Regression Discontinuity II: Extensions

Lecture 9 - Introduction to Causal Inference

Kevin Li

Fuzzy Regression Discontinuity

What if we have a cutoff $X = c$ that determines treatment D for most units, but not all.

- There are some people that should be treated based on the cutoff, but are "not complying" with the cutoff.

We can instead use a **Fuzzy Regression Discontinuity**.

- Basic idea: Use the cutoff $X = c$ as a **instrumental variable** for the actual treatment D.
- As long as the cutoff makes it more likely for someone to get treatment D, then the cutoff should be correlated with D (relevance).

Example and Setup

Example: scoring over 90% makes you **eligible** for a advanced-level class. However, you do not have to take the class.

- Issue: some people eligible for the advanced-class will decide not to take it.

Our research design will be as follows:

- Cutoff $X = c$ perfectly determines the instrument Z - whether you are "encouraged" to take the treatment D (the advanced class).
- Treatment D - whether someone actually takes the advanced class. Not all individuals "encouraged" in variable Z (above the cutoff) will take it, but being above the cutoff is correlated with taking the class.

Estimation

Stage 1: Estimate the causal effect of Z on Y, which we call τ_{ITT}

- This is done by running a Sharp regression discontinuity of cutoff $X = c$ with outcome variable Y (like last lecture).

Stage 2: Calculate Pr(compliers), the causal effect of Z on D:

- This is done by running a Sharp regression discontinuity of cutoff $X = c$ with **outcome variable** D**.**

Stage 3: Calculate τ_{LATE} using formula from lecture 7:

$$\tau_{\text{LATE}} = \frac{\tau_{\text{ITT}}}{Pr(\text{compliers})}$$

Note: **rdrobust** package in **R** does all three steps together, which makes things very simple.

Additional Notes on Fuzzy RDD

The τ_{LATE} of fuzzy RDD is a very narrow interpretation.

- It is the causal effect of only **compliers** who are near the cutoff $X = c$.
- Basically a combination of τ_{LATE} interpretation from IV and RDD.

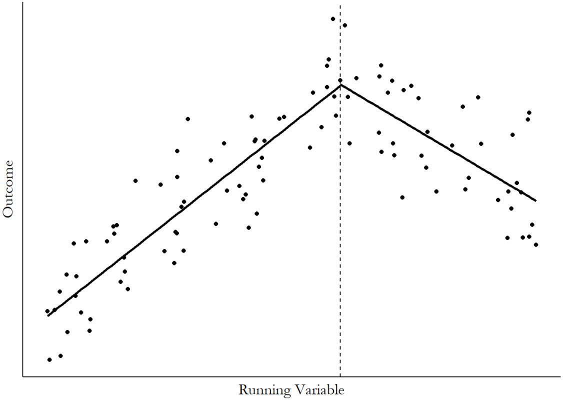
Assumptions also combine RDD with IV.

- We need continuity of potential outcomes of Y. We also need continuity in likelihood of being treated D (for the second stage RDD).
- We need IV assumptions of exclusions, relevance, and relevance.

Regression Kink

Regression Discontinuity focuses on if there is a "jump"/"discontinuity" in Y at some cutoff.

What if there isn't a "jump", but a change in the relationship (slope) between the running variable and outcome?



Regression Kink Setup

Our setup for regression kink is very similar to standard RDD:

- We have a treatment D and a outcome Y.
- Treatment D is assigned based on some **running** variable X. Some cutoff $X = c$ determines if a unit is treated or not treated.

Difference: Instead of looking for a jump in Y, we will look for a slope shift in the relationship between X and Y.

Thus, our goal is not to find the LATE, but instead, to find the **local average response**.

- Local average response is how much the relationship between X and Y changes at the cutoff $X = c$.

Derivatives and Discontinuity

Our goal in regression kink is to find a change in the slope. Let us first define the relationship between Y and X as a function:

$$f(X) = Y$$

- We will not specify any function form for now - this is just for illustration purposes.

What is the slope between X and Y? Well, it is the first derivative of the function $f(X) = Y$.

This implies that our goal in regression kink is to **find a discontinuity in the first derivative** $f'(X)$ at the cutoff point $X = c$.

Assumption: Continuity of Derivative

In regression discontinuity, one of the assumptions was that if there was no treatment at $X = c$, then there would be no "jump"/discontinuity in Y (so $Y_i(0)$ is continuous).

Our goal in regression kink is to find a discontinuity in the first derivative $f'(X)$ at the cutoff point $X = c$.

- Thus, our identification assumption in regression kink is that if there was no treatment, $f'(X)$ is continuous at the cutoff.
- Or in terms of potential outcomes, if $f(X) = Y_i(0)$, then $f'(X)$ must be continuous at the cutoff $c = 0$.

If this is not true (i.e. there is a "jump"/"discontinuity" in the slope/derivative at $X = c$ even without treatment), then we do not know if the "jump"/"discontinuity" we observe is a result of the treatment D.

Estimation

Estimation is done in a similar way as sharp regression discontinuity:

- Fit a best-fit line on both sides of the cutoff.
- Then, find the derivative of the best-fit line.
- Finally, check for any discontinuity in the derivatives.

The recommended procedure is to use the local linear regression method, just like in regression discontinuity.

- The **rdrobust** package allows for a simple option to turn a regression discontinuity into a regression kink.

We can also estimate this with a parametric model (linear, quadratic), but again, this is not recommended unless your sample size is very small.