

Spontaneous CP Violation in SUSY $SO(10)$

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Abstract

A scenario is suggested for spontaneous CP violation in non- $SUSY$ and $SUSY$ $SO(10)$. The idea is to have a scalar potential which generates spontaneously a phase, at the high scale, in the VEV that gives a mass to the RH neutrinos. As a possible realization the case of the minimal renormalizable $SUSY$ $SO(10)$ is discussed in detail and one finds that a phase is induced in the CKM matrix. It is also pointed out that, in these models, the scales of Baryogenesis, Seesaw, Spontaneous CP violation and Spontaneous $U(1)_{PQ}$ breaking are all of the same order of magnitude.

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There are three manifestations of CP violation in Nature:

- 1) *Fermi scale CP violation* as is observed in the K and B decays [1]. This violation is induced predominantly by a complex mixing matrix of the quarks (CKM).
- 2) *The cosmological matter antimatter asymmetry (BAU)* is an indication for high scale CP violation [2]. In particular, it's most popular explanation via leptogenesis [3] requires CP breaking decays of the heavy right-handed (RH) neutrinos.
- 3) *The strong CP problem* called also the QCD Θ problem [4] lies in the non-observation of CP breaking in the strong interactions while there is an observed CP violation in the interaction of quarks.

It is still not clear if there is one origin to those CP breaking manifestations. What is the nature of the violation of CP ? Is it intrinsic in terms of complex Yukawa couplings or due to spontaneous generation of phases in the Higgs VEVs ?

Spontaneous violation of CP [5] is more difficult to realize but has advantages with respect to the intrinsic ones:

- 1) It is more elegant and involves less parameters. The intrinsic breaking becomes quite arbitrary in the framework of $SUSY$ and GUT theories.
- 2) It solves the $SUSY$ CP violation problem (too many potentially complex parameters) as all parameters are real.
- 3) It leads to the vanishing of Θ_{QCD} (but not ArgDetM) at the tree level. This can be used as a first step towards solving the CP problem by adding extra symmetries and exotic quarks [6][7] [8].

For good recent discussion of spontaneous CP violation ($SCPV$), with many references, see Branco and Mohapatra [9].

It is preferable to break CP at a high scale. This is what we need for the BAU . Especially if this is due to leptogenesis i.e. CP violating decays of heavy neutrinos, it is mandatory. This is also needed to cure the domain wall problem [10].

Also, $SCPV$ cannot take place in the standard model (SM) because of gauge invariance. Additional Higgs bosons must be considered and those lead generally to flavor changing neutral currents. The best way to avoid these is to make the additional scalars heavy [9].

In this case, the scale of CP violation can be related to the seesaw [11] scale as well as to the $U(1)_{PQ}$ [12] breaking scale, i.e. the “axion window” [4].

Branco, Parada and Rebelo discussed in their paper [8] also the possibility of a common origin to all CP violations. Their model however is in the framework of a non-SUSY Standard Model (SM), extended by a heavy complex singlet Higgs and an exotic vectorlike quark.

I would like to suggest in this letter a scenario, along this line [13] [14], for $SVCP$ in SUSY GUTs by giving an explicit realization in the framework of the *minimal renormalizable SUSY $SO(10)$* [15] (without the need for exotic fermions).

As an introduction let me start by revising the *renormalizable non-SUSY $SO(10)$* and a possible $SCPV$ [13].

non-SUSY GUTs require intermediate gauge symmetry breaking (I_i) [16] to have gauge coupling unification.

$$GUT \longrightarrow I_i \longrightarrow SM = SU_C(3) \times SU_L(2) \times U_Y(1) . \quad (1)$$

Most models involve an intermediate scale at $\approx 10^{12-13} GeV$ which is also that of breaking of $B - L$, the masses of RH neutrinos and the CP violation responsible for leptogenesis (BAU).

$SO(10)$ fermions are in three **16** representations: $\Psi_i(\mathbf{16})$.

$$\mathbf{16} \times \mathbf{16} = (\mathbf{10} + \mathbf{126})_S + \mathbf{120}_{AS} . \quad (2)$$

Hence, only $H(\mathbf{10})$, $\overline{\Sigma}(\overline{\mathbf{126}})$ and $D(\mathbf{120})$ can contribute directly to Yukawa couplings and fermion masses. Additional Higgs representations are needed for the gauge symmetry breaking.

One and only one VEV $\overline{\Delta} = \langle \overline{\Sigma}(1, 1, 0) \rangle$ can give a (large) mass to the RH neutrinos via

$$Y_\ell^{ij} \nu_R^i \overline{\Delta} \nu_R^j \quad (3)$$

and so induces the seesaw mechanism. It breaks also $B - L$ and $SO(10) \rightarrow SU(5)$.

To generate $SCPV$ in conventional $SO(10)$ one can use the fact that $\overline{\Sigma}(\overline{\mathbf{126}})$ is the only relevant complex Higgs representation. Its other special property is that $(\overline{\Sigma})_S^4$ is invariant in $SO(10)$ [17]. This allows for a $SCPV$ at the high scale, using the scalar potential [13]:

$$V = V_0 + \lambda_1 (H)_S^2 [(\overline{\Sigma})_S^2 + (\overline{\Sigma}^*)_S^2] + \lambda_2 [(\overline{\Sigma})_S^4 + (\overline{\Sigma}^*)_S^4] . \quad (4)$$

Inserting the VEV s

$$\langle H(1, 2, -1/2) \rangle = \frac{v}{\sqrt{2}} \quad \overline{\Delta} = \frac{\sigma}{\sqrt{2}} e^{i\alpha} \quad (5)$$

in the neutral components, the scalar potential reads

$$V(v, \sigma, \alpha) = A \cos(2\alpha) + B \cos(4\alpha) . \quad (6)$$

For B positive and $|A| > 4B$ the absolute minimum of the potential requires

$$\alpha = \frac{1}{2} \arccos \left(\frac{A}{4B} \right). \quad (7)$$

This ensures the spontaneous breaking of CP [8].

It is not possible to realize the above scenario in renormalizable SUSY theories, as Φ^4 cannot be generated from the superpotential in this case. A different approach is needed and this is the aim of this paper.

I will present in the following a possible scenario for $SCPV$ in renormalizable SUSY $SO(10)$ models [18] [19] [20] [21]. This will be done by giving an explicit realization in terms of the so called *the minimal renormalizable SUSY $SO(10)$ model* [15]. The model became very popular recently due to its simplicity, predictability and automatic R -parity invariance (i.e. a dark matter candidate).

It includes the following Higgs representations

$$H(\mathbf{10}), \quad \Phi(\mathbf{210}), \quad \Sigma(\mathbf{126}) \oplus \bar{\Sigma}(\overline{\mathbf{126}}). \quad (8)$$

Both Σ and $\bar{\Sigma}$ are required to avoid high scale $SUSY$ breaking (D -flatness) and $\Phi(\mathbf{210})$ is needed for the gauge breaking.

The properties of the model are dictated by the superpotential. This involves all possible renormalizable products of the superfields

$$W = M_\Phi \Phi^2 + \lambda_\Phi \Phi^3 + M_\Sigma \Sigma \bar{\Sigma} + \lambda_\Sigma \Phi \Sigma \bar{\Sigma} + M_H H^2 + \Phi H (\kappa \Sigma + \bar{\kappa} \bar{\Sigma}) + \Psi_i (Y_{10}^{ij} H + Y_{\overline{126}}^{ij} \bar{\Sigma}) \Psi_j \quad (9)$$

(One can, however, add discrete symmetries or $U(1)_{PQ}$ invariance etc. on top of $SO(10)$).

We take all coupling constants real and positive, also in the soft $SUSY$ breaking terms.

The symmetry breaking goes in two steps

$$SUSY SO(10) \xrightarrow{\text{strong gauge breaking}} MSSM \xrightarrow{SUSY \text{ breaking}} SM \quad (10)$$

The F and D -terms must vanish during the strong gauge breaking to avoid high scale $SUSY$ breakdown ("F,D flatness").

D-flatness: only $\Sigma, \bar{\Sigma}$ are relevant therefore

$$|\Delta| = |\bar{\Delta}| \quad \text{i.e.} \quad \sigma = \bar{\sigma}. \quad (11)$$

The situation with *F-flatness* is more complicated.

The strong breaking is dictated by the *VEV*s that are *SM* singlets.

Those are in the $SU_C(4) \times SU_L(2) \times SU_R(2)$ notation :

$$\begin{aligned} \phi_1 = < \Phi(1, 1, 1) > \quad \phi_2 = < \Phi(15, 1, 1) > \quad \phi_3 = < \Phi(15, 1, 3) > \\ \Delta = < \Sigma(\overline{10}, 1, 3) > \quad \bar{\Delta} = < \bar{\Sigma}(10, 1, 3) > . \end{aligned}$$

The strong breaking superpotential in terms of those *VEV*'s is then

$$\begin{aligned} W_H = & M_\phi(\phi_1^2 + 3\phi_2^2 + 6\phi_3^2) + 2\lambda_\phi(\phi_1^3 + 3\phi_1\phi_2^2 + 6\phi_2\phi_3^2) \\ & + M_W\Delta\bar{\Delta} + \lambda_\Sigma\Delta\bar{\Delta}(\phi_1 + 3\phi_2 + 6\phi_3). \end{aligned} \quad (12)$$

$|\frac{\partial W_H}{\partial v_i}|^2=0$ gives a set of equations. Their solutions dictate the details of the strong symmetry breaking.

One chooses the parameters such that the breaking

$$SUSY SO(10) \longrightarrow MSSM$$

will be achieved [22] [23]. *SUSY* is broken by the soft *SUSY* breaking terms. The gauge *MSSM* breaking is induced by the *VEV*'s of the *SM* doublet $\phi^{u,d}(1, 2, \pm 1/2)$ components of the Higgs representations.

The mass matrices of the Higgs are then as follows

$$M_{ij}^u = \left[\frac{\partial^2 W}{\partial \phi_i^u \partial \phi_j^u} \right]_{\phi_i = < \phi_i >} \quad M_{ij}^d = \left[\frac{\partial^2 W}{\partial \phi_i^d \partial \phi_j^d} \right]_{\phi_i = < \phi_i >} . \quad (13)$$

The requirement

$$\det(M_{ij}^u) \approx 0 \quad \det(M_{ij}^d) \approx 0 \quad (14)$$

leaves only two light combinations of doublet components and those play the role of the bidoublets h_u, h_d of the *MSSM*. (This also is discussed in detail in the papers of [22] [23].)

We will come back to h_u, h_d later but let me discuss the *SCPV* first.

As in the non-SUSY case, we conjecture that Δ and $\bar{\Delta}$, and only those, acquire a phase at the tree level

$$< \Sigma(1, 1, 0) > \equiv \Delta = \sigma e^{i\alpha} \quad < \bar{\Sigma}(1, 1, 0) > \equiv \bar{\Delta} = \sigma e^{i\bar{\alpha}}. \quad (15)$$

Let me show that this is a minimum of the scalar potential in a certain region of the parameter space.

To do this we collect all terms with $\Delta, \bar{\Delta}$ in the superpotential. Those involve the VEV 's that are non-singlets under the SM . I.e. the SM doublet components of the Higgs representations.

$$\begin{aligned}
\phi^u &= \langle \Phi(1, 2, 1/2) \rangle & \phi^d &= \langle \Phi(1, 2, -1/2) \rangle \\
H^u &= \langle H(1, 2, 1/2) \rangle & H^d &= \langle H(1, 2, -1/2) \rangle \\
\Delta^u &= \langle \Sigma(1, 2, 1/2) \rangle & \Delta^d &= \langle \Sigma(1, 2, -1/2) \rangle \\
\bar{\Delta}^u &= \langle \bar{\Sigma}(1, 2, 1/2) \rangle & \bar{\Delta}^d &= \langle \bar{\Sigma}(1, 2, -1/2) \rangle
\end{aligned} \tag{16}$$

The relevant terms are:

$$\begin{aligned}
W_\Delta &= M_\Sigma \Delta \bar{\Delta} + \frac{\lambda_\Sigma}{10} (\phi^u \Delta^d \bar{\Delta} + \phi^d \bar{\Delta}^u \Delta) \\
&+ \left(\frac{\lambda_\Sigma}{10} \left(\frac{1}{\sqrt{6}} \phi_1 \Delta \bar{\Delta} + \frac{1}{\sqrt{2}} \phi_2 \Delta \bar{\Delta} + \phi_3 \Delta \bar{\Delta} \right) \right. \\
&+ \left. \frac{\lambda_\Sigma \sqrt{2}}{15} \phi_2 \bar{\Delta}^u \Delta^d - \frac{\kappa}{\sqrt{5}} \phi^d H^u \Delta - \frac{\bar{\kappa}}{\sqrt{5}} \phi^u H^d \bar{\Delta} \right)
\end{aligned} \tag{17}$$

using [22] [23].

One can then calculate the corresponding scalar potential

$$V(\alpha, \bar{\alpha}, M_\Sigma, \lambda_\Sigma, \kappa, \bar{\kappa}, v_i) = \sum_i \left| \frac{\partial W_\Sigma}{\partial v_i} \right|^2. \tag{18}$$

Noting that $|A + Be^{i\alpha}|^2 = A^2 + B^2 + 2AB \cos \alpha$

and $|K + P\Delta\bar{\Delta}|^2 = K^2 + P^2\sigma\bar{\sigma} + 2KP\sigma\bar{\sigma} \cos(\alpha + \bar{\alpha})$,

one finds that

$$V = A(M_\Sigma, \lambda_\Sigma, \kappa, \bar{\kappa}, v_i) + B(M_\Sigma, \lambda_\Sigma, \kappa, \bar{\kappa}, v_i) \cos \alpha +$$

$$D(M_\Sigma, \lambda_\Sigma, \kappa, \bar{\kappa}, v_i) \cos \bar{\alpha} + E(M_\Sigma, \lambda_\Sigma, \kappa, \bar{\kappa}, v_i) \cos(\alpha + \bar{\alpha}).$$

For explicit expressions of the coefficients see the Appendix.

The minimalization under $\alpha, \bar{\alpha}$ requires

$$\begin{aligned}
\frac{\partial V(\alpha)}{\partial \alpha} &= -B \sin \alpha - E \sin(\alpha + \bar{\alpha}) = 0 \\
\frac{\partial V(\bar{\alpha})}{\partial \bar{\alpha}} &= -D \sin \bar{\alpha} - E \sin(\alpha + \bar{\alpha}) = 0
\end{aligned} \tag{19}$$

This gives the equations

$$\sin \bar{\alpha} = \frac{B}{D} \sin \alpha$$

$$B \sin \alpha + E(\sin \alpha \cos \bar{\alpha} + \sin \bar{\alpha} \cos \alpha) = 0$$

and the solutions are

$$\begin{aligned} \cos \alpha &= \frac{ED}{2} \left(\frac{1}{B^2} - \frac{1}{D^2} - \frac{1}{E^2} \right) \\ \cos \bar{\alpha} &= \frac{EB}{2} \left(\frac{1}{D^2} - \frac{1}{B^2} - \frac{1}{E^2} \right). \end{aligned}$$

We have clearly a minimum for a certain range of parameters, with non trivial values of α , $\bar{\alpha}$. This means that CP is broken spontaneously.

CP is broken at the high scale, it is transferred however to the Fermi scale via the mixing of the Higgs representations which obey the restrictions (14). The $MSSM$ bi-doublets h_u, h_d are then (linear) combinations of the Higgs representations doublet components. The expressions involve quite a few parameters, are very complicated and model dependent. The details are out of the scope of this paper and I refer the reader to the papers [22] [23]. The only important relevant fact for us is, that in all variants, the coefficients of those combinations involve Δ and $\bar{\Delta}$ (and a possibly complex parameter x that fixes the local symmetry breaking [22]) so that the $VEVs$ $\langle h_u \rangle, \langle h_d \rangle$ are complex.

H and $\bar{\Sigma}$ which come in the Yukawa coupling and contribute to the mass matrices

$$M^i = Y_{10}^i H + Y_{\overline{126}}^i \bar{\Sigma}$$

are given in terms of the physical $h_{u,d}$ as follows (the heavy combinations decouple):

$$\begin{aligned} H_{u,d} &= a_u h_u + a_d h_d + \dots \text{ decoupled} \\ \bar{\Sigma}_{u,d} &= b_u h_u + b_d h_d + \dots \text{ decoupled} \end{aligned} \quad (20)$$

The mass matrices are expressed then in terms of $\langle h_{u,d} \rangle$

$$M_u = (a_u Y_{10} + b_u Y_{\overline{126}}) \langle h_u \rangle \quad (21)$$

$$M_d = (a_d Y_{10} + b_d Y_{\overline{126}}) \langle h_d \rangle \quad (22)$$

$$M_\ell = (a_d Y_{10} - 3b_d Y_{\overline{126}}) \langle h_d \rangle \quad (23)$$

$$M_\nu^D = (a_u Y_{10} - 3b_u Y_{\overline{126}}) \langle h_u \rangle \quad (24)$$

$$M_{\nu_R} = Y_{\overline{126}} \bar{\Delta} \quad (25)$$

The mass matrices of the quarks and also leptons are therefore complex and lead to a complex CKM matrix as well as a complex $PNMS$ leptonic one.

What was presented in this paper is only a possible realization of the scenario. To have a complete model, the free parameters must be fixed by fitting to the experimental data. For the minimal renormalizable SUSY $SO(10)$, it was observed [22][23][24] that when CP violation as well as the soft SUSY breaking terms are disregarded the model cannot be fully realistic. The main difficulty lies in the fact that to get the right absolute masses of the neutrinos one needs an intermediate symmetry breaking scale. This may cause problems in particular for the gauge coupling unification. Recently suggested solutions involve adding the $D(\mathbf{120})$ Higgs representation [25], adding type II seesaw [27], considering possible contribution from soft SUSY breaking terms [26] or adding warped extra dimensions [28]. Our scenario is applicable in those cases also. It requires additional parameters and the superpotential is more complicated, yet the conjecture (15) leads to $SCPV$.

Recently, Grimus and Kühböck [29]¹ were able, by adding $D(\mathbf{120})$, to fit correctly the fermionic masses and mixing, including the CKM phase. Using a \mathbf{Z}_2 -symmetry and specific requirements they reduced the number of free parameters. They assumed also that the Yukawa couplings are real but did not explain how the complex VEVs are spontaneously generated. Applying here our scenario one can explicitly relate the high scale CP violation to the CKM one [31].

What about *the strong CP problem*?

To solve the QCD Θ problem in the framework of $SCPV$ one must add not only extra symmetries but also exotic fermions [6][7][8], hence, to go beyond $SO(10)$. The simplest solution, in the framework of the renormalizable $SO(10)$, is to require global $U(1)_{PQ}$ [12] invariance with the invisible axion scenario [32] [33]. It is interesting then to observe that the energy range of our $SCPV$ lies within the invisible axion window [4]

$$10^9 GeV \lesssim f_a \lesssim 10^{12} GeV , \quad (26)$$

where f_a is the axion decay constant.

This can be applied to $SUSY SO(10)$ as well. The minimal renormalizable $SUSY SO(10) \times U(1)_{PQ}$ was discussed recently in a paper by Fukuyama and Kikuchi [34]. The requirement of $U(1)_{PQ}$ invariance using the PQ charges

$$\begin{aligned} PQ(\Psi) &= -1, & PQ(H) &= 2, \\ PQ(\Sigma) &= -2, & PQ(\bar{\Sigma}) &= 2, & PQ(\Phi) &= 0 \end{aligned}$$

forbids only two terms in the superpotential

$$\begin{aligned} W_{PQ} &= M\Phi^2 + \lambda_\Phi \Phi^3 + M_\Sigma \Sigma \bar{\Sigma} + \lambda_\Sigma \Phi \Sigma \bar{\Sigma} \\ &+ \mathcal{K} \Phi \Sigma H + \Psi_i (Y_{10}^{ij} H + Y_{126}^{ij} \bar{\Sigma}) \Psi_j . \end{aligned} \quad (27)$$

¹See also Aulakh and Garg [30]

Hence, our scenario for *SCPV* is still intact (although with different phases).

The breaking of local $B - L$ via the *VEVs* of $\overline{\Sigma}(126)$ and $\Sigma(126)$ will also break spontaneously the global $U(1)_{PQ}$ and explain the coincidence of the scales of the axion window and the seesaw one. In our scenario it will also coincide with the scale of *SCPV* and that of leptogenesis.

Conclusions

This paper is a version for publication of ref. [14]. I presented a scenario for *SCPV* in both non-*SUSY* and *SUSY* $SO(10)$. *CP* is broken spontaneously at the scale of the *RH* neutrinos but a phase is generated also in the *CKM* low energy mixing matrix. We have therefore *CP* violation at low and high energies as is required experimentally.

To the best of my knowledge there are no SUSY-GUT models that really discuss the way the phases are generated spontaneously. *SCPV* is induced in most models by giving ad-hoc phases by hand to some *VEVs*.

If $U(1)_{PQ}$ invariance is also used, one finds the interesting situation that the scales of Baryogenesis, Seesaw, *SCPV* and the breaking of $U(1)_{PQ}$ are all at the same order of magnitude.

Appendix: the parameters of the scalar potential

$$\frac{\partial W_\Delta}{\partial \phi^u}, \quad \frac{\partial W_\Delta}{\partial \phi^d}, \quad \frac{\partial W_\Delta}{\partial \phi_1}, \quad \frac{\partial W_\Delta}{\partial \phi_3}, \quad \frac{\partial W_\Delta}{\partial H^u}, \quad \frac{\partial W_\Delta}{\partial H^d}$$

do not give terms with a phase.

α dependent terms are obtained from $\frac{\partial W_\Delta}{\partial \Delta}$ and $\frac{\partial W_\Delta}{\partial \Delta^u}$ i.e.

$$\left| \frac{\partial W_\Delta}{\partial \Delta} \right|^2 + \left| \frac{\partial W_\Delta}{\partial \Delta^u} \right|^2 = \text{constant} + B \cos \alpha$$

Therefore,

$$B = 2\sigma\phi^u \left[M_\Sigma + \frac{\lambda_\Sigma}{10} \left(\frac{1}{\sqrt{6}}\phi_1 + \frac{1}{\sqrt{2}}\phi_2 + \phi_3 \right) \right] \left[\frac{\lambda_\Sigma}{10}\Delta^d - \frac{\bar{\kappa}}{\sqrt{5}}H^d \right] + \frac{\sqrt{2}}{75}\sigma\lambda_\Sigma^2\phi^d\phi_2\Delta^d =$$

$$B(M_\Sigma, \lambda_\Sigma, \bar{\kappa}, \phi_i, \phi^u, \phi^d, \Delta^d, H^d) .$$

In the same way

$$D = 2\sigma\phi^d \left[M_\Sigma + \frac{\lambda_\Sigma}{10} \left(\frac{1}{\sqrt{6}}\phi_1 + \frac{1}{\sqrt{2}}\phi_2 + \phi_3 \right) \right] \left[\frac{\lambda_\Sigma}{10}\Delta^u - \frac{\bar{\kappa}}{\sqrt{5}}H^u \right] + \frac{\sqrt{2}}{75}\sigma\lambda_\Sigma^2\phi^u\phi_2\Delta^u =$$

$$D(M_\Sigma, \lambda_\Sigma, \kappa, \phi_i, \phi^u, \phi^d, \Delta^u, H^u) .$$

A term proportional to $\cos(\alpha + \bar{\alpha})$ is generated only by $\frac{\partial W_\Delta}{\partial \phi_2}$.

Hence,

$$E = \frac{1}{75}\lambda_\Sigma^2\bar{\Delta}^u\Delta^d\sigma^2 = E(\lambda_\Sigma, \sigma, \bar{\Delta}^u, \Delta^d) .$$

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