Chapter 0.2 Discussion
Estimation, Units, Dimensional Analysis Name: Kevin Lee

1. In Wagner's opera *Das Rheingold*, the goddess Freia is ransomed for a pile of gold just tall enough and wide enough to hide her from sight. Estimate the monetary value of this pile.

$\rho_{Gold} = 10g/cm^3$, Cost : \$10/1g

density of pile : $10g \times 2\times10^5 = 2\times10^6$

monetary value of pile : $\boxed{\$2\times10^7}$

$V = 200\times100\times10\ cm^3$
$= 2\times10^5\ cm^3$

2. How many times does a typical person blink her eyes in a lifetime?

1blink/10s ⇒ 6blink/1min → 6×60/1hr ⇒ $6\times60\times24$/1Day → $365\times6\times60\times24$/1year

Average year of the person lifetime is 100 years, $365\times6\times60\times24\times100$ S/100year

$\boxed{\therefore 3.15\times10^8 \text{ times}}$

3. Given the quantities $a = 9.7$ m, $b = 4.2$ s, $c = 69$ m/s, what is the value of the quantity $d = a^3/(cb^2)$?

$$d = \frac{(9.7m)^3}{(69m/s)(4.2S)^2} = \frac{912.673\,m^3}{(69m)((4.2)^2S)} = \frac{912.673\,m^3}{1217.165}$$

$\boxed{\therefore 0.75\,m^2/s}$

4. At a resting pulse rate of 75 beats per minute, the human heart typically pumps about 70 mL of blood per beat. Blood has a density of 1060 kg/m^3. Circulating all of the blood in the body through the heart takes about 1 min in a person at rest.

(a) How much blood (in L and m^3), is in the body? 5.25L, $5.25\times10^{-3}m^3$

(b) On average, what mass of blood (in g and kg) does the heart pump each beat? 742×10^{-4} kg, 742×10^{-1}g

75 beats/1min

blood in body in 1min = 75×70 = 5250 mL
= 5.25 L

pumps : 70mL/1beat

ρ_{blood} : 1060kg/m^3

$1000L = 1m^3$
$1L = 10^{-3}m^3$
$5.25L = \underline{5.25\times10^{-3}m^3}$

1 beat pumps 70mL of blood

70mL ⇒ 0.7×10^{-1} L ⇒ $0.7\times10^{-4}m^3$

So mass = 70 mL × 1060 kg/m^3
= $0.7\times10^{-4}m^3 \times 1060kg/m^3$ = 742×10^{-4} kg

5. The acceleration of a falling object near a planet is given by the following equation: $g = GM/R^2$. If the planet's mass M is expressed in kg and the distance of the object to the planet's center R is expressed in meters, determine the units of the gravitational constant G. The acceleration g must have units of $\mathrm{m/s^2}$.

$\boxed{\therefore [G] \Rightarrow \mathrm{m^3/kg\cdot s^2}}$

$g = GM/R^2$, $M \Rightarrow \mathrm{kg}$

$R \Rightarrow \mathrm{m}$

acceleration $g \Rightarrow \mathrm{m/s^2}$

$$\frac{m}{s^2} = \square \times \frac{kg}{1} \times \frac{1}{m^2}$$

$$\square = \frac{m}{s^2} \times \frac{1}{kg} \times \frac{m^2}{1} = \frac{m^3}{kg \cdot s^2}$$

6. The air bubble formed by an explosion underwater undergoes oscillations with time period T, which depends on pressure p, density ρ, and on the energy of the explosion E. Establish a relation between T, p, E, and ρ.

$$\boxed{\therefore T \propto \frac{1}{p} \propto \rho \propto E}$$

(propotional to)

$[T] = T$

$[p] = \frac{ML}{T^2}/L^2$

$[\rho] = \frac{M}{L^3}$

$[E] = M\frac{L^2}{T^2}$

$$T = k p^x \rho^y E^z$$

$$T = \left(\frac{M}{LT^2}\right)^x \left(\frac{M}{L^3}\right)^y \left(\frac{ML^2}{T^2}\right)^z$$

$M^0 = M^{x+y+z} \dashrightarrow x+y+z=0$

$L^0 = L^{-x-3y+2z} \dashrightarrow -x-3y+2z=0$

$T^1 = T^{-2x-2z} \dashrightarrow -2x-2z=1$

$3x+3y+3z=0$

$+\ -x-3y+2z=0$

$2x \quad +5z=0$

$2x+5z=0$

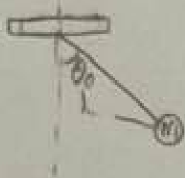
$+\ -2x-2z=1$

$3z=1$

$$\boxed{x = -\frac{5}{6},\ y = \frac{1}{2},\ z = \frac{1}{3}}$$

$-2x - \frac{2}{3} = 1$

7. We wish to calculate the period T of a pendulum of length ℓ, mass m, and initial angle of displacement θ_0, released from rest under the influence of Earth's gravitational field g. How does T depend on these quantities?



$$T = l^x m^y g^z \times \theta_0$$

$$T = L^x \times M^y \times \left(\frac{L}{T^2}\right)^z$$

$[T] = T$
$[l] = L$
$[m] = M$
$[g] = \frac{L}{T^2}$

z has to be $-\frac{1}{2}$ to make T^1
x has to be $\frac{1}{2}$ to get rid of L
y " 0 " M

$$\Rightarrow T = l^{\frac{1}{2}} m^0 g^{-\frac{1}{2}} \times \theta_0$$

$$\therefore T \propto \sqrt{\frac{l}{g}} \times \theta_0$$

when initial angle gets larger, T will become also larger.

8. Consider a wire of length ℓ vibrating with amplitude A. It has a linear mass density μ and is under tension F_T. How does the energy E of the vibration depend on these parameters? Are all present? Is the answer completely determined or would further experiment be necessary?

$[l]$ L
$[A]$ L
$[\mu]$ $\frac{M}{L}$
$[F_T]$ $\frac{ML}{T}$
$[E] = \frac{M \cdot L^2}{T^2}$

[M, L, T] which is all fundamental dimensions are presented.

So the E of the vibration can be defined based on these parameters.