Singular Lorentz Transformations and Pure Radiation Fields

Kevin Maguire

April 21, 2014



- Introduction: Lorentz Transformations
- Strange Minkowskian Line Element
- Singular Lorentz transformation
- \bigcirc $SL(2,\mathbb{C})$ Matrices of the Lorentz Transformation



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Layout



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- **3** $SL(2,\mathbb{C})$ Matrices of the Lorentz Transformation



Singular Lorentz Transformations and Pure Radiation Fields

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Layout

a Introduction: Lorentz Transformations
Strange Minkowskian Line Element
Singular Lorentz transformation
SIGL2 CI Milkstran of the Lorentz Transformation

 A Lorentz transformation is defined by the preservation of the quadratic form

$$x'^2 + y'^2 + z'^2 - t'^2 = x^2 + y^2 + z^2 - t^2$$

in the transformation

$$(x, y, z, t) \rightarrow (x', y', z', t')$$

- Take the Proper Orthochronous Lorentz Transformations(POLTs) which form the restricted Lorentz group SO⁺(1,3)
- In general lorentz transformations

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Singular Lorentz Transformations and Pure Radiation Fields

Introduction: Lorentz Transformations



stroduction: Lorentz Transformations

- -Proper is det 1 . preserves the orientation of spacial axes, preserves handedness
- –orthochronous means time is always positive and the direction of time is preserve
- ullet —Think of the standard Lornetz transformation, always two null directions at $x\pm$

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Singular Lorentz Transformations and Pure Radiation Fields

Introduction: Lorentz Transformations



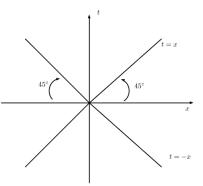
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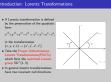




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Introduction: Lorentz Transformations



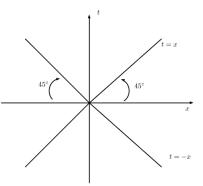
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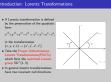




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add in the contents



Singular Lorentz Transformations and Pure Radiation Fields Layout

- derive a strange minkowskian line element
- making a complicated transformation that keeps a single null geodesic fixed look trivial

Start with the Schwarzschild solution

$$\epsilon \mathrm{d}s^2 = \left(1 - \frac{2m}{r}\right)^{-1} \mathrm{d}r^2 + r^2 \left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2\right) - \left(1 - \frac{2m}{r}\right) \mathrm{d}t^2.$$

• Make the Eddington-Finkelstein coordinate transformation [2]

$$u = t - r - 2m \ln(r - 2m)$$

• Make further coordinate transformations to obtain

$$\epsilon ds^2 = \frac{r^2}{\cosh^2 \mu \xi} (d\xi^2 + d\eta^2) - 2dudr - \left(\mu^2 - \frac{2k}{r}\right) du^2.$$

ullet Taking the limit as the energy, $\mu o 0$ gives The Kasner Solution

$$eds^{2} = r^{2}(d\xi^{2} + d\eta^{2}) - 2dudr - \frac{2k}{r}du^{2}$$



Singular Lorentz Transformations and Pure Radiation Fields

Strange Minkowskian Line Element

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- to remove the coordinate singularity in the Schwarzchild solution
- \bullet These transformations put the line element in a form where we can take the limit the energy goes to 0
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• Then with m = 0 the strange Minkowskian line element is obtained

$$\epsilon ds^2 = r^2 (d\xi^2 + d\eta^2) - 2du dr.$$

• It is easily shown that r = 0 gives

$$\epsilon ds^2 = 0.$$

and thus is a single null geodesic.



Singular Lorentz Transformations and Pure Radiation Fields

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add in the contents



Singular Lorentz Transformations and Pure Radiation Fields	Layout	
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Layout	add in the contents	

LTs that leave one null invariant direction are constructed

• Define an arbitrary complex parameter $\zeta := \xi + i\eta$, to get the new line element[3]

$$\epsilon ds^2 = r^2 d\zeta d\overline{\zeta} - 2dudr.$$

- The transformation $\zeta \to \zeta + w$, where $w \in \mathbb{C}$ is then trivial and leaves the single null geodesic r = 0 invariant.
- In Cartesian coordinates this transformation becomes

$$x' + iy' = x + iy + w(t - z),$$

 $z' - t' = -r = z - t,$
 $z' + t' = z + t + w(x - iy) + w(x + iy) + w\bar{w}(t - z).$

• Addition of complex numbers is commutative, and w has two parameters, so the singular Lorentz transformations form a 2-parameter abelian subgroup of the Lorentz group.



Singular Lorentz Transformations and Pure Radiation Fields

Singular Lorentz Transformation

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element[3]	arameter $\zeta := \xi + i\eta$, to get the new line $e^2 = e^2 d\zeta d\zeta^2 - 2dudr$.	

- This is what we want, An LT which leaves one null invariant.
- The use in the previous coord transforms was to make this transformation look trivial
- So this is what the seemingly trivial transformation looks like in cartesians
- Again its clear that r = 0 keeps one direction fixed, as then z=t
- but it doesn't work both ways, not all 2 parameter abelian subgroups are singular lorentz transformations

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add in the contents



- Singular Lorentz Transformations and Pure Radiation Fields

 Layout

 add in the carbons
 - shown here that there is a 2 to 1 correspondence between SL(2,C) and POLTs
 - first show there 1 to 1 correspondence between points in Minkowskian space time and Hermitian matrices

- There is a one to one correspondence between points in Minkowskian space-time and Hermitian matrices
- Contruct the following matrix

$$A = \left(\begin{array}{cc} t - z & x + iy \\ x - iy & t + z \end{array}\right)$$

• This is useful as its determinant is the Lorentz quadratic form modulo a sign

$$\det(A(\vec{x})) = t^2 - x^2 - v^2 - z^2$$

It is also closely related to spinors

$$A = [t\mathbb{I}_2 - \vec{x} \cdot \vec{\sigma}].$$

• Construct the transformation $A(\vec{x}') = UA(\vec{x})U^{\dagger}$, where

$$U = \left(\begin{array}{cc} \alpha & \beta \\ \gamma & \delta \end{array}\right),$$

is an element of $SI(2, \mathbb{C})$



Singular Lorentz Transformations and Pure Radiation Fields

 $-SL(2,\mathbb{C})$ Matrices of the POLT



- Complex Hermitian matrices have 4 independant components, so the element of such a matrix can be used to represent points in Minkowskian space-time.
- Where σ are the pauli matirces which form a basis for the Lie algebra of SU(2)
- where $\alpha, \beta, \gamma, \delta$ are complex its an element of the special linear group. This mean it has determinant 1. **write it on the board**

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Singular Lorentz Transformations and Pure Radiation Fields



- Complex Hermitian matrices have 4 independant components, so the element of such a matrix can be used to represent points in Minkowskian space-time.
- Where σ are the pauli matirces which form a basis for the Lie algebra of SU(2)
- where $\alpha, \beta, \gamma, \delta$ are complex its an element of the special linear group. This mean it has determinant 1. **write it on the board**

- There is a one to one correspondence between points in Minkowskian space-time and Hermitian matrices
- Contruct the following matrix

$$A = \left(\begin{array}{cc} t - z & x + iy \\ x - iy & t + z \end{array}\right),$$

• This is useful as its determinant is the Lorentz quadratic form modulo a sign

$$\det(A(\vec{x})) = t^2 - x^2 - y^2 - z^2.$$

• It is also closely related to spinors

$$A = [t\mathbb{I}_2 - \vec{x} \cdot \vec{\sigma}].$$

• Construct the transformation $A(\vec{x}') = UA(\vec{x})U^{\dagger}$, where

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Singular Lorentz Transformations and Pure Radiation Fields

 $SL(2,\mathbb{C})$ Matrices of the POLT



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Singular Lorentz Transformations and Pure Radiation Fields

L $SL(2,\mathbb{C})$ Matrices of the POLT

serves the Lorentz quadratic form, thus n a \(\) Lorentz transformation. The this transformation component wise $\begin{pmatrix} z' - z' & x' + \hat{p}' \\ z' - \hat{p}' & z' + \hat{x}' \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} z - x + \hat{p}' \\ x - \hat{p}' & z' + z' \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} z - \hat{p} + (x + \hat{p}) \hat{\beta} \\ z - \hat{p} + (x + \hat{p}) \hat{\beta} \end{pmatrix} \begin{pmatrix} z - \hat{p} + (x + \hat{p}) \hat{\beta} \\ z - \hat{p} + (x + \hat{p}) \hat{\beta} \end{pmatrix} \begin{pmatrix} z - \hat{p} + (x + \hat{p}) \hat{\beta} \\ z - \hat{p} + (x + \hat{p}) \hat{\beta} \end{pmatrix} \begin{pmatrix} z - \hat{p} + (x + \hat{p}) \hat{\beta} \\ z - \hat{p} + (x + \hat{p}) \hat{\beta} \end{pmatrix} \begin{pmatrix} z - \hat{p} + (x + \hat{p}) \hat{\beta} \\ z - \hat{p} + (x + \hat{p}) \hat{\beta} \end{pmatrix} \begin{pmatrix} z - \hat{p} + (x + \hat{p}) \hat{\beta} \\ z - \hat{p} + (x + \hat{p}) \hat{\beta} \end{pmatrix} \begin{pmatrix} z - \hat{p} + (x + \hat{p}) \hat{\beta} \\ z - \hat{p} + (x + \hat{p}) \hat{\beta} \end{pmatrix} \begin{pmatrix} z - \hat{p} + (x + \hat{p}) \hat{\beta} \\ z - \hat{p} + (x + \hat{p}) \hat{\beta} \end{pmatrix} \begin{pmatrix} z - \hat{p} + (x + \hat{p}) \hat{\beta} \\ z - \hat{p} + (x + \hat{p}) \hat{\beta} \end{pmatrix} \begin{pmatrix} z - \hat{p} + (x + \hat{p}) \hat{\beta} \\ z - \hat{p} + (x + \hat{p}) \hat{\beta} \end{pmatrix} \begin{pmatrix} z - \hat{p} + (x + \hat{p}) \hat{\beta} \\ z - \hat{p} + (x + \hat{p}) \hat{\beta} \end{pmatrix} \begin{pmatrix} z - \hat{p} + (x + \hat{p}) \hat{\beta} \\ z - \hat{p} + (x + \hat{p}) \hat{\beta} \end{pmatrix} \begin{pmatrix} z - \hat{p} + (x + \hat{p}) \hat{\beta} \\ z - \hat{p} + (x + \hat{p}) \hat{\beta} \end{pmatrix} \begin{pmatrix} z - \hat{p} + (x + \hat{p}) \hat{\beta} \\ z - \hat{p} + (x + \hat{p}) \hat{\beta} \end{pmatrix} \begin{pmatrix} z - \hat{p} + (x + \hat{p}) \hat{\beta} \\ z - \hat{p} + (x + \hat{p}) \hat{\beta} \end{pmatrix} \begin{pmatrix} z - \hat{p} + (x + \hat{p}) \hat{\beta} \\ z - \hat{p} + (x + \hat{p}) \hat{\beta} \end{pmatrix} \begin{pmatrix} z - \hat{p} + (x + \hat{p}) \hat{\beta} \\ z - \hat{p} + (x + \hat{p}) \hat{\beta} \end{pmatrix} \begin{pmatrix} z - \hat{p} + (x + \hat{p}) \hat{\beta} \\ z - \hat{p} + (x + \hat{p}) \hat{\beta} \end{pmatrix} \begin{pmatrix} z - \hat{p} + (x + \hat{p}) \hat{\beta} \\ z - \hat{p} + (x + \hat{p}) \hat{\beta} \end{pmatrix} \begin{pmatrix} z - \hat{p} + (x + \hat{p}) \hat{\beta} \\ z - \hat{p} + (x + \hat{p}) \hat{\beta} \end{pmatrix} \begin{pmatrix} z - \hat{p} + (x + \hat{p}) \hat{\beta} \\ z - \hat{p} + (x + \hat{p}) \hat{\beta} \end{pmatrix} \begin{pmatrix} z - \hat{p} + (x + \hat{p}) \hat{\beta} \\ z - \hat{p} + (x + \hat{p}) \hat{\beta} \end{pmatrix} \begin{pmatrix} z - \hat{p} + (x + \hat{p}) \hat{\beta} \\ z - \hat{p} + (x + \hat{p}) \hat{\beta} \end{pmatrix} \begin{pmatrix} z - \hat{p} + (x + \hat{p}) \hat{\beta} \\ z - \hat{p} + (x + \hat{p}) \hat{\beta} \end{pmatrix} \begin{pmatrix} z - \hat{p} + (x + \hat{p}) \hat{\beta} \\ z - \hat{p} + (x + \hat{p}) \hat{\beta} \end{pmatrix} \begin{pmatrix} z - \hat{p} + (x + \hat{p}) \hat{\beta} \\ z - \hat{p} + (x + \hat{p}) \hat{\beta} \end{pmatrix} \begin{pmatrix} z - \hat{p} + (x + \hat{p}) \hat{\beta} \\ z - \hat{p} + (x + \hat{p}) \hat{\beta} \end{pmatrix} \begin{pmatrix} z - \hat{p} + (x + \hat{p}) \hat{\beta} \\ z - \hat{p} + (x + \hat{p}) \hat{\beta} \end{pmatrix} \begin{pmatrix} z - \hat{p} + (x + \hat{p}) \hat{\beta} \\ z - \hat{p} + (x + \hat{p}) \hat{\beta} \end{pmatrix} \begin{pmatrix} z - \hat{p} + (x + \hat{p}) \hat{\beta} \\ z - \hat{p} + (x + \hat{p}) \hat{\beta} \end{pmatrix}$

SL(2 C) Matrices of the POLT

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Singular Lorentz Transformations and Pure Radiation Fields

 $\cup SL(2,\mathbb{C})$ Matrices of the POLT



SL(2 C) Matrices of the POLT

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$$t' - z' = t - z,$$

 $x' + iy' = x + iy + w(t - z),$
 $t' + z' = t + z + w(x - iy) + \bar{w}(x + iy) + w\bar{w}(t - z).$

• Equate coefficients on the RHS of this equation with the RHS of the general relations on the previous slide to obtain

$$\alpha = \pm 1, \qquad \beta = 0$$
 $\gamma = \bar{w}a, \qquad \delta = \alpha$

So there are always two possible choices of U

$$U=\pm \left(egin{array}{cc} 1 & 0 \ ar{w} & 1 \end{array}
ight)$$

• Thus there is a 2 to 1 correspondence between elements of $SL(2,\mathbb{C})$ and POLTs



Singular Lorentz Transformations and Pure Radiation Fields

Example: Singular Lorentz transformation

Value the singular Lorentz transformation from earlier t'-z'=t-z, $x'+\dot{y}''=x+\dot{y}+w(t-x),$ $t'+x'=t+x+w(x-\dot{y})+\ddot{w}(x+\dot{y})+w\ddot{w}(t-x).$

xample: Singular Lorentz transformation

- Where we have also used det(U) = 1
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Singular Lorentz Transformations and Pure Radiation Fields

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 $\sqrt{1}$ bit the singular Lorentz transformation from earlier $t'-t'=t-x, \\ x'+y''=x+iy+w(t-x), \\ t'+x''=x+iy+w(t-x), \\ t'+x''=x+ix+w(t-x), \\ t'+x''=x+ix+w(t-x), \\ u'''=x+ix+u'''=x+ix+u''''=x+ix+u''''=x+ix+u''''=x+ix+u'''=x+ix+u'''=x+ix+u'''=x+ix+u'''=x+ix+u'''=x+ix+u''=x+i$

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Singular Lorentz Transformations and Pure Radiation Fields

Example: Singular Lorentz transformation

$$\label{eq:problem} \begin{split} \nabla \text{Take the singular Lorentz transformation from earlier} & i'-z' = t-z,\\ & x' + p' = x+p' + w(t-z),\\ & i'+z' = t+z+w(x-p') + \delta(x+p') + w\delta(t-z). \end{split}$$
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Singular Lorentz Transformations and Pure Radiation Fields

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Singular Lorentz Transformations and Pure Radiation Fields

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Singular Lorentz Transformations and Pure Radiation Fields

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Singular Lorentz Transformations and Pure Radiation Fields

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