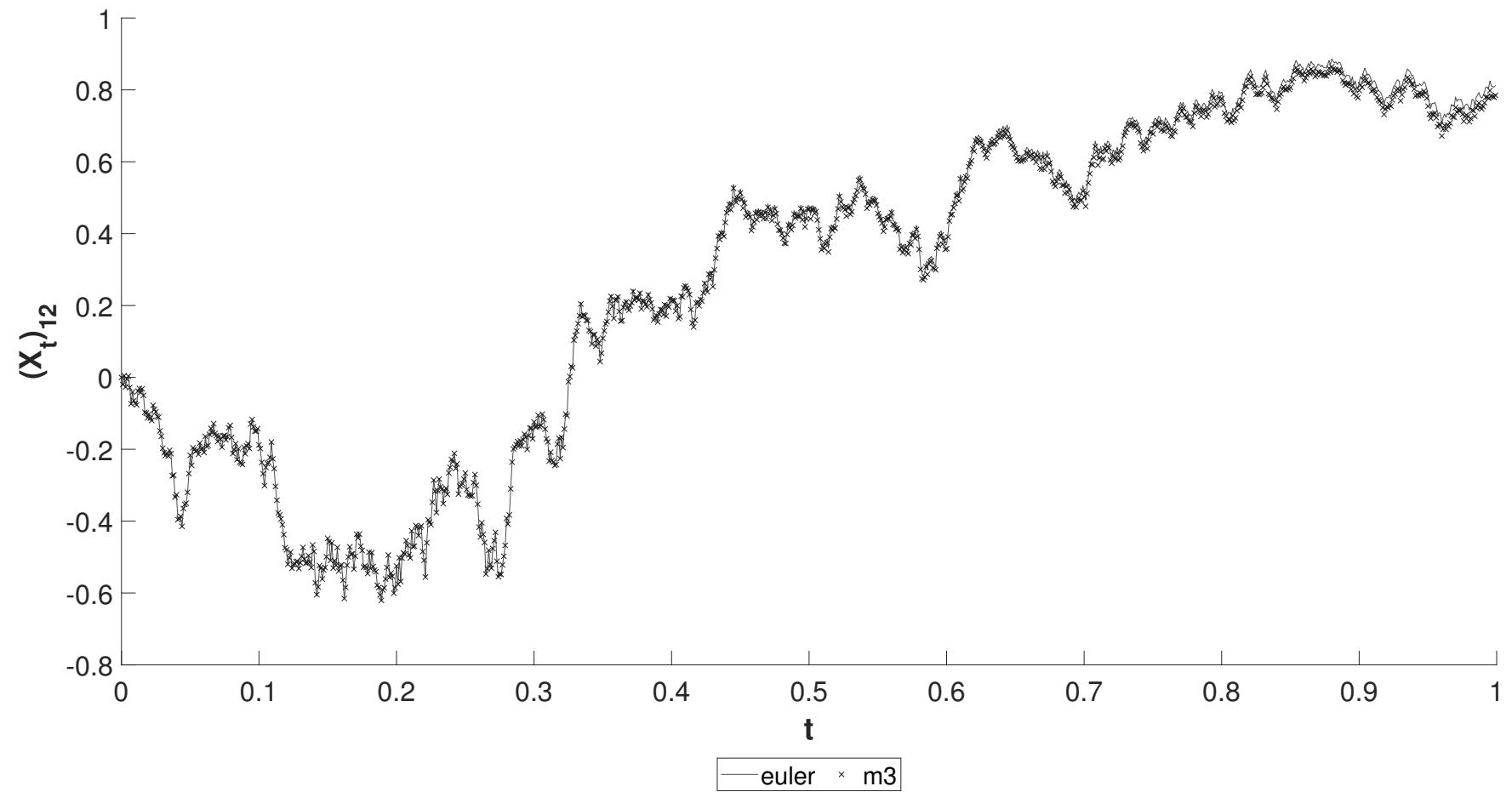


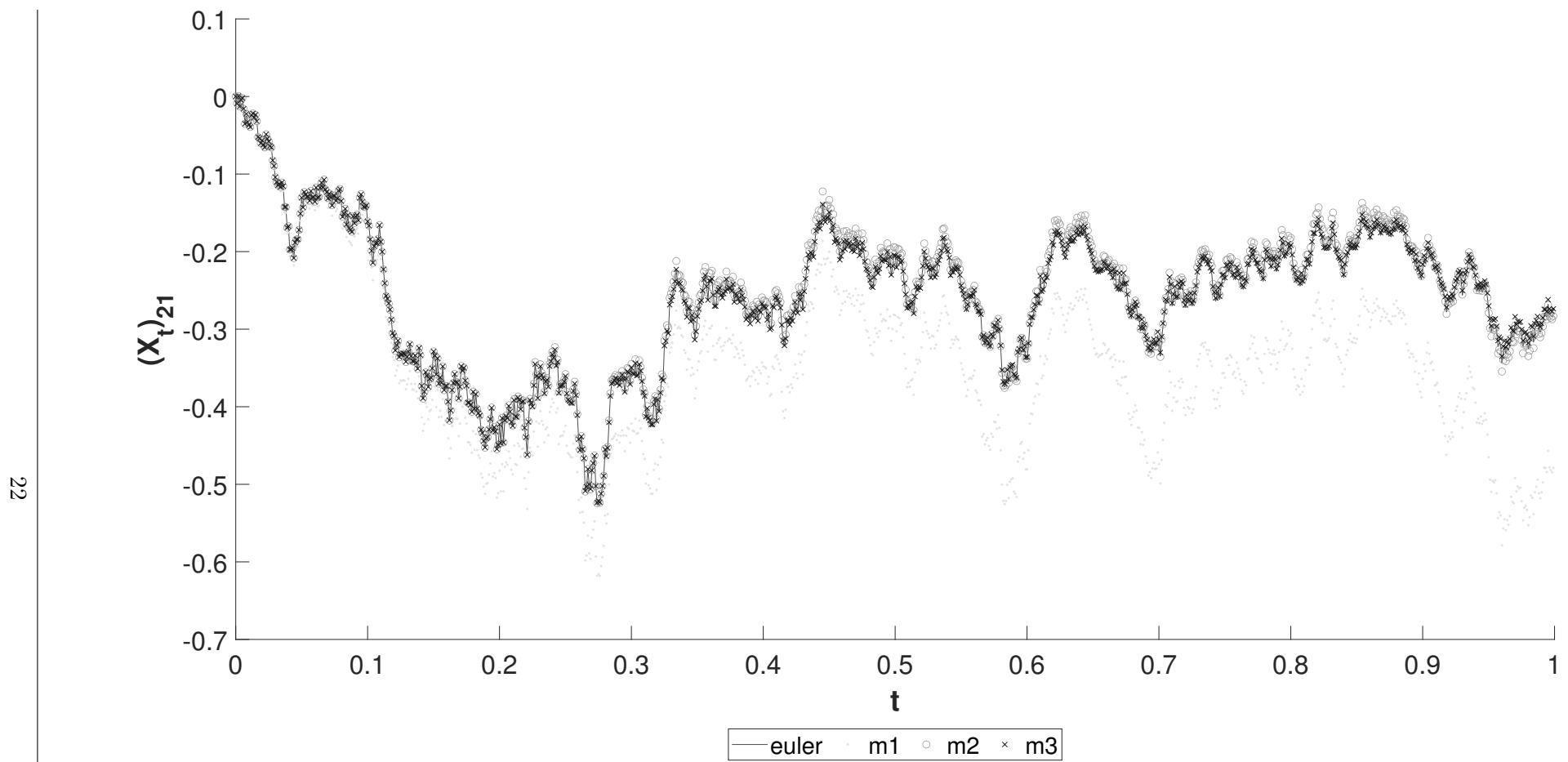


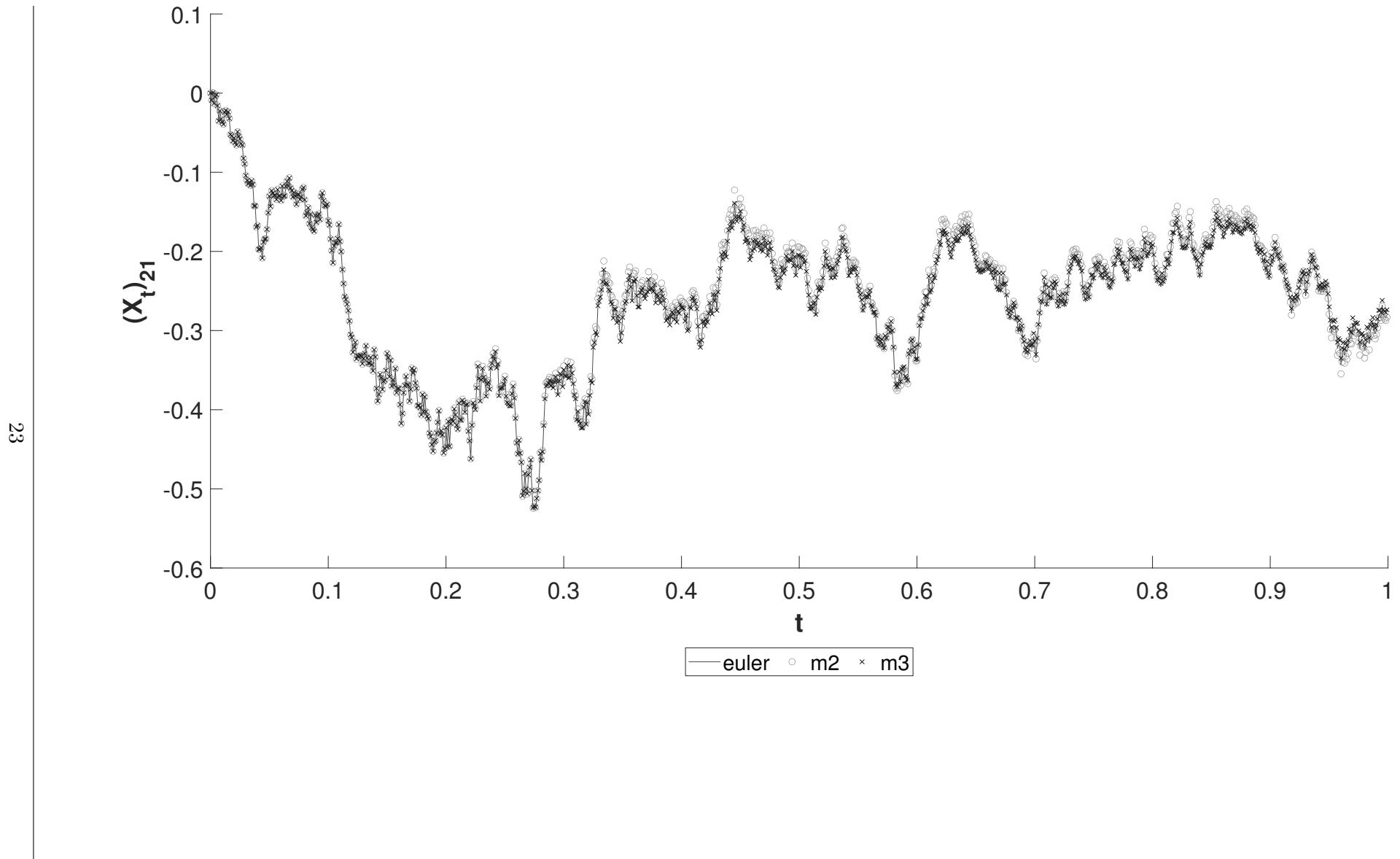


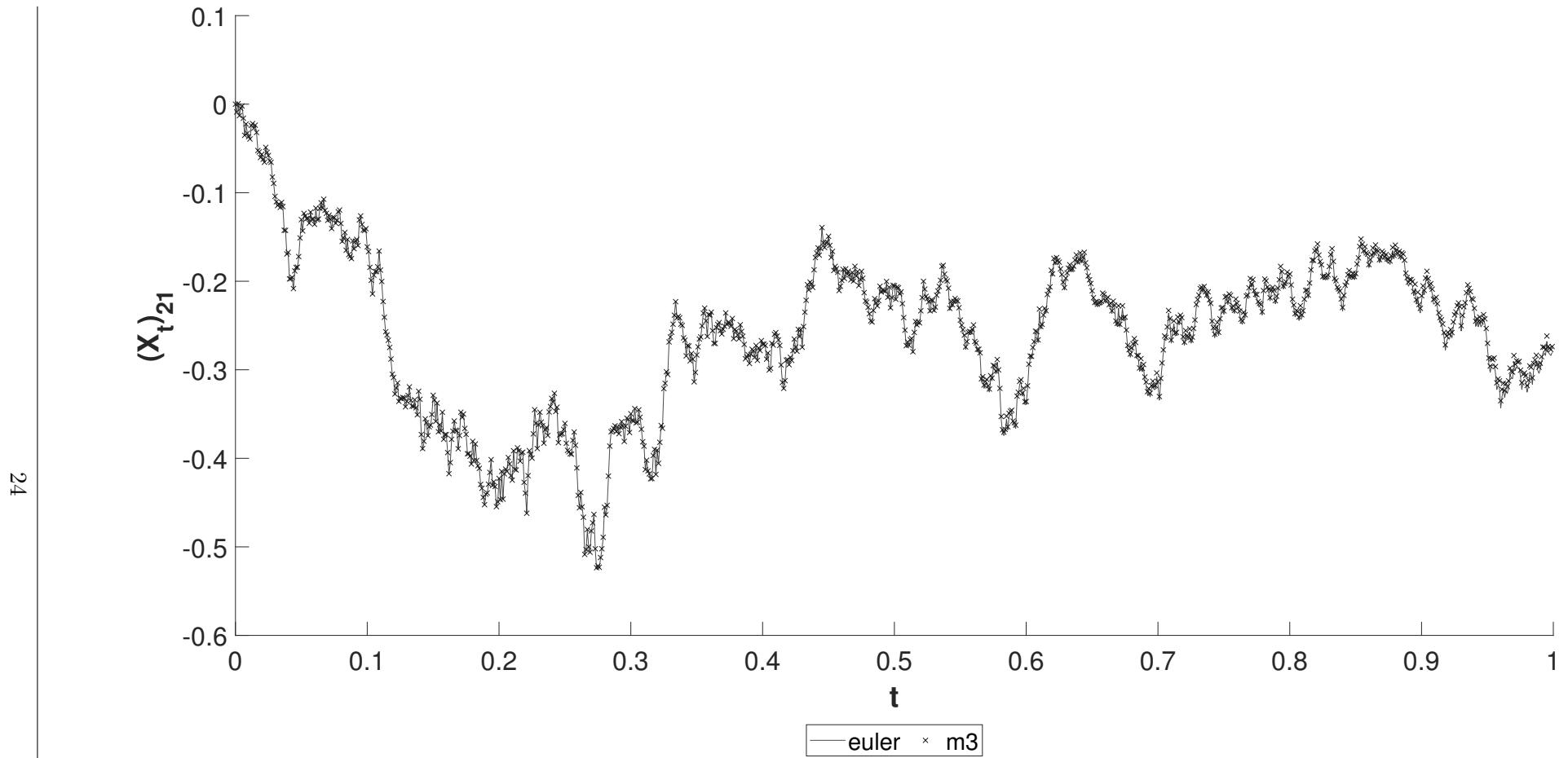


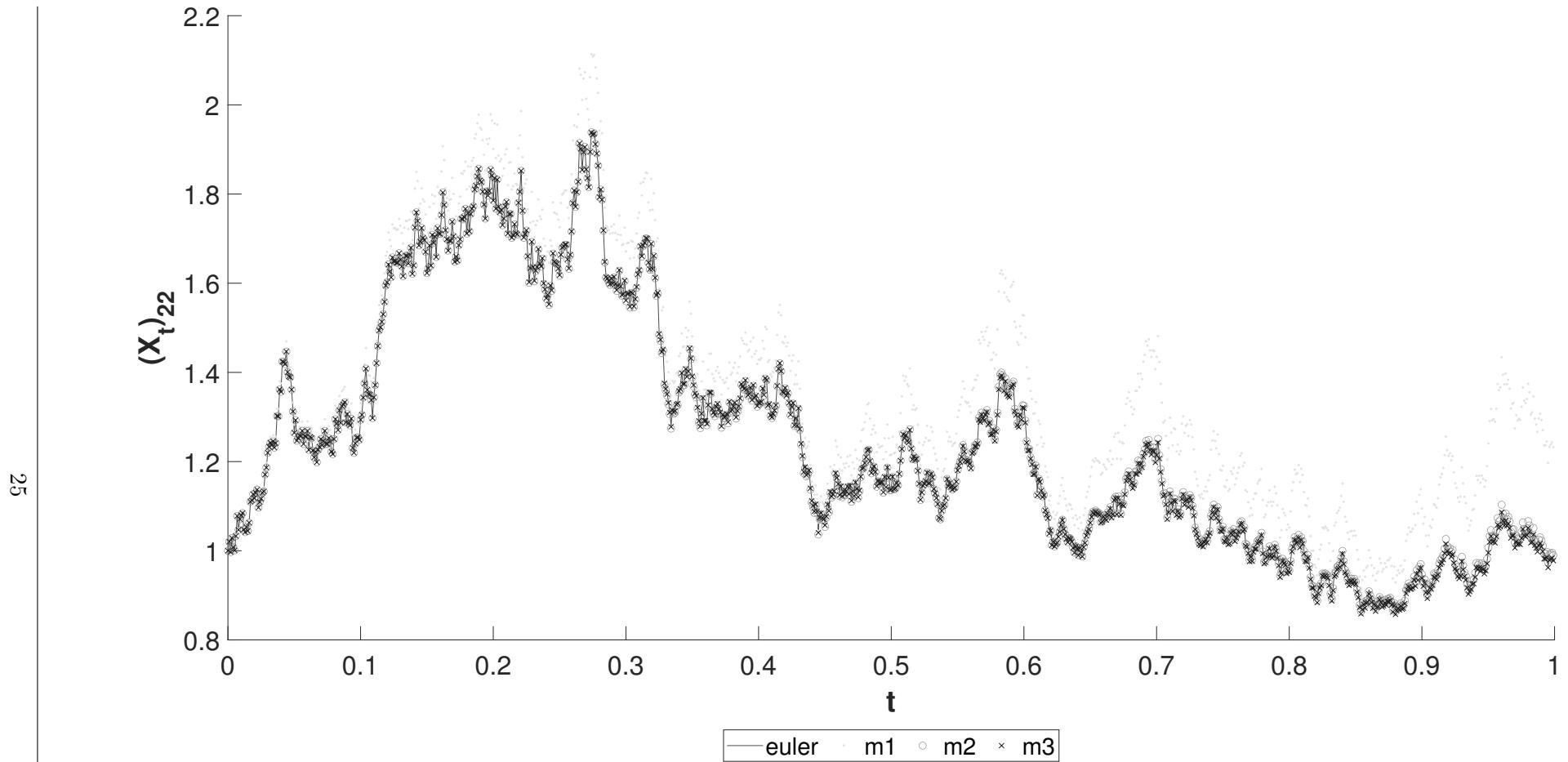


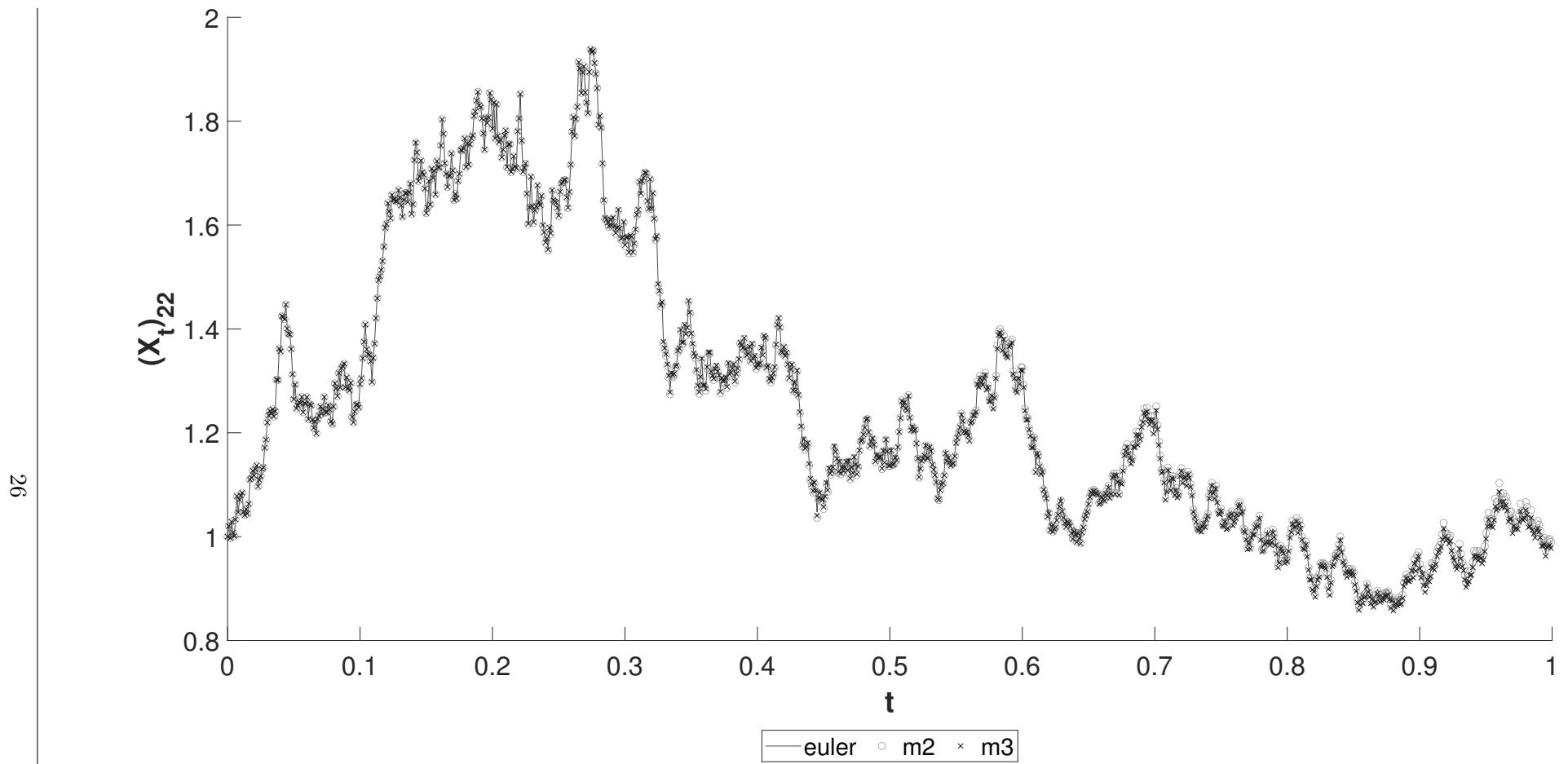


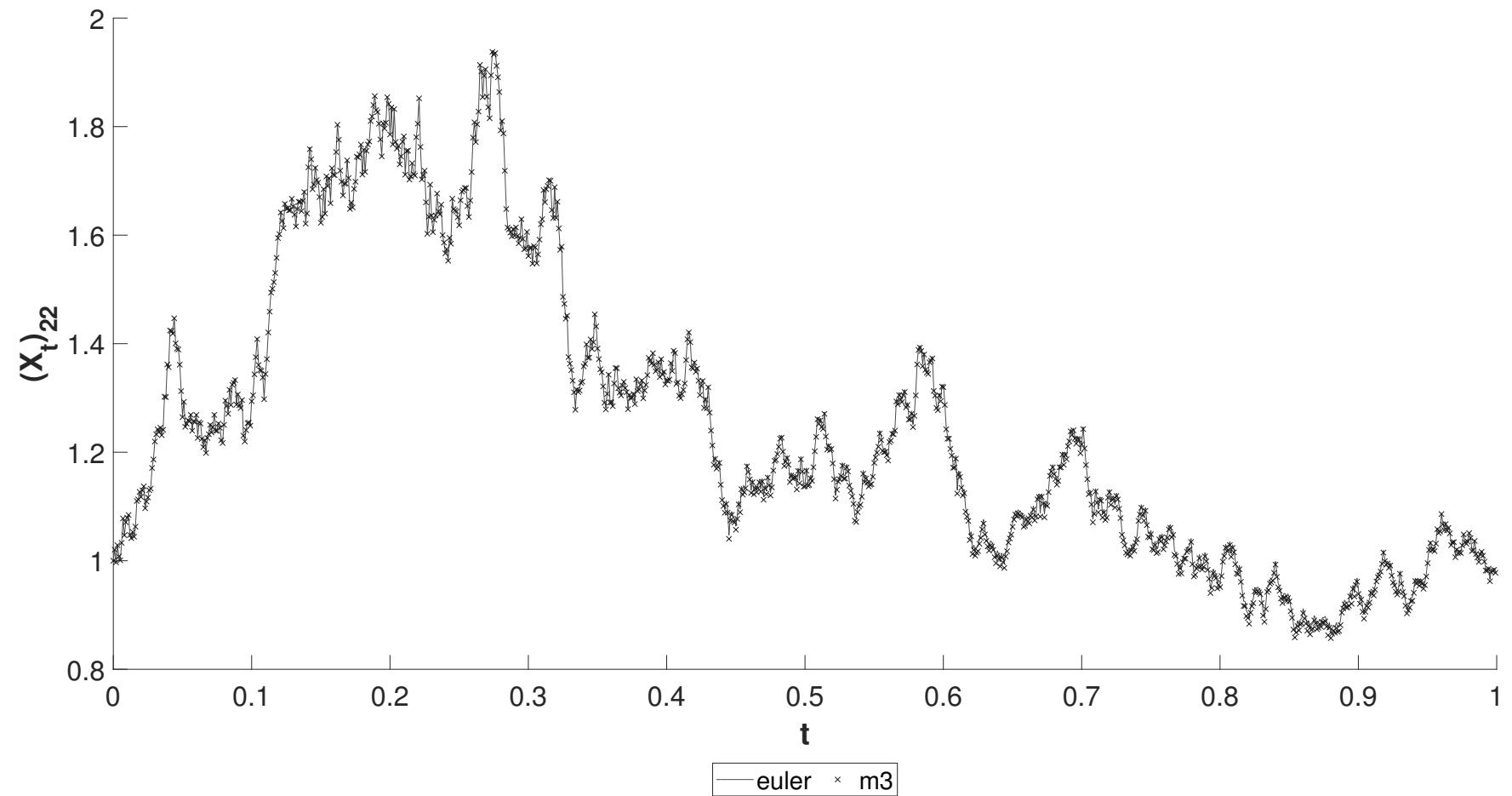












1.5 Error Plots

