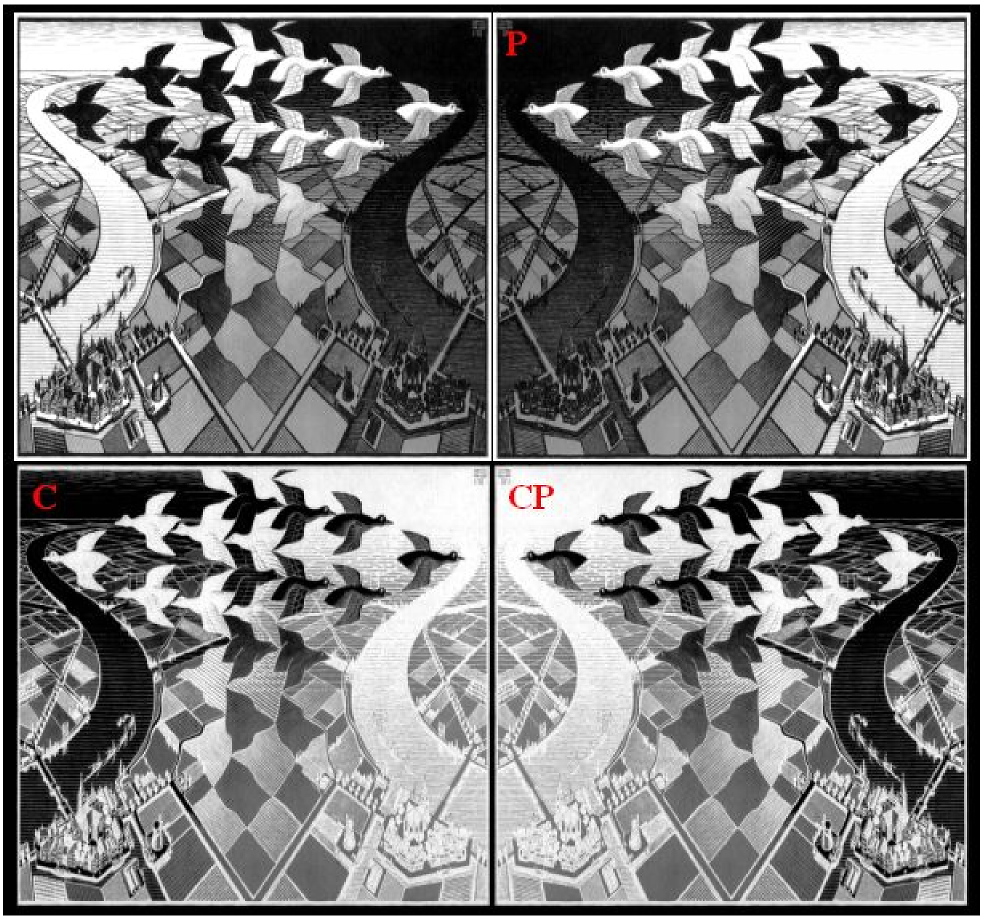
Review of CP Violation, with Group Formalism and the Universe

John Ronayne (10318997), Kevin Maguire (10318135), Sinead Hales(10318221), Dudley Grant (10275291)
(Dated: November 21, 2013)

Discussed in this review is the theoretical background for CP violation. This is conventionally explained through the mechanism of the CKM matrix. The mixing of quarks leads to complex phases that contribute to the so called unitary triangles. The three angles α ,β and γ that these triangles are composed of are measurable and provide information on the scale of CP violation. The CKM mechanism is a basis for calculations of Weak interactions and developments of deeper understandings of CP violation in the Standard Model. Presented are the current and expected experimental verifications in three systems of particles, neutral Kaons, B mesons and in the charm sector with D mesons. In order to explain the observed antimatter asymmetry of the universe various new models of CP violation have been created, the most popular being Super-symmetric or Spontaneous CP violation. The gauge-symmetries of these theories can be examined abstractly to determine which types of CP violation are available.



Contents

I. Introduction 2
A. The Parity Operator 2
B. The Charge Operator 3
C. The Time Reversal Operator 4
D. CPT Theorem 5
E. CP and Conservation 5

II. The CKM Mechanism 6

I. INTRODUCTION

A. The Parity Operator

Dudley Grant

The **parity operator**, $\hat{P}$, refers to a specific spatial reflection defined for a single particle by

$$\hat{P}\psi(\mathbf{r},t) = P\psi(-\mathbf{r},t)$$

Where $\psi(\mathbf{r},t)$ is the spatial representation of the time-evolving state $|\psi(t)\rangle$. The spatial reflection in Cartesian coordinates has matrix representation

$$\mathsf{M}_{\mathsf{Ref}}\mathsf{x} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix}$$

This corresponds to reflection along the plane orthogonal to the vector $\mathbf{r} = (x, y, z)$ and centred at the origin. It shall be shown $\mathsf{M}_{\mathsf{Ref}} \in \mathrm{O}(3)$, and it forms finite a symmetry subgroup $\{\mathsf{I}, \mathsf{M}_{\mathsf{ref}}\}$, where I is the identity matrix. This natural mathematical framework gives initial meaning to the parity operator forming a symmetry group, although really it shall be a symmetry of a Lagrangian. In this group $\mathsf{M}_{\mathsf{Ref}}$ must be self-inverse as can easily be understood by ${\mathsf{M}_{\mathsf{Ref}}}^2 = \mathsf{I}$. Using this

$$\begin{aligned} \hat{P}^2\phi(\mathbf{r},t) &= P\hat{P}\phi(\mathsf{M}_{\mathsf{Ref}}\mathbf{r},t) \\ &= P^2\phi({\mathsf{M}_{\mathsf{Ref}}}^2\mathbf{r},t) \\ &= P^2\phi(\mathsf{I}\mathbf{r},t) \\ &= P^2\phi(\mathbf{r},t) \end{aligned}$$

$P^2 = 1$ since normalisation is desired and $\phi(\mathbf{r},t)$ is normalised. Since $\hat{P}$ is chosen to be a Hermitian operator because we want to be able to observe its eigenvalues, its eigenvalues are real. So for an eigenstate, $\hat{P}\,|\phi\rangle = P\,|\phi\rangle$, $P = \pm 1$. For a system of n particles each in state $|\psi_i\rangle$ this definition can be extended naturally by

$$\hat{P}\,(|\psi_1\rangle \otimes |\psi_2\rangle \otimes ... \otimes |\psi_n\rangle) := (\hat{P}\,|\psi_1\rangle) \otimes (\hat{P}\,|\psi_2\rangle) \otimes ... \otimes (\hat{P}\,|\psi_n\rangle)$$

In position representation this reads more intuitively as

$$\hat{P}\psi(\mathbf{r}_1, \mathbf{r}_2, ..., \mathbf{r}_n, t) = P_1 P_2 ... P_n \psi(-\mathbf{r}_1, -\mathbf{r}_2, ..., -\mathbf{r}_n, t)$$

Where $\mathbf{r}_i$ correspond to the spatial positions of each particle. If a Lagrangian is invariant under $\hat{P}$, that is $\mathcal{L}(\psi, \nabla\psi, x^i)$ returns the same solution as $\mathcal{L}(\hat{P}\psi, \nabla\hat{P}\psi, x^i)$, then parity is said to be conserved. This is not the case for the Standard Model. Parity may not be conserved in weak interactions.

Looking at energy eigenstates in spherical coordinates spherical harmonics, $Y_l^m(\theta,\phi)$, may be used. This gives $\phi_{nlm}(\mathbf{r}) = R_{nl}(|\mathbf{r}|)Y_l^m(\theta,\phi)$. These are essentially the fourier modes in spherical coordinates. The parity transformation in spherical coordinates does not effect the radial distance, only the two angles. Some geometric reasoning shows that

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \mapsto \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix} \qquad \Leftrightarrow \qquad \begin{pmatrix} r \\ \theta \\ \phi \end{pmatrix} \mapsto \begin{pmatrix} r \\ \pi-\theta \\ \pi+\phi \end{pmatrix}$$

From a standard textbook on special functions it can be shown

$$Y_m^l(\theta,\phi) \mapsto Y_m^l(\pi-\theta, \pi+\phi) = (-1)^l Y_m^l(\theta,\phi)$$

For a free particle this representation is of no use. For bound systems, such as the hydrogen atom or mesons, it greatly simplifies calculation. Consider a system composed of two particles and write the effect of the parity operator on its spherical Fourier modes.

$$\begin{aligned} \hat{P}(|\phi_1\rangle \otimes |\phi_2\rangle) &= (\hat{P}\,|\phi_1\rangle) \otimes (\hat{P}\,|\phi_2\rangle) \\ &= (\hat{P}R_{n_1l_1}(r_1)Y_{m_1}^{l_1}(\theta_1,\phi_1)) \otimes (\hat{P}R_{n_2l_2}(r_2)Y_{m_2}^{l_2}(\theta_2,\phi_2)) \\ &= ((-1)^{l_1}R_{n_1l_1}(r_1)Y_{m_1}^{l_1}(\theta_1,\phi_1)) \otimes ((-1)^{l_2}R_{n_2l_2}(r_2)Y_{m_2}^{l_2}(\theta_2,\phi_2)) \\ &= (-1)^{l_1+l_2}\,|\phi_1\rangle \otimes |\phi_2\rangle \end{aligned}$$

So the parity of a spherical harmonic mode may be deduced by the total angular momentum of the system. What is left is to relate this to particle physics. This may be done by defining instrinsic parity.

A Fourier mode in Cartesian coordinates may be written in position representation as follows

$$\psi_{\mathbf{p}}(\mathbf{r},t) = e^{\frac{i}{\hbar}(\mathbf{p}\cdot\mathbf{r} - iEt)}$$

This state is not physical for it is non-normalisable. $\mathbf{p}$ has interpretation as momentum of the particle. This can be checked by applying the momentum operator. Consider the parity operator's effect

$$\begin{aligned}\hat{P}\psi_{\mathbf{p}}(\mathbf{r},t) &= Pe^{\frac{i}{\hbar}(\mathbf{p}\cdot(-\mathbf{r}) - iEt)}\\ &= Pe^{\frac{i}{\hbar}((-\mathbf{p})\cdot\mathbf{r} - iEt)}\\ &= P\psi_{-\mathbf{p}}(\mathbf{r},t)\end{aligned}$$

For $\mathbf{p} = 0$ this is an eigenvalue equation. In that case, P is called the **intrinsic parity** of a particle. $\mathbf{p} = 0$ may be interpreted as the particle being at rest, but as this is a non-normalisable mode it does not make sense: By the Heisenberg uncertainty principle, a quantum mechanical particle can not have an exact momentum.

In order for the Dirac equation to be symmetric under $\hat{P}$ it turns out that for an electron-positron system

$$\hat{P}(\psi_{e^-}(\mathbf{r}_-) \otimes \psi_{e^+}(\mathbf{r}_+)) = -1(\psi_{e^-}(-\mathbf{r}_-) \otimes \psi_{e^+}(-\mathbf{r}_+))$$

Now as

$$\begin{aligned}\hat{P}(\psi_{e^-}(\mathbf{r}_-) \otimes \psi_{e^+}(\mathbf{r}_+)) &= (\hat{P}\psi_{e^-}(\mathbf{r}_-)) \otimes (\hat{P}\psi_{e^+}(\mathbf{r}_+))\\ &= (P_{e^-}\psi_{e^-}(-\mathbf{r}_-)) \otimes (P_{e^+}\psi_{e^+}(-\mathbf{r}_+))\\ &= P_{e^-}P_{e^+}(\psi_{e^-}(-\mathbf{r}_-) \otimes \psi_{e^+}(-\mathbf{r}_+))\end{aligned}$$

This implies $P_{e^-}P_{e^+} = -1$, so depending on convention $P_{e^-} = \pm 1$ and $P_{e^+} = \mp 1$. The standard convention is to denote matter parity by 1 and antimatter parity by -1, so $P_{e^-} = +1$ and $P_{e^+} = -1$. It turns out this holds for any spin-$\frac{1}{2}$ particles. Using this and the spherical harmonics defined above an equation for a bound system of particles may be deduced.

First define a function $\mathsf{P} : \mathcal{H} \to \{-1, +1\}$. This function takes a state from the overall Hilbert-space of the system considered and returns the parity. For example in the electron-positron system $\mathsf{P}(|e^-\rangle \otimes |e^+\rangle) = -1$. Often this is written with the same P as the parity operator as an accepted abuse of notation. In general for a bound system of n states $|\psi_i\rangle$

$$\mathsf{P}\left(\bigotimes_{i=1}^{n} |\psi_i\rangle\right) = (-1)^{\sum_i l_i} \prod_{i=1}^{n} \mathsf{P}(|\psi_i\rangle)$$

where $\bigotimes_i |\psi_i\rangle$ is shorthand for $|\psi_1\rangle \otimes |\psi_2\rangle \otimes ... \otimes |\psi_n\rangle$. Now, denoting the total angular momentum $\sum_{i=1}^{n} l_i$ by l and omitting the tensor product of states of particles this reduces to

$$\mathsf{P}\left(|p_1 p_2 ... p_n\rangle\right) = (-1)^l \prod_{i=1}^{n} \mathsf{P}(|p_i\rangle)$$

Where p_i is short for the state of the i^{th} particle. Note that this is only true of bound states, just as free states of the hydrogen atom exist with non-discrete possible orbital angular momentum.

B. The Charge Operator

Dudley Grant

The **charge operator**, $\hat{C}$, changes a particle to its antiparticle. Intuitively, picture a particle in a certain state in the Hilbert space of states. Classically this is analogous to phase space which is the geometric space of all possible positions and velocities the particle can take, $(q^i, \dot{q}^i)$. When changed to a positron, an electron moving in phase space keeps the same position and momentum, but simply changes its sign. In other words classically all $\hat{C}$ changes the charge of a particle, but the particle keeps its direction of motion.

Consider a free electron orbiting a central positive charge, freeze this at one instant and replace with a positron. What happens? The positron accelerates away from the centre but it still keeps the tangent velocity it had originally.

In Hilbert space it is quite similar except with the addition of probabilities. If the particle is very likely to move in the $\mathbf{e_x}$ direction and $\hat{C}$ is applied, then at that instant the antiparticle is very likely to move in the $\mathbf{e_x}$ direction.

Let p represent a particle that is its own antiparticle, like the photon. Let q represent a particle that is not its own antiparticle, like a positron. The effect of $\hat{C}$ can then be described quite easily

$$\hat{C}\,|p\rangle = C_p\,|p\rangle \qquad\qquad \hat{C}\,|q\rangle = |\bar{q}\rangle$$

the effect of $\hat{C}^2$ should be $\hat{I}$ as changing from antiparticle and back should be invariant. This gives $C_p = \pm 1$. The reason that there is no C_q factor is: If it were introduced it does not correspond to any eigenvalue of $\hat{C}$, for antiparticles are different eigenstates. This means it cannot be measured since the definition of a quantum mechanical observable states that the observed values are eigenvalues of a hermitian operator. The arbitrary nature of C_q leads to the freedom to choose $C_q = 1$.

Generalising to a system of particles

$$\begin{aligned}\hat{C}(|p_1\rangle \otimes ... \otimes |p_n\rangle \otimes |q_1\rangle \otimes ... \otimes |q_m\rangle) &:= (\hat{C}\,|p_1\rangle) \otimes ... \otimes (\hat{C}\,|p_n\rangle) \otimes (\hat{C}\,|q_1\rangle) \otimes ... \otimes (\hat{C}\,|q_m\rangle)\\ &= (C_{p_1}\,|p_1\rangle) \otimes ... \otimes (C_{p_n}\,|p_n\rangle) \otimes |\bar{q}_1\rangle \otimes ... \otimes |\bar{q}_m\rangle\\ &= C_{p_1}...C_{p_n}\,|p_1\rangle \otimes ... \otimes |p_n\rangle \otimes |\bar{q}_1\rangle \otimes ... \otimes |\bar{q}_m\rangle\end{aligned}$$

In other words

$$\hat{C}\left(\bigotimes_{i=1}^{n}|p_i\rangle \otimes \bigotimes_{j=1}^{m}|q_j\rangle\right) = \prod_{i=1}^{n} C_{p_i}\left(\bigotimes_{i=1}^{n}|p_i\rangle \otimes \bigotimes_{j=1}^{m}|\bar{q}_j\rangle\right)$$

In a simplified notation this reads

$$\hat{C}\,|p_1...p_n q_1...q_m\rangle = C_{p_1}...C_{p_n}\,|p_1...p_n\bar{q}_1...\bar{q}_m\rangle$$

Like the parity operator $\hat{C}$ is also conserved under electromagnetic and strong interactions but in general is not under weak interactions.

C. The Time Reversal Operator

Dudley Grant

Time reversal is simply a reflection of the time coordinate. If the laws of physics are preserved under time reversal then while watching a video it would be impossible to tell if it were going forward and backward.

Newton's laws for conservative forces are preserved under time reversal as

$$\begin{aligned}m\mathbf{x}''(t) &= F(\mathbf{x})\\ m\partial_t\partial_t\mathbf{x}(t) &= F(\mathbf{x})\end{aligned}$$

Change time coordinate by reflection $t \mapsto \tilde{t} := -t$, this implies $\partial_t = \frac{\partial\tilde{t}}{\partial t}\partial_{\tilde{t}} = -\partial_{\tilde{t}}$. Note $\mathbf{x}(t)$ is written in one coordinate system for time, it can also be written as $\tilde{\mathbf{x}}(\tilde{t})$ where $\tilde{\mathbf{x}}(\tilde{t}(t)) = \tilde{\mathbf{x}}(-t) = \mathbf{x}(t)$ so,

$$\begin{aligned}m(-\partial_{\tilde{t}})(-\partial_{\tilde{t}})\mathbf{x}(-\tilde{t}) &= F(\mathbf{x})\\ m(-1)^2\partial_{\tilde{t}}\partial_{\tilde{t}}\tilde{\mathbf{x}}(\tilde{t}) &= F(\mathbf{x})\\ m\tilde{\mathbf{x}}''(\tilde{t}) &= F(\mathbf{x})\end{aligned}$$

As the equation of motion is the same for going backward in time the symmetry has been shown.

Now in quantum mechanics consider a Fourier mode

$$\psi_{\mathbf{p}}(\mathbf{r}, t) = e^{\frac{i}{\hbar}(\mathbf{p}\cdot\mathbf{r} - Et)}$$

In classical mechanics the direction of momentum changes under time reversal as

$$\mathbf{p} = m\partial_t\mathbf{x}(t) \mapsto m(-\partial_{\tilde{t}})\mathbf{x}(-t) = -m\tilde{\mathbf{x}}(\tilde{t}) = -\tilde{\mathbf{p}}$$

In quantum mechanics it must be similar

$$\begin{aligned}\psi_{\mathbf{p}}(\mathbf{r},t) &\mapsto \psi_{-\mathbf{p}}(\mathbf{r},-t)\\ &= e^{\frac{i}{\hbar}((-\mathbf{p})\cdot\mathbf{r}-E(-t))}\\ &= \psi^*_{\mathbf{p}}(\mathbf{r},t)\end{aligned}$$

So time reversal may be represented by complex conjugate. Normalisation is preserved and so are Hermitian observables

$$\begin{aligned}|\psi(\mathbf{r},t)| \mapsto |\psi(\mathbf{r},-t)| &= |\psi^*(\mathbf{r},t)| = |\psi(\mathbf{r},t)|\\ \langle\hat{H}\rangle = \langle\psi(\mathbf{r},\mathbf{t})|\hat{H}\psi(\mathbf{r},t)\rangle &\mapsto \langle\psi^*(\mathbf{r},\mathbf{t})|\hat{H}\psi^*(\mathbf{r},t)\rangle\\ &= \langle\hat{H}^*\psi^*(\mathbf{r},\mathbf{t})|\psi^*(\mathbf{r},t)\rangle\\ &= \langle\hat{H}\psi(\mathbf{r},\mathbf{t})|\psi(\mathbf{r},t)\rangle^*\\ &= \langle\psi(\mathbf{r},\mathbf{t})|\hat{H}\psi(\mathbf{r},t)\rangle^*\\ &= \langle\psi(\mathbf{r},\mathbf{t})|\hat{H}\psi(\mathbf{r},t)\rangle\\ &= \langle\hat{H}\rangle\end{aligned}$$

Making use of the Hermitian operator's definition and that they have only real eigenvalues.

Time reversal may be a symmetry but it does not give a conservation law. For $\hat{C}$ and $\hat{P}$ it was required that they were Hermitian so their eigenvalues, $\{-1,1\}$, could be measured. One cannot define a Hermitian operator that produces the desired effects of time reversal as

$$\hat{T}\left(|\alpha\psi(t)\rangle + \beta\,|\psi(t)\rangle\right) = \alpha^*\hat{T}\,|\psi(t)\rangle + \beta^*\hat{T}\,|\psi(t)\rangle \neq \alpha\hat{T}\,|\psi(t)\rangle + \beta\hat{T}\,|\psi(t)\rangle$$

That is, $\hat{T}$ is not linear. Not only is it not hermitian it is also not an operator. Where operator is defined as a linear functional of the Hilbert space $\mathcal{H}$. As an abuse of notation it is commonly still written as $\hat{T}$.

D. CPT Theorem

Dudley Grant

The **CPT** Theorem says that any relativistic theory is symmetric under the generalised operator $\hat{C}\hat{P}\hat{T}$. In quantum theories the topic of anti-unitary operators must be introduced and explored. Essentially introducing anti-unitary operators allows to speak of "eigenvalues" of $\hat{T}$ which are $\{-1,1\}$. The details of the proof are beyond the writer's current ability and can be found here [**?**].

Although CPV occurs, CPT is always conserved for any physical phenomena, for a very general relativistic theory. This is the use of the theorem.

For a more detailed treatment of $\hat{C}$, $\hat{P}$ and $\hat{T}$ see [**?**].

E. CP and Conservation

Kevin Maguire

It has been shown that the violation of C and P are large effects. In fact, they are both maximally violated by the weak force. CP symmetry was proposed to reconcile these two quantities. This new symmtery was of course conserved by the strong and EM forces and seemed to be conserved in the weak force. There is good evidence to suggest that CP is conserved in weak decays of leptons and the helicity of neutrinos.

In considering decays of polarized muons to electron final states:

$$\mu^- \to e^-\bar{\nu}_e\nu_\mu \qquad\qquad \mu^+ \to e^+\nu_e\bar{\nu}_\mu$$

It is found that the electrons are emitted more frequently in certain directions. Thus there is an asymmetry, $\xi_\pm$ in the emission, with the two decays having different asymmetries. Under the transformation $\hat{C}$ it is expected that these

two decays would have the same asymmetry. Similarly, by the $\hat{P}$ transformation it is expected that the distributions should be completely uniform and favour no direction. It was clear that C and P were being violated, but it was also noted that μ^+ and μ^- have the exact same lifetimes. For this system CP conservation implies that the probability of an electron in one direction should be equal to the probability of a positron being emitted in the opposite direction. Using the experimentally determined formula for muon decay assymetries it is found that CP conservation implies the $\xi_+ = -\xi_-$. Experimentally these values are measured as $\xi_+ = -\xi_- = -1.00 \pm 0.04$ [**?**]. Thus it is found that CP is conserved in this leptonic system. In fact, there is no experimental evidence for the the violation of CP in weak leptonic decays.
The action of $\hat{C}$ and $\hat{P}$ on neutrinoes with definite handedness is discussed here. Handedness, also known as helicity is defined as right if the projection of a partilces spin, m_l, is in the same direction as the particles motion. This corresponds to m_l having the same sign as the particles momentum. Similarly a left handed particle has the projection of its spin in the opposite direction to its momentum. The $\hat{C}$ operator changes a particle into its anti-particle and thus a left handed neutrino transforms to a left handed anti-neutrino. The $\hat{P}$ operator reverses a particles momentum and thus changes its handedness, so a left handed neutrino goes to a right handed neutrino. However, in nautre it is found that only left handed neutrinos and right handed anti-neutrinos are observed. C and P predict that decays involving lefthanded neutrinos and right handed anti-neutrinos should behave the same as decays involving right handed neutrinos and left handed anti-neutrinoes. Thus if none of the latter are observed it is clear that these particles are not treated the same by nature, showing that C and P are violated. The solution here is that the combined operation of $\hat{C}\hat{P}$ converts a left handed neutrino to a right handed anti-neutrino, thus we see that CP conservation requires that only these two definite handed neutrinos are observed in nature.

This review is, of course, not on CP conservation. As stated, CP is conserved in weak leptonic decays, but this is certainly not the case in hadroic or even semi-leptonic decays. CP violation (CPV) was first observed in the mixing of neutral K-mesons by Christenson, Cronin, Fitch and Turlay in 1964 [**?**]. They observed the $CP = -1$ state K_L^0 decaying to 2 pions, a state with $CP = 1$. Although the fraction of K_L^0 decays violating CP in this way is tiny, the discovery was significant.

II. THE CKM MECHANISM

John Ronayne

The weak force allows the change of flavor of say an up quark to a down quark. A deeper connection in the standard model can be made when we relate this to the electron and electron neutrino transitions [**?**]. It was originally noted by Nicola Cabibbo that the strengths of these process were remarkably similar to within 4%. This discrepancy however bore some real consequences. It was the assumption of the charmed and 3rd generation of quarks by Kobayashi and Maskawa that noted this 4% uncertainty had some real significance and this difference didn't simply disappear with more accurate readings. The ability of the Weak force to decay between the generations explained this reduction in the strength of the decay amplitude. This has some rather interesting features. One can make use of Pythagoras's theorem to determine a unique angle between each decay path, known as the Cabibbo angle. If two generations were the full story we would envision that Figure 1 would be the principal triangle.

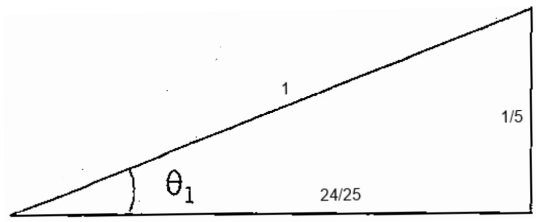


FIG. 1: Cabibbo angle θ_c. Lengths represent decay amplitudes

Only one thousandth of the 4% deviation here is accountable from the 3rd generation but for the moment lets look a bit more into the first two. It shall be shown that these subtle effects arising from the 3rd generation are where the theory upon CP violation developed from. From Figure 1 it is seen that the transition within a generation $u \to d$ is calculated as $\cos\theta$ and across the generation as $\sin\theta$ which actually corresponds to $u \to s$. Naively presuming only two generations of matter existed, constructing an amplitude matrix of the corresponding transitions based on this would be appropriate, as will be demonstrated in Eqn.(1). Note that, for the moment, anti-particles have amplitudes that are the same as their matter counterparts [**?**].

$$\begin{pmatrix} A_{ud} & A_{us} \\ A_{cd} & A_{cs} \end{pmatrix} = \begin{pmatrix} \cos\theta_1 & \sin\theta_1 \\ -\sin\theta_1 & \cos\theta_1 \end{pmatrix} \tag{1}$$

From Figure 1 it is calculated that $\theta_1 \sim 12^\circ$ experimentally this is measured $\theta_1 = 13.1^\circ$ [**?**].The impact of this was that the decay rate of many hadronic particles could be calculated, akin to lepton decays, with the additional factor of $\cos\theta_1$or $sin\theta_c$ in the matrix element. At a quick glance of the Weak Lagrangian,

$$L_{weak} = i\bar{\psi}\gamma^\mu(1-\gamma^5)\partial^\mu\psi - q\sum\bar{\psi}\gamma^\mu\sigma_i(1-\gamma^5)\psi A_{\mu i} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{2}$$

the interaction term (the second term in Eqn.(2)) is a key element in the vertexes of the Feynman diagram describing the interactions. An element of this the γ signifies axial vector coupling with properties which contribute to CPV [**?**].

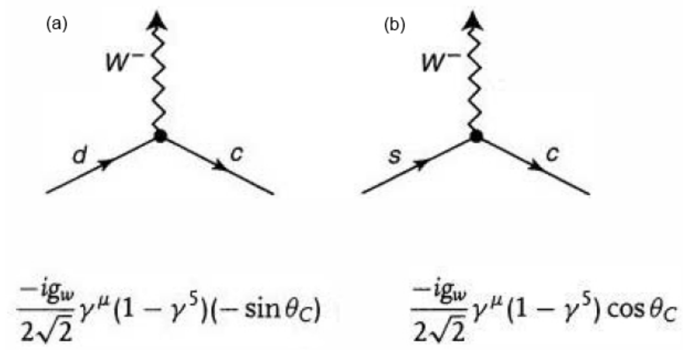


FIG. 2: Decay amplitudes across generations (Note the Cabbibo angle $\theta_c = \theta_1$). Left $d \to c$ and right $s \to c$.

To progress onto a mechanism for mixing 3 generations of quarks, the first steps must look further into what sets the Weak interacting quarks apart from the quarks we associate with electromagnetic and strong interactions. To begin let us look at an example of the Kaon decay into two Muons.

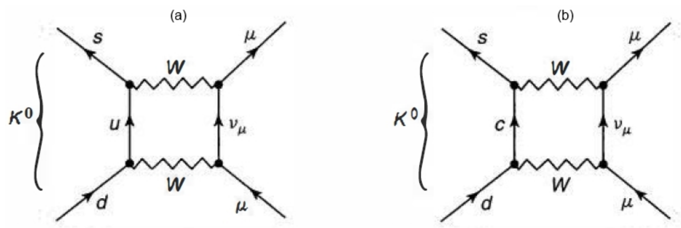


FIG. 3: Kaon Interfering Feynamn diagrams illustrating GIM mechanism

So for a decay scheme as the Kaon in Figure 3.a , the virtual up quark is what will be transmitted between the down and s. This is what would be know as a second order diagram as the direct decay to a W boson is forbidden. When the amplitudes are found the branching ratio between that and the $K^+ \to \mu\nu$ is calculated to be,

$$\frac{K^0 \to \mu\mu}{K^+ \to \mu\nu} = 10^{-8}. \tag{3}$$

However experimentally this value is found to be too high. What could also be possible is the diagram in Figure 3.b where the virtual quark is now charm. When taking into account these two process we find that in Figure 3.a the amplitude is proportional to $\sin\theta_1\cos\theta_1$ and in Figure 3.b the amplitude is proportional to $-\sin\theta_1\cos\theta_1$ on account of A_{cd} in our simple matrix above [**?**]. In 1970, what is called the (Glashow, Iliopoulos and Maiani) GIM mechanism was responsible for a solution [**?**], it proposed that ,through the interference with another possible decay process (or diagram) there would be a near cancellation. The remaining value came from the difference in mass between the up and charmed quark. Using the experimental Amplitudes this allowed calculations and clear predictions for the mass of the charmed quark of about $1.5GeV$. It was successfully discovered in 1974 which then ushered what was known as the November Revolution [?].

Cabibbo's theory of mixing together with the GIM mechanism allows for an insightful view of quarks from a different perspective. Instead of one quark that feels the strong, electromagnetic and weak force, there is a sort of mixed phase of quarks involved in weak interactions. So an incognito weak d and s are given by,

$$d' = d\cos\theta_1 + s\sin\theta_1 \tag{4}$$

and

$$s' = s\cos\theta_1 - sin\theta_1. \tag{5}$$

This can then formulate the matrix,

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} cos\theta_1 & sin\theta_1 \\ -sin\theta_1 & cos\theta_1 \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix} \tag{6}$$

Now the following doublets are found like in leptons using an analogous Cabibbo rotated states,

$$\begin{pmatrix} u \\ d' \end{pmatrix} = \begin{pmatrix} u \\ d\cos\theta_1 + s\sin\theta_1 \end{pmatrix} \text{ and } \begin{pmatrix} c \\ s' \end{pmatrix} = \begin{pmatrix} c \\ s\cos\theta_1 - d\sin\theta_1 \end{pmatrix} \tag{7}$$

Moving onto a third generation of quarks, the method of finding mixing angles from the Amplitude triangles is repeated.

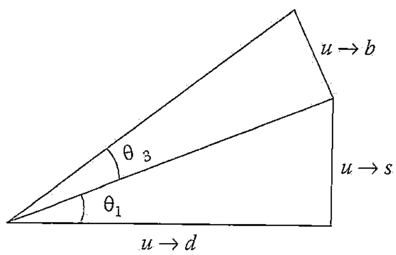


FIG. 4: Mixing triangle across 3 generations

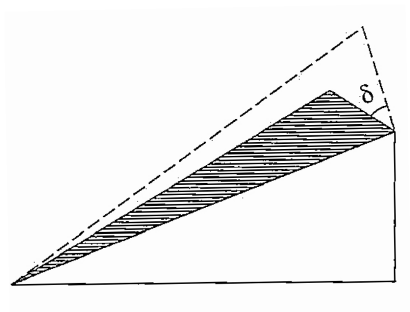


FIG. 5: Complex phase in the 2nd to 3rd generation transitions

In Figure 4 what now is in place is an additional triangle with its base atop the hypotenuse of the 1st to 2nd generation triangle. The transition now facilitates the Amplitude of the up quark transitioning towards the bottom quark path. This gives rise to the angle θ_3 a mixing angle between the 1st and 2nd generation and proceeding with this convention the mixing angle between the 2nd and 3rd is naturally θ_2. The pictorial representation has another hidden feature, Figure 6. The plane the triangle sits in can be thought of as the matter-antimatter mirror with the addition freedom of the upper right-angle triangle to swing in and out of the page at an angle δ. It is called the Kobayashi-Maskawa phase, where $\delta = 0$ is on the plane of the triangle below. This parameter can set a difference between preferred taste of flavor (quarks and anti-quarks) which will later be shown in Section[IV] and may be the clue to the source of CP violation in nature and perhaps even the structure of the universe itself, but that remains to be seen since in Section[VI] and Section[VII] while still abiding to the conditions that are in place to allow for CP violation , $\delta \neq 0,\pi$ or $\theta_i \neq 0, \frac{\pi}{2}$ [**?**] . With a full catalog of mixing between quarks, scaling the prior models to a 3x3 matrix is a task performed in [**?**] , it incorporates every type of quark mixing in the original formalism.

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ -s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix}. \tag{8}$$

In order to take advantage of the CKM matrix and illuminate CP violating decays, two weak amplitudes with complex phase components must exist. An example of such would be the model 4 quark system [**?**]. Consider two up quarks i and k and two down quarks j and l. Its possible to find that the matrix element is,

$$M = (V_{ij} V_{kl}) A_1 e^{i\delta_1} + (V_{il} V_{kj}) A_2 e^{i\delta_2} \tag{9}$$

Where A_1 and A_2 are real value amplitudes and each one represents a unique initial state transitioning to the same final states .The δ_1 and δ_2 are the phases due to higher order processes. The difference between them may be defined as the $\Delta\delta = \delta_1 - \delta_2$ and is known as the CP-even phase. Performing the $\bar{C}\bar{P}$ operation on this matrix element a new one is obtain,

$$\overline{M} = (V_{ij} V_{kl})^* A_1 e^{i\delta_1} + (V_{il} V_{kj})^* A_2 e^{i\delta_2} \tag{10}$$

The two individual amplitudes A_1 and A_2 in each of the Matrix elements interfere with each other, so to simplify this down for a moment let us absorb the coefficients into A_1 and A_2 and solve for their decay rates [**?**]. Let,

$$|A|^2 = |A_1 + A_2|^2 = |A_1|^2 + |A_2|^2 + 2Re\,|A_2^* A_1| \tag{11}$$

$$= |A_1|^2 + |A_2|^2 + 2\,|A_1 A_2|\, cos(\Delta\phi - \Delta\delta), \tag{12}$$

and

$$\left|\overline{A}\right|^2 = |A_1|^2 + |A_2|^2 + 2\,|A_1 A_2|\, cos(\Delta\phi - \Delta\delta), \tag{13}$$

The ϕ in this case is the CP-even phase. Now defining the CP asymmetry as,

$$\mathbf{A}_{CP} = \frac{|A|^2 - \left|\overline{A}\right|^2}{|A|^2 + \left|\overline{A}\right|^2} \tag{14}$$

Apply equation 14 to 9 and 10 to find the Asymmetry in our four quark system we solve for the matrix elements. The result is,

$$\mathbf{M}_{CP} = \frac{2Im(\,V_{ij}\,V_{kl}\,V_{kj}^*\,V_{il}^*)\sin(\Delta\delta)A_1\,A_2}{|V_{ij}V_{kl}|^2\,A_1^2 + |V_{kj}V_{il}|^2\,A_2^2 + 2Re(\,V_{ij}\,V_{kl}\,V_{kj}^*\,V_{il}^*)\cos(\Delta\delta)A_1\,A_2} \tag{15}$$

CP violation in this respect is proportional to $2Im(\,V_{ij}\,V_{kl}\,V_{kj}^*\,V_{il}^*)$ which is called $\mathcal{J}$, the Jacobian. The Jacobian is just the gradient of the scalar valued CKM matrix it also is subject to the same CP violating conditions as what δ boasted.

So now it may same possible to be stuck with the eternal burden of having a theory with an unknown number of free parameters to test with. To fix this we would like our CKM matrix to be unitary i.e. that itself by it's complex conjugate produces the Identity matrix. A quick glance and a brief frown reveals that the matrix thus far bare no hope unless our off-diagonal elements are relatively small. Hence since CP violation turns out to be very small experimentally and these off diagonal elements are in turn correlated to CPV it has been constructed, in close approximation, a unitary matrix as such. Adopting the Wolfenstein parametrization [**?**] where we expand on a small parameter $\lambda = 0.22$ to a power series,

$$V_W = \begin{vmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\nu) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\nu) & -A\lambda^2 & 1 \end{vmatrix} + \mathcal{O}(\lambda^4) \tag{16}$$

Looking back at the original CKM matrix in 8,

$$\lambda = s_1 \ , \ A = \frac{s_2}{s_1^2} \ , \ \rho = \frac{s_3}{s_1 s_2}\cos\delta \text{ and } \nu = \frac{s_3}{s_1}s_2\sin\delta.$$

Comparing this to experimentally measured values CKM matrix elements we have pretty close agreement.

$$V_{exp} = \begin{pmatrix} 0.9739 - 0.975 & 0.221 - 0.227 & 0.0029 - 0.0045 \\ 0.221 - 0.227 & 0.9730 - 0.9744 & 0.039 - 0.044 \\ 0.0048 - 0.01 & 0.037 - 0.043 & 0.9990 - 0.9992 \end{pmatrix} + \mathcal{O}(\lambda^4) \tag{17}$$

As you can see there is a dependence on experimental data but to what extent is it needed. An $n \times n$ complex matrix will have n^2 real and complex parameters while unitarity meaning we have n^2 constrains. Since we have 6 quarks ($2n$) which can all have independent phases we have $2n$ fewer parameters. Fixing one phase we then have $n^2 - (2n-1)$. In the real unitary matrix we have n dimensions and $\frac{n(n-1)}{2}$ free parameters. Thus the total imaginary parameters in the CKM matrix is,

$$n^2 - (2n-1) - \frac{n(n-1)}{2} = \frac{(n-1)(n-2)}{2} \tag{18}$$

which for n=3 is 1. Hence we have 4 unknown parameters in total, which is why the values of the CKM matrix depend on experimental constraints [**?**]

As found in [**?**], 9 constraints are needed, 6 of which are the sum of complex terms which are zero by orthogonality. Here are three of these unitary relation equations,

$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0, \tag{19}$$

$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0, \tag{20}$$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0, \tag{21}$$

A way to visualize these is as the Unitary triangles in the complex plane and have an surface area of $\frac{|\mathcal{J}|}{2}$ [**?**]. The area is then non-zero for CP violating weak interactions.

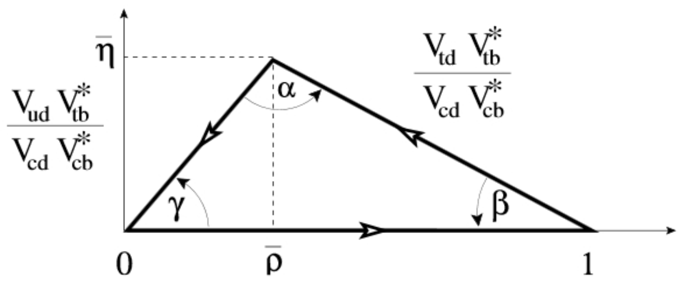


FIG. 6: Unitary Triangle with angles α, β and γ

Now to look a bit closer at the unitary equation 21 to construct the appropriate triangle. In Figure 6 a triangle on the basis (x,y) with corners at (0,0), (1,0) and $(\overline{\rho}, \overline{\nu})$ is formed, where the reparameterizations are,

$$\overline{\rho} = \rho\left(1 - \frac{\lambda^2}{2}\right) \text{ and } \overline{\nu} = \nu\left(1 - \frac{\lambda^2}{2}\right) \tag{22}$$

The three angles in this diagram α, β and γ are defined as,

$$\alpha \equiv arg\left(-\frac{V_{tb}V_{tb}^*}{V_{ud}V_{ub}^*}\right) = \frac{1}{2}\sin^{-1}\left(\frac{2\overline{\nu}(\overline{\nu}^2 + \overline{\rho}^2 - \overline{\rho})}{(\overline{\rho}^2 + \overline{\nu}^2)((1-\overline{\nu})^2 + \overline{\nu}^2)}\right). \tag{23}$$

$$\beta \equiv argl\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) = \frac{1}{2}\sin^{-1}\left(\frac{2\overline{\nu}(1-\overline{\rho})}{(1-\overline{\rho})^2 + \overline{\rho}^2}\right). \tag{24}$$

$$\gamma \equiv arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right) = \frac{1}{2}\sin^{-1}\left(\frac{2\overline{\rho\nu}}{\overline{\rho}^2 + \overline{\nu}^2}\right). \tag{25}$$

and $\alpha + \beta + \gamma = 180°$. Direct measurement of these angles is performed by observations of CP violations in B meson decays which shall be covered in the following section [**?**]