Chapter 0.4 Discussion
Math Review

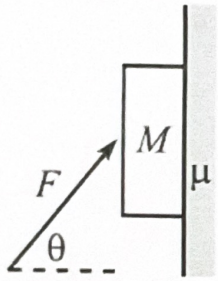
Name: Kevin Lee

1. A mass M is being held at rest against a wall with coefficient of static friction μ by a large force F at angle θ, as shown. A free-body diagram analysis yields the following three relations:

$$N - F\cos\theta = 0, \quad F\sin\theta - f_s - Mg = 0, \quad f_s \le \mu N.$$

$$\boxed{\therefore F \le \frac{Mg}{\sin\theta - \mu\cos\theta}}$$

Solve this system for F by eliminating the unknown forces N and f_s.



$N = F\cos\theta$

$f_s = F\sin\theta - Mg$ $\rightarrow$ $f_s \le \mu N$

$\Downarrow$

$F\sin\theta - Mg \le \mu F\cos\theta$

$F\sin\theta - \mu F\cos\theta \le Mg$

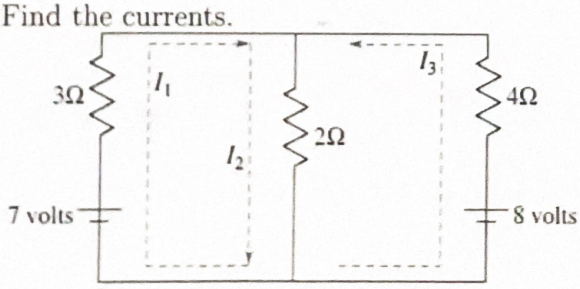
$F(\sin\theta - \mu\cos\theta) \le Mg$

2. Applying Kirchhoff's laws to the electrical network shown below, the currents I_1, I_2, and I_3 are solutions to the system of equations

$$I_1 - I_2 + I_3 = 0$$
$$3I_1 + 2I_2 = 7$$
$$2I_2 + 4I_3 = 8$$

$$\boxed{\therefore I_1 = 1, \quad I_2 = 2, \quad I_3 = 1}$$

Find the currents.



$I_1 + I_3 = I_2$

$3I_1 + 2(I_1 + I_3) = 7 \cdots ①$

$2(I_1 + I_3) + 4I_3 = 8 \cdots ②$

① $\cdots$ $5I_1 + 2I_3 = 7$ $\cdots$ $15I_1 + 6I_3 = 21$

② $\cdots$ $2I_1 + 6I_3 = 8$ $-)$ $2I_1 + 6I_3 = 8$

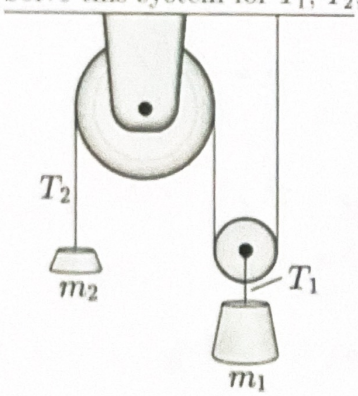
$13I_1 = 13$

$I_1 = 1$, $2 + 6I_3 = 8$ $I_3 = 1$

3. A system of massless pulleys is loaded with masses m_1 and m_2 as shown. The tensions T_1 and T_2 in the ropes and the accelerations a_1 and a_2 of the masses are found by solving the system

$$2T_2 - T_1 = 0$$
$$a_2 + 2a_1 = 0$$
$$T_1 - m_1 g = m_1 a_1$$
$$T_2 - m_2 g = m_2 a_2$$

Solve this system for T_1, T_2, a_1, and a_2 in terms of the masses and g.



$2T_2 = T_1$

$a_2 = -2a_1$

① ··· $T_1 - m_1 g = m_1 a_1$, $T_1 = m_1 a_1 + m_1 g$

② ··· $T_2 - m_2 g = m_2 a_2$, $T_2 = m_2 a_2 + m_2 g$

① ··· $2T_2 = m_1 a_1 + m_1 g$

② ··· $T_2 = m_2 a_2 + m_2 g$

$m_1 a_1 + m_1 g = 2(m_2 a_2 + m_2 g)$

$m_1 a_1 + m_1 g = 2m_2 a_2 + 2m_2 g$

$m_1 a_1 + m_1 g = 2m_2(-2a_1) + 2m_2 g$

$m_1 a_1 + m_1 g = -4m_2 a_1 + 2m_2 g$

$m_1 a_1 + 4m_2 a_1 = 2m_2 g - m_1 g$

$a_1(m_1 + 4m_2) = g(2m_2 - m_1)$

$a_1 = \frac{(2m_2 - m_1)g}{m_1 + 4m_2}$

$\therefore a_1 = \left(\frac{2m_2 - m_1}{m_1 + 4m_2}\right) g$

$a_2 = -2\left(\frac{2m_2 - m_1}{m_1 + 4m_2}\right) g$

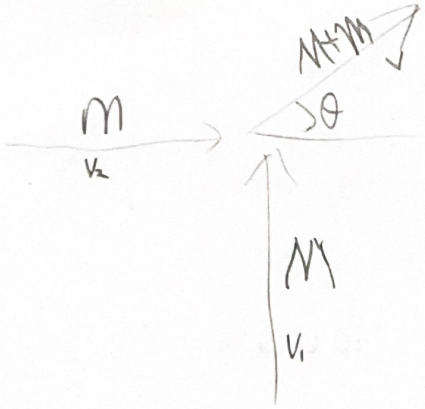
$T_1 = m_1 g\left(\frac{2m_2 - m_1}{m_1 + 4m_2} + 1\right)$

$T_2 = m_2 g\left(\frac{2m_1 - 4m_2}{m_1 + 4m_2} + 1\right)$

4. A truck with mass M and speed v_1 moving north collides with a car of mass m and speed v_2 moving east. The vehicles merge into one piece of wreckage with total mass $M+m$ and speed v moving in a northeasterly direction at an angle θ from due east. An analysis of this problem using conservation of momentum yields the following equations:

$$mv_2 = (M+m)v\cos\theta, \quad Mv_1 = (M+m)v\sin\theta.$$

Solve this system of equations for θ using $\tan\theta = \sin\theta/\cos\theta$ to eliminate v. Then, *starting with the same two equations*, solve for v using the relation $\sin^2\theta + \cos^2\theta = 1$ to eliminate θ.



$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$Mv_1 = (M+m)v\sin\theta$$

$$\div \quad mv_2 = (M+m)v\cos\theta$$

$$\frac{Mv_1}{mv_2} = \frac{(M+m)\cancel{v}\sin\theta}{(M+m)\cancel{v}\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$\boxed{\therefore \frac{Mv_1}{mv_2} = \tan\theta}$$

$$Mv_1 = (M+m)v\sin\theta \quad \cdots ①$$

$$mv_2 = (M+m)v\cos\theta$$

$$① \cdots \sin\theta = \frac{Mv_1}{(M+m)v}$$

$$② \cdots \cos\theta = \frac{mv_2}{(M+m)v}$$

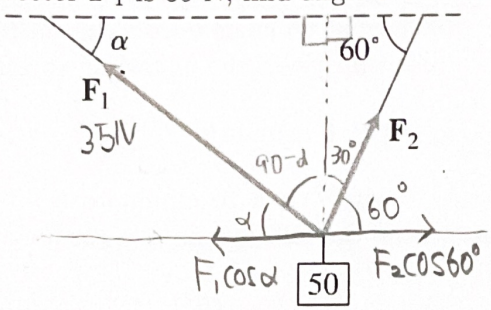
$$①^2 + ②^2 = \sin^2\theta + \cos^2\theta = \left(\frac{Mv_1}{(M+m)v}\right)^2 + \left(\frac{mv_2}{(M+m)v}\right)^2 = 1$$

$$= \frac{(Mv_1)^2 + (mv_2)^2}{(M+m)^2 v^2} = 1$$

$$\frac{(Mv_1)^2 + (mv_2)^2}{(M+m)^2} = v^2$$

$$\boxed{\therefore \frac{\sqrt{(Mv_1)^2 + (mv_2)^2}}{M+m} = v}$$

5. Consider a 50 N weight suspended by two wires as shown below. If the magnitude of vector $\boldsymbol{F}_1$ is 35 N, find angle α and the magnitude of vector $\boldsymbol{F}_2$.



$F_1 \sin\alpha \uparrow\uparrow F_2 \sin 60°$

$\boxed{\therefore \alpha = 74° \quad F_2 = 19N}$

① ... $F_1 \cos\alpha = F_2 \cos 60°$ stable horizontally

② ... $F_1 \sin\alpha + F_2 \sin 60° = 50N$ Stable vertically

① ... $35\cos\alpha = F_2\left(\frac{1}{2}\right)$, $70\cos\alpha = F_2$

② ... $35\sin\alpha + F_2\left(\frac{\sqrt{3}}{2}\right) = 50$ plug

$35\sin\alpha + 70\cos\alpha\left(\frac{\sqrt{3}}{2}\right) = 50$

$35(\sin\alpha + \sqrt{3}\cos\alpha) = 50$

$\sin\alpha + \sqrt{3}\cos\alpha = \frac{10}{7}$

$\sqrt{3}\cos\alpha = \frac{10}{7} - \sin\alpha$

$3\cos^2\alpha = \left(\frac{10}{7} - \sin\alpha\right)^2$ $\cos^2\alpha = 1 - \sin^2\alpha$

$3(1-\sin^2\alpha) = \frac{100}{49} + \sin^2\alpha - \frac{20}{7}\sin\alpha$

$3 - 3\sin^2\alpha = \frac{100}{49} + \sin^2\alpha - \frac{20}{7}\sin\alpha$

$4\sin^2\alpha - \frac{20}{7}\sin\alpha - \frac{47}{49} = 0$

$196\sin^2\alpha - 140\sin\alpha - 47 = 0$

$\boxed{\alpha = 74°}$

$70\cos\alpha = F_2$

$70\cos(74.4°) = F_2$

$\boxed{F_2 = 19N}$