Concepts of Error and Uncertainty

Welcome to the study of physics! For many of you this will be your first experience learning about physics in the classroom. The goal of every physics laboratory experiment is to use some equipment to test, measure, and/or predict the behavior of a physical system or principle. For example, today you will use a stop watch, a meterstick, and a ball to measure how long it takes the ball to fall to the ground. If you do this for many different heights, you can measure the effect gravity has on falling objects.

Every time you perform a laboratory experiment, you will be given some equipment to use and asked to perform the experiment as outlined by your instructor. The quality of the measurements that you get will depend on your own personal skills and technique using the equipment. For this reason, it is very important that you take the time to learn how to use your equipment before performing the actual experiment. You may even perform the experiment several times as **trial runs**, without recording this data. These trial runs are a time for you to practice using the equipment, to check if you understand what is going on, and to refine your skills using this equipment. It may seem like this just takes extra time, but the lab groups that learn how to use their equipment often finish the labs first, and almost always have great data.

Today we are going to take the time to practice using our equipment and learn some basic concepts of measurements and error. To begin with, here are some basic definitions and concepts:

- **Measurement** (x_{exp}): The value that you experimentally determine for some quantity. This measurement is not necessarily the true value of that quantity, it is just what you measured, and your measurement may be flawed.
- **True/Expected/Theoretical Value** (x_{th}): When you measure a physical quantity, that quantity has a true value that exists in nature. For example, if I drop a ball from a height of one meter (at Earth's surface with no drag), it will take 0.45 seconds to reach the ground, regardless of what measurement I obtained for this time.
- **Error** ($E_x = x_{\text{exp}} - x_{\text{th}}$): How far off your measurement is from the true value. For instance, if I drop a ball from one meter and record a time of 0.62 seconds for the ball to fall, my error is 0.17 seconds. We usually don't know the true value for something we are measuring, so the error is often unknown.
- **Uncertainty** (δx): The uncertainty is a statement about your belief in the size of the possible error. You want to state an uncertainty where you are 95% certain that the actual error is less than your estimated uncertainty. For instance, when I recorded a time of 0.62 seconds for the ball to drop, I might estimate an uncertainty of 0.20 seconds. In this case, I was correct – the error was less than the uncertainty. If I repeated the experiment 100 times, I would expect that about 95 times the error would be less than the uncertainty, but for 5 or so times the error would actually be larger than the uncertainty.
- **Average/Mean** ($\bar{x}$): If you measure the same value several times, you may average the measurements. When you average many measurements of the same quantity, the error of the average ($\delta\bar{x}$) tends to be smaller than the errors in the individual measurements. This is because some of the measurements were too small and some too big.

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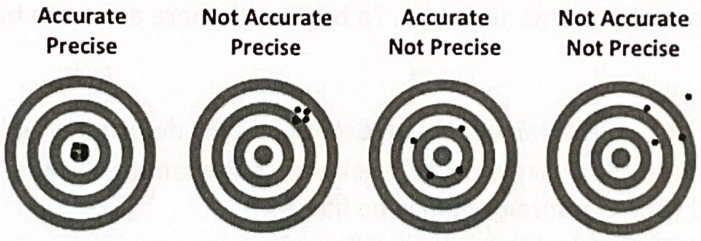
- **Instrumental Uncertainty** (δx_{inst}): The uncertainty introduced by the finite precision of your instrument. It's usually taken to be equal to the smallest (or half the smallest) increment measurable with the instrument. For instance, the metersticks used here have an instrumental uncertainty of 1 mm.
- **Scatter/Statistical Uncertainty** (δx_{stat}): When you take many measurements of the same physical quantity, ideally you would expect every measurement to be exactly the same. Because of the uncertainty in the measuring process, the various measurements will probably be different. The scatter is a statement about how different the values are. For instance, the five measurements 0.40s, 0.39s, 0.42s, 0.40s and 0.41s have a small scatter ($\delta x_{\text{stat}} = 0.02$), while the five measurements 0.32s, 0.47s, 0.41s, 0.36s and 0.52s have a much larger scatter ($\delta x_{\text{stat}} = 0.1$).
- **Precision**: The precision of a measurement is a statement about the number of decimal places in the measurement. A measurement that has many decimal places (small uncertainty) specified is said to be very precise. Note that a very precise measurement may still have a large error.
- **Accuracy:** The accuracy of a measurement is a statement about how small the error is. A measurement with a very small error is said to be very accurate but may still be imprecise.



- **Relative Uncertainty** – The relative or fractional uncertainty in a measurement of a value x is simply the uncertainty divided by the value (mean) itself: $\delta x/x$. The relative uncertainty represents how large the uncertainty is in comparison with the measured value.
- **Percent uncertainty** – This is simply the relative uncertainty expressed as a percentage

$$\%x = \frac{\delta x}{x} \times 100$$

- **Percent Error** – This is the percent difference between the experimental value, and the theoretically expected value.

$$\%E = \frac{x_{\text{ex}} - x_{\text{th}}}{x_{\text{th}}} \times 100$$

- **Percent Difference** – If we wish to compare two measurements without either being considered the "true" or "known" value, we divide their difference by the average:

$$\%\Delta x = \frac{|x_1 - x_2|}{\frac{1}{2}(x_1 + x_2)} \times 100$$

Your instructor will now lead you through a series of exercises. When you have read up to this point in the hand out, place a pencil or pen on the lab bench directly in front of you so the instructor will know when you are ready to go on with the lab.

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0.05

0.605

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Part A – Measuring the Time of Descent from 2 Meters

Your instructor will drop a ball from a height of 2 meters. Try to measure the time the ball takes to fall to the ground.

Your time: 0.87 Group average time: 0.79 Class average time: 0.64

Do you think the scatter is large? Yes

Which measurement, yours or the class average, do you think is best? Explain why.

I think mine is more accurate because the experiment was performed right infront of me.

Do you think this is a "good" measurement? What could you do to make it better?

using different equipement, make more trials, build a huge data sets.

New Concepts:

- **Parallax uncertainty**: If a gauge or ruler is measured from anything but a straight-on view, the reading may be read as too high or too low.
- **Reaction time uncertainty**: When you wait for something to occur and then make a time measurement based on seeing or hearing that event, you are always late by 0.15 to 0.25 seconds. The reason is that a nerve impulse travels from your eyes to your brain and then back to your hand for you to start or stop a stop watch. This takes time. Great athletes tend to have faster reaction times and may have reaction time uncertainties as low as 0.10 seconds.
- **Anticipation time uncertainties**: When you try to anticipate some event and start or stop a timer at the same time as the event, you will sometimes start the timer too early and sometimes too late. This is the anticipation uncertainty. How large or small this is depends on how much you have practiced anticipating this type of event and how alert you are. Anticipation uncertainties are typically 0.05 seconds to 0.15 seconds, depending on the skill and alertness of the person taking the measurement.
- **Self-synchronization timing uncertainties**: The uncertainty associated with trying to do two actions at the exact same instant. For instance, you may try to drop a ball with your left hand and start a timer with your right hand at the same time. The uncertainties associated with this are typically 0.05 s or less, and again the size depends on practice.

Part B – Practicing Measuring Times with a Pendulum

Now your instructor will use a pendulum and a photogate to help you practice making time measurements. The photogate will use fancy electronics to time the pendulum swing so you can see how close you were to the true value. Your instructor will explain how it works and what to do.

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681/50

Your times:

Time 1: 1.53 Time 2: 1.34 Time 3: 1.40 Time 4: 1.41 total Group

Time 5: 1.28 Time 6: 1.38 Time 7: 1.30 Time 8: 1.35 average = 1.36

Time 9: 1.37 Time 10: 1.38 Group Average of Time 10: 1.38

Did your measurements get better with practice? Yes

Was the class average of time 10 very close to the photogate value? Yes (1.3698)s

Your time for 10 swings of the pendulum: 13.74

Divide this value by 10: 1.37

Percent error : $\frac{1.38 - 1.3698}{1.3698} \times 100$

= 0.74 %

New Concepts: ten time at a row : 1.56

- **Counting uncertainty**: When you start timing a number of events, you have to say "zero" when you press start. If you say "one" when you press start, your count will be off by one, and this is called the counting uncertainty.
- **Measuring a larger value can give a smaller percent uncertainty**: If you measure some interval of time, you will have the same uncertainties pressing start and stop whether you measure a small time or a large time. This fixed uncertainty will be a smaller percentage of a larger time interval. This is why measuring 10 swings of the pendulum and dividing by 10 has less uncertainty than just measuring one swing of the pendulum.

Part C – Revisiting the Time of Descent Experiment

Now your instructor will repeat Part A, but this time count to three and drop the golf ball on three. This should allow you to anticipate when the ball is dropped. By watching it fall to the ground you can also practice anticipating when it hits the ground.

Your times:

Time 1: 0.5 Time 2: 0.53 Time 3: 0.6 Time 4: 0.55

Did your measurements get better with practice? Yes

Try to estimate a value for the uncertainty of time 4. What are the sources of uncertainty in your experiment? How much uncertainty did each source introduce? To get the total uncertainty, you need to add up all of the sources of uncertainty: ①

①: reaction time uncertainty : 0.2

: instrumental uncertainty : 0.005

Anticipation uncertainty : 0.05

Statistical uncertainty : 0.16

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$\frac{2\sqrt{5}}{7}$

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Source of uncertainty:	The value of that uncertainty:	
reaction time uncertainty	0.2	
instrumental uncertainty	0.005	calculation
Anticipation uncertainty	0.05	
Statistical uncertainty	0.16	$\frac{max - min}{2}$

The total uncertainty is: 0.4

Explain how you decided the values of each of the sources of uncertainty.

I decided reaction time and anticipation uncertainty depend on written at the front page and calculator only has only one-hundredth of digit so 0.005 for instrumental and for statistical I used the formula $\frac{max-min}{2}$.

Class Average of Time 4: 0.53

Also try to estimate and uncertainty for the class average value of time 4. Uncertainty: 0.16

Why did you choose this guess for the uncertainty of the class average time?

we can use half-rang approximation which is $\frac{max\, x_i - min\, x_i}{2}$

we can figure out the uncertainty of class average by half-range approximation.

The theoretical time of decent for a ball dropped from height h above the surface of the Earth is

$$t_{th} = \sqrt{2h/g}, \qquad g = 9.80 \text{ m/s}^2$$

The error for your measured time 4 is just the difference between the expected value and the measured value. Similarly, the error in the class average is the difference between the expected value and the class average. Calculate these errors. 0.64

Error for your time 4 measurement: 0.09 s

Error for the class average: 0.11 s

As discussed on page one of this handout, you want the estimated uncertainty to be bigger than the actual error.

Was the uncertainty for time 4 bigger than its error? yes

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Was the uncertainty for the class average bigger than its error? Yes

How did the results from Part A compare to those of Part C? Why do you think this happened?

Part C should be more accurate and precise than Part A. The difference between Part A and C is whe we do Part A we didn't count for the ball drop and also there was only one time so no practice. As a result, the percen error for part C must be lower than Part A.

New Concepts:

- **Estimating the total uncertainty**: When you take a measurement and you need to guess an uncertainty, you need to add up all of the possible uncertainties that can have an effect. Often the person or people involved in the measurement introduce the largest uncertainty. For a digital device, the equipment typically introduces an uncertainty of ±1 in the smallest decimal place.

Part D – Do It Yourself!

0.005 mm.
instrumental uncertainty = 0.05 cm

You now have 20 minutes to measure the time it takes a ball to drop 150 cm. You may work in pairs for this exercise. You will also need to think about how you use a ruler and how you estimate distance uncertainties. You are free to try different methods of timing and dropping. Just be sure to decide when you are taking real data and explain the actual method you use to take the data that you present. Also explain why you chose the uncertainties that you chose.

1st : 0.34
2nd : 0.51
3 : 0.45
4 = 0.53
5 = 0.51

Time: 0.51s

Uncertainty in time: 0.255 s
Reaction time: 0.2
Anticipation time: 0.05
instrumental 0.005 s

Distance: 150 cm

Uncertainty in distance: 0.05 cm

What procedure did you use to get the data reported above?

First, we measured the diameter and the weight of the golf ball. Next, we put the ruler on the wall to make sure that the ruler is perfectly perpendicular to the surface of the floor. Then we tried 4 times to practice performing the experiment accurately. Lastly, we got 0.51sec which we did on the 5th attempt.

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How did you choose the uncertainty for the time measurement?

I chose 3 types of uncertainty for time measurment. Reaction time uncertainty and anticipation uncertainty are inevitable when human checks the data of an experiment. The last one is instrumental uncertainty because of the stopwatch. we used that display only one-hundredth of the digit on the screen so there must be an uncertainty of 0.005 sec

How did you choose the uncertainty for the distance measurement?

First I considered two instrumental uncertainty. One from the ruler which is ±0.05cm and another from the Vernier Calipers. But I decide to neglect the uncertainty of the Vernier Calipers because the uncertainty which is 0.005mm is so small that It wou not have an effect on the result. So there is only one uncertainty for the distance measurement.

Write your final experimental values here:

Value and uncertainty: $\bar{t}_{ex} \pm \delta t =$ 0.51 ± 0.26s ±0.05

$t_{known} = 0.55s = \sqrt{\frac{2 \times 1.5m}{9.8 m/s^2}}$

Error: $E_t =$ −0.04 sec

Percent error: $\%E =$ −7.3%

Also state the mass and diameter of the golf ball (using the digital scale and Vernier calipers):

$m \pm \delta m =$ 46.0 ± 0.05g

$D \pm \delta D =$ 42.76 ± 0.005mm

Turn this hand out in to your instructor when you are finished.

$\frac{0.04}{0.55} \times 100 = 7.3\%$

$\sqrt{\frac{2 \times 1.5}{9.8}} = 0.55$

$\frac{\sqrt{15}}{7}$

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