

# CP Violation in the Neutral Kaon System

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## Abstract

$CP$  violation is an essential aspect of our understanding of the matter-antimatter asymmetry in the Universe. A natural question is whether the Standard Model (SM) of particle physics can provide necessary  $CP$  violation mechanism. In this paper, I will introduce the  $CP$  violation in the neutral kaon system and discuss the CKM matrix in SM which can provide explanation to experimental observations.

## 1 Introduction

As we know, under parity  $P$ , the spatial coordinates are inversed ( $\vec{x} \rightarrow -\vec{x}$ ); under the charge conjugation  $C$ , particle and antiparticle are interchanged, e.g.  $e^- \rightarrow e^+$ . The  $CP$  transformation combines operations of parity  $P$  and charge conjugation  $C$ . For example, under  $CP$ , a left-handed electron  $e_L^-$  is transformed into a right-handed  $e_R^+$ .

Both  $C$  and  $P$  are conserved in the strong and electromagnetic interactions (thus  $CP$  is conserved), however, the weak interactions violate  $C$  and  $P$ . For example, the nuclear  $\beta$  decay and muon decay  $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$ . Although weak interactions violate  $C$  and  $P$  separately,  $CP$  is still preserved in most weak processes. If  $CP$  is preserved rigidly, then the law of nature should be exactly the same both for matter and anti-matter. However, the  $CP$  symmetry is found to be violated in neutral  $K$  system where the  $K$  mesons are in the form of two isospin ( $I_3 = \pm\frac{1}{2}$ ) doublets with strangeness  $S = \pm 1$ , see Table 1 for detailed information.

	Quark combination	$I$	$I_3$	$S$	Mass(MeV)
$K^+$	$u\bar{s}$	1/2	1/2	1	494
$K^0$	$d\bar{s}$	1/2	-1/2	1	498
$K^-$	$\bar{u}s$	1/2	-1/2	-1	494
$\bar{K}^0$	$\bar{d}s$	1/2	1/2	-1	498

Table 1: Kaon (pseudoscalar) meson states as quark-antiquark combinations

## 2 The Neutral Kaon System

Considering the neutral components of  $K$  mesons,  $K^0$  ( $d\bar{s}$ ) and  $\bar{K}^0$  ( $\bar{d}s$ ) form particle and antiparticle counterparts with strangeness  $S = 1$  and  $-1$ , respectively. In comparison, neutron and anti-neutron are also neutral with baryon number  $B = 1$  and  $-1$ . However, those two are completely different since baryon number  $B$  is a rigidly conserved number while  $S$  is only conserved in strong interaction and electromagnetic interaction. In a weak process, a decay mode with  $\Delta S = 1$  can exist:

$$K^0 \rightarrow 2\pi, \quad \bar{K}^0 \rightarrow 2\pi$$

The reverse process of the second one:  $2\pi \rightarrow \bar{K}^0$  is also possible. Thus, mixing can occur via virtual intermediate  $\pi$  states:

$$K^0 \rightarrow 2\pi \rightarrow \bar{K}^0$$

From the above discussion we can see particle and anti-particle can mix via weak interaction (here  $\Delta S = 1$ ), however, no mixing will occur between neutron and anti-neutron since  $B$  is rigidly conserved. The transition from  $K^0$  to  $\bar{K}^0$  has  $\Delta S = 2$  and thus a second-order weak interaction. It implies a pure  $K^0$  state at  $t = 0$  will become a superposition of  $K^0$  and  $\bar{K}^0$  at a later time  $t$ .

Assume the decay amplitude of  $K^0 \rightarrow 2\pi$  is  $A$ , then the decay amplitude of  $\bar{K}^0 \rightarrow 2\pi$  is  $-A$  according to  $CPT$  theorem. Define the following linear combination:

$$|K_1^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle) \quad (1)$$

$$|K_2^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle) \quad (2)$$

Thus the decay amplitudes of  $K_1^0 \rightarrow 2\pi$  and  $K_2^0 \rightarrow 2\pi$  are  $\sqrt{2}A$  and  $0$ , respectively. Since  $2\pi$  is the easiest decay mode of  $K$  meson, Eqns.(1) and (2) imply  $K_2^0$  has a long life-time while  $K_1^0$  has a short life-time, and  $K_2^0$ s

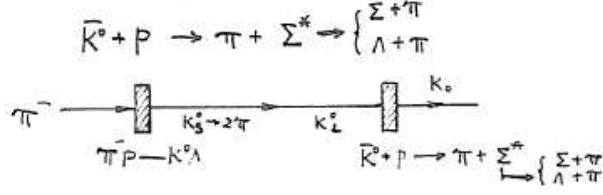


Figure 1:  $K^0$  regeneration.

decay via other channels (mainly through  $3\pi$ ). With a smaller phase space compared with  $2\pi$  mode, the life-time of  $K_2^0$  is thus longer. We can take  $K_S^0 = K_1^0$  and  $K_L^0 = K_2^0$ , thus

$$|K_S^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle) \quad (3)$$

$$|K_L^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle) \quad (4)$$

$K_1^0$  ( $K_S^0$ ) and  $K_2^0$  ( $K_L^0$ ) are two different states. They have definite life-time, but uncertain  $S$  value (since  $S$  is not conserved in weak interaction). In comparison,  $K^0$  and  $\bar{K}^0$  are states generated through strong process.

Experiments based on this physical picture can be done. Considering the generation of  $K^0$  through  $\pi^- p$  scattering:

$$\pi^- + p \rightarrow \Lambda + K^0$$

From Eqns. (3) and (4), we have

$$|K^0\rangle = \frac{1}{\sqrt{2}}(|K_S^0\rangle + |K_L^0\rangle) \quad (5)$$

where  $K_S^0$  component will disappear due to decaying into  $2\pi$  and  $K_L^0$  component is left. Notice from Eqn. (4) that  $K_L^0$  is the equal-amount mixing of  $K^0$  and  $\bar{K}^0$ , i.e., a portion of  $K^0$  has been converted into  $\bar{K}^0$ . To test the existence of  $\bar{K}^0$ , we can check via the following interaction:

$$\bar{K}^0 + p \rightarrow \pi + \Sigma^* \rightarrow \begin{cases} \Sigma + \pi \\ \Lambda + \pi \end{cases}$$

Since  $\Sigma^*$  cannot be generated from  $K^0$  (strangeness number for  $\Sigma^*$  and  $K^0$  are  $S = -1$  and  $1$ , respectively), if  $(\Sigma + \pi)$  or  $(\Lambda + \pi)$  can be detected,  $\bar{K}^0$  can thus be verified to exist in the beam. Experiments have shown, after going through the second slab,  $\bar{K}^0$  is absorbed and  $K^0$  is left to continue propagation. This process is called  $K^0$  **regeneration**.

## 2.1 Strageness Oscillations (neglecting CP violation)

The wavefunction of a non-stationary decaying state with frequency  $\omega_R = E_R/\hbar$  ( $E_R$  is the resonance energy) and lifetime  $\Gamma = \frac{\hbar}{\tau}$  can be expressed as (in units of  $\hbar = c = 1$ ):

$$\Psi(t) = \Psi(0)e^{-i\omega_R t}e^{-t/2\tau} = \Psi(0)e^{-t(iE_R + \Gamma/2)} \quad (6)$$

If measured in the particle's rest frame, given the mass of  $K_S^0$  is  $m_S$  and the width is  $\Gamma_S$ , we have the amplitude of  $K_S^0$

$$\Psi(K_S^0, t) = \Psi(K_S^0, 0)e^{-(im_S + \Gamma_S/2)t} \quad (7)$$

Similarly, the amplitude of  $K_L^0$  is:

$$\Psi(K_L^0, t) = \Psi(K_L^0, 0)e^{-(im_L + \Gamma_L/2)t} \quad (8)$$

According to Eqn. (3) and (4), the amplitude of  $\bar{K}^0$  at time  $t$  is:

$$\begin{aligned} \Psi(\bar{K}^0, t) &= \frac{1}{\sqrt{2}}[\Psi(K_L^0, t) - \Psi(K_S^0, t)] \\ &= \frac{1}{\sqrt{2}}[\Psi(K_L^0, 0)e^{-(im_L + \Gamma_L/2)t} - \Psi(K_S^0, 0)e^{-(im_S + \Gamma_S/2)t}] \end{aligned} \quad (9)$$

Assume at  $t = 0$ ,  $K^0$  is generated from  $\pi^-p$  scattering. Since  $K^0$  is the equal-amount mixing of  $K_S^0$  and  $K_L^0$ , i.e.  $\Psi(K_S^0, 0) = \Psi(K_L^0, 0) = \frac{1}{\sqrt{2}}$ , we thus have

$$\Psi(\bar{K}^0, t) = \frac{1}{2}[e^{-(im_L + \Gamma_L/2)t} - e^{-(im_S + \Gamma_S/2)t}] \quad (10)$$

The probability to find  $\bar{K}^0$  at time  $t$  (or intensity) is

$$\begin{aligned} \Gamma(K_{t=0}^0 \rightarrow \bar{K}^0) &= |\Psi(\bar{K}^0, t)|^2 \\ &= \frac{1}{4}[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2\cos(\Delta m \cdot t)e^{-\frac{1}{2}(\Gamma_S + \Gamma_L)t}] \end{aligned} \quad (11)$$

Using the same method to get

$$\Gamma(K_{t=0}^0 \rightarrow K^0) = \frac{1}{4}[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2\cos(\Delta m \cdot t)e^{-\frac{1}{2}(\Gamma_S + \Gamma_L)t}] \quad (12)$$

where  $\Delta m = m_L - m_S$  (since  $K_L^0$  and  $K_S^0$  are not particle-antiparticle pairs, they do not need to have the same mass). We can further infer that the number of  $\bar{K}^0$  in the beam is sensitively dependent on  $\Delta m$ . Current experiments give  $\Delta m = m_{K_L^0} - m_{K_S^0} = (3.483 \pm 0.006) \times 10^{-12} \text{MeV}$  [1] (assuming *CPT*),

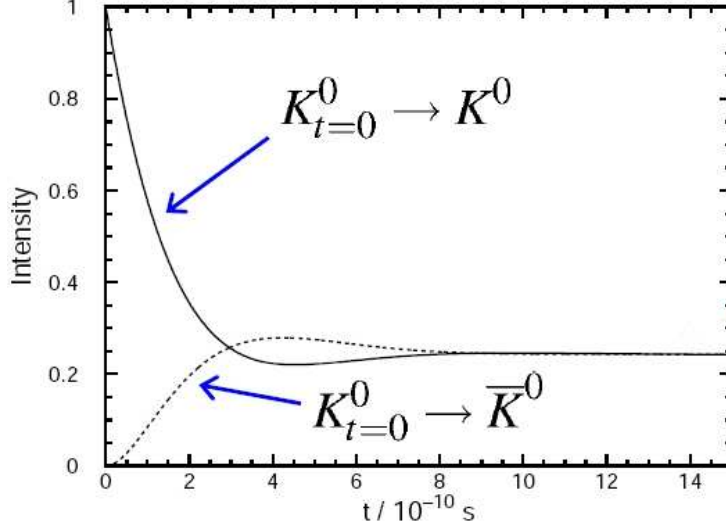


Figure 2: Oscillations in  $K^0$  and  $\bar{K}^0$  intensities with time. From Ref. [3]

which shows  $K_L^0$  is heavier than  $K_S^0$  and implies the correspondence to an oscillation period of

$$T_{oscillation} = \frac{2\pi\hbar}{\Delta m} \approx 1.2 \times 10^{-9} s.$$

Compared with current experimental results[1]:

$$\tau_S = (0.8953 \pm 0.0005) \times 10^{-10} s$$

$$\tau_L = (5.114 \pm 0.021) \times 10^{-8} s$$

we can see the oscillation period is longer than the  $K_S^0$  lifetime and in consequence strong oscillations are not seen (Figure 2).

## 2.2 Decays of CP Eigenstates (neglecting CP violation)

Charge conjugate  $C$  operation

$$CK^0 = \bar{K}^0, C\bar{K}^0 = K^0$$

Parity  $P$  operation

$$PK^0 = -K^0, P\bar{K}^0 = -\bar{K}^0$$

Table 2: C, P and CP value

	P	C	CP
$K_S^0(K_2)$	−	+	−
$K_L^0(K_1)$	−	−	+
$(\pi^+\pi^-)_{l=0}$	+	+	+
$(\pi^+\pi^-\pi^0)_{l=0}$	−	+	−

Under combined  $C$  and  $P$  ( $CP$ ) transformation

$$CPK^0 = -\bar{K}^0, CP\bar{K}^0 = -K^0$$

Thus we have

$$CPK_S^0 = CP\frac{1}{\sqrt{2}}(K^0 - \bar{K}^0) = \frac{1}{\sqrt{2}}(-\bar{K}^0 + K^0) = +K_S^0$$

$$CPK_L^0 = CP\frac{1}{\sqrt{2}}(K^0 + \bar{K}^0) = \frac{1}{\sqrt{2}}(-\bar{K}^0 - K^0) = -K_L^0$$

When considering the  $2\pi$  and  $3\pi$  decay modes:

- $\pi^0\pi^0, \pi^+\pi^-$ . The total wavefunction must be symmetric under interchange of the two particles according to Bose symmetry, thus  $CP = +1$
- $\pi^+\pi^-\pi^0$ . For  $l = 0$ : since  $CP$  of  $\pi^+\pi^-$  is  $+1$ , with the information that  $\pi^0$  has  $C = 1$  ( $\pi^0 \rightarrow \gamma\gamma$ ) and intrinsic parity  $P = -1$ , we can see  $CP$  of  $\pi^+\pi^-\pi^0$  is  $-1$ . For  $l > 0$ :  $CP$  of the system can be both  $+$  and  $-$ , however such modes are highly suppressed.

- $\pi^0\pi^0\pi^0$ . No matter what  $l$  value is,  $CP = -1$

See Table 2 for the summary of  $C$ ,  $P$  and  $CP$  values. According to the above analysis,  $K_S^0 \rightarrow 2\pi$  and  $K_L^0 \rightarrow (3\pi)_{l=0}$  are favored while  $K_L^0 \rightarrow 2\pi$  is strictly forbidden.

### 3 CP violation: $\pi\pi$ decays

However in 1964, Cronin *et al.* [4] observed  $K_L^0 \rightarrow \pi^+\pi^-$  decays with

$$\frac{\Gamma(K_L^0 \rightarrow \pi^+\pi^-)}{\Gamma(K_L^0 \rightarrow all)} = (2.3 \pm 0.3) \times 10^{-3}$$

This is the phenomenon of  $CP$  violation which implies weak interaction violates  $CP$ . In the neutral kaon system,  $\Delta S = 2$  dynamics transform the flavor eigenstates  $K^0$  and  $\bar{K}^0$  into mass eigenstates  $K_S^0$  and  $K_L^0$ , while  $\Delta S = 1$  arouses the decay into pions,

$$K^0 \longleftrightarrow \bar{K}^0 (\Delta S = 2) \implies K_L^0 \rightarrow \pi\pi (\Delta S = 1)$$

and both of them can contribute to the  $CP$  violation. In other words, this means there are two possible explanations of  $CP$  violation in the kaon system:

1). The long-lived  $K_L^0$  and short-lived  $K_S^0$  are admixture of  $K_1^0$  and  $K_2^0$  which are  $CP$  eigenstates. It implies we cannot simply take  $K_S^0 = K_2^0$  and  $K_L^0 = K_1^0$ , but take

$$K_L^0 = \frac{K_2^0 + \varepsilon_1 K_1^0}{\sqrt{1 + |\varepsilon_1|^2}}, \quad K_S^0 = \frac{K_1^0 + \varepsilon_2 K_2^0}{\sqrt{1 + |\varepsilon_2|^2}} \quad (13)$$

where  $\varepsilon_1$  and  $\varepsilon_2$  are small complex numbers (of the order  $10^{-3}$ ). Under  $CPT$  theorem,  $\varepsilon_1 = \varepsilon_2 \equiv \varepsilon$ .

2).  $CP$  is violated in the decay process and is parameterised by  $\varepsilon'$

$$|K_L^0\rangle = |K_2\rangle_{CP=-1} \rightarrow \begin{cases} \pi\pi\pi & CP = -1 \\ \pi\pi & CP = +1 \end{cases}$$

Current studies show both 1) and 2) contribute to  $CP$  violation in the Kaon system while mechanism 1) dominates ( $\varepsilon'/\varepsilon \sim 10^{-3}$ ).

In quantum mechanics, the weak interaction connecting  $K^0$  and  $\bar{K}^0$  can be parametrized by Hamiltonian  $H_{eff}$  which is described by the combination of two Hermitian  $2 \times 2$  matrices  $M$  and  $\Gamma$ ,

$$H_{eff} = M - i\frac{\Gamma}{2} = \begin{pmatrix} M_{11} - i\Gamma_{11}/2 & M_{12} - i\Gamma_{12}/2 \\ M_{21}^* - i\Gamma_{21}^*/2 & M_{22} - i\Gamma_{22}/2 \end{pmatrix}$$

where  $\Gamma$  is related to the life-times, and  $M$  is mass matrix. In the basis of  $(K^0, \bar{K}^0)$ , the diagonal element  $M_{11}$ ,  $M_{22}$  are masses of  $K^0$  and  $\bar{K}^0$  and  $\Gamma_{11}$ ,  $\Gamma_{22}$  are lifetimes of  $K^0$  and  $\bar{K}^0$ . Since the best limit on  $CPT$  symmetry (according to PDG 2006) is  $|m_{\bar{K}^0} - m_{K^0}|/m_{average} < 10^{-18}$  at 90% confident level, we view  $CPT$  as an exact symmetry, which leads to  $M_{11} = M_{22}$ ,  $\Gamma_{11} = \Gamma_{22}$  and  $\varepsilon_1 = \varepsilon_2 \equiv \varepsilon$ . The mass and lifetime eigenvalues obtained through solving Schrodinger equation are given by

$$M_S - \frac{i}{2}\Gamma_S = M_{11} - \frac{i}{2}\Gamma_{11} - \sqrt{(M_{12} - \frac{i}{2}\Gamma_{12})(M_{12}^* - \frac{i}{2}\Gamma_{12}^*)} \quad (14)$$

$$M_L - \frac{i}{2}\Gamma_L = M_{11} - \frac{i}{2}\Gamma_{11} + \sqrt{(M_{12} - \frac{i}{2}\Gamma_{12})(M_{12}^* - \frac{i}{2}\Gamma_{12}^*)} \quad (15)$$

$$\varepsilon = \frac{\sqrt{M_{12} - i\Gamma_{12}/2} - \sqrt{M_{12}^* - i\Gamma_{12}^*/2}}{\sqrt{M_{12} - i\Gamma_{12}/2} + \sqrt{M_{12}^* - i\Gamma_{12}^*/2}} \approx \frac{iIm(M_{12}) + Im(\Gamma_{12}/2)}{\Delta m_{L-S} + i\Delta\Gamma_{S-L}/2} \quad (16)$$

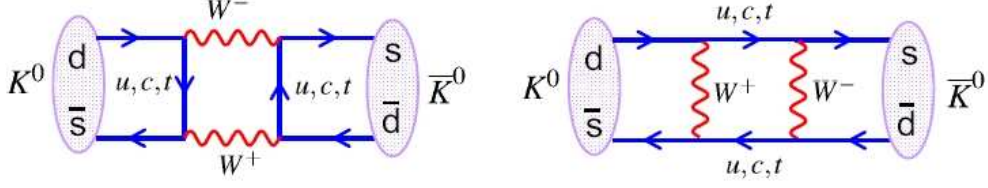


Figure 3: Box diagram for  $K^0 \rightarrow \bar{K}^0$  transition. From Ref. [3]

with  $\Delta m_{L-S} = m_L - m_S$  and  $\Delta \Gamma_{S-L} = \Gamma_S - \Gamma_L$ . Using  $\Delta m_{L-S} \approx \Delta \Gamma_{S-L}/2$  and  $Im(\Gamma_{12}) \ll Im(M_{12})$  give[2]

$$\varepsilon \approx \frac{Im(M_{12})}{\sqrt{2}\Delta m_{L-S}} e^{i\phi_\varepsilon} \quad (17)$$

Here  $\varepsilon$  is a small parameter quantifying the  $CP$  violation. The following five variables are measured ( $|\eta_\pm|$ ,  $\phi_\pm$ ,  $|\eta_{00}|$ ,  $\phi_{00}$  and the semileptonic asymmetry  $\Delta$ ):

$$\eta_\pm = \frac{\langle \pi^+ \pi^- | H_{eff} | K_L^0 \rangle}{\langle \pi^+ \pi^- | H_{eff} | K_S^0 \rangle} = |\eta_\pm| e^{i\phi_\pm} \simeq \varepsilon + \varepsilon' \quad (18)$$

$$\eta_{00} = \frac{\langle \pi^0 \pi^0 | H_{eff} | K_L^0 \rangle}{\langle \pi^0 \pi^0 | H_{eff} | K_S^0 \rangle} = |\eta_{00}| e^{i\phi_{00}} \simeq \varepsilon - 2\varepsilon' \quad (19)$$

$$\Delta = \frac{\Gamma(K_L \rightarrow \pi^- l^+ \nu_e) - \Gamma(K_L \rightarrow \pi^+ l^- \bar{\nu}_e)}{\Gamma(K_L \rightarrow \pi^- l^+ \nu_e) + \Gamma(K_L \rightarrow \pi^+ l^- \bar{\nu}_e)} \simeq 2Re\varepsilon \quad (20)$$

where

$$\varepsilon' = \frac{1}{\sqrt{2}} Im\left(\frac{A_2}{A_0}\right) e^{i(\delta_2 - \delta_0)}$$

with  $A_0, A_2$  and  $\delta_0, \delta_2$  being the decay amplitudes and phase angles of  $I = 0$  and  $I = 2$ , respectively.

Both  $\eta_\pm, \eta_{00} \neq 0$  show  $CP$  violation.  $\varepsilon$ , which is common to both  $\eta_\pm$  and  $\eta_{00}$ , is related to  $\Delta S = 2$  process and reflects the portion of  $CP$  from the state mixing via the box diagram of mixing in Figure 3. This is called **Indirect  $CP$  violation**;  $\varepsilon'$ , however, represents **Direct  $CP$  violation** in  $\Delta S = 1$  weak decay process. If  $\varepsilon \neq 0$ , then indirect  $CP$  violation exists; if  $\varepsilon' \neq 0$ , direct  $CP$  violation exists. From Equation (11) we expect the ratio of the intensity to be

$$\frac{\Gamma_{2\pi}(t)}{\Gamma_{2\pi}(0)} = e^{-\Gamma_S t} + |\eta_\pm|^2 e^{-\Gamma_L t} + 2|\eta_\pm| e^{-[(\Gamma_L + \Gamma_S)/2]t} \cos(\Delta m t + \phi_\pm) \quad (21)$$

where  $\phi_\pm$  is a phase angle between the amplitudes of  $K_S^0 \rightarrow 2\pi$  and  $K_L^0 \rightarrow 2\pi$ . Figure 4 shows the effect when the interference term is added and not



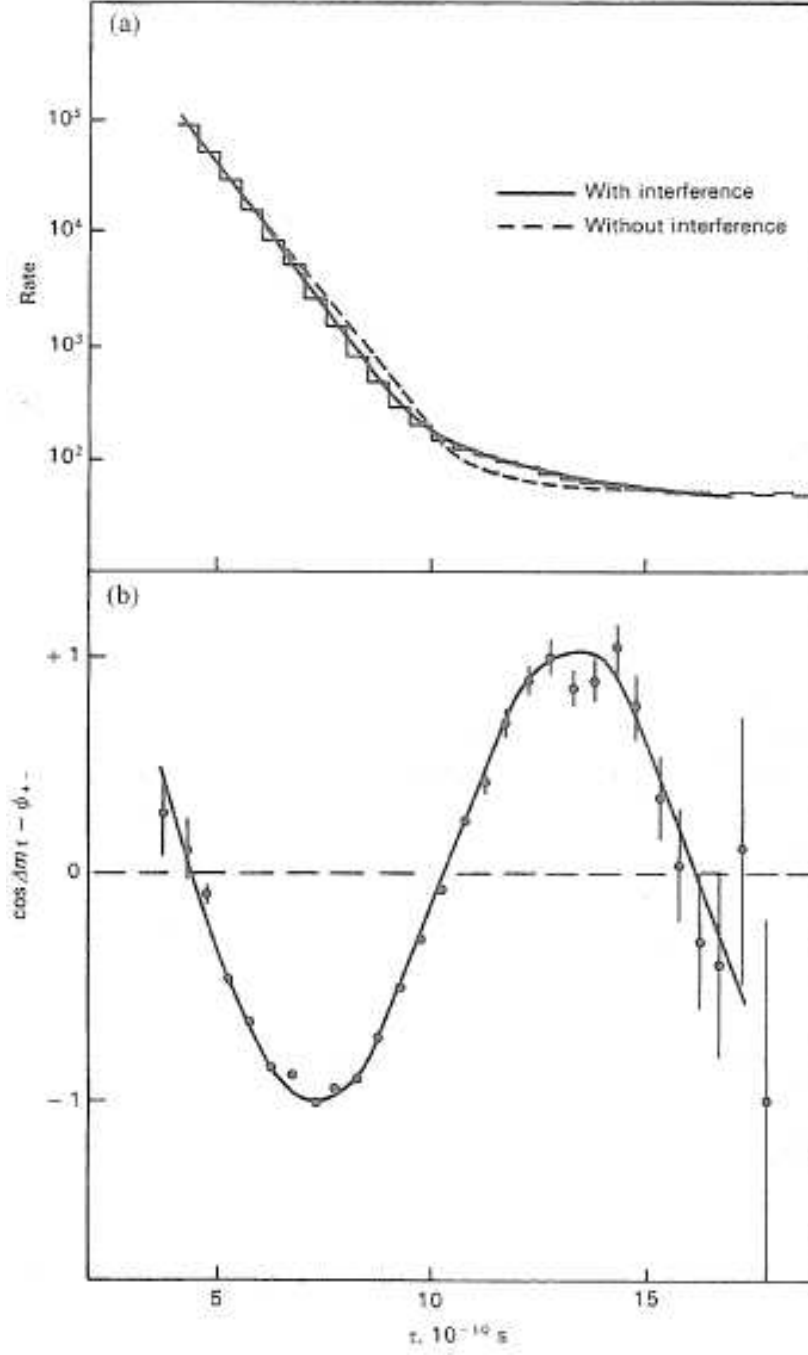


Figure 4: (a) Event rates for  $K^0 \rightarrow \pi^+\pi^-$  as a function of time. The best fit shows the existence of interference between  $K_S^0$  and  $K_L^0$  amplitudes. (b)  $\Delta m_{K_L-K_S}$  and phase angle  $\phi_{\pm}$  can be exacted from the fit. From Ref. [5]

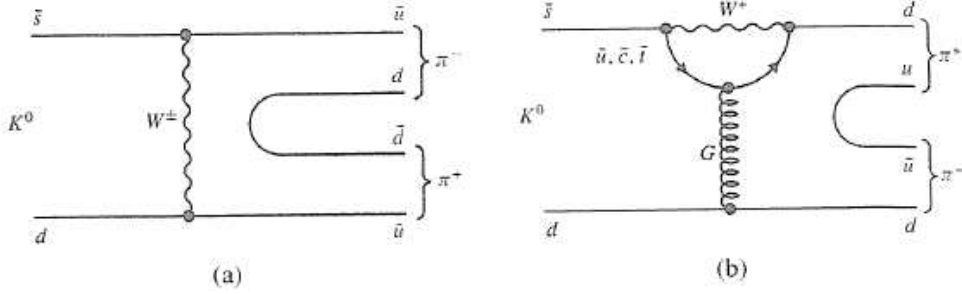


Figure 5: (a) Tree diagram for  $K^0 \rightarrow \pi^+\pi^-$  by exchanging  $W^\pm$ . (b) Penguin diagram for  $K^0 \rightarrow \pi^+\pi^-$  via quark states. The interference between (a) and (b) introduces direct  $CP$  violation in the decay process. From Ref. [5]

added. The asymmetry between  $K^0 \rightarrow \pi^+\pi^-$  and  $\bar{K}^0 \rightarrow \pi^+\pi^-$  can be measured by using Equation (21)

$$A_\pm(t) = \frac{2|\eta_\pm|e^{[(\Gamma_S-\Gamma_L)/2]t}\cos(\Delta mt + \phi_\pm)}{1 + |\eta_\pm|^2e^{(\Gamma_S-\Gamma_L)t}} \quad (22)$$

The values of  $\phi_\pm$  and  $\eta_\pm$  can be extracted from the above measurement if  $\Delta m$  is given as input parameter from other measurements. Latest experimental results are shown as follows:

$$|\eta_\pm| = (2.236 \pm 0.007) \times 10^{-3}, \quad \phi_{00} = 43.52^\circ \pm 0.05^\circ$$

$$|\eta_{00}| = (2.225 \pm 0.007) \times 10^{-3}, \quad \phi_\pm = 43.50^\circ \pm 0.06^\circ$$

$$\left| \frac{\eta_{00}}{\eta_\pm} \right| = 0.9950 \pm 0.0008$$

$$|\varepsilon| = (2.232 \pm 0.007) \times 10^{-3}$$

$$Re(\varepsilon'/\varepsilon) = (1.66 \pm 0.26) \times 10^{-3}$$

$$Im(\varepsilon'/\varepsilon) = (-3.3 \pm 4.4) \times 10^{-5}$$

$$\phi_\varepsilon = 43.51^\circ \pm 0.05^\circ$$

Based on those experimental observations, both indirect  $CP$  violation and direct  $CP$  violation appears to have been established. In the Standard Model, the source of direct  $CP$  violation is the interference between the tree diagram and 'penguin' diagram, see Figure 5. The value of  $\varepsilon'$  is a very small number since it inversely dependent on the mass of top quark ( $\sim 175 GeV$ ).

### 3.1 $\varepsilon'/\varepsilon$ : Comparison with the CKM Model

It is convenient to use Wolfenstein parameterization of matrix  $V_{CKM}$  given by[6]

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

To discuss the comparison of  $CP$  violation with the CKM model, we need to keep higher order  $\lambda$  terms and add  $-A^2\lambda^5(\rho + i\eta)$  and  $-A\lambda^4(\rho + i\eta)$  to  $V_{cd}$  and  $V_{ts}$ , respectively. Here,  $\lambda = \sin\theta_c = 0.22 \pm 0.002$ ,  $A = 0.8 \pm 0.1$  and  $CP$ -violating parameter is characterized by a non-zero  $\eta$ .

Before extracting the parameter  $\eta$  from the measured value of  $\varepsilon/\varepsilon'$ , information from many other elements such as matrix elements of some weak operators, values of  $A$  and  $\rho$ , top quark mass  $m_t$  etc. need to be known very precisely. Since the varying accuracies in those related quantities, the CKM model can only be tested in a more qualitative way than a quantitative way. Theoretical calculations [7] of  $\varepsilon'/\varepsilon$  using  $m_c = 1.3\text{GeV}$ ,  $\Lambda_{QCD} = 325\text{MeV}$ ,  $m_t = 170\text{GeV}$  and  $m_s(m_c) = (150 \pm 20)\text{MeV}$  give the following results:

$$-2.1 \times 10^{-4} \leq \frac{\varepsilon'}{\varepsilon} \leq 13.3 \times 10^{-4}$$

which is consistent with experimental measurements.

### 3.2 Explanation of $\Gamma(\bar{K}_{t=0}^0 \rightarrow K^0) \neq \Gamma(K_{t=0}^0 \rightarrow \bar{K}^0)$ using the CKM matrix

Previously in Eqns. (11) and (12) we obtained expressions for strangeness oscillations without considering  $CP$  violation. If we include the effects of  $CP$  violation,

$$\Gamma(\bar{K}_{t=0}^0 \rightarrow K^0) \propto \frac{1}{4}(1+4\text{Re}(\varepsilon))[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2\cos(\Delta m \cdot t)e^{-\frac{1}{2}(\Gamma_S + \Gamma_L)t}] \quad (23)$$

$$\Gamma(K_{t=0}^0 \rightarrow \bar{K}^0) \propto \frac{1}{4}(1-4\text{Re}(\varepsilon))[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2\cos(\Delta m \cdot t)e^{-\frac{1}{2}(\Gamma_S + \Gamma_L)t}] \quad (24)$$

we will find that  $\Gamma(\bar{K}_{t=0}^0 \rightarrow K^0) \neq \Gamma(K_{t=0}^0 \rightarrow \bar{K}^0)$ . To explain this phenomenon in terms of the CKM matrix, consider the box diagram (Figure. 3), where we need to sum over all possible quark exchanges. Consider just one diagram for the concern of simplicity, see Figure 6. The difference in

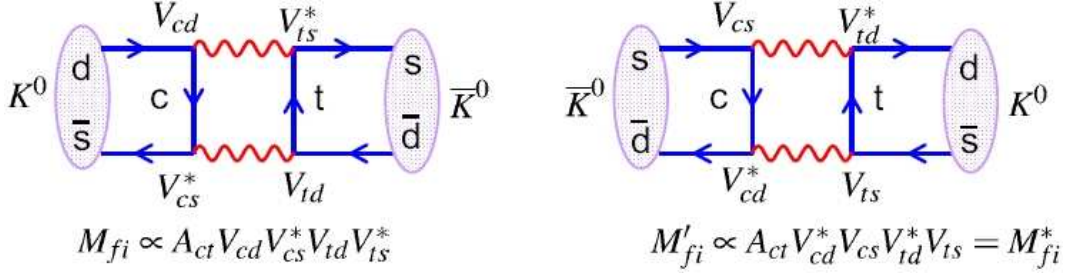


Figure 6: Comparison of equivalent diagram for  $K^0 \rightarrow \bar{K}^0$  and  $\bar{K}^0 \rightarrow K^0$  transition. From Ref. [3]

rates is

$$\Gamma(K_{t=0}^0 \rightarrow \bar{K}^0) - \Gamma(\bar{K}_{t=0}^0 \rightarrow K^0) \propto M_{fi} - M_{fi}^* = 2\text{Im}(M_{fi})$$

thus the rates can be different only when the CKM matrix is imaginary

$$|\varepsilon| \propto \text{Im}(M_{fi})$$

We can show in the kaon system

$$|\varepsilon| \propto A_{ut} \cdot \text{Im}(V_{ud} V_{us}^* V_{td} V_{ts}^*) + A_{ct} \cdot \text{Im}(V_{cd} V_{cs}^* V_{td} V_{ts}^*) + A_{tt} \cdot \text{Im}(V_{td} V_{ts}^* V_{td} V_{ts}^*)$$

In terms of the Wolfenstein parameterisation, the above relationship from kaon system is equivalent to

$$|\varepsilon| = \eta[1 - \rho + \text{const.}]$$

In the CKM matrix,  $CP$  violation term enters as

$$e^{i\delta} = \frac{\rho + i\eta}{\sqrt{\rho^2 + \eta^2}}$$

Together with constraints provided by other experiments, we can get  $(\eta, \rho)$  plot (see Figure 7). Studies show all experimental results are consistent with a single value for  $(\eta, \rho)$  and there is no evidence for  $CP$  violation other than in the Standard Model.

## 4 Experiments

**KTeV** experiment is located at Fermilab near Chicago whose goal is to extract the value of the  $CP$ -violation parameter  $\text{Re}(\varepsilon'/\varepsilon)$  with an uncertainty

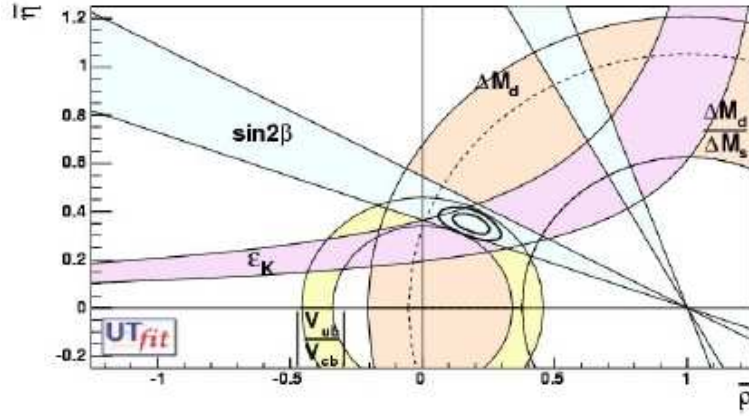


Figure 7: Relationship between  $\bar{\eta}$  and  $\bar{\rho}$ . From Ref. [3]

of  $1 \times 10^{-4}$ . Another goal of the experiment is to observe 4-body  $K_L^0$  decays with branching ratio  $\sim 10^{-11}$ . The KeV detector can simultaneously measure the charged and neutral decay of  $K_L^0$  and  $K_S^0$ . The momentum resolution for a charged particle is  $\sigma_p/p = 0.17\% \oplus 0.0071\% \times p(\text{in GeV}/c)$ , and the average inefficiency for a hit pair reconstruction is  $\sim 3.7\%$ . The energy resolution of calorimeter is  $\sigma_E/E = 2\%/\sqrt{E} \oplus 0.4\%$ ; the average resolution of position is  $\sim 1.2\text{mm}$  and  $\sim 2.4\text{mm}$  for small crystals and large crystals, respectively[8].

**CPLEAR** experiment ran from 1990 to 1996 at CERN. The detector measures decays from kaons produced via  $p\bar{p}$  reactions:  $p\bar{p} \rightarrow K^-\pi^+K^0$  and  $p\bar{p} \rightarrow K^+\pi^-\bar{K}^0$ . Particles produced in those processes are almost at rest due to low energy, thus production process and decay process are observed in the same detector. Charge of  $K^\pm\pi^\mp$  in the production is used to tag the initial neutral kaon as  $K^0$  or  $\bar{K}^0$  while charge of decay products tags the decay as being either  $K^0$  or  $\bar{K}^0$ . The momentum resolution of  $K^0$  ( $\sigma_{p_t}/p_t$ ) was about 0.25% if kinematically constrained fits are applied. The energy resolution for the electromagnetic calorimeter is about  $\sigma_E/E \approx 13\%/\sqrt{E}$  and the position resolution is about 5mm[8].

Other experiments like E731, NA31, NA48 are introduced in Ref. [8].

## 5 Summary

In the neutral kaon system, contributions of  $CP$  violations have been shown to come from three parts: the mixing ( $\varepsilon$ , indirect violation) which is the dominant source, the decay ( $\varepsilon'/\varepsilon$ , direct violation) and the interference between mixing and decay ( $\text{Im}(\varepsilon)$ ). All observations are consistent with the CKM model, where the  $CP$  violation enters via a non-trivial complex num-

ber in the  $3 \times 3$  matrix. This  $CP$  violation in the quark sector, together with other contributions, can principally result in the observed matter-antimatter asymmetry. However, further studies need to be done to check that.

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