Chapter 0.4b Discussion
Math Review (Calculus)

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Complete only the 3 question group corresponding to the level of mathematics you are currently taking (e.g., if you are now enrolled in MATH-192, complete group A). If you are not currently enrolled in one of these math classes, complete the 3 question group of the most recently taken class.

Group A - MATH-192 (Calculus I)

1. Cars 1 and 2 travel in a straight line and both start from the origin $x_0 = 0$. The positions of the two cars are given by the functions

$$x_1(t) = \alpha_1 t + \beta_1 t^2, \quad x_2(t) = \beta_2 t^2 - \gamma_2 t^3,$$

where

$$\alpha_1 = 2.60 \text{ m/s}, \quad \beta_1 = 1.20 \text{ m/s}^2, \quad \beta_2 = 2.80 \text{ m/s}^2, \quad \gamma_2 = 0.20 \text{ m/s}^3$$

are constants.

(a) At what time(s) are the cars at the same point?

(b) At what time(s) are the cars moving with the same velocity?

(c) At what time(s) do the cars have the same acceleration?

$\frac{d}{dx} \ln x = \frac{1}{x}$

$\frac{d}{dx} \ln x =$

2. An imperfect spring requires a force of magnitude

$$F_s = kx + \beta x^3$$

to compress or stretch from equilibrium by a distance x, where $k = 100$ N/m and $\beta = 12$ N/m^3. If the spring is compressed 2.0 m, what speed will it give to a 3.5 kg ball that is held against it and then released?

3. A particle of mass m at rest is subjected to a force $F(t) = F_0 \sin(\omega t)$ from time $t = 0$ to $t = \pi/\omega$, where F_0 and ω are constants. Find the speed of the particle at time $t = \pi/\omega$.

Group B - MATH-193 (Calculus II)

4. During a training exercise, the position of a helicopter of mass $m = 2.8 \times 10^4$ kg is given by

$$\boldsymbol{r}(t) = \alpha t^3 \hat{x} + \beta t \hat{y} - \gamma t^2 \hat{z},$$

where

$$\alpha = 0.020 \text{ m/s}^3, \quad \beta = 2.2 \text{ m/s}, \quad \gamma = 0.060 \text{ m/s}^2$$

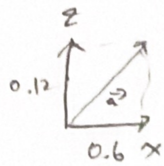
are constants. Find the net force (vector) on the helicopter at $t = 5.0$ s.

$r(t) = (0.02)t^3\hat{x} + 2.2t\,\hat{y} - (0.06)t^2\,\hat{z}$

$v(t) = (0.06)t^2\hat{x} + 2.2\hat{y} - 0.12t\,\hat{z}$

$a(t) = 0.12t\,\hat{x} - 0.12\,\hat{z}$

$a(5.0) = 0.6\hat{x} - 0.12\hat{z}$



$\vec{a} = \sqrt{(0.6)^2 + (0.12)^2} = 0.612 \text{ m/s}^2$

$\Sigma F = ma = (2.8\times10^4)\times(0.612) = \boxed{17136 \text{ N}}$

5. A block of mass m is at rest at the origin at $t = 0$. It is then pushed with a constant force F_0 from $x = 0$ to $x = L$ across a horizontal surface whose coefficient of kinetic friction is $\mu_k = \mu_0(1 - x/L)$ (the roughness decreases from μ_0 at the origin to zero at $x = L$).

(a) Use the chain rule to show that

$$a_x = v_x \frac{dv_x}{dx}.$$

derivative of x in terms of time is velocity : v

So, $\frac{dx}{dt} = v$

$v = v_0 + at$

$\frac{dv}{dt} = a$ → Chain rule : $\boxed{\underbrace{\frac{dv}{dx}}_{\frac{dv}{dx}} \cdot \underbrace{\frac{dx}{dt}}_{v} = \underbrace{\frac{dv}{dt}}_{a}}$

(b) Find the speed of the block when it reaches $x = L$.

$F_{net} = F_0 - \mu_k mg$

$F_{net} = F_0 - \mu_0(1 - x/L)mg = ma$

$m\left(v\frac{dv}{dx}\right) = F_0 - \mu_0\left(1 - \frac{x}{L}\right)mg$

$\int_0^v m(v)\,dv = \int_0^L \left(F_0 - \mu_0\left(1 - \frac{x}{L}\right)mg\right)dx$

$\frac{mv^2}{2} = F_0 x - \mu_0\left(x - \frac{x^2}{2L}\right)mg \Big|_0^L$

$= F_0 L - \mu_0\left(L - \frac{L^2}{2L}\right)mg$

$= F_0 L - \mu_0\left(\frac{L}{2}\right)mg$

$\cancel{m}v^2 = \frac{2F_0L}{m} - \mu_0 L \cancel{m}g$

$\boxed{\therefore v = \sqrt{\frac{2F_0L}{m} - \mu_0 L g}}$

6. A disk of radius R and thickness w has a mass density that increases from the center outward, given by $\rho = \rho_0 r/R$, where r is the distance from the disk axis.

   (a) Calculate the disk's total mass M.

$\rho = \rho_0 r/R$

$$\boxed{\therefore \frac{2\pi\rho_0 w R^2}{3} = M}$$

$\frac{dm}{dV} = \rho$, $dm = \rho dV$

$dm = (\rho_0 r/R)(2\pi r\,dr)w$

$$M = \int dm = \int (\rho_0 r/R)(2\pi r\,dr)w = \frac{2\pi\rho_0 w}{R}\int_0^R r^2\,dr = \frac{2\pi\rho_0 w}{R} \times \left[\frac{r^3}{3}\right]_0^R$$

   (b) Calculate its rotational inertia about its axis in terms of M and R. Compare with the results for a solid disk of uniform density and for a ring.

$$I = \int_0^R r^2\,dm$$

$$= \int_0^R r^2 \left(\frac{\rho_0 r}{R}\right)(2\pi r\,dr)w$$

$$= \frac{2\pi\rho_0 w}{R}\int_0^R r^4\,dr$$

$$= \frac{2\pi\rho_0 w}{R}\left[\frac{r^5}{5}\right]_0^R$$

$$\boxed{= \frac{2\pi\rho_0 w R^4}{5}}$$