Candidate Number: AE05750, Advanced Game Theory Project

Full Solution Guide

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Standard Parameters Game (p=0.5).

Strategy profiles:

- Player 1: given (High quality, Low quality)
  - WW
  - WN
  - NW
  - NN
- Player 2: given (Warranty, No Warranty)
  - BB
  - BR
  - RB
  - RR

Filling in Payoffs

- General Formula:
  - Player 1: *(p)(a1) + (1-p)(b1) = u1*
  - Player 2: *(p)(a2) + (1-p)(b2) = u2*
- WWBB and WWBR (both branches W,B)
  - Player 1: (0.5)(1)+(0.5)(1.5)=1.25
  - Player 2: (0.5)(3)+(0.5)(-2)=0.5
- WWRB and WWRR (both branches W,R)
  - Player 1: (0.5)(-2.5)+(0.5)(-2)=-2.25
  - Player 2: (0.5)(-2)+(0.5)(2)=0
- NNBB and NNRB (both branches N,B)
  - Player 1: (0.5)(2)+(0.5)(3)= 2.5
  - Player 2: (0.5)(2)+(0.5)(-1)= 0.5
- NNBR and NNRR (both branches N, R)
  - Player 1: (0.5)(-2)+(0.5)(-1)= -1.5
  - Player 2: (0.5)(-1)+(0.5)(1)= 0
- WNBB (H branch W,B; L branch N,B)
  - Player 1: (0.5)(1)+(0.5)(3)=2
  - Player 2: (0.5)(3)+(0.5)(-1)=1
- WNRR (H branch W, R; L Branch N, R)
  - Player 1: (0.5)(-2.5)+(0.5)(-1)=0.25
  - Player 2: (0.5)(-2)+(0.5)(1)=-1.5

- NWBB (H branch N, B; L Branch W, B)
  - Player 1: (0.5)(2)+(0.5)(1.5)=1.75
  - Player 2: (0.5)(2)+(0.5)(-2)=0
- NWRR (H branch N, R; L Branch W, R)
  - Player 1: (0.5)(-2)+(0.5)(-2)=-2
  - Player 2: (0.5)(-1)+(0.5)(2)=0.5
- WNBR (H branch W, B; L branch N, R)
  - Player 1: (0.5)(1)+(0.5)(-1)=0
  - Player 2: (0.5)(3)+(0.5)(1)=2
- WNRB (H branch W, R; L branch N, B)
  - Player 1: (0.5)(-2.5)+(0.5)(3)=0.25
  - Player 2: (0.5)(-2)+(0.5)(-1)=-1.5
- NWBR (H branch N, R; L branch W, B)
  - Player 1: (0.5)(-2)+(0.5)(1.5)=-0.25
  - Player 2: (0.5)(-1)+(0.5)(-2)=-1.5
- NWRB (H branch N, B; L branch W, R)
  - Player 1: (0.5)(2)+(0.5)(-2)=0
  - Player 2: (0.5)(2)+(0.5)(2)=2

Normal Form and Nash Equilibriums

Normal Form

		2			
		BB	BR	RB	RR
1	WW	1.25, **0.5**	**1.25**, **0.5**	-2.25, 0	-2.25, 0
	WN	2,1	0, **2**	0.25, -1.5	-1.75, -0.5
	NW	1.75, 0	-0.25, -1.5	0, **2**	-2, 0.5
	NN	**2.5**, **0.5**	-1.5, 0	**2.5**, **0.5**	-**1.5**, 0

Nash equilibriums:

- NNBB (for both types P1 choses N, P2 choses B)
- NNRB (Same as NNBB) (for both types P1 choses N, P2 choses B)
- WWBR (for both types P1 choses W, P2 choses B).

Pooling Equilibrium NN

Now, will analyse pooling equilibrium given $S_1^{t1} = S_1^{t2} = N$

- That means, μ and (1- μ) are on the equilibrium path, and β and (1- β) are off the equilibrium path.
- Requirement 4: Because β and (1- β) are off the equilibrium path, it can be anything between 0 and 1: $\beta \in [0, 1]$
- Requirement 3: Given the probabilities defined in the problem (p=0.5), we can deduce that μ=0.5, and thus, (1- μ) = 0.5
    - Because if we look at the player 2 nodes on the right side (given player 1 plays N like proscribed), the chance at arriving at the top node is the same as the probability of type 1 (high quality) occurring.
- Requirement 2: What is the best response for player 2, given player 1 choses N?
    - Since we know the probability of arriving at the two right nodes, we can calculate the expected value for playing each strategy.
    - Strategy B: (0.5)(2) + (0.5)(-1) = 0.5
    - Strategy R: (0.5)(-1) + (0.5)(1) = 0
    - Thus, player 2 would choose strategy B.
    - Thus, $S_2^2 = B$
- Player 1 deviation possibilities:
    - If player 1 deviates to W, would he get a higher payoff?
    - If quality is high, and player one choses N, we know player 2 will chose B (as defined above), which yields a payoff of "2" for player 1. If player 1 decides to play w when quality is high, neither option results in a higher payoff, even if β is at the maximum value of 1. Thus, player 1 will not deviate to W in the type "high quality"
    - If quality is low, and player one choses N, we know player 2 will chose B (as defined above), which yields a payoff of "3" for player 1. If player 1 decides to play W when quality is low, neither option results in a higher payoff, even if β is at the maximum value of 1. Thus, player 1 will not deviate to W in the type "low quality"
    - Thus, playing N for player 1 is a best response as well.
- So thus, a Perfect Bayesian Equilibrium will have:
    - Player 1 choosing N no mater the type
    - Player 2 choosing B

BUT, we are not done.

- Perfect Bayesian Equilibrium is defined as $[(S_1^{t1}, S_1^{t2}), (S_2^1, S_2^2), \mu, \beta]$

- So far, we have [(N, N), (S_2^1, B), μ = 0.5, β]. We are missing S_2^1 and β.

Finding S_2^1 and β (On the non-equilibrium path, so they do not change the outcome)

- What is the expected utility of player 2, given playing strategy B, and some value μ? Calculate the conditional probabilities:
  - Expected: 3(β) + -2(1- β) = 3(β) – 2 + 2(β) = 5(β) – 2.
- What is the expected utility of player 2, given playing strategy R, and some value μ? Calculate the conditional probabilities:
  - Expected: -2(β) + 2(1- β) = -2(β) + 2 - 2(β) = -4(β) + 2
- B is a best response if:
  - P2 Expected payoff of B ≥ P2 Expected payoff of R
  - 5(β) – 2 ≥ -4(β) + 2
  - 9(β) – 2 ≥ 2
  - 9(β) ≥ 4
  - β ≥ 4/9

THUS, Perfect Bayesian Equilibrium is:

- There are infinitely many pooling perfect Bayesian equilibrium, [(N, N), (S_2^1,B), μ=0.5, β],
  - where S_2^1 = B and β ≥ 4/9
  - where S_2^1 = R and β ≤ 4/9
  - Where no matter the value of S_2^1 and β, the outcomes are equivalent, as S_2^1 and β are off the equilibrium path.

Pooling Equilibrium WW

Now, will analyse pooling equilibrium given S_1^{t1}= S_1^{t2} = W

- That means, β and (1- β) are on the equilibrium path, and and (1- μ) are off the equilibrium path.
- Requirement 4: Because μ and (1- μ) are off the equilibrium path, it can be anything between 0 and 1: μ ∈ [0, 1]
- Requirement 3: Given the probabilities defined in the problem (p=0.5), we can deduce that β=0.5, and thus, (1- β) = 0.5
  - Because if we look at the player 2 nodes on the left side (given player 1 plays W like proscribed), the chance at arriving at the top node is the same as the probability of type 1 (high quality) occurring.
- Requirement 2: What is the best response for player 2, given player 1 choses W?

  - Since we know the probability of arriving at the two right nodes, we can calculate the expected value for playing each strategy.
  - Strategy B: (0.5)(3) + (0.5)(-2) = 0.5
  - Strategy R: (0.5)(-2) + (0.5)(2) = 0
  - Thus, player 2 would choose strategy B.
  - Thus, S_2^1 = B
- Player 1 deviation possibilities:
  - If player 1 deviates to N, would he get a higher payoff?
  - If quality is high, and player one choses W, we know player 2 will chose B (as defined above), which yields a payoff of "1" for player 1. If player 1 decides to play N when quality is high, it is, as of this moment, not possible to know if he can improve his payoff, as the off the equilibrium path is undetermined, and the potential payoffs could be higher (2) or lower (-2).
  - If quality is low, and player one choses W, we know player 2 will chose B (as defined above), which yields a payoff of "1.5" for player 1. If player 1 decides to play N when quality is high, it is, as of this moment, not possible to know if he can improve his payoff, as the off the equilibrium path is undetermined, and the potential payoffs could be higher (3) or lower (-1).
  - THUS, since the off the equilibrium path will determine if deviation for player 1 is possible or not, we can satisfy the pooling PBE, if we find the values of μ and (1- μ) that result in player 1 not having a profitable deviation.
  - Player 1 only has a profitable deviation when switching to N, if player 2 plays B. Thus, we have to find the values of μ and (1- μ) where player 2 will not play B, thus ensuring that player 1 will not have a profitable deviation.
  - Thus, the expected utility of player 2 when playing R must be greater than playing B, as expressed by: $2(\mu) + -1(1-\mu) < -1(\mu)+1(1-\mu)$
    - $2(\mu) + -1(1-\mu) < -1(\mu)+1(1-\mu)$
    - $2\mu - 1(1-\mu) < -1\mu + 1(1-\mu)$
    - $2\mu - 1 + \mu < -\mu + 1 - \mu$
    - $3\mu - 1 < -2\mu + 1$
    - $5\mu < 2$
    - $\mu < 2/5$
  - So, when $\mu < 2/5$, PBE is satisfied
- So thus, a Perfect Bayesian Equilibrium will have:
  - Player 1 choosing W no mater the type
  - Player 2 choosing B

THUS, second Perfect Bayesian Equilibrium is:

- There are infinitely many pooling perfect Bayesian equilibrium, [(W, W), (B, R), $\mu<2/5$, $\beta<0.5$],

Final Solutions for Standard Parameters (p=0.5) Game:

- There are infinitely many pooling perfect Bayesian equilibrium, [(N, N), (S_2^1,B), $\mu=0.5$, β],
    - where S_2^1 = B and $\beta \geq 4/9$
    - where S_2^1 = R and $\beta \leq 4/9$
    - Where no matter the value of S_2^1 and β, the outcomes on the equilibrium-path are equivalent, as S_2^1 and β are off the equilibrium path.
- There are also infinitely many pooling perfect Bayesian equilibrium, [(W, W), (B, R), $\mu<2/5$, $\beta=0.5$],

High-Reputation Parameters Game (p=0.9).

Strategy profiles:

- Player 1: given (High quality, Low quality)
  - WW
  - WN
  - NW
  - NN
- Player 2: given (Warranty, No Warranty)
  - BB
  - BR
  - RB
  - RR

Filling in Payoffs

- General Formula:
  - Player 1: *(0.9)(a1) + (0.1)(b1) = u1*
  - Player 2: *(0.9)(a2) + (0.1)(b2) = u2*
- WWBB and WWBR (both branches wb)
  - Player 1: *(0.9)(1) + (0.1)(1.5) = 1.05*
  - Player 2: *(0.9)(3) + (0.1)(-2) = 2.5*
- WWRB and WWRR (Both branches WR)
  - Player 1: *(0.9)(-2.5) + (0.1)(-2) = -2.45*
  - Player 2: *(0.9)(-2) + (0.1)(2) = -1.6*
- NNBB and NNRB (Both branches NB)
  - Player 1: *(0.9)(2) + (0.1)(3) = 2.1*
  - Player 2: *(0.9)(2) + (0.1)(-1) = 1.7*
- NNBR and NNRR (Both branches NR)
  - Player 1: *(0.9)(-2) + (0.1)(-1) = -1.9*
  - Player 2: *(0.9)(-1) + (0.1)(1) = -0.8*
- WNBB (H branch WB, L branch NB)
  - Player 1: *(0.9)(1) + (0.1)(3) = 1.2*
  - Player 2: *(0.9)(3) + (0.1)(-1) = 2.6*
- WNBR (H branch WB, L branch NR)
  - Player 1: *(0.9)(1) + (0.1)(-1) = 0.8*
  - Player 2: *(0.9)(3) + (0.1)(1) = 2.8*

- WNRB (H branch WR, L branch NB)
    - Player 1: *(0.9)(-2.5) + (0.1)(3) = -1.95*
    - Player 2: *(0.9)(-2) + (0.1)(-1) = -1.9*
- WNRR (H branch WR, L branch NR)
    - Player 1: *(0.9)(-2.5) + (0.1)(-1) = -2.35*
    - Player 2: *(0.9)(-2) + (0.1)(1) = -1.7*
- NWBB (H branch NB, L Branch WB)
    - Player 1: *(0.9)(2) + (0.1)(1.5) = 1.95*
    - Player 2: *(0.9)(2) + (0.1)(-2) = 1.6*
- NWBR (H branch NR, L branch WB)
    - Player 1: *(0.9)(-2) + (0.1)(1.5) = -1.65*
    - Player 2: *(0.9)(-1) + (0.1)(-2) = -1.1*
- NWRB (H branch NB, L branch WR)
    - Player 1: *(0.9)(2) + (0.1)(-2) = 1.6*
    - Player 2: *(0.9)(2) + (0.1)(2) = 2*
- NWRR (H branch NR, L branch WR)
    - Player 1: *(0.9)(-2) + (0.1)(-2) = -2*
    - Player 2: *(0.9)(-1) + (0.1)(2) = -0.7*

Normal Form and Nash Equilibriums

Normal Form

	BB	BR	RB	RR
WW	1.05. **2.5**	**1.05**, **2.5**	-2.45, -1.6	-2.45, -1.6
WN	1.2, 2.6	0.8, **2.8**	-1.95, -1.9	-2.35, -1.7
NW	1.95, 1.6	-1.65, -1.1	1.6, **2**	-2, -0.7
NN	**2.1**, **1.7**	-1.9, -0.8	**2.1**, **1.7**	**-1.9**, -0.8

Nash Equilibriums

- NNBB (for both types P1 choses N, P2 choses B)
- NNRB (Same as NNBB) (for both types P1 choses N, P2 choses B)
- WWBR (for both types P1 choses W, P2 choses B).

Pooling Equilibrium NN

Now, will analyse pooling equilibrium given $S_1^{t1} = S_1^{t2} = N$

- That means, μ and (1- μ) are on the equilibrium path, and β and (1- β) are off the equilibrium path.
- Requirement 4: Because β and (1- β) are off the equilibrium path, it can be anything between 0 and 1: $\beta \in [0, 1]$
- Requirement 3: Given the probabilities defined in the problem (p=0.5), we can deduce that μ=0.9, and thus, (1- μ) = 0.1
    - Because if we look at the player 2 nodes on the right side (given player 1 plays N like proscribed), the chance at arriving at the top node is the same as the probability of type 1 (high quality) occurring.
- Requirement 2: What is the best response for player 2, given player 1 choses N?
    - Since we know the probability of arriving at the two right nodes, we can calculate the expected value for playing each strategy.
    - Strategy B: (0.9)(2) + (0.1)(-1) = 1.7
    - Strategy R: (0.9)(-1) + (0.1)(1) = -0.8
    - Thus, player 2 would choose strategy B.
    - Thus, $S_2^2 = B$
- Player 1 deviation possibilities:
    - If player 1 deviates to W, would he get a higher payoff?
    - If quality is high, and player one choses N, we know player 2 will chose B (as defined above), which yields a payoff of “2” for player 1. If player 1 decides to play w when quality is high, neither option results in a higher payoff, even if β is at the maximum value of 1. Thus, player 1 will not deviate to W in the type “high quality”
    - If quality is low, and player one choses N, we know player 2 will chose B (as defined above), which yields a payoff of “3” for player 1. If player 1 decides to play W when quality is low, neither option results in a higher payoff, even if β is at the maximum value of 1. Thus, player 1 will not deviate to W in the type “low quality”
    - Thus, playing N for player 1 is a best response as well.
- So thus, a Perfect Bayesian Equilibrium will have:
    - Player 1 choosing N no mater the type
    - Player 2 choosing B

BUT, we are not done.

- Perfect Bayesian Equilibrium is defined as $[(S_1^{t1}, S_1^{t2}), (S_2^1, S_2^2), \mu, \beta]$

- So far, we have [(N, N), (S_2^1, B), μ = 0.9, β]. We are missing S_2^1 and β.

Finding S_2^1 and β (On the non-equilibrium path, so they do not change the outcome)

- What is the expected utility of player 2, given playing strategy B, and some value μ? Calculate the conditional probabilities:
  - Expected: $3(\beta) + -2(1-\beta) = 3(\beta) - 2 + 2(\beta) = 5(\beta) - 2$.
- What is the expected utility of player 2, given playing strategy R, and some value μ? Calculate the conditional probabilities:
  - Expected: $-2(\beta) + 2(1-\beta) = -2(\beta) + 2 - 2(\beta) = -4(\beta) + 2$
- B is a best response if:
  - P2 Expected payoff of B ≥ P2 Expected payoff of R
  - $5(\beta) - 2 \geq -4(\beta) + 2$
  - $9(\beta) - 2 \geq 2$
  - $9(\beta) \geq 4$
  - $\beta \geq 4/9$

THUS, Perfect Bayesian Equilibrium is:

- There are infinitely many pooling perfect Bayesian equilibrium, [(N, N), (S_2^1,B), μ=0.9, β],
  - where S_2^1 = B and $\beta \geq 4/9$
  - where S_2^1 = R and $\beta \leq 4/9$
  - Where no matter the value of S_2^1 and β, the outcomes are equivalent, as S_2^1 and β are off the equilibrium path.

Pooling Equilibrium WW

Now, will analyse pooling equilibrium given $S_1^{t1} = S_1^{t2}$ = W

- That means, β and (1- β) are on the equilibrium path, and and (1- μ) are off the equilibrium path.
- Requirement 4: Because μ and (1- μ) are off the equilibrium path, it can be anything between 0 and 1: $\mu \in [0, 1]$
- Requirement 3: Given the probabilities defined in the problem (p=0.5), we can deduce that β=0.9, and thus, (1- β) = 0.1
  - Because if we look at the player 2 nodes on the left side (given player 1 plays W like proscribed), the chance at arriving at the top node is the same as the probability of type 1 (high quality) occurring.
- Requirement 2: What is the best response for player 2, given player 1 choses W?

- Since we know the probability of arriving at the two right nodes, we can calculate the expected value for playing each strategy.
- Strategy B: (0.9)(3) + (0.1)(-2) = 2.5
- Strategy R: (0.9)(-2) + (0.1)(2) = -1.6
- Thus, player 2 would choose strategy B.
- Thus, $S_2^1 = B$

- Player 1 deviation possibilities:
  - If player 1 deviates to N, would he get a higher payoff?
  - If quality is high, and player one choses W, we know player 2 will chose B (as defined above), which yields a payoff of "1" for player 1. If player 1 decides to play N when quality is high, it is, as of this moment, not possible to know if he can improve his payoff, as the off the equilibrium path is undetermined, and the potential payoffs could be higher (2) or lower (-2).
  - If quality is low, and player one choses W, we know player 2 will chose B (as defined above), which yields a payoff of "1.5" for player 1. If player 1 decides to play N when quality is high, it is, as of this moment, not possible to know if he can improve his payoff, as the off the equilibrium path is undetermined, and the potential payoffs could be higher (3) or lower (-1).
  - THUS, since the off the equilibrium path will determine if deviation for player 1 is possible or not, we can satisfy the pooling PBE, if we find the values of μ and (1-μ) that result in player 1 not having a profitable deviation.
  - Player 1 only has a profitable deviation when switching to N, if player 2 plays B. Thus, we have to find the values of μ and (1- μ) where player 2 will not play B, thus ensuring that player 1 will not have a profitable deviation.
  - Thus, the expected utility of player 2 when playing R must be greater than playing B, as expressed by: $2(\mu) + -1(1-\mu) < -1(\mu)+1(1-\mu)$
    - $2(\mu) + -1(1-\mu) < -1(\mu)+1(1-\mu)$
    - $2\mu - 1(1-\mu) < -1\mu + 1(1-\mu)$
    - $2\mu - 1 + \mu < -\mu + 1 - \mu$
    - $3\mu - 1 < -2\mu + 1$
    - $5\mu < 2$
    - $\mu < 2/5$
  - So, when $\mu < 2/5$, PBE is satisfied
- So thus, a Perfect Bayesian Equilibrium will have:
  - Player 1 choosing W no mater the type
  - Player 2 choosing B

THUS, second Perfect Bayesian Equilibrium is:

- There are infinitely many pooling perfect Bayesian equilibrium, [(W, W), (B, R), $\mu<2/5$, $\beta=0.9$],

Final Solutions for High-Reputation Parameters (p=0.5) Game:

- There are infinitely many pooling perfect Bayesian equilibrium, [(N, N), (S_2^1,B), $\mu=0.9$, β],
    - where S_2^1 = B and $\beta \geq 4/9$
    - where S_2^1 = R and $\beta \leq 4/9$
    - Where no matter the value of S_2^1 and β, the outcomes on the equilibrium-path are equivalent, as S_2^1 and β are off the equilibrium path.
- There are also infinitely many pooling perfect Bayesian equilibrium, [(W, W), (B, R), $\mu<2/5$, $\beta=0.9$],

Poor-Reputation Parameters Game (p=0.1).

Strategy profiles:

- Player 1: given (High quality, Low quality)
  - WW
  - WN
  - NW
  - NN
- Player 2: given (Warranty, No Warranty)
  - BB
  - BR
  - RB
  - RR

Filling in Payoffs

- General Formula:
  - Player 1: *(0.1)(a1) + (0.9)(b1) = u1*
  - Player 2: *(0.1)(a2) + (0.9)(b2) = u2*
- WWBB and WWBR (both branches WB)
  - Player 1: *(0.1)(1) + (0.9)(1.5) = 1.45*
  - Player 2: *(0.1)(3) + (0.9)(-2) = -1.5*
- *WWRB and WWRR (both branches WR)*
  - Player 1: *(0.1)(-2.5) + (0.9)(-2) = -2.05*
  - Player 2: *(0.1)(-2) + (0.9)(2) = 1.6*
- NNBB and NNRB (both branches NB)
  - Player 1: *(0.1)(2) + (0.9)(3) = 2.9*
  - Player 2: *(0.1)(2) + (0.9)(-1) = -0.7*
- NNBR and NNRR (both branches NR)
  - Player 1: *(0.1)(-2) + (0.9)(-1) = -1.1*
  - Player 2: *(0.1)(-1) + (0.9)(1) = 0.8*
- WNBB (H branch WB, L branch NB)
  - Player 1: *(0.1)(1) + (0.9)(3) = 2.8*
  - Player 2: *(0.1)(3) + (0.9)(-1) = -0.6*
- WNBR (H branch WB, L branch NR)
  - Player 1: *(0.1)(1) + (0.9)(-1) = -0.8*
  - Player 2: *(0.1)(3) + (0.9)(1) = 1.2*
- *WNRB (H branch WR, L branch NB)*

  - Player 1: *(0.1)(-2.5) + (0.9)(3) = 2.45*
  - Player 2: *(0.1)(-2) + (0.9)(-1) = -1.1*
- WNRR (H branch WR, L branch NR)
  - Player 1: *(0.1)(-2.5) + (0.9)(-1) = -1.15*
  - Player 2: *(0.1)(-2) + (0.9)(1) = 0.7*
- NWBB (H branch NB, L branch WB)
  - Player 1: *(0.1)(2) + (0.9)(1.5) = 1.55*
  - Player 2: *(0.1)(2) + (0.9)(-2) = -1.6*
- NWBR (H branch NR, L branch WB)
  - Player 1: *(0.1)(-2) + (0.9)(1.5) = 1.15*
  - Player 2: *(0.1)(-1) + (0.9)(-2) = -1.9*
- NWRB (H branch NB, L branch WR)
  - Player 1: *(0.1)(2) + (0.9)(-2) = -1.6*
  - Player 2: *(0.1)(2) + (0.9)(2) = 2*
- NWRR (H branch NR, L branch WR)
  - Player 1: *(0.1)(-2) + (0.9)(-2) = -2*
  - Player 2: *(0.1)(-1) + (0.9)(2) = 1.7*

Normal Form and Nash Equilibriums

Normal Form

	BB	BR	RB	RR
WW	1.45, -1.5	**1.45**, -1.5	-2.05, **1.6**	-2.05, **1.6**
WN	2.8, -0.6	-0.8, **1.2**	2.45, -1.1	-1.15, 0.7
NW	1.55, -1.6	1.15, -1.9	-1.6, **2**	-2, 1.7
NN	**2.9**, -0.7	-1.1, **0.8**	**2.9**, -0.7	**-1.1**, **0.8**

Nash Equilibrium:

- NNRR (for both types P1 choses N, P2 choses R).

Pooling Equilibrium NN

Now, will analyse pooling equilibrium given $S_1^{t1} = S_1^{t2} = N$

- That means, μ and (1- μ) are on the equilibrium path, and β and (1- β) are off the equilibrium path.
- Requirement 4: Because β and (1- β) are off the equilibrium path, it can be anything between 0 and 1: $\beta \in [0, 1]$
- Requirement 3: Given the probabilities defined in the problem (p=0.5), we can deduce that μ=0.1, and thus, (1- μ) = 0.9
    - Because if we look at the player 2 nodes on the right side (given player 1 plays N like proscribed), the chance at arriving at the top node is the same as the probability of type 1 (high quality) occurring.
- Requirement 2: What is the best response for player 2, given player 1 choses N?
    - Since we know the probability of arriving at the two right nodes, we can calculate the expected value for playing each strategy.
    - Strategy B: (0.1)(2) + (0.9)(-1) = -0.7
    - Strategy R: (0.1)(-1) + (0.9)(1) = 0.8
    - Thus, player 2 would choose strategy R.
    - Thus, $S_2^2 = R$
- Player 1 Deviation possibilities:
    - If player 1 deviates to W, would he get a higher payoff?
    - If quality is high, and player one choses N, we know player 2 will chose R (as defined above), which yields a payoff of "-2" for player 1. If player 1 decides to play W when quality is high, it is, as of this moment, not possible to know if he can improve his payoff, as the off the equilibrium path is undetermined, and the potential payoffs could be higher (1) or lower (-2.5).
    - If quality is low, and player one choses N, we know player 2 will chose R (as defined above), which yields a payoff of "-1" for player 1. If player 1 decides to play w when quality is high, it is, as of this moment, not possible to know if he can improve his payoff, as the off the equilibrium path is undetermined, and the potential payoffs could be higher (1.5) or lower (-2).
    - THUS, since the off the equilibrium path will determine if deviation for player 1 is possible or not, we can satisfy the pooling PBE, if we find the values of β and (1-β) that result in player 1 not having a profitable deviation.
    - Player 1 only has a profitable deviation when switching to w, if player 2 plays B. Thus, we have to find the values of β and (1- β) where player 2 will not play B, thus ensuring that player 1 will not have a profitable deviation.

  - Thus, the expected utility of player 2 when playing R must be greater than playing B, as expressed by: $3(\beta) + -2(1- \beta) < -2(\beta)+2(1- \beta)$
    - $3(\beta) -2(1- \beta) < -2(\beta)+2(1- \beta)$
    - $3\beta -2 + 2\beta < -2\beta + 2 - 2\beta$
    - $5\beta - 2 < -4\beta + 2$
    - $9\beta < 4$
    - $\beta < 4/9$
  - So when $\beta < 4/9$, PBE is satisfied.
- So thus, a Perfect Bayesian Equilibrium will have:
  - Player 1 choosing N no mater the type
  - Player 2 choosing R
- There are infinitely many pooling perfect Bayesian equilibrium, [(N, N), (R, R), μ=0.1, β<4/9],