Population, Ideas, and the Speed of History*

Kevin Kuruc†

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Abstract

Scale effects have long been at the heart of endogenous theories of economic growth. Larger economies can allocate more resources to discover the (non-rival) ideas that drive long-run productivity improvements. This paper demonstrates that—in direct contrast with conventional wisdom—the non-rivalry of ideas implies nothing about the relationship between population size and individual economic outcomes. The reason is straightforward: A larger population increases the arrival rate of both ideas *and* people, potentially leaving each life unchanged. Instead, the population size governs the *speed of history*. To a first-order, lives of the same quality are lived, they just occur earlier in time if the population is larger. I then relax some standard simplifying assumptions to ask whether there are any conditions under which population size has an effect on individual outcomes when ideas are non-rival. I conclude that the key consideration is whether a larger per-period population leads to more lives ever being lived, putting new emphasis on existential risks when attempting to understand the implications of demographic change.

Keywords: Population economics, endogenous growth, scale effects, extinction risks, aggregate welfare.

JEL Codes: J11; I31; O41.

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†Population Wellbeing Initiative, University of Texas at Austin; Global Priorities Institute, University of Oxford. Contact: kevinkuruc@utexas.edu.

1 Introduction

Since Romer (1990), increasing returns to scale have been at the center of economic growth theory. Other things equal, an economy with more people and resources will discover more ideas for new products or production methods. Ideas are fundamentally non-rival, so this increase in the stock of aggregate ideas increases per capita incomes. It is a remarkably simple chain of logic: More people $\rightarrow$ more ideas $\rightarrow$ higher per capita incomes. The global fall in fertility rates and the prospect of declining populations has recently refocused attention on this relationship (Jones, 2022; Spears and Geruso, Forthcoming).

This paper demonstrates that the non-rivalry of ideas, in fact, implies nothing about the relationship between individual income levels and population growth. The reason is straightforward: Increasing the rate of population growth increases the arrival rate of both ideas and the people who would benefit from these ideas, offsetting the purported benefits. To illustrate how the argument will work, consider one of the original formulations of this scale-based reasoning in Phelps (1968).

> *If I could re-do the history of the world, halving population size each year from the beginning of time on some random basis, I would not do it for the fear of losing Mozart in the process.* (p. 512)

Suppose there is some fixed probability that each birth produces someone as talented as Mozart. It would be more accurate then to say that halving the historical population *delays* the expected date that someone as talented as Mozart is born.[1] At the same time, halving the historical population also halves the number of people living in these now Mozart-less years. To a first-order, these effects will exactly offset: the same number of people are expected to live prior to a Mozart regardless of per-period population sizes. Consequently, no lives are improved or worsened by scale here, even while recognizing that music is non-rival. Decreasing the population delays the arrival of a Mozart-like figure, but it delays all other lives too.

The first part of the paper formalizes this proposition in a standard Romer/Jones endogenous growth framework (Jones, 1995). I show analytically that the size of the population has no effect on the number of people-years lived prior to any given TFP level. The implication of ideas being non-rival is that cumulative TFP improvements depend on cumulative research effort over time. So, just as with Mozart, if history were re-ran with a smaller population, the expected number of people-

[1] Of course, it would not literally be Mozart because randomly halving the population size in all periods would almost certainly change the identity of everyone who lives afterwards—different events unfold, different adult pairs have children, etc. I am more precise about what an individual is in Section 2.

years lived prior to steam power, or the electric light bulb, or antibiotics would be unchanged. These discoveries would happen later, but proportionately fewer people-years would have been lived in those earlier years, leaving the quality of these people-years unchanged. Increases in economic growth attributable to increases in population growth cannot be *ceteris paribus*—the years in which lives are lived necessarily changes.

It is important to emphasize that this baseline proposition does not refute the claim that non-rival ideas lead to an aggregate economy with increasing returns to scale within periods. The novelty is a shift of focus, away from the income levels of each time period to the income level of people, where a person is identified by their order in history (e.g., person i =100 is the 100$^{\text{th}}$ person born). The positive economics from this vantage are crucially different: A larger population improves average incomes in each time period without changing the incomes of people. I show that these two facts can be simultaneously true because the years in which each of these people live will change with population growth.[2]

The second half of the paper studies whether there are model relaxations that interact with the non-rivalry of knowledge to rescue this intuition about the benefits of population size for per capita incomes when ideas drive economic growth. I consider four, three of which can plausibly, but not certainly, generate a positive relationship between population size and individual living standards. Even so, these effects are likely small and/or require a conceptual re-understanding of the mechanisms linking population size to individual incomes.

First, I endogenize research effort, rather than relying on a simple learning-by-doing assumption wherein each individual makes some fixed *ex-ante* contribution to TFP. Larger populations have larger consumer bases, which potentially changes the incentives to engage in R&D. This turns out to be a misleading line of thought. When the consumer base is larger, it is more profitable both to generate new ideas and to exploit existing ones. The benefits of engaging in research increase, but so does the opportunity cost. Formally, the equilibrium condition determining the share of the population engaged in research is independent of the population size.[3] Therefore, standard market-size considerations do not overturn the main result.

Second, I relax a commonly employed linearity assumption in the aggregate R&D production

[2]This turns out to be a dynamic application of the *Will Rogers Phenomenon*, where the comedian joked that the average intelligence in both California and Oklahoma was raised when the 'Okies' migrated to California during the Dust Bowl.

[3]Population *growth* may change the exploit/explore trade-off for potential entrepreneurs, depending on the patent structure, which I discuss later. But that is a substantially different claim that is about relative population size between periods, not a 'bigger-is-better' scale effect.

function. Specifically, I allow for convexity (or concavity) in the returns to research effort within a period. If collaboration effects between researchers generate increasing returns, then larger per-period populations will improve individual outcomes. Unsurprisingly, it would be better for more people to live in a given year if there are increasing returns to scale in the research production function itself. On the other hand, if there are diminishing returns to researchers within a period, individual outcomes are improved by *smaller* populations. Akcigit and Kerr (2018) provide evidence that there are diminishing returns to innovation along the firm-size margin, and this is the more common assumption in the literature that calibrates long-run growth models (see e.g., Bloom et al., 2020; Jones, 2022). Overall, then, relaxing this linearity assumption provides reason to believe smaller populations improve each individual's living standards, an exact reversal of the lesson typically drawn from the fact that non-rival ideas are a driver of economic growth.

Third, I consider the implications of knowledge depreciation. This force, in isolation, unambiguously serves to rescue the original intuition that larger populations improve individual outcomes. The reason is as follows: If knowledge depreciates over time, the faster that people 'arrive' following the discovery of an idea, the more people benefit from that idea before it is lost. When time erodes the knowledge base, it is beneficial for more people to live over shorter periods of time. Depreciation of knowledge is essentially unstudied, so it is difficult to know what to make of the potential magnitude of this effect. However, in a simple quantitative exercise, I demonstrate that plausible values for knowledge depreciation are unlikely to make this force pivotal when considered alongside the assumption that there are diminishing returns to research effort. Taken together, the baseline finding of a null relationship between population size and individual incomes appears fairly robust as a baseline understanding of (semi-)endogenous models of economic growth.

Finally, in light of this null relationship, I instead focus on other reasons to prefer larger per-period population sizes when ideas drive growth. Namely, temporal discounting, and its interaction with the fact that larger populations accelerate the arrival of discoveries. Consider the two reasons that are traditionally raised for discounting experienced utility over time: (i) a pure rate of time preference and (ii) the probability of death/extinction.[4] The former is trivial—it is just a preference that things happen earlier in time. If the planner prefers the exact same events when they happen on an accelerated time horizon, then bringing forward people and ideas via population growth is welfare-improving. This is true even if the positive claim is correct that no individual person-year

[4]The third reason for discounting *consumption* over time is that future generations will be richer than present generations when economic growth is positive. That is not relevant here as I am holding fixed consumption levels and asking only about the implication of moving these lives to different periods.

is improved by population growth. In this case, population growth would be beneficial for the novel reason that it functionally serves to accelerate history.

Discounting because of death to the representative agent (i.e., existential risk) is more interesting. In an expected value sense, future utility should be discounted by the probability it is experienced. Suppose there is some exogenous, unknown, date of extinction, T^X. This would give rise to potentially the most important effect of larger per-period populations: More lives are lived, and more ideas are discovered, during humanity's fixed temporal lifespan. Put differently, while the same number of people live prior to any given level of TFP (the first claim of this paper), if knowledge is accumulated earlier, more people may get to live between the time of a given discovery and T^X. It is still the case that no lives are improved *conditional on existing*, but more total lives are lived when the population is larger. This generates what I refer to as an *extensive margin* effect on per capita incomes—these additional lives have higher incomes than average. However, this result has very little to do with the fact that knowledge is non-rival. Once it is assumed that the planner puts value on the existence of these contingent individuals, nearly any modeling assumptions will generate the conclusion that larger per-period populations are socially beneficial.

To sharpen this point, I formalize an equivalence in this setting between (1) increasing the per-period population size and (2) delaying the date of extinction. Behind this equivalence is that fact that there is nothing, *per se*, important about the years in which these additional lives are lived. Once this is seen, it becomes clear that the relevant question is a normative one about the value of additional existences, not a positive question about the effect of population size on economic growth.

Modifying the model to take a richer view of extinction risks leads to even larger departures from conventional wisdom. For example, imagine that extinction is specified to be an endogenous outcome; perhaps the probability of a catastrophe is a function of the level of technology. The invention of nuclear weapons appears to now pose at least some risk to our existence. Looking forward, some claim that sufficiently powerful artificial intelligence may likewise be an existential threat (see e.g., Ord, 2020). Just as with Mozart, steam power and the electric light bulb, if ideas are accumulated in proportion to the population size, then a fixed number of people live prior to the invention of dangerous technologies. In this case, a larger population proportionately accelerates everything, *including the date of extinction*. The number of people who ever live is then invariant to per-period populations sizes, so any effect that population size could have is eliminated.[5]

[5] Interestingly, this claim does not rely on when, or how quantitatively likely it is that any given invention is dangerous rather than beneficial, it only relies on the assumption that *eventually*, ex-post, our demise is caused by the

In general, I show that a range of plausible ways of modeling existential risk and idea-generation can lead to a range of outcomes regarding the relationship between per-period population size and the quality and quantity of lives that are lived. This includes cases where economic growth is independent of population size, which generalizes the findings of this paper beyond the semi-endogenous growth literature. The time at which a dangerous technology is invented is independent of population size under this assumption, so the only effect of a larger population is to increase the number of lives that live prior to each threat. Interestingly, the case for a larger population is arguably stronger when population size does not accelerate innovation. This is not a mere technicality: It is a special case of the parameter values that appear to be most likely based on prevailing wisdom in these respective fields. Therefore, I conclude that a larger population size likely results in more people ever living; this has a weakly positive extensive-margin effect on per capita incomes, averaged over all of history. This is far from certain, however, and calls for more work assessing the joint effect of demographic change and existential threats.

This paper contributes to literatures at the intersection of scale economics and long-run economic growth. In particular, Jones (1995) builds on the Romer (1986, 1990) insight to highlight the importance of population size/growth in driving TFP improvements (see also Kremer, 1993; Jones, 2003, 2005, 2022). This counter-intuitive idea—that population size is the key driver of long-run improvements to living standards—sparked a series of papers attempting to eliminate these scale effects (Dinopoulos and Thompson, 1998; Segerstrom, 1998; Young, 1998). Jones (1999) shows these attempts fail; a larger population is still richer per capita each period, even if specific parametric assumptions can eliminate the result that faster population growth leads to faster economic growth. The link between the non-rivalry of ideas and increasing returns to scale has been seen as a deep feature of long-run growth since. Recent years have seen renewed attention and additional exploration of this finding as the world transitions to a regime of below-replacement fertility, with the possibility of long-term population decline (Jones, 2022; MacAskill, 2022; Spears and Geruso, Forthcoming).

The main contribution of this paper is to demonstrate that, contrary to conventional wisdom, these idea-based scale effects do not imply anything about whether larger populations improve individual lives. This literature has focused on whether balanced growth paths exhibit a positive relationship between income growth and TFP growth, or if each time period has higher levels of TFP when the population size is larger. This paper shows that increasing TFP in each per period does not mean that there is increased TFP per human life. Once this is established, I show that the

deployment of a sufficiently dangerous technology.

relevant question is only whether a larger population allows for more lives to ever be lived.

By studying long-run, aggregate welfare—that is, the quantity and quality of lives over long time horizons—this paper also intersects with a literature in applied welfare economics assessing changes in population sizes. Most relevant is Adhami et al. (2024), which decomposes the social welfare gains coming from population growth versus income growth over the last decades. They find that increases in the quantity of lives matters much more than the increase in per capita living standards. Theoretically, Pindyck (2024) studies the welfare properties over the long-run of a model with sustainable consumption and variable population when the planner values existences. Other papers in the literature of applied population ethics have similarly found the quantity of lives to be a first-order determinant of social welfare (see e.g., Lawson and Spears, 2023). These findings align with my own, wherein the quantity of people to ever live is the crucial consideration when determining whether larger per period populations have any positive or normative effects.

Finally, there is a recent literature interested in risks to humanity's survival that directly deals with the question of how many people will ever live. Jones (2016, 2024) studies problems where the risk of extinction endogenously evolves with technology, similar to what is assumed in some of the cases in Section 4. He instead studies the growth-safety trade-off, rather than anything related to population size. Aschenbrenner and Trammell (2024) likewise study a growth-safety trade-off faced by a planner who can use TFP improvements to improve safety or consumption and ask whether broad-based technological progress is beneficial for increasing humanity's long-run survival probability. Ord (2024b) develops a very general framework for assessing the long-run welfare implications of changes to the joint trajectory of life-quality and survival-probability of humanity over time; seen from this framework, population size is a method of speeding up events.[6]

The rest of the paper is structured as follows. Section 2 formalizes the main claim that the number of life-years lived prior to a given discovery is independent of population size in ideas-based models of economic growth. Section 3 shows this is robust to a number of relaxations of the standard (semi-)endogenous growth model. Section 4 then argues that the value of population size comes only through its effect on the number of people who ever live, and highlights the importance of assumptions about extinction risks. Section 5 concludes.

[6]See also Ord (2024a) for a more direct discussion of existential risk being pulled forward in time by increases to scientific progress.

2 The non-rivalry of ideas does not imply a positive relationship between population size and living standards

This goal of this section is to demonstrate that the non-rivalry of ideas does not, on its own, imply that larger populations raise individual living standards. The key is to look at outcomes from the perspective of people, rather than time periods. I begin by formalizing the main proposition using a standard (semi-)endogenous growth framework. I then discuss the intuition for this result, before assessing whether the standard intuition about the benefits of scale can be rescued with relaxations of the model in Section 3.

Equation (1) is a production function of productivity improvements (i.e., *ideas*) developed in the literature spawned by Jones (1995), which builds directly from Romer (1986, 1990). The key concept embedded in this formulation is that the production of ideas increases in the effort put towards discovering them, and that these discoveries improve aggregate productivity, A.

$$\frac{\dot{A}(t)}{A(t)} = \alpha(t)(s(t)N(t))^{\lambda}A(t)^{-\beta} \tag{1}$$

Time, t, is continuous; $\dot{A}$ is the instantaneous change in productivity. This depends on three factors. First, the number of individuals engaged in research. In (1) this is decomposed as the share of the population in research, s, multiplied by the population size, N. Second, the productivity of these researchers, α. Third, the accumulated knowledge stock, $A(t)$, which allows for non-linearity in idea production. There could be increasing returns to productivity gains ($\beta < 1$) if past inventions (e.g., the computer) help generate new inventions, or there could be diminishing returns to inventions ($\beta > 1$) if earlier discoveries are systematically easier to make. In practice, $\beta > 1$ matches the historical data better, both in aggregate and at the industry level (Jones, 1995; Bloom et al., 2020), though this parameter is not consequential for the results established here.

Notice that the number of researchers is raised to some power λ that determines whether there are diminishing returns to researchers within a period. There is considerable uncertainty about what value λ should take. One reason to believe that $\lambda < 1$ is *duplication*; people alive at the same time might solve the same problems, resulting in wasted effort. On the other hand, *collaboration* might be a reason that $\lambda > 1$; the ability to communicate with other researchers might result in faster progress than a counterfactual where those same researchers live in different years.[7]

[7] In reality, this relationship likely has an 'S'-shape, where collaboration benefits dominate at small populations, but duplication issues dominate for large populations. It is unclear which dominates for current population sizes.

For expositional simplicity, let $\lambda = 1$ and $s(t), \alpha(t)$ be some constants $\bar{s}, \bar{\alpha}$. I assume a constant α and s because the objective is to isolate the effects of population size. Of course, increasing α or s would also increase the production of ideas, but this is not the focus of this paper. The assumption of $\lambda = 1$ is more substantive. It imposes a constant marginal effect of population within a period (i.e., if the world population were twice as large, exactly twice as much progress would occur). To generate an analytical baseline result, I ignore possible curvature in this section. Section 3.2 relaxes this assumption to demonstrate how the implications of the model are affected.

With these assumptions, we can rewrite (1) as follows, where $\theta = \bar{\alpha}\bar{s}$.

$$\frac{\dot{A}(t)}{A(t)} = \theta N(t) A(t)^{-\beta} \tag{2}$$

Integrating this function delivers the following expression for the level of A in any given period.

$$A(t) = \left(\beta\theta \int_0^t N(\tau)d\tau + A_0^{\beta}\right)^{\frac{1}{\beta}} \tag{3}$$

Equation 3, quite intuitively, states that if idea discoveries per period are a function of per-period research effort, then cumulative discoveries over time will be a function of cumulative research effort over time. Formally, because the assumption is that people are the key input to idea discovery, cumulative research effort is just a scalar of cumulative people-years, $\int_0^t N(\tau)d\tau$. Notice that this does not need to be a linear relationship. In the case where $\beta \neq 1$, a person-year generates more or less knowledge depending on the size of the existing stock. This is not important for the argument that follows.

Substantively, what Equation 3 does is eliminate t as an independent variable. It is cumulative human effort, not time, that drives innovation. Our economic lives are richer than previous generations' not merely because time has passed, but because people have been working to improve productivity in that time. Therefore, time can be eliminated as an independent variable without losing anything of conceptual importance.[8]

The formal claim of this section is that the same number of people-years are lived prior to a given level of TFP, regardless of population sizes. This will follow by defining person-years lived by time t as:

$$i(t) = \int_0^t N(\tau)d\tau \in [0, I]. \tag{4}$$

[8]Of course, this could be contested. If experiments literally take time, then perhaps time *should* enter this function. I return to the potential direct effect of time in Section 3.3.

In continuous time, each i is an instantaneous person-experience, denoted in units of people-years. For example, if the population size were a constant 10 billion, and exactly two years pass, then, at that instant, the 20th billion person-year is being lived. This value ranges from 0 (the very first person-experience) to I, the number of people-years that humanity ever experiences. After formalizing the main proposition I will discuss the intuition of the result in detail, including how results in terms of these person-experiences map into the more familiar unit of people's lives.

Equation 3 implies that $A(t)$ is a function of people-years, which are now denoted $i(t)$. Therefore, the expression for $A(t)$ can be simply rewritten as a function of i.

$$\boxed{A(i) = \left(\beta\theta i + A_0^{\beta}\right)^{\frac{1}{\beta}}} \tag{5}$$

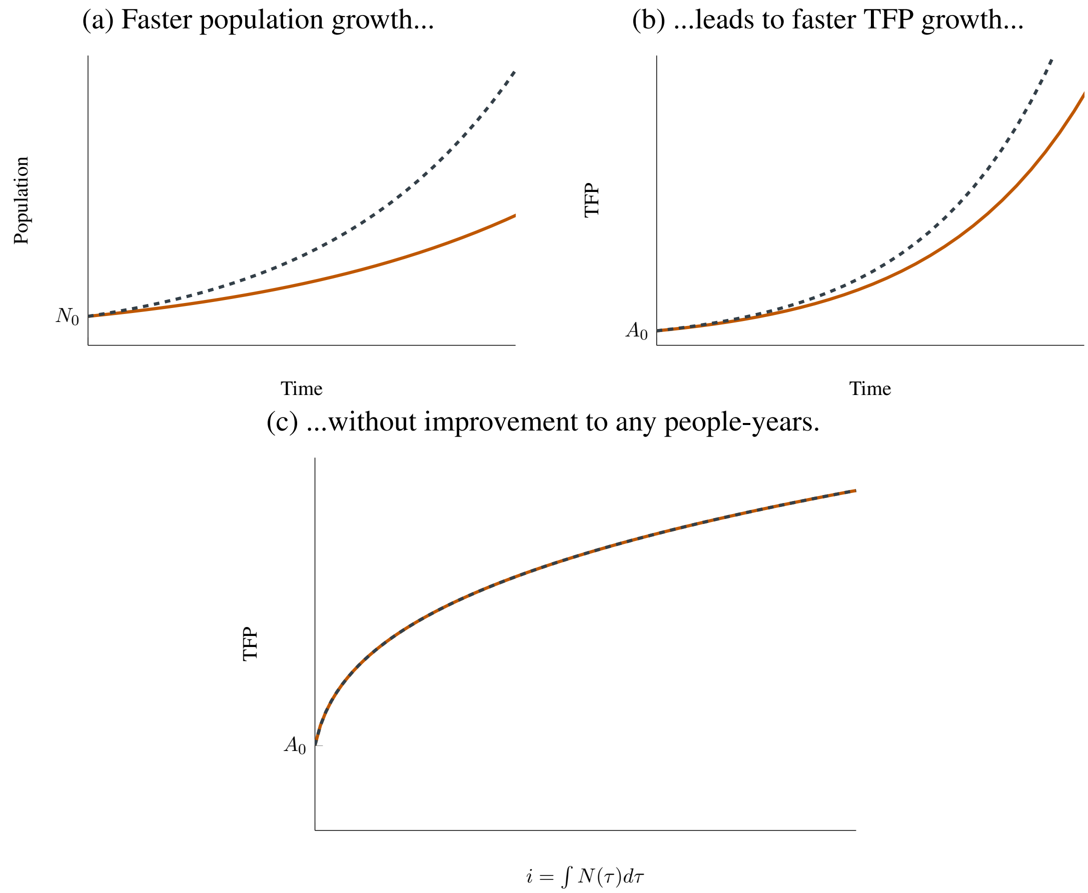
This result makes clear that the experience of each person-year i is only a function of i's order in history, conditional on θ, β. Time or population size no longer enter this simplified expression. If the population size is larger, each i happens earlier in time, but the ideas available to each i are not affected by population size.

Figure 1 demonstrates the mechanisms of this result. Panels (a) and (b) plot population and technology as they are usually plotted—against time. The history with faster population growth has faster TFP growth. Panel (c) eliminates time as a variable and plots them against one another. This is a plot of technology available to people, rather than time periods, which I take to be the more meaningful unit of analysis. Time periods do not have incomes or experience utility. People do. Panel (c) shows that TFP available to each person-year is unaffected by the size of the population; these curves exactly overlap. This is because the count of individual experiences is accelerated just as fast as TFP improvements are in the scenario with faster population growth.

To build up the intuition for this result, start by ignoring overlapping lives. Each i shows up, has access to the ideas discovered before her, contributes something to the knowledge stock, and then disappears. It is easy to see here why her order in history is all that matters: changing the speed at which humanity churns through these i's does not matter for how many people—and therefore, ideas—came before her.

The question at the heart of this paper is about different population sizes, so it is necessary to extend this intuition to cases of overlapping lives. What makes this more difficult to conceptualize is that the i-moments are now interspersed between different people. When ten people live for one year, it is easy enough to see that this equals 10 people-years. But the i's are defined to be ordered by time—recall from (4) that $i(t)$ is the integral of population size over time—so the first

Figure 1: Accelerating economic growth via population growth does not improve any life-years



Notes: Blue dotted line represents a history with faster population growth than solid orange line. Panel (b) illustrates that TFP growth would be faster in the history with faster population. Panel (c) demonstrates—by eliminating time as a variable—that the TFP available after a given number of people-years is invariant to the speed of population growth.

person-year in this ten-person world occurs after one-tenth of one year passes. It was collectively lived between these ten people, not by one person living one year. This complicates, but does not importantly change, the implications of the formal proposition.

To see this, consider another stylized example with overlapping lives. Suppose there will be 20 billion people who ever live, each with a 100 year lifespan. (Further issues are raised when the number of people to ever exist varies; I return to these in Section 4.) History could either be such that 10 billion lives are lived in first 100 years and then the next 10 billion live (call this History A), or it could be that all 20 billion live at once (History B). Ex-ante, it will take 1 trillion people-years of effort and economic activity to get to some key, life-improving technology. In History A, that

Table 1: With overlapping lives, population size improves some at the (offsetting) expense of others

	$N(t)$	Idea at:	People-years pre-idea	Entire lives lived w/out idea	Entire lives lived w/idea
History A	10 Bil.	$t = 100$	1 Tril.	10 Bil.	0
History B	20 Bil.	$t = 50$	1 Tril.	0	10 Bil.

Notes: Table cataloging a scenario with overlapping lives. A larger population implies that fewer people live their entire lives without a given discovery/idea/TFP-improvement. But it also means that more people live a fraction of their lives without this discovery. In terms of people-years, these effects cancel out.

invention comes precisely at the moment these generations are turning over, after 100 years have passed. In History B, it comes after only 50 years, because the population in the first century is twice as large. So, relative to History A, the half of the population that would have lived in the first century regardless has half of their life made better (they now live 50 years with this technology, whereas in History A they get 0 years with it); but the half of the population that would have lived in the second century has half of their life worsened (they now live half of their life without this technology, whereas in History A they live their entire lives with it).

In terms of people-years, these positive and negative effects exactly cancel out. In both cases—just as the main proposition indicated—1 trillion people-years are lived without this technology, 1 trillion are lived with it. These people-years are distributed across individuals differently, but this would not matter to a planner with a standard objective function that sums individual (time-separable) utility functions.[9] A planner with explicit egalitarian motives may prefer History B because the distribution of better and worse people-years are spread more evenly over individuals. But this is a normative claim about equality, not a claim that population increases non-rival ideas, and so unambiguously improves lives.

To make this even more concrete still, consider what this means for population growth today. You and I might personally hope for more population growth while we are alive. The result would be that more innovation happens during our finite lives that we benefit from. What is happening in the language of this paper is that population growth serves as a way of assigning my later years a higher-value of i. More people-years will be lived between now and 2050, for example, so the i I am personally experiencing in 2050 will be a larger value than it would have been if the population were smaller between now and then. But someone has to live those earlier i's. The trade-off is that the density of people is increased in the near-term, which is the worst-time to be alive conditional on not having lived until now.

[9] I.e., $\sum_{j \in N} \int_0^T u_j(t)dt$ is invariant to swapping utility moments, $u_j(t)$, between individual people, j.

Overall, this section demonstrates the main claim of the paper: the non-rivalry of ideas does not imply that population size improves the knowledge base accessible to, and therefore the quality of, any given person-year. Instead, the population size governs how fast human civilization moves through life-years of an *ex-ante* pre-determined quality. Section 3 shows that this baseline result is robust to relaxations of the simple semi-endogenous growth setting used here. Section 4 considers that a larger population in each period may lead to more people ever existing, which extends the implications of this finding in more realistic directions.

3 Main takeaway is robust to model enrichment

This section discusses potential modifications through which the importance of population size for individual living standards may emerge when ideas drive economic growth: endogenous research effort; non-linear returns to research effort within a period; and the depreciation of knowledge over time. In the latter two cases, the intuition that larger populations are beneficial because of the ideas they create can be rescued. However, these are, if anything, second-order effects that deserve more study. They fail to provide the same strong reasons that accounts for *a priori* confidence that population size increases the rate of economic growth per period.

3.1 Endogenous research effort

The results in Section 2 assume that s, the share of individuals contributing to the knowledge base, is fixed. However, larger markets may incentivize more R&D, providing additional reason to believe larger populations will lead to more idea creation. In a world where 10, rather than 1000, people have a rare disease in a given period, it seems less likely anyone will be incentivized to find a cure. This is true, but the causal mechanism works in a way that makes this irrelevant for the findings of this paper—endogenizing research effort in a straightforward way does not lead to the conclusion that larger populations improve outcomes for individual person-years.

With a fixed research share, s, the main results already accounted for the fact that there will be a larger number of researchers in a given period. The number of researchers scales linearly with N. So, for this endogenous research channel to deliver the result that a larger population generates more research per person-year, it must be accounted for by s increasing in N. That is, the composition of the workforce must also change with population size. A simple general equilibrium R&D model below shows that the standard market-size effect does not lead to this outcome.

Consider a one-period model where workers can choose before the period to either work or search for an idea and sell the patent as an entrepreneur. The aggregate production function is a CES aggregator of all goods that there is an idea for. In particular, the integral of possible intermediate goods to purchase has an upper-bound of A. When A is larger, the factors of production can be spread over more intermediate-goods, j, which increases Y through a love-of-variety channel. In short, the number of ways that resources can be used—and therefore aggregate consumption—is increasing in A.

$$Y = \Big[\int_0^A y_j^{\frac{\varepsilon-1}{\varepsilon}} dj\Big]^{\frac{\varepsilon}{\varepsilon-1}} \tag{6}$$

Each intermediate producer, j, chooses: whether to purchase a patent for an idea at price p_A; what price to charge for its good, p_j; and the quantity of its good to produce, y_j. It faces a linear production function, $y_j = l_j$, where l_j is labor in sector j. Labor is paid a wage w that is taken as given by each firm in the economy. I assume a free-entry condition, such that the fixed cost to operate will adjust to set profits to zero in equilibrium, despite the monopolistically competitive set up. In this set-up, the fixed operating cost is the purchase of a patent, so p_A is the price set in equilibrium by the free-entry condition. The demand curve faced by each intermediate firm is the result of a standard cost-minimization problem that the final good producer solves.

Overall, the problem for a firm that decides to purchase a patent and operate looks as follows (if the maximum profit that can be attained is less than zero, the firm will not purchase the patent and operate).

$$\max_{p_j, y_j, l_j} p_j y_j - w l_j - p_A \tag{7}$$

subject to:

$$y_j = \left(\frac{p_i}{P}\right)^{-\varepsilon} Y \tag{8}$$

$$y_j = l_j \tag{9}$$

Workers supply one unit of labor inelastically. They can either earn wage w, which they also take as given, or search for an idea. If they search for an idea, they are successful with probability ω and can sell the patent at price p_A. Therefore, the key equilibrium condition coming from the worker's problem is that the return to searching for an idea will be equal to the wage.

$$w = \omega p_A \tag{10}$$

The amount of knowledge in this economy is a function of how many researchers decide to search for ideas. Denote N as the population size and s as the share of individuals searching for ideas.

$$A = \omega s N \tag{11}$$

The full solution to the model is contained in Appendix A. What is relevant for this section is only the equilibrium condition that determines the share of individuals who decide to search for an idea.

$$\varepsilon - 1 = \frac{1-s}{s} \tag{12}$$

The share of the population working in research is a function only of the elasticity of substitution between goods—it is independent of the size of the population. The intuition for this result can be seen by considering the intermediate good producers problem. Intermediate firms need to purchase both a patent and labor to produce and sell into the (now) larger market. For the same reason that a patent becomes more valuable, so too does each unit of labor. The opportunity cost of searching for an idea grows alongside the value of a patent, leaving the share of individuals choosing to search for an idea unchanged.

Things become more complicated in cases where populations grow over time—this was a simple static setting designed to demonstrate why there is not good reason to believe the fraction of researchers is increasing in the population size. In Jones' classic (1995) paper highlighting the importance of population growth in R&D based models of economic growth, the equilibrium share of researchers is increasing in population growth, but not its size. The reason is that in his setting, patents have dynamic value. A population that is growing faster has a larger consumer base tomorrow than it has today, so the value of future production is higher than the value of current production. This tilts things in favor of researching, since a patent is an investment that pays flow dividends in proportion to future demand.

That population growth, not size, determines the share of the population is quite distinct.[10]

[10]This finding mirrors the results in Peters and Walsh (2021) and Karahan et al. (Forthcoming) where population growth contributes to firm dynamics because paying a fixed start up cost today is more worthwhile if future years will have increased levels of demand, and a larger workforce to hire.

A smaller population with faster population growth would have more research per capita than a larger population with slower growth, for example. Furthermore, the policy relevant question of the coming decades may be around what population size to stabilize at. Channels that rely only on growth-rate effects are irrelevant when comparing stable populations of different sizes. Nonetheless, it is interesting to note that the dynamic-structure of patents may interact with population dynamics to determine whether population growth is an important driver of innovation per person.

3.2 Increasing returns to research effort

In Section 2 the production function of ideas was assumed to be linear in research effort within a period. This allowed for the derivation of a clean analytical solution between cumulative people years and the ideas available to an individual for an arbitrary path of population. This linearity assumption may be too strict, and introducing curvature can indeed matter for the main takeaways. Equation (13) is the original production function of ideas, reintroducing the term λ which is not necessarily equal to one.

$$\frac{\dot{A}(t)}{A(t)} = \tilde{\theta} N(t)^{\lambda} A(t)^{-\beta} \tag{13}$$

This in turn produces the slightly modified version of the function determining cumulative research progress over time.

$$A(t) = \left(\beta\tilde{\theta} \int_0^t N(\tau)^{\lambda} d\tau + A_0^{\beta}\right)^{\frac{1}{\beta}} \tag{14}$$

This modification breaks the tight link between cumulative people-years and cumulative ideas produced. When $\lambda \neq 1$, how lives are spread out over time matters in addition to how many have been lived. For example, if $\lambda > 1$, for the same number of cumulative life-years, humanity discovers more ideas if those lives have more temporal overlap. If $\lambda < 1$, there are diminishing returns to research effort within a period, so less is discovered with more temporal overlap.

Somewhat trivially then, for $\lambda > 1$ the each person-year has more TFP accessible if historical populations were larger. The (fixed, by definition) i people-years that occur prior to i's existence would have been able to take advantage of more collaborative opportunities and discover more. Conversely, if there are diminishing returns to research effort within a period ($\lambda < 1$) the opposite will be true.

This can be seen formally by considering the special case where population sizes are a constant $\bar{N}$. The constant population case is one I will return to frequently for these more complicated cases because it allows for analytical solutions while continuing to isolate the core question of whether larger populations influence per person living standards.[11]

In the case of constant populations, the population integrals in Equation (14) can be rewritten.

$$A(t) = \left(\beta\tilde{\theta}\bar{N}^{\lambda}t + A_0^{\beta}\right)^{\frac{1}{\beta}} \tag{15}$$

$$i(t) = \bar{N}t \tag{16}$$

This implies:

$$A(i) = \left(\beta\tilde{\theta}\bar{N}^{\lambda-1}i + A_0^{\beta}\right)^{\frac{1}{\beta}} \Rightarrow \tag{17}$$

$$\frac{\partial A(i)}{\partial \bar{N}} = \tilde{\theta}(\lambda-1)\bar{N}^{\lambda-2}i\left(\beta\tilde{\theta}\bar{N}^{\lambda-1}i + A_0^{\beta}\right)^{\frac{1-\beta}{\beta}} \tag{18}$$

This derivative will be positive if and only if $\lambda > 1$. In the baseline case where $\lambda = 1$ it is easy to see the derivative is zero, generating the independence result of prior section.

In the literature that attempts to calibrate and/or estimate the effect of aggregate research effort on long-run growth, $\lambda < 1$ is the much more common deviation from a linearity assumption (see e.g., Bloom et al., 2020; Jones, 2022). If this is indeed the case, then the standard model of non-rival ideas may imply *decreasing* returns to scale along the margin that matters: each individual person-year is *worsened* if per-period populations are larger. It is easy to see this through the framing of the respective arrival rates of ideas and people. The arrival of ideas accelerates when the population is larger, but slower than the one-for-one rate that the arrival of people-years accelerates. Therefore, each person-year has less knowledge accessible to it.

Empirical evidence for this parameter is difficult to come by. And, in any case, a power function is almost certainly too simple.[12] The relevant question is whether on current margins congestion/duplication effects are dominated by collaboration effects. I know of no strong evidence on this question, so this does not appear to be a reason to be confident that a larger population improves individual living standards.

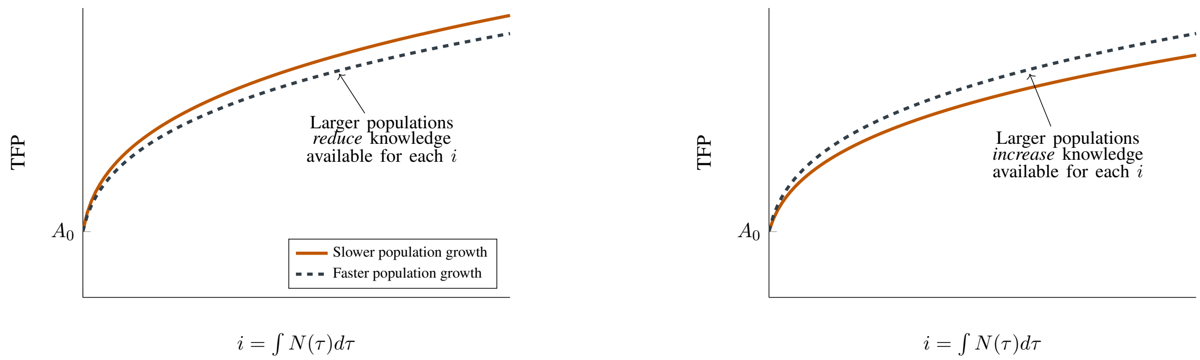
[11] Put differently, if a supposed scale effect does not show up in the case where populations are constant, it is hardly capturing what we mean by increasing returns to scale.

[12] If $\lambda < 1$ that would imply that spreading people out as much as possible over time maximizes research production per person; $\lambda > 1$ implies that bunching people all in the same period would maximize output per researcher. Neither of these extremes seems intuitively correct.

Figure 2: Second-order channels can generate positive or negative scale effects

(a) Decreasing returns to research effort

(b) Knowledge depreciation over time



Notes: Blue dotted line represents a history with faster population growth than solid orange line. Panel (a) illustrates that if there are decreasing returns to knowledge generation within a period, larger populations reduce the TFP available for each lifetime—a stark reversal of what non-rival ideas are thought to imply. Panel (b) illustrates that if knowledge depreciates over time, the result that larger populations improve individual outcomes is retained. It is unclear which of these is the dominant force.

3.3 Depreciation of knowledge

Knowledge depreciation leads to an unambiguous theoretical prediction: its existence works to rescue the link between non-rival ideas and the benefits of population size. Furthermore, and unlike the case of $\lambda > 1$, it does so for the original reason that knowledge is non-rival and can be freely used once it is discovered. The relevant question is whether the quantitative magnitude of knowledge depreciation is large enough to make this a significant force.

Consider the following implementation of depreciation, where, generally, it is a function of economic activity that period.

$$\frac{\dot{A}(t)}{A(t)} = \theta N(t)A(t)^{-\beta} - \delta_A(Y(t)) \tag{19}$$

This can only be integrated into a closed-form solution under the special case where depreciation is a constant $\bar{\delta}_A$ and $\beta = 0$. This is a reasonable baseline case for understanding how this force works. The value of $\beta = 0$ corresponds to the original Romer (1990) formulation, and nothing in the paper has depended on the choice of β, making it an inconsequential parameter for developing intuitions. Likewise, the assumption that δ_A is constant is the most natural starting point for depreciation, matching the standard assumption for how physical capital depreciates.

With these parameter values, and a constant population level, A follows a simple exponential growth process that depends on the size of the population. (In Appendix **??** I show that in another special case that produces an analytical solution, the same qualitative results are obtained.) I will again take advantage of the fact that with constant populations $\bar{N}t = i(t)$.

$$\begin{aligned} A(t) =& A_0 e^{\theta \bar{N} t} e^{-\delta_A t} \\ A(i) =& A_0 e^{\theta i} e^{-\delta_A t} \end{aligned} \tag{20}$$

For any i, the first two terms of (20) are fixed. The third term depends on when person-year i is lived. In particular, the earlier in time that i is lived, the larger is the knowledge base she has access to. To reduce the time-period in which each i lives, $\bar{N}$ has to be larger (e.g., humanity gets to its 100 billionth person-year earlier if annual populations are larger).

The intuition for this result is straightforward when thinking about the arrival rate of people-years accelerating or decelerating. For the ith person, there is now a drag on the knowledge accessible to them that is a function of time. If an idea was discovered long before the ith person-year, this person will have less access to it, in expectation. In the fixed number of life-years lived prior to i, there are a fixed number of discoveries (assuming $\lambda = 1$). It is therefore better if person i lives nearer in time to those discoveries. The way to promote person i living closer in time to more of her predecessors is for the population to be large, so that more births happen over a shorter time period.

A nice feature of this result is that the benefits of population size are retained for the 'right' reasons—knowledge is non-rival, so having more people around benefits everyone. Consider the limiting case where the depreciation rate on knowledge is one, so that the only knowledge accessible to person i is the knowledge discovered in the period in which she lives. Having many contemporaries benefits her because the ideas her contemporaries produce are non-rival and can be used by her. The mechanism in Section 2 that breaks the intuition about population size is that person i would have had access to all of her predecessors ideas, regardless of when they occur. If knowledge depreciates, this is no longer true.

Furthermore, the symmetric case of exogenous knowledge *appreciation* seems significantly less plausible. This is why its effect is less ambiguous, unlike the relaxation that $\lambda \neq 1$, which could work in either direction. It is not inconceivable that something isomorphic to knowledge appreciation exists—e.g., if some costless knowledge is accumulated from exogenous natural events—but this seems unlikely to be as important as the passing of time potentially eroding the knowledge base. So, the addition of a force capturing exogenous growth/decay serves to return the result that larger populations are beneficial because of non-rival knowledge.

Finally, it is important to note that the ways in which we might enrich the formulation of depreciation would serve to strengthen the takeaway. Namely, consider specifying knowledge depreciation as a function of economic activity within a period. It seems likely that in periods with more economic activity, fewer ideas go unused and forgotten.[13] Rather than being non-rival—where my use does not erode its availability—knowledge may be *amplifying*—my use increases its availability for others. Section 2 demonstrated that knowledge being non-rival was not enough to generate increasing returns to scale. But if knowledge is amplifying, larger populations will again generate better outcomes for everyone.

There is very little evidence on the functional form or magnitude of knowledge depreciation. Its existence is unlikely to change the balanced growth properties of models with exponential growth in populations, so it has received little attention within this literature to date. But, as noted in Jones (2022) and Eden and Kuruc (2023), it is a crucial parameter in models without long-term population growth. A future of zero or negative population growth appears to be the most likely demographic context of the coming decades, so more focus is likely to be paid to the rate at which knowledge depreciates. The result in this section furthers this emphasis on knowledge depreciation by emphasizing that this force is crucial for understanding how individuals are affected by idea-based growth, even in settings of population growth.

3.4 Summary of Negative Result: Population size has no clear intensive-margin effect on individual incomes

The lesson of Section 3 is that the main result of Section 2 cannot be easily avoided with simple model relaxations. I first showed that endogenizing research effort to account for market size effects does not affect the main lesson of Section 2, at least not without second-order reasons for why the share of the population researching should increase in a larger population. The takeaways of Sections 3.2 and 3.3 are less straightforward, especially when taken in conjunction.

Increasing returns to scale within the research production function, $\lambda > 1$, would generate the result that larger populations improve individual outcomes. However, that is arguably less relevant than the fact that if $\lambda < 1$ the standard intuition is reversed—smaller populations improve individual outcomes. Depreciation is less theoretically ambiguous. If it is the quantitatively significant

[13]This is an important conceptual distinction from capital. In models where capital utilization can vary over the business cycle, depreciation of capital increases when the economy is larger. The machines are worked harder and breakdown faster.

force, it generates the takeaway that larger populations improve individual outcomes. But we know little about its magnitude. The implicit consensus is that it is not an important force; nearly every study of ideas-based economic growth rounds it to zero.

Figure 3 demonstrates that it would be difficult to be confident in any particular deviation from the first-order result that larger populations have no effect on individual living standards. It plots the relationship of interest—TFP per person-year—for values of λ and δ_A. Namely, I begin with $\lambda = 0.75$, following Bloom et al. (2020), Jones (2022). For δ_A, there is less guidance. I plot the cases of both $\delta_A = 0.1\%, 0.3\%$ for two reasons. First, this seems a plausible order of magnitude; anything larger would imply that depreciation is a 50% drag on growth.[14] Second, these values happen to generate different directional conclusions, so they helpfully demonstrate how these forces quantitatively trade-off. If $\lambda = 0.75$ and $\delta_A = 0.1\%$, the relationship between population sizes and each person's income is negative; if $\lambda = 0.75$ and $\delta_A = 0.3\%$, the relationship between population sizes and each person's income is positive. Distinguishing between these cases with our current body of evidence is nearly impossible. While the exact analytical results in Section 2 are contingent on specific assumptions, the qualitative point that there is no clear relationship between population sizes and individual incomes generated by the existence of non-rival ideas is robust.

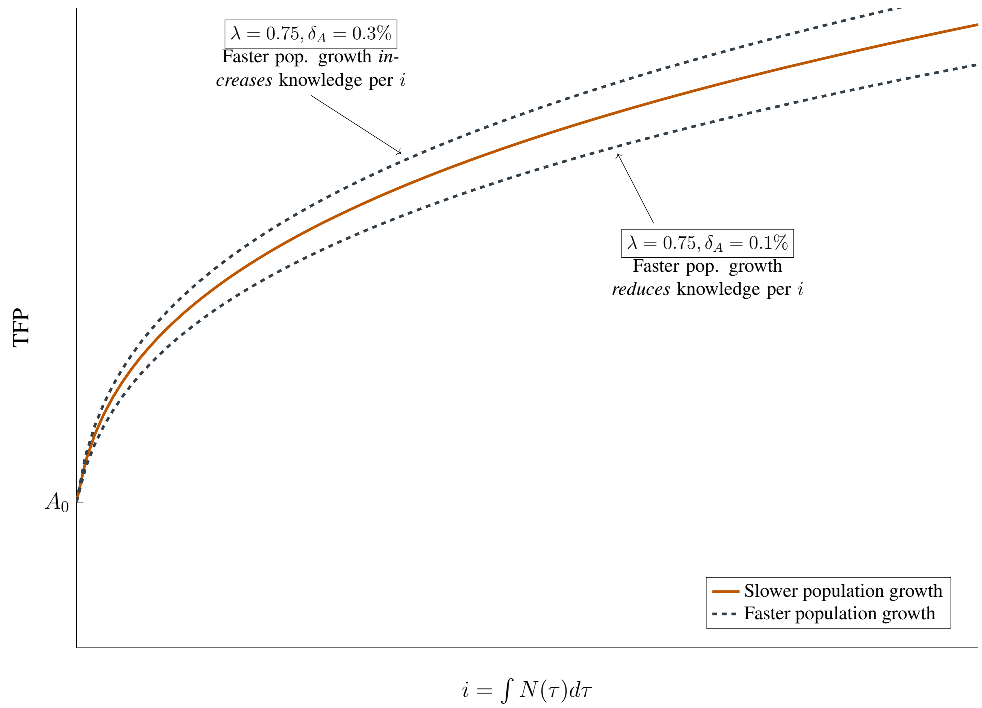
This is what I will call the intensive-margin result: For a fixed number of people-years lived, per capita incomes are, to a first-order, invariant to population size. However, this does not rule out an extensive margin effect of population size. A larger per period population may cause more lives to ever be lived; if those lives are above average, per capita incomes can be increased by population size. This is, conceptually, a quite different channel by which per capita incomes can increase. I turn to it next.

4 The quantity of people-years, discounting & extinction risk

So far, I have argued that standard endogenous growth frameworks imply that the size of the population governs the speed of historical events—the arrival and experience of people-years and discoveries—not the quality or content of those people-years. This section will explore the extensive-margin effect that a larger population can have. More people will plausibly live in histories with larger per-period populations. I show that, if this is the case, then the extensive margin

[14] If TFP growth is measured at 1-2% and δ_A =1%, then we would be adding 2-3% and losing 1%. That seems intuitively much too large.

Figure 3: Quantitatively, second-order effects have ambiguous effect when considered jointly



Notes: Blue dotted lines represent a history with faster population growth than solid orange line, under different parameterizations for λ, δ_A. When $\lambda = 0.75$, the directional effect depends on small differences in δ_A, leaving little reason to deviate from the baseline results of a null effect.

effect will be positive. This result ends up being related to temporal discounting, so I will also discuss the interaction of this force with the finding that population size governs how fast events unfold.

There are two reasons typically given for discounting utility over time; one normative, one positive. The normative one is simple. The planner may simply prefer that things happen earlier in time, a *pure rate of time preference.* This is incorporated to reflect the fact that people appear to be impatient in a way that suggests utility experienced earlier in time is more valuable than utility experienced later. There is disagreement among economists about whether the planner should inherit this impatience, notably in the literature on climate change. The relevant thought experiment for this setting is straightforward: Does the planner have reason to prefer one of two histories with exactly identical person-years if one history happens over the course of 500,000 years rather than 1,000,000 years? If so, this is a novel reason to prefer larger populations. It is not that lives are, on-net, improved, it is that things happen earlier, and this is valuable to the planner. Resolving the normative debate about whether the rate of pure time preference should be greater than zero is beyond the scope of this paper, but it is worth noting how this normative parameter interacts with the positive findings here.

The positive case for discounting is much more interesting. It relies on an expected value argument about whether there will be future experiences at all. For an individual, this represents their probability of death. For society as a whole, this might represent the fact that each year carries some risk that humanity goes extinct. Acknowledging these probabilities implies discounting the utility that would be experienced in 500 years, conditional on existing, by the probability there has been an extinction event. This line of reasoning provides the strongest reason to prefer larger per-period populations, though it ends up having little-to-nothing to do with the original Romer (1990) point that ideas are non-rival. In fact, I show that ideas-based growth models may be one of a very small class of models in which the planner does not prefer larger populations, depending on how extinction risks are modeled.

Consider first a simple asteroid case, where each year has some fixed exogenous probability that we go extinct via a natural event. Regardless of the population size, there is some *ex-post* year in which this event occurs. Call this date of extinction T^X. The proposition in Section 2 indicated that the quality of people-years is identical conditional on that person-year happening—so, for a fixed number of total people-years, I, per capita income is unaffected. But it is easy to see that a larger per-period population will get through more people-years prior to T^X.

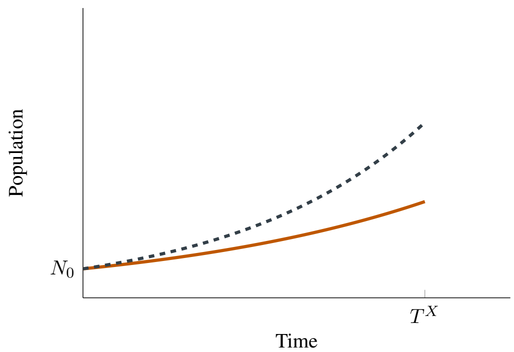
Furthermore, because the TFP available to a given person-year, $A(i)$, is increasing in i, these contingent people-years are the highest quality people-years. For example, if humanity makes it through 110 billion people-years, rather than 100 billion people-years, the marginal 10 billion people-years will have all the knowledge accessible to the 100 billion unconditional people-years and then some. As a result, total life-years lived and the average quality of these life-years are improved when per-period populations are larger and extinction happens at some exogenous date. There is a positive relationship between per capita outcomes and population size over the whole of human history, accounted for entirely by the extensive-margin effect that the additional lives are better than average.

Figure 4 depicts this case. It is organized just as Figure 1, the difference here is that I no longer assume that humanity goes on indefinitely. Panels (a) and (b) do not show anything surprising. If population and TFP growth are faster, higher levels of each are reached by the time of extinction.

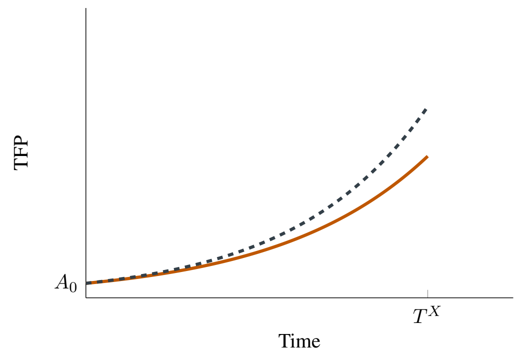
The substantive contribution is in panel (c). Recall that the x-axis here depicts the measure of people-years as humanity lives them. Let I_S be the number of people-years the small population gets through; I_L is the number of people-years the larger population gets through. As noted above,

Figure 4: Exogenous Extinction Event at T^X

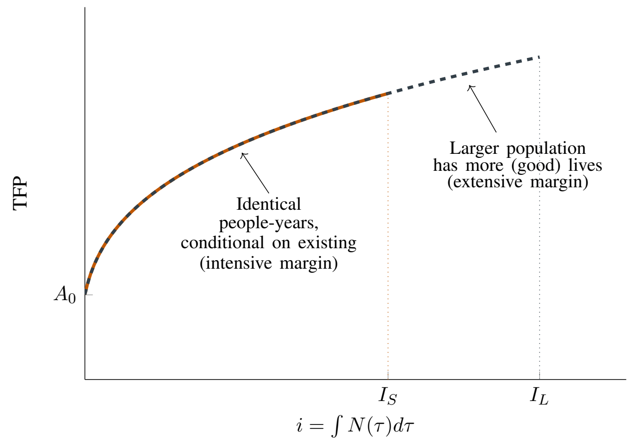
(a) Each population makes it to T^X.



(b) Faster growth $\rightarrow$ higher TFP reached.



(c) Larger populations 'get through' more history.



Notes: Dotted line represents a history with faster population growth than solid line. Panel (a) depicts how large these respective populations get by the extinction event at T^X. Panel (b) illustrates that TFP reaches higher levels in the history with larger populations. Panel (c) demonstrates, for the people-years lived in either history, there is no quality of life improvement. All potential benefits come along the extensive margin; marginal people-years are the highest quality, because TFP accumulates with people-years.

conditional on reaching a specific person-year the TFP available during that person-year is unaffected. This is the intensive margin result showing up in the case where total existences differ. The extensive margin is captured by the incomes of the people-years that are contingent on whether the high- or low-population history is realized. Figure 4 demonstrates that any effect of population size must come via the extensive margin.

In this simple setting where the knowledge accessible to an individual is directly linked to cumulative people-years, there is an equivalence between increasing the population size in each period and delaying the date of extinction. What matters is only the number of people-years lived, not when they are lived, so a $z\%$ increase in per-period populations has the same effect as extending our lifespan long enough to generate an additional $z\%$ of people existing.

This equivalence is helpful for seeing the results from a different angle that has nothing to do with endogenous growth models. Consider instead the question of how to value extending humanity's lifespan by some length of time. The first-order (perhaps only) question we would grapple with is how to value the lives of individuals who's existences are contingent on delaying extinction. Those who would get to live who otherwise wouldn't. It would also happen to be true that—if economic growth continues—those last-existing people would have the highest income levels, so including their life-years an average calculation would raise per capita incomes measured over all of human history. But that would likely seem trivial relative to the fact that those lives would be lived, and the question of how to account for that. The question of whether a larger population has valuable effects, even in (semi-)endogenous growth models, is a question about the quantity of lives that will be ever be lived. If the only thing that matters is how many people ever live, then modeling existential risk.

If so, even if the quantity of people-years is part of the planner's objective function, ideas-based growth models may be one of a small class of models that is indifferent to population sizes if extinction is endogenized. Research focused on the portfolio of existential threats believe that human-caused (anthropogenic) extinction—via dangerous technologies—is much more likely than naturally-caused extinction, via something like an asteroid (see e.g., Rees, 2003; Ord, 2020). A recent large-scale forecasting exercise—led by a research economist at the Federal Reserve Bank of Chicago—asked experts on particular risky technologies as well as "Superforecasters" attempted to aggregate probabilities on the chances of overall existential risk.[15] Median projections of existential risk this century were on the order of 1%, with more than 99% being anthropogenic (Karger et al., 2023).[16] When humanity entered the nuclear age, the consensus is that the annual probability of extinction dramatically increased; many forecast that the same may be true for future advances in artificial intelligence or bioengineering.

As a stylized case, endogenous extinction might be modeled as in Jones (2016): each new idea has some probability of ending humanity (Jones aptly calls this a model of "Russian Roulette" growth). Denote the ex-post level of technology that ends humanity as A^X. As should be clear by this point in the paper, A^X is reached once some number of cumulative people-years have been lived. Consequently, the number of people-years lived is independent of the size or growth rate of

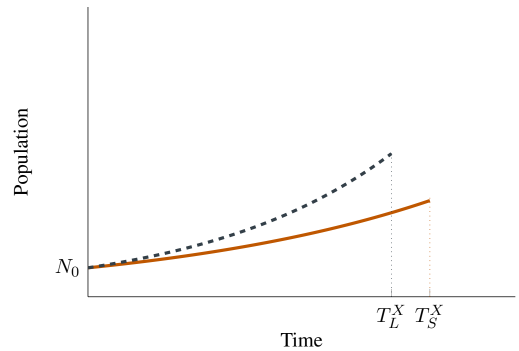
[15] *Superforecasters* are generalists who have displayed a track record of exceptionally good probabilistic calibrations on a range of geopolitical events (Tetlock and Gardner, 2016).

[16] Between groups, Superforecasters predicted a 1% chance of extinction by 2100, with 0.004% coming from non-anthropogenic risks; domain experts in risky technologies (e.g., AI, bioweapons, nuclear weapons) estimated a 6% chance of extinction, with a similar 0.004% non-anthropogenic risk.

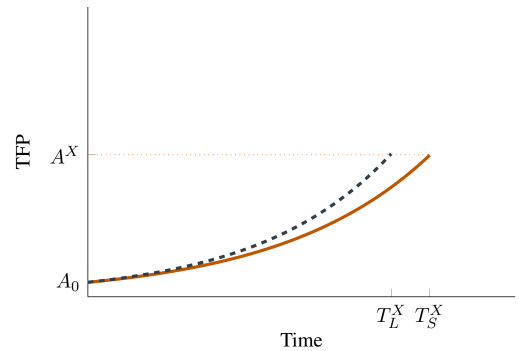
the population when extinction is endogenized under the baseline assumptions in this paper.

Figure 5: Endogenous Extinction Event at A^X

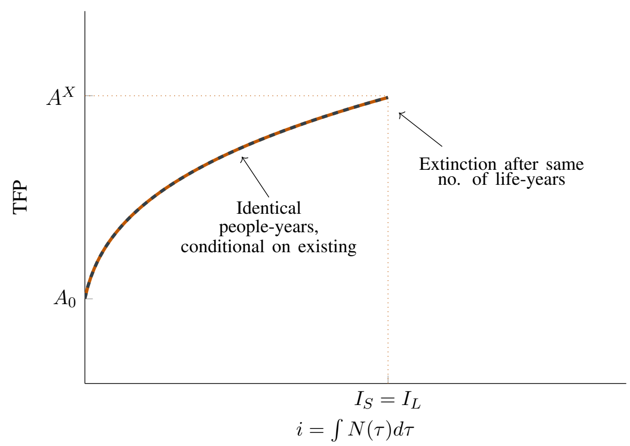
(a) A^X occurs later with small populations.



(b) Extinction happens at A^X.



(c) The same number of people-years are lived.



Notes: Dotted line represents a history with faster population growth than solid line. Panel (a) depicts how large these respective populations get by the time each reaches A^X. Panel (b) illustrates how fast each reaches the existential technology. Panel (c) demonstrates that the exact same number of lives are lived, at the exact same quality, prior to reaching A^X.

Figure 5 illustrates this case using the same three graphs as in Figures 1 and 4. The difference here is that time itself is not the causal variable that ends humanity, A is. A^X acts like an upper-bound on TFP that determines the time at which humanity goes extinct, as well as how many people live prior to extinction. As can be seen in panel (c), the intensive margin remains identical across population histories, but there is no longer an extensive margin effect from the population being larger. This eliminates any difference between histories, aside from the fact that one occurs on a condensed time horizon.

The discussion of existential risk will be further enriched in a moment, but first pause to appreciate how deeply this result contrasts with the lessons drawn from Romer (1990) and the literature it has spawned. Population size (and its growth rate) have been at the center of studies of long-run growth in living standards and economic outcomes; scale effects are nearly impossible to avoid when ideas are non-rival. This would seem to quickly lead to the conclusion that larger populations improve social outcomes. But here, closing the model with a simple and plausible assumption about extinction eliminates any effect of population: both the quantity and quality of life-years ever lived are independent of per-period population size. This is far too simple of a model to be confident in the complete neutrality of population size or growth, but it makes clear that the effect of population on *anything* is contingent on assumptions that are not typically probed.

To emphasize the contingency of these population-related effects, Table 2 mixes assumptions from earlier subsections with assumptions about existential risks to demonstrate how the implications can vary. To keep things tractable and isolate the effects of scale, I will compare two constant population sizes, L and S where $L > S$.[17] The rows represent different stylized examples for extinction scenarios. The first two have already been discussed above: exogenous extinction (e.g., an asteroid strikes) and endogenous extinction by the invention of a dangerous technology.

We could also imagine the probability of extinction being endogenized in other ways. The third row represents a case where each individual life-year poses some risk of ending humanity (a caricature of this case could be that every person-year poses some risk of generating a radicalized individual who wants to end humanity). That may seem unrealistic, but some believe that technological advances in AI or bioengineering may make it possible for a single individual to end humanity in the near future. The fourth row represents the case where technology instead improves our safety each period. For example, suppose that new game-theoretic solutions for nuclear deterrent are discovered, or our ability to withdraw carbon from the atmosphere in a cost-effective way reduces the probability of a warming-related catastrophe. This could also capture an endogenous social response to spend more on preserving the future as wealth increases, as in Jones (2016); Aschenbrenner and Trammell (2024). Finally, the fifth row represents the case where the number of people on the planet *in a given period* increases the chances of existential risk—perhaps because we push ecological boundaries too far.

The columns represent different assumptions for the law of motion for knowledge. Earlier it was discussed that λ becomes a particularly important term because it governs whether collaboration benefits ($\lambda > 1$) exceed duplication/congestion costs ($\lambda < 1$) of population size within a period. Introducing knowledge depreciation has similar implications to λ exceeding one, it gen-

[17] Again, this assumption of constant long-run population sizes isolates size effects from growth-rate effects, which may be important for transitory dynamics.

Table 2: The effects of a larger population depend on extinction and TFP growth assumptions

	$\lambda < 1$ (& $\delta_A = 0$)	$\lambda = 1$ (& $\delta_A = 0$)	$\lambda > 1$ (or $\delta_A > 0$)
T^X causes extinction (e.g., asteroid)		$\bar{y}_L \geq \bar{y}_S$ $I_L > I_S$	
A^X causes extinction (e.g., advanced A.I.)	$I_L > I_S$	$\bar{y}_L = \bar{y}_S$ $I_L = I_S$	$I_L < I_S$
I^X causes extinction (e.g., rogue actor)	$\bar{y}_L < \bar{y}_S$	$\bar{y}_L = \bar{y}_S$ $I_L = I_S$	$\bar{y}_L > \bar{y}_S$
A decreases risk (e.g., carbon capture)		$\bar{y}_L > \bar{y}_S$ $I_L > I_S$	
$\bar{N}$ causes extinction (e.g., ecological collapse)		$\bar{y}_L(?)\bar{y}_S$ $I_L(?)I_S$	

Notes: The implication of larger populations, L, relative to smaller populations, S on per capita outcomes $\bar{y}_j$, and number of individuals to ever live, I_j, under different modeling assumptions. For example, the top-row indicates that per capita outcomes and the number of people to ever live are both higher when populations are larger ($\bar{y}_L > \bar{y}_S, I_L > I_S$) if extinction is exogenous, regardless of the assumption on idea-generation. The second row indicates that per capita incomes are invariant to population size if A causes extinction, but the relationship between the number of people who ever live and per period population size depends on assumptions about knowledge generation.

erates a positive intensive margin effect of population size (e.g., it rescues the scales effects on individual outcomes). As discussed in Section 3.4, there are cases where the effect on the intensive margin is ambiguous—for example, $\lambda < 1$ and $\delta_A > 0$—which makes a qualitative prediction of their effect infeasible. The left-column can be thought of as cases where the intensive margin is negative; the right-column are cases where the intensive-margin effect is positive.

It is helpful to keep in mind the limiting case where $\lambda = 0$ corresponds to cases where TFP growth is independent of population size. This nests endogenous growth models that succeed in fully eliminating scale effects as well as simple models of exogenous economic growth. Recall that the intensive margin is negative because accelerating the arrival of people-years does not affect the arrival of ideas, so more years are lived with TFP lower than if these life-years came later in time.

Consider first the top row of Table 2, the case of exogenous extinction at some unknown time T^X. Here, larger populations increase both the knowledge base accessible to the average person and the number of people to ever live. The case for $\lambda = 1$ is already discussed in detail in Figure 4. If $\lambda > 1$, this means larger per period populations produce even more ideas than they otherwise would have, giving them a further advantage over the case when $\lambda = 1$. When $\lambda < 1$, the larger population still gets through more people-years prior to T^X, so it still reaches a higher level of TFP at the time of extinction as long as population size has any effect on knowledge accumulation. With $\lambda = 0$ (TFP growth independent of population) then the exact same level of TFP is reached with a fixed T^X constraining humanity. Reaching a higher level will correspond to the average level being higher because I have restricted focus to cases where the population is constant over time for simplicity.

Moving to the second row, this is the case where A^X functionally serves as an upper-bound on TFP. It is detailed for $\lambda = 1$ in Figure 5; it is the case where a sufficiently powerful and dangerous technology, once invented, leads quickly to humanity's extinction. When $\lambda = 1$ the result is that there is no effect on the quantity or quality of lives of larger populations. Economic growth and extinction are directly tied to people-years, so the arrival of each is accelerated in proportion to people-years when the population is increased.

The cases where $\lambda \neq 1$ no longer retain this independence result, but the independence breaks in an unintuitive way. Consider $\lambda > 1$. This is where idea-generation benefits from collaboration, and hence has increasing returns to scale. If the population were to be a bit larger, the increase in idea-generation would be more than one-for-one. This implies that *fewer* people-years are lived when populations are larger, because A^X is reached dispropotionately faster. At the same time, because the same level of TFP is reached, per capita incomes are not improved. Fewer people live with unchanged average life-quality when collaboration effects make people important for research progress, even if all inventions but the final one improve lives. [18] The reverse case works in exactly the opposite manner, and can be easiest to conceptualize for the particular case where TFP growth is unaffected by population growth ($\lambda = 0$). Here, increasing the population size results in more lives being lived prior to reaching A^X; when $\lambda = 0$ the time at which A^X is reached is unaffected by population size. But again, the average level of TFP is unaffected because A^X is a common upper bound. As a result, if something like Jones (2016)'s Russian Roulette model of growth is correct, the case for larger populations is stronger when idea-generation is *less* affected by population size.

In the third row, the causal driver of extinction is directly people-years. One example of this is use of a finite non-renewable resource: only a certain number of people can ever live prior to using up this resource, so the model is closed with an assumption directly on people-years (Greaves, 2019). Alternatively, it could be that every persons-year has some probability of making contact with a novel pathogen that could cause an existential pandemic. When $\lambda = 1$ this is identical to the case that A^X causes extinction because cumulative people-years increase one-for-one with TFP

[18] This particular case has close similarities to the result in Ord (2024a).

levels. However, when $\lambda \neq 1$ the opposite conclusions are generated. If history ends with some fixed number of people-years, then having larger populations lets those fixed number of people collaborate to produce more TFP. In an (unrealistic) limit, having all I individuals live at the same time would maximize the TFP achieved. Conversely, spreading these lives out over time would increase per capita incomes if $\lambda < 1$.[19]

In the fourth row, we have a case where A decreases risk. This works similarly to the case where extinction is exogenous. Partly, that is because some of these cases are enriched versions of purely exogenous risk. If asteroid detection-and-deflection technology can reduce the exogenous risks we face, the case for population size becomes even stronger. So if the dominant effect of technological progress is to reduce the probability of extinction in any given period, achieving these gains through a larger population increases the expected number of people to ever live and their per capita incomes.

Finally, for completeness I include cases where the population size in a given period increases the probability of existential risk. This results in an ambiguous effect on the number of people to live, and therefore the average quality of life because it depends on the elasticity of existential risk with respect to population sizes. The expected number of people to ever live is $\frac{1}{p_x}\bar{N}$. If the relationship is less than unit elasticity, a larger population gets through more life-years and discoveries (making this again analogous to the exogenous extinction case). If the relationship has unit elasticity, the expected number of people to live is invariant to per-period population sizes, which makes this equivalent the case where some level I^X leads to extinction. If the elasticity is more than one, then smaller per period populations lead to more people ever living. Interestingly, this demonstrates that even if ecologists are correct about being near planetary boundaries, for the recommendation to follow that population sizes ought to be shrunk it must be the case the elasticity of extinction risk per-period with respect to population size is greater than one.

It is difficult to know what to take away from this table—and that is precisely the point. It is premature to take confident stances on which of these rows will be the dominant force, or even which column of the idea-generation function is most likely. The results of this subsection demonstrate that the effect of increasing population sizes can depend entirely on these assumptions. Based on the consensus that anthropogenic risks are much more significant than natural risks, and the baseline results that $\lambda = 1$ is a reasonable guess, the single most likely model formulation might be the

[19] Again, suppose $\lambda = 0$, so that idea-generation happens independent to the population size. Then the planner would want those life-years to happen as slowly as possible. In the constant population case, that means the smallest per-period populations.

middle row, middle column. This was the case of Figure 5, where the per-period population size has no affect on the quality or quantity of life-years lived.

5 Conclusion

The question of whether a declining population will lead to counterfactual losses in living standards is complicated. Alongside the production of non-rival ideas, there are a host of issues that may depend on the size or age structure of the population: environmental pressures, human capital investments, pension financing, business formation decisions, etc. However, given the seeming importance of non-rival ideas in explaining past economic growth, this channel has received substantial attention among economists.

This paper shows this may be a mistake. The joint-distribution of people and the stock of ideas available during their lifetime is invariant to population size, at least without assuming further, second-order forces beyond the non-rivalry of ideas. The population size and its effect on the speed at which non-rival ideas are generated may be important for explaining rates of economic growth, but I show there is not a strong reason to believe this makes lives any better. Indeed, I show that our intuitions can be completely reversed under plausible parameter values: if idea-creation within a period has diminishing returns to researchers, each life is worsened by the population size being larger, even in a model where non-rival ideas are the only relevant force.

Going beyond this negative result, the key consideration turns out to be whether a larger population allows more people to ever live, over the whole of human history. If so, and if population size has any effect on non-rival idea creation, larger timeless populations raise total and average utility experienced over the whole of history. However, this points to the critical nature of assumptions about existential threats. When the variable that matters is the expected number of people to ever live, it is not surprising that model results turn on the assumption about what constrains the number of people who will ever live.

Unsurprisingly, the importance of extinction assumptions makes it difficult to be confident in the takeaways. So, one conclusion of this paper is that economists interested in the effects of population dynamics and economic growth must grapple with the issue of extinction. Based on available theories and commonly assumed parameters, I conclude (for now) that larger per period populations increase the expected number of people to ever live, raising the total and average welfare experienced by people.

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Appendix

A Solution to the Model of Endogenous Research Effort

Aggregate output is a CES aggregator of intermediate goods, each with the respective production functions, where l_j is the labor alloccted to sector j.

$$
\begin{aligned}
Y =& \Big[\int_0^A y_j^{\frac{\varepsilon-1}{\varepsilon}} dj \Big]^{\frac{\varepsilon}{\varepsilon-1}} \\
y_j =& l_j
\end{aligned}
$$

A is the measure of ideas in this economy, endogenously produced by potential entrepreneurs. Denoting the share of people in this economy who search for ideas s, we have a production function of A that is:

$$A = \omega \times s \times N.$$

That is, each individual has some probability ω of discovering a profitable idea. I assume that each worker inelastically supplies one unit of labor, so that the market clearing condition dictates that the sum of wage earners plus entrepenuers is the total population, N.

Let us first focus on each intermediate firm's production decision, conditional on operating (that is, $j \in [0, A]$). The demand curve for each y_j can be solved through a standard cost-minimization problem by the final good firm that aggregates these products.

$$\min_{y_j} \int_0^A y_j p_j \text{ such that } \Big[\int_0^A y_j^{\frac{\varepsilon-1}{\varepsilon}} dj \Big]^{\frac{\varepsilon}{\varepsilon-1}} \geq Y \tag{21}$$

This delivers the demand curve:

$$y_j = \left(\frac{p_j}{P}\right)^{-\varepsilon} Y, \tag{22}$$

where I have defined P, the aggergate price level, as the Lagrange multiplier.[20]

Each intermediate producer, j, faces a problem of whether to purchase a patent at price p_A, what price to charge for its good, p_j, and how much of its good to produce with a linear production function, $y_j = l_j$, where l_j is labor in sector j that gets paid a wage w that is taken as given by each small firm. I assume a free-entry condition, such that the fixed cost to operate, p_A, will be equal to

[20] This is because the lagrange multiplier captures the cost of having the constraint tightened, on the margin. Here, the constraint is producing one more unit of the final good. The marginal cost of producing Y will equal the price of Y in a competitive equilibrium.

the profit this monopolistically competitive firm can earn in equilibrium. The problem for a firm that decides to purchase a patent and operate looks as follows (if the maximum profit that can be attained is less than zero, the firm will not operate).

$$\max_{p_j, y_j, l_j} p_j y_j - w l_j - p_A \tag{23}$$

subject to:

$$y_j = \left(\frac{p_i}{P}\right)^{-\varepsilon} Y \tag{24}$$

$$y_j = l_j \tag{25}$$

Subbing these constraints directly into the problem and taking a first-order condition, we obtain the usual result that CES monopolistic competition delivers a price that is a constant mark-up over marginal costs (here, the wage). This implies a value for p_A that set profits to zero, by the free entry condition.

$$p_j = -\frac{\varepsilon}{1-\varepsilon} w \tag{26}$$

$$p_A = (-\frac{\varepsilon}{1-\varepsilon} - 1) w y_j \tag{27}$$

Also note that $\omega p_A = w$ in equilibrium, as the expected value of being an entrepreneur must be equal to the wage. We can use this fact to substitute w from the right hand side of (27), and simplify the expression.

$$p_A = \left(\frac{1}{\varepsilon - 1}\right) \omega p_A y_j \Rightarrow$$

$$\frac{(\varepsilon - 1)}{\omega} = y_j$$

Finally, notice that in a symmetric equilibrium each y_j will be identical. Furthermore, each $y_j = l_j$, the amount of labor supplied in that industry. Recalling that the share of the population earning wages is $(1-s)$, this implies that there are $(1-s)N$ total workers split across the A industries. So, each $l_j = \frac{(1-s)N}{A}$. However, we also know in equilibrium that $A = \omega \times s \times N$. This lets us sub in our new expression for l_j for y_j. Simplifying, we get:

$$\frac{(\varepsilon - 1)}{\omega} = \frac{(1-s)N}{\omega s N} \Rightarrow$$
$$\boxed{(\varepsilon - 1) = \frac{1-s}{s}} \tag{28}$$

First, verify that the intuition of this result makes sense. As $\varepsilon \to 1$, $s \to 1$. This is the case of perfect complements: the love-of-variety is so extreme that nearly everyone is employed in research to expand the line of product varieties. When $\varepsilon \to \infty$, $s \to 0$. This is the case of perfect substitutes. Consumers do not care about which products they consume, so everyone is employed in the production sector rather than expanding the variety of products.

But what is most important for the application here is that this function is independent of N. It is not the case that a larger market endogenously allocates a larger share of the population to research. So, the assumption employed throughout the paper that s is independent of N seems to be, at least, a good baseline assumption.