Population, Ideas, and the Speed of History*

Kevin Kuruc†

February 2025

Link to most recent version.

Abstract

The global decline in fertility has refocused attention on scale effects in long-run growth models. A smaller population will produce fewer (non-rival) ideas, slowing economic progress. This paper argues that this reasoning overlooks a fundamental feature of population growth: the rate at which individuals are born into the economy to benefit from new ideas also slows, offsetting or even reversing the per capita losses from slower idea generation. Formally, I show that under standard implementations of idea-based growth models, a smaller population leaves no individual worse off. The producers and the beneficiaries of new ideas are, in expectation, the same people; scaling the population cannot bring forward one group without the other. Instead, the primary role of population size in idea-based growth models is to govern the speed of history. Identical experiences unfold, but on an accelerated timeline in histories with larger populations. Relaxations of the standard model uncover previously unexamined channels—such as knowledge depreciation—through which population size influences per capita economic outcomes.

Keywords: Endogenous growth, scale effects, population economics, low fertility.

JEL Codes: J11; I31; O41.

*I am especially grateful to David Weil for his insightful discussion of an earlier version of this paper at the NBER's workshop on low fertility. I also thank Gustav Alexandrie, Eric Donald, Maya Eden, Jim Feyrer, John Firth, Loren Fryxell, Mike Geruso, Tom Houlden, Chad Jones, Antony Millner, Luis Mota, Toby Ord, Dean Spears, Christian Tarsney, Phil Trammell, Dietrich Vollrath, Anson Zhou, and other participants at the NBER's workshop on low fertility and Oxford's Global Priorities Workshops for comments that have greatly improved the paper. All remaining errors are my own.

†Population Wellbeing Initiative, University of Texas at Austin; Global Priorities Institute, University of Oxford. Contact: kevinkuruc@utexas.edu.

1 Introduction

Since Romer (1990), increasing returns to scale have been at the center of growth theory. Other things equal, an economy with more people and resources will discover more productivity-improving ideas. Knowledge is fundamentally non-rival, so this increase in the stock of aggregate ideas increases per capita incomes. It is a remarkably simple chain of logic: More people $\rightarrow$ more ideas $\rightarrow$ higher per capita incomes. The global decline in fertility has resulted in renewed attention to this relationship as economists attempt to understand the costs and benefits of this demographic shift (Jones, 2022a,b; Peters, 2022; Weil, 2023; Spears and Geruso, Forthcoming).

This paper demonstrates that, in contrast with conventional wisdom, the non-rivalry of ideas implies nothing about the relationship between individual income levels and population. This is because increasing the rate of population growth increases the arrival rate of both ideas and the people who stand to benefit from these ideas, potentially offsetting the purported benefits. To illustrate how the argument will work, consider an early articulation of this scale-based reasoning in Phelps (1968).

> *If I could re-do the history of the world, halving population size each year from the beginning of time on some random basis, I would not do it for the fear of losing Mozart in the process.* (p. 512)

Suppose there is some fixed probability that each birth produces someone as talented as Mozart. It would then be more accurate to say that halving the historical population *delays* the expected date that someone as talented as Mozart is born. At the same time, halving the historical population also halves the number of people living in these now Mozart-less years. To a first-order, these effects will exactly offset: the same number of people are expected to live prior to a Mozart regardless of per-period population sizes. Consequently, no lives are improved or worsened by scale here, even while recognizing that music is non-rival. Decreasing the population delays the arrival of a Mozart-like figure, but it delays all other lives too.

The first part of the paper formalizes this proposition in a standard Romer/Jones endogenous growth framework (Jones, 1995). I show analytically that the size of the population has no effect on the number of person-years lived prior to any given TFP level. A smaller future population would not change the number of person-years lived without a cure for cancer, commercial supersonic air travel, quantum computing, or other future technologies. These discoveries occur later, but with proportionately fewer people living in each of those years, leaving the average quality of these lives unchanged. This finding does not refute the standard claim that non-rival ideas lead to an aggregate economy with increasing returns to scale. The novelty is a shift of focus, away from the income levels of time periods to the income levels of people. The positive economics from this vantage look crucially different.

After establishing the cleanest version of the core idea in a standard endogenous growth setting, I then relax the model. I show that the exact neutrality between population size and individual outcomes holds only under a knife's edge condition. These generalizations do not point in one consistent direction, making it uncertain which side of the knife's edge we in fact live on and pointing to previously understudied parameters as being crucial for this question.

The first modification is to endogenize research effort, rather than relying on an assumption wherein each individual makes some fixed *ex-ante* contribution to TFP. Larger populations have larger consumer bases, which potentially changes the incentives to engage in R&D. This reasoning turns out to be misleading. When the consumer base is larger, it is more profitable both to generate new ideas and to exploit existing ones. The benefits of engaging in research increase, but so does the opportunity cost. Formally, the equilibrium condition determining the share of the population engaged in research is independent of the population size.[1] Therefore, standard market-size considerations do not overturn the main result, nor do other ways of endogenizing research effort.

Second, I relax a linearity assumption in the R&D production function. Specifically, I allow for convexity (or concavity) in the returns to research effort within a period. If collaboration effects between researchers generate increasing returns, then larger per-period populations will improve individual outcomes. Unsurprisingly, it would be better for more people to live in a given year if there are increasing returns to scale in the research production function itself. On the other hand, if there are diminishing returns to researchers within a period, individual outcomes are improved by *smaller* populations. Akcigit and Kerr (2018) provide recent evidence of diminishing returns in the research production function, and this is the (much) more common assumption in the literature that calibrates/estimates long-run growth models (see e.g., Bloom et al., 2020; Jones, 2022a). Relaxing this linearity assumption provides some reason to believe that smaller populations improve each individual's living standard, a reversal of the lesson typically drawn from the fact that non-rival ideas are a driver of economic growth.

Third, I consider the implications of knowledge depreciation. This force, in isolation, unambiguously serves to rescue the original intuition that larger populations improve individual out-

[1] Population *growth* may change the exploit/explore trade-off for potential entrepreneurs (see e.g., Peters and Walsh, 2021; Karahan et al., Forthcoming), which I discuss in Section 3.1. This is a substantially different claim about the incentives arising from differences in relative population sizes across time, not a 'bigger-is-better' scale effect.

comes. The reason is as follows: If knowledge depreciates over time, the faster that people 'arrive' following the discovery of an idea, the more people benefit from that idea before it is lost. When time erodes the knowledge base, it is beneficial for more people to live overlapping lives. Depreciation of knowledge is essentially unstudied, so it is difficult to know what to make of the potential magnitude of this effect. However, in a simple quantitative exercise, I demonstrate that plausible values for knowledge depreciation appear unlikely to make this force pivotal when considered alongside the assumption that there are diminishing returns to research effort.

Overall, the policy-relevant takeaway is that idea-discovery, on its own, is not a reason to prefer larger populations. A smaller future population, generating less per-period technological progress, can result in worse, unchanged, or even improved outcomes for each individual. So, while the exact independence result relies on a knife's edge condition, the absence of evidence for meaningful deviations from this baseline suggests that a null relationship between population size and individual outcomes is a useful and novel re-understanding of (semi-)endogenous models of economic growth.

In light of this potential neutrality, the last section of the paper focuses on other reasons larger per-period populations may be preferred when economic growth comes from ideas. Namely, temporal discounting, and its interaction with the fact that larger populations accelerate the arrival of discoveries. Consider the reasons that are traditionally raised for discounting utility over time: (i) a rate of pure time preference and (ii) the probability of death/extinction.[2] The former is trivial—it is a preference that benefits accrue earlier in time. If the planner prefers the exact same events when they happen on an accelerated time horizon, then bringing forward people and discoveries via population growth is welfare-improving. This is true even if the strongest version of the positive claim of this paper is correct. In this case, population growth would be beneficial for the reason that it functionally serves to accelerate history.

Discounting because of death to the representative agent (i.e., existential risk) is more interesting. In an expected value sense, future utility should be discounted by the probability it is not experienced. Suppose there is some date when a large asteroid will strike earth, ending humanity. This would give rise to a plausibly important effect of larger per-period populations: More lives are lived, and more ideas are discovered, during humanity's fixed temporal lifespan. It remains the case that no experiences are improved *conditional on occurring*, but more total life-years are

[2]The third reason for discounting *consumption* over time is that future generations will be richer than present generations when economic growth is positive. That is not relevant here: I am asking about the implication of moving lives with fixed utility to different periods, not about transferring resources between periods.

lived. This generates what I refer to as an *extensive margin* effect on per capita incomes—these additional existences are lived with access to more knowledge than average. However, this has very little to do with the fact that ideas are non-rival. Once it is assumed that the planner values the existence of additional lives, most modeling assumptions will generate the conclusion that larger per-period populations are beneficial.

To sharpen this point, I formalize an equivalence in this setting between (1) increasing the per-period population size and (2) delaying the date of extinction. Underpinning this equivalence is the fact that I assume there is nothing *per se* important about the years in which these additional lives are lived. Once this is seen, it becomes clear that the only relevant question—even in idea-based growth models—is whether a larger per-period population increases the number of people that ever live. I conclude the paper by showing that a range of plausible ways of modeling existential risk and idea-generation can lead to a range of outcomes regarding the relationship between per-period population size and the quality and quantity of lives that are lived.

This paper contributes to literatures at the intersection of scale economics and long-run economic growth. In particular, Jones (1995) builds on the Romer (1986, 1990) insight to highlight the importance of population size/growth in driving TFP improvements (see also Kremer, 1993; Galor and Weil, 2000; Jones, 2003, 2005). This counter-intuitive idea—that population size is a key driver of long-run improvements to living standards—sparked a series of papers attempting to eliminate these scale effects (Dinopoulos and Thompson, 1998; Segerstrom, 1998; Young, 1998). Jones (1999) shows these attempts largely fail; a larger population is still richer per capita each period in these models. Recent years have seen renewed attention and additional exploration of this finding as the world transitions to a regime of below-replacement fertility, with the possibility of long-term population decline (Jones, 2022a; MacAskill, 2022; Peters, 2022; Spears and Geruso, Forthcoming).

The main contribution of this paper is to demonstrate that, contrary to conventional wisdom, these idea-based scale effects do not imply anything about whether larger populations improve individual lives. In one way, it can be seen as offering a resolution to earlier debates about scale effects in growth models. It can be true both that (i) a larger population improves TFP per period and (ii) idea-based growth does not imply that larger populations improve per capita outcomes. What has been overlooked is that TFP improvements *per period* systematically differ from TFP improvements *per human life* in cases where the temporal distribution of population changes.

Once this is established, I show that the relevant question is instead whether a larger population allows for more lives to ever be lived. By studying long-run, aggregate welfare—that is, the quantity and quality of lives over long time horizons—this paper also intersects with a literature in applied welfare economics assessing changes in population sizes. Most recent and relevant is Adhami et al. (2024), which decomposes the social welfare gains coming from population growth versus income growth over the last six decades, finding that increases in the quantity of lives matters much more than the increase in per capita living standards. Likewise, there is a recent literature interested in risks to humanity's survival that considers the question of how many people will ever live. Aschenbrenner and Trammell (2024), Jones (2016, 2022a, 2024), and Peretto and Valente (2015, 2023) examine growth models with explicit extinction outcomes, either by way of dangerous technological development or persistent below-replacement fertility.[3]

Finally, this paper contributes to an active literature assessing the overall effects of demographic change. Vollrath (2020), Fernández-Villaverde et al. (2023), and Maestas et al. (2023) highlight the effect of a change in the composition of the workforce. Peters and Walsh (2021), Hopenhayn et al. (2022), and Karahan et al. (Forthcoming) study the relationship between business dynamism and population growth. Henderson et al. (2024) discusses population pressure on fixed natural resources. Galor (2022) and Weil (2023) give useful overviews of many important aspects of demographic change over longer time horizons. The broader lessons of this paper with respect to these debates about population size are two-fold. First, idea-based growth may not be an important consideration as previously thought. And second, framing outcomes per life, rather than per time period, may be useful in better understanding the implications of some of these other channels.

The rest of the paper is structured as follows. Section 2 formalizes the main claim that the number of life-years lived prior to a given discovery is independent of population size in idea-based models of economic growth. Section 3 shows this is robust to a number of relaxations of the standard (semi-)endogenous growth model. Section 4 then argues that the value of population size comes only through its effect on the number of people who ever live, and highlights the importance of assumptions about extinction risks. Section 5 concludes.

[3] In the existential risk literature, Ord (2024a,b) develops and applies a general framework for assessing the long-run welfare implications of changes to the joint trajectory of life-quality and survival-probability of humanity over time; my results can be seen as a special and particularly relevant case of accelerating progress. Additionally, Greaves (2019) similarly leverages the *per human life* emphasis of this paper to draw novel insights about environmental challenges and population size.

2 The non-rivalry of ideas does not imply a positive relationship between population size and individual outcomes

This goal of this section is to demonstrate that the non-rivalry of ideas does not, on its own, imply that larger populations raise individual living standards. This will follow from looking at outcomes from the perspective of people, rather than time periods. I begin by formalizing the main proposition using a standard (semi-)endogenous growth framework. I then discuss the intuition for this result, before assessing whether how model relaxations affect the main result in Section 3.

Equation (1) is a production function of productivity improvements (i.e., *ideas*) developed in the literature spawned by Jones (1995), buildling directly from Romer (1986, 1990). The key concepts embedded in this formulation are that (i) the production of ideas increases in the effort put towards discovering them, and (ii) that these discoveries improve aggregate productivity, A.

$$\frac{\dot{A}(t)}{A(t)} = \alpha(t)(s(t)N(t))^{\lambda}A(t)^{-\beta} \tag{1}$$

Time, t, is continuous; $\dot{A}$ is the instantaneous change in productivity. This depends on three factors. The first is the number of individuals engaged in research. In (1) this is decomposed as the share of the population in research, s, multiplied by the population size, N. Second, the productivity of these researchers, α. Third, the accumulated knowledge stock, $A(t)$, which allows for non-linearity in idea production. There could be increasing returns to productivity gains ($\beta < 1$) if past inventions (e.g., the computer) help generate new inventions, or there could be diminishing returns to inventions ($\beta > 1$) if earlier discoveries are systematically easier to make. In practice, $\beta > 1$ matches the historical data better, both in aggregate and at the industry level (Jones, 1995; Bloom et al., 2020), though this parameter is inconsequential for the results established here.

The number of researchers is raised to some power λ that pins down the degree of returns to researchers within a period. There is considerable uncertainty about what value λ should take. One reason to believe that $\lambda < 1$ is *duplication*; people alive at the same time might solve the same problems, resulting in wasted effort. On the other hand, *collaboration* might be a reason that $\lambda > 1$; the ability to communicate with other researchers might result in faster progress than a counterfactual where those same researchers live in different years.[4]

For expositional simplicity in this section, let $\lambda = 1$ and $s(t), \alpha(t)$ be some constants $\bar{s}, \bar{\alpha}$. I

[4]In reality, this relationship likely has an 'S'-shape, where collaboration-benefits dominate at small populations, but duplication and congestion dominate for large populations.

assume a constant α and s because the objective is to isolate the effects of population size. Of course, increasing α or s would also increase the production of ideas, but this is not the focus of this paper. The assumption of $\lambda = 1$ is more substantive. It imposes a fixed marginal effect of population within a period—if the world population were twice as large, exactly twice as much technological progress would occur. To generate an analytical baseline result, I ignore possible curvature for now. Section 3.2 relaxes this assumption.

With these assumptions, we can rewrite (1) as follows, where $\theta = \bar{\alpha}\bar{s}$.

$$\frac{\dot{A}(t)}{A(t)} = \theta N(t) A(t)^{-\beta} \tag{2}$$

Integrating this function delivers the following expression for the level of A in period t.

$$A(t) = \left(\beta\theta \int_0^t N(\tau) d\tau + A_0^{\beta}\right)^{\frac{1}{\beta}} \tag{3}$$

Equation 3 reflects the fact that if TFP improvements per period are a function of per-period research effort, then cumulative TFP over time will be a function of cumulative research effort over time. Formally, because the assumption is that people are the input to these improvements, cumulative research effort is a scalar of cumulative person-years, $\int_0^t N(\tau) d\tau$.

Notice that I have not imposed a linear relationship between cumulative person-years and cumulative idea production, which would be a stronger assumption than within-period linearity. If $\beta \neq 1$, a person-year generates more or less knowledge depending on the size of the existing knowledge stock. The argument that follows does not depend on the value of β, or even the power function structure. Likewise, I have not imposed that every individual, in fact, contributes to knowledge production. I have only assumed that the share of individuals contributing to research is constant, not that it is equal to one (and even this will be relaxed in Section 3.1). Every person, *ex-ante*, contributes to TFP progress, but not necessarily *ex-post*.

Substantively, what Equation 3 does is eliminate t as an independent variable. It is cumulative human effort, not time, that drives innovation. Our economic lives are richer than previous generations' not merely because time has passed, but because people have been working to improve productivity in that time. Therefore, time can be eliminated as an independent variable without losing anything of conceptual importance.[5]

[5] Of course, this could be contested. If experiments literally take time, or exogenous natural events deliver knowledge, then perhaps time *should* enter this function. I return to the potential direct effect of time in Section 3.3.

The formal claim of this section is that the same number of person-years are lived prior to a given level of TFP, regardless of population sizes. This will follow by defining person-years lived by time t as:

$$i(t) = \int_0^t N(\tau)d\tau \in [0, I]. \tag{4}$$

In continuous time, each i is an instantaneous person-experience, denoted in units of person-years. For example, if the population size were a constant 10 billion, and exactly two years pass, then, at that instant, the 20 billionth person-year is being lived. This value ranges from 0, the very first person-experience, to some finite I, the number of person-years that humanity collectively ever lives. After formally stating the main proposition, I will discuss the intuition in detail, making explicit how to understand the result in terms of the more familiar unit of people's lives.

Equation 3 implies that $A(t)$ is a function of the integral of population size over time, which we have now denoted $i(t)$. Therefore, the expression for $A(t)$ can be simply rewritten as a function of i.

Proposition 1. *The TFP accessible to an individual person-year, i, is a function only of its order in the history of person-years that will be lived.*

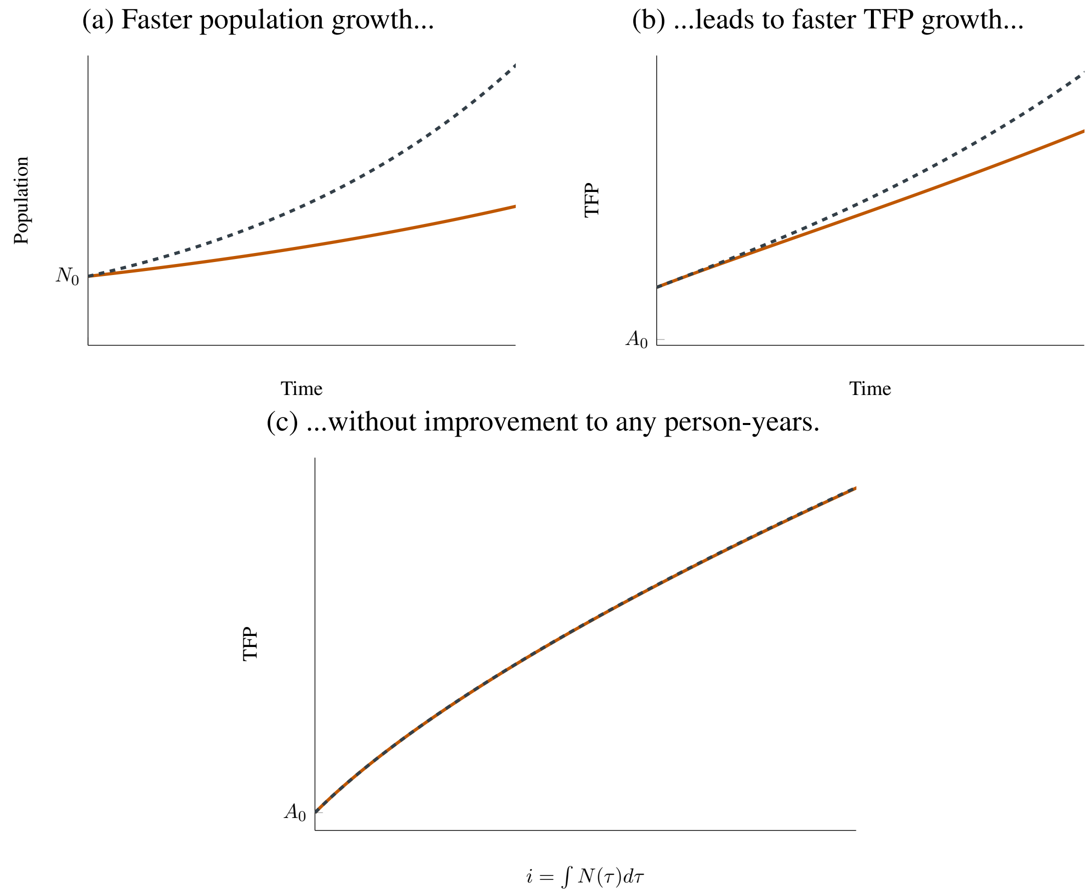
$$A(i) = \left(\beta\theta i + A_0^\beta\right)^{\frac{1}{\beta}}$$

It immediately follows that, conditional on β, θ, a larger population does not improve economic outcomes for any individual i.

$A(i)$ represents the TFP available during a given person-year, i. Time and population size no longer enter this simplified expression. The substitution of i into this equation follows from the fact that, *ex-ante*, the producers and the beneficiaries of ideas are the same people. Simply scaling the population brings forward the consumers and the creators of ideas, proportionately.

Figure 1 demonstrates this result graphically. Panels (a) and (b) plot population and technology as they are usually plotted—against time. Panel (c) eliminates time as a variable and plots TFP against person-years. I take this to be the more meaningful unit of analysis: Time periods do not have incomes or experience utility, people do. Panel (c) shows that TFP available to each person-year is unaffected by the size of the population. The curves exactly overlap. This is because the count of individual experiences is accelerated just as fast as TFP improvements in the scenario with faster population growth.

Figure 1: Accelerating economic growth via population growth does not improve any life-years.



Notes: Blue dotted line represents a history with faster population growth than solid orange line. Panel (b) illustrates that TFP growth would be faster in the history with faster population. Panel (c) demonstrates—by eliminating time as a variable—that the TFP available after a given number of person-years is invariant to the speed of population growth. These values come from a simple simulation of Equation 2 with $\beta = 1.5$ and θ set so that $\dot{A}/A = 2\%$ at $t = 0$. $N(0), A(0)$ are normalized to one; population growth rates are 0.5% and 1% per period, respectively.

To build up the intuition of this result, start by setting aside overlapping lives. Imagine that each individual is born, has access to the ideas discovered before her, contributes something to the knowledge stock, and then disappears. It is easy to see why her order in history is all that matters: Changing the speed at which humanity churns through these i's does not matter for how many people—and therefore, ideas—came before her.

The question at the heart of this paper, though, is whether larger per-period population sizes benefit individuals through non-rival idea creation. To be relevant for this question, it is necessary to extend the intuition to cases of overlapping lives. What makes this case less straightforward is that the i-moments are interspersed between different people. When ten people live for one year, ten person-years have been lived in total. However, the i's are defined to be ordered by time—recall from (4) that $i(t)$ is the integral of population size over time—so the first person-year in this ten-person world occurs after one-tenth of one temporal year passes. It is collectively lived between these ten people, not by one person living one year. This complicates, but does not negate, the implications of the formal proposition.

Table 1 presents a stylized example with overlapping lives to illustrate the implication of this feature. Without loss of generality, suppose we are considering the quality of life for the next 20 billion people to live.[6] Assume that each individual has a 100-year lifespan for simplicity. We will compare two hypothetical discrete options for the population size: 10B per year or 20B per year. Moreover, imagine that it will take 1 trillion person-years of effort and economic activity to get to some crucial life-improving technology. If the population is 10B per year, that invention comes after 100 years, precisely at the moment these generations are turning over; if the population is 20B, it comes after only 50 years, because the population in each period is twice as large.

Relative to the history with 10B people, the half of the population that would have lived first each has their life made better. When the population is 20B, they live the latter 50 years of their lives with this technology; if the population were only 10B per year they would have lived none of their life with it. However, the other half of the population has half of their life worsened. If the population were only 10B per year, they would have lived their entire life with this technology; when the population is 20B they live half their life without it.

In terms of person-years, these positive and negative effects exactly cancel out, just as Proposition 1 indicated. In both cases, 1 trillion person-years are lived without this technology, 1 trillion person-year are lived with it. What is substantively interesting about this case is that these person-years are distributed differently across individual lives. However, this rearranging across people is only normatively relevant if the planner has explicit egalitarian motives. In the larger population history the distribution of better and worse person-years are spread more evenly over individuals. And indeed, this would represent a novel reason to prefer accelerated population growth. But this is a normative claim about the value of equality, not the standard positive claim that larger populations are unambiguously beneficial because they result in more non-rival ideas.

To make this more applicable, consider what this means for population growth today. You and I might personally hope for more population growth while we are alive. This would result in more

[6] *Without loss of generality* in the sense that we could consider any (finite) number of future lives, with any number of years prior to a technological discovery, etc. The numbers of the case are not special, just simple.

Table 1: With overlapping lives, population size improves some at the (offsetting) expense of others

	$N(t)$	Idea at:	person-years pre-idea	Entire lives lived w/out idea	Entire lives lived w/idea
History A	10 Bil.	$t = 100$	1 Tril.	10 Bil.	0
History B	20 Bil.	$t = 50$	1 Tril.	0	0

Notes: Table cataloging a scenario with overlapping lives. A larger population implies that fewer people live their entire lives without a given discovery/idea/TFP-improvement. But it also means that more people live a fraction of their lives without this discovery. In terms of person-years, these effects cancel out.

innovation happening during our lives. In the language and notation of this paper, increasing the next generation's size is a way of assigning myself larger values of i later in my life. More person-years will be lived between now and 2050, for example, so the i that I am personally experiencing in 2050 will be a larger value than it would have been if the next cohort of workers and innovators were smaller. But I am implicitly swapping those i values with someone else, offsetting the benefits that I receive.

To conclude this section, let me restate the main claim of the paper that was drawn out here: The non-rivalry of ideas does not imply that population size improves the knowledge base accessible to any given person-year. Instead, the population size governs how fast society moves through life-years of an *ex-ante* pre-determined quality. Section 3 shows that how this clean analytical result is affected by relaxations to the simple setting employed here. Section 4 considers that a larger population in each period may lead to more people ever existing, which extends the implications of this finding in more realistic and policy-relevant directions.

3 The relationship between population size and individual outcomes depends on understudied parameters

This section discusses potential modifications through which the importance of population size for individual living standards may emerge when ideas drive economic growth: endogenous research effort/productivity; non-linear returns to research effort within a period; and the depreciation of knowledge over time. The latter two cases make clear that the main analytical result relies on a knife's edge condition. However, these relaxations can result in the world being on either side of this edge, leaving the neutrality result as a reasonable working approximation and identifying the key (understudied) parameters that can generate a relationship in one direction or the other.

3.1 Endogenous research effort

The results in Section 2 assume that s, the share of individuals contributing to the knowledge base, is fixed with respect to population size. However, larger markets may incentivize more R&D, providing additional reason to believe larger populations will lead to more idea creation. In a world where 10, rather than 1000, people have a rare disease in a given period, it seems less likely that anyone will be incentivized to find a cure. This is true, but the causal mechanism works in a way that makes this irrelevant for the findings of this paper. Overall, this section illustrates that endogenizing research effort in straightforward ways does not lead to the conclusion that larger populations improve outcomes for individual person-years.

By holding fixed s, the main result in Section 2 already accounts for the fact that there will be a larger number of researchers in a given period when N is larger. For this endogenous research channel to deliver the result that a larger population generates more research per person-year, it must be driven by s increasing with N. That is, the composition of the workforce must also change with population size. A simple general equilibrium R&D model below shows that the standard market-size effect does not lead to this outcome.

Consider a one-period model where workers can choose before the period to either work or search for an idea and sell the patent as an entrepreneur. The aggregate production function is a CES aggregator of all goods that there is an idea for. In particular, the integral of possible intermediate goods to purchase has an upper-bound of A. When A is larger, the factors of production can be spread over more intermediate-goods, j, which increases Y through a love-of-variety channel. In short, the number of ways that resources can be used—and therefore aggregate consumption—is increasing in A.

$$Y = \left[\int_0^A y_j^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{5}$$

Each intermediate producer, j, chooses: whether to purchase a patent for an idea at price p_A; what price to charge for its good, p_j; and the quantity of its good to produce, y_j. It faces a linear production function, $y_j = l_j$, where l_j is labor in sector j. Labor is paid a wage w that is taken as given by each firm in the economy. I assume a free-entry condition, such that the fixed cost to operate will adjust to set profits to zero in equilibrium, despite the monopolistically competitive environment. In this set-up, the fixed operating cost is the purchase of a patent, making p_A the price determined by the free-entry condition. The demand curve faced by each intermediate firm is the result of a standard cost-minimization problem that the final good producer solves.

Overall, the problem for a firm that decides to purchase a patent and operate looks as follows (if the maximum profit that can be attained is less than zero, the firm will not purchase the patent and operate).

$$\max_{p_j, y_j, l_j} p_j y_j - w l_j - p_A \tag{6}$$

subject to:

$$y_j = \left(\frac{p_i}{P}\right)^{-\varepsilon} Y \tag{7}$$

$$y_j = l_j \tag{8}$$

Workers supply one unit of labor inelastically. They can either earn wage w, which they take as given, or search for an idea. If they search for an idea, they are successful with probability ω and can sell the patent at price p_A. Therefore, the equilibrium condition coming from the worker's problem is that the return to searching for an idea will be equal to the wage.

$$w = \omega p_A \tag{9}$$

The amount of knowledge in this economy is a function of how many researchers decide to search for ideas. Denote N as the population size and s as the share of individuals searching for ideas.

$$A = \omega s N \tag{10}$$

The full solution to the model is contained in Appendix A. What is relevant for this section is only the equilibrium condition that determines the share of individuals who decide to search for an idea.

$$\varepsilon - 1 = \frac{1-s}{s} \tag{11}$$

The share of the population working in research is a function only of the elasticity of substitution between goods—it is independent of the size of the population. The intuition for this result can be seen by considering the intermediate good producers' problem. Intermediate firms need to purchase both a patent and labor to produce and sell into the (now) larger market. For the same reason that a patent becomes more valuable, so too does each unit of labor. The opportunity cost of searching for an idea grows alongside the value of a patent, leaving the share of individuals choosing to search for an idea unchanged.

Things become more complicated in cases where populations grow over time—this was a simple static setting designed to demonstrate why there is not good reason to believe the fraction of people researching is increasing in the population size. In Jones' classic (1995) paper highlighting the importance of population growth in R&D based models of economic growth, the equilibrium share of researchers/entrepreneurs is increasing in population growth, but not its size (see also Peters and Walsh, 2021; Karahan et al., Forthcoming). The reason is that in these settings, patents and business formation have dynamic value. A population that is growing faster has a larger consumer base tomorrow than it has today, making the value of future goods higher than the value of current goods. This tilts things in favor of researching, since these activities are investments that pay flow dividends in proportion to future demand.

That population growth, not size, determines the share of the population in R&D is distinct. A smaller population with faster population growth would have more research per capita than a larger population with slower growth, for example. Furthermore, the policy relevant questions of the coming decades may concern what population size to stabilize at (Eden and Kuruc, 2023; Spears and Geruso, Forthcoming). Channels that rely only on growth-rate effects are irrelevant when comparing stable populations of different sizes. Nonetheless, it is interesting to note that the dynamic structure of patents may interact with population dynamics to determine whether population growth is an important driver of innovation per person.

A related concern about the baseline model environment is that s may depend directly on the productivity of the economy. For example, if wealthy economies dedicate more per capita effort to research, the dynamics in this model get more complicated. Or, if one of the productivity-improving ideas is an idea that also causally results in a higher fraction of individuals having the ability to generate ideas. Improvements in agricultural yields, for example, both increase production per period *and* increase the fraction of people who are in a position to generate new knowledge (e.g., because of reductions in malnutrition).

This too ends up being unimportant for the main result. Appendix B discusses this case in detail. This relaxation makes the dynamics more complicated, but it does not create a channel through which population size impacts the living standards of any given person-year. Suppose that

s is written as a function of A.[7]

$$\begin{aligned}\frac{\dot{A}}{A} =& \theta s(A) N A^{-\beta} \\ =& \theta N g(A)\end{aligned} \tag{12}$$

This is just a relaxation of the functional form by which A influences TFP production. For the same reason that the value of β does not matter, the structure of $g(A)$ is not critical. Intuitively, the logic of Section 2 can be iterated one more level up. It still takes some X person-years to get to the level of A that increases s. This implies that all $i < X$ will have the same idea-generation productivity, regardless of per period population size. Attempting to get to an intermediate invention that increases s by increasing the population runs into exactly the same conceptual problem as attempting to directly achieve a higher level of TFP through a larger population.

There is not a simple mechanism by which endogenizing research effort influences the main takeaway.

3.2 Increasing returns to research effort

In Section 2 the production function of ideas was assumed to be linear in research effort within a period. This allowed for the derivation of a clean analytical solution between cumulative person-years and the ideas available to an individual for an arbitrary path of population. This linearity assumption may be too strict, and introducing the possibility of curvature makes clear that the analytical results only arise under a knife's edge condition. However, realistic deviations from this knife's edge do not appear undermine the main takeaway in a straightforward way, and may even take us further from conventional wisdom.

Equation (13) is the original production function of ideas, reintroducing the term λ which is no longer imposed to equal to one.

$$\frac{\dot{A}(t)}{A(t)} = \tilde{\theta} N(t)^{\lambda} A(t)^{-\beta} \tag{13}$$

This relaxation results in a slightly modified version of the function determining cumulative research progress over time.

$$A(t) = \left(\beta\tilde{\theta}\int_0^t N(\tau)^{\lambda} d\tau + A_0^{\beta}\right)^{\frac{1}{\beta}} \tag{14}$$

[7] Equivalently, s could be a function of per capita income, if per capita income depends only on A as in standard growth settings.

This modification breaks the tight link between cumulative person-years and cumulative TFP. When $\lambda \neq 1$, in addition to their total number, how lives are spread out over time matters. For example, if $\lambda > 1$, for the same number of cumulative life-years, humanity discovers more ideas if those lives have more temporal overlap. If $\lambda < 1$, there are diminishing returns to research effort within a period, so less is discovered with more temporal overlap.

Somewhat trivially then, for $\lambda > 1$ each person-year has more TFP accessible if historical populations were larger. The (fixed, by definition) i person-years that occur prior to i's existence would have been able to take advantage of more collaborative opportunities and discover more. Conversely, if there are diminishing returns to research effort within a period ($\lambda < 1$) the opposite will be true.

This can be seen formally by considering the special case where population sizes are a constant $\bar{N}$. The constant population case is one I will return to frequently for these more complicated cases because it allows for analytical solutions while continuing to isolate the core question of whether larger populations influence per-person living standards.

In the case of constant populations, the relevant integrals can be simplified as follows.

$$A(t) = \Big(\beta\tilde{\theta}\bar{N}^{\lambda}t + A_0^{\beta}\Big)^{\frac{1}{\beta}} \tag{15}$$

$$i(t) = \bar{N}t \tag{16}$$

This implies:

$$A(i) = \Big(\beta\tilde{\theta}\bar{N}^{\lambda-1}i + A_0^{\beta}\Big)^{\frac{1}{\beta}} \Rightarrow \tag{17}$$

$$\frac{\partial A(i)}{\partial \bar{N}} = \tilde{\theta}(\lambda-1)\bar{N}^{\lambda-2}i\Big(\beta\tilde{\theta}\bar{N}^{\lambda-1}i + A_0^{\beta}\Big)^{\frac{1-\beta}{\beta}} \tag{18}$$

This derivative will be positive if and only if $\lambda > 1$. In the baseline case where $\lambda = 1$ it is easy to see that the derivative is zero, generating the independence result of the prior section.

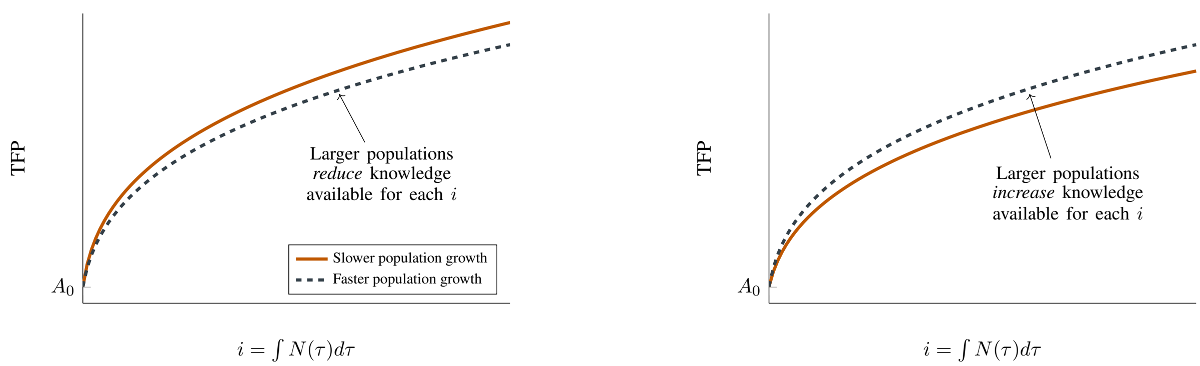
In the literature that attempts to calibrate and/or estimate the effect of aggregate research effort on long-run growth, $\lambda < 1$ is the much more common deviation from a linearity assumption (see e.g., Bloom et al., 2020; Jones, 2022a). If this is indeed the case, then the standard model of non-rival ideas implies *decreasing* returns to scale along the margin that matters: Each individual person-year is made *worse* by per-period populations being larger. It is easy to see this through the framing of the respective arrival rates of ideas and people. The arrival of ideas accelerates when the population is larger, but more slowly than the one-for-one rate that the arrival of person-years accelerates with a larger population. A 1% increase in population size accelerates the arrival of the beneficiaries of ideas by 1%, but only accelerates the arrival of new ideas by $\lambda\%$. Therefore, each person-year has less knowledge accessible to it.

Empirical evidence for this parameter is difficult to come by. Even the evidence in Akcigit and Kerr (2018) is of limited value here. Arguments based on firms scaling up within an economy—or a comparative advantage argument that applies to a fixed pool of individuals (Ekerdt and Wu, 2023)—do not necessarily indicate there would be diminishing returns to scaling the entire population and economy. And, in any case, a power function is almost certainly too simple.[8] The relevant question is whether on current margins congestion/duplication effects are dominated by collaboration effects. I know of no strong evidence on this question. This relaxation certainly does not appear to be a reason to be confident that a larger population improves individual lives, and may well be a channel by which a larger population can worsen individual lives.

Figure 2: Second-order channels can generate effect of either sign

(a) Decreasing returns to research effort

(b) Knowledge depreciation over time



Notes: Blue dotted line represents a history with faster population growth than solid orange line. Panel (a) illustrates that if there are decreasing returns to knowledge generation within a period, larger populations reduce the TFP available for each lifetime—a stark reversal of what non-rival ideas are thought to imply. Panel (b) illustrates that if knowledge depreciates over time, the result that larger populations improve individual outcomes is retained.

[8] If $\lambda < 1$ that would imply that spreading people out as much as possible over time maximizes research production per person; $\lambda > 1$ implies that bunching people all in the same period would maximize output per researcher. Neither of these extremes seems intuitively correct.

3.3 Depreciation of knowledge

Knowledge depreciation leads to an unambiguous theoretical prediction—its existence works to rescue the link between non-rival ideas and the benefits of population size. Furthermore, and unlike the potential case of $\lambda > 1$, it does so for the original reason that knowledge is non-rival and can be freely used once it is discovered. The relevant question is whether the quantitative magnitude of knowledge depreciation makes this a significant force.

Consider the following implementation of depreciation, where, generally, it is a function of economic activity that period.

$$\frac{\dot{A}(t)}{A(t)} = \theta N(t) A(t)^{-\beta} - \delta_A(Y(t)) \tag{19}$$

This can be integrated into a closed-form solution under the special cases where depreciation is a constant $\bar{\delta}_A$ and $\beta = 0$. This value of β corresponds to the original Romer (1990) formulation, and nothing in the paper has depended on its value, making it an inconsequential parameter for developing intuitions. The assumption that δ_A is constant is the most natural starting point for depreciation, matching the standard assumption for how physical capital is commonly assumed to depreciate.

With these parameter values, and a constant population level, A follows a simple exponential growth process that depends on the size of the population. I will again take advantage of the fact that with constant populations $\bar{N}t = i(t)$.

$$\begin{aligned} A(t) =& A_0 e^{\theta \bar{N} t} e^{-\delta_A t} \\ A(i,t) =& A_0 e^{\theta i} e^{-\delta_A t} \end{aligned} \tag{20}$$

For each i, the first two terms of (20) are fixed. The third term depends on when person-year i is lived. In particular, the earlier in time that i is lived, the larger is the knowledge base she has access to. Reducing the time period in which each i lives, amounts to increasing $\bar{N}$ (e.g., humanity gets to its 100 billionth person-year earlier in time if annual populations are larger).

The intuition for this result is straightforward when again thinking about the arrival rate of person-years. For the ith person, there is now a drag on the knowledge accessible to them that is a function of time. If an idea was discovered long before the ith person-year, this person will have less access to it, in expectation. In the fixed number of life-years lived prior to i, there are a fixed number of discoveries (assuming $\lambda = 1$). It is therefore better if person i lives nearer in time to those discoveries. The way to promote person i living closer in time to more of her predecessors is for the population to be large, so that more births happen over a shorter time period.

An appealing feature of this result is that the benefits of population size are retained for the 'right' reasons—knowledge is non-rival, so having more people around benefits everyone. Consider the limiting case where the depreciation rate on knowledge is one, so that the only knowledge accessible to person i is the knowledge discovered in the period in which she lives. Having many contemporaries benefits her because the ideas her contemporaries produce are non-rival. The mechanism in Section 2 that breaks the intuition about population sizes is that person i will have access to all of her predecessors ideas, regardless of when they are discovered. If knowledge depreciates, this is no longer true and it becomes beneficial to live alongside others.

Furthermore, the symmetric case of exogenous knowledge *appreciation* seems significantly less plausible. This is why its effect is less ambiguous than the relaxation that $\lambda \neq 1$, which could work in either direction. It is not inconceivable that something isomorphic to knowledge appreciation exists—e.g., if some costless knowledge is accumulated from exogenous natural events—but this seems unlikely to be as important as the passing of time eroding the knowledge base. The addition of a force capturing exogenous growth/decay seems to unambiguously return the result that larger populations are beneficial because of non-rival knowledge.

As a final conceptual point, it is important to note that the ways in which we might enrich the formulation of depreciation would serve to strengthen this takeaway. Namely, consider specifying knowledge depreciation as a function of economic activity within a period. It seems likely that in periods with more economic activity, fewer valuable ideas go unused and forgotten.[9] Rather than being non-rival—where my use does not erode its availability—knowledge may be *amplifying*—my use increases its availability for others. Section 2 demonstrated that knowledge being non-rival was not enough to generate increasing returns to scale. But if knowledge is amplifying, larger populations will again generate better outcomes for everyone.

There is very little evidence on the functional form or magnitude of knowledge depreciation. Its existence is unlikely to change the balanced growth properties of models with exponential growth in populations, so it has received little attention within this literature to date. But, as noted in Jones (2022a) and Eden and Kuruc (2023), it is a crucial parameter in models without long-term population growth. A future of zero or negative population growth appears to be the most

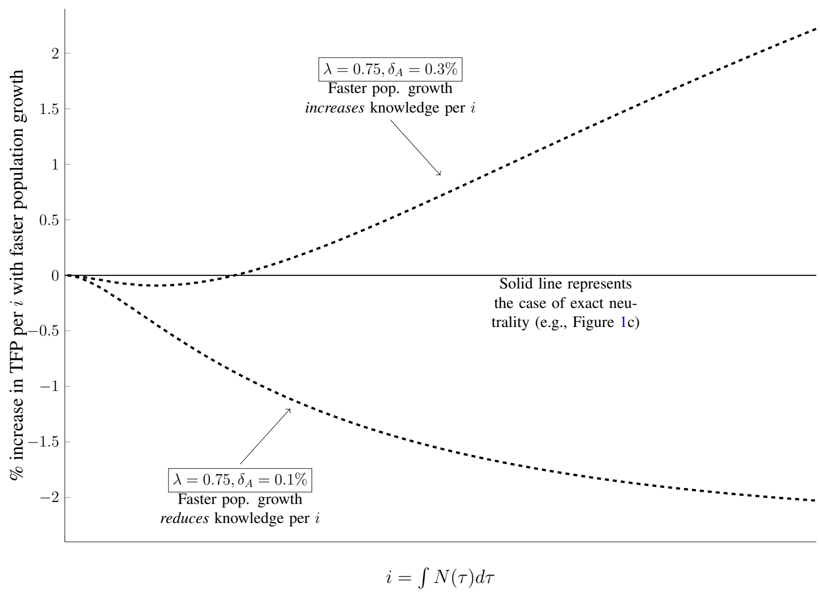
[9]This is an important conceptual distinction from capital. In models where capital utilization can vary over the business cycle, depreciation of capital increases when the economy is running hot, reducing the value of this excess production. The machines are worked harder and breakdown faster.

likely demographic context of the coming decades, so more focus is likely to be paid to the rate at which knowledge depreciates. The result in this section furthers this emphasis on knowledge depreciation, even for settings of population growth.

3.4 Taken together: Population size has no clear *intensive-margin* effect on individual incomes

This section explored three channels by which the exact neutrality result could be broken, uncovering the importance of variables that have not gotten much attention in this literature. I first showed that endogenizing research effort does not affect the main lesson of Section 2, at least in standard models where the size of the population does not affect the composition of the workforce. The takeaways of Sections 3.2 and 3.3 are less straightforward, especially when taken in conjunction.

Figure 3: Quantitatively, second-order effects have ambiguous effect when considered jointly



Notes: Dotted lines represent a history with faster population growth, under different parameterizations for λ, δ_A. When $\lambda = 0.75$, the sign of the effect of faster population growth depends on small differences in δ_A. These values come from a simulating TFP growth per period directly, with the same values used to generate Figure 1 aside from λ, δ. The initial drop for the $\delta = .3\%$ case is due to the fact that δ takes time to influence A, whereas λ affects the dynamics as soon as N differs.

Increasing returns to scale within the research production function, $\lambda > 1$, would generate the result that larger populations improve individual outcomes. However, that is arguably less relevant than the fact that if $\lambda < 1$ the standard intuition is reversed—smaller populations improve individual outcomes. Knowledge depreciation is less theoretically ambiguous. If it is the dominant force, it generates the takeaway that larger populations improve individual outcomes. But we know little about its magnitude. The implicit consensus is that it is not an important force; nearly every study of idea-based economic growth rounds it to zero.

Figure 3 demonstrates that, given this uncertainty, it is plausible that we live on either side of the knife's-edge neutrality result. It plots the relationship of interest—TFP per person-year—for values of λ and δ_A. I begin with $\lambda = 0.75$, following Bloom et al. (2020) and Jones (2022a). For δ_A, there is less guidance. I plot the cases of both $\delta_A = 0.1\%, 0.3\%$ for two reasons. First, this seems a plausible order of magnitude; anything larger would imply that depreciation is a 50% drag on TFP growth, which seems *a priori* too large. Second, these values happen to generate different directional conclusions, so they helpfully demonstrate how these forces quantitatively trade-off. If $\lambda = 0.75$ and $\delta_A = 0.1\%$, the relationship between population sizes and each person's income is negative; if $\lambda = 0.75$ and $\delta_A = 0.3\%$, the relationship between population sizes and each person's income is positive. Distinguishing between these cases with our current body of evidence is infeasible. While the exact analytical results in Section 2 are contingent on specific assumptions, the qualitative point that there is no clear relationship between population sizes and individual incomes generated by the existence of non-rival ideas is robust.

This is what I will call the *intensive* margin, in the sense that this proposition conditions on the quantity of person-years.

Proposition 2. *Let per capita economic outcomes, for a set of I person-years, be defined simply as:*

$$\bar{y}(I) = \int_0^I \xi A(i) di \tag{21}$$

If $A(i)$ is independent of per-period population size, it immediately follows that $\bar{y}(I)$ is also independent of per-period population size. Therefore, per capita outcomes are fixed with respect to population size conditional on I.

It is easy to see why this is true. Each i's experience is invariant to population sizes, so unless I changes, the average experience will remain fixed. However, if larger populations cause more people to ever exist, this could be an *extensive margin* through which average experiences are improved or worsened. Changing the number of good or bad lives is a conceptually different channel by which we would think of per capita outcomes being improved or worsened. I turn to this consideration in the next section.

4 The quantity of person-years, discounting, & extinction risk

Sections 2 and 3 argued that the first-order effect of population size is to govern when each life is lived, not how good it is. This section will explore the extensive margin: In histories with larger populations—that are unfolding faster—more people will plausibly ever live, and more discoveries will ever happen. I show that this extensive margin effect is likely to be positive, but it points to a novel reunderstanding of the implications of growth models, with new areas of inquiry. These results are closely related to temporal discounting; I begin this section by first discussing the interaction of discounting with the finding that population size governs when events and lives happen.

There are two reasons typically given for discounting utility over time; one normative, one positive. The normative one is simple. The planner may inherit the public's preference for benefits to accrue earlier in time. This is a *rate of pure time preference*, and serves to assign more weight to events happening earlier in time. The relevant thought experiment for this setting is straightforward: Does the planner prefer one of two histories with exactly identical person-years, if one history happens over the course of 500,000 years rather than, say, 1,000,000 years? If so, this is a novel reason to prefer larger populations. It is not that lives are improved, it is that there is value in having those lives earlier.

The positive case for discounting is more interesting in this context. It relies on an expected value argument about whether there will be future experiences at all. For an individual, this represents their probability of death. For society as a whole, this might represent the fact that each year carries some risk that humanity goes extinct. Acknowledging this implies a need to discount the utility that would be experienced at some future date by the probability there has been an extinction event by then. This extinction line of reasoning provides the strongest reason to prefer larger per-period populations, though it ends up having little-to-nothing to do with the original Romer (1990) point that ideas are non-rival. In fact, depending on how extinction risks are modeled, I show that idea-based growth models may be one of a small class of models in which per-period population sizes have no effect on the total number of people to ever live.

Consider first a simple asteroid case, where each year carries some fixed exogenous probability of extinction via a natural event. Regardless of the per-period population size, there is some *ex-post* year in which this event occurs. Call this date of extinction T^X. The proposition in Section 2 indicated that the quality of a given person-year is invariant to population size, conditional on that person-year happening. But it is easy to see here that a larger per-period population will affect how many lives are lived.

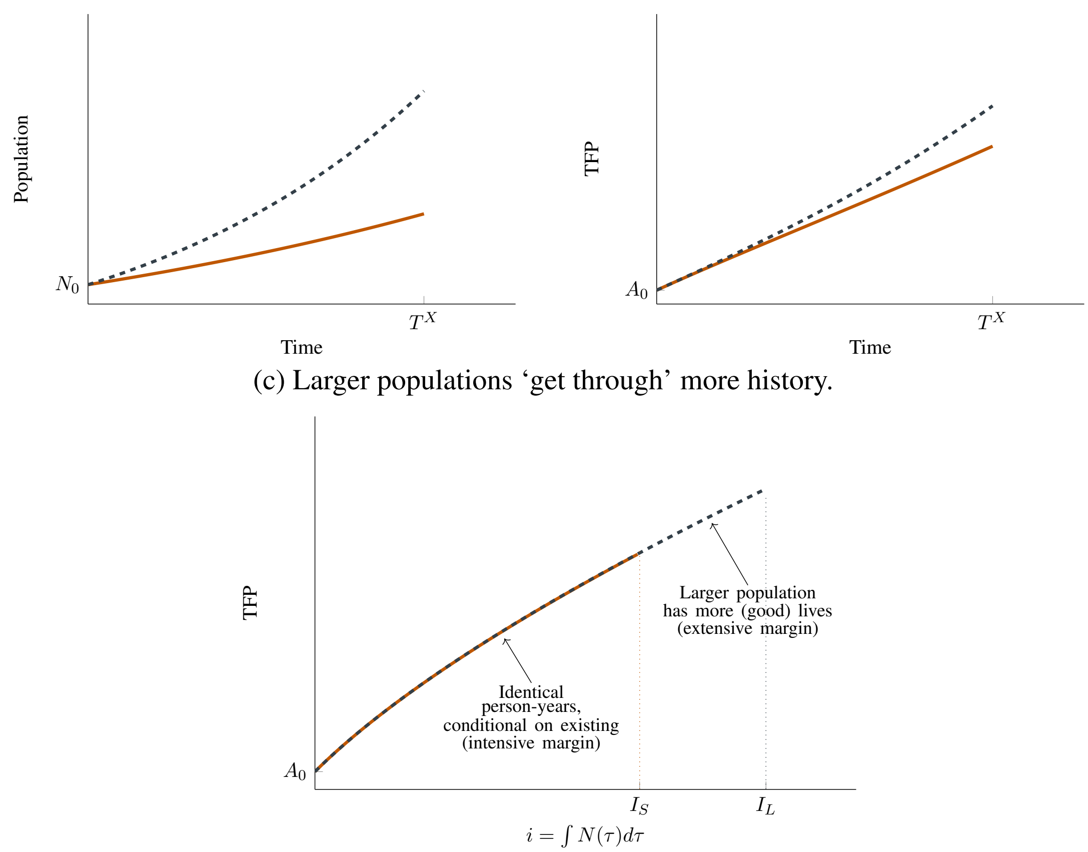
Furthermore, because the TFP available to a given person-year, $A(i)$, is increasing in i, these contingent person-years are the highest quality person-years. If humanity makes it through 110 billion more person-years, rather than 100 billion more person-years, the marginal 10 billion person-years will have all the knowledge accessible to the 100 billion unconditional person-years, and then some. As a result, total life-years lived and the average quality of life-years are both improved when per-period populations are larger, under the assumption that extinction happens at some exogenous date. There is a positive relationship between per capita outcomes and the number of people to live over the whole of human history, accounted for entirely by this extensive margin.

Figure 4: Exogenous Extinction Event at T^X

(a) Each population makes it to T^X.

(b) Faster growth $\rightarrow$ higher TFP reached.

(c) Larger populations 'get through' more history.



Notes: Dotted line represents a history with faster population growth than solid line. Panel (a) depicts how large these respective populations get by the extinction event at T^X. Panel (b) illustrates that TFP reaches higher levels in the history with larger populations. Panel (c) demonstrates, for the person-years lived in either history, there is no quality of life improvement. All potential benefits come along the extensive margin; marginal person-years are the highest quality, because TFP accumulates with person-years.

Figure 4 depicts this case. It is organized just as Figure 1, the difference here is that I no longer assume that humanity goes on indefinitely. Panels (a) and (b) do not show anything surprising. If population and TFP growth are faster, higher levels of each are reached by the time of extinction.

The substantive contribution lies in panel (c). Recall that the x-axis here depicts the measure of person-years as humanity lives them. Let I_S be the number of person-years the small population gets through; I_L is the number of person-years the larger population gets through. As noted above, conditional on reaching a specific person-year the TFP available during that person-year is unaffected. This is the intensive margin result showing up in the case where total existences differ. The extensive margin is captured by the TFP available to the person-years that are contingent on whether the high- or low-population history is realized. Figure 4 demonstrates that any effect of population size must come via the extensive margin.

In this simple setting there is an equivalence between increasing the population size in each period and delaying the date of extinction. What matters is only the number of person-years ever lived, not when they are lived, so a $z\%$ increase in per-period populations has the same effect as extending humanity's lifespan in a way that generates an additional $z\%$ of people existing.

This equivalence is helpful for seeing the results from a different angle that has nothing to do with endogenous growth models. Consider instead the question of how to value extending humanity's lifespan by some length of time. The first-order (and perhaps only) question we would grapple with is how to value the lives of individuals who's existences are contingent on delaying extinction. Those who would get to live, but otherwise would not. It would also be true that—if economic growth continues—those last-existing people would have the highest income levels, so including their life-years would serve to increase average human living standards. But that would seem trivial, or entirely irrelevant, relative to the fact that those lives would be lived. By way of analogy, even in (semi-)endogenous growth models, the relevant demographic question is how to maximize the number of people who will ever live.

If we ask *this* question directly—does a larger per-period population lead to more individuals ever living—the departures from conventional wisdom can get even more extreme. When population size increases innovation rates, this extremely intuitive claim may be overturned. Consider the possibility of *endogenous* extinction. Research focused on the portfolio of existential threats argues that human-caused (anthropogenic) extinction, via dangerous technologies, is more likely than naturally-caused (exogenous) extinction (see e.g., Rees, 2003; Ord, 2020). A recent large-scale forecasting exercise—led by an economist at the Federal Reserve Bank of Chicago—elicited extinction probabilities from experts on individually risky technologies as well as "Superforecasters" (Karger et al., 2023).[10] Median projections of existential risk this century were on the order of 1%, with more than 99% being anthropogenic.[11] When humanity entered the nuclear age, the consensus is that the annual probability of extinction dramatically increased.

As a stylized case, endogenous extinction might be modeled as in Jones (2016): each new idea has some probability of ending humanity (Jones aptly calls this a model of "Russian Roulette" growth). Denote the *ex-post* level of technology that ends humanity as A^X. As should be clear by this point in the paper, A^X is reached once some number of cumulative person-years have been lived. Consequently, the number of person-years lived is independent of the size or growth rate of the population when extinction is endogenized under the baseline assumptions in this paper.

Figure 5 illustrates this case using the same three panels as in Figures 1 and 4. The difference here is that time itself is not the causal variable that ends humanity, A is. A^X acts like an upper-bound on TFP. As can be seen in panel (c), the intensive margin remains identical across population histories. However, there is no longer an extensive margin effect from the population being larger. This eliminates any difference between histories, aside from the fact that one occurs on a condensed time horizon.

The discussion of existential risk will be further enriched in a moment, but first pause to appreciate how deeply this result contrasts with the lessons drawn from Romer (1990) and the literature it has spawned. Population size (and its growth rate) have been at the center of studies of long-run growth in living standards and economic outcomes; scale effects are nearly impossible to avoid when ideas are non-rival. This would seem to quickly lead to the conclusion that larger populations improve social outcomes. But here, closing the model with a simple and plausible assumption about extinction eliminates any effect of population: Both the quantity and quality of life-years ever lived are independent of per-period population size. This is far too simple of a model to be confident in the complete neutrality of population size or growth, but it makes clear that the effects of population are contingent on assumptions that are not typically probed.

To emphasize the contingency of these population-related effects, Table 2 mixes and matches assumptions from earlier subsections with assumptions about existential risks to demonstrate how the implications can vary. To keep things tractable and isolate the effects of scale, for all of these

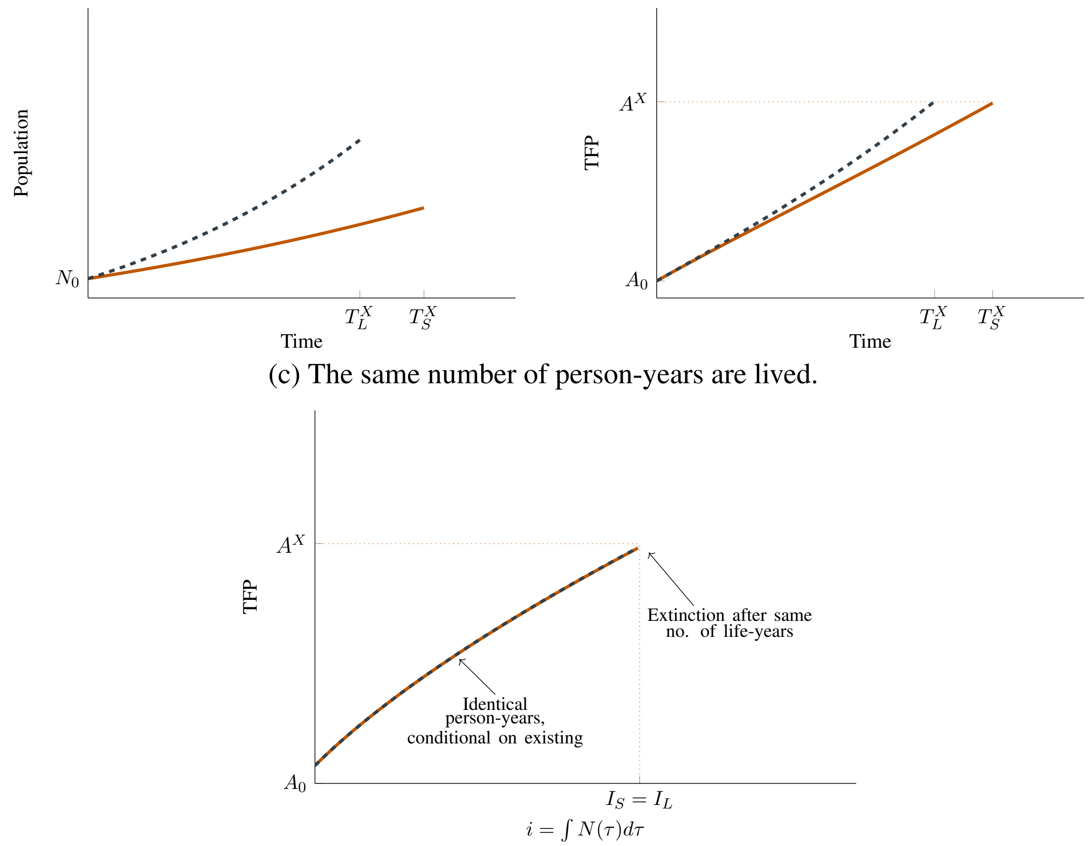
[10] *Superforecasters* are generalists who have displayed a track record of exceptionally good probabilistic calibration and accuracy on a range of geopolitical events (Tetlock and Gardner, 2016). The domain experts were from areas such as pandemics/bioweapons, artificial intelligence and nuclear weapons.

[11] Between groups, Superforecasters predicted a 1% chance of extinction by 2100, with 0.004% coming from non-anthropogenic risks; domain experts estimated a 6% chance of extinction, with a similar 0.004% non-anthropogenic risk.

Figure 5: Endogenous Extinction Event at A^X

(a) A^X occurs later with small populations. (b) Extinction happens at A^X.

(c) The same number of person-years are lived.



Notes: Dotted line represents a history with faster population growth than solid line. Panel (a) depicts how large these respective populations get by the time each reaches A^X. Panel (b) illustrates how fast each reaches the existential technology. Panel (c) demonstrates that the exact same number of lives are lived, at the exact same quality, prior to reaching A^X.

cases but one I conceptualize the comparison as being between two different constant population sizes.[12] The rows represent different stylized examples for extinction scenarios. The first two have already been discussed; exogenous extinction (e.g., the asteroid case) and endogenous extinction by the invention of a dangerous technology.

We could also imagine the probability of extinction being endogenized in other ways. The third row represents a case where each individual life-year poses some risk of ending humanity (a caricature of this case could be that every person-year poses some risk of generating a radicalized individual who wants to end humanity). The fourth row represents the case where technology instead improves our safety each period. For example, suppose that new game-theoretic solutions

[12] Again, this assumption of constant long-run population sizes isolates size effects from growth-rate effects, which may be important for transitory dynamics.

for nuclear deterrent are discovered, or our ability to withdraw carbon from the atmosphere in a cost-effective way reduces the probability of a warming-related catastrophe. The fifth row represents the case where the number of people on the planet *in a given period* increases the chances of existential risk—perhaps because ecological boundaries are pushed too far. Finally, the sixth row considers that below-replacement fertility rates may themselves be an extinction assumption—sustained exponential decay in population sizes eventually results in an "Empty Planet" (Jones, 2022a; Spears and Geruso, Forthcoming; Peretto and Valente, 2023).

The columns represent different assumptions for the law of motion for knowledge. Earlier it was discussed that λ becomes a particularly important term because it governs whether collaboration benefits ($\lambda > 1$) exceed duplication/congestion costs ($\lambda < 1$) of population size within a period. Introducing knowledge depreciation has similar implications to λ exceeding one. As discussed in Section 3.4, there are cases where the effect on the intensive margin is ambiguous—for example, $\lambda < 1$ and $\delta_A > 0$. The left-column generally represents cases where the intensive margin is negative; the right-column are cases where the intensive-margin effect is positive.

It is helpful to keep in mind that the limiting case where $\lambda = 0$ corresponds to cases where TFP is independent of population size. This nests endogenous growth models that fully eliminate scale effects, as well as simple models of exogenous economic growth. The intensive margin is negative here because accelerating the arrival of person-years does not affect the arrival of ideas. If the population is larger, more person-years are lived in states of relative technological immaturity, making each i worse off than they would otherwise be.[13]

Consider the top row of Table 2, the case of exogenous extinction at some unknown time T^X. Here, larger populations increase both the knowledge base accessible to the average person and the number of people to ever live. The case for $\lambda = 1$ is already detailed in Figure 4. If $\lambda > 1$, this would imply that larger per-period populations produce even more knowledge than they otherwise would, giving large populations a further advantage over the case when $\lambda = 1$. When $\lambda < 1$, the larger population continues to get through more person-years prior to T^X. As long as $\lambda > 0$, histories with more total person-years end up accumulating more knowledge (recall Equation 14). Furthermore, reaching a higher level of TFP implies the average level per person will be higher.[14] With $\lambda = 0$ (where TFP growth is independent of population) the exact same level of TFP is reached when a fixed T^X constrains humanity's temporal lifespan.

[13] In a completely unrealistic case, if growth in per capita living standards were truly independent of population size, the planner would want to back-load humanity to have it's mass distributed as late as possible.

[14] This is because I have restricted attention to constant population cases. Here, the average $A(i)$ is increasing in the average $A(t)$ (since the same number of people are alive at each t).

Table 2: The effects of a larger population depend on extinction and TFP growth assumptions

	$\lambda < 1$ (& $\delta_A = 0$)	$\lambda = 1$ (& $\delta_A = 0$)	$\lambda > 1$ (or $\delta_A > 0$)
T^X causes extinction (e.g., asteroid)		$\bar{y}_L \geq \bar{y}_S$ $I_L > I_S$	
A^X causes extinction (e.g., advanced A.I.)	$I_L > I_S$	$\bar{y}_L = \bar{y}_S$ $I_L = I_S$	$I_L < I_S$
I^X causes extinction (e.g., rogue actor)	$\bar{y}_L < \bar{y}_S$	$\bar{y}_L = \bar{y}_S$ $I_L = I_S$	$\bar{y}_L > \bar{y}_S$
A decreases risk (e.g., carbon capture)		$\bar{y}_L > \bar{y}_S$ $I_L > I_S$	
$\bar{N}$ causes extinction (e.g., ecological collapse)		$\bar{y}_L (?) \bar{y}_S$ $I_L (?) I_S$	
Sustained below-replacement fertility		$\bar{y}_L > \bar{y}_S$ $I_L > I_S$	

Notes: The implication of larger populations, L, relative to smaller populations, S on per capita outcomes $\bar{y}_j$, and number of individuals to ever live, I_j, under different modeling assumptions. For example, the top-row indicates that per capita outcomes and the number of people to ever live are both higher when populations are larger ($\bar{y}_L > \bar{y}_S, I_L > I_S$) if extinction is exogenous, regardless of the assumption on idea-generation. The second row indicates that per capita incomes are invariant to population size if A causes extinction, but the relationship between the number of people who ever live and per period population size depends on assumptions about knowledge generation.

The second row represents the case where A^X functionally serves as an upper-bound on TFP, as was detailed for $\lambda = 1$ in Figure 5. This is the case where a dangerous technology, once invented, leads quickly to humanity's extinction. When $\lambda = 1$ the result is that both the quantity and quality lives are invariant to per-period population sizes. Economic growth and extinction are directly tied to person-years, so the arrival of each is accelerated in proportion to person-years when the population is increased.

The cases where $\lambda \neq 1$ break this independence result, but in unintuitive ways. Consider $\lambda > 1$ (or equivalently, a large value of δ_A). This is where idea-generation benefits from collaboration, and hence has increasing returns. If the population were to be a bit larger, the acceleration in idea-generation is faster than the acceleration in person-years. This implies that *fewer* person-years are lived when populations are larger, because A^X is reached dispropotionately faster. At the same time, because an identical "upper-bound" on TFP is reached, per capita incomes are not improved. Under these assumptions, the aggregate effect of a larger per-period population is for fewer people to live lives with a constant average quality. Strikingly, this is true even if every invention but the final A^X improves lives.[15] The reverse case works in exactly the opposite manner, and can be easiest to conceptualize for the limiting case where TFP growth is unaffected by population growth ($\lambda = 0$). Here, increasing the population size simply results in more lives being lived prior to reaching A^X. When $\lambda = 0$, the time at which A^X is reached is unaffected by population size, so it is easy to see why more people live prior to A^X if per-period populations are large. Again, the average level of TFP per person is unaffected because A^X remains the upper bound. As a result, if something like Jones (2016)'s Russian Roulette model of growth is correct, the case for larger populations may be stronger when idea-generation is *less* affected by population size.[16]

In the third row, the causal driver of extinction is directly person-years. One example of this is use of a finite non-renewable resource: Only a certain number of people can ever live prior to using up this resource, so the model is closed with an assumption that directly constrains person-years (Greaves, 2019). Alternatively, it could be that every person-year has some probability of making contact with a novel pathogen that could cause an existential pandemic. When $\lambda = 1$ this is identical to the case that A^X causes extinction because cumulative person-years are tied directly to TFP levels. However, when $\lambda \neq 1$ conclusions in the opposite direction are generated. Collaboration effects or knowledge depreciation would imply that larger populations are indeed

[15]This particular case has close similarities to the result in Ord (2024a).

[16]Though, this is not true if societies optimally choose to forgo growth as they may in an optimal policy framework with sufficiently dangerous technologies. In this case, the endogenous threat disappears and we are back in an exogenous-threat long-run (where population size has positive effects).

better, as the same fixed number of I person-years benefit from having more overlap with one another. Conversely, spreading this fixed number of lives out over time would increase per capita incomes if $\lambda < 1$.[17]

In the fourth row, I have assumed that A decreases risk. This works similarly to the case where extinction is exogenous. Partly, that is because many of these cases are enriched versions of purely exogenous risk. If asteroid detection-and-deflection technology can reduce the risks we face, the case for population size becomes even stronger (see e.g., Spears and Geruso, Forthcoming). If the dominant effect of technological progress is to reduce the probability of extinction in any given period, achieving these gains through a larger population increases the expected number of people to ever live, which then positively influences per capita outcomes through the extensive margin.

Cases in the fifth row are those where the population size in a given period increases the probability of existential risk. As Weil (2023) points out, this results in an ambiguous relationship between per-period population sizes and the number of people to ever live. When combined with the endogenous growth insights, this also implies that the effect on the average quality of life is ambiguous. The reason this case would not uniformally prefer smaller populations is because there is a trade-off between the probability of extinction per period and the number of people who live per period. Formally, the expected number of people to ever live is:

$$\mathbb{E}(I) = \frac{1}{p_x(\bar{N})}\bar{N}, \tag{22}$$

with $p_x(\bar{N})$ being the per-period risk of extinction for a given population size. If $p_X(\bar{N})$ has less than unit elasticity with respect to $\bar{N}$, a larger population gets through more life-years and discoveries in expectation (making this similar to the exogenous extinction case). If the relationship has unit elasticity, the expected number of people to live is invariant to per-period population sizes, which makes this equivalent the case where some I^X leads to extinction. If the elasticity is more than one, then smaller per-period populations lead to more people ever living. Interestingly, this demonstrates that even if ecologists are correct about being near planetary boundaries, for the recommendation to follow that population sizes ought to be shrunk, it must be the case the elasticity of extinction risk per-period with respect to population size is greater than one. Little is known about this elasticity.

Finally, consider the last case where sustained below-replacement fertility is the existential

[17] Again, suppose $\lambda = 0$, so that idea-generation happens independent to the population size. Then the planner would want those life-years to happen as slowly as possible. In the constant population case, that means the smallest per-period populations.

threat. Jones (2022a) calls an equilibrium with this property an "Empty Planet" outcome; Peretto and Valente (2023) directly calls it an extinction steady-state. The intuition here cannot be developed for constant population cases, as the entire concern is a sustained negative growth rate. Instead, conceptualize the larger population case one where the growth rate is less negative. This has similarities to the exogenous extinction case because population decay with certainty is expectationally equivalent to a stable population facing a constant hazard rate. Reducing the rate of population decay, then, is equivalent to reducing the probability of extinction per period (an unambiguous good). So, just as Jones (2022a)'s findings suggest, slower population decay is better for per capita incomes. What the analysis here adds is to recognize that this result is driven entirely by an extensive margin—reducing the rate of (sustained) population decay simply leads to more people ever being born. Without a change in the number of people to ever live, per capita incomes would be unaffected.

It is difficult to know what to take away from this table—that is the point of presenting it. It is premature to take confident stances on which of these rows represents the right stylized model of extinction, or even which column of the idea-generation function is most likely to be true. The results of this section demonstrate that the effect of increasing population sizes depend entirely on these assumptions. Based on the consensus that (i) anthropogenic risks are much more significant than natural risks (Karger et al., 2023) and (ii) $\lambda = 1$ is a reasonable approximation, the single most likely model formulation might be the middle column of row two or three. This would indicate that a current best guess is that population size truly does nothing but accelerate history. On the other hand, many of these cases indicate population does affect the number of individuals to ever live, so in expectation positive effects of population may remain. The broader point is that future research on demographic change needs to think seriously about how the model is closed, recognizing that changes in population size today may influence the quantity and quality of lives ever lived in unintuitive ways.

5 Conclusion

The question of whether a declining population will lead to counterfactual losses in living standards is complicated. Alongside the production of non-rival ideas, there are a host of issues that may depend on the size or age structure of the population: environmental pressures, human capital investments, pension financing, business formation decisions, etc. However, given the importance of non-rival ideas in explaining past economic growth, this channel has received substantial attention among those assessing the effects of population decline (MacAskill, 2022; Spears and Geruso, Forthcoming).

This paper shows this may be a mistake. The stock of ideas available during an individual's lifetime is invariant to population size, at least without assuming an interaction with forces beyond the non-rivalry of ideas. The population size and its effect on the speed at which non-rival ideas are generated may be important for influencing economic growth per period, but I show this is importantly different from influencing economic growth per human life. Indeed, I show that our intuitions can be completely reversed under plausible parameter values: If idea-creation within a period has diminishing returns to researchers, each life is worsened by the population size being larger, even in a simple model where non-rival ideas are the only determinant of living standards.

The key consideration is whether a larger per-period population allows more people to ever live, over the whole of human history. If so, and if population size has any effect on non-rival idea creation, larger populations raise total and average utility experienced over the whole of history. However, this points to the critical nature of assumptions about existential threats. When the variable that matters is the expected number of people to ever live, it is not surprising that model results turn on assumptions about extinction.

The importance of extinction assumptions makes it difficult to be confident in any particular takeaway. One conclusion of this paper is that economists interested in the effects of population dynamics and economic growth must grapple with these issues, echoing a point made in Jones (2016) for technological progress. I tentatively conclude that larger per-period populations are likely to increase the expected number of people to ever live, raising the total and average utility.

References

Adhami, Mohamad, Mark Bils, Charles I Jones, and Peter Klenow (2024) "Population and Welfare: Measuring Growth when Life is Worth Living," Available at: https://web.stanford.edu/~chadj/popwelfare.pdf.

Akcigit, Ufuk and William R Kerr (2018) "Growth through Heterogeneous Innovations," *Journal of Political Economy*, 126 (4), 1374–1443.

Aschenbrenner, Leopold and Philip Trammell (2024) "Existential Risk and Growth," *Global Priorities Institute Working Paper*, Available at: https://philiptrammell.com/static/Existential_Risk_and_Growth.pdf.

Bloom, Nicholas, Charles I Jones, John Van Reenen, and Michael Webb (2020) "Are Ideas Getting Harder to Find?" *American Economic Review*, 110 (4), 1104–44.

Dinopoulos, Elias and Peter Thompson (1998) "Schumpeterian Growth Without Scale Effects," *Journal of Economic Growth*, 3 (4), 313–335.

Eden, Maya and Kevin Kuruc (2023) "The Long-run Relationship Between Population Size and Per Capita Incomes," *CEPR Discussion Paper DP19353*.

Ekerdt, Lorenz and Kai-Jie Wu (2023) "Self-Selection and the Diminishing Returns to Research," *Working Paper*, Available at https://sites.google.com/view/lorenz-ekerdt/research.

Fernández-Villaverde, Jesús, Gustavo Ventura, and Wen Yao (2023) "The Wealth of Working Nations," *NBER Working Paper No. 31914*.

Galor, Oded (2022) *The Journey of Humanity: The origins of wealth and inequality*: Penguin.

Galor, Oded and David N Weil (2000) "Population, Technology, and Growth: From Malthusian Stagnation to the Demographic Transition and Beyond," *American Economic Review*, 90 (4), 806–828.

Greaves, Hilary (2019) "Climate change and optimum population," *The Monist*, 102 (1), 42–65.

Henderson, J Vernon, Bo Yeon Jang, Adam Storeygard, and David N. Weil (2024) "Climate Change, Population Growth, and Population Pressure," *NBER Working Paper 32145*.

Hopenhayn, Hugo, Julian Neira, and Rish Singhania (2022) "From population growth to firm demographics: Implications for concentration, entrepreneurship and the labor share," *Econometrica*, 90 (4), 1879–1914.

Jones, Charles I (1995) "R&D-Based Models of Economics Growth," *Journal of Political Economy*, 103 (4).

——— (1999) "Growth: With or Without Scale Effects?" *American Economic Review*, 89 (2), 139–144.

——— (2003) "Population and Ideas: A Theory of Endogenous Growth," in Aghion, Phillipe, Roman Frydman, Joseph Stiglitz, and Michael Woodford eds. *Knowledge, Information, and Expectations in Macroeconomics (in honor of Edmund S. Phelps)*, 498–521: Princeton University Press.

——— (2005) "Growth and Ideas," in Aghion, Philippe and Steven N Durlauf eds. *Handbook of Economic Growth*, 1B, 1063–1111: Elsevier.

——— (2016) "Life and Growth," *Journal of Political Economy*, 124 (2).

——— (2022a) "The End of Economic Growth? Unintended Consequences of a Declining Population," *American Economic Review*, 112 (11), 3489–3527.

——— (2022b) "The past and future of economic growth: A semi-endogenous perspective," *Annual Review of Economics*, 14 (1), 125–152.

——— (2024) "The A.I. Dilemma: Growth versus Existential Risk," *American Economic Review: Insights*, Available at: https://web.stanford.edu/~chadj/existentialrisk.pdf.

Karahan, Fatih, Benjamin Pugsley, and Ayşegül Şahin (Forthcoming) "Demographic origins of the startup deficit," *American Economic Review*.

Karger, Ezra, Josh Rosenberg, Zachary Jacobs et al. (2023) "Forecasting Existential Risks: Evidence from a Long-Run Forecasting Tournament," *Forecasting Research Institute WP 1*, Available at: https://static1.squarespace.com/static/635693acf15a3e2a14a56a4a/t/64f0a7838ccbf43b6b5ee40c/1693493128111/XPT.pdf.

Kremer, Michael (1993) "Population Growth and Technological Change: One Million B.C. to 1990," *The Quarterly Journal of Economics*, 108 (3), 681–716.

MacAskill, William (2022) *What We Owe The Future*: Simon and Schuster.

Maestas, Nicole, Kathleen J Mullen, and David Powell (2023) "The Effect of Population Aging on Economic Growth, the Labor Force, and Productivity," *American Economic Journal: Macroeconomics*, 15 (2), 306–332.

Ord, Toby (2020) *The Precipice*: Hatchette Books.

——— (2024a) "On the Value of Advancing Progress," Available at: http://files.tobyord.com/progress.pdf.

——— (2024b) "Shaping Humanity's Longterm Trajectory," *Essays on Longtermism*, Available at: http://files.tobyord.com/shaping-humanity's-longterm-trajectory.pdf.

Peretto, Pietro F and Simone Valente (2015) "Growth on a finite planet: resources, technology and population in the long run," *Journal of Economic Growth*, 20, 305–331.

——— (2023) "Growth with Deadly Spillovers," *Working Paper*.

Peters, Michael (2022) "Market Size and Spatial Growth – Evidence from Germany's Post-War Population Expulsions," *Econometrica*.

Peters, Michael and Conor Walsh (2021) "Population Growth and Firm Dynamics," *National Bureau of Economic Research Working Paper*, No. 29424.

Phelps, Edmund S (1968) "Population Increase," *The Canadian Journal of Economics*, 1 (3), 497–518.

Rees, Martin J (2003) *Our final hour: A scientist's warning: how terror, error, and environmental disaster threaten humankind's future in this century–on earth and beyond*: Basic Books.

Romer, Paul M (1986) "Increasing Returns and Long-Run Growth," *Journal of Political Economy*, 94 (5), 1002–1037.

——— (1990) "Endogenous Technological Change," *Journal of Political Economy*, 98 (5, Part 2), S71–S102.

Segerstrom, Paul S (1998) "Endogenous Growth Without Scale Effects," *American Economic Review*, 1290–1310.

Spears, Dean and Michael Geruso (Forthcoming) *After the Spike: Population, Progress, and the Case for People*: Simon & Schuster.

Tetlock, Philip E and Dan Gardner (2016) *Superforecasting: The art and science of prediction*: Random House.

Vollrath, Dietrich (2020) *Fully grown*: University of Chicago Press.

Weil, David (2023) "Replacement Fertility is Neither Natural nor Optimal nor Likely," *Working Paper*, Available at: https://conference.nber.org/conf_papers/f179025.pdf.

Young, Alwyn (1998) "Growth Without Scale Effects," *Journal of Political Economy*, 106 (1), 41–63.

Appendix

A Solution to the Model of Endogenous Research Effort

Aggregate output is a CES aggregator of intermediate goods, each with the respective production functions, where l_j is the labor allocted to sector j.

$$
\begin{aligned}
Y =& \Big[\int_0^A y_j^{\frac{\varepsilon-1}{\varepsilon}} dj\Big]^{\frac{\varepsilon}{\varepsilon-1}} \\
y_j =& l_j
\end{aligned}
$$

A is the measure of ideas in this economy, endogenously produced by potential entrepreneurs. Denoting the share of people in this economy who search for ideas s, we have a production function of A that is:

$$A = \omega \times s \times N.$$

That is, each individual has some probability ω of discovering a profitable idea. I assume that each worker inelastically supplies one unit of labor, so that the market clearing condition dictates that the sum of wage earners plus entrepenuers is the total population, N.

Let us first focus on each intermediate firm's production decision, conditional on operating (that is, $j \in [0, A]$). The demand curve for each y_j can be solved through a standard cost-minimization problem by the final good firm that aggregates these products.

$$\min_{y_j} \int_0^A y_j p_j \text{ such that } \Big[\int_0^A y_j^{\frac{\varepsilon-1}{\varepsilon}} dj\Big]^{\frac{\varepsilon}{\varepsilon-1}} \geq Y \tag{23}$$

This delivers the demand curve:

$$y_j = \left(\frac{p_j}{P}\right)^{-\varepsilon} Y, \tag{24}$$

where I have defined P, the aggergate price level, as the Lagrange multiplier.[18]

Each intermediate producer, j, faces a problem of whether to purchase a patent at price p_A, what price to charge for its good, p_j, and how much of its good to produce with a linear production function, $y_j = l_j$, where l_j is labor in sector j that gets paid a wage w that is taken as given by each small firm. I assume a free-entry condition, such that the fixed cost to operate, p_A, will be equal to

[18] This is because the lagrange multiplier captures the cost of having the constraint tightened, on the margin. Here, the constraint is producing one more unit of the final good. The marginal cost of producing Y will equal the price of Y in a competitive equilibrium.

the profit this monopolistically competitive firm can earn in equilibrium. The problem for a firm that decides to purchase a patent and operate looks as follows (if the maximum profit that can be attained is less than zero, the firm will not operate).

$$\max_{p_j, y_j, l_j} p_j y_j - w l_j - p_A \tag{25}$$

subject to:

$$y_j = \left(\frac{p_i}{P}\right)^{-\varepsilon} Y \tag{26}$$

$$y_j = l_j \tag{27}$$

Subbing these constraints directly into the problem and taking a first-order condition, we obtain the usual result that CES monopolistic competition delivers a price that is a constant mark-up over marginal costs (here, the wage). This implies a value for p_A that set profits to zero, by the free entry condition.

$$p_j = -\frac{\varepsilon}{1-\varepsilon} w \tag{28}$$

$$p_A = (-\frac{\varepsilon}{1-\varepsilon} - 1) w y_j \tag{29}$$

Also note that $\omega p_A = w$ in equilibrium, as the expected value of being an entrepreneur must be equal to the wage. We can use this fact to substitute w from the right hand side of (29), and simplify the expression.

$$p_A = \left(\frac{1}{\varepsilon - 1}\right) \omega p_A y_j \Rightarrow$$
$$\frac{(\varepsilon - 1)}{\omega} = y_j$$

Finally, notice that in a symmetric equilibrium each y_j will be identical. Furthermore, each $y_j = l_j$, the amount of labor supplied in that industry. Recalling that the share of the population earning wages is $(1-s)$, this implies that there are $(1-s)N$ total workers split across the A industries. So, each $l_j = \frac{(1-s)N}{A}$. However, we also know in equilibrium that $A = \omega \times s \times N$. This lets us sub in our new expression for l_j for y_j. Simplifying, we get:

$$\frac{(\varepsilon-1)}{\omega} = \frac{(1-s)N}{\omega s N} \Rightarrow$$
$$\boxed{(\varepsilon-1) = \frac{1-s}{s}} \tag{30}$$

First, verify that the intuition of this result makes sense. As $\varepsilon \to 1$, $s \to 1$. This is the case of perfect complements: the love-of-variety is so extreme that nearly everyone is employed in research to expand the line of product varieties. When $\varepsilon \to \infty$, $s \to 0$. This is the case of perfect substitutes. Consumers do not care about which products they consume, so everyone is employed in the production sector rather than expanding the variety of products.

But what is most important for the application here is that this function is independent of N. It is not the case that a larger market endogenously allocates a larger share of the population to research. So, the assumption employed throughout the paper that s is independent of N seems to be, at least, a good baseline assumption.

B Endognizing s as a function of A

This appendix section demonstrates that endogenizing s as a function of A has no effect on the main result. This specification allows the fraction of the population that, *ex-ante*, contributes to TFP improvements to vary with s. For example, if one social innovation is universal is universal public high-school, the fraction of people with the skills and ability to contribute to research will increase.

Generally, the function of TFP improvements (with $\lambda = 1$, $\delta_A = 0$) looks as follows.

$$\frac{\dot{A}}{A} = \theta s(A) N A^{-\beta} \tag{31}$$

This function cannot be analytically integrated, in general. Instead, I will numerically demonstrate that this relaxation has no effect on the main result. Let $s(A)$ be a simple logistic function:

$$s(A) = \frac{1}{1+e^{A}}. \tag{32}$$

I will not attempt to calibrate this or take a stance on its realism. The point is to demonstrate that the precise neutrality result holds for a function with the straightforward properties that $s'(A) > 0$ and $s(A) \in [0, 1]$.

Figure A1 demonstrates this point: The relationship between i and $A(i)$ is invariant to population sizes when s is relaxed to be non-constant and a function of A. The reason—as described in Section 3—is that there is still some fixed number of people who contribute to TFP by a given i. For example, it will still take X person-years to get to universal public schooling whether the population is large or small. That implies that all $i < X$ will live in an environment without public high-school, regardless of per-period population sizes. The result is that each i's research productivity remains invariant to per-period population size, which is what would need to change for the total TFP-improvements by person i's life to be affected by per period population size.

Figure A1: Endgonizing s does not break neutrality result