Schedule Design and Container Routing in Liner Shipping

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A liner shipping company seeks to provide liner services with shorter transit time compared with the benchmark of market-level transit time because of the ever-increasing competition. When the itineraries of its liner service routes are determined, the liner shipping company designs the schedules of the liner routes such that the wait time at transshipment ports is minimized. As a result of transshipment, multiple paths are available for delivering containers from the origin port to the destination port. Therefore, the medium-term (3 to 6 months) schedule design problem and the operational-level container-routing problem must be investigated simultaneously. The schedule design and container-routing problems were formulated by minimization of the sum of the total transshipment cost and penalty cost associated with longer transit time than the market-level transit time, minus the bonus for shorter transit time. The formulation is nonlinear, noncontinuous, and nonconvex. A genetic local search approach was developed to find good solutions to the problem. The proposed solution method was applied to optimize the Asia–Europe–Oceania liner shipping services of a global liner company.

Maritime transportation is the backbone of international trade. Despite the global economic downturn and sharp decline in world merchandise trade in the last quarter of 2008, United Nations Conference on Trade and Development (*1*) estimated the 2008 international seaborne trade at 8.17 billion tons of goods loaded, an increase of 3.6% over 2007. Accounting for approximately 16% of world goods loaded in volume terms (tons), container trade volumes increased to 137 million 20-ft equivalent units (TEUs) in 2008 (*1*). Containerization not only protects cargo from damage and pilferage but also facilitates the handling operations. A direct result of this improved handling efficiency is transshipment. Transshipment enables cargo consolidation for the deployment of large container ships. Transshipment also expands the service scope of liner shipping companies, because any port-to-port delivery service is feasible even if there is no liner route connecting these two ports. Transshipment is an important feature that must be considered in all levels of planning problems. For example, at Singapore port, one of the world's busiest ports, more than 80% of containers handled are transshipped containers (*2*). Approximately 28% of the world's port container throughput is transshipped containers (*3*).

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Offering short transit time is a competitive factor for liner shipping companies, especially when the goods involved are time sensitive. Typical examples are perishable goods and consumer goods with a short life cycle or elevated economic and technical depreciation, such as fashion and computers (*4*). As a result of transshipment, the origin–destination (O-D) transit time of containers on a port-to-port basis depends not only on the time onboard ships (port-to-port sailing time) but also on the wait time at the transshipment port(s) during the trip, if any. The time buffers liner shipping companies build in their service schedules are typically very low (*4*). Therefore, the port-to-port sailing time is determined once a liner route is designed. To shorten the O-D transit time, it is important to reduce the possible wait time at the transshipment port, especially for those O-Ds with fierce competition (i.e., O-Ds for which many liner shipping companies are providing services). The wait time depends on the schedules of the liner routes, namely, the departure time from each port calls. This schedule design problem occurs at a medium-term planning level (3–6 months). The designed schedules, together with the port calls of the routes, are published in advance to attract potential container shipment demand.

Ideally, the schedules should be designed such that two ships arrive at the transshipment port at the same time, and thus containers can be transshipped from one ship to another without any delay. However, this problem is complicated in that containers can be transshipped between two liner routes at any common port of call. Transshipment further complicates the problem by enabling multiple routing possibilities for delivering containers from the origin port to the destination. Hence, the ideal zero delay at all transshipment ports cannot be achieved. Therefore, the medium-term schedule design problem is associated with the operational-level container-routing problem, namely, how to transport containers from the origin to the destination with the given liner services. Different container-routing plans involve different transshipment costs and O-D transit times. The joint optimization of liner shipping schedule design and container routing to minimize the transshipment cost and transit time is a practical research topic arising in the liner shipping industry.

RELEVANT PAST RESEARCH

Most of the literature on liner route design (*5–11*) or fleet deployment (*12–17*) has not dealt with the schedule design problem, because researchers assume that containers are transported from origin to destination with direct liner shipping services. Because no transshipment is allowed in these models, the schedules are designed such that the time interval between two voyages of a liner route is uniform in the planning horizon.

Agarwal and Ergun investigated liner shipping schedules in a space–time network. They sought to minimize the total operating cost, including the transshipment cost, whereas the transit time was not taken into account (*18*). Meng and Wang designed a liner shipping service network without considering transit time or schedules (*19*). Mourão et al. studied a simple hub-and-spoke network, which consists of one main liner route, one feeder liner route, and a single hub port (*20*). The feeder route has two possible schedules: Tuesday's and Thursday's departures from the hub. The objective is to minimize the total operating cost and inventory cost. This simplified model cannot address practical liner shipping schedule design problems with various transshipment possibilities. Other researchers have also considered the inventory cost in liner shipping: Lane et al. initiated a six-port liner shipping network design problem with the holding cost of cargo inventoried from the time of cargo's arrival at the origin port until loaded on vessels (*21*). Leachman presented an economic optimization model of waterborne containerized imports from Asia to the United States in which imports were allocated to alternative ports and logistics channels so as to minimize total transportation and inventory costs for each importer (*22*). Chen et al. investigated a two-port vessel scheduling problem to satisfy container shipment demand at minimum cost, including the inventory cost of laden containers and empty containers (*23*). Besides not accounting for container transshipment between ships at ports, the inventory cost approach has the following disadvantages: (*a*) liner shipping companies are normally indiscriminate with the cargo in the containers, and they are not concerned with the inventory cost; and (*b*) shippers have to accept the transit time provided in the liner shipping market, because one shipper's goods are far less than a shipload (*20–23*). A liner shipping company cares more about the market level transit time: it gains more market share when providing a shorter transit time; otherwise it loses market share.

RESEARCH OBJECTIVE

According to the literature review, the schedule design and container routing problem (SDCRP) as a practical liner shipping decision problem has not been addressed to its generality. The decision making on the medium-term schedule design must take into account the operational-level routing of containers. This makes the SDCRP inherently nonlinear and brings a great challenge to designing the solution algorithm. The objective of this paper is to properly formulate the SDCRP and to develop an efficient solution method for it.

SCHEDULE DESIGN AND CONTAINER-ROUTING PROBLEM

Consider a liner shipping company that operates a group of liner routes, denoted by set $\mathcal{R}$, over a number of ports, denoted by set $\mathcal{P}$, to meet a given container shipment demand. It is assumed that each liner route is deployed with a string of homogeneous ships sailing at a weekly service frequency within the medium-term planning horizon (3 to 6 months). Because of the high operating cost of ships, the schedule of a liner route is usually very tight with little slack. The sailing time between two adjacent ports and the time spent at a port on each liner route are determined.

The itinerary of a liner route $r \in \mathcal{R}$ forms a loop and can be expressed by the port calling sequence:

$$p_{r1} \rightarrow p_{r2} \rightarrow \cdots \rightarrow p_{rN_r} \rightarrow p_{r1} \tag{1}$$

where N_r is the number of port calls on route r, and p_{ri} is the ith port call on the route. For example, Route 1, Route 2, and Route 3, respectively, in Figure 1 can be expressed by the following port calling sequence:

$$p_{r_1 1}(\text{HK}) \rightarrow p_{r_1 2}(\text{JK}) \rightarrow p_{r_1 3}(\text{SG}) \rightarrow p_{r_1 1}(\text{HK}) \tag{2}$$

$$p_{r_2 1}(\text{HK}) \rightarrow p_{r_2 2}(\text{XM}) \rightarrow p_{r_2 3}(\text{SG}) \rightarrow p_{r_2 4}(\text{CB}) \rightarrow p_{r_2 5}(\text{SG}) \rightarrow p_{r_2 1}(\text{HK}) \tag{3}$$

$$p_{r_3 1}(\text{CB}) \rightarrow p_{r_3 2}(\text{CN}) \rightarrow p_{r_3 3}(\text{CC}) \rightarrow p_{r_3 1}(\text{CB}) \tag{4}$$

where

HK = Hong Kong;
JK = Jakarta, Indonesia;
SG = Singapore;
XM = Xiamen, China;
CB = Colombo, Sri Lanka;
CN = Chennai, India; and
CC = Cochin, China.

Define I_r as an index set: $I_r = \{1, 2, \ldots, N_r\}$. Two consecutive ports p_{ri} and $p_{r,i+1}$ on liner route $r \in \mathcal{R}$ is referred to as leg i ($i \in \{1, 2, \ldots, N_r - 1\}$), denoted by the pair of ordered ports $< p_{ri}, p_{r,i+1} >$. Leg N_r is $< p_{rN_r}, P_{r1} >$.

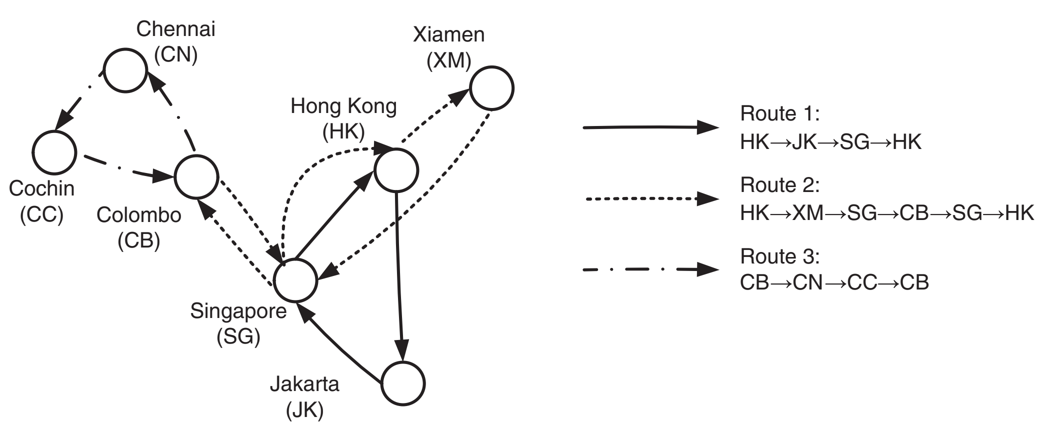

FIGURE 1 Network with three liner routes.

TABLE 1 Examples of Container Shipment Plans

O-D	Notation	Container Shipment Plan
SG-HK	$h_1 \in \mathcal{H}_{\mathrm{SG,HK}}$	$< p_{r_2 5}, p_{r_2 1} >$
	$h_2 \in \mathcal{H}_{\mathrm{SG,HK}}$	$< p_{r_1 3}, p_{r_1 1} >$
HK-CN	$h_3 \in \mathcal{H}_{\mathrm{HK,CN}}$	$< p_{r_2 1}, p_{r_2 2} > + < p_{r_2 2}, p_{r_2 3} > + < p_{r_2 3}, p_{r_2 4} > + < p_{r_3 1}, p_{r_3 2} >$
	$h_4 \in \mathcal{H}_{\mathrm{HK,CN}}$	$< p_{r_1 1}, p_{r_1 2} > + < p_{r_1 2}, p_{r_1 3} > + < p_{r_2 3}, p_{r_2 4} > + < p_{r_3 1}, p_{r_3 2} >$

Container Shipment Plan

Let n_{pq} (TEUs/week) be the weekly volume of containers to be transported from port $p \in \mathcal{P}$ to port $q \in \mathcal{P}$ within the medium-term planning horizon; $n_{pq} = 0$ if there is no container shipment demand from port p to port q. Containers can be transported from one port to another even if there are no liner routes calling at both ports, because containers can be transshipped between ships serving different liner routes. Transshipment not only expands the service scope of the liner shipping company but also enables multiple container shipment plans. A container shipment plan, denoted by h, is a definition of how containers are transported from origin to destination, including the liner route(s) involved and the transshipment ports, if any. A container shipment plan can be uniquely defined by a sequence of legs on which containers are transported.

Let $\mathcal{H}_{pq}$ be a set of all the container shipment plans for containers from port $p \in \mathcal{P}$ to port $q \in \mathcal{P}$. Table 1 shows some examples of container shipment plans for the network in Figure 1. Furthermore, $\mathcal{H}$ is the set of all container shipment plans, that is, $\mathcal{H} = \cup_{p\in\mathcal{P},q\in\mathcal{P}} \mathcal{H}_{pq}$. Theoretically, feasible container shipment plans may not be used in practice, such as container shipment plans that consist of transshipments at too many ports or involve much longer journeys than necessary. This research assumed that the set of practical container shipment plans are given, and in the model only the practical container shipment plans are considered. Note that different container shipment plans have different transshipment costs. For instance, in Table 1, although h_3 and h_4 are both for containers from Hong Kong to Chennai, h_3 incurs the transshipment cost at Colombo, whereas h_4 is associated with transshipments at both Singapore and Colombo.

Schedule Design

Each O-D pair of ports $p \in \mathcal{P}$ to $q \in \mathcal{P}$ has a market level transit time, denoted by T_{pq} (h). A penalty $\bar{c}_{pq}$ (US\$/TEU · h) is incurred when the real transit time exceeds the market level transit time. Analogously, a bonus $\tilde{c}_{pq}$ (US\$/TEU · h) is gained when the real transit time is shorter than T_{pq}. The market level transit time approach nests the inventory cost model as a special case. That is, to consider the inventory cost rather than the market level transit time, $T_{pq} = 0$ and set $\bar{c}_{pq}$ = inventory cost.

As each liner route maintains a weekly service frequency, once the entry time of one port is determined, the entry and departure times of all ports of call are also known. Note that buffer time is allowed for the arrivals of container ships in practical operations because of port congestion, bad weather conditions, mechanical problems, and strikes at ports. This study focused on the medium-term schedule design problem, and hence the average buffer time was assigned to the time spent at port and the buffer time was assumed to be 0. If 00:00 on Sunday is defined as the base time 0 (h), the schedule design problem is to determine the arrival time x_r (h) at the first port call on each liner route $r \in \mathcal{R}$, and

$$0 \le x_r < 168 \qquad \forall r \in \mathcal{R} \tag{5}$$

where 168 is the number of hours in a week. The schedules for Route 2 and Route 3 when $x_{r_2} = 0$ and $x_{r_3} = 0$ are shown in Figure 2. Four ships are required to maintain the weekly frequency on Route 2 because the round-trip journey time is 4 weeks. Ships on Route 2 call at Colombo port at 10:00 on Tuesday each week. Note that a particular ship on Route 2 calls at Hong Kong port and Colombo port in different weeks.

The real O-D transit time here is defined as the time interval between the departure time at the origin port and the entry time at the destination port. For those container shipment plans with no transshipments, the real transit time is determined and independent of the schedules. For instance, as can be seen from Figure 2, the transit time of container shipment plan h_1 in Table 1 is 120 h. By contrast, shipment plan h_3 involves the transshipment at Colombo port. At a port $p \in \mathcal{P}$, there is a minimum time interval requirement for transshipment denoted by $\hat{t}_p$ (h). If the entry time of the incoming ship is earlier than the departure time of the outgoing one by at least $\hat{t}_p$ h, containers can be transshipped from the former ship to the latter; otherwise containers have to wait at port p till the next week. Given the schedules shown in Figure 2, when a ship on Route 2 arrives at

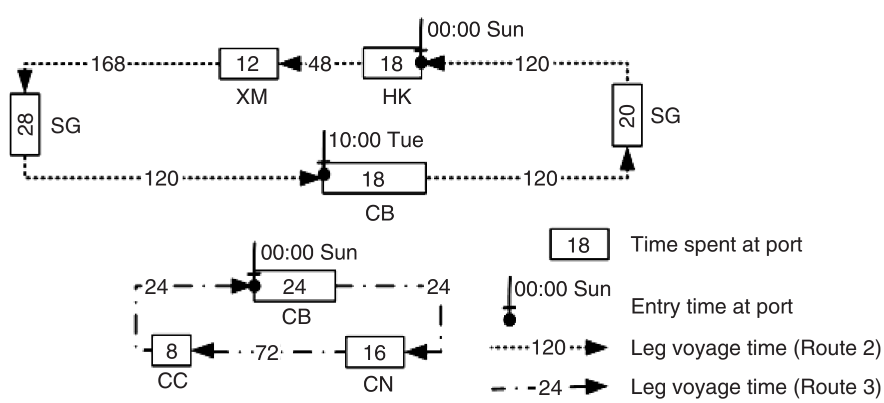

FIGURE 2 Schedules of Route 2 and Route 3.

Colombo port, containers are discharged and have to wait at Colombo port until the next Sunday. The time containers spend at a transshipment port is referred to as the wait time at port, which is from the entry time of the incoming ship to the departure time of the outgoing ship. Therefore, the transit time of container shipment plan h_3 is the sum of the time from Hong Kong port to Colombo on Route 2 (394 h), the wait time at Colombo port (134 h), and the time from Colombo port to Chennai port on Route 3 (24 h).

Denote by y_h (TEUs) the number of containers transported according to container shipment plan $h \in \mathcal{H}$. Define vector $\mathbf{x} = (x_{r_1}, x_{r_2}, \ldots, x_{r_{|\mathcal{R}|}})$ and $\mathbf{y} = (y_{h_1}, y_{h_2}, \ldots, y_{h_{|\mathcal{H}|}})$. The SDCRP is hence to determine the schedule x_r of each liner route $r \in \mathcal{R}$ and the number of containers delivered according to each container shipment plan $h \in \mathcal{H}$ to minimize the transshipment cost and the penalty cost for longer transit time minus the bonus for shorter transit time.

MATHEMATICAL MODEL

To formulate the SDCRP, let c_h^{trs} be the fixed transshipment cost (U.S. dollars/TEU) and t_h (h) be the variable representing the real transit-time of container shipment plan h with the schedule $\mathbf{x}$. The objective function of the SDCRP can be formulated as follows:

[SDCRP]:

$$\min_{\mathbf{x},\mathbf{y}} \sum_{p\in\mathcal{P}}\sum_{q\in\mathcal{P}}\sum_{h\in\mathcal{H}_{pq}} y_h\left(c_h^{\mathrm{trs}} + \bar{c}_{pq}\max\left\{0, t_h - T_{pq}\right\} - \tilde{c}_{pq}\max\left\{0, T_{pq} - t_h\right\}\right) \tag{6}$$

Next, the real transit time t_h with the schedules $\mathbf{x}$ of the liner routes is formulated. For liner route $r \in \mathcal{R}$ with the schedule x_r, the entry and departure times at the ith port call can be denoted by $t_{ri}^{\mathrm{en}} + x_r$ and $t_{ri}^{\mathrm{de}} + x_r$, respectively, in which t_{ri}^{en} (h) and t_{ri}^{de} (h) are constants and correspond to the time intervals from the arrival at the first port to the arrival at the ith port and the departure from the ith port, respectively. $t_{r1}^{\mathrm{en}} = 0, r \in \mathcal{R}$. If two liner routes r and s have a common port call p, namely $p = p_{ri} = p_{sj}$, it is possible that containers are transshipped from ships on route r when they call at the ith port call to ships on route s when they call at the jth port call. The wait time at port p under this circumstance, denoted by t_{rsij}^{w}, can be calculated as follows:

$$\begin{aligned} &t_{rsij}^{w} = 168k + t_{sj}^{\mathrm{de}} + x_s - \left(t_{ri}^{\mathrm{en}} + x_r\right) \\ &k = \arg\min\left\{k \middle| 168k + t_{sj}^{\mathrm{de}} + x_s - \left(t_{ri}^{\mathrm{en}} + x_r\right) \ge \hat{t}_{p_{ri}}, k \in \mathbb{Z}\right\} \\ &\qquad \forall r, s \in \mathcal{R}, i \in \mathcal{I}_r, j \in \mathcal{I}_s, p_{ri} = p_{sj} \end{aligned} \tag{7}$$

where $\mathbb{Z}$ denotes the set of integers and k is an auxiliary variable introduced by virtue of the weekly service frequency of each liner service route. For the ease of presentation, let $\mathcal{Q}$ denote the set of all the quadruplets $<r, s, i, j>$ that represent possible transshipments, namely,

$$\mathcal{Q} = \left\{<r, s, i, j> \middle| \forall r, s \in \mathcal{R}, i \in \mathcal{I}_r, j \in \mathcal{I}_s, p_{ri} = p_{sj}\right\} \tag{8}$$

Then Equation 7 can be restated as follows:

$$\begin{aligned} &t_{rsij}^{w} = 168k + t_{sj}^{\mathrm{de}} + x_s - \left(t_{ri}^{\mathrm{en}} + x_r\right) \\ &k = \arg\min\left\{k \middle| 168k + t_{sj}^{\mathrm{de}} + x_s - \left(t_{ri}^{\mathrm{en}} + x_r\right) \ge \hat{t}_{p_{ri}}, k \in \mathbb{Z}\right\} \\ &\qquad \forall <r, s, i, j> \in \mathcal{Q} \end{aligned} \tag{9}$$

The real transit time t_h of container shipment plan h consists of a fixed time (time stowed onboard ships) denoted by t_h^{fix} (h) and wait time at transshipment ports, if any. A container shipment plan explicitly defines how containers are transshipped during the trip from origin to destination. Let δ_{hrsij} be a binary coefficient, which takes a value of 1 if container shipment plan h contains the transshipment represented by $<r, s, i, j>$ and 0 otherwise. Thus,

$$t_h = t_h^{\mathrm{fix}} + \sum_{<r,s,i,j>\in\mathcal{Q}} \delta_{hrsij} t_{rsij}^{w} \qquad \forall h \in \mathcal{H} \tag{10}$$

The container shipment demand must be fulfilled, namely

$$\sum_{p\in\mathcal{P}}\sum_{q\in\mathcal{P}}\sum_{h\in\mathcal{H}_{pq}} y_h = n_{pq} \qquad \forall p \in \mathcal{P} \qquad \forall q \in \mathcal{P} \tag{11}$$

The ship capacity constraint on each leg of each liner route has to be respected. To formulate this constraint, let ρ_{hri} be a binary coefficient that takes a value of 1 if containers of the shipment plan h are transported on the ith leg of liner route r and 0 otherwise, and let V_r be the load capacity (TEUs) of ships deployed on liner route r, and the capacity constraint is as follows:

$$\sum_{h\in\mathcal{H}} \rho_{hri} y_h \le V_r \qquad \forall r \in \mathcal{R}, i \in \mathcal{I}_r \tag{12}$$

y_h are nonnegative variables:

$$y_h \ge 0 \qquad \forall h \in \mathcal{H} \tag{13}$$

The SDCRP is now formulated as a nonlinear programming model of Equations 5, 6, and 9 to 13.

GENETIC LOCAL SEARCH ALGORITHM

The nonlinear [SDCRP] is also noncontinuous and nonconvex, and it is very difficult (if not impossible) to design an exact solution method. Nevertheless, this problem has a special structure in that both the schedule design subproblem with fixed container routing decisions and the container routing subproblem with determined schedules can be formulated as linear programming models. This feature was exploited and an efficient heuristic method was designed.

Schedule Design with Fixed Container-Routing Decisions

In [SDCRP], once the routing variable $\mathbf{y}$ is fixed, the transshipment costs are also determined and the schedule design problem (SDP) is to minimize the cost associated with transit time. Denoted by vector $\mathbf{y}^i = (y_{h_1}^i, y_{h_2}^i, \ldots, y_{h_{|\mathcal{H}|}}^i)$ the routing variables, SDP can be formulated as follows:

[SDP($\mathbf{y}^i$)]:

$$\begin{aligned} \bar{c}\left(\mathbf{y}^i\right) = \min_{\mathbf{x}} &\sum_{p\in\mathcal{P}}\sum_{q\in\mathcal{P}}\sum_{h\in\mathcal{H}_{pq}} y_h^i\left(\bar{c}_{pq}\max\left\{0, t_h - T_{pq}\right\} - \tilde{c}_{pq}\max\left\{0, T_{pq} - t_h\right\}\right) \\ &+ \sum_{h\in\mathcal{H}} y_h^i c_h^{\mathrm{trs}} \end{aligned} \tag{14}$$

In Equation 14, the first summation is the cost associated with transit time, and the last summation is the constant transshipment cost. [SDP($\mathbf{y}^i$)] has the constraints from Equations 5, 9, and 10 and is still a nonlinear program. However, this nonlinear program can be transformed into an equivalent mixed integer linear program as follows. First, to reformulate the nonlinear Constraint 9, define the auxiliary constants:

$$t_{rsij}^{w0} = 168k + t_{sj}^{\text{de}} - t_{ri}^{\text{en}}$$
$$k = \arg\min\left\{168k + t_{sj}^{\text{de}} - t_{ri}^{\text{en}} \ge \hat{t}_{p_{ri}}, k \in \mathbb{Z}\right\} \qquad \forall < r,s,i,j > \in Q \tag{15}$$

and Constraint 9 can be restated as follows:

$$t_{rsij}^{w} = 168k + t_{rsij}^{w0} + \left(x_s - x_r\right)$$
$$k = \arg\min\left\{168k + t_{rsij}^{w0} + \left(x_s - x_r\right) \ge \hat{t}_{p_{ri}}, k \in \mathbb{Z}\right\} \qquad \forall < r,s,i,j > \in Q \tag{16}$$

It is clear that $\hat{t}_{p_{ri}} \le t_{rsij}^{w0} < 168 + \hat{t}_{p_{ri}}$, $-168 < x_s - x_r < 168$. Thus, the value of k in Equation 16 can only be 1, 0, or −1. Consequently, Constraint 16 can be restated as follows:

$$t_{rsij}^{w} = 168z_{rsij}^{+} - 168z_{rsij}^{-} + t_{rsij}^{w0} + \left(x_s - x_r\right) \qquad \forall < r,s,i,j > \in Q \tag{17}$$

$$168z_{rsij}^{+} + t_{rsij}^{w0} + \left(x_s - x_r\right) \ge \hat{t}_{p_{ri}} \qquad \forall < r,s,i,j > \in Q \tag{18}$$

$$t_{rsij}^{w0} + \left(x_s - x_r\right) - 168z_{rsij}^{-} \ge \hat{t}_{p_{ri}} \qquad \forall < r,s,i,j > \in Q \tag{19}$$

$$z_{rsij}^{+}, z_{rsij}^{-} \in \{0,1\} \qquad \forall < r,s,i,j > \in Q \tag{20}$$

where z_{rsij}^{+} and z_{rsij}^{-} are auxiliary binary variables introduced by virtue of the weekly service frequency of each liner service route.

Second, to transform the nonlinear objective Function 14 of [SDP($\mathbf{y}^i$)] into a linear one, divide the set of O-D port pairs into three disjoint sets:

$$\mathcal{B}^{+} = \left\{(p,q) \middle| \forall p \in \mathcal{P}\ \forall q \in \mathcal{P}, \bar{c}_{pq} > \tilde{c}_{pq}\right\} \tag{21}$$

$$\mathcal{B}^{0} = \left\{(p,q) \middle| \forall p \in \mathcal{P}, \forall q \in \mathcal{P}, \bar{c}_{pq} = \tilde{c}_{pq}\right\} \tag{22}$$

$$\mathcal{B}^{-} = \left\{(p,q) \middle| \forall p \in \mathcal{P}, \forall q \in \mathcal{P}, \bar{c}_{pq} < \tilde{c}_{pq}\right\} \tag{23}$$

$\mathcal{B}^{+}$, $\mathcal{B}^{0}$, and $\mathcal{B}^{-}$ are mutually exclusive and collectively exhaustive subsets of O-D port pairs. The objective Function 14 can now be restated as follows:

$$\bar{c}\left(\mathbf{y}^i\right) = \min_{\mathbf{x}}\left\{\left(c^{+} + c^{0} + c^{-}\right) + \sum_{h\in\mathcal{H}} y_h^i c_h^{\text{trs}}\right\} \tag{24}$$

$$c^{+} = \sum_{(p,q)\in\mathcal{B}^{+}} \sum_{h\in\mathcal{H}_{pq}} y_h^i\left(\bar{c}_{pq}\max\left\{0, t_h - T_{pq}\right\} - \tilde{c}_{pq}\max\left\{0, T_{pq} - t_h\right\}\right) \tag{25}$$

$$c^{0} = \sum_{(p,q)\in\mathcal{B}^{0}} \sum_{h\in\mathcal{H}_{pq}} y_h^i\left(\bar{c}_{pq}\max\left\{0, t_h - T_{pq}\right\} - \tilde{c}_{pq}\max\left\{0, T_{pq} - t_h\right\}\right) \tag{26}$$

$$c^{-} = \sum_{(p,q)\in\mathcal{B}^{-}} \sum_{h\in\mathcal{H}_{pq}} y_h^i\left(\bar{c}_{pq}\max\left\{0, t_h - T_{pq}\right\} - \tilde{c}_{pq}\max\left\{0, T_{pq} - t_h\right\}\right) \tag{27}$$

Constraint 25 can be restated as follows:

$$c^{+} = \sum_{(p,q)\in\mathcal{B}^{+}} \sum_{h\in\mathcal{H}_{pq}} y_h^i\left(\bar{c}_{pq}t_h^1 - \tilde{c}_{pq}t_h^2\right) \tag{28}$$

$$t_h^1 - t_h^2 = t_h - T_{pq} \qquad \forall(p,q)\in\mathcal{B}^{+} \qquad \forall h \in \mathcal{H}_{pq} \tag{29}$$

$$t_h^1, t_h^2 \ge 0 \qquad \forall(p,q)\in\mathcal{B}^{+} \qquad \forall h \in \mathcal{H}_{pq} \tag{30}$$

Constraint 26 can be reformulated as follows:

$$c^{0} = \sum_{(p,q)\in\mathcal{B}^{0}} \sum_{h\in\mathcal{H}_{pq}} y_h^i\left(\bar{c}_{pq}t_h - \tilde{c}_{pq}T_{pq}\right) \tag{31}$$

Let M be a huge number and z_h be an auxiliary variable to linearize Equation 27, and Constraint 27 can be transformed to

$$c^{-} = \sum_{(p,q)\in\mathcal{B}^{-}} \sum_{h\in\mathcal{H}_{pq}} y_h^i c_h^{-} \tag{32}$$

$$c_h^{-} = \bar{c}_{pq}t_h^1 - \tilde{c}_{pq}T_{pq}z_h + \tilde{c}_{pq}t_h^2 \tag{33}$$

$$t_h^1 \ge t_h - T_{pq} \qquad \forall(p,q)\in\mathcal{B}^{-} \qquad \forall h \in \mathcal{H}_{pq} \tag{34}$$

$$Mz_h \ge T_{pq} - t_h \qquad \forall(p,q)\in\mathcal{B}^{-} \qquad \forall h \in \mathcal{H}_{pq} \tag{35}$$

$$M\left(1 - z_h\right) + t_h^2 \ge t_h \qquad \forall(p,q)\in\mathcal{B}^{-} \qquad \forall h \in \mathcal{H}_{pq} \tag{36}$$

$$t_h^1, t_h^2 \ge 0 \qquad \forall(p,q)\in\mathcal{B}^{-} \qquad \forall h \in \mathcal{H}_{pq} \tag{37}$$

$$z_h \in \left\{0,1\right\} \qquad \forall(p,q)\in\mathcal{B}^{-} \qquad \forall h \in \mathcal{H}_{pq} \tag{38}$$

To eliminate the symmetric solutions of the schedules, arbitrarily stipulate that the schedule of the first liner route is fixed at time 0,

$$x_{r_1} = 0 \tag{39}$$

The feasible set of x_r defined by Equation 5, is an open set. This open set can be replaced with one of its closed subsets from the proposition that follows:

Proposition. If the values of t_{ri}^{en}, t_{ri}^{de}, $\hat{t}_p$, and T_{pq} for any $r \in \mathcal{R}$, $i \in I_r$, $p \in \mathcal{P}$, and $q \in \mathcal{P}$ are all integers, at least in one of the optimal solutions of [SDP($\mathbf{y}^i$)] all the x_r, $r \in \mathcal{R}$ take integer values.

Proof. Suppose that in an optimal solution, some x_r are not integers. Derive another optimal solution where all x_r are integers from this solution as follows: Rearrange those noninteger x_r according to the sequence $\bar{x}_{r_1'} - \lfloor \bar{x}_{r_1'} \rfloor \le \bar{x}_{r_2'} - \lfloor \bar{x}_{r_2'} \rfloor \le \cdots \le \bar{x}_{r_m'} - \lfloor \bar{x}_{r_m'} \rfloor$, where $\lfloor x \rfloor$ is the largest integer not more than x, and m is the number of noninteger x_r.

1. If $\bar{x}_{r_1'} - \lfloor \bar{x}_{r_1'} \rfloor < \bar{x}_{r_2'} - \lfloor \bar{x}_{r_2'} \rfloor$, because of the integrality property of t_{ri}^{en}, t_{ri}^{de}, $\hat{t}_p$, and T_{pq}, the objective cost value would be either asymptotically increasing or asymptotically decreasing when $\bar{x}_{r_1'}$ increases from $\lfloor \bar{x}_{r_1'} \rfloor$ to $\lfloor \bar{x}_{r_1'} \rfloor + \bar{x}_{r_2'} - \lfloor \bar{x}_{r_2'} \rfloor$. In the former case, $\lfloor \bar{x}_{r_1'} \rfloor$ could be used in place of $\bar{x}_{r_1'}$, and the solution is as least as good. In the latter case, replace $\bar{x}_{r_1'}$ with $\lfloor \bar{x}_{r_1'} \rfloor + \bar{x}_{r_2'} - \lfloor \bar{x}_{r_2'} \rfloor$, and the solution is at least as good.
2. If $\bar{x}_{r_1'} - \lfloor \bar{x}_{r_1'} \rfloor = \bar{x}_{r_2'} - \lfloor \bar{x}_{r_2'} \rfloor < \bar{x}_{r_3'} - \lfloor \bar{x}_{r_3'} \rfloor$, reset $\bar{x}_{r_1'}$ and $\bar{x}_{r_2'}$ at $\lfloor \bar{x}_{r_1'} \rfloor$ and $\lfloor \bar{x}_{r_1'} \rfloor$, respectively, or reset $\bar{x}_{r_1'}$ and $\bar{x}_{r_2'}$ at $\lfloor \bar{x}_{r_1'} \rfloor + \bar{x}_{r_3'} - \lfloor \bar{x}_{r_3'} \rfloor$ and

$\lfloor \bar{x}_{r'_2} \rfloor + \bar{x}_{r'_3} - \lfloor \bar{x}_{r'_3} \rfloor$, respectively, depending on the change of the objective value.

3. Other cases can be handled similarly. By repeating this procedure, another optimal solution will be obtained in which all x_r are integers.■

From the previously mentioned proposition, Constraint 5 could be replaced with the following:

$$0 \le x_r \le 167 \qquad \forall r \in \mathcal{R} \tag{40}$$

Now [SDP($\mathbf{y}^i$)] defined by Equations 17 to 20, 24, 28 to 40 is a mixed integer linear program and can be solved efficiently by CPLEX.

Container Routing with Determined Schedules

In [SDCRP], once the schedule variables x_r are fixed, the coefficients of y_h in the Objective Function 14 are also determined, and [SDCRP] becomes a simple linear programming model with exclusively continuous variables y_h. Let the vector $\mathbf{x}^i$ denote the values of the fixed schedule variables, and define the coefficients:

$$c_h(\mathbf{x}^i) = c_h^{\text{trs}} + \bar{c}_{pq} \max\{0, t_h - T_{pq}\} - \tilde{c}_{pq} \max\{0, T_{pq} - t_h\}$$
$$\forall p \in \mathcal{P} \qquad \forall q \in \mathcal{P} \qquad \forall h \in \mathcal{H}_{pq} \tag{41}$$

The container routing problem (CRP) is formulated as follows:

[CRP($\mathbf{x}^i$)]:

$$\bar{c}\left(\mathbf{x}^i\right) = \min_{\mathbf{y}} \sum_{p \in \mathcal{P}} \sum_{q \in \mathcal{P}} \sum_{h \in \mathcal{H}_{pq}} y_h c_h\left(\mathbf{x}^i\right) \tag{42}$$

subject to Constraints 11 through 13.

Genetic Local Search Heuristic

On the basis of the two linear programming subproblems SDP and CRP, a genetic local search heuristic was designed for the SDCRP. The chromosome of a candidate solution is the direct coding of the schedule variables, namely $(x_{r_1}, x_{r_2}, x_{r_3}, \ldots, x_{r_{|\mathcal{R}|}})$ in which $x_{r_1} = 0$. The routing variables y_h are not required. The schedule variables were chosen rather than the routing variables because there are fewer schedule variables and the feasible set of the schedule variables is easily obtainable from Equation 40.

A fitness value reflects the goodness of an individual compared with the other individuals in the population. Before determining the fitness value of a chromosome, start from it and find a locally optimal solution and then replace the original chromosome with the locally optimal one. Here, a locally optimal solution is defined as one dominating all solutions that differ from it either in the schedule variables or in the routing variables, but not both. The search process from the current solution to a locally optimal one is as follows: denote the schedule decisions in the chromosome by $\mathbf{x}^1$ and solve [CRP($\mathbf{x}^1$)]. Let $\bar{c}(\mathbf{x}^1)$ be the optimal objective value and the optimal solution is $\mathbf{y}^1$. Then solve [SDP($\mathbf{y}^1$)] and the optimal objective value and optimal solution are $\bar{c}(\mathbf{y}^1)$ and $\mathbf{x}^2$, respectively. It is clear that $\bar{c}(\mathbf{y}^1) \le \bar{c}(\mathbf{x}^1)$. If $\bar{c}(\mathbf{y}^1) = \bar{c}(\mathbf{x}^1)$, the local optimum has been found. The objective value of [SDCRP] cannot be improved by changing only the schedule variables or by changing only the routing variables. Otherwise, repeat this procedure from $\mathbf{x}^2$ until the locally optimal solution $\mathbf{x}^*$ and $\mathbf{y}^*$ is found. Then replace the original chromosome with the local optimal one and the fitness value is defined as $1/\bar{c}(\mathbf{x}^*)$.

The selection process is used for choosing two parents to apply the crossover operator. For two parents with schedule variables $\mathbf{x}^a$ and $\mathbf{x}^b$, the crossover procedure simply generates a random variable $\lambda \in [0, 1]$, and the offspring are $\lambda\mathbf{x}^a + (1 - \lambda)\mathbf{x}^b$ and $(1 - \lambda)\mathbf{x}^a + \lambda\mathbf{x}^b$. The mutation step randomly chooses a liner route $r \in \mathcal{R} \setminus \{r_1\}$ and sets its schedule at a random number between 0 and 167 from Equation 40.

APPLICATION TO ASIA–EUROPE–OCEANIA LINER SHIPPING NETWORK

The proposed model and solution algorithm were applied to the Asia–Europe–Oceania liner shipping network of a global liner company, which consists of 46 ports, as shown in Figure 3. Table 2 shows the 11 liner shipping routes that connect these ports, the entry and departure time (shown in parentheses) at each port call, as well as the type and number of ships deployed. For all the 46 ports $p \in \mathcal{P}$, the minimum time interval for transshipment $\hat{t}_p = 6$ (h), and the transshipment cost $\hat{c}_p = 200$ (US$/TEU). Container shipment demand n_{pq} are derived from realistic demand data of the liner shipping company. Market level transit time T_{pq}, penalty $\bar{c}_{pq}$ and bonus $\tilde{c}_{pq}$ are estimated by shipping network designers. The data reported here were modified from the real data for confidentiality. However, the problem size and characteristics remain the same.

The program is coded with C++ and the linear models [SDP($\mathbf{y}^i$)] and [CRP($\mathbf{x}^i$)] are solved by CPLEX-12.1. The heuristic algorithm runs on a 3 GHz Dual Core PC with 4 GB of RAM. The settings of the heuristic algorithm are as follows: the population size is 20, crossover rate is 0.4, mutation rate is 0.2, and after 100 generations, the algorithm stops. The results of 10 runs with $\bar{c}_{pq} = \tilde{c}_{pq} = 1$ (US$/TEU · h) are depicted in Figure 4. There were many local optima in the solution space. Each step in Figure 4 denotes a local optimum. In the heuristic search process, the solution improves very fast. The average relative gap between the solution in the 60th generation and the 100th generation is only 0.007%. That is why we run the heuristic algorithm for only 100 generations.

To justify the efforts for solving the nonlinear formulation [SDCRP], the solution of the genetic local search heuristic with the solution that minimizes the transshipment cost was compared. The latter is obtained as follows. First, solve the cargo routing problem [CRP($\mathbf{x}^i$)] while in the objective function only the transshipment cost is considered, namely, let $c_h(\mathbf{x}^i) = c_h^{trs}$ in place of Equation 41. Consequently, obtain the cargo routing decision that minimizes the transshipment cost. Then use this cargo routing result as an input for [SDP($\mathbf{y}^i$)] and compute the optimal schedules and the total cost. The comparison of the two methods with $\bar{c}_{pq} = \tilde{c}_{pq}$ varying from 0.6 to 1.6 (US$/TEU · h) is shown in Figure 5. In particular, divide the total cost into two parts: transshipment cost and the penalty cost minus the bonus. When minimizing the transshipment cost is taken as the primary objective, the total cost is larger than the solution of the genetic local search heuristic, especially when the penalty (bonus) is large. Another interesting result from Figure 5 is that in the solution of the genetic local search heuristic, the transshipment cost

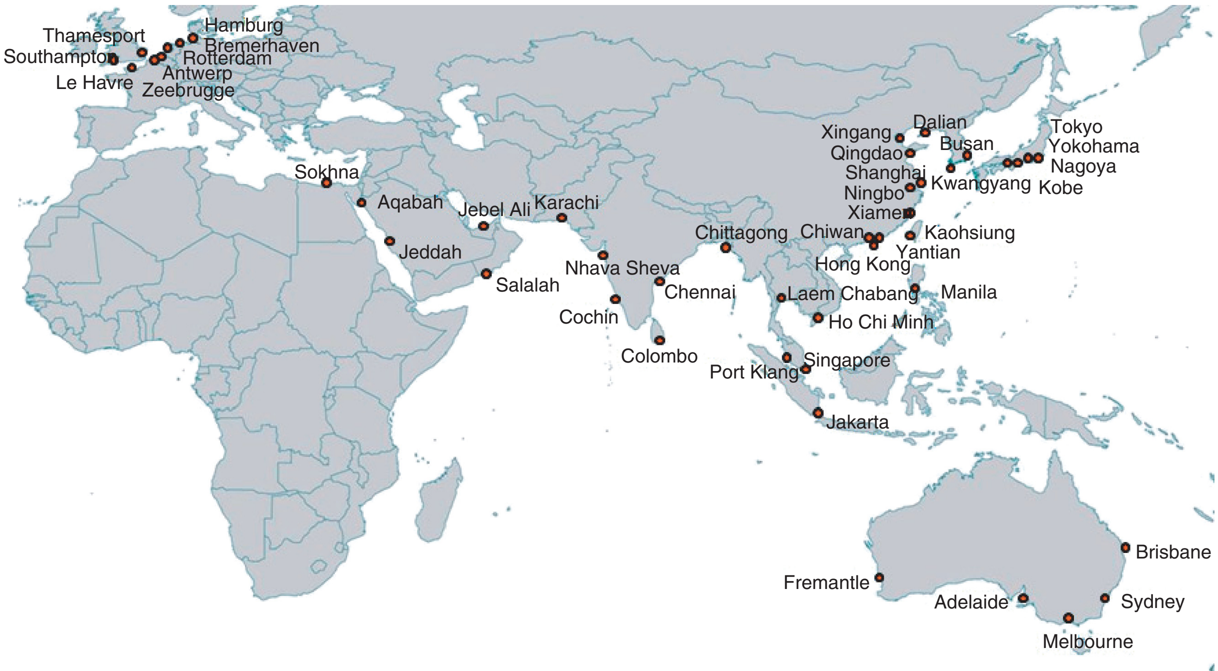

FIGURE 3 Asia–Europe–Oceania liner shipping network.

TABLE 2 Liner Shipping Routes in Asia–Europe–Oceania Network

No.	Ship Type and No.	Portcalls (Entry and Departure Time)
1	10,000 TEU, 7	Southampton, United Kingdom (0,16)→Sokhna, Egypt (142,158)→Salalah, Dhofar, Oman (233,249) →Colombo (314,330)→Singapore (392,408)→Hong Kong (464,480) →Xiamen (490,506)→Shanghai, China (525,541)→Busan, South Korea (558,574) →Dalian, China (593,609)→Xingang, China (616,632)→Qingdao, China (647,663) →Shanghai (675,691)→Hong Kong (720,736)→Singapore (792,808) →Colombo (870,886)→Salalah (951,967)→Southampton
2	5,000 TEU, 3	Singapore (0,20)→Brisbane, Australia (182,202)→Sydney, Australia (220,240) →Melbourne, Australia (262,282)→Adelaide, Australia (302,322) →Fremantle, Australia (380,400)→Singapore
3	5,000 TEU, 5	Xiamen (0,24)→Chiwan, China (36,60)→Hong Kong (60,84) →Singapore (146,170)→Port Klang, Malaysia (178,202)→Salalah (335,359) →Jeddah, Saudi Arabia (415,439) →Aqaba, Jordan (463,487)→Salalah (567,591) →Singapore (732,756)→Xiamen
4	3,000 TEU, 2	Yokohama, Japan (0,44)→Tokyo (44,88)→Nagoya, Japan (97,141)→Kobe, Japan (151,195) →Shanghai (233,277)→Yokohama
5	3,000 TEU, 2	Ho Chi Minh, Vietnam (0,52)→Laem Chabang, Thailand (83,135)→Singapore (174,226) →Port Klang (235,287)→Ho Chi Minh
6	3,000 TEU, 4	Brisbane (0,31)→Sydney (53,84)→Melbourne (111,142) →Adelaide (166,197)→Fremantle (267,298)→Jakarta (389,420) →Singapore (445,476)→Brisbane
7	3,000 TEU, 2	Manila, Philippines (0,35)→Kaohsiung, Taiwan, China (62,97)→Xiamen (105,140) →Hong Kong (153,188)→Yantian, China (188,223)→Chiwan (224,259) →Hong Kong (259,294)→Manila
8	3,000 TEU, 2	Dalian (0,28)→Xingang (37,65)→Qingdao (85,113) →Shanghai (129,157)→Ningbo, China (161,189)→Shanghai (193,221) →Kwangyang, South Korea (241,269)→Busan (272,300)→Dalian
9	3,000 TEU, 3	Chittagong, Bangladesh (0,30)→Chennai (76,106)→Colombo (136,166) →Cochin (182,212)→Nhava Sheva, India (250,280)→Cochin (318,348) →Colombo (364,394)→Chennai (424,454)→Chittagong
10	5,000 TEU, 3	Sokhna (0,31)→Aqaba (42,73)→Jeddah (97,128)→Salalah(184,215) →Karachi, Pakistan (254,285)→Jebel Ali, United Arab Emirates (315,346) →Salalah (384,415)→Sokhna
11	10,000 TEU, 1	Southampton (0,15)→Thamesport, United Kingdom (21,36)→Hamburg, Germany (51,66) →Bremerhaven, Germany (69,84)→Rotterdam, Netherlands (91,106) →Antwerp, Belgium (107,122)→Zeebrugge, Belgium (124,139) →Le Havre, France (145,160)→Southampton

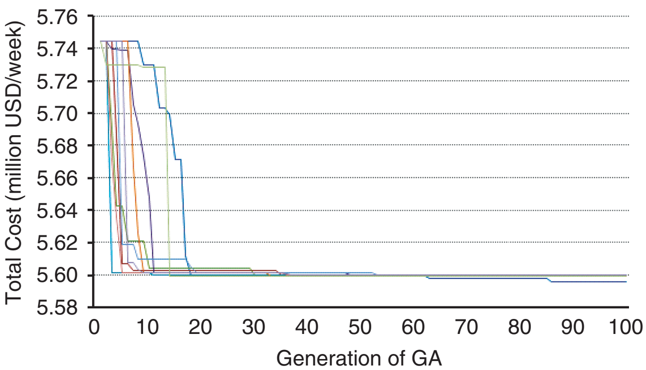

FIGURE 4 Results of 10 runs of genetic local search heuristic (GA = genetic algorithm).

increases with the penalty (bonus). When the penalty (bonus) is large, it is more important to shorten the transit time than to reduce the transshipment cost.

More experiments were carried out to analyze the sensitivity of the solution with the penalty and bonus and the market level transit time. First, 25 experiments were implemented with different combinations of penalty and bonus for the same market level transit time as cases in Figure 5 and the results are reported in Figure 6*a*. Surprisingly, the total cost decreases almost linearly with the decrease of penalty and increase of bonus. It should be mentioned that the total cost is more sensitive to the penalty than the bonus. With the penalty increasing from 0.6 to 1.4, an additional \$1.7 million/week is incurred, whereas the total cost increases by only \$1 million/week when the bonus decreases from 1.4 to 0.6. This is because in the current market level transit time settings, the total penalty is larger than the total bonus (which can also be seen from Figure 5 that the penalty minus bonus is positive). In other words, more O-D port pairs or more containers cannot maintain the market level transit time. Hence, the variation of penalty has more significant impacts on the total cost than the bonus. Cases with different market level transit times were also tested. On the basis of the market level transit time used for cases in Figure 5, the time was reduced by 1 day for all the O-D port pairs or increased by 1, 2, or 3 days. The results are shown in Figure 6*b*. Again, the total cost decreases almost linearly with the increase of market level transit time. The total cost is very sensitive to the market level transit time: the decrease of the market level transit time by one day means a \$0.5 million/week increase of the total cost.

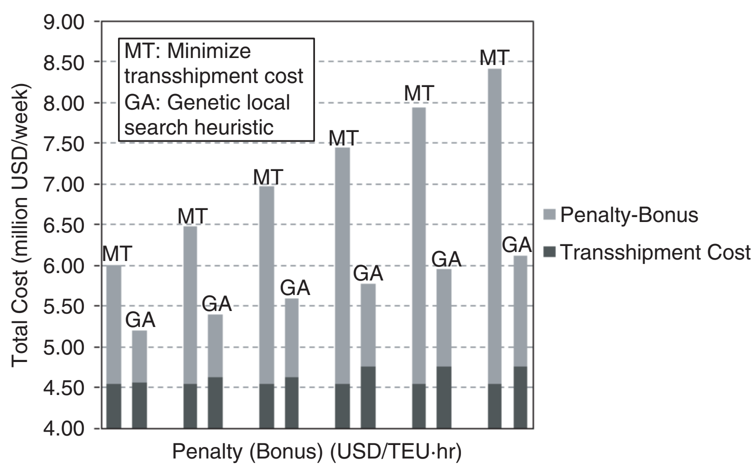

FIGURE 5 Comparison of transshipment cost minimization method and genetic local search heuristic.

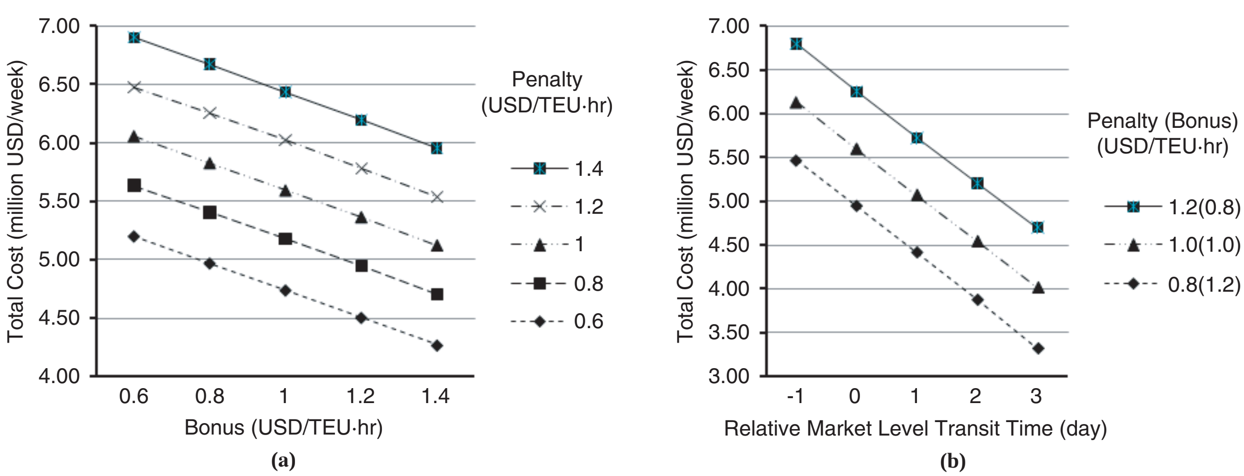

FIGURE 6 Sensitivity analysis for penalty (bonus) and market level transit time.

CONCLUSIONS

Motivated by industrial practice, the schedule design, and container routing problem in liner shipping were investigated. The concept of market-level transit time was introduced, which is one major concern of liner service designers. The formulation for this problem is nonlinear, noncontinuous, and nonconvex. An efficient genetic local search heuristic was developed. Computational results show that the genetic local search heuristic can efficiently find good quality solutions. It should be mentioned that the real container shipment demand may be a little different from the predicted one used for designing the schedules because of the cancellation of contracted shipping orders and the fluctuation of the spot shipping market. Therefore, the model for the container routing subproblem can be separately used to optimize the day-to-day container routing decisions for the real container shipment demand after the schedules have been designed.

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