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Review on CP Violation

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Abstract

A phenomenological description of the neutral K and B meson systems is presented. The situation of the current CP violation experiments is described and a detailed discussion of their results is given, followed by some future prospects.

1   Introduction

Symmetries are one of the most fundamental concepts in the laws of nature leading to conserving quantities. Unexpected violations of symmetries indicate some dynamical mechanism beyond the current understanding of physics.

Observed P and C violations are well understood in the framework of the standard model. They are naturally generated by left-handed charged weak currents. On the other hand, the origin of CP violation is still not explained. The standard electroweak theory can accommodate CP violation with some complex elements in the quark mass mixing matrix [1]. However, a possibility that CP violation originates from some effect at a much higher energy scale is not excluded [2]. Interest in possible CPT violation has been revived by string theory [3].

Since the discovery of CP violating K_L decays [4], the neutral Kaon system is still the only system known to exhibit violation of this symmetry. In this article, we recapitulate the description of the neutral kaon system in order to outline necessary experimental measurements which are still missing [5]. It is followed by a review on the current experimental situation and on future prospects.

B-mesons appear to be the most attractive place to study CP violation in a quantitative manner [6]. After a comparison of the K and B meson systems, a short discussion of the experimental prospects in the B sector is presented.

2   Neutral K-Meson System

2.1   Basic Formalism

Let $|K^0\rangle$ and $|\overline{K}^0\rangle$ be the stationary states of the K^0-meson and its antiparticle $\overline{K}^0$, respectively. Both states are eigenstates of the strong and electromagnetic interaction Hamiltonian, i.e.

$$(H_{\text{st}} + H_{\text{em}})\,|K^0\rangle = m_0|K^0\rangle \;\; \text{and} \;\; (H_{\text{st}} + H_{\text{em}})\,|\overline{K}^0\rangle = \overline{m}_0|\overline{K}^0\rangle$$

where m_0 and $\overline{m}_0$ are the rest masses of K^0 and $\overline{K}^0$, respectively. The K^0 and $\overline{K}^0$ states are connected through CP transformations. For stationary states, T does not alter them with the exception of an arbitrary phase. In summary, we obtain

$$
\begin{aligned}
CP\,|K^0\rangle = e^{i\,\theta_{\text{CP}}}|\overline{K}^0\rangle \;\; &\text{and} \;\; CP\,|\overline{K}^0\rangle = e^{-\,i\,\theta_{\text{CP}}}|K^0\rangle \\
T\,|K^0\rangle = e^{i\,\theta_{\text{T}}}|K^0\rangle \;\; &\text{and} \;\; T\,|\overline{K}^0\rangle = e^{i\,\overline{\theta}_{\text{T}}}|\overline{K}^0\rangle
\end{aligned}
\tag{1}
$$

where θ's are arbitrary phases and it follows that

$$2\,\theta_{\text{CP}} = \overline{\theta}_{\text{T}} \,-\, \theta_{\text{T}} \;.$$

by assuming $CPT\,|K^0\rangle = TCP\,|K^0\rangle$.

If strong and electromagnetic interactions are invariant under CPT transformation, which is assumed throughout this paper, it follows that $m_0 = \overline{m}_0$.

Next, we introduce a new interaction, V, violates strangeness conservation. Through such interactions, the K-mesons can decay into final states with no strangeness ($|\Delta S| = 1$) and K^0 and $\overline{K}^0$ can oscillate to each other ($|\Delta S| = 2$). Thus, a general state $|\psi(t)\rangle$ which is a solution of the Schrödinger equation

$$i\,\frac{\partial}{\partial t}|\psi(t)\rangle = (H_{\text{st}} + H_{\text{em}} + V)\,|\psi(t)\rangle \tag{2}$$

can be written as

$$|\psi(t)\rangle = a(t)|K^0\rangle + b(t)|\overline{K}^0\rangle + \sum_{\text{f}} c_{\text{f}}(t)|\text{f}\rangle$$

where $a(t)$, $b(t)$ and $c_f(t)$ are time dependent functions. For a new interaction which is much weaker than strong and electromagnetic interactions, perturbation theory and the Wigner-Weisskopf method [7] can be applied to solve equation 2. We obtain

$$i\frac{\partial}{\partial t}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = \boldsymbol{\Lambda}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = \left(\boldsymbol{M} - i\frac{\boldsymbol{\Gamma}}{2}\right)\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} \tag{3}$$

where the 2×2 matrices $\boldsymbol{M}$ and $\boldsymbol{\Gamma}$ are often referred to as the mass and decay matrices.

The elements of the mass matrix are given as

$$M_{ij} = m_0\,\delta_{ij} + \langle i|V|j\rangle + \sum_f \mathcal{P}\left(\frac{\langle i|V|f\rangle\langle f|V|j\rangle}{m_0 - E_f}\right) \tag{4}$$

where $\mathcal{P}$ stands for the principal part and the index $i = 1(2)$ denotes $K^0(\overline{K}^0)$. Let us split the Hamiltonian V into the known weak interaction part H_{weak} and a hypothetical superweak interaction [2], H_{sw}, i.e. $V = H_{weak} + H_{sw}$. Since ordinary weak interactions do not produce a direct K^0-$\overline{K}^0$ transition, the second term of equation 4 applies only for the superweak interaction for $i \neq j$. The third term is dominated by the weak interaction since the second order superweak interaction must be negligible. It follows that

$$M_{ij} = m_0\,\delta_{ij} + \langle i|H_{sw}|j\rangle + \sum_f \mathcal{P}\left(\frac{\langle i|H_{weak}|f\rangle\langle f|H_{weak}|j\rangle}{m_0 - E_f}\right) . \tag{5}$$

Note that the sum is taken over *all possible intermediate states* common to K^0 and $\overline{K}^0$ for $i \neq j$.

The elements of the decay matrix are given by

$$\Gamma_{ij} = 2\pi \sum_f \langle i|H_{weak}|f\rangle\langle f|H_{weak}|j\rangle\delta(m_0 - E_f) \tag{6}$$

The sum is taken over only *real final states* common to K^0 and $\overline{K}^0$ for $i \neq j$. Since Γ_{ij} starts from second order, the superweak Hamiltonian can be neglected.

If Hamiltonians are not Hermitian, transition probabilities are not conserved in decays or oscillations, i.e. the number of initial particles is not identical to the number of final particles. This is also referred as break down of unitarity. **We assume from now on that all the Hamiltonians are Hermitian**.

If V is Hermitian and invariant under T, CPT or CP transformations, the mass and decay matrices must satisfy the following conditions;

$$\begin{array}{lll} \text{T} & : & \left|M_{12} - i\dfrac{\Gamma_{12}}{2}\right| = \left|M_{12}^* - i\dfrac{\Gamma_{12}^*}{2}\right| \\[2ex] \text{CPT} & : & M_{11} = M_{22},\ \Gamma_{11} = \Gamma_{22} \\[2ex] \text{CP} & : & \left|M_{12} - i\dfrac{\Gamma_{12}}{2}\right| = \left|M_{12}^* - i\dfrac{\Gamma_{12}^*}{2}\right|,\ M_{11} = M_{22},\ \Gamma_{11} = \Gamma_{22} \end{array}$$

where equations 1, 5 and 6 are used. It follows that

$$\begin{array}{l} \bullet\text{if } M_{11} \neq M_{22} \text{ or } \Gamma_{11} \neq \Gamma_{22}\ : \\ \quad\textbf{CPT and CP are violated} \\[2ex] \bullet\text{if } \sin(\varphi_\Gamma - \varphi_M) \neq 0\ : \\ \quad\textbf{T (or unitarity) and CP are violated .} \end{array} \tag{7}$$

where $\varphi_M = \arg(M_{12})$ and $\varphi_\Gamma = \arg(\Gamma_{12})$.

Note that CP is not conserved in both above cases; i.e. CP violation in the mass and decay matrices cannot be separated from CPT violation or T violation.

Solutions of equation 3 for initially pure K^0 and $\overline{K}^0$ states are given by

$$
\begin{aligned}
|K^0(t)\rangle &= [\, f_+(t) - 2\,\varepsilon_{\text{CPT}}\, f_-(t)\,]\,|K^0\rangle + (\,1 - 2\,\varepsilon_{\text{T}}\,)\, e^{-i\,\varphi_\Gamma}\, f_-(t)\,|\overline{K}^0\rangle \quad (8) \\
&= \frac{1}{\sqrt{2}}\left(\,|K_S\rangle\, e^{-i\,\lambda_S\, t} + |K_L\rangle\, e^{-i\,\lambda_L\, t}\,\right) \quad (9)
\end{aligned}
$$

and

$$
\begin{aligned}
|\overline{K}^0(t)\rangle &= (\,1 + 2\,\varepsilon_{\text{T}}\,)\, e^{i\,\varphi_\Gamma}\, f_-(t)\,|K^0\rangle + [\, f_+(t) + 2\,\varepsilon_{\text{CPT}}\, f_-(t)\,]\,|\overline{K}^0\rangle \quad (10) \\
&= \frac{1 + 2\,\varepsilon_{\text{T}}}{\sqrt{2}}\, e^{i\,\varphi_\Gamma} \\
&\quad \times \left[\,(1 + 2\,\varepsilon_{\text{CPT}})\,|K_S\rangle\, e^{-i\,\lambda_S\, t} - (1 - 2\,\varepsilon_{\text{CPT}})\,|K_L\rangle\, e^{-i\,\lambda_L\, t}\,\right]
\end{aligned}
$$

where

$$
f_\pm(t) = \frac{1}{2}\left(\,e^{-i\,\lambda_S\, t} \pm e^{-i\,\lambda_L\, t}\,\right) \ .
$$

The parameters λ_S and λ_L are eigenvalues of $\boldsymbol{\Lambda}$, and K_S and K_L are the corresponding eigenstates given by

$$
\begin{aligned}
|K_S\rangle &= \frac{1}{\sqrt{2}}\left[\,(\,1 - 2\,\varepsilon_{\text{CPT}}\,)\,|K^0\rangle + (\,1 - 2\,\varepsilon_{\text{T}}\,)\, e^{-i\,\varphi_\Gamma}\,|\overline{K}^0\rangle\,\right] \\
|K_L\rangle &= \frac{1}{\sqrt{2}}\left[\,(\,1 + 2\,\varepsilon_{\text{CPT}}\,)\,|K^0\rangle - (\,1 - 2\,\varepsilon_{\text{T}}\,)\, e^{-i\,\varphi_\Gamma}\,|\overline{K}^0\rangle\,\right] \ .
\end{aligned} \quad (11)
$$

They have definite masses and decay widths given by λ_S and λ_L as

$$
\lambda_{S(L)} = m_{S(L)} - i\,\frac{\Gamma_{S(L)}}{2}
$$

with

$$
\begin{aligned}
m_{S(L)} &= \frac{M_{11} + M_{22}}{2} + (-)\,\Re\left(\sqrt{\Lambda_{12}\,\Lambda_{21}}\right) \\
&= \frac{M_{11} + M_{22}}{2} - (+)\,|M_{12}|
\end{aligned}
$$

and

$$
\begin{aligned}
\Gamma_{S(L)} &= \frac{\Gamma_{11} + \Gamma_{22}}{2} - (+)\,2\,\Im\left(\sqrt{\Lambda_{12}\,\Lambda_{21}}\right) \\
&= \frac{\Gamma_{11} + \Gamma_{22}}{2} + (-)\,|\Gamma_{12}|
\end{aligned}
$$

where we used

$$
\varphi_\Gamma - \varphi_M = \pi - \delta\varphi, \quad |\delta\varphi| \ll 1
$$

and

$$
|\Lambda_{22} - \Lambda_{11}| \ll 1
$$

which are derived from empirical facts, $m_L > m_S$, $\Gamma_S > \Gamma_L$ and small CP violation. Note that the current values given by Particle Data Group [8] are

$$
\begin{aligned}
\Delta m &\equiv m_L - m_S = 2\,|M_{12}| = (0.5351 \pm 0.0024)\times 10^{10}\ \hbar\text{s}^{-1} \\
\Delta\Gamma &\equiv \Gamma_S - \Gamma_L = 2\,|\Gamma_{12}| = (1.1189 \pm 0.0025)\times 10^{10}\ \text{s}^{-1}\ .
\end{aligned}
$$

The two CP violation parameters ε_{T} and $\varepsilon_{\mathrm{CPT}}$ are given by

$$\varepsilon_{\mathrm{T}} = \frac{\Delta m\,\Delta\Gamma}{4\,\Delta m^2 + \Delta\Gamma^2}\left(1 + i\,\frac{2\,\Delta m}{\Delta\Gamma}\right)\delta\varphi$$

$$\varepsilon_{\mathrm{CPT}} = \frac{i\,2\,\Delta\Gamma}{4\,\Delta m^2 + \Delta\Gamma^2}\left(1 + i\,\frac{2\,\Delta m}{\Delta\Gamma}\right)(\Lambda_{22} - \Lambda_{11})\ .$$

As seen from the statements 7, $\varepsilon_{\mathrm{T}} \neq 0$ implies CP and T violation, and $\varepsilon_{\mathrm{CPT}} \neq 0$ means CP and CPT violation. It should be noted that both ε_{T} and $\varepsilon_{\mathrm{CPT}}$ **do not depend on any phase convention**. The phase of ε_{T} is given by the $\mathrm{K_S}$-$\mathrm{K_L}$ mass and decay width differences which are not related to CP violation. This phase is often referred to as "superweak" phase:

$$\phi_{\mathrm{sw}} = \arg\left(\varepsilon_{\mathrm{T}}\right) = \tan^{-1}\left(\frac{2\,\Delta m}{\Delta\Gamma}\right)\ .$$

If we assume that ordinary weak interactions conserve CPT, i.e. $\Gamma_{11} = \Gamma_{22}$, the phase of the CP and CPT violation parameter $\varepsilon_{\mathrm{CPT}}$ is given by

$$\arg\left(\varepsilon_{\mathrm{CPT}}\right) = \phi_{\mathrm{sw}} + \frac{\pi}{2}\ .$$

2.2 CP Violation and Semileptonic Decays

The instantaneous decay amplitudes for K^0 and $\overline{\mathrm{K}}^0 \to \ell^+\pi^-\nu$ are given by

$$A_+ = \langle\pi^-(\vec{p}_\pi), \ell^+(\vec{p}_\ell, \vec{s}), \nu(\vec{p}_\nu)|H_{\mathrm{weak}}|\mathrm{K}^0\rangle$$

and

$$\overline{A}_+ = \langle\pi^-(\vec{p}_\pi), \ell^+(\vec{p}_\ell, \vec{s}), \nu(\vec{p}_\nu)|H_{\mathrm{weak}}|\overline{\mathrm{K}}^0\rangle\ .$$

Parameters $\vec{p}$ and $\vec{s}$ are the momentum vectors and spin, respectively. In the standard model, $\overline{A}_+$ is very strongly suppressed compared with A_+ ($\Delta Q = \Delta S$ rule). Similarly, we can define decay amplitudes for K^0 and $\overline{\mathrm{K}}^0 \to \ell^-\pi^+\overline{\nu}$ to be

$$A_- = \langle\pi^+(\vec{p}_\pi), \ell^-(\vec{p}_\ell, -\vec{s}), \overline{\nu}(\vec{p}_\nu)|H_{\mathrm{weak}}|\mathrm{K}^0\rangle$$

and

$$\overline{A}_- = \langle\pi^+(\vec{p}_\pi), \ell^-(\vec{p}_\ell, -\vec{s}), \overline{\nu}(\vec{p}_\nu)|H_{\mathrm{weak}}|\overline{\mathrm{K}}^0\rangle\ .$$

Note that the spin of the lepton is reversed. The decay amplitude A_- violates the $\Delta Q = \Delta S$ rule.

If we assume that CPT is conserved in the ordinary weak interaction, the following relations can be obtained;

$$|A_+| = \left|\overline{A}_-\right|,\ \ |A_-| = \left|\overline{A}_+\right|$$

and

$$A_+\,\overline{A}_+^* = A_-\,\overline{A}_-^*\ .$$

These relations do not depend on any phase convention.

The parameter x defined as

$$x = \frac{\sum_{\vec{s}}\int\mathrm{d}\Omega\,A_+^*\,\overline{A}_+\,e^{-i\varphi_\Gamma}}{\sum_{\vec{s}}\int\mathrm{d}\Omega\,|A_+|^2} = \frac{\sum_{\vec{s}}\int\mathrm{d}\Omega\,A_-^*\,\overline{A}_-\,e^{-i\varphi_\Gamma}}{\sum_{\vec{s}}\int\mathrm{d}\Omega\,\left|\overline{A}_-\right|^2} \tag{12}$$

is a measure for the violation of $\Delta Q = \Delta S$. Note that x **does not depend on any phase convention**.

Using equations 11, the charge asymmetry in the semileptonic K_L decay is now given as

$$\delta_\ell \;=\; \frac{\Gamma(K_L \to \pi^- \ell^+ \nu) \;-\; \Gamma(K_L \to \pi^+ \ell^- \overline{\nu})}{\Gamma(K_L \to \pi^- \ell^+ \nu) \;+\; \Gamma(K_L \to \pi^+ \ell^- \overline{\nu})}$$

$$=\; 2\left[\Re\left(\varepsilon_{\mathrm{CPT}}\right) + \Re\left(\varepsilon_{\mathrm{T}}\right)\right].$$

where $\mathcal{O}(\varepsilon_{\mathrm{CPT}}) \approx \mathcal{O}(\varepsilon_{\mathrm{T}}) \approx x \ll 1$ is assumed. It has to be noted that the experimentally well established CP violation effect [8]

$$\delta_\ell = (0.327 \pm 0.012)\times 10^{-2}$$

cannot tell whether it is violation of CP and CPT or CP and T.

The separation of CP and CPT violating processes from CP and T violating processes can be done if we start from the initially pure K^0 and $\overline{K}^0$ states [9]. Let us consider the following four time dependent decay rates:

- $R_+(t)$: K^0 at $t = 0$ decaying into $\ell^+ \pi^- \nu$ at time t

- $R_-(t)$: K^0 at $t = 0$ decaying into $\ell^- \pi^+ \overline{\nu}$ at time t

- $\overline{R}_+(t)$: $\overline{K}^0$ at $t = 0$ decaying into $\ell^+ \pi^- \nu$ at time t

- $\overline{R}_-(t)$: $\overline{K}^0$ at $t = 0$ decaying into $\ell^- \pi^+ \overline{\nu}$ at time t.

These four rates can be obtained using equations 8 and 10 as

$$
\begin{aligned}
R_+(t)\ \left(\overline{R}_-(t)\right)\ \propto\ & \left[1 -(+)\,4\,\Re(\varepsilon_{\mathrm{CPT}}) + 2\,\Re(x)\right]e^{-i\,\Gamma_{\mathrm{S}}\,t} \\
& + \left[1 +(-)\,4\,\Re(\varepsilon_{\mathrm{CPT}}) - 2\,\Re(x)\right]e^{-i\,\Gamma_{\mathrm{L}}\,t} \\
& + 2\,e^{-\overline{\Gamma}\,t}\cos(\Delta m\,t) \\
& +(-)\,4\left[2\,\Im(\varepsilon_{\mathrm{CPT}}) - \Im(x)\right]e^{-\overline{\Gamma}\,t}\sin(\Delta m\,t)
\end{aligned}
$$

and

$$
\begin{aligned}
R_-(t)\ \left(\overline{R}_+(t)\right)\ \propto\ & \left[1 -(+)\,4\,\Re(\varepsilon_{\mathrm{T}}) + 2\,\Re(x)\right]e^{-i\,\Gamma_{\mathrm{S}}\,t} \\
& + \left[1 -(+)\,4\,\Re(\varepsilon_{\mathrm{T}}) - 2\,\Re(x)\right]e^{-i\,\Gamma_{\mathrm{L}}\,t} \\
& - 2\left[1 -(+)\,4\,\Re(\varepsilon_{\mathrm{T}})\right]e^{-\overline{\Gamma}\,t}\cos(\Delta m\,t) \\
& -(+)\,4\,\Im(x)\,e^{-\overline{\Gamma}\,t}\sin(\Delta m\,t)
\end{aligned}
$$

For simplicity, let us assume that the $\Delta Q = \Delta S$ rule holds for the moment. Then, $R_+(t)$ and $\overline{R}_-(t)$ are due to the processes where K^0 remains K^0 and $\overline{K}^0$ remains $\overline{K}^0$, respectively. The two processes are CP and CPT conjugate to each other. Therefore, $R_+(t) \neq \overline{R}_-(t)$ must be a sign of CP and CPT violation. Indeed, a time dependent CP asymmetry $A_{\mathrm{CPT}}(t)$ given as

$$
\begin{aligned}
A_{\mathrm{CPT}}(t)\ =\ & \frac{\overline{R}_-(t) - R_+(t)}{\overline{R}_-(t) + R_+(t)} \\
=\ & 4\,\frac{\Re\left(\varepsilon_{\mathrm{CPT}}\right)\left(e^{-\Gamma_{\mathrm{S}}\,t} - e^{-\Gamma_{\mathrm{L}}\,t}\right) - \Im\left(2\,\varepsilon_{\mathrm{CPT}} - x\right)e^{-\overline{\Gamma}\,t}\sin\left(\Delta m\,t\right)}{e^{-\Gamma_{\mathrm{S}}\,t} + e^{-\Gamma_{\mathrm{L}}\,t} + 2\,e^{-\overline{\Gamma}\,t}\cos\left(\Delta m\,t\right)}
\end{aligned}
$$

depends only on $\varepsilon_{\mathrm{CPT}}$ but not on ε_{T}. Note that $\overline{\Gamma}$ is the average decay width $(\Gamma_{\mathrm{L}} + \Gamma_{\mathrm{S}})/2$. For a large decay time t, we obtain

$$\lim_{t \to \infty} A_{\mathrm{CPT}}(t) = -4\,\Re\left(\varepsilon_{\mathrm{CPT}}\right)\ .$$

5

$R_-(t)$ and $\overline{R}_+(t)$ are due to oscillations of K^0 into $\overline{K}^0$ and $\overline{K}^0$ into K^0 respectively, which are CP and T conjugate to each other. The CP asymmetry defined as

$$
\begin{aligned}
A_{\mathrm{T}}(t) &= \frac{\overline{R}_+(t) - R_-(t)}{\overline{R}_+(t) + R_-(t)} \\
&= 4\,\Re\,(\varepsilon_{\mathrm{T}}) - \frac{4\,\Im\,(x)\,e^{-\overline{\Gamma} t}\sin(\Delta m\,t)}{e^{-\Gamma_{\mathrm{S}} t} + e^{-\Gamma_{\mathrm{L}} t} - 2\,e^{-\overline{\Gamma} t}\cos(\Delta m\,t)}
\end{aligned}
$$

is sensitive only to CP and T violation with

$$
\lim_{t\to\infty} A_{\mathrm{T}}(t) = 4\,\Re\,(\varepsilon_{\mathrm{T}}) \ .
$$

It must be noted that A_{T} depends only on the real part of ε_{T} if the $\Delta Q = \Delta S$ rule is valid. We conclude that CP violation in oscillations which is generated by the interference between the mass and decay matrices contributes only to the **real part** of ε_{T}.

2.3 CP Violation and Two-Pion Decays

Two-pion final states are common to both K^0 and $\overline{K}^0$ decays. Since they must be eigenstates of electromagnetic and strong interactions, we consider them to be in the isospin eigenstates. The total angular momentum of the two-pion final state is 0 and only isospin states $I = 0$ and $I = 2$ are allowed due to Bose statistics. The amplitude of a K^0 decaying into $2\pi(I = 0)$ is given by

$$
A_{I=0} = {}_{\mathrm{out}}\langle 2\pi(I=0)|\,H_{\mathrm{weak}}\,|K^0\rangle
$$

where $|2\pi(I = 0)\rangle_{\mathrm{out}}$ denotes the state for out-going two-pions with $I = 0$. By assuming that the weak interaction is invariant under CPT transformation and that the S-matrix for the two-pion state is unitary and diagonal, we can relate $A_{I=0}$ to the $\overline{K}^0$ decay amplitude by

$$
\overline{A}_{I=0} = {}_{\mathrm{out}}\langle 2\pi(I=0)|\,H_{\mathrm{weak}}\,|\overline{K}^0\rangle = A^*_{I=0}\,e^{i\,2\delta_0}\,e^{i\left(\theta_{\mathrm{CP}} - \overline{\theta}_{\mathrm{T}}\right)}
$$

where δ_0 is the strong interaction phase shift measured in the π-$\pi(I = 0)$ elastic scattering at $\sqrt{s} = m_{K^0}$. The arbitrary phases, θ's are defined in equations 1. It is common to write

$$
\begin{aligned}
A_{I=0} &= a_0\,e^{i\,\delta_0} \\
\overline{A}_{I=0} &= a_0^*\,e^{i\left(\delta_0 + \theta_{\mathrm{CP}} - \overline{\theta}_{\mathrm{T}}\right)}
\end{aligned}
\tag{13}
$$

where a_0 contains only the weak interaction part. Similarly for the $2\pi(I = 2)$ state, we have

$$
\begin{aligned}
A_{I=2} &= a_2\,e^{i\,\delta_2} \\
\overline{A}_{I=2} &= a_2^*\,e^{i\left(\delta_2 + \theta_{\mathrm{CP}} - \overline{\theta}_{\mathrm{T}}\right)} \ .
\end{aligned}
$$

Experimentally we have [10]

$$
\left|\frac{a_2}{a_0}\right| \approx 0.045
$$

i.e. the decay into the $I = 2$ state is suppressed ($\Delta I = 1/2$ rule).

Physical two-pion states are $\pi^+\pi^-$ and $\pi^0\pi^0$. The instantaneous K^0 decay width into $\pi^+\pi^-$ is given by

$$
\Gamma(K^0 \to \pi^+\pi^-) = \left|\sqrt{\frac{2}{3}}\,A_{I=0} + \sqrt{\frac{1}{3}}\,A_{I=2}\right|^2
$$

Since $K^0 \to \pi^+ \pi^-$ and $\overline{K}^0 \to \pi^+ \pi^-$ are CP conjugate processes, the ratio

$$
\frac{\Gamma(\overline{K}^0 \to \pi^+ \pi^-) - \Gamma(K^0 \to \pi^+ \pi^-)}{\Gamma(\overline{K}^0 \to \pi^+ \pi^-) + \Gamma(K^0 \to \pi^+ \pi^-)} = \sqrt{2}\, \Im\left(\frac{a_2}{a_0}\right) \sin(\delta_2 - \delta_0)
$$
$$
\equiv -2\,\Re\left(\varepsilon'\right) \tag{14}
$$

where

$$
\varepsilon' = \frac{i}{\sqrt{2}}\, \Im\left(\frac{a_2}{a_0}\right) e^{i\,(\delta_2 - \delta_0)}
$$

is a measure of CP violation. A non-zero value of $\Re\left(\varepsilon'\right)$ implies CP violation in the decay amplitude ($|\Delta S| = 1$) which is often referred to as "direct CP violation" . This requires that the strong interaction phase shifts for $I = 0$ and $I = 2$ states and the phases of $I = 0$ and $I = 2$ weak decay amplitudes are **both** different. The former is experimentally measured to be [11]

$$
\delta_0 - \delta_2 = (47 \pm 6)^\circ \ ,
$$

If CP violation is due to the superweak interaction H_{sw}, no CP violation is expected in the decay amplitudes and $\varepsilon' = 0$. On the other hand, we do expect some phase difference between the $I = 0$ and $I = 2$ weak decay amplitudes if CP violation is due to the weak interaction.

It must be noted again that CP violation in the decay amplitude, which is generated by the interference between two decay amplitudes contributing to the same final state, depends only on the **real part** of ε' as seen from equation 14.

Since K^0 and $\overline{K}^0$ are not mass eigenstates, it is more suitable to discuss K_S and K_L decays. The two-pion final states are CP eigenstates with an eigenvalue of $+1$. In the limit of CP conservation, we have $\varepsilon_{\text{CPT}} = 0$, $\varepsilon_{\text{T}} = 0$ and $\varphi_\Gamma = \theta_{\text{CP}}$ where θ_{CP} is an arbitrary CP phase of the neutral kaon system defined in equations 1. In this limit, the mass eigenstates K_S and K_L become also CP eigenstates with eigenvalues $+1$ and -1 respectively. Then, a parameter defined as

$$
\eta_{I=0} = \frac{\langle 2\,\pi(I=0)\,|H_{\text{weak}}|\,K_L\rangle}{\langle 2\,\pi(I=0)\,|H_{\text{weak}}|\,K_S\rangle}
$$

is a measure of CP violation. Using equations 11 and 13, it follows that

$$
\eta_{I=0} = \varepsilon_{\text{CPT}} + \varepsilon_{\text{T}} + \frac{i}{2}\left(2\,\varphi_0 + \varphi_\Gamma + \overline{\theta}_{\text{T}} - \theta_{\text{CP}}\right) \tag{15}
$$

where $\varphi_0 = \arg(a_0)$. Note that both ε_{CPT} and ε_{T} contribute to $\eta_{I=0}$. The third term is usually considered to be 0. This will be examined in detail later. A commonly used parameter ε, which is somewhat ambiguous due to the phase convention, is here given by

$$
\varepsilon = \varepsilon_{\text{T}} + \frac{i}{2}\left(2\,\varphi_0 + \varphi_\Gamma + \overline{\theta}_{\text{T}} - \theta_{\text{CP}}\right) \ .
$$

The **imaginary part** of $\eta_{I=0}$ is interpreted as CP violation generated by the interference between oscillations and decays.

The CP violation parameters for the $I = 2$ state is given by

$$
\eta_{I=2} = \varepsilon_{\text{CPT}} + \varepsilon_{\text{T}} + \frac{i}{2}\left(2\,\varphi_2 + \varphi_\Gamma + \overline{\theta}_{\text{T}} - \theta_{\text{CP}}\right)
$$

where $\varphi_2 = \arg(a_2)$. If CP violation in the decay amplitude is present, $\varphi_0 \neq \varphi_2$ thus we expect $\Im\left(\eta_{I=0}\right) \neq \Im\left(\eta_{I=2}\right)$.

CP violation parameters measured directly in the experiment are

$$\eta_{+-} = \frac{\langle \pi^+ \pi^- | H_{\mathrm{weak}} | \mathrm{K_L} \rangle}{\langle \pi^+ \pi^- | H_{\mathrm{weak}} | \mathrm{K_S} \rangle}$$
$$= \eta_{I=0} + \varepsilon' \tag{16}$$

and

$$\eta_{00} = \frac{\langle \pi^0 \pi^0 | H_{\mathrm{weak}} | \mathrm{K_L} \rangle}{\langle \pi^0 \pi^0 | H_{\mathrm{weak}} | \mathrm{K_S} \rangle}$$
$$= \eta_{I=0} - 2\varepsilon' . \tag{17}$$

With the presence of direct CP violation, we expect $\eta_{+-} \neq \eta_{00}$.

Since ε' is measured to be very small and $\arg(\varepsilon') \approx \arg(\varepsilon_\mathrm{T}) \approx 45°$, the phase of η_{+-}, ϕ_{+-} is essentially identical to the phase of $\eta_{I=0}$, ϕ_{00}.

2.4 CP Violation and Three-Pion Decays

The CP eigenvalue of the three-pion final state depends on the angular momentum configuration. For $\pi^+ \pi^- \pi^0$ mode, we denote $l_{\pi^+ \pi^-}$ to be the angular momentum between π^+ and π^-. The CP eigenvalue of the $\pi^+ \pi^-$ pair is always $+1$. Then the CP eigenvalue of the $\pi^+ \pi^- \pi^0$ system is given by $-(-1)^{l'}$ where l' is the relative angular momentum between the $\pi^+ \pi^-$ pair and the remaining π^0 (see figure 1). Due to conservation of angular momentum, we have $l_{\pi^+ \pi^-} = l'$ for the neutral K-meson final state: i.e.

$$CP(\pi^+ \pi^- \pi^0) = -(-1)^{l_{\pi^+ \pi^-}} .$$

Since the kaon mass is very close to the three-pion mass, higher angular momentum states are suppressed due to the centrifugal barrier. Therefore, we consider only $l_{\pi^+ \pi^-} = 0$ or 1. Note that the angular momentum part of the wave function is symmetric for the $l_{\pi^+ \pi^-} = 0$ state and antisymmetric for the $l_{\pi^+ \pi^-} = 1$ state over the exchange of π^+ and π^-.

The $\pi^+ \pi^-$ subsystem can have an isospin of $I_{\pi^+ \pi^-} = 0$, 1 or 2. The isospin part of the wave function is symmetric for $I_{\pi^+ \pi^-} = 0$ and 2 over the exchange of π^+ and π^-. The $I_{\pi^+ \pi^-} = 1$ state is antisymmetric. The total isospin of the $\pi^+ \pi^- \pi^0$ system is $I_{3\pi} = 0$, 1, 2 or 3 where $I_{\pi^+ \pi^-} = 0$ contributes to $I_{3\pi} = 1$, $I_{\pi^+ \pi^-} = 1$ to $I_{3\pi} = 0$ and 2 and $I_{\pi^+ \pi^-} = 2$ to $I_{3\pi} = 3$.

Since the total wave function must be symmetric over the exchange of π^+ and π^-, the $l_{\pi^+ \pi^-} = 1$ state has $I_{3\pi} = 0$ or 2 and the $l_{\pi^+ \pi^-} = 0$ state $I_{3\pi} = 1$ or 3. Table 1 summarises the allowed $\pi^+ \pi^- \pi^0$ configuration

The $I_{3\pi} = 3$ state is expected to be suppressed compared with $I_{3\pi} = 1$ state due to the $\Delta I = 1/2$ rule. The $I_{3\pi} = 0$ state is suppressed since the isospin part of the wave function must be totally antisymmetric over the exchange of π^+, π^- and π^0 which requires that it must be at least in the third order of the pion kinetic energies [12]. In conclusion, we have

- $\pi^+ \pi^- \pi^0 (CP = -1)$
  $l_{\pi^+ \pi^-} = l' = 0$, $I_{3\pi} = 1$

- $\pi^+ \pi^- \pi^0 (CP = +1)$
  $l_{\pi^+ \pi^-} = l' = 1$, $I_{3\pi} = 2$

Using the invariant masses of $\pi^+ \pi^0$ and $\pi^- \pi^0$ pairs denoted by m_{+0} and m_{-0} respectively, the decay amplitudes are given as

$$A(\mathrm{K}^0 \to \pi^+ \pi^- \pi^0) = a(I_{3\pi}{=}2, m_{+0}, m_{-0}) \, e^{i\,\delta_{3\pi}(I=2)}$$
$$+ a(I_{3\pi}{=}1, m_{+0}, m_{-0}) \, e^{i\,\delta_{3\pi}(I=1)} \tag{18}$$

and

$$A(\overline{K}^0 \to \pi^+\pi^-\pi^0) = a^*(I_{3\pi}=2, m_{+0}, m_{-0}) e^{i\left(\delta_{3\pi\,(I=2)}+\theta_{\mathrm{CP}}-\overline{\theta}_{\mathrm{T}}\right)}$$
$$-a^*(I_{3\pi}=1, m_{+0}, m_{-0}) e^{i\left(\delta_{3\pi\,(I=1)}+\theta_{\mathrm{CP}}-\overline{\theta}_{\mathrm{T}}\right)} \qquad (19)$$

where the δ's are strong interaction phase shifts for the corresponding isospin states. Note that the phase space integrations give

$$\int\limits_{m_{+0}>m_{-0}} \mathrm{d}\Omega\, a(I_{3\pi}=2, m_{+0}, m_{-0}) = -\int\limits_{m_{+0}<m_{-0}} \mathrm{d}\Omega\, a(I_{3\pi}=2, m_{+0}, m_{-0})$$

as seen from the table 1 so that

$$\int\limits_{\mathrm{all}} \mathrm{d}\Omega\, a(I_{3\pi}=2, m_{+0}, m_{-0}) = 0 \ . \qquad (20)$$

The K_S and K_L decay amplitudes can be obtained using equations 11, 18 and 19. The K_S decay amplitude consists of an $I_{3\pi}=2$ part which is suppressed by the centrifugal effect, and an $I_{3\pi}=1$ part which is suppressed by CP violation, i.e.

$$A_S^{+-0} = A_S(I_{3\pi}=2, m_{+0}, m_{-0}) + A_S(I_{3\pi}=1, m_{+0}, m_{-0}) \ .$$

Using equation 20, we obtain

$$\int\limits_{\mathrm{all}} \mathrm{d}\Omega\, A_S(I_{3\pi}=2, m_{+0}, m_{-0}) = 0 \ . \qquad (21)$$

The decay amplitude $a(I_{3\pi}=2)$ can be neglected in the K_L decay amplitude, since it is doubly suppressed by the centrifugal effect and CP violation:

$$A_L^{+-0} = A_L(I_{3\pi}=1, m_{+0}, m_{-0}) \ .$$

Therefore, both K_S decay amplitudes, i.e. the CP allowed $A_S(I_{3\pi}=2)$ and the CP violating $A_S(I_{3\pi}=1)$ can interfere with the CP allowed K_L decay amplitude $A_L(I_{3\pi}=1)$. However, the contribution from the CP allowed K_S decay amplitude vanishes in the interference once it is integrated over the entire three-pion phase space as seen from equation 21. The strong phase shifts $\delta_{3\pi(I=1)}$ and $\delta_{3\pi(I=2)}$ are both expected to be small due to small kinetic energies available for the pions. It follows that the parameter

$$\eta_{+-0} = \frac{\int\limits_{\mathrm{all}} \mathrm{d}\Omega\, A_S(I_{3\pi}=1, m_{+0}, m_{-0})\, A_L^*(I_{3\pi}=1, m_{+0}, m_{-0})}{\int\limits_{\mathrm{all}} \mathrm{d}\Omega\, |A_L(I_{3\pi}=1, m_{+0}, m_{-0})|^2}$$

$$\approx -\varepsilon_{\mathrm{CPT}} + \varepsilon_{\mathrm{T}} + \frac{i}{2}\left(2\,\varphi_{3\pi(I=1)} + \varphi_\Gamma + \overline{\theta}_{\mathrm{T}} - \theta_{\mathrm{CP}}\right) \qquad (22)$$

is completely due to CP violation where $\varphi_{3\pi(I=1)}$ is the phase of the weak decay amplitude $a(I_{3\pi}=1)$ at $m_{+0}=m_{-0}$.

The contribution from the CP allowed part of the $K_S \to \pi^+\pi^-\pi^0$ decay remains in the interference term if the decays are studied for $m_{+0} > m_{-0}$ and $m_{+0} < m_{-0}$ separately. It follows that

$$\eta_{+-0}^{m_{+0}>m_{-0}} = \eta_{+-0} + \rho$$

and

$$\eta_{+-0}^{m_{+0}<m_{-0}} = \eta_{+-0} - \rho$$

where

$$\rho = \frac{\displaystyle\int_{m_{+0}>m_{-0}} d\Omega\, A_{\mathrm{S}}(I_{3\pi}=2, m_{+0}, m_{-0}) A_{\mathrm{L}}^*(I_{3\pi}=1, m_{+0}, m_{-0})}{\displaystyle\int_{m_{+0}>m_{-0}} d\Omega\, |A_{\mathrm{L}}(I_{3\pi}=1, m_{+0}, m_{-0})|^2} \tag{23}$$

From various isospin studies of all kaon decays [10, 13], we expect that ρ is real and about 20 times larger than $\Re(\eta_{+-0})$.

The $\mathrm{K_S}$-$\mathrm{K_L}$ interference term is best studied using the time dependent rate for initially pure $\mathrm{K^0}$'s decaying into $\pi^+\pi^-\pi^0$. When the rate is measured over all the three-pion phase space, we obtain from equation 9

$$R_{+-0}(t) \quad \propto \quad e^{-\Gamma_{\mathrm{L}} t} + \frac{\Gamma(\mathrm{K_S} \to \pi^+\pi^-\pi^0)}{\Gamma(\mathrm{K_L} \to \pi^+\pi^-\pi^0)} e^{-\Gamma_{\mathrm{S}} t}$$
$$+ 2\,|\eta_{+-0}|\, e^{-\overline{\Gamma} t} \cos(\Delta m\, t + \phi_{+-0})\ .$$

Note that ρ can be also obtained from $\eta_{+-0}^{m_{+0}>m_{-0}}$, if the events are restricted only to $m_{+0} > m_{-0}$,

2.5 Phase of Γ_{12}

The decay matrix element Γ_{12} is given by equation 6. When unitarity is valid, $\langle \mathrm{f}|H_{\mathrm{weak}}|i\rangle$ represents the physical decay amplitude. Common final states between $\mathrm{K^0}$ and $\overline{\mathrm{K}}^0$ that should be considered are $2\pi\ 3\pi$, $2\pi\gamma$ and $\ell\pi\nu$ produced by violating the $\Delta Q = \Delta S$ rule. The other decay modes have negligible branching fractions. It follows that [14]

$$\Gamma_{12} \quad \approx \quad A_0^*\overline{A}_0 + A_2^*\overline{A}_2 + \int d\Omega \left[A_{2\pi\gamma(\mathrm{IB})}^*\overline{A}_{2\pi\gamma(\mathrm{IB})} + A_{2\pi\gamma(\mathrm{E1})}^*\overline{A}_{2\pi\gamma(\mathrm{E1})} \right.$$
$$\left. + A_{2\pi\gamma(\mathrm{M1})}^*\overline{A}_{2\pi\gamma(\mathrm{M1})} \right] + \int d\Omega \left[A_{3\pi(I=1)}^*\overline{A}_{3\pi(I=1)} \right.$$
$$\left. + A_{3\pi(I=2)}^*\overline{A}_{3\pi(I=2)} \right] + \int d\Omega \left[A_+^*\overline{A}_+ + A_-^*\overline{A}_- \right], \tag{24}$$

where IB, E1 and M1 denote the amplitudes for $\mathrm{K^0}$ decaying into $2\pi\gamma$ via inner bremsstrahlung, direct emission with electric dipole transition and direct emission with magnetic dipole transition, respectively. The integrations are done over all necessary phase space including spin.

Compared with the $\mathrm{K^0} \to 3\pi(I=1)$ decay amplitude, the $\mathrm{K^0} \to 3\pi(I=2)$ decay amplitude can be safely neglected here as seen in the previous section [10, 13]. It can be shown [15] that the direct emission part of the $\mathrm{K^0} \to \pi\pi\gamma$ decay amplitude is small enough to be ignored in this discussion. The phase of the inner bremsstrahlung decay amplitude is almost identical to that of a_0.

Equation 24 is now further approximated as

$$\Gamma_{12} \quad \approx \quad \frac{1}{2}\,\Gamma_{\mathrm{S}} e^{i\left(\theta_{\mathrm{CP}} - \overline{\theta}_{\mathrm{T}} - 2\varphi_0\right)} \left\{ 1 + B(\mathrm{K_S} \to 2\pi\gamma) \right.$$
$$\left. + \frac{\Gamma_{\mathrm{L}}}{\Gamma_{\mathrm{S}}} \left[8B(\mathrm{K_L} \to \ell^+\nu\pi^-)\,x - B(\mathrm{K_L} \to 3\pi) e^{i\,2\left(\varphi_0 - \varphi_{3\pi(I=1)}\right)} \right] \right\} \tag{25}$$

where B's are the relevant branching ratios and equation 12 is used.

Using equations 25 and 22, the phase of Γ_{12} is determined to be [1]

$$\varphi_\Gamma \quad \approx \quad -2\varphi_0 - \overline{\theta}_{\mathrm{T}} + \theta_{\mathrm{CP}}$$
$$+2\frac{\Gamma_{\mathrm{L}}}{\Gamma_{\mathrm{S}}} \left[4\,B(\mathrm{K_L} \to \ell^+\pi^-\nu)\,\Im(x) - B(\mathrm{K_L} \to 3\pi)\,\Im(\eta_{I=0} - \eta_{3\pi}) \right].$$

---

[1]This is identical to determine the phase of ε using the Bell-Steinberger relation [16].

Using current experimental values of [8]

$$\Im(x) = -0.003 \pm 0.026$$

and [17, 18, 19]

$$\Im(\eta_{3\pi}) = 0.02 \pm 0.12$$

we conclude

$$\varphi_\Gamma = -2\,\varphi_0 - \overline{\theta}_\mathrm{T} + \theta_\mathrm{CP} \pm 1.8{\times}10^{-4} \ .$$

Due to this uncertainty, the imaginary part of $\eta_{I=0}$ can deviate from $\Im(\varepsilon_\mathrm{T})$ as much as $1.8{\times}10^{-4}$ even in the absence of CPT violation as seen from equation 15. Since the real part of ε_T is $\sim 10^{-3}$, this uncertainty in the phase of $\eta_{I=0}$ corresponds to $1.6°$.

Measurements of ϕ_{+-} with an uncertainty of much better than $1°$ are soon to be expected. The standard model prediction is

$$\varphi_\Gamma = -2\,\varphi_0 - \overline{\theta}_\mathrm{T} + \theta_\mathrm{CP} \pm \mathcal{O}\left(< 10^{-7}\right),$$

thus $\arg(\eta_{I=0}) = \arg(\varepsilon_\mathrm{T}) \equiv \phi_\mathrm{sw}$ if CPT is conserved. However, much better measurements of $\Im(x)$ and $\Im(\eta_{3\pi})$ are needed in order to perform a strictly **experimental** test of CPT by comparing the experimentally measured ϕ_{+-} and ϕ_sw.

2.6 Summary

Let us now summarise important experimental questions on CP violation in the neutral kaon system discussed in the next chapter:

- Is the observed CP violation through K^0-$\overline{\mathrm{K}}^0$ oscillations accompanied by T violation or CPT violation?
  This can be studied by measuring non-zero values for A_T or A_CPT using the semileptonic decays or comparing ϕ_{+-} with ϕ_sw. Independent of T or CPT violation, we expect $\phi_{+-} = \phi_{00}$.

- Is there CP violation in the decay amplitude?
  This is the case if we measure a non-zero value for $1 - |\eta_{00}/\eta_{+-}|$.

- Is the phase of "ε" obtained from unitarity identical to $\phi_\mathrm{sw} = \tan^{-1}(2\,\Delta m/\Delta\Gamma)$?
  This requires more precise measurements on $\Im(x)$ and $\Im(\eta_{3\pi})$.

If CP violation is generated in the ordinary weak interaction, CP violation in the decay amplitude is expected to appear at some level. On the other hand, if CP violation occurs only at the higher mass scale, CP violation can appear only through particle-antiparticle oscillations.

3 CP Violation Experiments

3.1 CPLEAR

The simplest way to demonstrate CP violation is to compare the decay of an initially pure K^0 into a final state "f" with its CP conjugate process, i.e. an initially pure $\overline{\mathrm{K}}^0$ decaying into "$\overline{\mathrm{f}}$" where $\overline{\mathrm{f}}$ is the CP conjugate state of f. CPLEAR is an experiment exactly doing this.

The K^0 and $\overline{\mathrm{K}}^0$ are produced by $\mathrm{p}\overline{\mathrm{p}}$ annihilation at rest:

$$\mathrm{p}\overline{\mathrm{p}} \rightarrow \mathrm{K}^0\mathrm{K}^-\pi^+ \text{ or } \overline{\mathrm{K}}^0\mathrm{K}^+\pi^- \quad (\text{branching ratio}: \ \sim 0.2\% \text{ each}) \ .$$

Antiprotons with a flux of $\sim 10^6$/s and a momentum of 200 MeV/c extracted from the Low Energy Antiproton Ring (LEAR) at CERN are stopped in a gaseous hydrogen target with 16 bar placed in the center of the CPLEAR detector. The target is surrounded by layers of cylindrical tracking chambers followed by a particle identification system consisting of a threshold Cherenkov counter sandwiched by two scintillation counters. The last component of the detector is a gas sampling electromagnetic calorimeter with lead converters. All sub-detectors were inserted into of a solenoidal magnet providing 0.44 T field.

The primary particles, $K^\pm \pi^\mp$ provide the initial flavour of the neutral kaon and its momentum. The neutral kaon decays into various final states such as $\pi^+\pi^-$, $\pi^0\pi^0$, $\pi^+\pi^-\pi^0$ and $e^\pm\pi^\mp\nu$.

The experiment is now taking data with its full capability and the results presented are based on the data taken up to 1992. Data taking should continue till 1995.

Early results for $K \to \pi^+\pi^-$ have been published [20]. Figure 2 shows the time dependent CP asymmetry obtained from recent data. The asymmetry is defined as

$$A_{+-}(t) = \frac{\overline{R}_{+-}(t) - R_{+-}(t)}{\overline{R}_{+-}(t) + R_{+-}(t)}$$

where $R(\overline{R})_{+-}(t)$ is the time dependent rate for the initially pure $K^0(\overline{K}^0)$ decaying into $\pi^+\pi^-$. A clear signal of the K_L-K_S interference is visible. By fitting the data with the expected distribution

$$A_{+-}(t) = \Re(\epsilon_{\mathrm{CPT}} + \epsilon_{\mathrm{T}}) - \frac{2\,|\eta_{+-}|\,e^{\Delta\Gamma t/2}\cos(\Delta m\,t - \phi_{+-})}{1 + |\eta_{+-}|^2\,e^{\Delta\Gamma t}}$$

which is valid for $t \le 20\,\tau_S$ where τ_S is the K_S lifetime, one obtains [21]

$$\left.\begin{array}{rcl} |\eta_{+-}| & = & [2.25 \pm 0.07(\mathrm{sta.})] \times 10^{-3} \\ \phi_{+-} & = & 44.7^\circ \pm 1.3^\circ(\mathrm{sta.}) \end{array}\right\} \quad \mathrm{CPLEAR}$$

where the final systematic errors are not yet attributed[2] In the fit, the value of Δm given by the Particle Data Group (PDG) [8] was used.

For the $\pi^+\pi^-\pi^0$ final state, from the time dependent asymmetry

$$A_{+-0}(t) = \frac{\overline{R}_{+-0}(t) - R_{+-0}(t)}{\overline{R}_{+-0}(t) + R_{+-0}(t)}$$

the result is [21]

$$\left.\begin{array}{rcl} \Re\,(\eta_{+-0}) & = & 0.002 \pm 0.016(\mathrm{sta.}) \\ \Im\,(\eta_{+-0}) & = & 0.044 \pm 0.026(\mathrm{sta.}) \end{array}\right\} \quad \mathrm{CPLEAR}$$

where again the systematic errors are under study but claimed to be much smaller than the statistical errors. Figure 3 shows η_{+-0} measured by two previous experiments [17, 19] together with the results from CPLEAR and E621. The two new results improve the error by a factor of ten.

CPLEAR measured the CP allowed $K_S \to \pi^+\pi^-\pi^0$ decay amplitude through the interference between the CP allowed K_L and K_S decay amplitudes. This was done by studying CP asymmetries for events with $m_{+0} > m_{-0}$ and $m_{+0} < m_{-0}$ separately where $m_{+(-)0}$ denotes the invariant mass between $\pi^+(\pi^-)$ and π^0. As

---

[2]Preliminary systematic errors excluding the effect due to the uncertainty in Δm are given [21] to be $\pm 0.02 \times 10^{-3}$ for η_{+-} and $\pm 0.4^\circ$ for ϕ_{+-}.

Figure 2: Time dependent CP asymmetry for K→ $\pi^+\pi^-$ decays measured by CPLEAR. The solid (dotted) line is the result of a fit with (without) taking the residual three-body background into account.
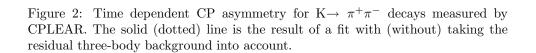
Figure 3: Measured CP violation parameters for K→ $\pi^+\pi^-\pi^0$ decays.

explained in the previous chapter, the signs of the interference are opposite for the two regions.

Figure 4 shows [21] the measured time dependent CP asymmetry for the two different regions. By fitting the expected asymmetry

$$
\begin{aligned}
A_{+-0}^{m_{+0}>(<)m_{-0}}(t) \;=\;& 2\,\Re\left(-\varepsilon_{\mathrm{CPT}}+\varepsilon_{\mathrm{T}}\right) \\
& -2\,e^{-\Delta\Gamma t/2}\left\{\left[\Re\left(\eta_{+-0}\right)+(-)\rho\right]\cos\Delta m\,t-\Im\left(\eta_{+-0}\right)\sin\Delta m\,t\right\}
\end{aligned}
$$

a value [21] of ρ

$$
\rho = 0.037 \pm 0.011(\text{sta.}) \quad \text{CPLEAR}
$$

was obtained, where ρ given by equation 23 is due to the interference between CP allowed K_L and K_S decay amplitudes into the $\pi^+\pi^-\pi^0$ final state. This is the first time that the CP allowed $K_S \to \pi^+\pi^-\pi^0$ decay amplitude has been seen.

The study of semileptonic decays provides different information. Using the time dependent decay rates for the initially pure $K^0(\overline{K}^0)$ decaying into $e^{\pm}\pi^{\mp}\nu$, $R(\overline{R})_{\pm}(t)$, the following time dependent asymmetries are studied:

$$
A_1 = \frac{\left[\overline{R}_-(t)+R_+(t)\right]-\left[\overline{R}_+(t)+R_-(t)\right]}{\left[\overline{R}_-(t)+R_+(t)\right]+\left[\overline{R}_+(t)+R_-(t)\right]}
$$

for Δm and $\Re(x)$,

$$
A_2 = \frac{\left[\overline{R}_-(t)+\overline{R}_+(t)\right]-\left[R_+(t)+R_-(t)\right]}{\left[\overline{R}_-(t)+\overline{R}_+(t)\right]+\left[R_+(t)+R_-(t)\right]}
$$

for $\Re\left(-\varepsilon_{\mathrm{CPT}}+\varepsilon_{\mathrm{T}}\right)$ and $\Im(x)$. Preliminary results [21] are

$$
\begin{aligned}
&\text{CPLEAR}\\
\Delta m &= \left[0.524\pm0.006(\text{sta.})\pm0.002(\text{sys.})\right]\times10^{10}\,\hbar\text{s}^{-1}\\
\Re(x) &= -0.024\pm0.020(\text{sta.})\pm0.005(\text{sys.})\\
\Im(x) &= 0.007\pm0.008(\text{sta.})\pm0.001(\text{sys.})\\
\Re\left(-\varepsilon_{\mathrm{CPT}}+\varepsilon_{\mathrm{T}}\right) &= -0.0005\pm0.0014(\text{sta.})\pm0.0023(\text{sys.})
\end{aligned}
\tag{26}
$$

The current world average of x given by PDG is [8]

$$
x = (0.006\pm0.018)+i\,(-0.003\pm0.026) \quad \text{PDG} .
$$

The error on $\Im(x)$ which is of particular interest as discussed in the previous chapter is improved by a factor of three. The asymmetry A_{T}, a direct signal for CP and T violation, was also studied and is given by

$$
A_{\mathrm{T}} = \frac{\overline{R}_+(t)-R_-(t)}{\overline{R}_+(t)+R_-(t)} = 0.000\pm0.004(\text{sta.})\pm0.008(\text{sys.}) \quad \text{CPLEAR}
$$

assuming a flat distribution of the decay time. If the observed CP violation is accompanied by T violation, we expect A_{T} to be $\sim 6.4\times10^{-3}$.

3.2 E621 Experiment

The experiment E621 is designed to measure CP violation in $K_S \to \pi^+\pi^-\pi^0$ decays through K_L-K_S interference. It is a fixed target experiment at FNAL using a ~ 60 m long magnetic spectrometer with a lead glass calorimeter. The experiment used two kaon beams of which one was produced very close to the experiment. The decay time distribution for the $\pi^+\pi^-\pi^0$ final states from this beam is given by

$$
\begin{aligned}
\frac{dn_{+-0}}{dt} \;=\;& \frac{N_{\mathrm{L}}\,B\left(K_L\to\pi^+\pi^-\pi^0\right)}{\tau_{\mathrm{L}}}\Big[e^{-\Gamma_{\mathrm{L}}t}\\
& +|\eta_{+-0}|^2\,e^{-\Gamma_{\mathrm{S}}t}+2\,D\,|\eta_{+-0}|\,e^{-\overline{\Gamma}t}\cos\left(\Delta m\,t+\phi_{+-0}\right)\Big]
\end{aligned}
$$

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where D is the fraction of K^0 defined as

$$D = \frac{N_{K^0} - N_{\overline{K^0}}}{N_{K^0} + N_{\overline{K^0}}} \ .$$

Since the proton beam produces more K^0 than $\overline{K}^0$, $D \neq 0$ and the K_L-K_S interference term remains. The other beam originated from the experiment delivering only K_L used to determine the normalization factor. A fit to the experimental distribution obtained from 18% of the data taken gives [22]

$$\left.\begin{array}{rcl} \Re\left(\eta_{+-0}\right) & = & 0.005 \pm 0.023 \\ \Im\left(\eta_{+-0}\right) & = & 0.031 \pm 0.030 \end{array}\right\} \quad \text{E621} \ .$$

The results are shown in figure 3. By applying a constraint of $\Re\left(\eta_{+-0}\right) = \Re\left(\eta_{+-}\right) = 0.0016$, η_{+-0} is obtained to be [22]

$$\Im\left(\eta_{+-0}\right) = 0.027 \pm 0.017\text{(sta.)} \pm 0.019\text{(sys.)} \quad \text{E621.}$$

3.3   Search for Direct CP Violation

As discussed in the previous chapter, CP violation in the decay amplitude leads to a nonvanishing ε'. Two experiments, E731 at FNAL and NA31 at CERN have recently reported their final analyses on the measurement of ε'. Figure 5 shows both detectors. Note that E731 used a magnetic spectrometer while the NA31 detector had no magnetic field.

Both experiments measure essentially the ratio $|\eta_{00}/\eta_{+-}|^2$ which is related to ε' as

$$\left|\frac{\eta_{00}}{\eta_{+-}}\right|^2 \approx 1 - 6\,\Re\left(\frac{\varepsilon'}{\epsilon}\right) \tag{27}$$

where equations 16 and 17 are used and $\varepsilon = \eta_{I=0}$ in our notation. Experimentally, this ratio is given as

$$\left|\frac{\eta_{00}}{\eta_{+-}}\right|^2 = \left(\frac{\epsilon_{00}^L n_{00}^L}{N_{00}^L} \frac{N_{00}^S}{\epsilon_{00}^S n_{00}^S}\right) \times \left(\frac{\epsilon_{+-}^S n_{+-}^S}{N_{+-}^S} \frac{N_{+-}^L}{\epsilon_{+-}^L n_{+-}^L}\right) \tag{28}$$

where $n_{00}^{S(L)}$ and $n_{+-}^{S(L)}$ are the numbers of observed $K_S(K_L) \to \pi^0\pi^0$ decays and $K_S(K_L) \to \pi^+\pi^-$ decays respectively, and N denotes the kaon flux for each detection mode. The experimental correction factors for each mode are represented by ϵ's.

NA31 measured K_S and K_L decays separately, but $\pi^+\pi^-$ and $\pi^0\pi^0$ final states simultaneously, i.e. $N_{+-}^L = N_{00}^L$ and $N_{+-}^S = N_{00}^S$; thus the kaon fluxes cancel completely in the ratio 28. E731 utilised two parallel K_L beams where one of them was regenerated to K_S. The regenerator was moved from one beam to the other in between each spill so that $N_{+-}^S \propto |\rho|^2 N_{+-}^L$ and $N_{+-}^S \propto |\rho|^2 N_{+-}^L$ where ρ is the regeneration amplitude. Then, the kaon fluxes cancel in the ratio 28 too.

The remaining problems are the correction factors. They can be divided into the following four effects:

1. geometrical acceptance

2. detector performance; i.e. trigger, energy-momentum resolution, calibration, detector stability etc.

3. accidentals; overlapping of background events

4. background from other decay modes

The first effect is given by the beam and detector geometries. The second correction depends on the beam condition, decay modes and detection time. The third and fourth corrections are due to the offline selection where background from the overlapping events or other decay modes were subtracted. The accidental correction depends on the beam condition and decay modes and the background correction depends on the beam condition, decay modes and the detection time.

E731 was designed such that the correction factors for 2, 3 and 4 cancel in the ratio $|\eta_{00}/\eta_{+-}|^2$. Having two K_L beams where one of them was regenerated to K_S allowed to take K_L and K_S decays simultaneously in the same experimental environment. This makes correction factors to be $\epsilon_{+-}^S \approx \epsilon_{+-}^L$ and $\epsilon_{00}^S \approx \epsilon_{00}^L$ **except for the geometrical acceptance**, hence they cancel in the ratio 28. Due to the difference in the lifetimes, K_S and K_L have different decay vertex distributions. The accepted decay regions for the $\pi^+\pi^-$ and $\pi^0\pi^0$ final states in the E731 detector are not identical. Therefore, the detector acceptances are completely different for all four modes and the geometrical correction factors do not cancel in the ratio 28.

The effect due to the geometrical acceptance was studied by simulation. A very detailed simulation programme was tuned using $K_L \rightarrow \pi^\pm e^\mp \nu$ events which were also accepted by the $\pi^+\pi^-$ trigger. The $K_L \rightarrow 3\pi^0$ events were used for tuning the neutral decay simulation. Figure 6 [23, 24] shows the comparison between the simulated and measured kaon decay vertex distribution for the $\pi^\pm e^\mp \nu$ events. The tuned simulation programme was used to correct the acceptance for $\pi^+\pi^-$ and $\pi^0\pi^0$ events.

NA31 optimised their detector to cancel the effect from the geometrical acceptance. The K_S beam was produced by a moving target which simulated the decay point distribution of the K_L beam. The incident proton momenta were chosen such that the K_L and K_S beams had similar momentum spectra. Thus, the geometrical acceptances are almost identical between K_L and $K_S \rightarrow \pi^+\pi^-$ and between K_L and $K_S \rightarrow \pi^0\pi^0$. Figure 7 [25] shows the energy spectra and the weighted vertex distributions for the K_L and K_S beams. All the other effects in the correction factors, in principle, do not cancel in the ratio $|\eta_{00}/\eta_{+-}|^2$ since K_L and K_S decays were taken at different time with different conditions. The intensities of the primary protons were adjusted to achieve similar background conditions between K_S and K_L beams in order to minimise the difference.

Table 2 summarises the statistics of the two experiments after background subtraction [23, 25, 26]. NA31 took data in 1986, 1988 and 1989. The result from the 1986 data has been published [26]. Some detector improvements were made for the 1988 and 1989 data taking. E731 took all the data during the run from 1987 to 1988. Results obtained from part of the data has been published previously [27]. Statistical limitations come from the K_L decays in both experiments (for E731, it was the $\pi^+\pi^-$ mode and for NA31 it was the $\pi^0\pi^0$ decay mode).

Table 3 summarises the subtracted background [23, 25, 26]. In addition to the usual three body decays of kaons, E731 studied additional sources of background. One is due to the incoherent regeneration of the K_L beam in the regenerator. This has to be subtracted in the K_S decays. For the $\pi^0\pi^0$ decay mode, the incoherent regeneration introduces a background in the K_L decays as well. Due to the worse vertex resolution for the $\pi^0\pi^0$ decays, some of the reconstructed K_S decay vertices fall on the K_L beam spot taken as K_L decays. Another source of background was the trigger plane for the charged decay mode which affected only the $\pi^0\pi^0$ decays (see figure 5).

Table 4 lists systematic uncertainties in the ratio $|\eta_{00}/\eta_{+-}|^2$ for the two experiments [23, 25, 26]. Figure 8 shows $|\eta_{00}/\eta_{+-}|$ for E731(final) and NA31 results with 1986 data and with 1987+1988 data [23, 25, 26]. The statistic and systematic errors are separately drawn indicating that NA31 is limited by systematics and E731 is

statistically limited.

The parameter $\Re(\varepsilon'/\varepsilon)$ is determined from equation 27. For NA31, the common systematic errors in the 1986 and 1988+1989 data have been taken into account when the two results were combined [25]. It follows that

$$\Re\left(\frac{\varepsilon'}{\varepsilon}\right) = \begin{cases} (23.0 \pm 6.5) \times 10^{-4} & \text{NA31}(1986+1988+1989) \\ (7.4 \pm 5.9) \times 10^{-4} & \text{E731} \end{cases}$$

Taken Individually, the two experiments may lead to two different conclusions. The NA31 result can be interpreted that $\Re(\varepsilon'/\varepsilon) > 0$ with a significance of more than three standard deviations, a strong indication of CP violation in the decay amplitude. The result from E731 is compatible with $\Re(\varepsilon'/\varepsilon) = 0$, thus with no direct CP violation. However, the probability for the two results being statistically compatible is 7.6% which is not negligible. If we combine the two results and scale the combined error in the way done by the Particle Data Group [8], we obtain

$$\Re\left(\frac{\varepsilon'}{\varepsilon}\right) = (14.4 \pm 7.8) \times 10^{-4} \quad \text{E731 + NA31}$$

which gives a significance of less than two standard deviations for a non-zero value for $\Re(\varepsilon'/\varepsilon)$.

We conclude that CP violation in the decay amplitude is not yet established from the measured $\Re(\varepsilon'/\varepsilon)$. Experiments with a better sensitivity are required in order to resolve the discrepancy between NA31 and E731 which deviate by 1.8 standard deviations.

3.4   CPT Test by E731

As described in the previous chapter, the phases of η_{+-} and η_{00} are expected to be almost identical to

$$\phi_{\mathrm{sw}} = \tan^{-1}\left(\frac{2\,\Delta m}{\Delta\Gamma}\right) = 43.7^\circ \pm 0.2^\circ \ ,$$

if CPT is conserved in K^0-$\overline{\mathrm{K}}^0$ oscillations. We used the current average value[3] of Δm [28, 29, 30]

$$\Delta m = (0.5352 \pm 0.0031) \times 10^{10}\,\hbar\,\mathrm{s}^{-1} \tag{29}$$

and the PDG value [8]

$$\tau_{\mathrm{S}} = (0.8922 \pm 0.0020) \times 10^{-10}\,\mathrm{s} \quad \text{PDG} \tag{30}$$

for obtaining $\Delta\Gamma$.

The current world average value for ϕ_{+-} given by PDG [8] is

$$\phi_{+-} = 46.5^\circ \pm 1.2^\circ \quad \text{PDG,}$$

which is 2.3 standard deviations apart from ϕ_{sw}.

Using the same data sample as in the ε' analysis, E731 asked whether the data were consistent with CPT conservation [31]. This requires to measure Δm, Γ_{S}, ϕ_{+-} and the phase difference $\phi_{00} - \phi_{+-}$. Since $\Gamma_{\mathrm{S}} \gg \Gamma_{\mathrm{L}}$, the value of Γ_{L} is much less important. The following describes how the data were analysed.

After correcting for the detector acceptance, the decay rates of the K_{L} beam without regenerator and that behind the regenerator with a given momentum p and at a decay position z are given by

$$\frac{d^2 n_{\mathrm{L}}}{dz\,dp} = F(p)\,|\eta_{\mathrm{f}}|^2\,e^{-\Gamma_{\mathrm{L}}\,z/\beta\,\gamma\,c} \tag{31}$$

---

[3]The error on Δm is scaled by 1.24 applying the PDG recipe [8].

and

$$\frac{d^2 n_{\text{reg}}}{dz\,dp} \;=\; t\,F(p)\left[|\rho(p)|^2\,e^{-\,\Gamma_{\text{S}}\,z/\beta\,\gamma\,c} + |\eta_{\text{f}}|^2\,e^{-\,\Gamma_{\text{L}}\,z/\beta\,\gamma\,c}\right.$$
$$\left. +\, 2\,|\eta_{\text{f}}\rho(p)|\,e^{-\,\overline{\Gamma}\,z/\beta\,\gamma\,c}\cos\left(\Delta m\,z/\beta\,\gamma\,c - \phi_{\text{f}} + \phi_{\rho}\right)\right] \qquad (32)$$

respectively, where the final state "f" represents both $\pi^+\pi^-$ and $\pi^0\pi^0$ and η_{f} is the CP violation parameter. The parameter t is the relative transmission of the kaon to the K_{L} beam without regenerator, which was constrained to the value measured by $\pi^+\pi^-\pi^0$ and $3\pi^0$ final states. $F(p)$ is the kaon flux, $\beta\gamma$ is the Lorentz boost factor and $\rho(p)$ and ϕ_{ρ} are the regeneration amplitude and its phase respectively.

Apart from the trivial factor coming from the propagation of the kaon in the regenerator, the modulus of the regeneration amplitude is assumed to be given by a power of the K_{S} momentum as

$$|\rho(p)| \propto b\left(\frac{p}{70\text{GeV}/c}\right)^a$$

where a and b are free parameters[4]. This assumption is based on the observation that a single Regge trajectory (ω) is dominant in the regeneration process at high energies. The validity of this power law with a single exponent was tested with the data and quoted errors include a possible deviation. Then, the phase of the regeneration amplitude is given by the exponent a as

$$\phi_{\rho} = \frac{a-1}{2}\,\pi + \phi_{\text{G}}$$

due to analyticity where ϕ_{G} is coming from the propagation of the kaon in the regenerator. With this relation, the regeneration phase is determined from the term proportional to $|\rho|^2$ in equation 32. E731 tested the validity of the analyticity relation experimentally and found it to be better than the other experimental uncertainties.

The experimentally observed decays were binned in various z and p values and equations 31 and 32 were used to fit the data. First, the phases of the CP violation parameters are fixed to

$$\phi_{+-} = \phi_{00} = \tan^{-1}\left(\frac{2\,\Delta m}{\Delta\Gamma}\right)$$

and $\pi^+\pi^-$ and $\pi^0\pi^0$ decay modes are fitted separately with a, b, Γ_{S} and Δm as free parameters with the η's constrained to their known values. This is the first time that Δm and τ_{S} are measured using the $\pi^0\pi^0$ decay mode with a comparable precision as the results obtained from the $\pi^+\pi^-$ decay mode. Combining the results from the $\pi^+\pi^-$ and $\pi^0\pi^0$ decay modes, it follows that [31]

$$\left.\begin{array}{rcl}\tau_{\text{S}} &=& \Gamma_{\text{S}}^{-1} = (0.8929 \pm 0.0016)\times 10^{-10}\,\text{s} \\ \Delta m &=& (0.5286 \pm 0.0028)\times 10^{10}\,\hbar\,\text{s}^{-1}\end{array}\right\}\text{E731}\;. \qquad (33)$$

The E731 result on the K_{S} lifetime is in perfect agreement with the PDG value given in equation 30. The central value of Δm obtained by E731 is 2.2 standard deviations smaller than the world average in equation 29, although the two values are only 1.6 standard deviations apart.

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[4]Detailed discussions on this subject are found in [24] and [31]

The phase difference $\phi_{00} - \phi_{+-}$ is measured by fitting both $\pi^+\pi^-$ and $\pi^0\pi^0$ decay modes simultaneously while fixing Γ_S and Δm to their measured values and with a, b, ϕ_{+-}, $\phi_{00} - \phi_{+-}$ and $\Re(\varepsilon'/\varepsilon)$ as free parameters. The result is [31]

$$\phi_{00} - \phi_{+-} = -1.6° \pm 1.2° \quad \text{E731}$$

which provides a better limit on the phase difference than from the current PDG average [8]

$$\phi_{00} - \phi_{+-} = -0.1° \pm 2.0° \quad \text{PDG}.$$

Lastly, Δm was treated also as a free parameter in the fit to extract ϕ_{+-} and the fit gives [31]

$$\phi_{+-} = 42.2° \pm 1.4° \quad \text{E731}.$$

At the same time, the fit gives $\Delta m = (0.5257 \pm 0.0049) \times 10^{10}\, \hbar\, \text{s}^{-1}$ [24] which is in good agreement with their own value shown in 33. With the E731 values for Γ_S and Δm, ϕ_{sw} is determined to be

$$\phi_{sw} = 43.4° \pm 0.2° \quad \text{E731} .$$

In summary, E731 concludes that their data are indeed compatible with CPT conservation and

$$\phi_{+-} \approx \phi_{00} \approx \phi_{sw} \quad \text{E731}.$$

How can we accommodate this new results with previous measurements? The E731 data clearly prefers a lower value of Δm as shown from the third analysis. The value of ϕ_{+-} is extracted from the K_L-K_S interference term proportional to $\cos[\Delta m\, t - \phi_{+-}(+\phi_\rho)]$. If there were equal numbers of events at all decay times, Δm and ϕ_{+-} could be decoupled by measuring more than half of the oscillation length since Δm gives the frequency and the ϕ_{+-} gives the phase of the oscillation. However, more weight is given to earlier decay times due to the exponential decay law introducing a positive correlation between the measured Δm and ϕ_{+-}. Figure 9 shows the extracted ϕ_{+-} as a function of Δm for two experiments [32, 33]. The differences in the slopes are due to different methods, one using vacuum regeneration and the other using a regenerator. Also shown is the superweak phase.

Curiously, the two experimental results and the superweak phase seem to agree at Δm which is somewhat smaller than the current world average. This conclusion remains valid even including more results of ϕ_{+-} [34, 35], which are less accurate.

Figure 10 shows three previous Δm measurements which are used to obtain the world average [28, 29, 30]. Two experiments, C. Geweniger et al. and S. Sjesdal et al. used the same detector with an additional common systematic error of 0.0015. The third experiment, M. Cullen et al. gives a higher value than the other two. The E731 result is also shown.

Whether the true value of Δm is indeed about two standard deviations lower than the current value needs an independent confirmation from other experiments, in particular those without regenerator. CPLEAR measuring simultaneously Δm using semileptonic decays and ϕ_{+-} from the $\pi^+\pi^-$ decay modes will make an important contribution. It is interesting to note that its preliminary value of Δm given in equations 26 is also smaller than the world average.

The possibility that a wrong value of Γ_S is responsible for this problem is not likely. Firstly, E731 gives a value of Γ_S which is perfectly consistent with the world average. Secondly, a different value of Γ_S does not give a consistent answer. In order to demonstrate this, let us consider the two experiments which give explicit

Δm and Γ_S dependences of their extracted ϕ_{+-} values [33, 35]

$$\phi_{+-}=\begin{cases} 46.9° \pm 1.6° + \left(\dfrac{\tau_\mathrm{S}}{0.8922} - 1\right) \times 270° + \left(\dfrac{\Delta m}{0.5351} - 1\right) \times 310° & \text{NA31} \\[2ex] 42.21° \pm 0.9° - \left(\dfrac{\tau_\mathrm{S}}{0.8922} - 1\right) \times 410° + \left(\dfrac{\Delta m}{0.5286} - 1\right) \times 100° & \text{E731} \end{cases}$$

and as the third condition, we take the CPT invariance, i.e.

$$\phi_{+-} = \tan^{-1}\left(\frac{2\,\Delta m}{\Delta \Gamma}\right) \ .$$

The three equations give a solution

$$\begin{aligned} \tau_\mathrm{S} &= 0.8900 \times 10^{-10}\,\mathrm{s} \\ \Delta m &= 0.5303 \times 10^{10}\,\hbar\,\mathrm{s}^{-1} \\ \phi_{+-} &= \phi_\mathrm{sw} = 43.4° \end{aligned}$$

which could be an "educated guess".

3.5 Summary

CP violation is still observed only in the neutral kaon system, with four different processes:

1. Charge asymmetry in the semileptonic K_L decays.

2. CP violating $K_L \to \pi^+\pi^-$ decay amplitude.

3. CP violating $K_L \to \pi^0\pi^0$ decay amplitude.

4. CP violating $K_L \to \pi^+\pi^-\gamma$ decay amplitude recently observed by E731[36].

CP violation in the oscillation must be accompanied with either T violation or CPT violation or both. The preliminary measurement of A_T with 0.000 ± 0.009 (sensitive only to CP+T violation) by CPLEAR does not yet show a direct signal of T violation with the present statistics. However, it should be seen with the expected increase in statistics in the near future.

In order to test CPT indirectly from the phase of η_{+-}, the phase of "ε" must be calculated using the unitarity relation. With new values for $\Im(x)$ measured by CPLEAR and η_{+-0} measured by E621 and CPLEAR, the phase of "ε" is equal to ϕ_sw within $\sim 0.4°$. Further improvements on x and η_{+-0} are still needed for future CPT test.

New measurements on τ_S, Δm, ϕ_{+-} and $\phi_{00} - \phi_{+-}$ by E731 are in perfect agreement with CPT conservation. The discrepancy between ϕ_{+-} and ϕ_sw with previous measurements at a level of 2.3 standard deviations can be explained by a new value of Δm which would be about one standard deviation lower than the current average value given by the Particle Data Group. Future measurements will answer this question.

NA31 measured a positive value of $\Re(\varepsilon'/\varepsilon) = 2.3 \times 10^{-3}$ with a significance of more than three standard deviations. E731 gives $\Re(\varepsilon'/\varepsilon) = 7.4 \times 10^{-4}$ with a one standard deviation significance. By combining the two results, we conclude that a non-zero value of $\Re(\varepsilon'/\varepsilon)$ is not yet established, i.e. no sign of direct CP violation. Most recent theoretical calculations for $\Re(\varepsilon'/\varepsilon)$ [37] tend to favour the value obtained by E731. However, uncertainties in the hadronic matrix elements involving the penguin diagram are still too large to be reliable.

4 Future Prospects

4.1 Neutral Kaon System

Experimentally, the kaon system will continue to dominate the field of CP violation for a while. CPLEAR plans to improve the statistical errors by roughly a factor of four by 1995 [21]. This will for example allow to measure ϕ_{+-} to better than $0.5°$ and Δm within $0.002 \times 10^{10} \, \hbar \, \text{s}^{-1}$. These two measurements will improve the indirect CPT test in the K^0-$\overline{K}^0$ oscillation. An expected error of A_T of 0.001 will clearly establish T violation in the K^0-$\overline{K}^0$ oscillation. Although CPLEAR will not reach the sensitivity to observe CP violation in the $K \to \pi^+\pi^-\pi^0$ decay, a better limit on η_{+-0} together with an improved limit on $\Im(x)$ will lower the uncertainty in the phase value of "ε" using the unitarity relation. Without this, measuring ϕ_{+-} with a precision better than $0.5°$ does not really provide a rigorous test of CPT.

E773 is a dedicated experiment at FNAL to measure the phases of the η's with the E731 detector. Results are expected on ϕ_{+-} and $\phi_{00} - \phi_{+-}$ very soon with errors smaller than $1°$ and $0.5°$ respectively [38].

In the field of the three-pion decay mode, E621 will present a better limit on η_{+-0} with full statistics. E621 should be able to measure the CP allowed $K_S \to \pi^+\pi^-\pi^0$ decay amplitude as well.

Next generation $\Re(\varepsilon'/\varepsilon)$ experiments are already under construction: NA48 [39] at CERN and E832(KTEV) at FNAL [40]. Both experiments use magnetic spectrometers with very good electromagnetic calorimeters: CsI crystals for KTEV and liquid Krypton for NA48. Both have K_S-K_L double beams: Two different targets with tag counters for NA48 and two K_L beams with a much improved regenerator for KTEV. Both will be able to take all four decay modes $K_L, K_S \to \pi^+\pi^-, \pi^0\pi^0$ at the same time reducing systematic uncertainties considerably. Both experiments expect to start data taking in 1995 and the goal is to achieve a sensitivity of less that 10^{-4}.

Various rare kaon decay experiments being continued at BNL and KEK are to some extent also sensitive to CP violation in other decay channels. Further discussion can be found in the contribution from J. Ritchie in these proceedings.

A new ϕ factory operating at $\sqrt{s} \approx 1\text{GeV}$ under construction in Frascati (DAΦNE) [41] with a designed luminosity of $\sim 10^{32} \, \text{cm}^{-2} \, \text{s}^{-1}$ will provide 5×10^9 tagged K_S and K_L (also $\sim 2 \times 10^{10}$ tagged $K^\pm$) in one year. This is well suited to make global studies of CP, T and CPT violation, extending CPLEAR results [42]. A unique feature of the ϕ decays producing pure K_S-K_L (or K^0-$\overline{K}^0$) initial states[5] will allow to test quantum mechanics. This will be only the place able to study rare K_S decays, e.g. CP violating $K_S \to 3\pi^0$ decays (~ 30 decays in one year). A sensitivity of $\sim 10^{-4}$ for measuring the real and imaginary part of ε'/ε with the KLOE detector seems possible. More details and further references can be found in [41].

The Main Injector ring at FNAL can provide very intensive kaon beams which may allow to construct a detector capable of measuring $\Re(\varepsilon'/\varepsilon)$ with a sensitivity much better than 10^{-4} [43].

4.2 Charged Kaons

In the standard model, a CP violation effect larger than ε' is expected in the charged kaon decay rates asymmetry

$$\frac{\Gamma_{K^+ \to \pi^+\pi^+\pi^-} - \Gamma_{K^- \to \pi^+\pi^-\pi^-}}{\Gamma_{K^+ \to \pi^+\pi^+\pi^-} + \Gamma_{K^- \to \pi^+\pi^-\pi^-}} \quad .$$

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[5]A similar situation occurs in the $\Upsilon(4S)$ decays into $B\overline{B}$.

The slope parameter in the Dalitz plot is also expected to be different in this decay mode between K^+ and K^-. With DAΦNE or at BNL [44], such an effect could be investigated.

4.3 The Λ System

A CP asymmetry in the decay asymmetries of α for $\Lambda \to \mathrm{p}\pi^-$ and $\overline{\alpha}$ for $\overline{\Lambda} \to \overline{\mathrm{p}}\pi^+$ given by

$$A_\alpha = \frac{\alpha + \overline{\alpha}}{\alpha - \overline{\alpha}}$$

is non-zero if there exists an interference between the proton-pion s- and p-wave amplitudes. Such an interference could be possible in the standard model where the penguin diagrams contribute differently to the s- and p-waves. This effect is expected to be less than 10^{-4} [45]. The experiment PS185 at CERN measured [46]

$$A_\alpha = -0.07 \pm 0.09$$

using Λ-$\overline{\Lambda}$ pairs produced by $\mathrm{p}\overline{\mathrm{p}}$ annihilations in flight. A proposal submitted to FNAL [47] to measure this asymmetry using Λ and $\overline{\Lambda}$ from hyperon decays hopes for a sensitivity of 10^{-4}. Possible new facilities such as Super LEAR or a Tau-charm factory can provide a clean and intense source of Λ-$\overline{\Lambda}$.

5 B-Meson System

The Physics of B-mesons are discussed by A. Sanda in these proceedings.

5.1 Formalism

We just present a short description of the neutral B-meson system in order to make a comparison to the neutral kaon system. For simplicity, we assume that CPT is conserved [48]. The two mass eigenstates are given as

$$|\mathrm{B_l}\rangle = \frac{1}{\sqrt{2}}\left[|\mathrm{B}^0\rangle - (1 - 2\varepsilon_\mathrm{B})\,e^{-i\,\varphi_M}|\overline{\mathrm{B}}^0\rangle\right]$$

$$|\mathrm{B_h}\rangle = \frac{1}{\sqrt{2}}\left[|\mathrm{B}^0\rangle + (1 - 2\varepsilon_\mathrm{B})\,e^{-i\,\varphi_M}|\overline{\mathrm{B}}^0\rangle\right]$$

where $\mathrm{B_l}$ and $\mathrm{B_h}$ denote the light and heavy neutral B-mesons respectively and φ_M is the phase of M_{12}. The CP violation parameter ε_B is

$$\varepsilon_\mathrm{B} = -\frac{1}{4}\left|\frac{\Gamma_{12}}{M_{12}}\right|\sin\left(\delta\varphi\right)$$

with

$$\delta\varphi = \varphi_M - \varphi_\Gamma + \pi$$

as in the kaon system. We use $\Delta\Gamma \ll \Delta m$, i.e. $|\Gamma_{12}| \ll |M_{12}|$ which is different from the kaon system. Note that ε_B is real which also differs from the kaon system. The standard model predicts $\varepsilon_\mathrm{B} \approx \mathcal{O}(10^{-3})$.

In the limit of CP conservation, we have $\varphi_M = -\theta_\mathrm{CP} + \pi$ **and** $\varepsilon_\mathrm{B} = 0$ so that

$$CP|\mathrm{B_l}\rangle = +|\mathrm{B_l}\rangle \quad \text{and} \quad CP|\mathrm{B_h}\rangle = -|\mathrm{B_h}\rangle$$

where we assumed that the B parameter in the calculation of the box diagram is positive [49]. In this case, the $CP = +1$ state is lighter than the $CP = -1$ state which is identical to the kaon system.

The CP violation parameter for a decay into a CP eigenstate "f" with $CP = +1$ is then defined as

$$\eta_{\rm f} = \frac{\langle {\rm f}|H_{\rm weak}|{\rm B_h}\rangle}{\langle {\rm f}|H_{\rm weak}|{\rm B_l}\rangle} \ . \tag{34}$$

In the absence of CP violation in the $B^0\overline{B}^0$ oscillation, i.e. $\varepsilon_{\rm B} = 0$, B_l and B_h are still not CP eigenstates. In the absence of CP violation in the decay amplitude, we have

$$|\langle {\rm f}|H_{\rm weak}|{\rm B}^0\rangle| = |\langle {\rm f}|H_{\rm weak}|\overline{\rm B}^0\rangle| \ .$$

Using these relations, equation 34 can be written as

$$\eta_{\rm f} = -\,i\,\frac{\sin\left(2\,\varphi_{\rm f} + \varphi_M + \overline{\theta}_{\rm T} - \theta_{\rm CP}\right)}{1 - \cos\left(2\,\varphi_{\rm f} + \varphi_M + \overline{\theta}_{\rm T} - \theta_{\rm CP}\right)} \tag{35}$$

which is pure imaginary, where $\varphi_{\rm f}$ is the phase of the $B^0 \to$ f decay amplitude. Note that this corresponds more or less to the third term in equation 15 in the case of the kaon system, i.e. CP violation due to interference between decay and oscillation. In the kaon system, the phase of Γ_{12} given by the unitarity relation is dominated by the $2\pi(I = 0)$ decay amplitude. In the B-meson system, there exists no dominant decay amplitude and φ_M (in the absence of CP violation in the oscillation this is identical to $\varphi_\Gamma + \pi$) can be very different from $-2\,\varphi_{\rm f} - \overline{\theta}_{\rm T} + \theta_{\rm CP}$, thus introducing a large CP violation in $\eta_{\rm f}$. In the kaon system, this is typically $\Im(\eta_{I=0}) \approx \mathcal{O}(10^{-3})$.

In the B-meson system, the standard model can make a precise prediction for the CP violation given in equation 35. Since M_{12} is dominated by the top quark in the box diagram, φ_M is governed by the elements $V_{\rm tb}$ and $V_{\rm td}$ of the Cabibbo-Kobayashi-Maskawa (CKM) [50] quark mixing matrix. In the decays where only one quark diagram contributes, the phase of the decay amplitude is also given by the CKM-matrix elements. Well known final states are $J/\psi\,K_{\rm S}$ and $\pi^+\pi^-$. Once the CKM matrix elements $V_{\rm cb}$, $V_{\rm ub}$ and $V_{\rm td}$ become known, the CP violation parameters for these two channels can be predicted with the help of the unitarity of the CKM matrix [51].

In the B-meson system (also in the D-meson system), some final states such as $D^+\,\pi^-$ can be produced from both B^0 and $\overline{B}^0$ decays although they are not CP eigenstates. The CP conjugated state $D^-\,\pi^+$ is also the decay final state of B^0 and $\overline{B}^0$. In this case, the same formalism used to describe the semileptonic decay of kaons including a possible violation of the $\Delta Q = \Delta S$ rule can be applied.

CP violation can be studied by comparing an initially pure B^0 decaying into $D^+\,\pi^-$ with an initially pure $\overline{B}^0$ decaying into $D^-\,\pi^+$. The imaginary part of the phase convention invariant parameter

$$x = -\frac{\langle D^-\,\pi^+|H_{\rm weak}|\overline{\rm B}^0\rangle}{\langle D^-\,\pi^+|H_{\rm weak}|{\rm B}^0\rangle}e^{i\,\varphi_{\rm M}} = -\frac{\langle D^+\,\pi^-|H_{\rm weak}|{\rm B}^0\rangle}{\langle D^+\,\pi^-|H_{\rm weak}|\overline{\rm B}^0\rangle}e^{i\,\varphi_{\rm M}}$$

generates CP violation (see $A_{\rm CPT}$ and $A_{\rm T}$ in the kaon system), by assuming no CP violation in decay amplitudes and in oscillations. Note that $|x| \ll 1$ since the decay amplitudes for $B^0 \to D^+\pi^-$ and $\overline{B}^0 \to D^-\pi^+$ are proportional to $|V_{\rm ub}|\times|V_{\rm cd}|$ which is very small.

Since many decay modes are available for the B-meson system, CP violation is expected in many different final states and could be large. This allows us to make a consistency test by checking whether all the observed CP violation follows the pattern predicted by the standard model. This is essential, since other models of CP violation can also generate large CP violation in B-meson decays although with different patterns.

The branching ratios for the interesting decay modes are all small requiring high statistics. More details on various strategies to study CP violation can be found in [6].

5.2 Experimental Considerations

Data on b-hadrons are so far dominated by results from e^+e^- machines running at the $\Upsilon(4S)$ resonance and at Z^0. Although high energy hadron machines have a promising potential due to the large number of produced b-hadrons, there is still much work needed to prove experimental capabilities to reconstruct final states without J/ψ.

The observation of CP violation, even in the most promising channel $B \to J/\psi\,K_S$ requires a new generation of e^+e^- machine in order to reach the necessary luminosities with an asymmetric beam energy configuration when working at $\Upsilon(4S)$. Details can be found in numerous B-meson factory proposals [52].

The rate estimates in these studies should be quite reliable due to the small background and the long experience in doing experiments with B-mesons using such machines. The most crucial point is whether an e^+e^- collider working with an asymmetric beam energy configuration can achieve the required luminosity of $\mathcal{L} \geq 3 \times 10^{33}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$.

A B-meson factory working with the designed luminosity can exploit physics requiring 10^8 B-$\overline{\mathrm{B}}$ pairs. According to the most resent standard model predictions [51], CP violation in the $B \to J/\psi\,K_S$ decay is within reach with such a statistics. Its clean environment would also help to study final states of more complex nature such as $D^0\,\overline{D}^0$ and $\psi'\,K_S$.

CP violation study involving $b \to u + W^-$ decays, such as the decay final state $\pi^+\pi^-$, needs significantly more B-mesons. Current standard model predictions [51] do not exclude the possibility that CP violation is very small for this channel. Recent CLEO results indicate a branching ratio of $\sim 10^{-5}$ for this decay mode [53].

CP violation studies in flavour specific B-meson decays provide a clean way to eliminate the superweak model. They need also more B-mesons. Very optimistically, the effect of CP violation could be $\sim 10\%$ for $B \to K^*K$ with a branching ratio of $\sim 10^{-5}$ [54]. This already shows that more than 10^9 B-mesons are needed.

CP violation experiments at hadron machines may become variable if they can fully exploit the large number of produced B-mesons [55]. Various projects are being considered based on both existing machines such as Tevatron at FNAL and HERA at DESY and future machines such as RHIC at BNL, LHC and SSC [55].

Unlike for e^+e^- B-meson factories, here the issue is the detector capability. The high interaction rate (1 to 50 MHz depending on $\sqrt{s}$) necessary to produce a sufficient number of B-mesons of $> 10^{10}$ can be achieved with little problems. Whether trigger and detector components can cope with such a high rate must be carefully looked at. Due to high background, rate estimates will never be certain. One can hardly generate enough simulated minimum bias events to study the background. Since we are looking for events with branching ratios of $\sim 10^{-5}$, any tail in various distributions can influence the background estimation. Such a tail cannot be reproduced with a simulation. To design an ultimate CP experiment at a hadron machine may need more experiences from existing hadron machines.

6   T Violation

There are experiments addressing the violation of T invariance without referring to C (or CP). A typical example is the electric dipole moment of the neutron [56] and electron [57]. There are other experiments such as the scattering of polarised protons where a triple vector product in the final state can be used to test T violation. The standard model either does not produce any T violation or produces very little signal and present experiments are not sensitive enough. However, physics beyond the standard model can produce a substantial T violation signal [58] which can be

tested by these experiments.

7 Conclusions

Present experimental efforts to solve the mystery of CP violation are large and will remain so. We are all eager to know whether CP violation can be fully accommodated in the standard model. The first experimental step is to observe CP violation in the decay amplitude. This would exclude the superweak model. Then, we have to make a quantitative comparison between the observation and the standard model prediction.

Experiments with kaons will still dominate the field of CP violation for some time. However, the smallness of CP violation in the decay amplitude may forbid any discovery. Theoretical difficulties related to low energy QCD may further hinder quantitative comparisons.

In the long therm, the best place to make a systematic study is in the B-meson system. CP violation is expected in many different decay channels and reliable standard model predictions can be made for some of the decay modes once all four parameters of the Cabibbo-Kobayashi-Maskawa quark mixing matrix are obtained.

CP violation effects in the weak decay of the kaon are in fact not that small compared with CP violation in strong interactions which is naturally expected in the standard model but not observed so far. It may be indeed that CP violation is a reflection of physics at a much higher energy scale.

Acknowledgement

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FIG. 1. Angular momentum configuration of the $\pi^+\pi^-\pi^0$ final state.

Table 1: Allowed isospin and angular momentum configurations for the $\pi^+\pi^-\pi^0$ system. S and AS stand for symmetric and antisymmetric.

$CP(\pi^+\pi^-\pi^0)$	+1		-1	
$l_{\pi^+\pi^-}$	1		0	
l-Sym.$(\pi^+ \leftrightarrow \pi^-)$	AS		S	
$I_{3\pi}$	0	2	1	3
$I_{\pi^+\pi^-}$	1	1	0 or 2	2
I-Sym.$(\pi^+ \leftrightarrow \pi^-)$	AS	AS	S	S

Figure 4: Time dependent CP asymmetries for K$\to \pi^+\pi^-\pi^0$ decays for events with $m_{+0} > m_{-0}$ and $m_{+0} < m_{-0}$ where $m_{+(-)0}$ is the $\pi^+(\pi^-)\pi^0$ invariant mass of the corresponding decay pions.

Table 2: Event statistics for NA31 and E731 after subtracting all the background.

Decay Mode	NA31		E731
	1986	1988+1989	
$K_L \to \pi^0\pi^0$	109×10^3	319×10^3	410×10^3
$K_L \to \pi^+\pi^-$	295×10^3	847×10^3	327×10^3
$K_S \to \pi^0\pi^0$	932×10^3	1322×10^3	800×10^3
$K_S \to \pi^+\pi^-$	2300×10^3	3241×10^3	1061×10^3

Figure 5: Detector layouts for E731 and NA31 experiments.

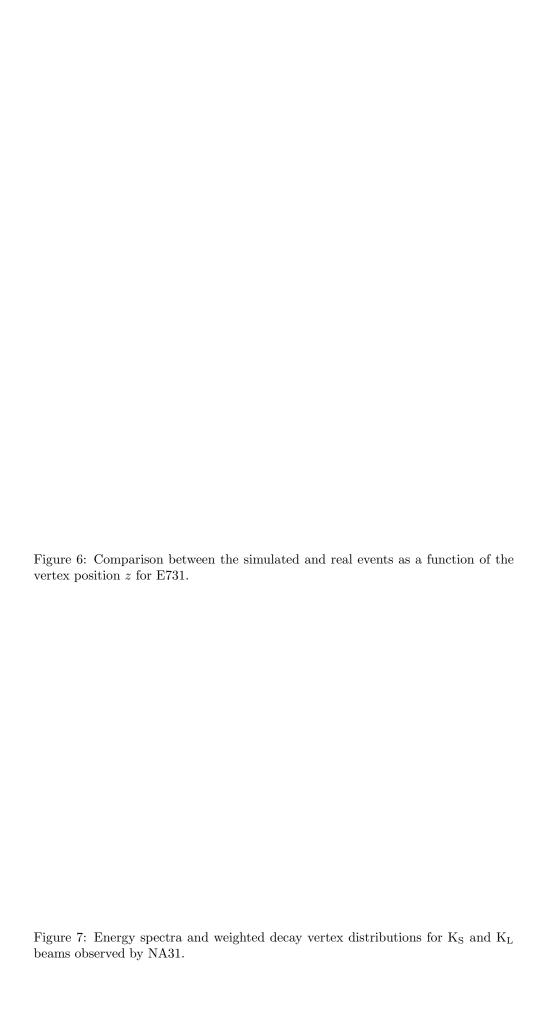Figure 6: Comparison between the simulated and real events as a function of the vertex position z for E731.

Figure 7: Energy spectra and weighted decay vertex distributions for K_S and K_L beams observed by NA31.

Table 3: Subtracted background.

Decay Modes	NA31		E731
	1986	1988+1989	
three-body decay background			
$K_L \to \pi^0\pi^0$	4.0%	2.67%	1.78%
$K_L \to \pi^+\pi^-$	0.6%	0.63%	0.32%
$K_S \to \pi^0\pi^0$	< 0.1%	0.07%	0.049%
effect due to the regenerator			
$K_L \to \pi^0\pi^0$	-	-	2.26%
$K_S \to \pi^0\pi^0$	-	-	2.53%
$K_S \to \pi^+\pi^-$	-	-	0.155%
effect due to the trigger plane			
$K_L \to \pi^0\pi^0$	-	-	1.12%
$K_S \to \pi^0\pi^0$	-	-	0.264%

Table 4: List of systematic uncertainties in $|\eta_{0\,0}/\eta_{+\,-}|^2$.

	NA31		E731
	1986	1988+1989	
energy calibration	3.0×10^{-3}	1.3×10^{-3}	0.96×10^{-3}
accidental correction	2.0×10^{-3}	1.4×10^{-3}	0.64×10^{-3}
acceptance	1.7×10^{-3}	1.0×10^{-3}	0.71×10^{-3}
background			
$\quad K_L \to \pi^0\pi^0$	2.0×10^{-3}	1.3×10^{-3}	0.36×10^{-3}
$\quad K_L \to \pi^+\pi^-$	2.0×10^{-3}	1.0×10^{-3}	0.17×10^{-3}
trigger and anti K_S counter			
$\quad$ inefficiencies	1.0×10^{-3}	0.9×10^{-3}	-
wire chamber inefficiencies	-	1.0×10^{-3}	-
regenerator related corrections	-	-	0.59×10^{-3}
trigger plane efficiency	-	-	0.73×10^{-3}
beam scattering	-	-	0.18×10^{-3}
total	5.0×10^{-3}	3.0×10^{-3}	1.71×10^{-3}

Figure 8: The measured ratio $|\eta_{0\,0}/\eta_{+\,-}|$ by NA31 and E731.

Figure 9: The Δm dependence of the extracted phase $\phi_{+\,-}$ from the observed K_L-K_S interference for three different experiments.

Figure 10: The K_L-K_S mass difference measured by various experiments used by the Particle Data Group to obtain the world average and the E731 result.

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