Chapter 7 Discussion
Newton's 3rd Law

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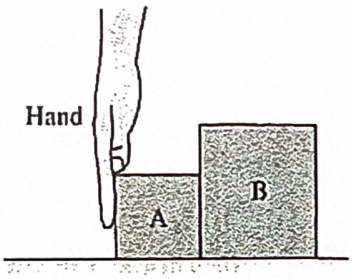
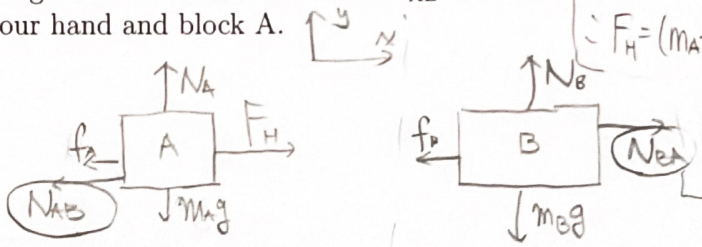
For each of the situations below, draw a free-body diagram for each relevant mass in the problem, find the net force on each mass in each relevant coordinate direction, and apply Newton's 2nd Law. On your FBDs, circle any action-reaction pairs due to Newton's 3rd Law, then solve for the indicated quantity or quantities, keeping in mind any constraints on accelerations, tensions, etc.

1. Blocks A and B (with $m_B > m_A$) are placed on a rough surface as shown, and accelerated to the right at rate a by your hand H. Both blocks have the same coefficient of kinetic friction μ_k. Determine the magnitude of the normal force N_{AB} between the two blocks, and the force F_H between your hand and block A.





$\therefore F_H = (m_A+m_B)(a+\mu_k g)$

$N_{AB} = m_B a + \mu_k m_B g$

$\Sigma F_x = F_H - f_A - N_{AB} = m_A a_x$

$\Sigma F_y = N_A - m_A g = m_A a_y = 0$

$\Sigma F_x = N_{BA} - f_B = m_B a_x$

$\Sigma F_y = N_B - m_B g = m_B a_y = 0$

Since acceleration is same,

$N_{AB} = N_{BA}$ (Newton 3rd Law)

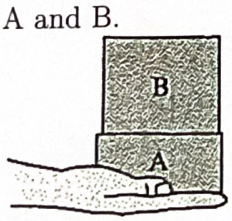
$= N$

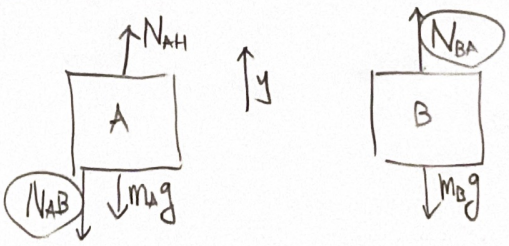
$F_H - \mu_k m_A g - N = m_A a$

$N - \mu_k m_B g = m_B a$ +

$F_H = (m_A+m_B)(a+\mu_k g)$

2. Blocks A and B are held in your outstretched hand H as you lift them upward at constant speed. Assume $m_B > m_A$ and calculate the normal forces between H and A and between A and B.





$\therefore N_{BA} = m_B g$

$N_{HA} = (m_A - m_B)g$

Since it is constant speed, Net force $= 0$

$N_{AB} = N_{BA} = N$ (Newton 3rd Law)

$N_{AH} + N - m_A g = +N - m_B g$

$N_{AH} = m_A g - m_B g$

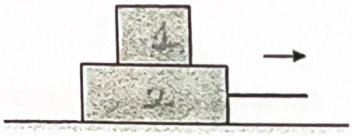
$\Sigma F_{yA} = N_{AH} - N_{AB} - m_A g = m_A a = 0$

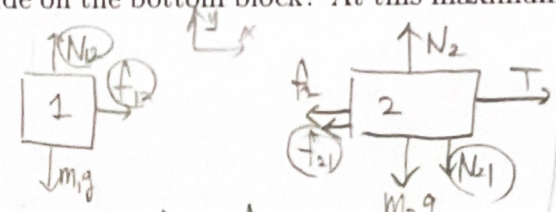
$\Sigma F_{yB} = N_{BA} - m_B g = m_B a = 0$ +

$N_{AH} - (m_A + m_B)g = (m_A + m_B)a$

$N_{AH} = (m_A+m_B)a + (m_A+m_B)g$

3. Block 1 sits on block 2 as shown and block 2 is accelerated to the left with a rope of tension T. There is friction between the blocks and between block 2 and the ground. If the coefficient of kinetic friction between block 2 and the ground is μ_k and that of static friction between the blocks is μ_s, what is the maximum acceleration of the system so that the top block does not begin to slide on the bottom block? At this maximum value, what is the tension in the rope?





$\therefore a = \frac{\mu_s \cdot m_1 g}{m_2}$

$T = \mu_k(m_2g + N_{21}) + \mu_s m_1 g + m_1 a$

$\Sigma F_{x1} = f_{12} = m_1 a_x$

$\Sigma F_{y1} = N_{12} - m_1 g = m_1 a_y = 0$

$\Sigma F_{x2} = T - f_2 - f_{21} = m_2 a_x$

$\Sigma F_{y2} = N_2 - N_{21} - m_2 g = m_2 a_y = 0$

$f_1 = \text{Static} \le \mu_s \cdot m_1 g$

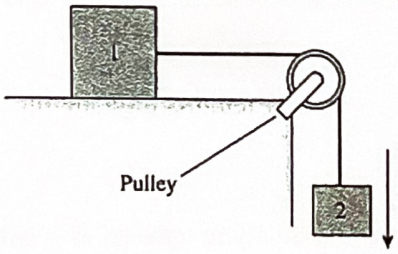
$N_2 = m_2 g + N_{21}$

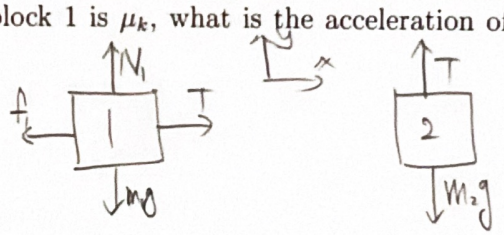
$T = f_2 + m_2 a_x + f_{21}$

$= \mu_k(m_2 g + N_{21}) + \mu_s \cdot m_1 g + m_1 a_x$

maximum $= m_2 a_x < \mu_s \cdot m_1 g \Rightarrow a_x < \frac{\mu_s \cdot m_1 g}{m_2}$

4. Mass 1 is sliding on a rough surface due to the tension supplied by hanging mass 2. The pulley is frictionless and the string and pulley are massless. If the coefficient of kinetic friction between the surface and block 1 is μ_k, what is the acceleration of the blocks? What is the tension in the string?





$\Sigma F_{x1} = T - f_1 = m_1 a_x$

$\Sigma F_{y1} = N_1 - m_1 g = m_1 a_y = 0$

$\Sigma F_{y2} = T - m_2 g = m_2 a_y$

$T = m_2 a + m_2 g$

$\therefore T = m_2\left(\frac{m_2 g - \mu_k m_1 g}{m_1 + m_2}\right) + m_2 g$

Two boxes are constraint, $a_x = -a_y = a$

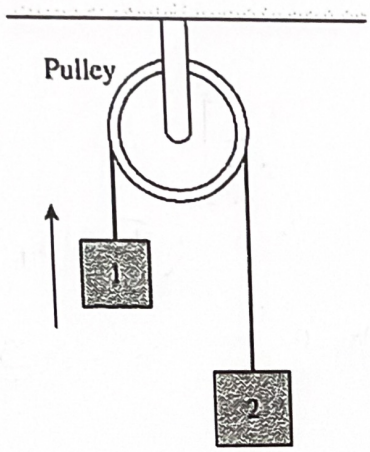
$T - \mu_k m_1 g = m_1 a$

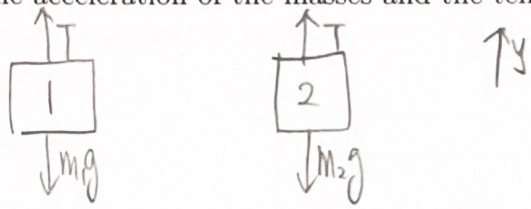
$-\ T - m_2 g = -m_2 a$

$m_2 g - \mu_k m_1 g = (m_1 + m_2) a$

$\therefore a = \frac{m_2 g - \mu_k m_1 g}{m_1 + m_2}$

5. Two masses $m_2 > m_1$ are connected by a massless string over a frictionless, massless pulley (*Atwood's machine*). Calculate the acceleration of the masses and the tension in the string.





$\Sigma F_{y_1} = T - m_1 g = m_1 a_{y_1}$; $\Sigma F_{y_2} = T - m_2 g = m_2 a_{y_2}$

$a_{y_1} = -a_{y_2}$

$T - m_1 g = m_1 a$

$T - m_2 g = m_2(-a)$

$m_1 g + m_1 a - m_2 g = -a m_2$

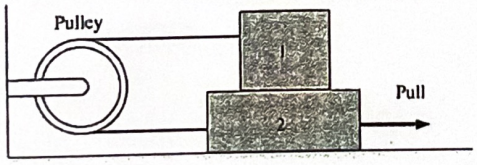
$m_1 a + m_2 a = m_2 g - m_1 g$

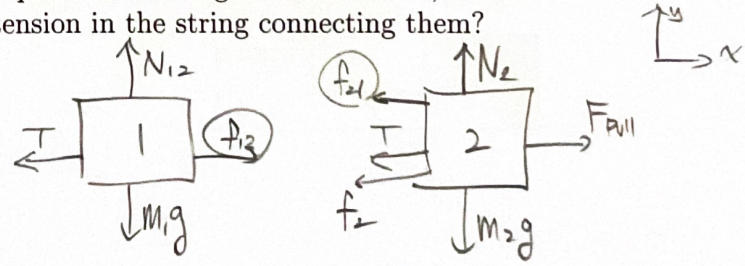
$a = \frac{(m_2 - m_1)g}{m_1 + m_2}$

$\therefore\ T = m_1 g + m_1 \frac{(m_2 - m_1)g}{(m_1 + m_2)}$

$a = \frac{(m_2 - m_1)g}{m_1 + m_2}$

6. Block 1 sits on block 2, and block 2 sits on the ground. All flat surfaces have coefficients of kinetic friction μ_k between them. The blocks are connected by a string wrapped around a frictionless pulley. If block 2 is pulled to the right with force F, what is the acceleration of the blocks? What is the tension in the string connecting them?





$\Sigma F_{x_1} = f_{12} - T = m_1 a_x$ | $\Sigma F_{x_2} = F_{pull} - f_{21} - f_2 - T = m_2 a_x$

$\Sigma F_{y_1} = N_{12} - m_1 g = m_1 a_y = 0$ | $\Sigma F_{y_2} = N_2 - m_2 g = m_2 a_y = 0$

$a = \frac{\mu_k m_1 g - T}{m_1}$

$T = \frac{F_{pull} - \frac{f_{12}}{m_1} m_2 - f_2 - f_{21}}{\left(1 - \frac{m_2}{m_1}\right)}$

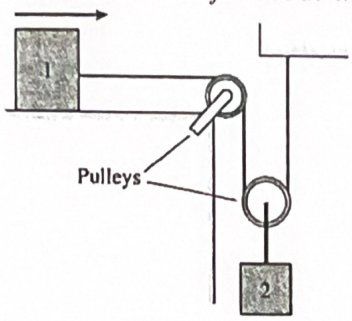
$F_{pull} - f_{21} - f_2 - T = m_2 \left(\frac{f_{12} - T}{m_1}\right)$

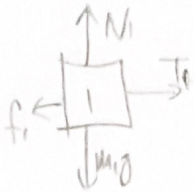
$F_{pull} = m_2 \left(\frac{f_{12} - T}{m_1}\right) + T + f_2 + f_{21}$

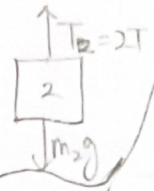
$\left(\frac{f_{12}}{m_1}\right) m_2 - \frac{T}{m_1} m_2 + T + f_2 + f_{21}$

$F_{pull} - \frac{f_{12}}{m_1} m_2 - f_2 - f_{21} = T\left(1 - \frac{m_2}{m_1}\right)$

7. Block 1 is being pulled along a rough surface (coefficient μ_k) by block 2 hanging from a pulley. Find the accelerations of the blocks and the tension in the string connected to block 1. *Be careful about the acceleration constraint!*







$$a = \frac{2m_2g - 4\mu_k m_1 g}{4m_1 + m_2}$$

$$T = \mu_k m_1 g + m_1\left(\frac{2m_2g - 4\mu_k m_1 g}{4m_1 + m_2}\right)$$

$$\Sigma F_x = T_1 - \mu_k m_1 g = m_1 a$$

$$\Sigma F_y = N_1 - m_1 g = m_1 a_y = 0$$

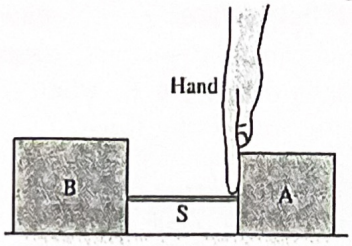
$$\Sigma F_y = m_2 g - 2T = m_2 \frac{a}{2}$$

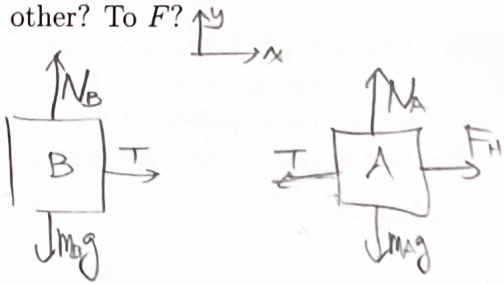
$$2m_2 g - 4T = m_2 a$$

$$+\ 4T - 4\mu_k m_1 g = 4m_1 a$$

$$2m_2 g - 4\mu_k m_1 g = (4m_1 + m_2)a$$

8. You push block A with force F on a smooth table. Block A is connected to block B ($m_B > m_A$) with a *massive* string S. How do T_A and T_B (the force of A on the string [ie the tension at the right side of the string] and the force of the string on B [ie the tension at the left side of the string]) relate to each other? To F?





$$\Sigma F_{xB} = T = m_B a_x$$

$$\Sigma F_{yB} = N_B - m_B g = m_B a_y = 0$$

$$\Sigma F_{xA} = F_H - T = m_A a_x$$

$$\Sigma F_{yA} = N_A - m_A g = m_A a_y = 0$$

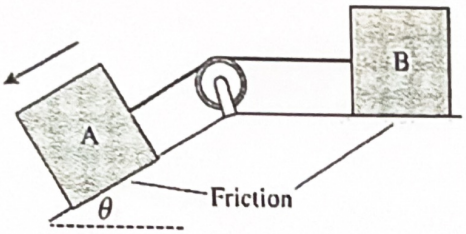
$$\boxed{\therefore F_H = \underbrace{\left(\frac{m_A}{m_B} + 1\right)}_{\text{relation}} T}$$

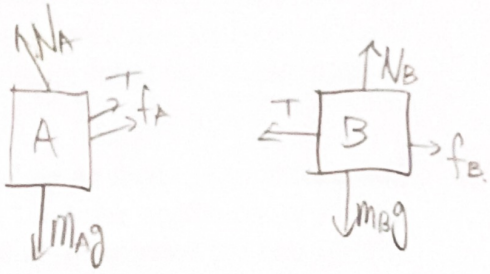
T_A and T_B are same due to Constraint.

$$\frac{T}{m_B} = a \quad , \quad F_H - T = m_A\left(\frac{T}{m_B}\right)$$

$$F_H = \left(\frac{m_A}{m_B} + 1\right) T$$

9. Blocks A and B are connected by a string over a frictionless pulley and are accelerating down a ramp of inclination angle θ. There is kinetic friction with coefficient μ_k between the blocks and the surfaces. Find the acceleration of the blocks and the tension in the string. Also find the reaction force on the (massless) pulley.





$\Sigma F_x = m_A g \sin\theta - T - \mu_k N_A = m_A a_x$

$\Sigma F_y = N_A - m_A g \cos\theta = m_A a_y = 0$

$\Sigma F_x = T - f_B = m_B a$

$\Sigma F_y = N_B - m_B g = m_B a_y = 0$

$m_A g \sin\theta - T - \mu_k m_A g \cos\theta = m_A a$

$+| \quad T - \mu_k m_B g = m_B a$

$\therefore a = \frac{T}{m_B} - \mu_k g$

$m_A g \sin\theta - \mu_k m_A g \cos\theta - \mu_k m_B g = (m_A + m_B) a$

$$\boxed{\frac{m_A g \sin\theta - \mu_k m_A g \cos\theta - \mu_k m_B g}{m_A + m_B} = a}$$

$$\boxed{T = m_B \left(\frac{m_A g \sin\theta - \mu_k m_A g \cos\theta - \mu_k m_B g}{m_A + m_B} \right) + \mu_k m_B g.}$$