HEPP-CPV-project

John Ronayne (10318997), Kevin Maguire (10318135), Sinead Hales(10318221), Dudley Grant (10275291)
(Dated: November 17, 2013)

I. INTRODUCTION

Short intro here

II. $\hat{P}$, $\hat{C}$ AND $\hat{C}\hat{P}\hat{T}$

III. CP VIOLATION

CP violation was first observed in the mixing of neutral K-mesons by Christenson, Cronin, Fitch and Turlay in 1964 [1]. They observed the $\hat{C}\hat{P} = -1$ state K_L^0 decaying to 2 pions, a state with $\hat{C}\hat{P} = 1$. Although the fraction of K_L^0 decays violating $\hat{C}\hat{P}$ in this way is tiny, the discovery was significant.

IV. $\hat{C}\hat{P}$V IN KAON SYSTEM

A. Neutral Kaon Mixing

As mentioned CPV was first observed in the neutral Kaon system. Direct and indirect CPV have been observed but it is found that the process is entirely dominated by the indirect method. Essential to these mechanisms is the mixing between the neutral Kaon and its anti-particle, corresponding to the states $|K^0\rangle$ and $|\bar{K}^0\rangle$. These have quark compositions of $d\bar{s}$ and $s\bar{d}$, respectively.

In interactions involving the strong or EM force, the quantum number strangeness, which tells us the number of strange quarks in a particle, must be conserved. For the weak force it is found that, like parity, this symmetry is not conserved. Due to this many processes forbidden for the strong and EM interactions are allowed through the weak force. This violation is what makes mixing possible. Mixing is the decay of a particle into its anti-particle and can only take place when a particle is its own anti-particle, or if the particles differ by a quantum number which is not conserved by some interaction. This is the case in neutral Kaon mixing, also know as Kaon oscillations. The neutral Kaon and its anti-particle have opposite strangeness but can decay into each other through the strangeness violating weak force. See Fig.(1).

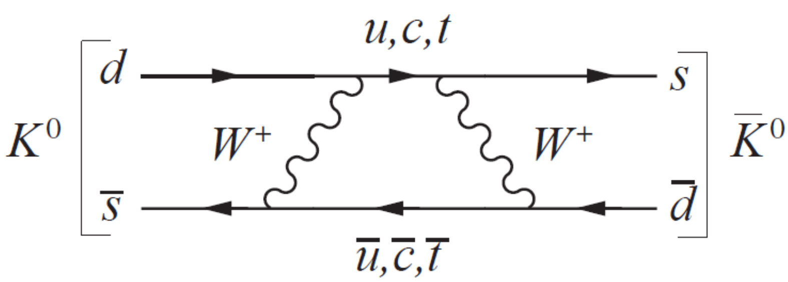

FIG. 1: *Feynmann diagram illustrating the process through which neutral Kaons decay into eachother*

Analogous to the mixing of mass eigenstate quarks to different quark flavours, it is found that the neutral Kaon flavour eigenstates do not correspond to eigenstates of the $\hat{C}\hat{P}$ operator. To show this first operate on the Kaon states with $\hat{C}$. Neglecting phase throughout and assuming no CPV for now, one obtains:

$$\hat{C}\,|K^0(d\bar{s})\rangle = (1)(-1)\,|\bar{K}^0(s\bar{d})\rangle = -\,|\bar{K}^0(s\bar{d})\rangle$$
$$\hat{C}\,|\bar{K}^0(s\bar{d})\rangle = (1)(-1)\,|K^0(d\bar{s})\rangle = -\,|K^0(d\bar{s})\rangle$$

Where we have used the convention that $C(q) = 1$ and $C(\bar{q}) = -1$. Also, the action of the $\hat{P}$ operator is given by:

$$\hat{P}\,|K^0(d\bar{s})\rangle = P(d)P(\bar{s})(-1)^l\,|K^0(d\bar{s})\rangle = (1)(-1)(-1)^0\,|K^0(d\bar{s})\rangle = -\,|K^0(d\bar{s})\rangle$$
$$\hat{P}\,|\bar{K}^0(s\bar{d})\rangle = P(s)P(\bar{d})(-1)^l\,|\bar{K}^0(s\bar{d})\rangle = (1)(-1)(-1)^0\,|\bar{K}^0(s\bar{d})\rangle = -\,|\bar{K}^0(s\bar{d})\rangle$$

Where we have used the convention $P(fermion) = 1$ and $P(anti-fermion) = -1$ as well as $l = 0$ because the Kaon is the lowest energy combination of these quarks and itself has a J^P of 0^-. Now the eigenstates of $\hat{C}\hat{P}$ can be determined:

$$\hat{C}\hat{P}\,|K^0\rangle = |\bar{K}^0\rangle$$
$$\hat{C}\hat{P}\,|\bar{K}^0\rangle = |K^0\rangle$$

So it is clear that any eigenfunction of the $\hat{C}\hat{P}$ operator will be a linear combination of the two Kaon states:

$$|K_1^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle) \tag{1}$$
$$|K_2^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle) \tag{2}$$

Where 1 and 2 are the usual labels given to these states. Now the action of $\hat{C}\hat{P}$ on these linear combinations can be determined:

$$\hat{C}\hat{P}\,|K_1^0\rangle = \frac{1}{2}(\hat{C}\hat{P}\,|K^0\rangle + \hat{C}\hat{P}\,|\bar{K}^0\rangle) = \frac{1}{2}(|\bar{K}^0\rangle + |K^0\rangle) = |K_1^0\rangle$$
$$\hat{C}\hat{P}\,|K_2^0\rangle = \frac{1}{2}(\hat{C}\hat{P}\,|K^0\rangle - \hat{C}\hat{P}\,|\bar{K}^0\rangle) = \frac{1}{2}(|\bar{K}^0\rangle - |K^0\rangle) = -\,|K_2^0\rangle$$

In experiment, two Kaon states are observed, a short lived state denoted by $|K_S^0\rangle$ and a relatively long lived state, $|K_L^0\rangle$. The lifetimes of these particles are $(8.954 \pm 0.004) \times 10^{-11}$ s and $(5.116 \pm 0.021) \times 10^{-8}$ s, respectively [2]. We make the natural assumption that these are the $\hat{C}\hat{P}$ eigenstates just derived and the identifications $|K_S^0\rangle = |K_1^0\rangle$ and $|K_L^0\rangle = |K_2^0\rangle$, to see what is predicted. If CP is conserved then all the decays of the $|K_S^0\rangle$ $(CP = 1)$ state must be to final products with $CP = 1$, and similarly, the decays of $|K_L^0\rangle$ $(CP = -1)$ must be to final products with $CP = -1$. The observed decays for these states are as follows [3, pg. 292]:

$$K_S^0 \to \pi^0\pi^0(B = 0.31), \quad K_S^0 \to \pi^+\pi^-(B = 0.69)$$
$$K_L^0 \to \pi^0\pi^0\pi^0(B = 0.20), \quad K_L^0 \to \pi^+\pi^-\pi^0(B = 0.13)$$

The reason for the difference in lifetimes of these two Kaon states is that the mass of the K_L^0 is not much bigger than the mass of three pions, thus it is relatively unlikey for it to undergo decay, compared to the K_S^0 which must only create energy to make two pions. The CP of these final states can now be determined. This is easy for the two pion final states. One finds:

$$P(\pi^0\pi^0) = (-1)(-1)(-1)^{l=0} = +1 \qquad \Rightarrow P = 1 \tag{3}$$
$$C(\pi^0\pi^0) = 1 \qquad \Rightarrow C = 1 \tag{4}$$
$$P(\pi^+\pi^-) = (-1)(-1)(-1)^{l=0} = +1 \qquad \Rightarrow P = 1 \tag{5}$$
$$C(\pi^+\pi^-) = (-1)^{l=0} \qquad \Rightarrow C = 1 \tag{6}$$

Thus $CP(\pi\pi) = 1$. For the three pion final state the second orbital angular momentum introduced by the third pion must be taken into account. The general formula for such a system is $P(ABC) = P(A)P(B)P(C)(-1)^{\mathbf{L}_{AB}}(-1)^{\mathbf{L}_{(AB)C}}$ where $\mathbf{L}_{AB}$ is the orbital angular momentum of the first two pions and $\mathbf{L}_{(AB)C}$ is the orbital angular momentum of the third pion with respect to the mutual centre of mass of the first two pions. The J^P of the Kaon is 0^-, thus the overall orbital angular momentum must be zero: $\mathbf{L} = \mathbf{L}_{AB} + \mathbf{L}_{(AB)C} = 0$. As this is angular momentum addition and $\mathbf{L}$ can only take positive values, it is clear that $L_{AB} = L_{(AB)C}$ so $L_{AB} + L_{(AB)C} = 2L$, which is an even number:

$$
\begin{aligned}
P(\pi^0\pi^0\pi^0) &= (-1)(-1)(-1)(-1)^{2L=even} = -1 && \Rightarrow P = -1 \\
C(\pi^0\pi^0\pi^0) &= (1)(1)(1) = 1 && \Rightarrow C = +1 \\
CP(\pi^0\pi^0\pi^0) &= -1
\end{aligned}
$$

For the $|\pi^+\pi^-\pi^0\rangle$ final state the parity is also -1, but the charge conjugation picks up an extra factor of $(-1)^l$ as in Eqn.(6). So if the centre of mass of pions A and B is taken to be the centre of mass between the π^+ and π^- one obtains:

$$
\begin{aligned}
C(\pi^+\pi^-\pi^0) &= C(\pi^0)(-1)^{L_{AB}} = 1 && \Rightarrow C = +1 \\
CP(\pi^0\pi^0\pi^0) &= -1
\end{aligned}
$$

Where we set $L_{AB} = 0$ as higher L values are much less likely [4]. Thus as long as the K_L^0 deacy to final states with three pions or other $CP = -1$ states and the K_S^0 only decay to two pion final states or other $CP = 1$ states, then CP is conserved.

This was thought to be the case until in 1964 when Christenson et al discovered the decay mode $K_L^0(CP = -1) \rightarrow \pi^+\pi^-(CP = 1)$ with a branching ratio of $(2.3 \pm 0.3) \times 10^{-3}$, thus discovering CP violation for the first time [1]. The experiment exploits the difference in lifetimes between K_S^0 and K_L^0. A 30GeV proton beam is incident on a metal target which creates a secondary beam of many different particles. The centre of mass energy for such an arrangement is 787 MeV, which is more than enough energy to produce a neutral Kaon having about a 497 MeV rest mass. The secondary beam is passed through a magnetic field to remove any charged particles and through a 4 cm thick block of lead to remove photons. At this point the beam contains both K_S^0 and K_L^0. The detecting aparatus is placed 18 m away from the metal target, so by the time the beam reaches it, all of the K_S^0 have decayed and only K_L^0 remain. The beam is further collimated and then undergoes collisions in a helium filled bag. Two arms containing a series of detectors are mounted symmetrically around the helium bag, so they both make the same angle with the horizontal. These arms consist of a spark chamber and magnet to determine the momentum and direction of an incident particle. Water Cherenkov and scintillation detectos act as a trigger by only recording events with two oppositely charged particles and a velocity of 0.75 c to eliminate background, see Fig.(2). The aim of the experiment is to measure the angular distribution of produced particles. The results of the experiment are shown in Fig.(3) where N is the number of counts and θ is the angle between the net momentum of the detected particles and the initial beam direction. These measurements were taken in various mass ranges, two are shown. If $K_L^0 \rightarrow \pi^+\pi^-$ is observed, the detected particles will have opposite signs, their invariant mass will match that of $K_L^0(497)$ and their net momentum will be in the same direction as the incident beam, hence the measured angle will be zero. The results show that a peak occurs at an angle of $0°$ in the correct mass range. This is clear evidence of the CP violating decay $K_L^0 \rightarrow \pi^+\pi^-$.

The results of the Christensen et al experiment implies, that the weak eigenstates $|K_S^0\rangle$ and $|K_L^0\rangle$ are not aligned with the true CP eigenstates $|K_1^0\rangle$ and $|K_2^0\rangle$. As in Eqn.(1) and (2) one can write:

$$|K_S^0\rangle = a\,|K_1^0\rangle + b\,|K_2^0\rangle \tag{7}$$

$$|K_L^0\rangle = a\,|K_1^0\rangle - b\,|K_2^0\rangle \tag{8}$$

Where a and b are complex numbers. The degree to which the states are not aligned is determined using the CP violation decay amplitudes and corresponding CP conserving amplitudes [6]:

$$
\begin{aligned}
\eta_{+-} &:= \frac{A(K_L^0 \rightarrow \pi^+\pi^-)}{A(K_S^0 \rightarrow \pi^+\pi^-)} = \epsilon + \epsilon' \\
\eta_{00} &:= \frac{A(K_L^0 \rightarrow \pi^0\pi^0)}{A(K_S^0 \rightarrow \pi^0\pi^0)} = \epsilon - 2\epsilon'
\end{aligned}
$$

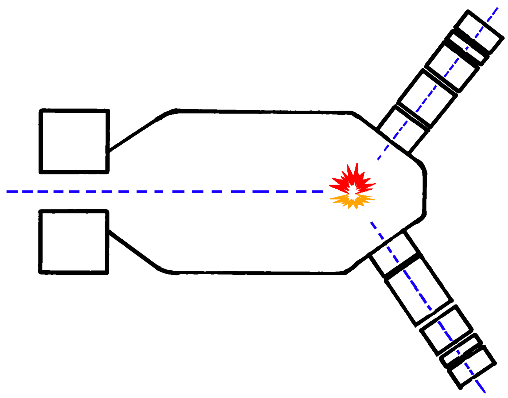

FIG. 2: *Apparatus used in the Christenson et al experiment [5]*

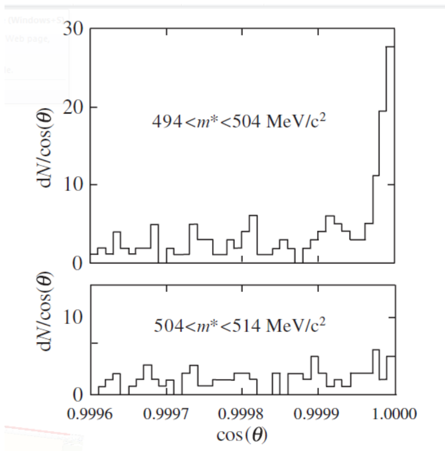

FIG. 3: *Results of the Christenson et al experiment [1]*

The two complex parameters ϵ and ϵ' determine the amount of indirect and direct CPV, respectively. The indirect CPV is due to the CP conserving decay of the $K_1^0(CP=1)$ component of the $K_L^0(CP=-1)$ to $CP=1$ final states, this is possible because of Kaon oscillations. The direct CPV is due to the CP violating decay of the $K_2^0(CP=-1)$ component of the $K_L^0(CP=-1)$ to $CP=1$ final states, this is possible due to interference between different decay methods with the same final state, as in Fig.(4).

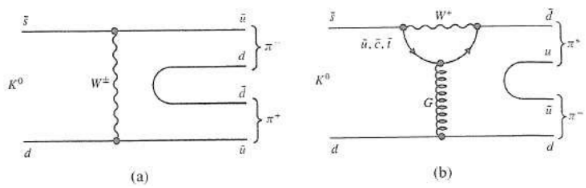

FIG. 4: *Two possible decay modes for $K^0 \to \pi^+\pi^-$. (a) Tree diagram for decay by exchanging a W boson (b) Penguin diagram for decay via quark states [7]*

However it is found that the direct CPV contribution is much smaller in this case. The indirect CPV almost completely dominates as can be seen from the similarity of the experimental values for $|\eta_{+-}|$ and $|\eta_{00}|$ [2]:

$$|\eta_{00}| = 0.002220 \pm 0.000011 \qquad |\eta_{+-}| = 0.002232 \pm 0.000011$$

If these values were significantly different it would suggest the amount of direct CPV would be comparable to the amount of indirect CPV, this of course is not the case. An experimentally determined value which illustrates this is the real part of the ratio of ϵ' to ϵ [2]:

$$\mathbb{R}\left(\frac{\epsilon'}{\epsilon}\right) = \left(1 - \left|\frac{\eta_{00}}{\eta{+-}}\right|\right)/3 = 0.00166 \pm 0.00023$$

$|\epsilon|$ can also be determined using:

$$|\epsilon| = \big(2|\eta_{+-}| + |\eta_{00}|\big)/3 = 0.002228 \pm 0.000011$$

If the direct CPV contributions are ignored one can write Eqn.(7) and (8) in terms of ϵ:

$$|K_L^0\rangle = \frac{1}{(1+|\epsilon|^2)^{1/2}}\Big[\epsilon\,|K_1^0\rangle + |K_2^0\rangle\Big] \tag{9}$$

$$|K_S^0\rangle = \frac{1}{(1+|\epsilon|^2)^{1/2}}\Big[|K_1^0\rangle - \epsilon\,|K_2^0\rangle\Big] \tag{10}$$

This linear combination shows the non-zero amplitude for weak eigenstate Kaons to oscillate between two different states with definite and opposite CP.

B. Semi-leptonic decays

Decays of neutral Kaons to products containing leptons can be used to verify Eqn.(9) and (10) as well as finding the asymmetry in the Kaon oscillation $K^0 \leftrightarrow \bar{K}^0$. First the selection rules that play an important role in these decays must be discussed.

The $\Delta S = \Delta Q$ selection rule is an empirical rule backed up by some theoretical approximations. This rules states that in decays involving strangeness(S) and leptons, the change in the charge(Q) of the hadrons must be the same as the change in strangeness which must have a value of ± 1. As an example consider semi-leptonic decays of the charged Σ baryon. Two semi-leptonic decays of this baryon are:

$$\Sigma^-(dds) \rightarrow n(udd) + e^- + \bar{\nu}_e \tag{11}$$

$$\Sigma^+(uus) \rightarrow n(udd) + e^+ + \nu_e \tag{12}$$

The feynmann diagram for decay (11) can be drawn as in Fig.(5), while decay (12) requires a diagram which must have at least two W bosons. It is clear that the diagram for Σ^- is quite likely as it contains the Cabbibo favoured quark coupling V_{ud} while any digram with two W bosons is unlikely, as for the Σ^+ decay. For this reason it is highly suppressed and has a braching ratio of $< (5 \times 10^{-6})$, which is consitent with it not existing in nature [2]. In comparrison the decay (11) has a branching ratio of $(1.017 \pm 0.034) \times 10^{-3}$. As there is no selection rule forbidding the second decay, the $\Delta S = \Delta Q$ rule was introduced to identify proceses like it. The change in strangeness and hadron charge for these decays can be found in Table.(I).

TABLE I: $\Delta S = \Delta Q$ *selection rule table for the decays (11) and (12)*

Shown decay of	ΔS	ΔQ	$\Delta S = \Delta Q$
Σ^-	+1	+1	yes
Σ^+	+1	-1	no

Where ΔA is the difference between the final and initial states of A such that $\Delta A = A_{final} - A_{initial}$. Also remember that the definition of strangeness assigns the strange quark a value of -1 and the anti-strange quark a value of $+1$.

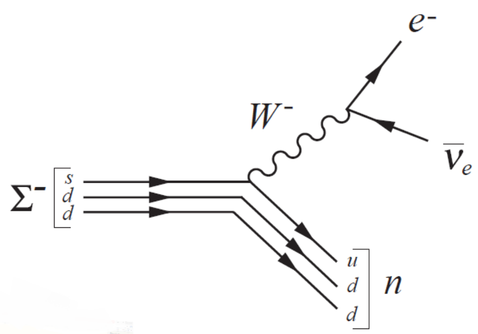

FIG. 5: *Feynmann diagram for the $\Delta s = \Delta Q$ allowed decay of the Σ^- boson to semi-leptonic final products*

Thus it is clear that the decay (12) violates the $\Delta S = \Delta Q$ selection rule. Similarly, a decay with $\Delta S = \pm 2$ will contain two W bosons and as a result will be very suppressed. In conclusion, $\Delta S = \Delta Q = \pm 1$ for an allowed process.

This can now be applied to semi-leptonic decays of Kaons of the form $K \to \pi l \nu_l$. There are four Kaon decays that have this form:

$$K^0(d\bar{s}) \to \pi^- l^+ \nu_l \tag{13}$$
$$\bar{K}^0(s\bar{d}) \to \pi^+ l^- \bar{\nu}_l \tag{14}$$
$$K^0(d\bar{s}) \to \pi^+ l^- \bar{\nu}_l \tag{15}$$
$$\bar{K}^0(s\bar{d}) \to \pi^- l^+ \nu_l \tag{16}$$

A similar table as before can now be constructed. See Table.(II), where the number shown refers to the equations above

TABLE II: $\Delta S = \Delta Q$ *selection rule table for the decays (13) - (16)*

Shown decay of	ΔS	ΔQ	$\Delta S = \Delta Q$
(13)	-1	-1	yes
(14)	+1	+1	yes
(15)	-1	+1	no
(16)	+1	-1	no

Thus it is clear that the only possible semi-leptonic decays of this form for K^0 and $\bar{K}^0$ are (13) and (14). As there is only one way for these processes to occur, there can be no interference between different processes and thus there can be no direct CP violation in the semi-leptonic decays of Kaons [8, pg. 10]. So it is clear that the amplitudes of the $\Delta S = \Delta Q$ violating decays are:

$$A(K^0 \to \pi^+ l^- \bar{\nu}_l) = A(\bar{K}^0 \to \pi^- l^+ \nu_l) = 0$$

It is possible now to write these amplitudes in terms of K^0_S and K^0_L. Using Eqn.(1),(2),(9) and (10) one finds[8, pg. 11]:

$$A(K^0_S \to \pi^+ l^- \bar{\nu}_l) = -A(K^0_L \to \pi^+ l^- \bar{\nu}_l) = \frac{1-\epsilon}{\sqrt{2}} A(\bar{K}^0 \to \pi^+ l^- \bar{\nu}_l)$$
$$A(K^0_S \to \pi^- l^+ \nu_l) = A(K^0_L \to \pi^- l^+ \nu_l) = \frac{1+\epsilon}{\sqrt{2}} A(K^0 \to \pi^- l^+ \nu_l)$$

Where terms of order $|\epsilon|^2$ have been neglected. The quantities $\delta_{L,S}$ can now be defined, which show the tendency for the oscillations $K^0 \leftrightarrow \bar{K}^0$ to favour the matter particle state. Thus making a very small contribution to the matter anti-matter assymmetry.

$$\delta_{L,S} = \frac{A(K^0_{L,S} \to \pi^- l^+ \nu_l) - A(K^0_{L,S} \to \pi^+ l^- \bar{\nu}_l)}{A(K^0_{L,S} \to \pi^- l^+ \nu_l) + A(K^0_{L,S} \to \pi^+ l^- \bar{\nu}_l)} := 2\mathbb{R}(\epsilon)$$

The experimental value for this quantity is $\delta_L = (3.27 \pm 0.12) \times 10^{-3}$. Which clearly indicates a small tendency to favour the matter particle in oscillations. This can also be illustrated graphically by measuring the relative number(N) of K^0 and $\bar{K}^0$ particles over time in a beam consisting initially of K^0. See Fig.(6)

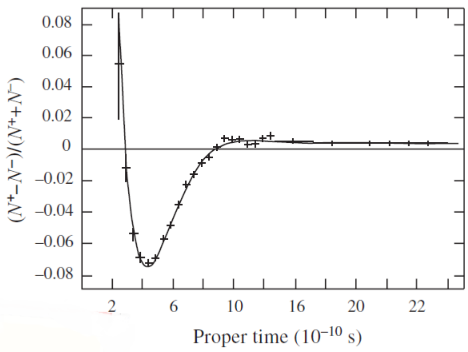

FIG. 6: *Meaurements of observed Kaon semi-leptonic decays from a beam initially consisting of K^0 mesons which shows the oscillation between the states K^0 and $\bar{K}^0$ as well as the small asymmetry, favouring the matter particle [9]*

C. Testing CPT conservation through Strangenss Oscillation

The CPT theorem links conservation of CPT with Lorentz invariance. Thus to preserve the fundamental Lorentz symmetry physicists desperately hope CPT is conserved. Here is presented some evidence that this is indeed the case. The theorem requires particles and their anti-particles to have the same masses and lifetimes. It has been stated previously that the K^0_L and K^0_S particles have very different lifetimes, but thankfully these are not a particle anti-particle pair. From earlier it is clear that K^0 and $\bar{K}^0$ are such a pair. Thus we aim to test CPT conservation by measuring their mass difference.

By investigating the time evolution of Kaon oscillations it is possible to measure their mass difference. By inverting the linear combinations in Eqn.(1) and (2) and neglecting the very small contribution of ϵ, the flavour eigenstates are described by:

$$|K^0(t)\rangle = \frac{1}{\sqrt{2}}(|K^0_S(t)\rangle + |K^0_L(t)\rangle)$$
$$|\bar{K}^0(t)\rangle = \frac{1}{\sqrt{2}}(|K^0_S(t)\rangle - |K^0_L(t)\rangle)$$

The time evolutions of the states are then written in terms of the mass and the lifetimes of the particles

$$|K^0_S(t)\rangle = |K^0_S(0)\rangle\, e^{-(im_S + \Gamma_s/2)t}$$
$$|K^0_L(t)\rangle = |K^0_L(0)\rangle\, e^{-(im_L + \Gamma_L/2)t}$$

Where the exponential factor is as a result of the particle oscillations with time, and the fact that the particle will decay in time. We do the calculation for the $\bar{K}^0$ and simply state the result for the K^0. The probability amplitude(A) for the oscillations and then the probability of decay are determined using the linear combination:

$$|\bar{K}^0(t)\rangle = \frac{1}{\sqrt{2}}(|K_L^0(0)\rangle\, e^{-(im_L+\Gamma_L/2)t} - |K_S^0(0)\rangle\, e^{-(im_S+\Gamma_s/2)t}) \tag{17}$$

$$\bar{A} = \frac{1}{2}(e^{-(im_L+\Gamma_L/2)t} - e^{-(im_S+\Gamma_s/2)t}) \tag{18}$$

$$P(\bar{K}^0) = |\bar{A}|^2 = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos(t\Delta m)\right] \tag{19}$$

$$\tag{20}$$

Where $\Delta m = |m_S - m_L|$. The extra factor of $1/\sqrt{2}$ comes from the initial condition that the experiment is started with a beam of K^0 particles which is equal parts K_L^0 and K_L^0. Thus $|K_L^0(t=0)\rangle = |K_S^0(t=0)\rangle = 1/\sqrt{2}$. The corresponding probability for K^0 is as follows:

$$P(K^0) = |A|^2 = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos(t\Delta m)\right] \tag{21}$$

For this experiment, the initial beam of Kaons is "flavour tagged". This is done by producing the K^0 particles in a strangeness conserving strong decay. The technique of tagging will be discussed further in section [JOHNS SECTION ON B]. The strangeness of the final state particles is then determined by looking for semi-leptonic decays discussed in section IV B. The oscillations in time are made clear by plotting Eqn.(19) and (21) in Fig.(7). The decay rates for K_S^0 and K_L^0 are known so the results of this experiment can be used to determine Δm for the weak eigenstate Kaons. This value is $\Delta m = (3.483 \pm 0.006) \times 10^{-12}$.

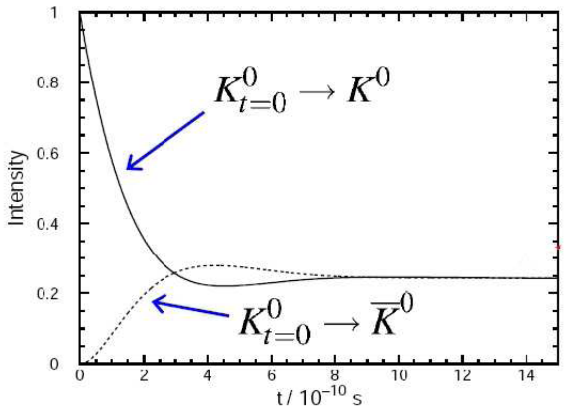

FIG. 7: *Theoretical predictions of the strangeness oscillations of a beam initially consisting of K^0 particles [10]*

To find $\Delta m_{flavour} = |m_{K^0} - m_{\bar{K}^0}|$ the link between Δm above must be determined. The time dependent asymmetry in this system is given by:

$$A_{CPT} = \frac{P[\bar{K}^0 \to \bar{K}^0(t)] - P[K^0 \to K^0(t)]}{P[\bar{K}^0 \to \bar{K}^0(t)] + P[K^0 \to K^0(t)]} = 4\mathbb{R}(\delta)$$

Where δ is a CPT violation parameter which can be written in terms of its projections parallel and perpendicular to the super weak direction $\phi_{SW} = \tan^{-1}(2\Delta m/\Delta\Gamma)$ [2]:

$$\delta_\parallel = \frac{1}{4}\frac{\Delta\Gamma_{flavour}}{\sqrt{\Delta m^2 + \left(\frac{\Delta\Gamma}{2}\right)^2}} \tag{22}$$

$$\delta_\perp = \frac{1}{2}\frac{\Delta m_{flavour}}{\sqrt{\Delta m^2 + \left(\frac{\Delta\Gamma}{2}\right)^2}} \tag{23}$$

Thus it is possible to determine $\mathbb{R}(\delta)$ in this experiment. Using other methods and other experiments $\mathbb{I}m(\delta)$ can be measured. So $\delta_{\parallel}$ and $\delta_{\perp}$ can be determined. Thus as shown in Eqn.(23) $\Delta m_{flavour}$ can be determined. The current best result for this quantity is [2]:

$$\frac{\Delta m_{flavour}}{m_{av}} < 6 \times 10^{-19}$$

which is consitent with zero. Thus this experiment gives some confidence to CPT conservation and the preservation of Lorentz invariance.

V. CPV IN D-MESON SYSTEM

The quark constituents of the $D^0(1865)$ and $\bar{D}^0(1865)$ mesons are $(c\bar{u})$ and $(u\bar{c})$, respectively. This system is unique as it is the only system which undergoes mixing and contains an up-type quark. As opposed to the K^0,B^0 and B_S, which contain down quarks. This results in different quarks in the mixing box diagrams of these processes, which are illustrated in Fig.(**??**) and (8). The rates for D^0 mixing are expected to be very small as the mixing process shown is suppressed in two ways. If the intermediate quark is a b, then the decay is doubly Cabbibo suppressed[explain? or has it been explained already?], while if the quark is a d or an s then the process is GIM suppressed[Reference John]. Other processes which may not have the same degree of suppression have been proposed, but there are large incertainties in the theoretical calculations of their decay rates [14].

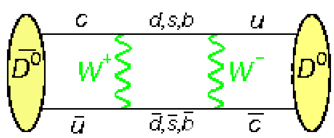

FIG. 8: *Feynmann diagram showing the process by which the two D^0 states mix. This process is the only known mixing process which contain the d,s,b quarks in this position [13]*

The first stage in detecting CPV in any system is to find mixing between a particle and its anti-particle. As in the case of Kaon and B-meson mixing we define the CP eigenstates of the D-meson to be linear combinations of flavour eigenstates.

$$|D^0_{1,2}\rangle = p\,|D^0\rangle \pm q\,|\bar{D}^0\rangle \tag{24}$$

Where for normailzation $|p|^2 + |q|^2 = 1$. In the absence of CPV the $|D_1\rangle$ state is a CP even state while the $|D_2\rangle$ state is a CP odd state. As expected, we will see CPV in mixing if $|p| \neq |q|$. Clear evidence for mixing between these states was announced in 2007 and published in 2008 by the BaBar collaboration, followed shortly by the Belle collaboration [11][12]. Results from both experiments show a small amount of D^0 mixing with 3.9 σ certainty, at a level which is consistent with SM predictions in the order of $|x|, |y| \leq \times 10^{-2}$, see Eqn.(25) [14]. However, measured CP violating parameters were consistent with zero, and thus with no CPV.

Two decays and their corresponding anti-particle decays are important for the measuremnt of mixing in the D-meson system. The doubly Cabbibo suppressed (DCS) $D^0 \to K^+\pi^-$ known as the wrong sign (WS) decay and $D^0 \to K^-\pi^+$ Cabbibo favoured (CF) decay called the right sign (RS), are used. Two parameters which determine the amount of mixing in a system are defined as:

$$x = \frac{\Delta M}{\Gamma} \qquad\qquad y = \frac{\Delta\Gamma}{2\Gamma} \tag{25}$$

where $M = (M_1 + M_2)/2$ is average mass,$\Gamma = (\Gamma_1 + \Gamma_2)/2$ is average lifetime and $\Delta A := A_2 - A_1$. An approximation to the time dependence of the WS decay in the absence of CPV is given by [11]:

$$\frac{T_{WS}(t)}{e^{-\Gamma t}} \propto R_D + \sqrt{R_D} y \Gamma t + \frac{x'^2 + y'^2}{4} (\Gamma t)^2$$
$$x' = x\cos(\delta) + y\sin(\delta)$$
$$y' = y\cos(\delta) - x\sin(\delta)$$

Where R_D is the ratio of the amplitudes of the DCS decay to to CF deacy and δ is the strong phase difference between the DCS and CF decays. If there is no CPV then we would expect $x' = x$ and $y' = y$. By measuring this time dependence and fitting the results to this formula, it is possible to compare predicted values of x' and y' from various theoretical models and see which best fits the data. Fig.(9) shows the results of the first experiment at BaBar while Fig.(10) shows more recent results from LHCb with a confidence of 5σ. Flavour tagging of the D^0 is used in these experiments. The sign of a pion known as the "slow pion" from the decay $D^{0*} \to D^0\pi_s^+$ is compared to the sign of the final product Kaon. Where D^{0*} is a heavier, and thus more energetic version of the D^0 meson. If the signs are the same, the decay is WS, if they are opposite then the decay is RS. Misidentifying a random pion - not from the D^{0*} decay - as the slow pion causes events which do not contain D^0 decays to be included in analysis. This creates a background which obstructs the signal data. Other sources of background are misreconstructed D^0 and combinatorial sources. Misreconstruction is due in part to semi-leptonics decays of the D^0 or $\bar{D}^0$ in which the detector has misidentified a lepton as a pion. Combinartorial background is caused by D-mesons being produced not from D^{0*}, but from various possible decays of a B-meson. All of these backgrounds are reduced and excluded from the signal decays by making offline cuts to various parameters. These parameters include the the χ^2 of the track, vertex and impact parameters of the partilces, the momentum and the mass as well as many others. Mass is plotted in two ways, the reconstructed D^0 mass distribution ($m_{K\pi}$) and the mass difference between the reconstructed D^{0*} and the D^0 mass (Δm) [15]. Signal events are identifed by a mass peak in the correct place in both $m_{K\pi}$ and (Δm), random pion background has a peak in $m_{K\pi}$ but no peak in (Δm), and vise versa for misreconstracted particles. Combinatorial background has no peak in either mass distribution. These techniques are of course universal to most particle physics experiemnts.

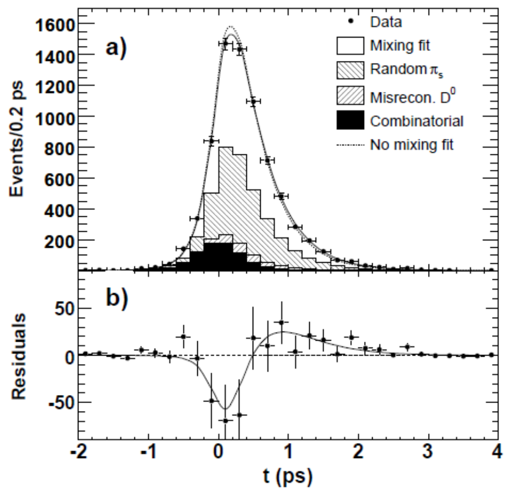

FIG. 9: *Results from the first D^0 mixing experiments at BaBar, which plots the time distribution of WS decays. It is clear that the data best fits the mixing hypothesis. Backround contributions from wrongly identified π_s^+, misconstructed D^0 decays and combinatorial contributions from D^0 production from B^0 decays are removed*

CPV in mixing in this system is described by the parameter A_m defined by:

$$A_m = \frac{|q/p|^2 - |p/q|^2}{|q/p|^2 + |p/q|^2}$$

Where p and q are the coeffiiencts in the linear combinations 24. If this value is found to not be zero then CPV in D-meson mixing will be proved. Similarly for direct CPV we define:

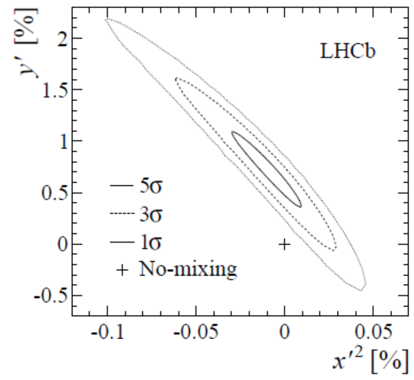

FIG. 10: *2013 Results from LHCb of D^0 mixing with a confidence of 5σ. This is the first conclusive evidence for D^0 made by one experiemnt. The cross marks the no-mixing values of x' and y'*

$$A_d = \frac{|A_f|^2 - |\bar{A}_f|^2}{|A_f|^2 + |\bar{A}_f|^2}$$

Where A_f is the decay amplitude of D^0 to some final state f, and $\bar{A}_f$ is the decay amplitude of $\bar{D}^0$ to the state $\bar{f}$. These two parameters can be combined to construct the quantity λ_f defined by [17]:

$$\lambda_f = \frac{q\bar{A}_f}{pA_f} = -\eta_{CP}\left|\frac{q}{p}\right|\left|\frac{\bar{A}_f}{A_f}\right|e^{i\phi}$$

Where η_{CP} is the CP eigenvalue of the state f and ϕ is the phase between q/p and $\bar{A}_f A_f$ and is chosen by convention so that $|D_1\rangle$ is an even CP eigenstate. To date there has been no evidence for CPV in the D-meson system. Measurements are ongoing at LHCb and if the addition of the 2013 data does not find evidence for CPV then we must wait till the restart of the LHC in 2015. The mush larger luminosity expected after the upgrade will hopefully supply the necessary statistics to conclusively measure a non-zerp value of one of the above parameters.

Appendix A: Appendix

Difficult calculations in here.

[1] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay (1964) Phys. Review letters, vol. 13, issue 4 - "Evidence for the 2π Decay of the K_2^0 Meson"
[2] J. Beringer et al. (Particle Data Group), Phys. Rev. D86, 010001 (2012) and 2013 partial update for the 2014 edition - "2013 Review of Particle Physics"
[3] B.R Martin, G. Shaw, Wiley(2008) - "Particle Physics, Third Edition"
[4] Tatsuya Nakada arXiv:hep-ph/9312290v1 15 Dec 1993 - "Review on CP Violation"
[5] http://large.stanford.edu/courses/2008/ph204/coleman1/
[6] KTeV Collaboration, arXiv:hep-ex/0208007v1 6 Aug 2002 - "Measurements of Direct CP Violation, CPT Symmetry, and Other Parameters in the Neutral Kaon System"
[7] Donald H. Perkins, Cambridge University Press, 4th edition - "Introduction to High Energy Physics"
[8] Giancarlo DAmbrosio and Gino Isidori, arXiv:hep-ph/9611284v1 8 Nov 1996 - "CP violation in Kaon Decays"
[9] Gjesdal, S. et al. Phys.Lett. B52 (1974) 113 Print-74-1358 (CERN) - "A Measurement of the K(L)-K(s) Mass Difference from the Charge Asymmetry in Semileptonic Kaon Decays"
[10] http://www.hep.phy.cam.ac.uk/ thomson/partIIIparticles/handouts/Handout_12_2011.pdf
[11] The BaBar Collaboration http://arxiv.org/abs/hep-ex/0703020v1 1 Apr 2007 - "Evidence for $D^0\bar{D}^0$ Mixing"

[12] The Belle Collaboration arXiv:hep-ex/0703036v2 1 Apr 2007 - "Evidence for $D^0\bar{D}^0$ Mixing"
[13] http://inspirehep.net/record/1209003/files/mix-rick.png
[14] Giulia Casarosa for the BABAR Collaboration - "Studies of CP Violation and Mixing in the D Mesons decays from BABAR" - Journal of Physics: Conference Series 335 (2011) 012043 doi:10.1088/1742-6596/335/1/012043
[15] Kevin Maguire, C. Parkes, M. Vesterinen - "New tagged $D^0 \to hh\pi^0$ stripping lines" - LHCb-INT-2013-049 October 1, 2013
[16] The BaBar Collaboration - "Improved measurement of the CKM angle γ in $B^{\pm} \to D^{(*)}K^{(*)\mp}$ decays with a Dalitz plot analysis of D decays to $K^0_S\pi^+\pi^-$ and $K^0_S K^+ K$" - arXiv:0804.2089v2 [hep-ex] 7 Aug 2008
[17] The LHCb Collaboration - "Measurements of indirect CP asymmetries in $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$"- arXiv:1310.7201v1 [hep-ex] 27 Oct 2013
[18] François Goffinet, CP3 Seminar, Unitè de Physique Thèorique et Mathèmatique, U.C.L., December 2003
[19] Yoav Achiman, "Spontaneous CP Violation in SUSY," Physics Letters B, March 2007
[20] Peter Szekeres, "A Course in Modern Mathematical Physics," Cambridge University Press, 2004
[21] John Baez and John Huerta, "The Algebra of Grand Unified Theories," Bulletin of the American Mathematical Society, vol 47, May 4 2010
[22] Howard E. Haber and Ze'ev Surujon, "Group-theoretic Condition for Spontaneous CP Violation," Physical Review D, Volume 86, Issue 7, October 2012
[23] Bernard de Wit and Eric Laenen, "Field Theory in Particle Physics" Lecture Notes, Universiteit Utrecht, 2009 http://www.staff.science.uu.nl/ wit00103/ftip/Ch11.pdf
[24] Paul H. Frampton and Masayasu Harada, "Kaon Spontaneous CP Violation Reevaluated," Physical Review D, Volume 59, Issue 1, March 1998