Chapter 8 Discussion
2D Dynamics

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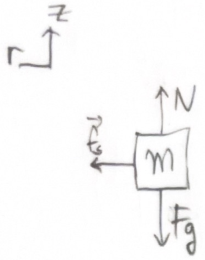
1. A coin of mass m is placed a distance r from the center of a turntable. The coefficient of static friction between the coin and the turntable is μ_s. Starting from rest, the turntable is gradually rotated faster and faster. At what angular velocity ω does the coin slip and fly off?



$$\Sigma F_z = N - mg = ma_z = 0$$

$$\Sigma F_r = f_s = ma_r = m\left(\frac{v^2}{R}\right) = m(w^2 R)$$

$$f_s \le \mu_s \cdot N \rightarrow m(w^2R) \le \mu_s \cdot mg$$

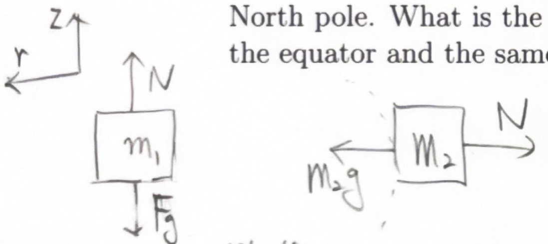
$$\rightarrow (w^2 R) \le \mu_s \cdot g$$

$$w^2 \le \frac{\mu_s \cdot g}{R}$$

$$w \le \sqrt{\frac{\mu_s \cdot g}{R}}$$

$$\boxed{\therefore \sqrt{\frac{\mu_s \cdot g}{R}}}$$

2. Assuming the Earth is a perfect rotating sphere, determine the apparent weight (ie, the normal force a scale would read) of a person of mass 70.0 kg at the equator and at the North pole. What is the percent difference between the apparent weight of a person on the equator and the same person at one of the poles?



$$\Sigma F_{z m_1} = N_1 - m_1 g = m_1 a_z = 0, \quad N = (70.0\,kg)(9.8\,m/s^2), \quad N_1 = 686N$$

$$\Sigma F_{r m_2} = m_2 g - N_2 = m a_r = m_2\left(\frac{v^2}{R}\right) = m_2(w^2R)$$

$$\%\Delta = \frac{|N_1 - N_2|}{\frac{1}{2}(N_1+N_2)} = \frac{686 - 683.64}{\frac{1}{2}(686+683.64)} = 3.446\times10^{-3}$$

$$w = \frac{1\,rev}{24\,hours} \times \left(\frac{2\pi\,rad}{1\,rev}\right) \times \left(\frac{1\,hour}{3600\,sec}\right) = \frac{\pi}{43200}\,rad/s = \frac{1}{86400}\,rad/s$$

$$N_2 = m_2 g - m_2(w^2R) = m_2(g - w^2R) = \boxed{683.64}$$

$\therefore$ North pole : 686N
equator : 683.64N
0.3%

3. The design of a new road includes a straight stretch that is horizontal and flat but that suddenly dips down a steep hill at 18°. The transition should be rounded with what minimum radius so that cars traveling 95 km/hr will not leave the road? *Hint*: Right when the car leaves the road, the normal force becomes zero.

$$F_r = mg\cos\theta - N = m\left(\frac{v^2}{r}\right)$$

$$N = mg\cos\theta - m\left(\frac{v^2}{r}\right)$$

(=0 @ contact loss)

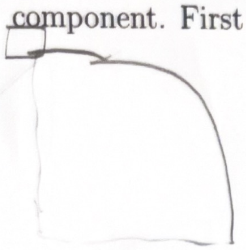
$$mg\cos 18^\circ = m\left(\frac{\left(\frac{475}{18}\right)^2}{r}\right)$$

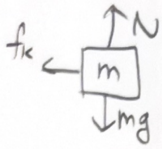
$$\frac{95\,km}{1\,hr} \times \frac{1\,hr}{3600\,s} \times \frac{1000\,m}{1\,km} = \frac{475}{18}\,m/s$$

$$r = \frac{\left(\frac{475}{18}\right)^2}{g\cos 18^\circ} = 74.72\,m$$

$$\boxed{\therefore 74.72\,m}$$

4. A car traveling on a flat (unbanked), circular track accelerates uniformly from rest with a tangential acceleration of a. The car makes it one-quarter of the way around the circle before it skids off the track. From these data, determine the coefficient of static friction between the car and the track in terms of a. *Hint*: Friction provides the only horizontal force, but the total acceleration will include both a tangential and centripetal component. First find the centripetal acceleration after traveling through 1/4 circle.





$$\Sigma F_y = N - mg = 0$$

$$\Sigma F_r = f_k = ma_c = m\frac{v^2}{R}$$

$$\mu_k mg = m$$

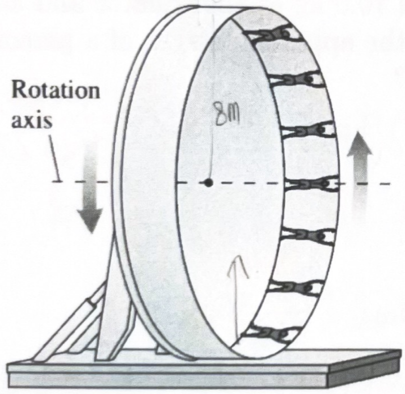
$$F_t = ma$$

$$\mu_s mg = \sqrt{F_t^2 + F_c^2} = \sqrt{(ma)^2 + \left(m\frac{v^2}{R}\right)^2}$$

$$\sqrt{m^2\left(a^2 + \frac{v^4}{R^2}\right)}$$

$$\boxed{\mu_s = \frac{\sqrt{a^2 + \frac{v^4}{R^2}}}{g}}$$

5. In an amusement park ride, passengers stand inside a 16 m diameter rotating ring. After the ring has acquired sufficient speed, it tilts into a vertical plane, as shown below.



$$N_{Top} = m(\omega^2 R) - mg = 318.8\,N$$

$$N_{Bottom} = mg + m(\omega^2 R) = 1396.8\,N$$

(a) Suppose the ring rotates once every 4.5 s. If a rider's mass is 55 kg, with how much force does the ring push on her at the top of the ride? At the bottom?

$$\omega = \frac{1\,rev}{4.5\,sec} \times \frac{2\pi\,rad}{1\,rev} = \frac{4}{9}\pi\,rad/sec$$

Bottom ($r\uparrow$): N, m, mg

$$\Sigma F_r = N - mg = ma_r = m(\omega^2 R)$$

$$N = mg + m(\omega^2 R)$$

Top ($\downarrow r$): m, N, mg

$$\Sigma F_r = N + mg = ma_r = m(\omega^2 R)$$

$$N = m(\omega^2 R) - mg$$

$$\boxed{\therefore\ N_{Top} = 318.8\,N \quad N_{Bottom} = 1396.8\,N}$$

(b) What is the longest rotation period of the wheel that will prevent the riders from falling off at the top?

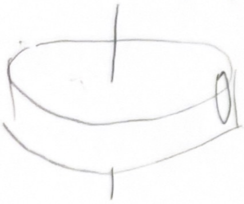
$$\boxed{\therefore\ 1.1068\,s}$$

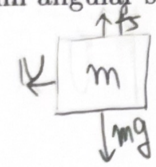
$$m(\omega^2 R) = mg$$

$$\omega = \sqrt{\frac{g}{R}} = \sqrt{\frac{9.8\,m/s^2}{8\,m}} = 1.1068\,s$$

6. In an amusement park ride, passengers stand inside a 5.0 m diameter hollow steel cylinder with their backs against the wall. The cylinder begins to rotate about a vertical axis. Then the floor on which the passengers are standing suddenly drops away! If all goes well, the passengers will "stick" to the wall and not slide. Clothing has a static coefficient of friction against steel in the range 0.60 to 1.0 and a kinetic coefficient in the range 0.40 to 0.70. A sign next to the entrance says "No children under 30 kg allowed." What is the minimum angular speed, in rpm, for which the ride is safe?

$\boxed{1.4 \text{ rpm}}$





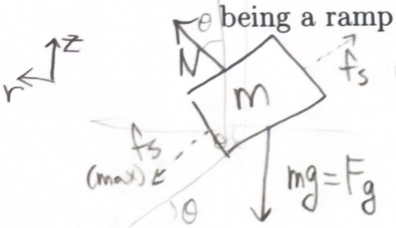
$\Sigma F_z = f_s - mg = 0$

$\Sigma F_r = N = m a_c = m\omega^2 R$

$\mu_s (m)\omega^2 R = mg \qquad \omega^2 = \frac{g}{R\mu_s} = \frac{9.8}{(5)(1.0)} = 1.96$

$\mu_s \omega^2 R = g$

7. A car rounds a curve of radius 50 m and bank angle 15°. The coefficient of static friction between the tires and the road is 0.80. What are the maximum and minimum speeds that the car can take around the curve so as not to slip against the road? *Hint:* Despite being a ramp problem, you should orient the coordinate axes in the usual way (why?)



$\Sigma F_z = N\cos\theta + f_s \cdot \sin\theta - F_g = ma_z = 0$

$\Sigma F_r = N\sin\theta - f_s\cos\theta = ma_r = m\left(\frac{v^2}{r}\right)$

$\left(N\cos\theta + f_s\sin\theta = mg\right) \times \cos$

$+\left(N\sin\theta - f_s\cos\theta = m\left(\frac{v^2}{r}\right)\right) \times \sin$

$0 + N = mg\cos\theta + m\left(\frac{v^2}{r}\right)\sin\theta$

$\left(N\cos\theta + f_s\cdot\sin\theta = mg\right) \times \sin\theta$

$+\left(N\sin\theta - f_s\cdot\cos\theta = m\left(\frac{v^2}{r}\right)\right) \times -\cos\theta$

$f_s\cdot\sin^2\theta + f_s\cdot\cos^2\theta$

$f_s = mg\sin\theta - m\left(\frac{v^2}{r}\right)\cos\theta$

$f_s \le \mu_s \cdot N$

$mg\sin\theta - m\left(\frac{v^2}{r}\right)(\cos\theta) \le \mu_s\left(mg\cos\theta + m\left(\frac{v^2}{r}\right)\sin\theta\right)$

$g\sin\theta - \mu_s g\cos\theta \le m\left(\frac{v^2}{r}\right)\sin\theta + m\left(\frac{v^2}{r}\right)\cos\theta$

$g\sin\theta - \mu_s g\cos\theta \le \left(\frac{v^2}{r}\right)\sin\theta + \left(\frac{v^2}{r}\right)\cos\theta$

$\frac{r(g\sin\theta - \mu_s g\cos\theta)}{\mu\sin\theta + \cos\theta} \ge v^2$ (min)

$\frac{r(g\sin\theta + \mu_s g\cos\theta)}{\cos\theta - \mu\sin\theta} \le v^2$ (max)