1 Sensitivity

In this section we discuss the sensitivity of POL-2 at 850μm for the Stokes Q and U maps and for the polarized intensity map, $PI = \sqrt{Q^2 + U^2}$. The noise sensitivity of the POL-2 maps has been analysed in the same way as was done for SCUBA-2. We looked at observations with point sources at the centre of the map, and

1. calculated a noise across the central circle of radius $3'$ for each map;
2. calculated the average exposure time per pixel across the central circle of $3'$ radius for each map;
3. used these results to derive an $NEFD$ (Noise Equivalent Flux Density) to transmission relation across all the maps;
4. used the average exposure time to derive an empirical relation between the exposure time in the central map area and the elapsed time of the map;
5. combined these relationships to get the empirical relationship between the elapsed time of an observation and its RMS, given a specific transmission.

These relationships have been incorporated into the SCUBA-2 ITC in Hedwig. The POL-2 calculations may be accessed by selecting **maptype = POL-2 Daisy** in the ITC.

The sensitivity analysis was conducted using observations of 3C273, 3C279, 3C84, CB68, and Uranus. A total of 82 850μm observations were analysed, producing a Q and U maps for each observation, which then were used to generate un-IP-corrected and un-de-biased PI maps.

1.1 Methods of measuring Noise

We use two methods to estimate the noise in each map:

1. **Variance map method:** Measure the standard deviation where $RMS = \sqrt{\sigma^2}$, where σ^2 a central circle of radius $3'$ of the variance map.
2. **Emission map method:** Measure the standard deviation in a circle of $3'$ radius from the emission map, while excluding a central circle of $30''$ radius (quasars) or $40''$ radius (Uranus) in order to mask out the point source at the centre of each map.

NOTE: We used a larger mask for Uranus since it is a much brighter source than the quasars. We used KAPPA `ardmask` to generate cutouts and mask out sources.

1.2 Sensitivity for point sources & blank regions

The $NEFD$ is calculated as:

$$NEFD = \sigma \sqrt{t_{\text{exposure}}} \quad (1)$$

where σ is the RMS noise for a given map area, and t_{exposure} is the exposure time for that same map area. We measure the uncertainty of the $NEFD$ by propagating the uncertainty in the RMSa and exposure time:

$$\begin{aligned}\delta NEFD &= \sqrt{\left(\frac{dNEFD}{d\sigma}\delta\sigma\right)^2 + \left(\frac{dNEFD}{dt_{\text{exposure}}}\delta t_{\text{exposure}}\right)^2} \\ &= NEFD\sqrt{\frac{1}{\sigma^2}\left(\delta\sigma\right)^2 + \frac{1}{4t^2_{\text{exposure}}}\left(\delta t_{\text{exposure}}\right)^2}\end{aligned} \tag{2}$$

The $NEFD$ can then be empirically related to the transmission at the time of observation. The figures below show this relation at 850μm for both noise methods for both the Q and U maps and the PI maps.

Upon fitting the $NEFD$s to all 82 850μm observations we find the equations relating the $NEFD$s to the 850μm transmission for both noise methods and for Stokes Q and U and PI maps are:

$$\begin{aligned}NEFD_{PI}^{\text{variance map}} &= \frac{226 \pm 21}{T_{850}} + (-58 \pm 31)\ \text{mJy} \cdot \sqrt{\text{s}} \\ NEFD_{PI}^{\text{emission map}} &= \frac{194 \pm 11}{T_{850}} + (12 \pm 17)\ \text{mJy} \cdot \sqrt{\text{s}} \\ NEFD_{Q\&U}^{\text{variance map}} &= \frac{304 \pm 14}{T_{850}} + (-3 \pm 21)\ \text{mJy} \cdot \sqrt{\text{s}} \\ NEFD_{Q\&U}^{\text{emission map}} &= \frac{298 \pm 13}{T_{850}} + (3 \pm 19)\ \text{mJy} \cdot \sqrt{\text{s}}\end{aligned} \tag{3}$$

The Variance map method and the Emission map method give nearly equivalent, reliable, and precise $NEFD$s for the Stokes Q and U maps. The uncertainties in the Stokes Q and U maps are what drive the uncertainties on the polarization angles. As a result one should only rely on the $NEFD$s derived to the Stokes Q and U maps.

We analysed the $NEFD$s derived from the PI maps in order to compare the noise levels. We know the noise on the signal in PI maps should be roughly the same as the noise in their Q and U maps. However, Rice Distributions tell us we'd expect to see $\sim$50% lower noise on a blank part of a PI map compared to their Q and U maps – which is exactly what we find. The noise on the signal in PI maps cannot be found by measuring the noise on a blank part of sky in PI maps.

We find that the Emission map method is a more reliable method for obtaining an $NEFD$ for PI maps. We find that the Variance map method results in a larger spread of data and therefore a larger uncertainty in the $NEFD$ equation for PI maps. Namely, the Variance map method known to systematically underestimate the noise for a bright source (e.g. Uranus) **reference**.

We find that CB68 does not behave well on the $NEFD$ curve. CB68 is embedded in a large scale, and somewhat faint, structure that covers a large part of POL-2 field of view. This may complicate the reduction of single observations for makemap, therefore the noise we measure fluctuates more from map to map.

A relationship between the average exposure time per 4″ pixel in a 3′ radius central circle of our reduced POL-2 850μm maps and the elapsed time of the each observation was derived in the form:

$$t_{\text{elapsed}} = \frac{t_{\text{exposure}}}{const} \tag{4}$$

giving:

$$
\begin{aligned}
t_{\text{elapsed}}^{\text{variance map}} &= \frac{t_{\text{exposure}}}{0.171 \pm 0.014} \\
t_{\text{elapsed}}^{\text{emission map}} &= \frac{t_{\text{exposure}}}{0.170 \pm 0.015}
\end{aligned}
\tag{5}
$$

In the figure bellow we present the difference between the actual elapsed time and the predicted elapsed time using the equations above for 850μm data:

Using the definition of $NEFD$, the empirical relation between $NEFD$ and transmission, and the empirical relation between elapsed time and exposure time, it is possible, for a given transmission, to predict the elapsed time required to yield a desired noise value across the same region of the POL-2 observation and at the same resolution (4″):

$$
t_{\text{elapsed}} = \frac{1}{const} \left(\frac{NEFD}{\sigma} \right)^2 \tag{6}
$$

for Stokes Q and U giving:

$$
\begin{aligned}
t_{\text{elapsed}}^{\text{variance map}} &= \frac{1}{0.171 \pm 0.014} \left[\left(\frac{304 \pm 14}{T_{850}} + (-3 \pm 21) \right) \frac{1}{\sigma_{\text{mJy}}} \right]^2 \\
t_{\text{elapsed}}^{\text{emission map}} &= \frac{1}{0.170 \pm 0.015} \left[\left(\frac{298 \pm 13}{T_{850}} + (3 \pm 19) \right) \frac{1}{\sigma_{\text{mJy}}} \right]^2
\end{aligned}
\tag{7}
$$

We may also relate the elapsed telescope time to a given 1-σ (mJy) depth for Stokes Q and U maps as:

$$
\begin{aligned}
t_{\text{elapsed}}^{\text{variance map}} &= \frac{1}{f} \left[\left(\frac{747 \pm 34}{T_{850}} + (-7 \pm 52) \right) \frac{1}{\sigma_{\text{mJy}}} \right]^2 \\
t_{\text{elapsed}}^{\text{emission map}} &= \frac{1}{f} \left[\left(\frac{732 \pm 32}{T_{850}} + (17 \pm 47) \right) \frac{1}{\sigma_{\text{mJy}}} \right]^2
\end{aligned}
\tag{8}
$$

where f is the sampling factor, taking into account the map pixel size relative to the defaults. It is defined as:

$$
f = (\text{output map pixel size requested/default pixel size})^2 \tag{9}
$$

At 850μm this becomes:

$$
f_{850} = (\text{output map pixel size requested}/4'')^2 \tag{10}
$$

These relationships have been incorporated into the SCUBA-2 ITC in Hedwig. The POL-2 calculations are available if **maptype = POL-2 Daisy** is selected.

2 Summary

Over a set of 82 850μm POL-2 observations we compute the noise sensitivity of POL-2 maps with two methods of noise estimates, 1. Variance map method and 2. Emission map method. We find both methods give reasonable and precise $NEFD$s for the Stokes Q and U maps (Equation 3). We are able to predict the elapsed time required to yield a desired noise value for the central 3′ radius of a POL-2 map (Equation 7).

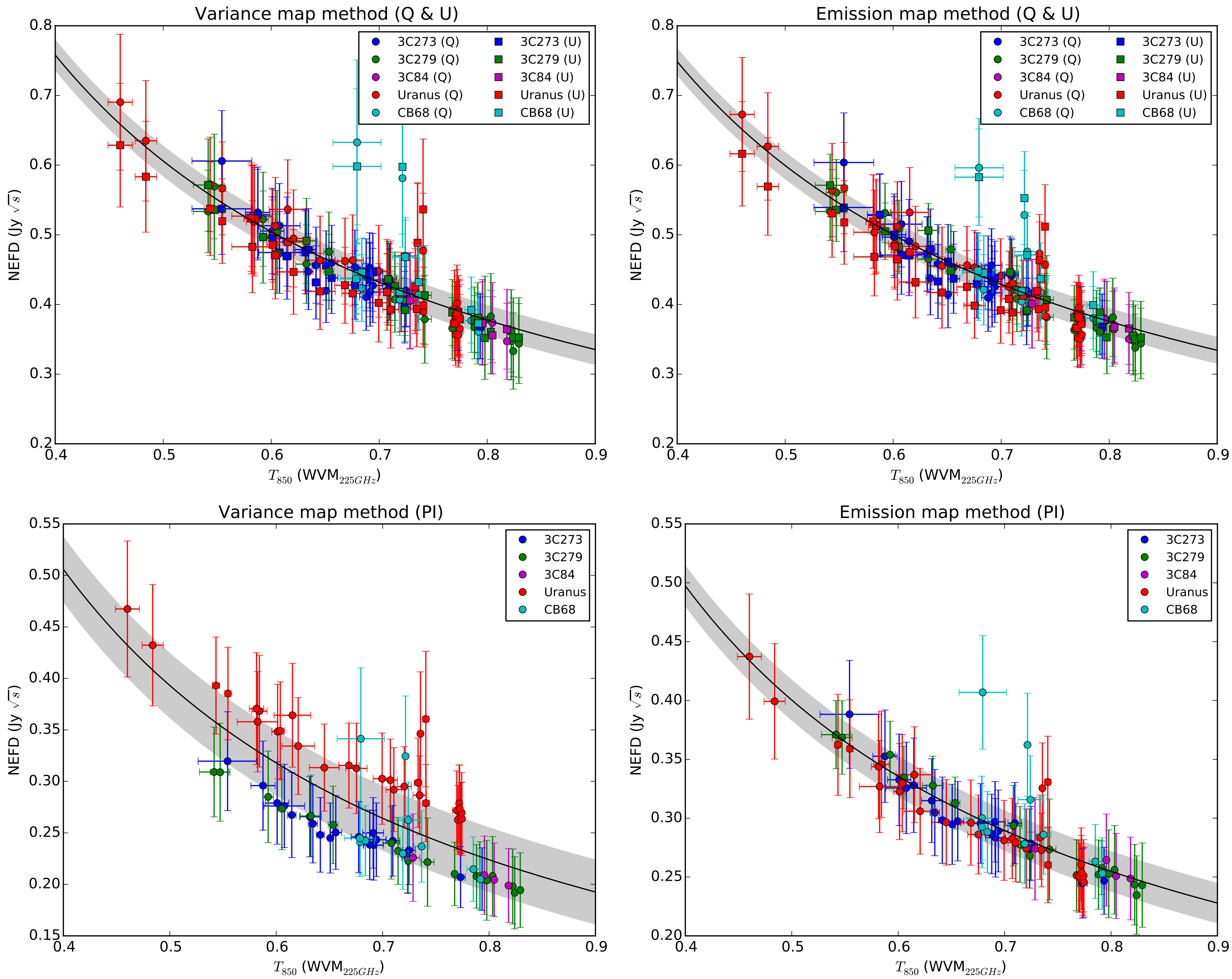

Figure 1: POL-2 $NEFD$s calculated from the noise and exposure time in 82 850μm maps, and plotted against the transmission at 850μm for each observation. **Top:** Stokes Q and U maps. **Bottom:** PI maps. **Left:** The noise and exposure time were calculated using the Variance map method. The vertical error is estimated by using Equation 2. **Right:** The noise and exposure time were calculated using the Emission map method. The vertical error is estimated by using Equation 2 where $\delta\sigma = 0$. For each method the horizontal error is estimated by the change in transmission throughout the observation. The black curves are the best fit to the data and are shown in Equation 3, and the grey track represents a $\pm 1\sigma$ uncertainty in the fit.

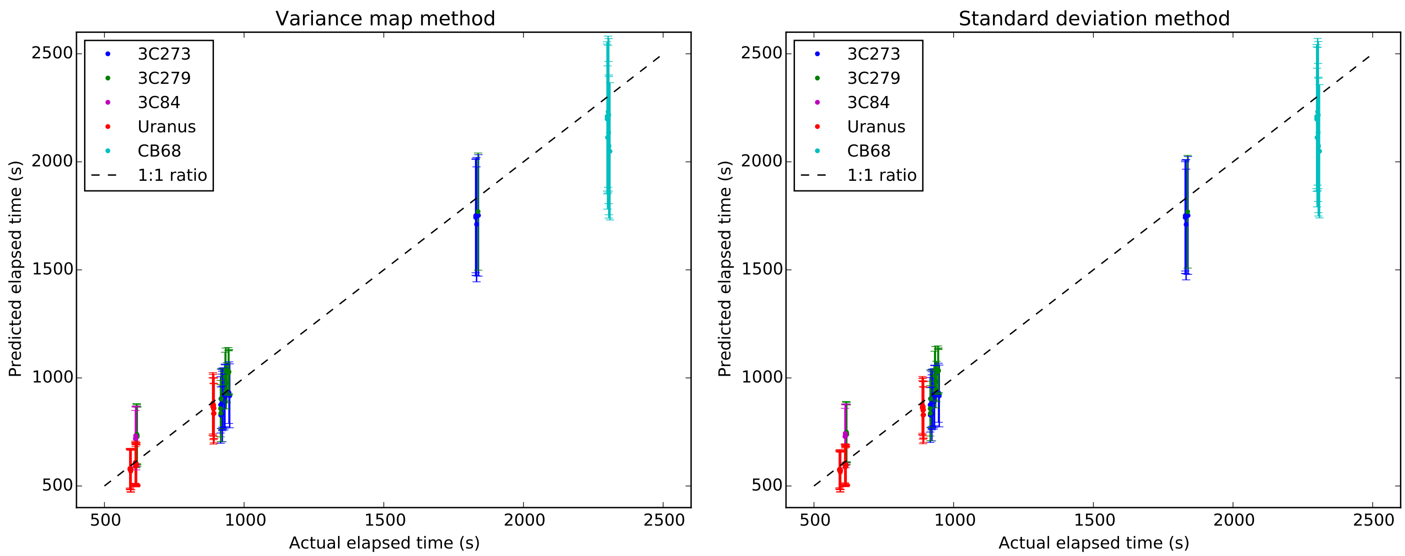

Figure 2: The measured elapsed time from 82 POL-2 850μm observations, compared with the predicted elapsed time using Equation 5. **Left:** The noise and exposure time were calculated using the Variance map method. **Right:** The noise and exposure time were calculated using the Emission map method. For each method the vertical error is estimated by the variance in average exposure time within a central circle of $3'$ radius.