Singular Lorentz Transformations and Pure Radiation Fields

Kevin Maguire

April 21, 2014



- Introduction: Lorentz Transformations
- Strange Minkowskian Line Element
- Singular Lorentz transformation

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Introduction: Lorentz Transformations
 Strange Minkowskian Line Element
 Singular Lorentz transformation

 A Lorentz transformation is defined by the preservation of the quadratic form

$$x'^2 + y'^2 + z'^2 - t'^2 = x^2 + y^2 + z^2 - t^2$$

in the transformation

$$(x, y, z, t) \rightarrow (x', y', z', t')$$

- Take the Proper Orthochronous Lorentz Transformations(POLTs) which form the restricted Lorentz group $SO^+(1,3)$
- In general lorentz transformations



Singular Lorentz Transformations and Pure Radiation Fields

Introduction: Lorentz Transformations



stroduction: Lorentz Transformations

- -Proper is det 1 . preserves the orientation of spacial axes, preserves handedness
- –orthochronous means time is always positive and the direction of time is preserve
- ullet —Think of the standard Lornetz transformation, always two null directions at $x\pm$

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Introduction: Lorentz Transformations



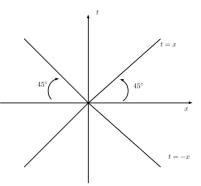
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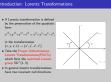
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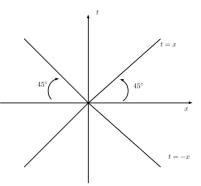
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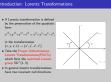
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- derive a strange minkowskian line element
- making a complicated transformation that keeps a single null geodesic fixed look trivial

Start with the Schwarzschild solution

$$\epsilon \mathrm{d}s^2 = \left(1 - \frac{2m}{r}\right)^{-1} \mathrm{d}r^2 + r^2 (\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2) - \left(1 - \frac{2m}{r}\right) \mathrm{d}t^2.$$

• Make the Eddington-Finkelstein coordinate transformation [2]

$$u = t - r - 2m \ln(r - 2m)$$

Make further coordinate transformations to obtain

$$\epsilon \mathrm{d}s^2 = \frac{r^2}{\cosh^2 \mu \epsilon} (\mathrm{d}\xi^2 + \mathrm{d}\eta^2) - 2\mathrm{d}u\mathrm{d}r - \left(\mu^2 - \frac{2k}{r}\right) \mathrm{d}u^2.$$

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Strange Minkowskian Line Element

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- to remove the coordinate singularity in the Schwarzchild solution
- \bullet These transformations put the line element in a form where we can take the limit the energy goes to 0
- It is easily shown with further coord transforms that this is Kasner, but it wont be done here

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$$\epsilon ds^2 = r^2 (d\xi^2 + d\eta^2) - 2du dr.$$

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and thus is a single null geodesic.



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LTs that leave one null invariant direction are constructed

Singular Lorentz Transformation

• Define an arbitrary complex parameter $\zeta = \xi + i\eta$, to get the new line element

$$\epsilon ds^2 = r^2 d\zeta d\overline{\zeta} - 2dudr.$$

- The transformation $\zeta \to \zeta + w$, where $w \in \mathbb{C}$ is then trivial and leave the single null geodesic r = 0 invariant.
- Now see what this looks like in Cartesian coordinates



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Singular Lorentz Transformation

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Singular Lorentz Transformation

- This is what we want, An LT which leaves one null invariant.
- The use in the previous coord transforms was to make this transformation look trivial

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Singular Lorentz Transformation

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Singular Lorentz Transformation

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