Jordan blocks of Hamiltonian Matrices with pure imaginary eigenvalues

Main result

H_∞

${}^*H+$ $G-$ $HB\Gamma^{-1}B^*H<0$

where A,B,G,Γ are given matrices of dimensions $n\times n, n\times m, n\times n$, and $m\times m$ respectively

G,Γ are Hermitian matrices and $\det\Gamma\neq 0$

H is Herimitian matrix, solution to the inequality

H_-

H_+

$A-B\Gamma^{-1}B^*H_-$

$-(A-B\Gamma^{-1}B^*H_+)^*$

$\square$

Γ

This lemma has been originally proved for finite dimensional case, and extended to infinite dimensional case

Γ

${}^*H+G-HB\Gamma^{-1}B^*H=0$

which is closely related to the existence and properties of maximal J-orthogonal invariant subspaces of Hamiltonian matrices

(A,B)

A

Γ

$$\begin{pmatrix} HA+A^*H+G & HB \\ B^*H & \Gamma \end{pmatrix}<0$$

This may be solved via the well known interior point method.

$\pi(i\omega) <$
$0,$

$\omega \in$
$[-\infty, \infty]$

$$\pi(\lambda) = \Gamma + B^*(\lambda I + A^*)^{-1} G (A - \lambda I)^{-1} B,$$

Γ
$\pi(i\omega) <$
0

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