Difference-in-Differences II: Generalised DiD

Lecture 4 - Introduction to Causal Inference

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More Time Periods

In Classical DiD, we have two time periods for units.

- $t = -1$ (pre-treatment) and $t = 0$ (post-treatment).

In Generalised DiD, we allow for more time periods.

- Multiple Pre-treatment periods $t = -1, -2, -3, ...$
- Multiple Post-treatment periods $t = 0, 1, 2, 3, ...$

For now, we assume panel data - i.e. the observed individuals/units in our data are observed throughout each time period.

- Ex. individual A has a value for each time period $..., -2, -1, 0, 1, 2, ...$

Relative Time Periods

In our data, treatment might start in any time-period, for example, the year $t = 2000$.

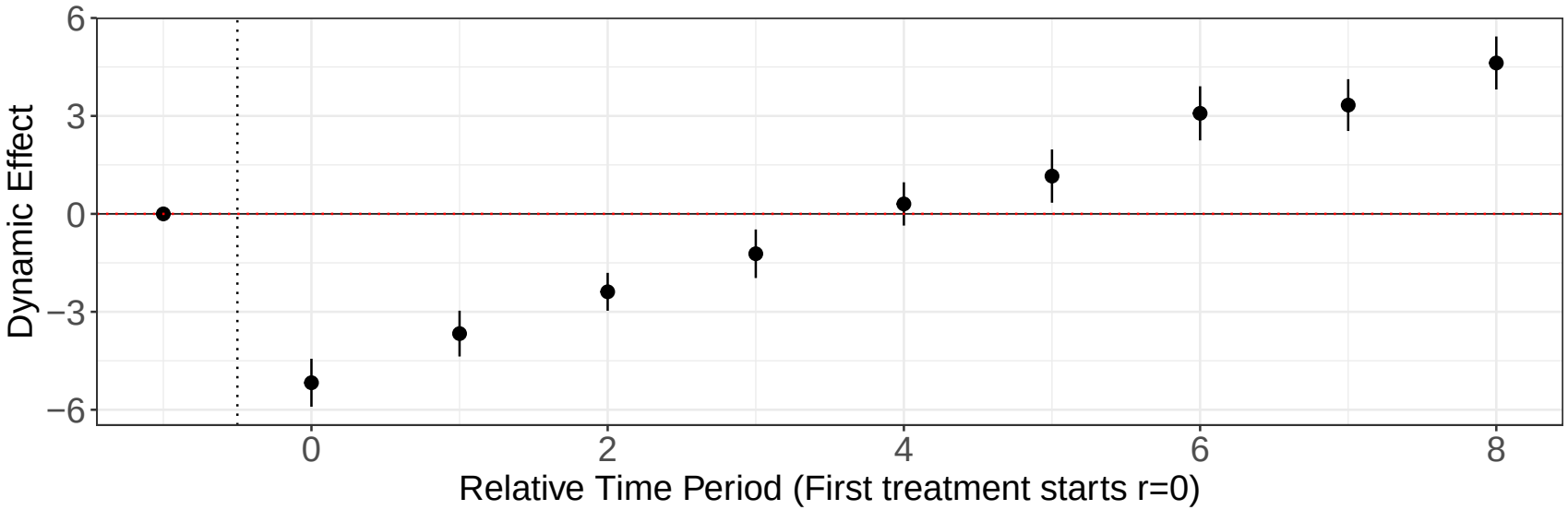
It is standard to create a new variable called **relative time** R - that indicates the time relative to initial treatment adoption for each cohort.

- The first treatment year in the relative time variable is always $R = 0$.
- Pre-treatment periods are always negative $R \leq -1$.
- Post-treatment periods are always positive $R \geq 0$.

For the control group who do not get treated ever, there is no R value for the observations.

Dynamic Treatment Effects

We can now estimate causal effects for each individual post-treatment time-period $t = 0, 1, 2, ...$



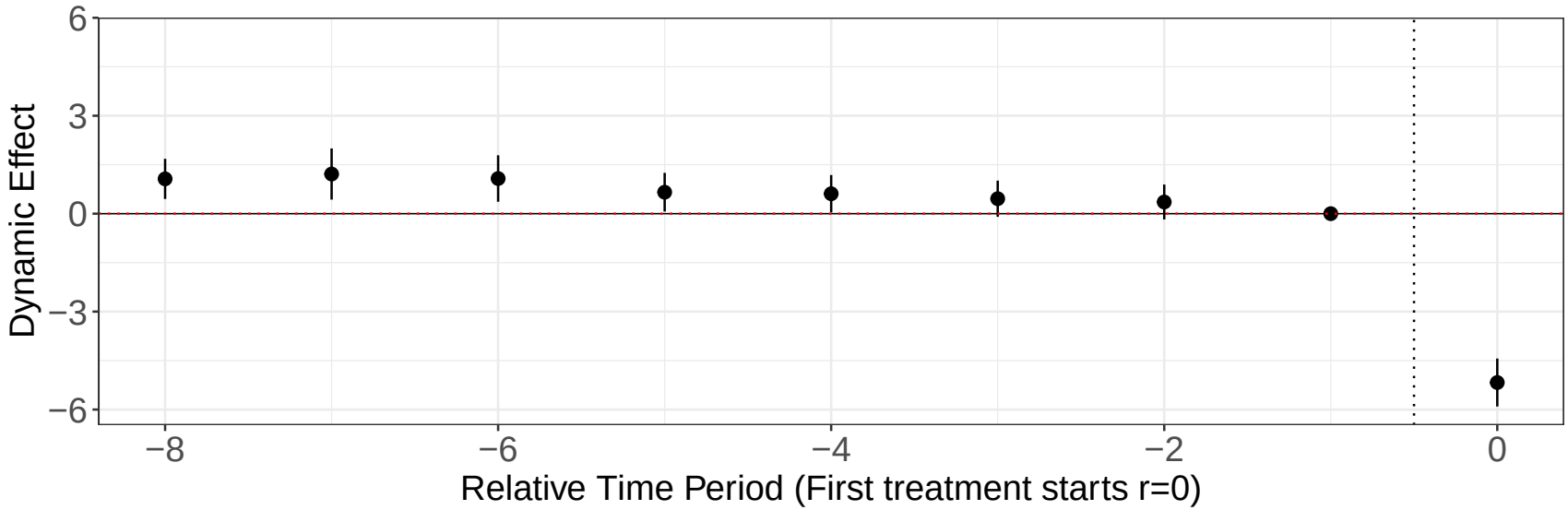
This allows us to see how the effect of treatment evolves over time.

- You can see treatment starts negative in earlier post-treatment periods, before becoming more positive over time.

Pre-Treatment Effects

We can also calculate estimated effects for pre-treatment periods $t = -1, -2, ...$ These test the **parallel trends assumption**.

- If the estimate is not equal to 0, or close to 0, that is evidence the parallel trends assumption is violated.



The more pre-treatment estimates being insignificant, the stronger evidence of meeting parallel trends.

Conditional Parallel Trends

Generalised DiD allows us to **control/condition** for potential variables that cause violations in parallel trends.

- Let us say we have some variable X, that is correlated with Y.
- If the trend of X values over time in the treated group and control group are different, it is likely that the trend in Y between the treated group and control group is different.
- Thus, we would want to condition on X - i.e. holding X constant, parallel trends is met.

This allows us to apply Generalised DiD to more situations than classical DiD.

Two-Way Fixed Effects

The estimator for Generalised DiD is the two-way fixed effects (TWFE) estimator. It is a linear regression:

$$\hat{Y}_{it} = \underbrace{\hat{\alpha}_i + \hat{\gamma}_t}_{\text{fixed effects}} + D_{it}\hat{\tau}_{\text{ATT}} + \mathbf{X}_{it}^{\top}\hat{\boldsymbol{\beta}}$$

- $\mathbf{X}_{it}^{\top}$ are the values of the variables we are using to condition for parallel trends. This is optional (only use if parallel trends is not met without them).
- $\hat{\alpha}_i$ and $\hat{\gamma}_t$ are unit and time fixed effects.

The estimated $\hat{\tau}_{\text{ATT}}$ from the regression is the overall causal estimate for all post-treatment periods. Standard errors should be clustered by units.

What are Fixed Effects

Unit fixed effects $\hat{\alpha}_i$ are intercepts in regression, like β_0. However, each unit i gets its own intercept value.

- These account for differences between units. Think of them as a control variable for units.

Time fixed effects $\hat{\gamma}_t$ are the same, but for time periods t. These account for differences between time periods.

The only remaining potential confounders is differences in trends over time between units. But if we meet the parallel trends assumption, this is also controlled for.

Thus, all confounders are controlled for, and thus, treatment D_{it} is exogenous. Thus, we can find the causal effect with TWFE.

TWFE In R

TWFE in R is as follows:

```
library(fixest)

model <- feols(
  fml  = Y ~ D + X1 + X2 | unit + time,
  data = df,
  vcov = ~unit
)
summary(model)
```

- X1 and X2 are confounders. If no confounders, just don't include any.

TWFE for Dynamic Treatment Effects

We can also use two-way fixed effects to estimate dynamic treatment effects (post-treatment) and pre-treatment estimates.

$$\hat{Y}_{it} = \underbrace{\hat{\alpha}_i + \hat{\gamma}_t}_{\text{fixed effects}} + \sum_{r \neq -1} I_{itr} G_i \hat{\tau}_r + \mathbf{X}_{it}^{\top} \hat{\boldsymbol{\beta}}$$

- r is the specific relative time period R of the observation in question (see slide 3).
- I_{itr} is an indicator, that takes value $I = 1$ if observation it is in relative time period $R = r$, and $I = 0$ otherwise. G_i is a treatment/control group indicator.
- The $\sum_r I_{itr} G_i$ basically says estimate a causal effect $\hat{\tau}_r$ for every relative time period r, except -1 (the last pre-treatment period).

Dynamic TWFE In R

```
library(fixest)
library(ggfixest)

model <- feols(
  fml  = Y ~ i(R, group, ref = -1) + X | unit + time,
  data = df,
  vcov = ~unit
)
ggiplot(model)
```

- R is the relative time period variable.
- group is a binary variable. Individuals that receive treatment at any time (treatment group) get value 1, individuals that never receive treatment (control group) get value 0.