Population, Ideas, and the Speed of History*

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Abstract

Scale effects have long been at the heart of endogenous theories of economic growth. Larger economies can allocate more resources to discover the non-rival ideas that drive long-run productivity improvements. This paper demonstrates that—in direct contrast with conventional wisdom—these idea-based scale effects imply nothing about the relationship between individual economic outcomes and population size. The reason is straightforward: A larger population increases the arrival rate of both ideas *and* people, potentially leaving each life unchanged. Instead, the population size governs the *speed of history*; to a first-order, the same events occur, they just occur earlier in time if the population is larger. I then relax the standard model to identify where idea-based scale effects—of either sign—are likely to arise. I conclude that the key consideration is whether a larger per period population leads to more individuals ever existing, putting new emphasis on existential risks when attempting to understand the implications of demographic change.

Keywords: Population economics, endogenous growth, scale effects, extinction risks, aggregate welfare.
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1 Introduction

Since Romer (1986, 1990), increasing returns to scale have been at the center of economic growth theory. Other things equal, an economy with more people and resources will discover more ideas for new products or production methods. Ideas are fundamentally non-rival, so this increase in the stock of aggregate ideas increases per capita incomes. It is a remarkably simple chain of logic: More people $\rightarrow$ more ideas $\rightarrow$ higher living standards. Conversely, recent attention has been paid to the fact that per capita income growth may slow or end in a future of low fertility and population decline (Jones, 2022; Geruso and Spears, Forthcoming).

This paper demonstrates that these idea-based scale effects, in fact, say nothing about the relationship between individual living standards and population growth. The reason is straightforward: Increasing the rate of population growth increases the arrival rate of *both* discoveries and the people who would benefit from these discoveries, offsetting the purported benefits. To fix ideas, consider one of the original formulations of this scale-based reasoning in Phelps (1968): "If I could re-do the history of the world, halving population size each year from the beginning of time on some random basis, I would not do it for the fear of losing Mozart in the process." Suppose there is some fixed probability that each birth produces someone as talented as Mozart. It would be more accurate then to say that halving the historical population *delays* the expected date that someone as talented as Mozart is born.[1] At the same time, halving the historical population also halves the number of people living in these now Mozart-less years. To a first-order, these effects will exactly offset: the same number of people are expected to live prior to a Mozart regardless of per-period population sizes. In other words, no individual lives are improved or worsened by scale, even while recognizing that music is non-rival. Decreasing the population delays the arrival of a Mozart-like figure, but it delays all other lives too.

The first part of the paper formalizes this logic in a standard Romer/Jones-style endogenous growth framework (Jones, 1995). Under simple linearity assumptions, I show analytically that the size of the population has no effect on the number of people-years lived prior to any given TFP level. These endogenous growth models, quite sensibly, imply that cumulative TFP improvements depend on cumulative research effort over time. So, just as with Mozart, if history were re-ran with a smaller population, the expected number of people-years lived prior to steam power, or the electric light bulb, or antibiotics would be unchanged. These discoveries would happen later,

[1] Of course, it would not literally be Mozart because any change to history affects the identities of everyone who lives afterwards—different events unfold, different adult pairs have children, etc.—so everything said throughout the paper is meant in expectation, unless otherwise stated.

but proportionately fewer people-years would have been lived in those earlier years, leaving the inventions available to a given individual life unchanged. Scale-based growth is not, and can not be, *ceteris paribus*—the distribution of population necessarily changes.

The second half of the paper studies whether there are channels whereby populations size can affect individual outcomes in these models, and whether the conditions that would rescue this intuition between non-rival ideas and the benefits of scale are likely to hold. I consider four, three of which can plausibly, but not certainly, generate a positive relationship between population size and individual living standards. Even so, these second-order channels likely generate small effects and require a substantial re-understanding of the mechanisms linking population size to incomes.

First, I endogenize research effort, in contrast to a simple learning-by-doing assumption where each individual has some fixed *ex-ante* contribution to TFP growth. Larger populations have larger consumer bases, potentially changing the incentives to engage in R&D. This turns out to be misleading. When the consumer base is larger, it is more profitable both to generate new ideas and to exploit existing ones. The benefits of engaging in research increase, but so does the opportunity cost. Formally, the equilibrium condition determining the share of researchers is independent of the population size.[2] Relaxing the simplifying assumption that per capita research effort is fixed does not change the main takeaway, at least not without further reasons that per capita research effort would change with population size.

Second, I relax a linearity assumption in the aggregate R&D equation used to generate exact analytical results. Specifically, I allow for convexity (or concavity) in the returns to research effort within a period. If collaboration effects between researchers generate increasing returns, then larger per-period populations will improve individual outcomes. Unsurprisingly, it would be better for more people to live in a given year if there are increasing returns to scale in the research production function itself. However, this would be true if there were increasing returns to scale in any sector, so this claim is no longer distinct to R&D models of growth. More importantly, if there are instead diminishing returns to researchers within a period, individual outcomes are improved by *smaller* populations. This concavity assumption is (much) more commonly applied. If anything, relaxing the linearity assumption on researcher productivity provides reason to believe smaller populations improve individual living standards, an exact reversal of the lesson typically drawn from idea-based models of economic growth.

[2] Population *growth* may change the exploit/explore trade-off for potential entrepreneurs, depending on the patent structure, which I discuss later. But that is a substantially different claim that is about relative population size between periods, not a 'bigger-is-better' scale effect.

Third, I consider knowledge depreciation. This force, in isolation, unambiguously serves to rescue the original intuition about scale effects. The reason is as follows: If knowledge depreciates over time, the faster that people 'arrive' following the discovery of an idea, the more people benefit from that idea before it is lost. When time erodes the knowledge base, the planner would prefer to pack humanity into fewer periods. Depreciation of knowledge is essentially unstudied, so it is difficult to know what to make of the potential magnitude of this effect. However, in a simple quantitative exercise, I demonstrate that plausible values for knowledge depreciation are unlikely to be pivotal when considered alongside a range of plausible values for λ. Therefore, taken in conjunction, these first three model relaxations provide little reason to deviate from the main result that there is no effect between population size and individual outcomes in idea-based growth models.

Finally, in light of the robustness of this null relationship, I instead focus on the fact that there may be other reasons to prefer larger per period population sizes in these models. Namely, temporal discounting. The main result is that larger populations accelerate the arrival of both discoveries and lives, so non-zero discounting will interact with the temporal aspect of this finding. Consider the two reasons that are traditionally raised for discounting experienced utility over time: (i) a pure rate of social time preference and (ii) to account for the probability of death/extinction.[3] The former is trivial—it is just a preference that things happen earlier in time. If the planner prefers the exact same events when they happen on an accelerated time horizon, bringing forward people and ideas via population growth is welfare-improving, even if the earlier, positive, claim that no individual person-year is improved by scale is correct. In this case, population growth would be beneficial for the novel reason that it functionally serves to accelerate history.

Discounting because of death to the representative agent (i.e., existential risk) is more interesting. In an expected value sense, future utility should be discounted by the probability it is in fact experienced. Here we find potentially the biggest impact of larger per period populations in idea-based growth models: A larger per-period population results in more lives being lived, and more ideas being discovered, during humanity's lifespan. Put differently, the main result above is that the same number of people live *prior* to any given level of TFP; but if discoveries happen earlier, more people may live after a discovery. It is still the case that no lives are made any better off *conditional on existing*, but more total lives are lived when the population is larger and ex-

[3] The third reason for discounting *consumption* over time is that future generations will be richer than present generations under positive economic growth. That is not relevant here as I am dealing directly with utility over time for individuals.

tinction happens at some exogenous date T^X. However, this too diminishes what is unique about scale effects in ideas-based growth theory. If the planner cares about the number of people who will ever live, nearly any model will generate the conclusion that larger per-period populations are socially beneficial. This may be true, but it is a normative question about the value of existing, not a positive question about the effect of population size on economic growth.

In fact, taking a richer view of extinction risks, idea-based growth models may be one of a small class that does *not* lead the planner to prefer larger populations, even if she cares about the number of existences. Imagine, ex-post, that humanity's extinction is endogenous; perhaps the probability of extinction is related to the level of technology. The invention of nuclear weapons, for example, appear to threaten our existence. Looking forward, some claim that sufficiently powerful artificial intelligence may likewise be a threat to civilization (see e.g., Ord, 2020). Just as with Mozart, steam power and the electric light bulb, these endogenous growth models predict that a fixed number of people live prior to the invention of dangerous technologies. In this case, a larger population proportionately accelerates everything, *including extinction*. The number of people who ever live is then invariant to per-period populations sizes, so any effect that population size could have is eliminated.[4]

In general, I show that a range of plausible ways of modeling existential risk and idea-generation can lead to a range of outcomes regarding the relationship between per period population size and the quality and quantity of lives that are lived. This includes cases where economic growth is independent of population size, to generalize the findings of this paper beyond the semi-endogenous growth literature. When TFP growth is independent of population size, larger populations allow more people to live prior to the invention of a dangerous technology. The development of something dangerous like (perhaps) artificial super-intelligence happens at the same date under this assumption, so the only effect of a larger population is to increase the number of lives that live prior to each extinction threat. Indeed, this is a special case of the parameter values that appear to be most likely based on prevailing wisdom in these respective fields. The conclusion I draw is that larger per period populations are likely better than smaller populations because more people ever live, which has a (weakly) positive effect on per capita incomes, averaged over all of history. This is far from settled, however, and calls for more work assessing the joint effect of demographic change and existential threats.

[4]Interestingly, this claim does not rely on when, or how quantitatively likely it is that any given invention is dangerous rather than beneficial, it only relies on the assumption that *eventually*, ex-post, our demise is caused by the deployment of a sufficiently dangerous technology.

This paper contributes to literatures at the intersection of scale economics and long-run economic growth. In particular, Jones (1995) builds on the Romer (1986, 1990) insight to highlight the importance of population size/growth in driving TFP improvements (see also Kremer, 1993; Jones, 2003, 2005, 2022). This counter-intuitive idea—that population size is the key driver of long-run improvements to living standards—sparked a series of papers attempting to eliminate these scale effects (Dinopoulos and Thompson, 1998; Segerstrom, 1998; Young, 1998). Jones (1999) shows these attempts fail; a larger population is still richer per capita each period, even if specific parametric assumptions can eliminate the result that faster population growth leads to faster economic growth. The link between the non-rivalry of ideas and increasing returns to scale has been seen as a deep feature of long-run growth since. Recent years have seen renewed attention and additional exploration of this finding as the world transitions to a regime of below-replacement fertility, with the possibility of long-term population decline (Jones, 2022; MacAskill, 2022; Geruso and Spears, Forthcoming).

The main contribution of this paper is to demonstrate that, contrary to conventional wisdom, that these idea-based scale effects do not imply anything about whether larger populations improve people's economic lives. It is critical to note that I take as given throughout the paper the positive claim that larger per period populations produce more non-rival knowledge. The novelty is a shift of focus, away from the income levels of time periods to the income levels of people. The positive economics from this vantage are crucially different: A larger population improves average incomes in each time period without changing the incomes of any people. Once this is established, the relevant question in these models is only whether a larger population allows for more lives to ever be lived.

By studying long-run, aggregate welfare—that is, the quantity and quality of lives over long time horizons—this paper also intersects with a literature in applied welfare economics assessing changes in population sizes. Most relevant is Adhami et al. (2024), which decomposes the social welfare gains coming from population growth versus income growth over the last decades. They find that increases in the quantity of lives matters much more than the increase in living standards over this period. Other papers in the literature of applied population ethics have similarly found the quantity of lives to be a first-order determinant of social welfare (see e.g., Lawson and Spears, 2023). These findings align with my own, wherein the quantity of people to ever live is a crucial component in determining whether larger per period populations have any positive or normative effects.

Finally, there is a recent literature interested in risks to humanity's survival that directly deals with the question of how many people will ever live. Greaves (2019) studies the case of climate change, where each human life has some non-zero carbon footprint in the long run. She shows that this implies that a fixed number of people will ever live: either the planet is warmed beyond a level that we could survive, or we run out of whatever the necessary dirty input is, and that happens after a fixed number of lives are lived. The present study works with a premise that is more relevant for economists (and more realistic), but generates analogous takeaways using a similar style of reasoning. Jones (2016, 2024) studies problems where the risk of extinction endogenously evolves with technology, similar to what is assumed in some of the cases in Section 4, but instead studies the growth-safety trade-off. Aschenbrenner and Trammell (2024) likewise study a growth-safety trade-off faced by a planner who can invest in safety technology as well as consumption enhancing technology, asking whether technological growth is beneficial for increasing humanity's long-run survival probability. Ord (2024b) develops a very general framework for assessing the long-run welfare implications of changes to the joint trajectory of life-quality and survival-probability of humanity over time.[5] Seen from Ord (2024b)'s framework, I show that population size is a method of speeding up events, coming to an analogous conclusion that the effect of accelerating progress importantly depends on assumptions about extinction.

2 Ideas-based growth models do not imply a positive relationship between population size and living standards

This goal of this section is to demonstrate that the non-rivalry of ideas does not, on its own, imply that larger populations raise individual living standards. The key is to look at outcomes from the perspective of people, rather than time periods. I begin by formalizing the main proposition using a standard (semi-)endogenous growth framework. I then discuss the intuition for this result, before assessing whether the standard intuition about the benefits of scale can be rescued with relaxations of the model in Section 3.

Equation (1) is a production function of productivity improvements (i.e., *ideas*) developed in the literature spawned by Jones (1995), which builds directly from Romer (1986, 1990). The key concept embedded in this formulation is that the production of ideas increases in the effort put

[5] See also Ord (2024a) for a more direct discussion of existential risk being pulled forward in time by increases to scientific progress.

towards discovering them, and that these discoveries improve aggregate productivity, A.

$$\frac{\dot{A}(t)}{A(t)} = \alpha(t)(s(t)N(t))^{\lambda}A(t)^{-\beta} \tag{1}$$

Time, t, is continuous; $\dot{A}$ is the instantaneous change in productivity. This depends on three factors. First, the number of individuals engaged in research. In (1) this is decomposed as the share of the population in research, s, multiplied by the population size, N. Second, the productivity of these researchers, α. Third, the accumulated knowledge stock, $A(t)$, which allows for non-linearity in idea production. There could be increasing returns to productivity gains ($\beta < 1$) if past inventions (e.g., the computer) help generate new inventions, or there could be diminishing returns to inventions ($\beta > 1$) if earlier discoveries are systematically easier to make. In practice, $\beta > 1$ matches the historical data better, both in aggregate and at the industry level (Jones, 1995; Bloom et al., 2020), though this parameter is not consequential for the results established here.

Notice that the number of researchers is raised to some power λ that determines whether there are diminishing returns to researchers within a period. There is considerable uncertainty about what value λ should take. One reason to believe that $\lambda < 1$ is *duplication*; people alive at the same time might solve the same problems, resulting in wasted effort. On the other hand, *collaboration* might be a reason that $\lambda > 1$; the ability to communicate with other researchers might result in faster progress than a counterfactual where those same researchers live in different years.[6]

For expositional simplicity, let $\lambda = 1$ and $s(t), \alpha(t)$ be some constants $\bar{s}, \bar{\alpha}$. The assumption of constant α, s, is employed because the objective is to isolate the effects that population size changes have. Of course it is true that increasing α or s would also increase the production of ideas, but this is not the focus of this paper. The assumption of $\lambda = 1$ is more substantive. It imposes a constant marginal effect of population within a period (i.e., if the world population were twice as large, exactly twice as much progress would occur). In Section 3.2 I relax this assumption and explore how the implications of the model are affected. For now, I ignore possible curvature to generate exact analytical solutions.

With these assumptions, we can rewrite (1) as follows, where $\theta = \bar{\alpha}\bar{s}$.

$$\frac{\dot{A}(t)}{A(t)} = \theta N(t)A(t)^{-\beta} \tag{2}$$

[6]In reality, this relationship likely has an ‘S’-shape, where collaboration benefits dominate at small populations, but duplication issues dominate for large populations. It is unclear which dominates for current population sizes.

Integrating this function delivers the following expression for the level of A in any given period.

$$A(t) = \left(\beta\theta \int_0^t N(\tau)d\tau + A_0^\beta\right)^{\frac{1}{\beta}} \tag{3}$$

Equation 3, quite intuitively, states that if idea discoveries per period are a function of per-period research effort, then cumulative discoveries will be a function of cumulative research effort. Formally, because the assumption is that people are the key input to idea discovery, cumulative research effort is just a scalar of cumulative people-years, $\int_0^t N(\tau)d\tau$. Notice that this does not need to be a linear relationship. In the case where $\beta \neq 1$, a person-year generates more or less knowledge depending on the size of the existing stock. This is not important for the argument that follows.

Substantively, what Equation 3 does is eliminate t as an independent variable. It is cumulative human effort, not time, that drives innovation. Our economic lives are richer than previous generations' not because time has passed, but because people have been working to improve productivity in that time. Therefore, time can be eliminated as an independent variable without losing anything of conceptual importance.[7]

The formal claim of this section is that the same number of people-years are lived prior to a given level of TFP, regardless of population sizes. This will follow immediately by defining person-years as:

$$i(t) = \int_0^t N(\tau)d\tau \in [0, I]. \tag{4}$$

This is a continuous value, so i is an instantaneous person-experience denoted in units of people-years. It captures that if the population size were 10 billion and exactly two years passed, then, at that instant, the 20th billion person-year is being lived. What I mean when I have said "individual" is technically each individual person-year, ordered from 0 until the number of people years humanity eventually experiences in total, I.[8]

Equation 3 implies that $A(t)$ is a function of people-years, which are now denoted $i(t)$. There-

[7]Of course, this could be contested. If experiments literally take time, then perhaps time *should* enter this function.

[8]You might be thinking that the i values associated with your life would change if the historical population were bigger or smaller. That is only true under very special circumstances: if historical populations were any different, "you" would almost certainly not come into existence. The events that led to the exact sperm-egg combination that produced you are essentially a probability zero event. This is why I think it makes sense to think of labeling experiences as i's without specifying whether the same people experience them.

fore, the expression for $A(t)$ can be simply rewritten as a function of i.

$$\boxed{A(i) = \left(\beta\theta i + A_0^{\beta}\right)^{\frac{1}{\beta}}} \tag{5}$$

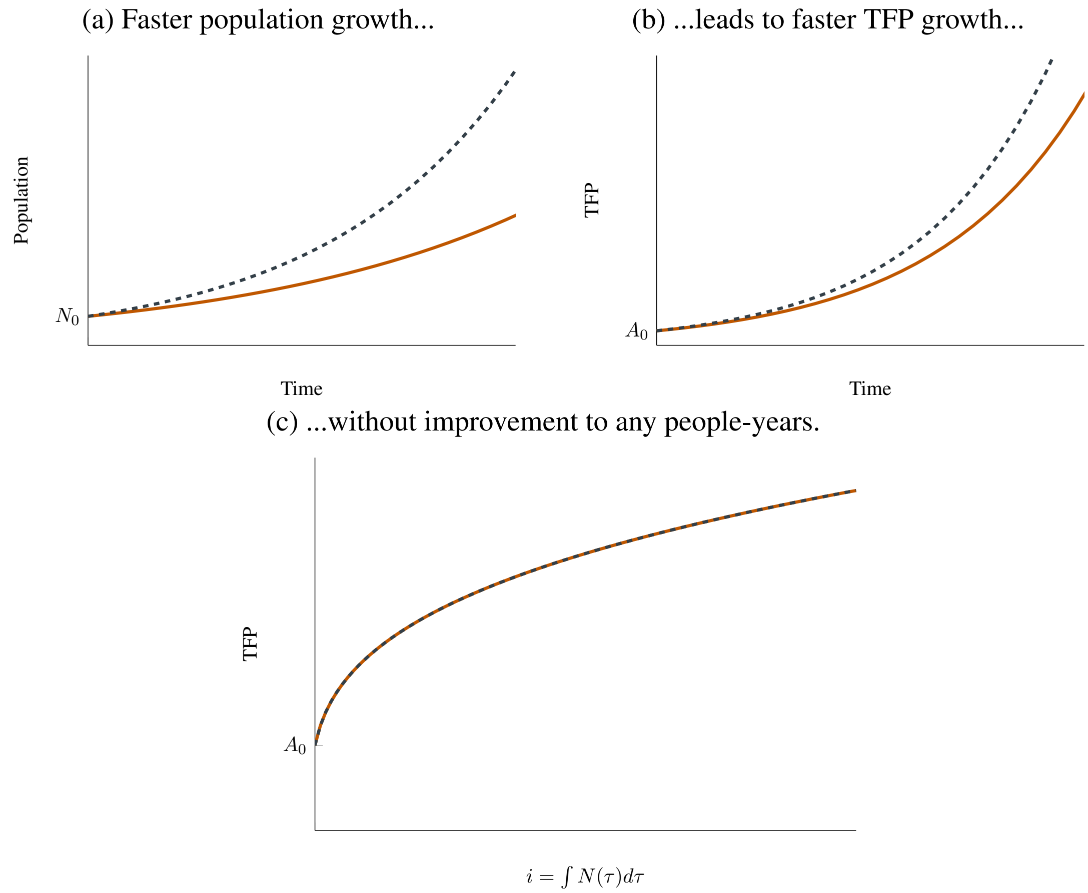
This result makes clear that the experience of each person-year i is pre-determined by i's order in history. The arrival of this experience can be accelerated by increasing the density (or arrival rate) of people-years, but the ideas available during each momentary experience cannot be improved via scale-based growth.

Figure 2 demonstrates the mechanisms of this result. Panels (a) and (b) plot population and technology as they are usually plotted—against time. The history with faster population growth has faster TFP growth. Panel (c) eliminates time as a variable and plots them against one another. This is a plot of technology available to people, rather than time periods, which I take to be the more meaningful unit of analysis. Time periods do not have incomes or experience utility. People do. Panel (c) shows that TFP available to each person-year is unaffected by the size of the population; these curves exactly overlap. This is because the count of individual experiences is accelerated just as fast as the TFP improvements are in the larger population growth scenario.

The intuition of this result is easiest to grasp if we zoom out and conceptualize each i as an individual life (i.e., ignore the consequences of overlapping lives for the moment). This amounts to assuming that an individual can live before steam power is invented, or after, but we're ignoring cases where their life overlaps with this invention. Equation 5 says that the number of people who live prior to steam power is not affected by population size, because the number of lives necessary to build up this knowledge base is fixed, conditional on θ, β.

The intuition in more realistic cases with overlapping lifetimes is slightly less straightforward, but the main result continues to hold. Start with another stylized example. Imagine there will be 20 billion people who ever live, each with a 100 year lifespan. (Having different numbers of people ever live raises further conceptual difficulties that I return to in Section 4 after establishing results for the individuals who live regardless of population sizes.) History could either be such that 10 billion lives are lived in first 100 years and then the next 10 billion live (History A), or it could be that all 20 billion live at once (History B). Suppose it will take 1 trillion people-years of effort and economic activity to get to some key, life-improving technology. In History A, that invention comes precisely at the moment that generations are turning over, after 100 years have passed. In History B, it comes after 50 years. So, relative to History A, the earlier-born half of the population has half of their life made better (they now live 50 years with this technology, whereas in History

Figure 1: Accelerating economic growth via population growth does not improve any life-years



Notes: Blue dotted line represents a history with faster population growth than solid orange line. Panel (b) illustrates that TFP growth would be faster in the history with faster population. Panel (c) demonstrates—by eliminating time as a variable—that the TFP available after a given number of people-years is invariant to the speed of population growth.

A they get 0 years with it); but the later-born half of the population has half of their life worsened (they now live half of their life without this technology, whereas in History A they live their entire lives with it). In terms of people-years, these effects exactly cancel out. In both cases 1 trillion people-years are lived without this technology, 1 trillion are lived with it. These people-years are distributed across individuals differently, so a planner that explicitly accounts for inequality may prefer History B, but strictly for inequality-aversion reasons, not because economic growth was accelerated.

To make this even more concrete, consider what this means for the generations who's size is in the process of being determined—our children and grandchildren. We might hope that others

Table 1: With overlapping lives, population size improves some at the (offsetting) expense of others

	$N(t)$	Idea at:	People-years pre-idea	Entire lives lived w/out idea	Entire lives lived w/idea
History A	10 Bil.	$t = 100$	1 Tril.	10 Bil.	0
History B	20 Bil.	$t = 50$	1 Tril.	0	10 Bil.

Notes: Table cataloging a scenario with overlapping lives. A larger population implies that fewer people live their entire lives without a given discovery/idea/TFP-improvement. But it also means that more people live a fraction of their lives without this discovery. In terms of people-years, these effects cancel out.

have more children, so that ours benefit from the knowledge of these individuals. The relevant trade-off here is that more people-years then need to live alongside your children, which is worse than people-years lived after their adult-lives and the TFP improvements they contribute. Again, ex-ante, these effects precisely cancel out under the linearity assumption on research effort applied above. You may self-interestedly retain a preference for population growth for these reasons, but an impartial planner without explicit egalitarian motives will be indifferent without further reasons to prefer population size or growth. These potential further reasons are what I turn to next.

3 Main takeaway is robust to model enrichment

This section discusses potential modifications through which scale effects in ideas-based growth models may emerge: endogenizing research effort; increasing returns to research effort within a period; and the depreciation of knowledge over time. In the latter two cases, the scale effects of ideas-based growth models can be rescued: Larger populations improve individual outcomes. However, these are, if anything, second-order effects that deserve more study. They fail to provide the same strong reasons we have for *a priori* confidence that population size increases the rate of economic growth.

3.1 Endogenous research effort

The results above assume that s, the share of individuals contributing to the knowledge base, is fixed. However, larger markets may incentivize more research and development, providing additional reason to believe larger populations will lead to more idea creation that is omitted from Section 2. In a world where only 10 people have a rare disease in a given period, rather than 1000, it seems straightforwardly less likely anyone will be incentivized to find a cure. This is true, but the causal mechanism works in a way that is misleading for the application of this paper—endogenizing research effort in a straightforward way does not rescue the intuition of scale effects.

With a fixed research share, s, the main results already accounted for the fact that there will be a larger number of researchers in a given period. The number of researchers scales linearly with N. So, for this endogenous research channel to deliver the result that a larger population generates more research per person-year, i, it must be accounted for by s itself increasing in N. That is, the composition of the population must also change with population size. A simple general equilibrium R&D model below shows that it is not at all obvious this is the case.

Consider a simple one-period model where workers can choose before the period to either work or search for an idea and sell the patent as an entrepreneur. The aggregate production function is a CES aggregator of all goods that there is an idea for. In particular, the integral of possible intermediate goods to purchase extends to A. When A is larger, the factors of production can be spread over more intermediate-goods, j, which increases Y through a love-of-variety channel. In short, the number of ways our resources can be used—and therefore aggregate consumption—is increasing in A.

$$Y = \left[\int_0^A y_j^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{6}$$

Each intermediate producer, j, chooses whether to purchase a patent at price p_A, the price to charge for its good, p_j, and the quantity of its good to produce, y_j. It faces a linear production function, $y_j = l_j$, where l_j is labor in sector j that gets paid a wage w that is taken as given by each small firm. I assume a free-entry condition, such that the fixed cost to operate, p_A, will adjust to set profits to zero in equilibrium, despite the monopolistically competitive set up. The demand curve faced by the firm comes from a standard cost-minimization problem that the final good's producer solves. Overall, the problem for a firm that decides to purchase a patent and operate looks as follows (if the maximum profit that can be attained is less than zero, the firm will not purchase the patent and operate).

$$\max_{p_j, y_j, l_j} p_j y_j - w l_j - p_A \tag{7}$$

subject to:

$$y_j = \left(\frac{p_i}{P}\right)^{-\varepsilon} Y \tag{8}$$

$$y_j = l_j \tag{9}$$

Workers supply one unit of labor inelastically. They can either earn wage w, which they take as given, or search for an idea. If they search for an idea, they are successful with probability ω and can sell the patent for profit p_A. Therefore, the key equilibrium condition coming from the worker's problem is that the return from searching for an idea will be equal to the wage.

$$w = \omega p_A \tag{10}$$

The amount of knowledge in this economy is a function of how many researchers decide to search for ideas. Denote N as the population size and s as the share of individuals searching for ideas.

$$A = \delta s N \tag{11}$$

The full solution to the model is contained in Appendix A. The relevant equilibrium condition is that:

$$1 - \varepsilon = \frac{1 - s}{s} \tag{12}$$

The share of the population working in research is a function only of the elasticity of substitution between goods—it is independent of the size of the population. The intuition for this result can be seen by considering the intermediate good producers problem. Intermediate firms need to purchase both a patent and labor to produce and sell into the larger market. For the same reason that a patent becomes more valuable, so too does each unit of labor. The opportunity cost of searching for an idea grows alongside the benefit of searching for an idea, leaving the share of individuals choosing to search for an idea unchanged.

Things become more complicated in cases where populations grow over time—this was a simple static setting designed to demonstrate why there is not good reason to believe the fraction of researchers is not increasing in the population size. In Jones' classic (1995) paper highlighting the importance of population growth in R&D based models of economic growth, the equilibrium share of researchers is increasing in population growth, but not its size. The reason is that in his setting, patents have dynamic value. A population that is growing faster has a larger consumer base tomorrow than it has today, so the value of future production is higher than the value of current production. This tilts things in favor of researching, since a patent is something of an investment that pays flow dividends in proportion to future demand for ideas.

That population growth, not size, determines the share of the population is quite distinct. It mirrors the results in Peters and Walsh (2021) and Karahan et al. (Forthcoming) where population growth contributes to firm dynamics because paying a fixed start up cost today is more worthwhile if future years will have increased levels of demand, and a larger workforce to hire. This sort of channel has implications that are much less clear for population sizes. A smaller population with faster population growth would have more research per capita than a larger population with slower growth. Furthermore, if populations stabilize in the coming century, for example, this channel is irrelevant for determining whether a larger world population is preferable to smaller population.

3.2 Increasing returns to research effort

In Section 2 the production function of ideas was assumed to be linear in research effort within a period. This allowed for the derivation of a clean analytical solution between cumulative people years and the ideas available to an individual for an arbitrary path of population. This linearity assumption may be too strict, and introducing curvature can indeed matter for the main takeaways. Equation 13 is the original production function of ideas, reintroducing the term λ which is no longer equal to one.

$$\frac{\dot{A}(t)}{A(t)} = \tilde{\theta} N(t)^{\lambda} A(t)^{-\beta} \tag{13}$$

This in turn produces the slightly modified version of cumulative returns to research.

$$A(t) = \left(\beta\tilde{\theta} \int_0^t N(\tau)^{\lambda} d\tau + A_0^{\beta}\right)^{\frac{1}{\beta}} \tag{14}$$

This modification breaks the tight link between cumulative people-years and cumulative ideas produced. When $\lambda \neq 1$ it also matters how lives are spread out over time. For example, if $\lambda > 1$, for the same number of cumulative life years, humanity discovers more ideas if those lives have more overlap. If $\lambda < 1$, there are diminishing returns to research effort within a period, so less is discovered if there is more overlap of a fixed number of life-years.

Somewhat trivially then, for $\lambda > 1$ the ith person-year—where recall, $i(t) = \int_0^t N(\tau)d\tau$—has more TFP accessible to it if historical populations were larger. The i people-years that occur prior to i's existence were able to take advantage of more collaborative opportunities and discover more. Conversely, if there are diminishing returns to research effort within a period ($\lambda < 1$) the opposite will be true.

This can be seen formally by considering the special case where population sizes are a constant $\bar{N}$. The constant population case is one I will return to frequently for these more complicated cases because it allows for analytical solutions while continuing to isolate the core question of whether larger populations improve individual outcomes in idea-based growth models.[9]

In the case of constant populations, the population integrals in Equation 14 can be rewritten.

$$A(t) = \left(\beta\tilde{\theta}\bar{N}^{\lambda}t + A_0^{\beta}\right)^{\frac{1}{\beta}} \tag{15}$$

$$i(t) = \bar{N}t \tag{16}$$

This implies:

$$A(i) = \left(\beta\tilde{\theta}\bar{N}^{\lambda-1}i + A_0^{\beta}\right)^{\frac{1}{\beta}} \Rightarrow \tag{17}$$

$$\frac{\partial A(i)}{\partial \bar{N}} = \tilde{\theta}(\lambda-1)\bar{N}^{\lambda-2}i\left(\beta\tilde{\theta}\bar{N}^{\lambda-1}i + A_0^{\beta}\right)^{\frac{1-\beta}{\beta}} \tag{18}$$

This term will be positive if and only if $\lambda > 1$. In the baseline case where $\lambda = 1$ it is easy to see the derivative here is zero, generating the independence result of prior section. This derivative will be positive only if collaboration effects dominate the drag coming from duplication.

Interestingly, $\lambda < 1$ is the standard assumption in the literature that attempts to calibrate and/or estimate the effect of aggregate research inputs on long-run growth (see e.g., Bloom et al., 2020; Jones, 2022). If this is indeed the case, then the standard model of non-rival ideas may imply *decreasing* returns to scale along the margin that matters: each individual person-year is *worsened* if per-period populations are larger. If the population size is increased, the arrival ideas increases, but at a slower rate than the arrival of people-years is increased. Therefore, each person-year has less knowledge accessible to it.

Empirical evidence for this parameter is difficult to come by. Within research teams there is good evidence that collaboration is important. Across research teams it is much less clear. Do the benefits of getting to email with others working in your field outweigh the chances you have each wasted time working out the same details? In reality, a global power function is almost certainly too simple.[10] The relevant question is whether on current margins congestion/duplication effects are dominated by collaboration effects. I know of no strong evidence either way on this question, so this does not appear to be a reason to be confident that ideas-based scale effects are important for living standards near current population sizes. And in any case, this is a second-order factor

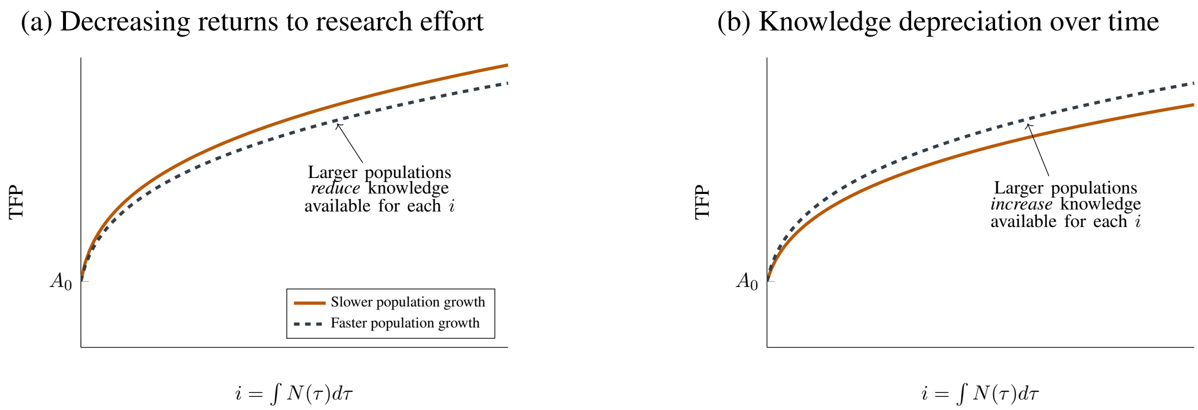
[9]Put differently, if a supposed scale effect does not show up even in the case where populations are constant, it is hardly capturing what we mean by increasing returns to scale.

[10]If $\lambda < 1$ that would imply that spreading people out as much as possible over time maximizes research production per person; $\lambda > 1$ implies that bunching people all in the same period would maximize output per researcher. Neither of these extremes seems intuitively correct.

that is unlikely to lead to important departures from the main result.

Finally, even if this channel did generate increasing returns to scale because $\lambda > 1$, there is no longer anything special about the non-rivalry of knowledge driving this result. If any sector in the economy has increasing returns within a period, while the rest are constant returns to scale, of course increasing returns to scale will be the result. So, while the benefits of larger populations would be retained if $\lambda > 1$, the deep conceptual link between non-rival ideas and scale economics would not be.

Figure 2: Second-order channels can generate positive or negative scale effects



Notes: Blue dotted line represents a history with faster population growth than solid orange line. Panel (a) illustrates that if there are decreasing returns to knowledge generation within a period, larger populations reduce the TFP available for each lifetime—a stark reversal of what non-rival ideas are thought to imply. Panel (b) illustrates that if knowledge depreciates over time, the result that larger populations improve individual outcomes is retained. It is unclear which of these is the dominant force.

3.3 Depreciation of knowledge

Knowledge depreciation leads to an unambiguous theoretical prediction: its existence works to rescue the increasing returns to scale intuition. Furthermore, and unlike the case of $\lambda > 1$, it does so for the original reason that knowledge is non-rival and can be freely used once it is discovered. The relevant question is whether the quantitative magnitude of knowledge depreciation is large enough to make this a significant force.

Consider the following implementation of depreciation, where, generally, it is a function of economic activity that period.

$$\frac{\dot{A}(t)}{A(t)} = \theta N(t) A(t)^{-\beta} - \delta_A(Y(t)) \tag{19}$$

This can only be integrated into a closed-form solution under the special case where depreciation is a constant $\bar{\delta}_A$ and $\beta = 0$. This is a reasonable baseline case for understanding how this force works. The value of $\beta = 0$ corresponds to the original Romer (1990) formulation, and nothing in the paper has depended on the choice of β, making it an inconsequential parameter for developing intuitions. Likewise, the assumption that δ_A is constant is the most natural starting point for depreciation, matching the standard assumption for how physical capital depreciates.

When $\beta = 0$, $\lambda = 1$, δ_A is fixed, and populations are constant, A follows a simple exponential growth process that depends on the size of the population. (The appendix shows that in another special case with an analytical solution, $\beta = 1$, the same qualitative results are obtained.) Also, I will again take advantage of the fact that with constant populations $Nt = i(t)$.

$$\begin{aligned} A(t) =& A_0 e^{\theta N t} e^{-\delta_A t} \\ A(i) =& A_0 e^{\theta i} e^{-\delta_A t} \end{aligned} \tag{20}$$

For any i, the first two terms are fixed. The third term depends on when person-year i is lived. In particular, the earlier in time that i is lived, the larger is the knowledge she has access to. The way to reduce the time-period in which each i lives is for the population to be larger (e.g., humanity gets to its 100 billionth person-year earlier if historical populations are larger).

The intuition for this result is straightforward when thinking about the arrival rate of people-years. For the ith person, there is now a drag on the knowledge accessible to them that is directly a function of time. If an idea was discovered long before the ith person-year, this person will have less access to it, in expectation. In the fixed number of life-years lived prior to i, there are a fixed number of discoveries (assuming $\lambda = 1$). It is therefore better if person i lives nearer in time to those discoveries. The way to promote person i living closer in time to more of her predecessors is for the population to be large, so that more births happen over a shorter time period.

What is nice about this result is that increasing returns to scale are retained for the 'right' reasons—knowledge is non-rival, so having more people around benefits everyone. Consider the limiting case where the depreciation rate on knowledge is one, so that the only knowledge accessible to person i is the knowledge discovered in the period in which she lives. Having many contemporaries benefits her because the ideas her contemporaries produce are non-rival and can be used by her. The mechanism in Section 2 that breaks the intuition about population size is that person i would have had access to all of her predecessors ideas, regardless of when they occur. If knowledge depreciates, this is no longer true.

Furthermore, the symmetric case of exogenous knowledge *appreciation* seems significantly less plausible. This is why its effect is unambigious, unlike the relaxation that $\lambda \neq 1$, which could work in either direction. It is not inconceivable that something like knowledge appreciation exists—e.g., if some costless knowledge is accumulated from exogenous natural events—but this seems unlikely to be as important as the passing of time eroding the knowledge base. So, the addition of a force capturing exogenous growth/decay serves to return the result that larger populations are beneficial because of non-rival knowledge.

Finally, it is important to note that the ways in which we might enrich the formulation of depreciation would serve to strengthen the takeaway. Namely, consider specifying knowledge depreciation as a function of economic activity within a period. It seems likely that in periods with more economic activity, fewer ideas go unused and forgotten.[11] Rather than being non-rival—where my use does not erode its availability—knowledge may be *amplifying*—my use increases its availability for others. Section 2 demonstrated that knowledge being non-rival was not enough to generate increasing returns to scale. But if knowledge is amplifying, larger populations will again generate better per capita outcomes.

There is very little evidence on the functional form or magnitude of knowledge depreciation. Its existence is unlikely to change the balanced growth properties of models with exponential growth in populations, so it has received little attention within this literature to date. But, as noted in Jones (2022) and Eden and Kuruc (2023), it is a crucial parameter in models without long-term population growth. As this is overwhelmingly likely to be the relevant demographic context moving forward, more focus is likely to be paid to the rate at which knowledge depreciates. The result in this section further this point by emphasizing that this force is crucial for understanding how individuals are affected by idea-based growth independent of population dynamics.

3.4 Summary of Negative Result: Population size has no clear intensive-margin effect on per capita incomes

The lesson of Section 3 is that the main result of Section 2 cannot be easily avoided to rescue the intuition that if ideas are non-rival, then larger populations improve individual outcomes. I first showed that endogenizing research effort does not deliver the result that market size effects rescues this intuition, at least not without second-order reasons for why the share of the population

[11] This is an important conceptual distinction from capital. In models where capital utilization can vary over the business cycle, depreciation of capital increases when the economy is larger. The machines are worked harder and breakdown faster.

researching should increase in a larger population. The takeaways of Sections 3.2 and 3.3 are less straightforward, especially when taken in conjunction.

Increasing returns to scale within the research production function, $\lambda > 1$, would generate the result that larger populations improve individual outcomes. However, that is arguably less relevant than the fact that $\lambda < 1$ reverses our standard intuition—smaller populations improve individual outcomes. Depreciation is less theoretically ambiguous. If it is quantitatively significant, it generates the takeaway that larger populations improve individual outcomes. But it is far from obvious whether it is a quantitatively meaningful force. The implicit consensus is that it is small; nearly every study modeling long-run economic growth rounds it to zero.

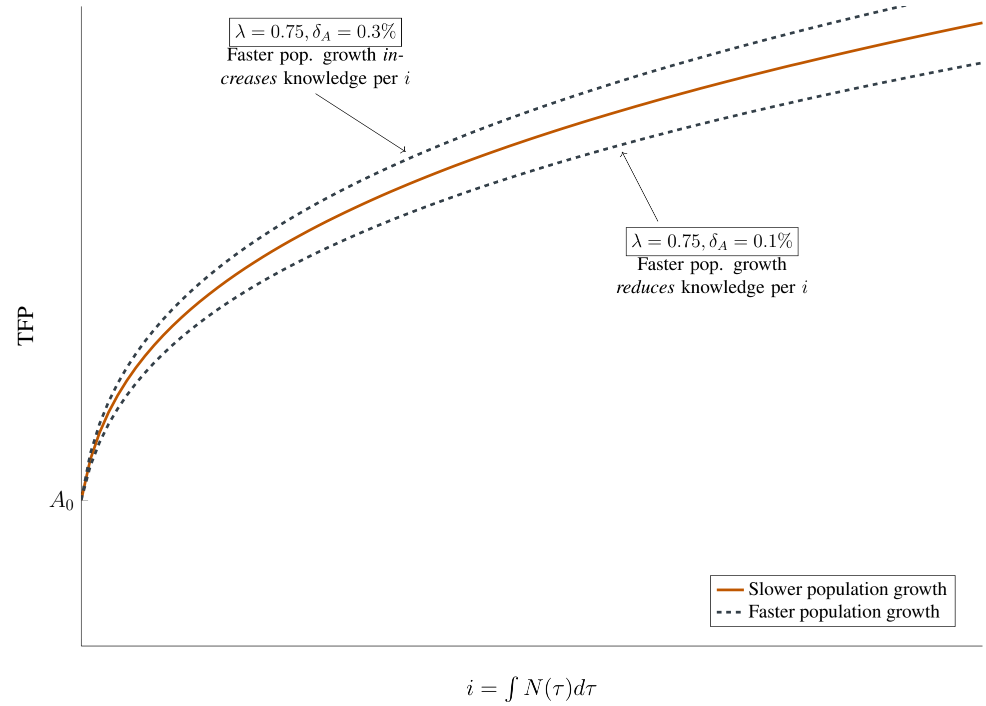
Figure 3 demonstrates that it would be difficult to be confident in any particular deviation from the first-order result that larger populations have no effect on individual living standards. It plots the relationship of interest—TFP per person-year—for values of λ and δ_A. Namely, I begin with $\lambda = 0.75$ following the baseline assumption in Bloom et al. (2020); Jones (2022). For δ_A, there is less guidance. I plot this function for both $\delta_A = 0.1\%, 0.3\%$ because (1) these seem a plausible order of magnitude and (2) they generate different conclusions, so they helpfully demonstrate the fragility of a directional result for plausible values. If $\lambda = 0.75$ and $\delta_A = 0.1\%$, the relationship between population sizes and per capita incomes is negative; if $\lambda = 0.75$ and $\delta_A = 0.3\%$, the relationship between population sizes and per capita incomes is positive. Distinguishing between these cases with our current body of evidence is nearly impossible. The overall takeaway is that while the exact analytical results in Section 2 are contingent on specific assumptions, the qualitative point that there is no clear relationship between population sizes and per capita incomes generated by non-rival ideas is robust.

This is what I will call the intensive-margin result: for a fixed number of people-years lived, per capita incomes are, to a first-order, invariant to population size. However, this does not rule out what I will call an extensive margin effect. A larger per period population may have more lives that are ever lived; if those lives are above average, per capita income can be increased by population size. This is a significantly different channel by which per capita incomes can increase, conceptually, so I discuss it separately in the next section.

4 The quantity of people years & existential risk

So far, I have argued that standard endogenous growth frameworks imply that the size of the population governs the speed of historical events—the arrival and experience of people-years and

Figure 3: Quantitatively, second-order effects have ambiguous effect in conjunction



Notes: Blue dotted lines represent a history with faster population growth than solid orange line, under different parameterizations for λ, δ_A. When $\lambda = 0.75$, the directional effect depends on small differences in δ_A, leaving little reason to deviate from the baseline results of a null effect.

discoveries—not the quality or content of those people-years. One way to immediately rescue scale effects would be to discount the future. If the planner prefers that things happen earlier, speeding up history would be straightforwardly beneficial.

There are two reasons typically given for discounting utility over time; one normative, one positive. The normative one is simple. The planner may simply prefer that things happen earlier in time, a *pure rate of time preference.* This is meant to reflect the fact that people appear to be impatient in a way that suggests utility experienced earlier in time is inherently more valuable than utility experienced later. There is disagreement among economists about whether the planner should inherit this impatience, notably in the literature on climate change. The relevant thought experiment for this setting is extremely straightforward: Does the planner have reason to prefer one of two histories with exactly identical quantity and quality of person-years if one happens over the course of 50 billion years rather than 100 billion years? Even if so, that is an importantly different reason to prefer larger populations. It is not that lives are, on-net, improved, it is that things happen earlier. There is not much more to say about the case of a positive pure rate of time preference.

The positive case for discounting is an expected value argument about whether there will be future people years. For example, if each year carries some risk that an asteroid eliminates life on earth, then events that will happen in 500 years ought to be discounted by the probability that humanity is still around. This is a significantly more interesting case, and provides what I believe is the strongest reason to prefer larger per-period populations. In this simple asteroid case, it remains true that the quality of people-years is identical conditional on that person-year happening, but humanity 'gets through' more of these existences prior to its extinction.

Figure 4 depicts this case for some exogenous extinction event that occurs at time T^X. I will take this baseline case to discuss the mechanisms of this modeling dimension before enriching the discussion of existential threats. Most of these lessons will carry over, at least in part, to more realistic cases.

Figure 4 is set-up just as Figure 2, the difference here is that instead of assuming humanity goes on indefinitely, we have closed the model with an end date. I have denoted the period in which this (ex-post) occurs as T^X, though within the model individuals would treat this as uncertain. Panels (a) and (b) do not show anything of note: if population and TFP growth are faster, higher levels of each are reached by the time of extinction.

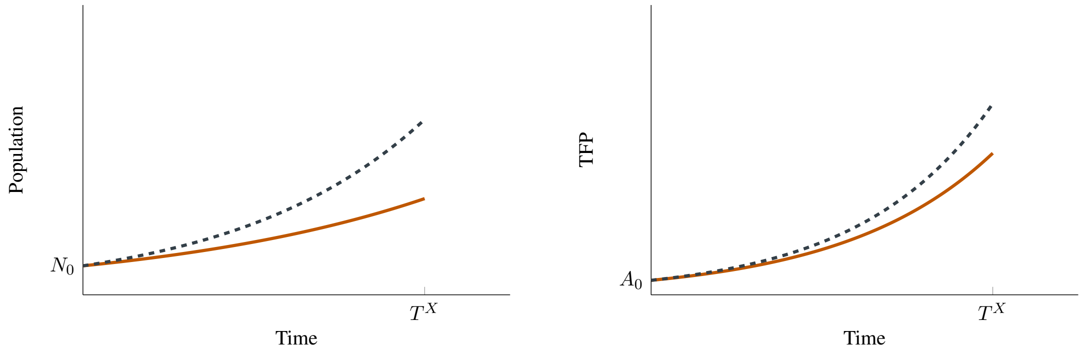
The substantive contribution lies in panel (c). Recall that the x-axis here depicts the measure of people-years as humanity lives them. Let I_S be the number of people-years the small population gets through; $I_L > I_S$ is the number of people-years the larger population gets through. The first thing to notice is that, conditional on reaching a specific person-year, $i \leq I_S$, the TFP available during that person-year is unaffected. This is the *intensive margin* result showing up in the case where total existences differ. The *extensive margin* is captured by the incomes of the people-years that are conditional on whether the high or low population history is realized.

What Figure 4 shows is that any effect of population size must come via the extensive margin. This is because, in ideas-based growth models, lives that come after other lives are better (under the standard assumption that TFP maps to economic well-being). In a history that includes more people, the higher living standards are experienced by those extra people. In other words, in this setting, it would be equivalent to increase the population size by 10% in each period or extend the lifetime of humanity enough that the total historical population is 10% larger.

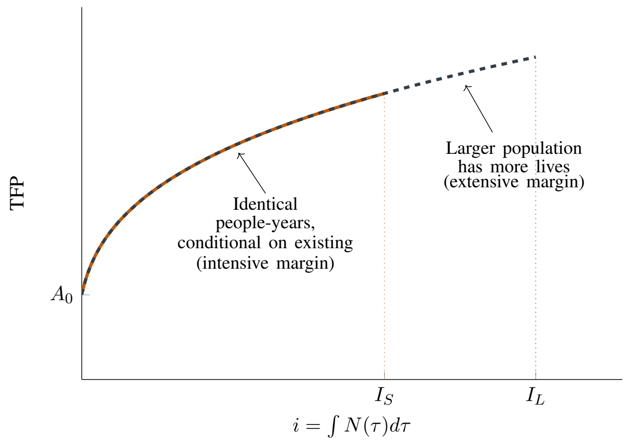
In some ways, this seems to straightforwardly rescue the scale effects of idea-based growth models. Per capita well-being—measured over the whole of history—increases in the size of the population that ever lives. However, (1) it is not standard to consider the value of marginal existences economic policy making and (2) even if these existences are counted, this interpretation

Figure 4: Exogenous Extinction Event at T^X

(a) Each population makes it to T^X. (b) Faster growth $\rightarrow$ higher TFP reached.



(c) Larger populations 'get through' more history.



Notes: Dotted line represents a history with faster population growth than solid line. Panel (a) depicts how large these respective populations get by the extinction event at T^X. Panel (b) illustrates that TFP reaches higher levels in the history with larger populations. Panel (c) demonstrates, for the people-years lived in either history, there is no quality of life improvement. All potential benefits come along the extensive margin; marginal people-years are the highest quality, because TFP accumulates with people-years.

eliminates much of what is distinct about ideas-based growth models. I will expand on each of these points in turn.

First, consider the difficult question of whether the value of existences should be counted. Here, per capita well-being only increases because the number of people who ever live increases. If economic analyses were to value this extensive margin, it would call for dramatic reevaluation of many policies: getting to exist would perhaps be the ultimate externality someone can impose on you! Valuing potential lives at the VSL, for example, would imply that incentivizing a birth could be worth upwards of 10 million dollars. Or, consider investments that might only marginally lower the probability of an existential catastrophe, such as pandemic preparedness investments. If trillions(?) of potential lives hang in the balance, optimal policy might require *enormous* sacrifices of current well-being.

These sorts of questions have only recently been taken up in earnest by economists and applied philosophers. It is surprisingly difficult to avoid counter-intuitive conclusions of this sort.[12] Indeed, the competing claim that what matters is per capita outcomes, or per capita outcomes conditional on existing, appear untenable.[13] So, it may be reasonable to conclude that larger populations are better because larger populations result in more good lives being lived over the course of history.

But notice that this interpretation eliminates anything that is distinct about the scale effects of idea-based growth theory. If larger populations are good because more people live good lives, then the positive fact that larger populations generate more TFP growth is doing no work here—the same conclusion would be drawn in a model with entirely exogenous TFP growth. The question to settle is the normative one of whether a additional existences should be treated as a social good.

Beyond this, even if the quantity of people-years is part of the planner's objective function, ideas-based growth models may be one of a small class of models that is indifferent to population sizes if extinction is endogenized. Scholars of existential risk appear to believe that human-caused extinction—-via dangerous technologies—is much more likely than naturally-caused extinction, via e.g. an asteroid (Ord, 2020). When humanity entered the nuclear age, many believe the annual probability of us going extinct increased dramatically; the same may be true for future advances in artificial intelligence or bioengineering.

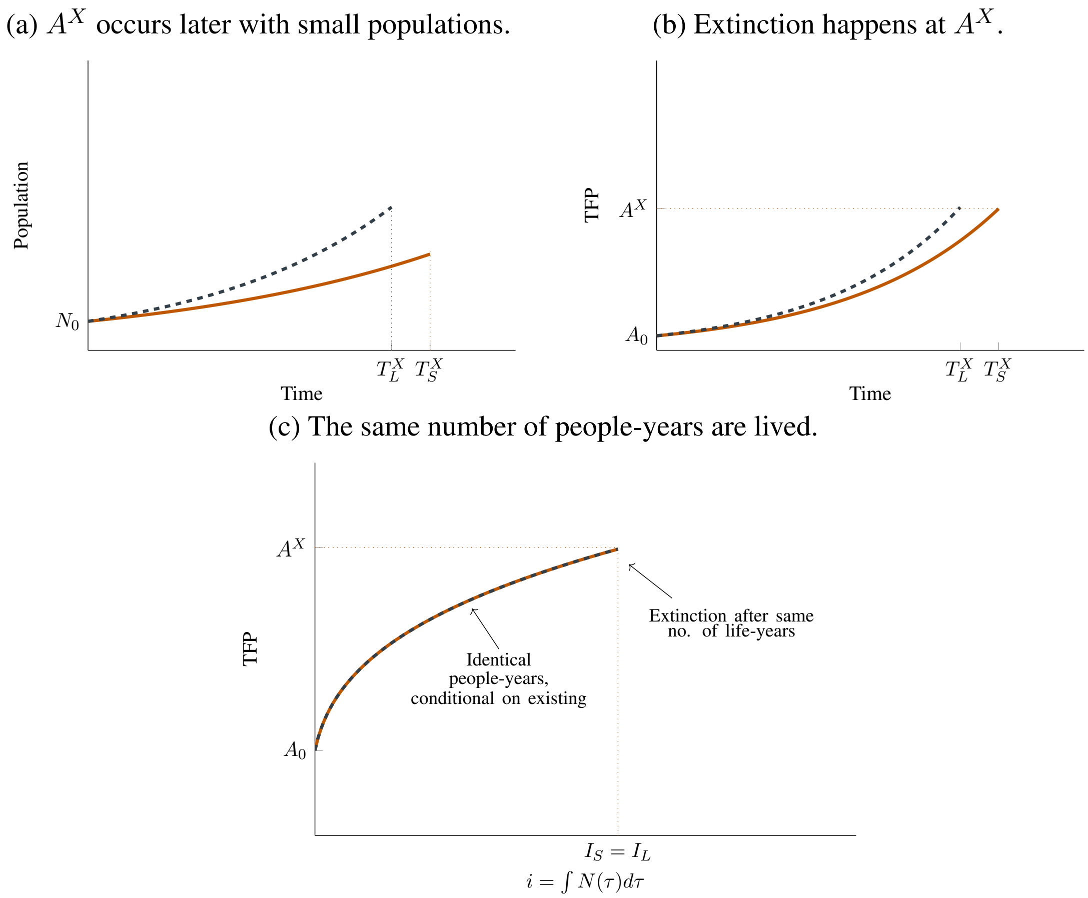
As a stylized case, endogenous extinction might be modeled as in Jones (2016): each new idea has some probability of ending humanity (Jones calls this a model of "Russian Roulette" growth). Denote the ex-post level of technology that ends humanity as A^X. As should be clear by this point in the paper, A^X is reached once some number of cumulative people-years have been lived. Consequently, the number of people-years lived is independent of the size or growth rate of the population when extinction is endogenized under these assumptions.

Figure 5 illustrates this case using the same three graphs as in the prior two figures. The difference here is that time itself is not the causal variable that ends humanity, A is. A^X acts like an upper-bound on TFP that determines the time at which humanity goes extinct, as well as how many people live prior to extinction. As can be seen in panel (c), the intensive margin remains

[12] See e.g., Ord (2020); Kuruc et al. (2022); MacAskill (2022); Lawson and Spears (2023); Weil (2023); Geruso and Spears (Forthcoming).

[13] Consider the question of an additional life being lived that is below-average, but on the whole good: the planner would value *preventing* that existence if per capita outcomes was her target. If, instead, we only considered the value of lives that happen regardless, then the planner would be wholly indifferent to preventing a life void of anything good, but full of misery and suffering. Neither seems plausible.

Figure 5: Endogenous Extinction Event at A^X



Notes: Dotted line represents a history with faster population growth than solid line. Panel (a) depicts how large these respective populations get by the time each reaches A^X. Panel (b) illustrates how fast each reaches the existential technology. Panel (c) demonstrates that the exact same number of lives are lived, at the exact same quality, prior to reaching A^X.

identical across population histories, but there is no longer an extensive margin effect from the population being larger. This eliminates any difference between histories, aside from the fact that one occurs on a condensed time horizon.

The discussion of existential risk will be enriched in a moment, but first lets pause to appreciate how deeply this result contrasts with the lessons drawn from Romer (1990) and the literature it has spawned. Population size and growth have been at the center of studies of long-run growth in living standards and economic outcomes; scale effects have been seen as essentially impossible to avoid. But here, we find that closing the model with a simple assumption about extinction entirely eliminates any effects of scale: both the quantity and quality of life-years ever lived are independent of per-period population size in Figure 5. This is far too simple of a model to be confident in this independence result, but it makes clear that scale effects can be eliminated under fairly innocuous assumptions.

To emphasize the contingency of these scale effects, Table 2 mixes assumptions from earlier subsections with assumptions about existential risks to demonstrate how the implications of population size are affected. To keep things tractable and isolate the effects of scale, I will compare two constant population sizes, $L > S$ (i.e., so that population *growth* is zero in both cases). The rows represent different stylized examples for extinction scenarios. The first two have already been discussed above: exogenous extinction (e.g., an asteroid strikes) and endogenous extinction by the invention of a dangerous technology. We could also imagine endogenous extinction of other sorts. The third row represents a case where each individual life-year poses some risk of ending humanity (a caricature of this case could be that every person-year poses some risk of generating a radicalized individual who wants to end humanity). That may seem unrealistic, but some believe that technological advances in AI or bioengineering may make it possible for a single individual to end humanity in the near future.

The columns represent different assumptions for the law of motion for knowledge. Earlier it was discussed that λ becomes a particularly important term because it governs whether collaboration benefits ($\lambda > 1$) exceed duplication/congestion costs ($\lambda < 1$) of population size within a period. Introducing knowledge depreciation has similar implications to λ exceeding one, it generates a positive intensive margin effect of population size (e.g., it rescues the scales effects on individual outcomes). There are cases where the effect on the intensive margin is ambiguous—for example, $\lambda < 1$ and $\delta_A > 0$—which makes a qualitative prediction of their effect infeasible.

Consider first the top row of Table 2, the case of exogenous extinction at some unknown time T^X. Here, larger populations dominate whether per capita outcomes are considered or total utility is considered (i.e., the sum of economic well-being over time). The case for $\lambda = 1$ is already discussed in detail in Figure 4. If $\lambda > 1$, this means larger per period populations produce even more ideas than they otherwise would have, giving them a further advantage over the case when $\lambda = 1$. When $\lambda < 1$, the larger population still gets through more people-years prior to T^X, so it still reaches a higher level of TFP at the time of extinction. Because I have restricted focus to cases where the population is constant over time, the density of people is equal at all times, and simply knowing that one history gets to a higher level of TFP is enough to conclude that the average experience is better.

Interestingly, this is a case where the intensive margin is *negative*. If $\lambda < 1$, then for any i, y^i is larger for smaller per period populations. The 100th person-year is improved when the first

Table 2: The effects of a larger population depend on extinction and TFP growth assumptions

	$\lambda < 1$ (& $\delta_A = 0$)	$\lambda = 1$ (& $\delta_A = 0$)	$\lambda > 1$ (or $\delta_A > 0$)
T^X causes extinction (e.g., asteroid)	$\bar{y}^L > \bar{y}^S$ $I_L > I_S$	$\bar{y}^L > \bar{y}^S$ $I_L > I_S$	$\bar{y}^L > \bar{y}^S$ $I_L > I_S$
A^X causes extinction (e.g., advanced A.I.)	$\bar{y}^L = \bar{y}^S$ $I_L > I_S$	$\bar{y}^L = \bar{y}^S$ $I_L = I_S$	$\bar{y}^L = \bar{y}^S$ $I_L < I_S$
I^X causes extinction (e.g., rogue actor)	$\bar{y}^L < \bar{y}^S$ $I_L = I_S$	$\bar{y}^L = \bar{y}^S$ $I_L = I_S$	$\bar{y}^L > \bar{y}^S$ $I_L = I_S$

Notes: The implication of larger populations, L, relative to smaller populations, S on per capita outcomes $\bar{y}^j$, and number of individuals to ever live, I_j, under different modeling assumptions. For example, the upper-left cell indicates that per capita outcomes and the number of people to ever live are both higher when populations are larger ($\bar{y}^L(i) > \bar{y}^S(i), I_L > I_S$) if extinction is exogenous and there are decreasing returns to people on idea-creation within a period.

100 people-years are spread over more time, because it reduces duplication. It is a case where the extensive margin has a larger effect on per capita outcomes than the intensive margin. This helpfully demonstrates the perverse recommendations that could be given by only looking at the intensive margin; in this case the intensive margin would recommend arbitrarily small populations whenever comparing two population sizes despite per capita and total wellbeing increasing in population sizes.

Moving to the second row, this is the case where A^X serves as something of an upper-bound on TFP detailed for $\lambda = 1$ in Figure 5. Once a sufficiently powerful and dangerous technology is invented, humanity goes extinct quickly afterwards. Nuclear weapons appear to have almost been such a technology. Some worry that advanced artificial super-intelligence could be the end of humanity (see e.g., Jones, 2024). The common thread is that there is some level of technology that, once reached, ends the model. When $\lambda = 1$ the result was that the planner is completely indifferent to the size of the population: larger populations, ex-ante, result in exactly the same quantity and quality of lives, because both economic growth and extinction are directly tied to people years.

The cases where $\lambda \neq 1$ no longer retain this independence result, but the independence breaks in an unintuitive way. Consider $\lambda > 1$. This is where idea-generation benefits from collaboration, and hence has increasing returns to scale. In other words, increasing the arrival rate of people-years (i.e., population density) increases the arrival rate of ideas even faster. The result is that fewer people years are lived prior to TFP reaching A^X. This is unambiguously bad: per capita outcomes have been unchanged—all of the same levels of TFP are eventually reached—but fewer people get to live.[14] The reverse case works in exactly the opposite manner, and can be easiest to conceptualize for the particular case where TFP growth is unaffected by population growth ($\lambda = 0$). Here, increasing the population size results in more lives being lived prior to reaching A^X; when $\lambda = 0$ the time at which A^X is reached is unaffected by population size. But again, the average level of TFP is unaffected because A^X is a common upper bound. So, if something like Jones (2016)'s Russian Roulette model of growth is correct, the case for larger populations is stronger when idea-generation is *less* affected by population size.

Finally, consider the bottom row, where the causal driver of extinction is people-years directly. One example of this is use of a finite non-renewable resource: only a certain number of people can ever live prior to using up this resource, so the model is closed with an assumption directly on people-years (Greaves, 2019). Alternatively, it could be that every persons-year has some probability of making contact with a novel pathogen that could destroy humanity, so that cumulative extinction risk is increasing in cumulative people-years. When $\lambda = 1$ this is identical to the case that A^X causes extinction because cumulative people-years increases one-for-one with TFP levels. However, when $\lambda \neq 1$ the opposite conclusions are generated. If history ends with some fixed number of people-years, then having larger populations lets those fixed number of people collaborate to produce more TFP. In the (unrealistic) limit, having all I individuals live at the same time would maximize the TFP achieved. Conversely, spreading them out would be best if $\lambda < 1$.[15]

It is difficult to know what to take away from this table—and that is precisely the point. Economists know very little about λ, δ_A, or the relatively likelihood of different extinction scenarios. Table 2 demonstrates that the effect of increasing population sizes can entirely depend on these assumptions. Based on the frequent semi-endogenous growth calibrations where $\lambda < 1$ (e.g., Bloom et al., 2020) and the widely held belief that man-made technologies are much more dangerous than natural threats (see e.g., Ord, 2020), the single most likely model formulation might be the middle row, left column. This is a model where the planner would prefer larger populations, but for the reason that people-years do not generate as much knowledge, so more live prior to reaching some sufficiently dangerous level of technology. Indeed, the planner prefers larger populations by a larger degree the less endogenous economic growth is, suggesting a significant reevaluation of what is good about population in endogenous growth models.

[14] This particular case has close similarities to the result in Ord (2024a).

[15] Again, suppose $\lambda = 0$, so that idea-generation happens independent to the population size. Then the planner would prefer back-loading the fixed population as far into history as possible.

5 Conclusion

The question of whether a declining population will lead to counterfactual losses in living standards is complicated. Alongside the production of non-rival ideas, there are a host of issues that may depend on the size or age structure of the population: environmental pressures, human capital investments, pension financing, business formation decisions, etc. However, given the seeming importance of non-rival ideas in explaining past economic growth, this channel has received substantial attention among economists.

This paper shows this may be a mistake. The joint-distribution of people and the stock of ideas available during their lifetime is invariant to population size, at least without assuming further, second-order forces beyond the non-rivalry of ideas. The population size and its effect on the speed at which non-rival ideas are generated may be important for explaining rates of economic growth, but I show there is not a strong reason to believe this makes lives any better. Indeed, I show that our intuitions can be completely reversed under plausible parameter values: if idea-creation within a period has diminishing returns to researchers, each life is worsened by the population size being larger, even in a model where non-rival ideas are the only relevant force.

Going beyond this negative result, the key consideration turns out to be whether a larger population allows more people to ever live, over the whole of human history. If so, and if population size has any effect on non-rival idea creation, larger timeless populations raise total and average utility experienced over the whole of history. However, this points to the critical nature of assumptions about existential threats. When the variable that matters is the expected number of people to ever live, it is not surprising that model results turn on the assumption about what constrains the number of people who will ever live.

Unsurprisingly, the importance of extinction assumptions makes it difficult to be confident in the takeaways. So, one conclusion of this paper is that economists interested in the effects of population dynamics and economic growth must grapple with the issue of extinction. Based on available theories and commonly assumed parameters, I conclude (for now) that larger per period populations increase the expected number of people to ever live, raising the total and average welfare experienced by people.

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Appendix

A Solution to the Model of Endogenous Research Effort

Aggregate output is a CES aggregator of intermediate goods, each with the respective production functions, where l_j is the labor allocted to sector j.

$$\begin{aligned} Y =& \Big[\int_0^A y_j^{\frac{\varepsilon-1}{\varepsilon}} dj\Big]^{\frac{\varepsilon}{\varepsilon-1}} \\ y_j =& l_j \end{aligned}$$

A is the measure of ideas in this economy, endogenously produced by potential entrepreneurs. Denoting the share of people in this economy who search for ideas s, we have a production function of A that is:

$$A = \omega \times s \times N.$$

That is, each individual has some probability ω of discovering a profitable idea. I assume that each worker inelastically supplies one unit of labor, so that the market clearing condition dictates that the sum of wage earners plus entrepenuers is the total population, N.

Let us first focus on each intermediate firm's production decision, conditional on operating (that is, $j \in [0, A]$). The demand curve for each y_j can be solved through a standard cost-minimization problem by the final good firm that aggregates these products.

$$\min_{y_j} \int_0^A y_j p_j \text{ such that } \Big[\int_0^A y_j^{\frac{\varepsilon-1}{\varepsilon}} dj\Big]^{\frac{\varepsilon}{\varepsilon-1}} \geq Y \tag{21}$$

This delivers the demand curve:

$$y_j = \left(\frac{p_j}{P}\right)^{-\varepsilon} Y, \tag{22}$$

where I have defined P, the aggergate price level, as the Lagrange multiplier.[16]

Each intermediate producer, j, faces a problem of whether to purchase a patent at price p_A, what price to charge for its good, p_j, and how much of its good to produce with a linear production function, $y_j = l_j$, where l_j is labor in sector j that gets paid a wage w that is taken as given by each small firm. I assume a free-entry condition, such that the fixed cost to operate, p_A, will be equal to

[16]This is because the lagrange multiplier captures the cost of having the constraint tightened, on the margin. Here, the constraint is producing one more unit of the final good. The marginal cost of producing Y will equal the price of Y in a competitive equilibrium.

the profit this monopolistically competitive firm can earn in equilibrium. The problem for a firm that decides to purchase a patent and operate looks as follows (if the maximum profit that can be attained is less than zero, the firm will not operate).

$$\max_{p_j, y_j, l_j} p_j y_j - w l_j - p_A \tag{23}$$

subject to:

$$y_j = \left(\frac{p_i}{P}\right)^{-\varepsilon} Y \tag{24}$$

$$y_j = l_j \tag{25}$$

Subbing these constraints directly into the problem and taking a first-order condition, we obtain the usual result that CES monopolistic competition delivers a price that is a constant mark-up over marginal costs (here, the wage). This implies a value for p_A that set profits to zero, by the free entry condition.

$$p_j = -\frac{\varepsilon}{1-\varepsilon} w \tag{26}$$

$$p_A = \left(-\frac{\varepsilon}{1-\varepsilon} - 1\right) w y_j \tag{27}$$

Also note that $\omega p_A = w$ in equilibrium, as the expected value of being an entrepreneur must be equal to the wage. We can use this fact to substitute w from the right hand side of (27), and simplify the expression.

$$p_A = \left(\frac{1}{\varepsilon - 1}\right) \omega p_A y_j \Rightarrow$$

$$\frac{(\varepsilon - 1)}{\omega} = y_j$$

Finally, notice that in a symmetric equilibrium each y_j will be identical. Furthermore, each $y_j = l_j$, the amount of labor supplied in that industry. Recalling that the share of the population earning wages is $(1-s)$, this implies that there are $(1-s)N$ total workers split across the A industries. So, each $l_j = \frac{(1-s)N}{A}$. However, we also know in equilibrium that $A = \omega \times s \times N$. This lets us sub in our new expression for l_j for y_j. Simplifying, we get:

$$\frac{(\varepsilon-1)}{\omega} = \frac{(1-s)N}{\omega s N} \Rightarrow$$
$$\boxed{(\varepsilon-1) = \frac{1-s}{s}} \tag{28}$$

First, verify that the intuition of this result makes sense. As $\varepsilon \to 1$, $s \to 1$. This is the case of perfect complements: the love-of-variety is so extreme that nearly everyone is employed in research to expand the line of product varieties. When $\varepsilon \to \infty$, $s \to 0$. This is the case of perfect substitutes. Consumers do not care about which products they consume, so everyone is employed in the production sector rather than expanding the variety of products.

But what is most important for the application here is that this function is independent of N. It is not the case that a larger market endogenously allocates a larger share of the population to research. So, the assumption employed throughout the paper that s is independent of N seems to be, at least, a good baseline assumption.