Quantitative Methods

Companian to the series: Introduction to Political Economics

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Preface

This book is one of the 3 **mathematical-companian-guides** of the part of a series: **Introduction to Political Economics**. These companian guides are meant to ensure that one has the proper mathematical training prior to studying Political Economics:

1. **Quantitative Methods** (This Book) introduces the essential mathematical concepts that are essential for studying Political Economics. Topics include single variable calculus, introductory linear algebra, and introductory probability and statistical theory.
2. **Further Quantitative Methods** expands on the topics taught in the previous book. While these topics are not absolutely essential for Political Economics, they can be highly useful for a deeper understanding of Political Economics. Topics include advanced linear algebra and multivariate calculus.
3. **Introductory Proofs and Analysis** is a high-level introduction to proofs in mathematics. This is not essential for studying Political Economics, but having a strong idea behind proofs is quite useful for more advanced Microeconomic Models that are used in the study of Political Economics.

This book, Quantitative Methods, is a collection of mathematical and statistical topics that I consider to be essential to understand prior to starting any serious instruction in Political Economics. The book assumes a solid understanding of high-school level algebra, and discusses topics in single variable calculus, introductory linear algebra, and probability and statistical theory. Note that this book is not a full mathematics course of those topics - topics have been selected based on what is commonly used in Political Economics and other social sciences.

Part I

Single Variable Calculus

Chapter 1

Pre-Calculus

1.1 Set Theory

A **set** is a collection of objects. The objects within a set are the **elements** of that set.

We can define a set by either listing every element out, or through describing the properties of the elements of the set.

- For example, $A = \{1, 2, 3, 4, 5\}$
- Or, $A = \{n | n \in \mathbb{Z}; 1 \leq n \leq 5\}$ - or in other words, A is the set of values n, *such that* n is in the set of all *integers* Z, and n is between 1 and 5 inclusive.

Here are some common notation for sets:

- If two sets A and B have the same elements, they are equal: $A = B$.
- If a is an element of set A, we use the notation: $a \in A$.
- If a is not an element of set A, we use the notation: $a \notin A$.
- If all elements of set A are also within set B, then it is considered a **subset**, and notated $A \subseteq B$.
- If $A \subseteq B$, but $A \neq B$, then A is a **proper subset** of B, notated: $A \subset B$.

There are a few common **operations** of sets:

- The **union** of sets A and B, notated $A \cup B$, is the set of elements that belong to either A <u>or</u> B. For example, if $A = \{1, 2, 3\}, B = \{3, 4, 5\}$, then $A \cup B = \{1, 2, 3, 4, 5\}$.
- The **intersection** of sets, notated $A \cap B$, is the set of elements that belong to A <u>and</u> B. For example, if $A = \{1, 2, 3\}, B = \{3, 4, 5\}$, then $A \cap B = \{3\}$.

The **cardinality** of a set A is the number of elements within set A, and is notated $|A|$. For example, if $A = \{1, 2, 3\}$, then $|A| = 3$. An **empty set** is a set with a cardinality of 0, or in other words, with no elements. It is notated with $\emptyset$

A **infinite set** is one with infinitely many elements. Common examples of infinite sets include:

- Set of all natural numbers $\mathbb{N} = \{1, 2, 3, ...\}$.

- Set of all integers $\mathbb{Z} = \{..., -2, -1, 0, 1, 2, ...\}$.
- Set of all rational numbers $\mathbb{Q}$, which are all numbers that can be written in a fraction (so excluding numbers like π).
- Set of all real numbers $\mathbb{R}$, which is any number on the real number line.

The set of all real numbers $\mathbb{R}$ is the most common used set. A subset of $\mathbb{R}$ is called an **interval**, and can be notated with either brackets or parentheses as follows:

- Closed interval: $[a, b] = \{x \in \mathbb{R} | a \leq x \leq b\}$
- Open interval: $(a, b) = \{x \in \mathbb{R} | a < x < b\}$
- We can also mix closed and open intervals: $[a, b) = \{x \in \mathbb{R} | a \leq x < b\}$

In all these intervals, a and b are called **endpoints** of the interval. Any point inside of an interval that is not an endpoint is an **interior point**.

1.2 Exponents, Logarithms, Sum and Product Notation

1.3 Functions and the Cartesian Plane

1.4 Special Types of Functions

1.5 Limits and Continuity

Chapter 2

Derivatives

2.1 Definition of a Derivative

2.2 Rules of Differentiation

2.3 Derivative of an Inverse Function

2.4 Partial Derivatives

2.5 Application: Linear Regression

Chapter 3

Optimisation

3.1 Local Extrema and Stationary Points

3.2 Concavity and Inflextion Points

3.3 Global Extrema

3.4 Optimisation Without Constraints

3.5 Application: Linear Regression Estimation

3.6 Simple Optimisation With Constraints

Chapter 4

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4.1 Indefinite Integrals

section 9.1 and 9.3

4.2 Definite Integrals and Riemann Sums

4.3 Fundamental Theorem of Calculus

4.4 Integration by Recognition

4.5 Integration by Substitution

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Introductory Linear Algebra

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topics 3.2-3.3

5.2 Inverse Matrix

5.3 Simple Linear Systems of Square Matrix

5.4 Elementary Row Operations and Reduced Row Echelon Form

5.5 Theorems on Matrix Invertibility

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Part III

Probability and Statistical Theory

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7.4 Law of Total Probability

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Chapter 8

Random Variables

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Chapter 9

Distributions

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9.2 Binomial and Poisson Distribution

9.3 Continous Uniform and Exponential Distribution

9.4 Normal (Gaussian) Distribution

9.5 Normal Approximation of the Binomial Distribution

Chapter 10

Multivariate Random Variables

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10.2 Marginal Distributions

10.3 Continuous Multivaraite Distributions

10.4 Conditional Distributions

10.5 Covariance and Correlation

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10.7 Sums and Products of Random Variables