# Difference-in-Differences I: Classical DiD

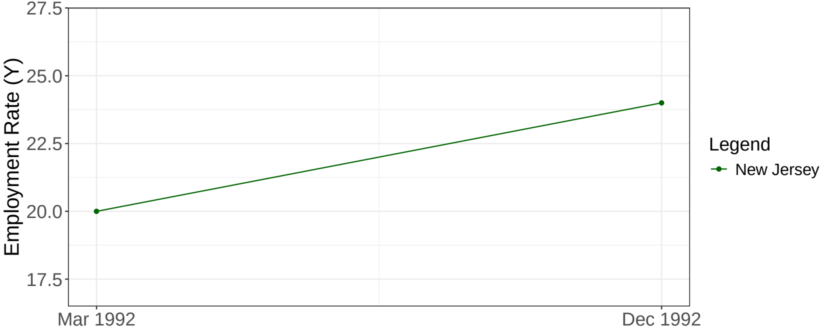
## Lecture 3 - Introduction to Causal Inference

Kevin Li

# New Jersey Employment

New Jersey implemented a minimum wage increase in April 1992. We want to find the causal effect of this on employment in fast food restaurants.

We have average data of all restaurant's employment rate from before and after the change.

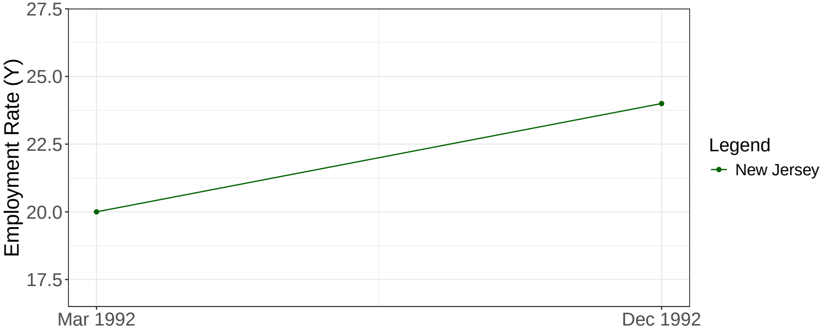


# Our Issue

Is our causal effect the Dec 1992 - Mar 1992? No!

Why? Something could have happened between March and December, like a recession or something else.

- How do we know if those factors are explaining the change in employment, or our treatment?

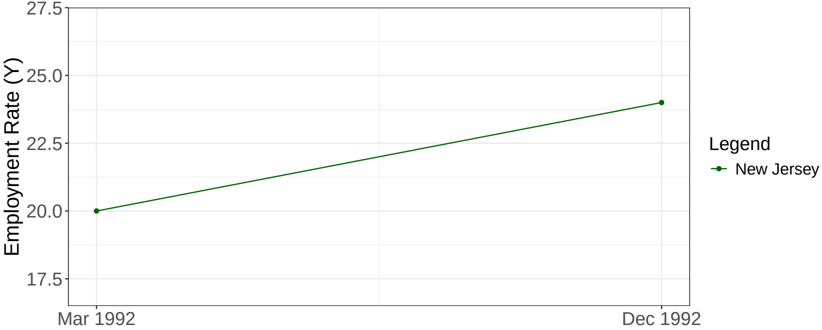


# Our Issue in Potential Outcomes

We can also think of this issue in terms of potential outcomes. Our causal effect in December 1992 is:

$$\text{Average } \tau_{\text{Dec}} = \overline{Y}_{\text{Dec}}(1) - \overline{Y}_{\text{Dec}}(0)$$

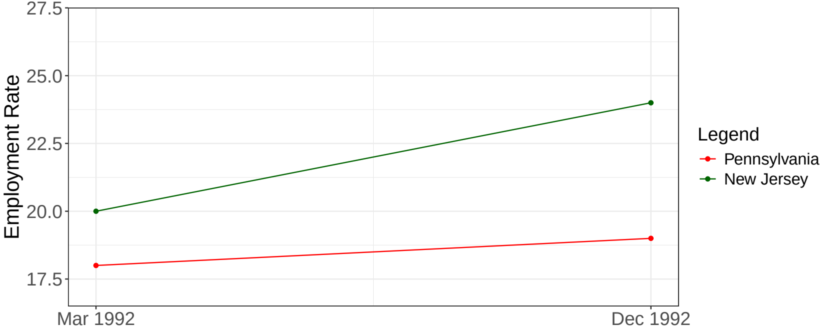
But we do not observe $\overline{Y}_{\text{Dec}}(0)$ since New Jersey is treated in Dec.



# Solution: Trend in Control Group

What if we looked at another state who did not have a minimum wage increase during this time, such as Pennsylvania?

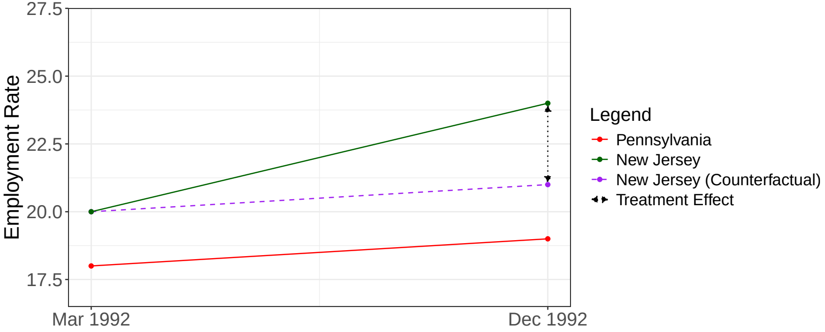
Pennsylvania tells us the **trend** in employment $Y$ if no treatment occurred.



# Solution: Parallel Trends

Then, we assume that if New Jersey had hypothetically not received a minimum wage increase, it would have followed the same trend as Pennsylvania.

- This allows us to estimate New Jersey's counterfactual, and find New Jersey's causal effect in December.



# Classical DiD

The example we just showed is an example of classical difference-in-differences. We have 2 groups of observations:

- A treated group $G = 1$. These are units that will receive treatment sometime in our study (like New Jersey).
- An untreated group $G = 0$. These are units that will not receive treatment (like Pennsylvania).

We have 2 time periods.

- In $t = 0$ (pre-treatment), both groups are untreated.
- In $t = 1$ (post-treatment), only the treated group is treated, while the control group remains untreated.

# Estimating Causal Effects

The trend in the untreated group $G = 0$ between time periods is:

$$\text{trend} = \overline{Y}_{t=1}^{(G=0)} - \overline{Y}_{t=0}^{(G=0)}$$

The estimated counterfactual for the treated group is thus:

$$\overline{Y}_{t=0}^{(G=1)} + (\text{trend}) = \overline{Y}_{t=1}^{(G=1)} + \left(\overline{Y}_{t=1}^{(G=0)} - \overline{Y}_{t=0}^{(G=0)}\right)$$

The estimated causal effect is thus:

$$\tau_{\text{ATT}} = \overline{Y}_{t=0}^{(G=1)} - \underbrace{\left[\overline{Y}_{t=1}^{(G=1)} + \left(\overline{Y}_{t=1}^{(G=0)} - \overline{Y}_{t=0}^{(G=0)}\right)\right]}_{\text{estimated counterfactual}}$$

# Why is it Called Difference-in-Differences?

Distribute out the negative sign, and you get:

$$\tau_{\mathsf{ATT}} = \underbrace{\left(\overline{Y}_{t=0}^{(G=1)} - \overline{Y}_{t=1}^{(G=1)}\right)}_{\Delta \text{ for } G=1} - \underbrace{\left(\overline{Y}_{t=1}^{(G=0)} - \overline{Y}_{t=0}^{(G=0)}\right)}_{\Delta \text{ for } G=0}$$

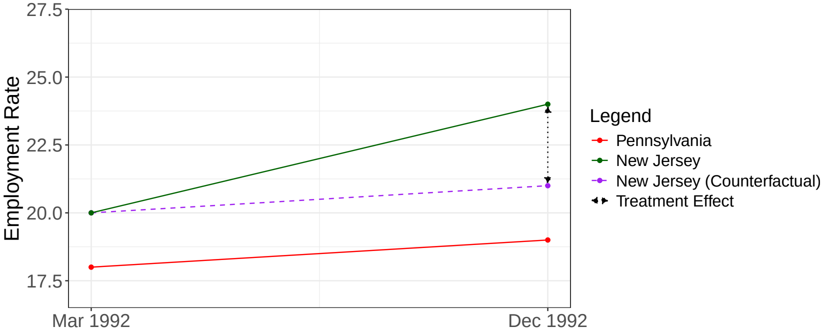
- The first part is the difference in average $Y$ between time periods for the treated group $G = 1$
- The second part is the difference in average $Y$ between time periods for the control group $G = 0$.

The total causal effect is the difference of these two differences.

# Parallel Trends Assumption

The causal estimate $\tau_{\text{ATT}}$ is only accurate if the parallel trends assumption is met.

- That if the treated group had really been untreated, it would have followed the same trend as the control group.



# Example of Violating Parallel Trends

Here, we can see the true counterfactual of New Jersey is not the same trend as Pennsylvania. So if we assumed the parallel trends assumption, our causal estimate would be very wrong.