Further Quantitative Methods

Companian to the series: Introduction to Political Economics

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Preface

This book is one of the 3 **mathematical-companian-guides** of the part of a series: **Introduction to Political Economics**. These companian guides are meant to ensure that one has the proper mathematical training prior to studying Political Economics:

1. **Quantitative Methods** introduces the essential mathematical concepts that are essential for studying Political Economics. Topics include algebra, single variable calculus, and probability and statistical theory.
2. **Further Quantitative Methods** (This Book) expands on the topics taught in the previous book. These topics are not "essential", but it is highly recommended to have some grasp of these topics. Topics include linear algebra and multivariate calculus.
3. **Introductory Proofs and Analysis** is a high-level introduction to proofs in mathematics. This is not essential for studying Political Economics, but having a strong idea behind proofs is quite useful for more advanced Microeconomic Models that are used in the study of Political Economics.

This book, Further Quantitative Methods, is a collection of mathematical and statistical topics that I consider to very useful, although not absolutely essential, before instruction of Political Economics. This book assumes a strong understanding of high-school level algebra, as well as the algebra and calculus topics from *Quantitative Methods*. In this book, we first discuss topics in linear algebra, before moving on to multivariate calculus.

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Basics of Probability and Statistics

Chapter 3

Basic Probability

See Mathematical Methods for Political Economy for more detailed explanations.

3.1 Sets and Set Operators

A **set** is the collection of objects, while the **elements** of the set are the specific objects within a set. A capital letter is used to represent a set, for example, set A. A lowercase letter represents an element within the set. For example, element a is a part of set A.

There are a few different set operators that are important to understand.

An **intersection** of sets A and B, formally notated $A \cap B$, indicates the elements that are both within A and B at the same time.

- For example, if $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$, then $A \cap B = \{2, 3\}$, since those are the elements that are contained in both A and B at the same time.

A **union** of sets A and B, formally notated as $A \cup B$, indicates elements that are in either A, B, or both A and B.

- For example, if $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$, then $A \cap B = \{1, 2, 3, 4\}$.
- $A \cup B = A + B - A \cap B$. We subtract $A \cap B$ since that part is counted twice in both A and B, so we need to get rid of it once to avoid over-counting.

The **complement** of set A is everything that is not in A, but still within the universal set. The complement is denoted as A' or A^c.

- For example, if the universal set contains $\{1, 2, 3, 4, 5\}$, and $A = \{1, 2\}$, then $A' = \{3, 4, 5\}$

A **subset** A has all its elements belonging to another set B. This is notated $A \subset B$.

- For example, if $A = \{1, 2\}$, and $B = \{1, 2, 3\}$, then A is a subset of B since all of A's elements belong to set B as well.

3.2 Basic Properties of Probability

Kolmogrov's Axioms are the key properties of probability:

1. For any event A, the probability of A occurring is between 0 and 1.
2. The probability of all events in the sample space S is 1. Mathematically: $Pr(S) = 1$. The sample space is the set of all possible events.
3. If we have a group of mutually exclusive events $A_1, A_2, ..., A_k$, then the probability of those events all occurring is the sum of their probabilities. Mathematically, $Pr\left(\bigcup A_i\right) = \sum Pr(A_i)$
    - Note: mutually exclusive events are events that cannot occur at the same time together.

Other important properties to note include:

- $Pr(A') = 1 - Pr(A)$ - the probability of the complement of A, is equal to 1 minus the probability of A
- $Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$ - this is because of a property of unions, as shown in section 2.2

3.3 Joint and Conditional Probability

Joint Probability is the probability of two or more events occurring simultaneously. The joint probability of events A and B is notated $Pr(A \cap B)$.

For example, in a deck of cards, A could be the event of drawing an ace, and B could be the event of drawing a spade. Thus, $Pr(A \cap B)$ would be the probability of drawing a card that was both an ace and a spade.

Conditional Probability is the probability of one event occurring, given another has already occurred. Probability of event A, given event B has occurred, is notated as $Pr(A|B)$

To calculate the conditional probability, we use the following formula:

$$Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)}$$

3.4 Bayes' Theorem

Bayes' Theorem states the following relationship is true:

$$Pr(A|B) = \frac{Pr(B|A) \times Pr(A)}{Pr(B)}$$

Each part of Bayes' Theorem has a name. They are commonly referenced, so it is useful to know their names:

- $Pr(A|B)$ is the conditional probability
- $Pr(B|A)$ is the posterior probability
- $Pr(A)$ is the prior probability
- $Pr(B)$ is the marginal probability

Chapter 4

Basic Statistics

See Essential Mathematics for Political Economy for more detailed explanations.

4.1 Random Variables

Random variables are variables that represent unobserved events that have some randomness - a set of potential outcomes, with each outcome having a probability of occurring.

For example, if you flip a coin 10 times, and count the number of heads you get, you could get 5 heads, 6 heads, 4 heads, or any amount between 0 and 10. We are not sure what will happen - however, some outcomes are more likely than others, because of the probabilities associated with each outcome.

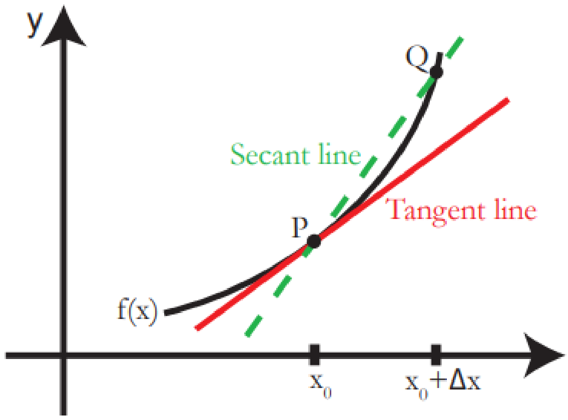
There are two types of random variables:

- **Discrete Random Variables** are random events which have a distinct number of outcomes. For example, rolling a dice has 6 outcomes.
- **Continuos Random Variables** are random events which have an infinite amount of outcomes. For example, my drive to work tomorrow could take 5 minutes, 5.123 minutes, 5.234237847 minutes, and so on… there is no distinct outcomes since you can continuously subdivide the gaps between outcomes by adding more decimal points.

4.2 Distributions and Probability Density Functions

Random variables are often called distributions - because there are a distribution of outcomes, with associated probabilities for each outcome. We can actually graph this - put potential outcomes on the x axis, and the probability that each outcome occurs on the y axis.

For example, take this probability distribution of a die - there are 6 sides that you could land on, and each has an equal probability of occurring:



The **probability mass/density function** $f(y)$ takes a potential outcome of an event as an input, and outputs the respective probability.

For example, the probability mass/density function of a dice is $f(y) = 1/6$. This is because every outcome y has the same probability of occurring: 1/6. So $f(1), f(2)... = 1/6$.

4.3 Expectation and Variance

Expectation and Variance are two ways we can summarise the distributions of random variables.

The **expectation**, often called the expected value or mean, is the best guess of an outcome of a random variable, given no other information except its distribution. We notate expected value of a variable Y as either $E[Y]$, $\bar{Y}$, or μ.

The expected value for discrete variables is calculated by multiplying each outcome value by its associated probability, then doing that for all outcomes, and summing everything together. In other words, it is a weighted average of the outcomes, with the weights being the probability of each outcome.

$$E[Y] = y_1 \times f(y_1) + y_2 \times f(y_2)... = \sum [y_j \times f(y_j)]$$

For a continuous random variable, it is a little more complicated. This is because continuous variables have an infinite number of potential outcomes. For example, if you drive to school, your driving time could be 23 minutes, or 23.12 minutes, or 23.123324 minutes... basically, an infinite amount. As a result, we have to alter the expected value formula a little:

$$E[Y] = \int_{-\infty}^{\infty} y \times f(y) dy$$

Variance σ^2 is a measure of how spread out our distribution is. Variance basically measures how far values are, on average, from the mean of the variable. Mathematically:

$$Var(X) = \sigma^2 = \frac{1}{n} \sum (X - \mu)^2 = E[(X - \mu)^2]$$