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$\Delta S=-\Delta Q$ Transition, $\Delta I=1/2$ Rule and CP Violation in Kaon Leptonic Decays

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A model for the strangeness changing leptonic decays is proposed, in which the presence of the $\Delta S=-\Delta Q$ transition is shown to be compatible with the predictions of the well-known $\Delta I=1/2$ rule for both K_{l3} and K_{l4} decays, provided that CP invariance is "maximally" violated in that transition. Some remarks on the sigma β decay are given.

§ 1. Introduction

The CP violating decay mode $K_L^0\rightarrow\pi^++\pi^-$ has been observed by several groups.[1] Subsequently many theoretical attempts have been made to explain this violation. In this connection, Sachs[2] suggested the possibility of maximal CP violation in the leptonic decay processes in the sense that the $\Delta S=-\Delta Q$ interaction is out of phase with the $\Delta S=\Delta Q$ interaction by $\pi/2$.

Recently, an experimental analysis[3] of the K_{e3} decays reported some results not inconsistent with Sachs's suggestion. If one takes these results seriously, one should take account of the $\Delta S=-\Delta Q$ transition at least for the processes where the vector current plays a dominant role.

On the other hand, as is well known, the $\Delta I=1/2$ rule holds with great accuracy in the K_{l3} modes.[4]

In this theoretical and experimental situation, it is important to reexamine the isospin structure of weak interactions and to look for the possible interrelation[5] of CP violation with it. The main purpose of the present work is to propose a model in which the presence of the $\Delta S=-\Delta Q$ transition is still compatible with the predictions of the usual $\Delta I=1/2$ rule for the leptonic decay modes of kaons. The model is essentially based upon Lee's version[6] of the intermediate vector boson theory of weak interactions, which is just designed to allow the $\Delta S=-\Delta Q$ transition.

In § 2, this model, combined with the assumption of maximal CP violation, is shown to predict the following relations for the K_{l3} decays:

$$R_{SL}\equiv\frac{\Gamma(K_S{}^0\rightarrow\pi^\pm l^\mp\nu)}{\Gamma(K_L{}^0\rightarrow\pi^\pm l^\mp\nu)}=1, \qquad (1\cdot1)$$

$$R_{+L}\equiv\frac{\Gamma(K^+\rightarrow\pi^0 l^+\nu)}{\Gamma(K_L{}^0\rightarrow\pi^\pm l^\mp\nu)}=\frac{1}{2}, \qquad (1\cdot2)$$

where l stands for e or μ and

$$\Gamma(K^0\to\pi^\pm l^\mp \bar{\nu})=\Gamma(K^0\to\pi^+ l^- \nu)+\Gamma(K^0\to\pi^- l^+ \nu).$$

Eqs. (1·1) and (1·2) are identical with those expected from the $\Delta I=1/2$ rule The model is then applied in § 3 to the K_{e4} mode, and relations between the various amplitudes are derived. In § 4, some remarks on the decay mode $\Sigma\to nl\nu$ are given.

CPT invariance is assumed throughout this paper.

§ 2. The model and the K_{l3} decays

Following Lee,[6] we assume*) that

i) weak interactions are mediated by the vector bosons, $\boldsymbol{W}$, which form an isotopic triplet with charge $+$, 0, $-$ respectively;

ii) the bosons interact with the strangeness changing currents according to the $\Delta I=1/2$ rule;

iii) only the charged bosons interact with the lepton currents.

Both $\Delta I=1/2$ and $\Delta I=3/2$ transitions are thus involved in the effective interaction for the strangeness changing leptonic decays in this model.

Now, let us first discuss the K_{l3} decays in this model.**)

The interaction for the process $K\to\pi+W$ has the form

$$A(\bar{x}\,\partial_\mu K)\boldsymbol{\pi}\cdot\boldsymbol{W}_\mu-iB(\bar{x}\,\boldsymbol{\tau}\partial_\mu K)\cdot\boldsymbol{\pi}\times\boldsymbol{W}_\mu, \tag{2}$$

where x denotes isospinor spurion, i.e. $\bar{x}=(0,1)$, and A and B are coupling constants (complex in general). The effective Hamiltonian for the over-all process, $K\to\pi l\nu$, is obtained from Eq. (2) (apart from a common factor) by the substitutions

$$\begin{aligned} W_\mu{}^+ &\to i\,\bar{l}\,\gamma_\mu(1+\gamma_5)\nu,\\ W_\mu{}^- &\to i\,\bar{\nu}\,\gamma_\mu(1+\gamma_5)l,\\ W_\mu{}^0 &\to 0. \end{aligned} \tag{3}$$

For the l^+ decays, the following effective interaction results:

$$\begin{aligned} H=i\,\bar{\nu}\,\gamma_\mu(1+\gamma_5)l[&(A+B)\pi^+\partial_\mu K^0\\ &+(A^*-B^*)\pi^+\partial_\mu \bar{K}^0+\sqrt{2}\,B\pi^0\partial_\mu K^+]. \end{aligned} \tag{4}$$

If we let f, g and h be the decay amplitudes of the processes $K^0\to\pi^- l^+\nu$,

*) We confine our discussion to the strangeness changing leptonic decays.

**) Lee's model[6] has been applied to the K_{l3} decays by Bell et al.[7] and by Gao,[8] in connection with the earlier experimental results[9] which show large deviations from the $\Delta I=1/2$ rule. Our resulting Hamiltonian (4) is similar to theirs.

$\bar{K}^0\to\pi^- l^+\nu$ and $K^+\to\pi^0 l^+\nu$ respectively, then it is easily seen from Eq. (4) that we have

$$h=\frac{1}{\sqrt{2}}(f-g^*). \tag{5}$$

Because of the small CP noninvariant admixture observed,[1] we assume the short and long lived neutral K mesons to be

$$K_S{}^0\approx\frac{1}{\sqrt{2}}(K^0+\bar{K}^0),$$

$$K_L{}^0\approx\frac{1}{\sqrt{2}}(K^0-\bar{K}^0) \tag{6}$$

with $\bar{K}^0\equiv CPK^0$.

From Eqs. (5) and (6) and CPT invariance, the ratios R_{SL} and R_{+L} defined in Eqs. (1·1) and (1.2) are given by

$$R_{SL}=\frac{|1/\sqrt{2}\cdot(f+g)|^2+|1/\sqrt{2}\cdot(g^*+f^*)|^2}{|1/\sqrt{2}\cdot(f-g)|^2+|1/\sqrt{2}\cdot(g^*-f^*)|^2} \tag{7·1}$$

and

$$R_{+L}=\frac{|1/\sqrt{2}\cdot(f-g^*)|^2}{|1/\sqrt{2}\cdot(f-g)|^2+|1/\sqrt{2}\cdot(g^*-f^*)|^2}. \tag{7·2}$$

One observes that the results (1·1) and (1·2) follow from Eqs. (7·1) and (7·2), if and only if the condition*)

(i) $g=0$

or

(ii)**) $f=$real, $g=$pure imaginary

holds.

Note that the condition (i) implies the $\Delta S=\Delta Q$ rule, and the condition (ii) is nothing but a special case of the "maximal" CP violation introduced by Sachs.[2] It should be also remarked that the result (1·1) is a general consequence obtained only from (i) or (ii) and is irrespective of the present special model.

§ 3. The K_{e4} decays

Next we turn to the K_{e4} decays.

*) There is, of course, another possibility, namely that $f=0$ or $f=$pure imaginary and $g=$real, which is neglected in the text.

**) The statement (ii) implies, in general, that f is almost real relative to $A(K^0\to\pi^+\pi^-)$, and fg^* is almost pure imaginary relative to $[A(K^0\to\pi^+\pi^-)]^2$. In the Sachs model,[2] the nonleptonic decays are assumed to be CP invariant so that $A(K^0\to\pi^+\pi^-)=$real.

The relevant decay modes and their decay amplitudes in the model, obtained by using the usual spurion technique, are listed in Table I.

Table I. Modes and amplitudes of the K_{e4} decays.

Modes	$\Delta S/\Delta Q$	Amplitudes*)	
		even 2π- states	odd 2π- states
$K^+\to\pi^+\pi^+e^-\bar{\nu}$	-1	$\sqrt{3}\ T_1{}^2$	
$K^+\to\pi^+\pi^-e^+\nu$	$+1$	$\sqrt{1/6}\ T_1{}^2+\sqrt{2/3}\ T_1{}^0$	$-\sqrt{2}\ T_1{}^1$
$K^+\to\pi^0\pi^0e^+\nu$	$+1$	$-\sqrt{1/3}\ T_1{}^2+\sqrt{1/3}\ T_1{}^0$	
$K^0\to\pi^+\pi^0e^-\bar{\nu}$	-1	$-\sqrt{3/4}\ T_1{}^2$	$-T_1{}^1+T_0{}^1$
$K^0\to\pi^-\pi^0e^+\nu$	$+1$	$-\sqrt{3/4}\ T_1{}^2$	$T_1{}^1+T_0{}^1$

*) The i and j in $T_j{}^i$ indicate the isospin of the final 2π state and of the total system of the 2π and virtual W respectively.

From Table I, one obtains the following relations among the amplitudes for the various decay modes:

$$A(K^+\to\pi^+\ \pi^-\ e^+\ \nu)-\sqrt{2}\ A(K^+\to\pi^0\ \pi^0\ e^+\ \nu)$$
$$=\frac{1}{\sqrt{2}}A(K^+\to\pi^+\ \pi^+\ e^-\ \bar{\nu}) \tag{8}$$
$$=-\sqrt{2}\ A(K^0\to\pi^+\ \pi^0\ e^-\ \bar{\nu})=-\sqrt{2}\ A(K^0\to\pi^-\ \pi^0\ e^+\ \nu)$$

for even angular momentum 2π states, and

$$A(K^+\to\pi^+\ \pi^-\ e^+\ \nu)$$
$$=-\frac{1}{\sqrt{2}}[A(K^0\to\pi^-\ \pi^0\ e^+\ \nu)-A(K^0\to\pi^+\ \pi^0\ e^-\ \bar{\nu})] \tag{9}$$

for odd 2π states. In the following, we consider two cases corresponding to the two cases, (i) and (ii), in the K_{l3} decays.

Case (i). The $\Delta S=\Delta Q$ rule holds exactly:

$$A(K^+\to\pi^+\ \pi^+\ e^-\ \bar{\nu})=A(K^0\to\pi^+\ \pi^0\ e^-\ \bar{\nu})=0. \tag{10}$$

Equations (8) and (9) now reduce respectively to

$$A(K^+\to\widehat{\pi^+\ \pi^-}\ e^+\nu)-\sqrt{2}\ A(K^+\to\widehat{\pi^0\ \pi^0}\ e^+\ \nu)$$
$$=-\sqrt{2}\ A(K^0\to\widehat{\pi^-\ \pi^0}\ e^+\ \nu)=0 \tag{8a}$$

and

$$A(K^+\to\overline{\pi^+\ \pi^-}\ e^+\ \nu)=-\frac{1}{\sqrt{2}}\ A(K^0\to\overline{\pi^-\ \pi^0}\ e^+\ \nu), \tag{9a}$$

where $\widehat{\pi\pi}$ and $\overline{\pi\pi}$ indicate even and odd angular momentum 2π states respectively.

Case (ii). *CP* invariance is violated maximally in the $\Delta S=-\Delta Q$ transition in the sense that, similarly to (ii) in § 2,

$$A(K^0\to\pi^-\,\pi^0\,e^+\,\nu)=\text{real},$$

$$A(K^0\to\pi^+\,\pi^0\,e^-\,\bar{\nu})=\text{pure imaginary}.$$

Since this restriction requires $T_1{}^2=0$, one finds that Eqs. (10) and (8a) hold, in this case also, only if $A(K^0\to\pi^+\,\pi^0\,e^-\,\bar{\nu})$ is replaced by $A(K^0\to\widehat{\pi^+\,\pi^0}\,e^-\,\bar{\nu})$.

For the above two cases, we obtain identical results for the decay rates, as follows:

$$\Gamma(K^+\to\pi^+\,\pi^+\,e^-\,\bar{\nu})=0, \tag{11·1}$$

$$\Gamma(K_S{}^0\to\pi^\pm\,\pi^0\,e^\mp\,\nu)=\Gamma(K_L{}^0\to\pi^\pm\,\pi^0\,e^\mp\,\nu), \tag{11·2}$$

$$\Gamma(K^+\to\pi^+\,\pi^-\,e^+\,\nu)=2\Gamma(K^+\to\pi^0\,\pi^0\,e^+\,\nu)+\frac{1}{2}\Gamma(K_L{}^0\to\pi^\pm\,\pi^0\,e^\mp\,\nu). \tag{11·3}$$

These relations as well as Eqs. (8a) and (9a) are the same as those expected from the usual $\Delta S=\Delta Q$ rule and/or the $\Delta I=1/2$ rule.[10] One of the above results, Eq. (11.1), is supported by the present experimental evidence.[11]

§ 4. Remarks on the decay mode $\Sigma\to n\,l\,\nu$

One may naturally raise a question about the compatibility of the presence of the $\Delta S=-\Delta Q$ transition with the experimental results on the suppression of the decay $\Sigma^+\to n\,l^+\,\nu$ compared to the decay $\Sigma^-\to n\,l^-\,\bar{\nu}$. In this connection, we shall add some remarks on this decay mode.

The first point is that there is, *a priori*, no reason to discuss the decays of baryons and of mesons on the same footing, i.e. they may have different decay mechanism.

Secondly, even when one takes the point of view that the hadronic currents responsible for the hyperon decays have the same isospin structure as for *K*-meson decays, the present experimental results on the $\Sigma\to n\,l\,\nu$ decays are still not inconsistent with the presence of the $\Delta S=-\Delta Q$ transitions in the *K*-meson decays, as will be illustrated below. To see this, it is sufficient to point out that all the arguments given in the preceding sections refer only to the phase of the amplitudes, not to their magnitude.

Let us try to give a numerical estimate. For the vector transition, we may take the value

$$\left|\frac{A(\bar{K}^0\to\pi^-\,e^+\,\nu)}{A(K^0\to\pi^-\,e^+\,\nu)}\right|=0.44 \tag{12}$$

as the relative magnitude of the $\Delta S=-\Delta Q$ amplitude to the $\Delta S=\Delta Q$ one. The corresponding ratio for the axial vector transition can be obtained from the K_{e4} decays, which, from our point of view, should be given by

$$\left|\frac{A(K^0\to\pi^+\,\pi^0\,e^-\,\bar{\nu})}{A(K^0\to\pi^-\,\pi^0\,e^+\,\nu)}\right|. \tag{13}$$

Since nothing about the ratio (13) is known, let us take, for the moment, the value[11)]

$$\left|\frac{A(\Delta S=-\Delta Q)}{A(\Delta S=\Delta Q)}\right|<0.25, \tag{14}$$

for our purpose; this is estimated*) from the decay rates of $K^+\to\pi^+\,\pi^+\,e^-\,\bar{\nu}$ and $K^+\to\pi^+\,\pi^-\,e^+\,\nu$. Using the values (12) and (14), we obtain

$$\frac{\Gamma(\Sigma^+\to n\,l^+\,\nu)}{\Gamma(\Sigma^-\to n\,l^-\,\bar{\nu})}=4.8\%\sim 9.5\%, \tag{15}$$

where the interaction for $\Sigma^-\to n\,l^-\,\bar{\nu}$ is assumed to be of the V-A type. The value obtained, Eq. (15), is below the experimental upper limit[12)] for the decay.

§ 5. Conclusion

We have proposed a model for the strangeness changing leptonic decays, in which the presence of the $\Delta S=-\Delta Q$ transition is shown to be compatible with the $\Delta I=1/2$ rule if CP invariance is maximally violated in that transition. The predicted relations, (1·1), (1·2) and (11·1), (11·2), (11·3), are the same as those expected from the usual $\Delta I=1/2$ rule and are thus all consistent with the present available experimental data.

Furthur experimental investigations on the K_{l3} and K_{l4} decays, as well as on the $\Sigma\to n\,l\,\nu$ and $\Xi\to\Sigma\,l\,\nu$ decays, are required to test this model and the assumption of the maximal CP violation.

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*) It should be recalled that the suppression of the mode $K^+\to\pi^+\pi^+e^-\bar{\nu}$ may be due to the maximal CP violation as discussed in § 3.

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