Two Particle Physics Models With Spontaneous CP Violation From Gauge Theory On Discrete Group

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Abstract

Based on the differential calculus and the gauge theory on discrete groups, we reconstruct two physics models with spontaneous CP violation: (1) The Georgi-Glashow-Lee model with two Higgs triplets; (2) The Weinberg-Salam-Branco model with three Higgs doublets and the natural flavor conservation (NFC). We focus on the Lagrangian terms containing the Higgs particles and show that with an appropriate choice of the discrete groups, we can obtain the physically meaningful Yukawa couplings and the Higgs potentials which lead to the spontaneous CP violation consequently.

1 Introduction

There were many mechanisms to explain the CP nonconservation after the theoretical prediction raised by Lee, Oheme and Yang as early as 1956 and the experimental support, ${K_L}^0 \to \pi^+\pi^-$ observed by Chritenson, Cronin, Fitch and Turlay in 1964 [1]. But it still remains an open question for none of the explanation could match all the experimental data fairly well. Among these mechanisms, the spontaneous CP violation (SCPV) first suggested by T.D. Lee [1] has drawn much attention because it seems natural and elegant, the total Lagrangian is assumed to satisfy invariance under CP and T transformation, renormalizability and invariance under certain weak- electromagnetic gauge transformations. Lee also generalized a more realistic physical model at that time, Georgi-Glashow Model to a version with SCPV. In this model, two Higgs triplets were introduced to provide SCPV and satisfy other physical condition. Having considered the natural flavor conservation (NFC), G.Branco gave a "minimal" model based on the standard $SU(2) \times U(1)$ gauge theory with three Higgs doublets [2], the CP nonconservation is entirely due to Higgs-boson exchange , not in the manner of Kobayashi-Maskawa [3]. Recently, Y-L Wu proposed a $SU(2)_L \times U(1)_Y$ gauge theory with two Higgs doublets which has both the SCPV and NFC, the Glashow- Weinberg criterion for NFC is sufficient but not necessary [4].

In these mechanisms, the Higgs particles play a key role to violate CP. But we did not understand the origin of Higgs particles very well. The development of non-commutative geometry (NCG) has made a great progress toward the understanding for the origin of Higgs particles [5-9]. According to NCG, the Higgs fields are gauge fields with respect to discrete symmetry, the Higgs potential can be viewed as Yang-Mills term in general gauge field theory and can be determined with less ambiguity. Moreover, the non-commutative Yang-Mills term together with fermions can reproduce many particle physics models with some constraints in free parameters. Lots of efforts have been made along this way.[10-18]

This paper is one of our series of work on developing the gauge theory on discrete group. We start with the NCG and take the discrete group to be $Z_2 \times Z_2$ and $Z_2 \times Z_2 \times Z_2$

respectively, then construct physics models with SCPV. Following the same way in [15] [18], we get the Yukawa coupling terms , Higgs kinetic terms as well as Higgs potential which is important in SCPV. The paper is arranged as follows: In section 2 we give a brief introduction of gauge theory on discrete groups. In section 3 we reconstruct the Georgi-Glashow-Lee model in the spirit of NCG. In section 4 we take the discrete group to be $Z_2 \times Z_2 \times Z_2$ and obtain Weinberg-Salam-Branco three Higgs model with SCPV, while the NFC are ensured naturally. At section 5 we end with some discussions and remarks.

2 The Gauge theory on the Discrete Group $Z_2 \times Z_2$

2.1 Differential Calculus on Discrete Groups G

Let G be a discrete group of size N_G, its elements are $\{e, g_1, g_2, \cdots, g_{N_G-1}\}$, and $\mathcal{A}$ the algebra of the all complex valued functions on G .The right and left multiplication on G induce natural automorphisms of $\mathcal{A}$, R_g and L_g respectively,

$$(R_h f)(g) = f(g \cdot h), (L_h f)(g) = f(h \cdot g) \tag{2.1}$$

The basis $\partial_i,\ (i = 1, \cdots, N_G - 1)$ of the left invariant vector space $\mathcal{F}$ on $\mathcal{A}$ are defined as

$$\partial_i f = f - R_i f, \quad \forall f \in \mathcal{A}, \tag{2.2}$$

∂_i also satisfies

$$\partial_i \partial_j = \sum_k C_{ij}^k \partial_k, \qquad C_{ij}^k = \delta_i^k + \delta_j^k - \delta_{i \cdot j}^k \tag{2.3}$$

where $i, j, \cdots, (i \cdot j)$ denote $g_i, g_j, \cdots, (g_i \cdot g_j)$ respectively. The Haar integral, which remains invariant under group action, is introduced as a complex valued linear functional on $\mathcal{A}$ as,

$$\int_G f = \frac{1}{N_G} \sum_{g \in G} f(g). \tag{2.4}$$

Having chosen the basis of $\mathcal{F}$ we can introduce the dual basis of $\mathcal{F}^*$ consisting of one forms, χ^i, satisfy

$$\chi^i(\partial_j) = \delta_j^i. \tag{2.5}$$

There exists exactly one linear operator $d, d : \Omega^n \rightarrow \Omega^{n+1}$, which is nilpotent, $d^2 = 0$, satisfies the graded Leibniz rule and for every $f \in \mathcal{A}$ and every vector field $v, df(v) = v(f)$, provided that χ^i satisfy the following two conditions[8]

$$\begin{aligned} \chi^i f = &\quad (R_i f)\chi^i, \quad \forall f \in \mathcal{A}, \\ d\chi^i = &\quad -\textstyle\sum_{j,k} C_{jk}^i \chi^j \otimes \chi^k. \end{aligned} \tag{2.6}$$

To construct a physical model we must define the involution operator * on A , which agrees with the complex conjugation on $\mathcal{A}$, takes the assumption that $(\chi^g)^* = -\chi^{g^{-1}}$, and (graded) commutes with d, i.e. $d(\omega^*) = (-1)^{deg\omega}(d\omega)^*$.

2.2 Gauge Theory on Discrete Group G

Using the differential calculus introduced in last section, we can construct the generalized gauge theory on finite groups. The gauge transformations group is often taken to be the group of unitary elements of zero-forms $\tilde{A}$,

$$\mathcal{H} = \mathcal{U}(\tilde{A}) = a \in \tilde{A} : aa^* = a^* a = 1 \tag{2.7}$$

Like the usual gauge theory, the $d+\phi$ is gauge covariant which requires the transformation of gauge field one form ϕ as,

$$\phi \to H\phi H^{-1} + HdH^{-1}. \tag{2.8}$$

If we write $\phi = \sum_g \phi_g \chi^g$, the coefficients ϕ_g transform as

$$\phi_g \to H\phi_g(R_g H^{-1}) + H\partial_g H^{-1}. \tag{2.9}$$

It is convenient to introduce a new field $\Phi_g = 1 - \phi_g$, then (2.10) is equivalent to

$$\Phi_g \to H\Phi_g(R_g H^{-1}). \tag{2.10}$$

The extended anti-hermitian condition $\phi^* = -\phi$ results in the following relations on its coefficients ϕ_g as well as Φ_g,

$$\phi_g^\dagger = R_g(\phi_{g^{-1}}), \qquad \Phi_g^\dagger = R_g(\Phi_{g^{-1}}) \tag{2.11}$$

It can be easily shown that the curvature two form $F = d\phi + \phi \otimes \phi$ is gauge covariant and can be written in terms of its coefficients

$$F = \sum_{g,h} F_{gh}\chi^g \otimes \chi^h \tag{2.12}$$

$$F_{gh} = \Phi_g R_g(\Phi_h) - \Phi_{h\cdot g}. \tag{2.13}$$

In order to construct the Lagrangian of this gauge theory on discrete groups we need to introduce a metric on the forms. Let us first define the metric η as a bilinear form on the bimodule Ω^1 valued in the algebra $\mathcal{A}$,

$$\eta : \quad \Omega^1 \otimes \Omega^1 \to \mathcal{A} \tag{2.14}$$

$$< \chi^g, \chi^h >= \eta^{gh}. \tag{2.15}$$

The gauge invariance requires that $\eta^{gh} \sim \delta^{gh^{-1}}$[16]. The metric on the two forms becomes,

$$< \chi^g \otimes \chi^h, \chi^p \otimes \chi^q >= \alpha\eta^{gh}\eta^{pq} + \beta\eta^{gq}\eta^{hp} + \gamma\eta^{gp}\eta^{hq}, \tag{2.16}$$

where the term proportional to γ is only appeared when G is commutative [8].In the next section ,We will get a constrain relation of α, β and γ if the spontaneous CP violation is considered. Then the most general Yang- Mills action is given,

$$\begin{aligned}
\mathcal{L} &= -\int_G < F, \overline{F} > \\
&= -\int_G \sum_{g,h,p,q} Tr(F_{gh}F_{pq}^\dagger) < \chi^g \otimes \chi^h, \chi^{g^{-1}} \otimes \chi^{p^{-1}} > \\
&= -\int_G \sum_{g,h,p,q} Tr(F_{gh}F_{pq}^\dagger)(\alpha\eta^{gh}\eta^{q^{-1}p^{-1}} + \beta\eta^{gp^{-1}}\eta^{hq^{-1}} + \gamma\eta^{gq^{-1}}\eta^{hp^{-1}}),
\end{aligned} \tag{2.17}$$

where we have used the involution relations

$$\overline{F} = (\chi^q)^* \otimes (\chi^p)^* F_{pq}^\dagger, \qquad (\chi^g)^* = -\chi^{g^{-1}}. \tag{2.18}$$

2.3 Gauge Field Theory on $M^4 \times G$

To deal with the model building in particle physics, we must include fermions and their Yukawa couplings to Higgs. A detailed construction of the gauge theory on discrete groups coupled to the fermions has given in [16] [15]. Here we only site some conclusion which is useful to the model building in the next section. M^4 is the four-dimension space-time.

We defined the exterior derivative operators d_M and d_G on M^4 and G respectively,i.e $d_M f = \partial_\mu f dx^\mu, \;\; d_G f = \partial_g f \chi^g$, then $d_{M^4\times G} = d_M + d_G$ can be extended to the covariant derivative

$$\begin{array}{ll} D_{M^4\times G} & = d_{M^4\times G} + A \\ d_{M^4\times G} f & = \partial_\mu f dx^\mu + \partial_r f x^r \\ A & = igA_\mu dx^\mu + \frac{1}{\mu}\phi_g \chi x^g \end{array} \tag{2.19}$$

The generalized curative two form:

$$\begin{array}{ll} F & = dA + A \otimes A \\ & = \frac{ig}{2} F_{\mu\nu} dx^\mu \wedge dx^\nu + \frac{1}{\mu} F_{g\mu} dx^\mu \otimes \chi^g + \frac{1}{\mu^2} F_{gh} \chi^g \otimes \chi^h \end{array} \tag{2.20}$$

where

$$\begin{array}{ll} F_{\mu\nu} & = \partial_\nu A_\mu - \partial_\nu A_\mu + ig[A_\mu, A_\nu] \\ F_{g\mu} & = \partial_\mu \phi_g + igA_\mu \phi_g - ig\phi_g R_g A_\mu \\ F_{gh} & = \Phi_g R_g(\Phi_h) - \Phi_{h\cdot g}. \end{array} \tag{2.21}$$

In next section we will find that $F_{\mu\nu}$ is the ordinary gauge field intense, $F_{g\mu}$ will lead to the kinetic term of Higgs, and F_{gh} lead to the Higgs potential.

The "local gauge invariant" fermion field Lagrangian on $M_4 \times G$ is written as,[15]

$$\mathcal{L} = \overline{\psi}(x,g)[i\gamma^\mu(\partial_\mu + A_\mu(x,g)) + \mu \sum_h (\partial_h + \frac{1}{\mu}\phi_h(x,g)R_h)]\psi(x,g) \tag{2.22}$$

3 The Georgi-Glashow-Lee Model on the Discrete Group $Z_2 \times Z_2$

ˇThis model is a generalization of the Georgi-Glashow model given by T.D. Lee. [1] The basic gauge group of the weak and electromagnetic interaction is SO_3. There is a triplet of spin 1 gauge field W_μ and at least eight quark-like hadron fields grouped into two triplets $\psi, \psi^{'}$, as well as two singlets $\chi, \chi^{'}$.

These fields can be expressed by the physical states p^+, n^0, λ^0, q^- and $p^{+'}, n^{0'}, \lambda^{0'}, q^{-'}$ but here we focused on two SO_3 triplets of spin 0 Hermitian field $\vec{\Phi}_R, \vec{\Phi}_I$, which play important roles in CP spontaneous violation of this model, Under the time reversal transformation T,

$$T\vec{\Phi}_R T^{-1} = +\vec{\Phi}_R, \quad but \quad T\vec{\Phi}_I T^{-1} = -\vec{\Phi}_I \tag{3.23}$$

The total Lagrangian density can be written as

$$\mathcal{L} = \mathcal{L}(W,\psi) + \mathcal{L}(\vec{\Phi}_R,\psi) + \mathcal{L}(\vec{\Phi}_I,\psi) + \mathcal{L}(W,\vec{\Phi}) \tag{3.24}$$

in which $\mathcal{L}(W,\psi)$ contain massive free fermions terms and their coupling with gauge fields W, and

$$\mathcal{L}(\vec{\Phi}_R,\psi) = -(g_R \bar{\psi}\vec{I}\psi + {g_R}^{'} \bar{\psi}^{'}\vec{I}\psi^{'}) \cdot \vec{\Phi}_R \tag{3.25}$$

$$\mathcal{L}(\vec{\Phi}_I,\psi) = -i(g_I \bar{\psi}\vec{I}\psi + {g_I}^{'} \bar{\psi}^{'}\vec{I}\psi^{'}) \cdot \vec{\Phi}_I \tag{3.26}$$

where g_R and g_I are real by hermiticity, and $\vec{I}$ is the 3×3 matrix representation of SO_3 generators. For simplicity, we omitted all other CP invariant and SO_3 invariant couplings between Φ_R and the hadron fields. We also assume Φ_R not directly interacting with the singlet fields χ and $\chi^{'}$.

$$\mathcal{L}(W,\vec{\Phi}) = -\frac{1}{4}\vec{W}_{\mu\nu}^2 - \frac{1}{2}\partial_\mu {\vec{\Phi}_R}^2 - \frac{1}{2}\partial_\mu {\vec{\Phi}_I}^2 - V(\Phi) \tag{3.27}$$

where

$$\vec{W}_{\mu\nu} = \partial_\mu \vec{W}_\nu - \partial_\nu \vec{W}_\mu + e(\vec{W}_\mu \times \vec{W}_\nu) \tag{3.28}$$

$$\partial_\mu \vec{\Phi}_i = (\partial_\mu + e\vec{W}_\mu \times)\vec{\Phi}_i \qquad (i = R, I) \tag{3.29}$$

$$V(\Phi) = -\frac{1}{2}\lambda_R \vec{\Phi}_R^2 - \frac{1}{2}\lambda_I \vec{\Phi}_I^2 + \frac{1}{4}A_R(\vec{\Phi}_R^2)^2 + \frac{1}{4}A_I(\vec{\Phi}_I^2)^2 + \frac{1}{4}B(\vec{\Phi}_I \cdot \vec{\Phi}_R)^2 - \frac{1}{4}C\vec{\Phi}_R^2\vec{\Phi}_I^2 \tag{3.30}$$

in which the constants $\lambda_R, \lambda_I, ..., C$ are real by Hermiticity. As in [1], to insure that $V(\Phi)$ has a minimum, we assume

$$A_R > 0, \quad A_I > 0 \quad and \quad A_I A_R > \frac{1}{4}(B - C)^2 \tag{3.31}$$

In order that the minimum of $V(\Phi)$ occurs at the values of Φ_R and Φ_I given by

$$< \vec{\Phi}_R >= \vec{\rho}_R \neq 0, \qquad < \vec{\Phi}_I >= \vec{\rho}_I \neq 0 \tag{3.32}$$

we assume

$$A_I \lambda_R > \frac{1}{2}(B - C)\lambda_I, \qquad A_R \lambda_I > \frac{1}{2}(B - C)\lambda_R \tag{3.33}$$

In addition we require $C > 0$ so that $\vec{\rho}_R \parallel \vec{\rho}_I$.

Now we apply the gauge theory on discrete group discussed in last section to the case of $G = Z_2 \times Z_2$. G is isomorphism to Klein group K_4 and has four elements, denoted by $\{e, a, b, ab\}$ or the tensor product forms of $Z_2 = \{e, r\}, r^2 = e$,

$$\begin{aligned} e &= e_1 \otimes e_2, \qquad & a = e_1 \otimes r_2, \\ b &= r_1 \otimes e_2, \qquad & ab = r_1 \otimes r_2. \end{aligned} \tag{3.34}$$

The multiplication $*$ is defined as ,

$$g * h = g_1 h_1 \otimes g_2 h_2, \quad g = g_1 \otimes g_2, \quad h = h_1 \otimes h_2 \tag{3.35}$$

There are three steps in our approach to reconstruct the physical model. First we give a classification for all the fields on $M_4 \times G$ according to their transformation characteristics with respect to the discrete symmetry, more precisely, we can define the right displacement R_g as follows,

$$R_g f = g_1 g_2 f g_2^{-1} g_1^{-1}, \qquad g = g_1 \otimes g_2 \tag{3.36}$$

If we take the $g_1 = (CPT)^2, g_2 = CPCP = PCPC$, where C, P, T denote electrical charge conjugation, parity, and time reversal transformations respectively, we get a classification of the fields on $M_4 \times G$. Then we apply the differential calculus and gauge theory on $M_4 \times G$ to this case, and choose a consistent arrangement for those fields. Finally we do calculation straightforward and compare our results with the physical model we discussed.

Following this way, we arrange the fermion fields as,

$$\begin{aligned} \psi(x,e) &= \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \qquad & \psi(x,a) = -\begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \\ \psi(x,b) &= -\begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \qquad & \psi(x,ab) = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \end{aligned} \tag{3.37}$$

and the gauge fields as,

$$W_\mu(x,h) = W_\mu(x) \tag{3.38}$$

We also take the following arrangement of Higgs fields,

$$\begin{aligned} \Phi_a(x,h) &= \begin{pmatrix} 0 & g_R \vec{I} \cdot \vec{\Phi}_R \\ g_R \vec{I} \cdot \vec{\Phi}_R & 0 \end{pmatrix}, \quad & \Phi_b(x,h) = \begin{pmatrix} 0 & -ig_I \vec{I} \cdot \vec{\Phi}_I \\ ig_I \vec{I} \cdot \vec{\Phi}_I & 0 \end{pmatrix}, \\ \Phi_e(x,h) &= 0, & \Phi_{ab}(x,h) = 0 \end{aligned} \tag{3.39}$$

where h is any element of G. It can be checked that these arrangements are consistent and satisfy the extended anti-Hermitian condition (2.11). According to the equations (2.22), the Yukawa coupling terms are,

$$
\begin{aligned}
\mathcal{L}_Y ukawa \;&\propto \overline{\psi}(x,g)\phi_h(x,g)R_h\psi(x,g) \\
&\propto \overline{\psi}(x,g)\phi_a(x,g)R_a\psi(x,g) + \overline{\psi}(x,g)\phi_b(x,g)R_b\psi(x,g) \\
&\propto -g_R\overline{\psi}_L\vec{I}\cdot\vec{\Phi}_R\psi_R - g_R\overline{\psi}_R\vec{I}\cdot\vec{\Phi}_R\psi_L - ig_I\overline{\psi}_L\vec{I}\cdot\vec{\Phi}_I\psi_R + ig_I\overline{\psi}_R\vec{I}\cdot\vec{\Phi}_I\psi_L \\
&\propto -g_R\overline{\psi}\vec{I}\cdot\vec{\Phi}_R\psi - ig_I\overline{\psi}\vec{I}\cdot\vec{\Phi}_I\gamma_5\psi
\end{aligned}
\tag{3.40}
$$

where $\gamma_5\psi_R = +\psi_R, \quad \gamma_5\psi_L = -\psi_L$ have been used. For anther triplet ψ', the similar procedure can be done.As mentioned in previous section, $F_{g\mu}$ lead to the Higgs kinetic terms which is exactly the equation (3.29). It can be easily calculated because on $M_4 \times G$ we have required the ordinary gauge fields keep the same.

Now we calculate the Higgs potential which is more complicated. The only nontrivial curvatures are,

$$
\begin{aligned}
F_{aa} &= \Phi_a \cdot R_a\Phi_a - 1, \qquad & F_{bb} = \Phi_b \cdot R_b\Phi_b - 1, \\
F_{ab} &= \Phi_a \cdot R_a\Phi_b, \qquad & F_{ba} = \Phi_b \cdot R_b\Phi_a.
\end{aligned}
\tag{3.41}
$$

If we take the matrix representation of the SO_3 generators $\vec{I}$ and rewrite $\vec{I}\cdot\vec{\Phi}_R$ and $\vec{I}\cdot\vec{\Phi}_I$ as follows,

$$
\vec{I}\cdot\vec{\Phi}_R = \begin{pmatrix} 0 & -i\Phi_{R3} & i\Phi_{R2} \\ i\Phi_{R3} & 0 & -i\Phi_{R1} \\ -i\Phi_{R2} & i\Phi_{R1} & 0 \end{pmatrix}, \quad \vec{I}\cdot\vec{\Phi}_I = \begin{pmatrix} 0 & -i\Phi_{I3} & i\Phi_{I2} \\ i\Phi_{I3} & 0 & -i\Phi_{I1} \\ -i\Phi_{I2} & i\Phi_{I1} & 0 \end{pmatrix}, \tag{3.42}
$$

we have

$$
\begin{aligned}
&Tr[(\vec{I}\cdot\vec{\Phi}_i)^2] = 2\vec{\Phi}_i^2 & &Tr[(\vec{I}\cdot\vec{\Phi}_i)^4] = 2(\vec{\Phi}_i^2)^2 \qquad i = R, I \\
&Tr[((\vec{I}\cdot\vec{\Phi}_R)(\vec{I}\cdot\vec{\Phi}_I))^2] = 2(\vec{\Phi}_R\cdot\vec{\Phi}_I)^2 & &Tr[(\vec{I}\cdot\vec{\Phi}_R)^2(\vec{I}\cdot\vec{\Phi}_I)^2] = (\vec{\Phi}_R\cdot\vec{\Phi}_I)^2 + \vec{\Phi}_R^2\vec{\Phi}_I^2
\end{aligned}
\tag{3.43}
$$

With the help of these identities and the equation (2.17), the "Yang-Mills term" can be calculated straightforward,

$$
\begin{aligned}
< F, \overline{F} > \; = Tr[\;& \alpha(E_a^2F_{aa} + 2E_aE_bF_{aa}F_{bb} + E_b^2F_{bb}) \\
& +\beta(E_a^2F_{aa} + 2E_aE_bF_{ab}F_{ba} + E_b^2F_{bb}) + \gamma(E_a^2F_{aa} + 2E_aE_bF_{ab}F_{ba} + E_b^2F_{bb} \\
= Tr[\;& (\alpha+\beta+\gamma)(E_a^2F_{aa} + E_b^2F_{bb}) + 2(\gamma-\beta)E_aE_bF_{ab}^2 + 2\alpha E_aE_bF_{aa}F_{bb}]
\end{aligned}
\tag{3.44}
$$

Then we have the following expression of Higgs potential similar with the one in [1], i.e (3.30),

$$
\begin{aligned}
V(\vec{\Phi}_R, \vec{\Phi}_I) = \; & 2(\alpha+\beta+\gamma)E_a^2g_R^4(\vec{\Phi}_R^2)^2 + 2(\alpha+\beta+\gamma)E_b^2g_I^4(\vec{\Phi}_I^2)^2 \\
& -4g_R^2[(\alpha+\beta+\gamma)E_a^2 + \alpha E_aE_b]\vec{\Phi}_R^2 - 4g_I^2[(\alpha+\beta+\gamma)E_b^2 + \alpha E_aE_b]\vec{\Phi}_I^2 \\
& +[4(\gamma-\beta) + 2\alpha]E_aE_bg_R^2g_I^2(\vec{\Phi}_I\cdot\vec{\Phi}_R)^2 + 2\alpha E_aE_bg_R^2g_I^2\vec{\Phi}_R^2\vec{\Phi}_I^2
\end{aligned}
\tag{3.45}
$$

If we absorbed the Yukawa coupling constants g_R, g_I into the Higgs fields , and require that,

$$
E_a > 0, \qquad E_b > 0 \tag{3.46}
$$

$$
\alpha > 0, \qquad \alpha+\beta+\gamma > 0, \qquad \beta-\gamma > \frac{\alpha}{2} \tag{3.47}
$$

The condition (3.31)-(3.33) are satisfied automatically.

4 Weinberg-Branco Model on the Discrete Group $Z_2 \times Z_2 \times Z_2$

It was shown that if one imposed NFC in the context of the standard $SU_2 \times U_1$ gauge theory, then at least three Higgs doublets are needed in order to violate CP. In [18] a three-Higgs toy model of SCPV was constructed via the gauge theory on $Z_2 \times Z_2 \times Z_2$, which allow each Higgs couple to all the quarks but ensure the NFC by imposing some constrains on the Yukawa coupling constant matrices. Here we choose another approach to ensure the NFC, i.e preventing the third Higgs doublet Φ_3 from coupling to quarks. As in [2] , we take the element of third Z_2 group to be a reflection symmetry$R' : \Phi_3 \to -\Phi_3$ while all other fields remain unchanged. The first Z_2 group is still generated by $\theta = (CPT)^2$, the second Z_2 group is generated by $R : \Phi_2 \to -\Phi_2, \quad D^i_R \to -D^i_R$ with all the other fields unchanged.

Following the method in last section, we construct the Weinberg-Salam-Branco model on the discrete group $\theta \times R \times R' = g_0, g_1, g_2, g_3, g_4, g_5, g_6, g_7$

$$\begin{aligned}
g_0 &= e_1 \otimes e_2 \otimes e_3, & g_1 &= r_1 \otimes e_2 \otimes e_3, \\
g_2 &= e_1 \otimes r_2 \otimes e_3, & g_3 &= e_1 \otimes e_2 \otimes r_3, \\
g_4 &= r_1 \otimes r_2 \otimes e_3, & g_5 &= r_1 \otimes e_2 \otimes r_3, \\
g_6 &= e_1 \otimes r_2 \otimes r_3, & g_7 &= r_1 \otimes r_2 \otimes r_3.
\end{aligned} \tag{4.48}$$

where e_i denotes identity, r_i denotes the transformation of each discrete group respectively. The multiplication $*$ is defined as ,

$$g * h = g_1h_1 \otimes g_2h_2 \otimes g_3h_3, \quad g = g_1 \otimes g_2 \otimes g_3, \quad h = h_1 \otimes h_2 \otimes h_3 \tag{4.49}$$

According to their transformation properties of the symmetryθ, R and R', i.e.$R_g f = g_1g_2g_3 f g_3^{-1} g_2^{-1} g_1^{-1}, g = g_1 \otimes g_2 \otimes g_3$ we take the following arrangement of the fermion fields , gauge fields and Higgs fields, (To compare with the model in [2], we only write down the quark fields. the inclusion of lepton fields is straightforward and given in [15] [18].)

$$\psi(x,g) = \begin{pmatrix} L(x,g) \\ R(x,g) \end{pmatrix}, \quad L(x,g) = \begin{pmatrix} U(x,g) \\ D(x,g) \end{pmatrix}_L, \quad R(x,g) = \begin{pmatrix} U(x,g) \\ D(x,g) \end{pmatrix}_R, \tag{4.50}$$

where $U = (u, c, t), \quad D = (d, s, b)$, with the left-handed components U^i_L, D^i_L forming SU_2 doublets (U^i_L, D^i_L), while the right-handed components are singlets.

$$\begin{aligned}
U^i(x,g) = \; & +U^i(x), \; if \;\; g = g_0, g_2, g_3, g_6. & D^i_R(x,g) = \; & +D^i_R(x), \; if \;\; g = g_0, g_3, g_4, g_7 \\
& -U^i(x), \; if \;\; g = g_1, g_4, g_5, g_7. & & -D^i_R(x), \; if \;\; g = g_1, g_2, g_5, g_8
\end{aligned} \tag{4.51}$$

and $D^i_L(x,g)$ is arranged as same as $U^i(x,g)$. and the gauge fields as,

$$A_\mu(x,g) = A_\mu(x), \quad for \quad g \in G \tag{4.52}$$

The Higgs fields,

$$\Phi_{g_1}(x,h) = \begin{pmatrix} 0 & \Phi_1(x) \\ \Phi_1^\dagger(x) & 0 \end{pmatrix}, \; for \quad g \in G \tag{4.53}$$

$$\begin{aligned}
\Phi_{g_3}(x,g) = \; & \begin{pmatrix} 0 & \Phi_2(x) \\ \Phi_2^\dagger(x) & 0 \end{pmatrix}, \; if \; g = g_0, g_1, g_3, g_5 \\
or \; - & \begin{pmatrix} 0 & \Phi_2(x) \\ \Phi_2^\dagger(x) & 0 \end{pmatrix}, \; if \; g = g_2, g_4, g_6, g_7
\end{aligned} \tag{4.54}$$

$$
\begin{aligned}
\Phi_{g_4}(x,g) = & \quad \begin{pmatrix} 0 & \Phi_3(x) \\ \Phi_3^\dagger(x) & 0 \end{pmatrix}, \; if \;\; g = g_0, g_1, g_2, g_4 \\
& or \;\; - \begin{pmatrix} 0 & \Phi_3(x) \\ \Phi_3^\dagger(x) & 0 \end{pmatrix}, \; if \;\; g = g_3, g_5, g_6, g_7
\end{aligned} \tag{4.55}
$$

$$
\Phi_{g_0}(x,g) = \Phi_{g_2}(x,g) = \Phi_{g_5}(x,g) = \Phi_{g_6}(x,g) = \Phi_{g_7}(x,g) = 0 \tag{4.56}
$$

where

$$
\Phi_n(x) = \begin{pmatrix} \phi_n^{0*} & \phi_n^+ \\ -\phi_n^{+*} & \phi_n^0 \end{pmatrix} \otimes I_i \; . \; \begin{pmatrix} \Gamma_n^U & \\ & \Gamma_n^D \end{pmatrix} \tag{4.57}
$$

Γ_n^U, Γ_n^D are the coefficients of Yukawa couplings.

The above arrangement of the quarks and Higgs fields differ from the arrangement in [18], because we took the second and the third Z_2 to be the R and R' symmetry respectively. This modification is important to prevent the third Higgs particle from coupling to the quarks, the Higgs Φ_1 from coupling to D_R^i as well as the Higgs Φ_2 from coupling to U_R^i. We have

$$
\begin{aligned}
\mathcal{L}_{Yukawa} &= \sum_g \sum_h \overline{\psi}(x,g)\phi_h(x,g)R_h\psi(x,g) \\
&= -\sum_{n=1,2} \Gamma_{2ij}^U(\bar{U}_L^i\phi_2^{0*} - \bar{D}_L^i\phi_2^{+*})U_R^j + \Gamma_{1ij}^D(\bar{U}_L^i\phi_1^+ + \bar{D}_L^i\phi_1^0)D_R^j + h.c. \\
&= \Gamma_{2ij}^U\bar{L}^i\pi_2 D_R^j + \Gamma_{1ij}^U\bar{L}^i\tilde{\pi}_1 U_R^j + h.c.
\end{aligned} \tag{4.58}
$$

where $L^i = (\bar{U}_L^i, \bar{D}_L^i), \pi_n = (\phi_n^+, \phi_n^0)^T$,and $\tilde{\pi}_1 = i\sigma_2\pi_1$.

Thus we obtained the Yukawa interactions discussed in [2]. On the other hand, as we expected, this modification does not impact on the Higgs kinetic terms and the Higgs potential, which still contain three Higgs as in [18]

$$
\begin{aligned}
V(\Phi_1,\Phi_2,\Phi_3) = \; & \alpha Tr[E_1(\Phi_1\Phi_1^\dagger - 1) + E_2(\Phi_2\Phi_2^\dagger - 1) + E_3(\Phi_3\Phi_3^\dagger - 1)]^2 \\
& +\beta[E_1^2 Tr(\Phi_1\Phi_1^\dagger - 1)^2 + E_2^2 Tr(\Phi_2\Phi_2^\dagger - 1)^2 + E_3^2 Tr(\Phi_3\Phi_3^\dagger - 1)^2 \\
& +2E_1E_2 Tr\Phi_1\Phi_2^\dagger\Phi_2\Phi_1^\dagger + 2E_1E_3 Tr\Phi_1\Phi_3^\dagger\Phi_3\Phi_1^\dagger + 2E_3E_2 Tr\Phi_3\Phi_2^\dagger\Phi_2\Phi_3^\dagger] \\
& +\gamma[E_1^2 Tr(\Phi_1\Phi_1^\dagger - 1)^2 + E_2^2 Tr(\Phi_2\Phi_2^\dagger - 1)^2 + E_3^2 Tr(\Phi_3\Phi_3^\dagger - 1)^2 \\
& +E_1E_2 Tr(\Phi_1\Phi_2^\dagger\Phi_1\Phi_2^\dagger + \Phi_2\Phi_1^\dagger\Phi_2\Phi_1^\dagger) \\
& +E_1E_3 Tr(\Phi_1\Phi_3^\dagger\Phi_1\Phi_3^\dagger + \Phi_3\Phi_1^\dagger\Phi_3\Phi_1^\dagger) \\
& +E_3E_2 Tr(\Phi_3\Phi_2^\dagger\Phi_3\Phi_2^\dagger + \Phi_2\Phi_3^\dagger\Phi_2\Phi_3^\dagger)]
\end{aligned} \tag{4.59}
$$

It is very important that the third Higgs isn't cancelled from the Higgs potential, for at least three Higgs are needed to violate CP in the Weinberg-Salam-Branco model.

$$
\begin{aligned}
Tr(\Phi_m\Phi_m^\dagger) &= 2Tr(\Gamma_m^U\Gamma_m^{U\dagger} + \Gamma_m^D\Gamma_m^{D\dagger})(\pi_m^\dagger\pi_m) \\
Tr(\Phi_m\Phi_m^\dagger\Phi_m\Phi_m^\dagger) &= 2Tr(\Gamma_m^U\Gamma_m^{U\dagger}\Gamma_m^U\Gamma_m^{U\dagger} + \Gamma_m^D\Gamma_m^{D\dagger}\Gamma_m^D\Gamma_m^{D\dagger})(\pi_m^\dagger\pi_m)(\pi_m^\dagger\pi_m) \\
Tr(\Phi_m\Phi_n^\dagger\Phi_m\Phi_n^\dagger) &= 2Tr(\Gamma_m^U\Gamma_n^{U\dagger}\Gamma_m^U\Gamma_n^{U\dagger})(\pi_n^\dagger\pi_m)(\pi_n^\dagger\pi_m) + 2Tr(\Gamma_m^D\Gamma_n^{D\dagger}\Gamma_m^D\Gamma_n^{D\dagger})(\pi_m^\dagger\pi_n)(\pi_m^\dagger\pi_n) \\
& \quad +4Tr(\Gamma_m^D\Gamma_n^{D\dagger}\Gamma_m^U\Gamma_n^{U\dagger})[(\pi_n^\dagger\pi_n)(\pi_n^\dagger\pi_m) - (\pi_m^\dagger\pi_m)(\pi_n^\dagger\pi_n)] \\
Tr(\Phi_m\Phi_n^\dagger\Phi_n\Phi_m^\dagger) &= 2Tr(\Gamma_m^U\Gamma_n^{U\dagger}\Gamma_n^U\Gamma_m^{U\dagger} + \Gamma_m^D\Gamma_n^{D\dagger}\Gamma_n^D\Gamma_m^{D\dagger})(\pi_m^\dagger\pi_m)(\pi_n^\dagger\pi_n) \\
Tr(\Phi_m\Phi_m^\dagger\Phi_n\Phi_n^\dagger) &= 2Tr[(\Gamma_m^U\Gamma_m^{U\dagger} + \Gamma_m^D\Gamma_m^{D\dagger})](\Gamma_n^U\Gamma_n^{U\dagger} + \Gamma_n^D\Gamma_n^{D\dagger})(\pi_m^\dagger\pi_n)(\pi_n^\dagger\pi_m) \\
& \quad -2Tr(\Gamma_m^D\Gamma_m^{D\dagger}\Gamma_n^U\Gamma_n^{U\dagger} + \Gamma_m^U\Gamma_m^{U\dagger}\Gamma_n^D \; G_n^{D\dagger})(\pi_m^\dagger\pi_m)(\pi_n^\dagger\pi_n)
\end{aligned} \tag{4.60}
$$

With the help of these identities, we obtained

$$\begin{aligned} V(\pi_1,\pi_2,\pi_3) &= \sum_{n=1}^{3}[a_{nn}(\pi_n^\dagger\pi_n)^2\pi_n + m_n(\pi_n^\dagger\pi_n)] + \sum_{n<m}\{a_{nm}(\pi_n^\dagger\pi_n)(\pi_m^\dagger\pi_m) \\ &+b_{nm}(\pi_n^\dagger\pi_m)(\pi_m^\dagger\pi_n) + [c_{nm}(\pi_n^\dagger\pi_m)(\pi_n^\dagger\pi_m) + h.c.]\} \end{aligned} \tag{4.61}$$

where

$$c_{nm} = 2\gamma E_n E_m Tr(\Gamma_m^U\Gamma_n^{U\dagger}\Gamma_m^U\Gamma_n^{U\dagger} + \Gamma_n^D\Gamma_m^{D\dagger}\Gamma_n^D\Gamma_m^{D\dagger}) \tag{4.62}$$

If we assume the minimum to be at $\pi_i = \frac{1}{\sqrt{2}}(0, v_i e^{i\theta_i})^T$ and set $\theta_2 = 0$, we can classify the solutions of stationarity condition [17] as in [2]

(1)CP- conserving solution:

$$\theta_1 = \frac{1}{2}n\pi, \quad \theta_3 = \frac{1}{2}m\pi, \quad m, n \in \mathcal{Z} \tag{4.63}$$

(2)CP- violating solution:

$$cos2\theta_1 = \frac{1}{2}\left(\frac{d_{13}d_{23}}{d_{12}^2} - \frac{d_{23}}{d_{13}} - \frac{d_{13}}{d_{23}}\right), \qquad cos2\theta_3 = \frac{1}{2}\left(\frac{d_{13}d_{12}}{d_{23}^2} - \frac{d_{12}}{d_{13}} - \frac{d_{13}}{d_{12}}\right) \tag{4.64}$$

where $d_{nm} = c_{nm}v_n^2 v_m^2$, c_{nm} are given by (4.62).

5 conclusion and remark

We have reconstructed Georgi-Glashow-Lee model and Weinberg-Salam-Branco model with SCPV, based on the differential calculus and gauge theory on discrete group. We showed that in these models, all the Lagrangian terms containing Higgs particles are related as a whole by the gauge theory with respect to certain discrete symmetry, which can determine the Yukawa couplings, the Higgs kinetic terms and the Higgs potential with less ambiguity. In the first model to insure that $V(\Phi)$ has a minimum and the minimum occurs at the vacuum expected value of Φ_R, Φ_I , the metric parameters must satisfy some condition. In the second model, to insure NFC we take the two Z_2 symmetries to be the R and R', find this modification also lead to SCPV. Thus we established a correlation between the physics model and the nontrivial geometry structure behind it. The constraints of NCG on the physics models are to be investigated further.

Acknowledgments

One of the authors (C.Xiong) would like to thank Doctor Bin Chen for his lecture on NCG. The helpful discussions with Professors Chao-Hsi chang and Yue-Liang Wu , Doctors Jianming Li and Wei Zhang are truly appreciated.

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