An Introduction to Political Economy

An Introduction to the application of Economic and Formal Mathematical Models to Political Phenomena

Kevin Lingfeng Li

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Preface

This book is an introduction to the field of Political Economy - the use of economic tools, such as econometrics and game theory, applied to the study of political topics.

This book in focuses on how we can use economic and formal mathematical models to explore political phenomena. The book starts out with an introduction to the field of Political Economy, as well as some neccesary probability theory. Then, I introduce the concept of economic modelling, also called formal mathematical modelling, game theory, or formal theory. Finally, I apply the models we have covered to selected topics in Political Economy research.

While this book goes over the creation of economic models of political situations, it does not go over how we use empirical data to prove/reject our models. The subsequent book, Introductory Econometrics for Political Economy, covers these methods in detail.

This book is meant to be a relatively approachable introduction to the field. However, as Political Economy depends on many economic tools, an rough understanding of Algebra, Single Variable Calculus, and simple Linear Algebra is required. You do not need to be a math wizard, or even good at solving mathematical problems - you simply need an understanding of the intuition behind some key techniques. This book comes with a companion manual - Essential Mathematics for Political Economy. It is recommended that anyone interested in Political Economy glance at the topics covered in the manual, to ensure that they have the mathematical background necessary to succeed.

I created this book as a way to revise for my exams, as well as provide a handy booklet where I could reference all the things I learned throughout my undergraduate and postgraduate degrees. I hope that this guide to Political Economy can be useful to not just me, but others also interested in the field.

Part I

Fundamentals and Background

Chapter 1

Introduction to Political Economy

Political Economy is a term that has many different uses. Historically, the term Political Economy was used to describe the field we know today as Economics. This was particularly the case prior to the mathematical turn that the field of Economics took starting in the 20th century. Famous writers such as Adam Smith, Karl Marx, and John Locke, often identified themselves as Political Economists.

Today, Political Economy still has many different meanings. There are currently three major approaches to Political Economy - who all agree that Political Economy is somewhere between Political Science and Economics - but disagree on the approaches to studying this intersection of disciplines. First, there is the more "economics" side of Political Economy, focusing on how government and power inequalities affect economics and the distribution of resources. Second, there is the field of International Political Economy, which studies how economics interacts with International Relations. Finally, there is the field that applies economic models and methods to the study of Political and Economic phenomena.

This book mainly focuses on the third approach of Political Economy. For this book, Political Economy is a field that uses tools from economics, primarily game theory and econometrics, and applies these tools to study how political institutions, actors, and choices affect political and economic outcomes.

However, what exactly does that mean? How do we use these methods?

In the Social Sciences, there are two parts to any research - the hypothesis/theory, and the empirical data that either supports or refutes the theory. Economic tools perfectly fit into this framework. Political Economists will often use game theory and formal mathematical models to make predictions about potential outcomes of political situations. Then, Political Economists will gather real-world data, and test their hypothesis with statistical/econometric methods.

This book focuses on the economic modelling and creating hypotheses part of Political Economy Research. I start with exploring the funamentals of Game Theory and Formal Mathematical Modelling. These include the properties of games, solution concepts, as well as different types of common games. After the introduction to economic modelling, we will apply what we have learned to model current issues in Political Economy reseearch.

While this book goes over the creation of economic models of political situations, it does not go over how we use empirical data to prove/reject our models. The subsequent book, Introductory Econometrics for Political Economy, covers these methods in detail.

Let us begin!

Chapter 2

Concepts in Probability

Game Theory and Mathematical Models depend on many aspects of probability. This chapter will go through what I consider the "essential" probability you must know to understand the later chapters. It is highly recommended to read more about probability theory (and Calculus), either through a math textbook, or through consulting the "Essential Mathematics for Political Economy" companion manual.

2.1 Basics of Sets

A **set** is the collection of objects, while the **elements** of the set are the specific objects within a set. A capital letter is used to represent a set, for example, set A. A lowercase letter represents an element within the set. For example, element a is a part of set A.

There are a few common types of sets we use often:

- N is the set of natural numbers - the numbers we use to count from 1: $\{1, 2, 3...\}$
- Z is the set of integers - non-decimal numbers both negative and positive: $\{..., -2, -1, 0, 1, 2...\}$
- R is the set of all real numbers - any numbers that are on the number line, including decimals

We can define sets in multiple ways.

- We can list out each element of a set. For example $A = \{1, 2, 4, 6\}$

- We can define them with an interval. For example, $A = [0, 1]$ means all values within the range of 0 to 1, including 0 and 1.
- We can define them in formal notation. For example, $A = \{x : 0 \leq x \leq 1, x \in R\}$. This literally means: x such that x is between 0 and 1, including 0 and 1, and x is in the set of all real numbers R.

Useful notation tips:

- The semicolon : means "such that", and the sign $\in$ means "in" or "belongs to".
- When we put absolute value bars around a set, like $|A|$, that refers to the number of elements within that set.

2.2 Set Operators

There are a few different set operators that are important to understand.

An **intersection** of sets A and B, formally notated $A \cap B$, indicates the elements that are both within A and B at the same time.

- For example, if $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$, then $A \cap B = \{2, 3\}$, since those are the elements that are contained in both A and B at the same time.

A **union** of sets A and B, formally notated as $A \cup B$, indicates elements that are in either A, B, or both A and B.

- For example, if $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$, then $A \cap B = \{1, 2, 3, 4\}$.
- $A \cup B = A + B - A \cap B$. We subtract $A \cap B$ since that part is counted twice in both A and B, so we need to get rid of it once to avoid over-counting.

The **complement** of set A is everything that is not in A, but still within the universal set. The complement is denoted as A' or A^c.

- For example, if the universal set contains $\{1, 2, 3, 4, 5\}$, and $A = \{1, 2\}$, then $A' = \{3, 4, 5\}$

A **subset** A has all its elements belonging to another set B. This is notated $A \subset B$.

- For example, if $A = \{1, 2\}$, and $B = \{1, 2, 3\}$, then A is a subset of B since all of A's elements belong to set B as well.

2.3 Basic Properties of Probability

Kolmogrov's Axioms are the key properties of probability:

1. For any event A, the probability of A occurring is between 0 and 1. Mathematically: $0 \leq Pr(A) \leq 1$
2. The probability of all events in the sample space S is 1. Mathematically: $Pr(S) = 1$. The sample space is the set of all possible events.
3. If we have a group of mutually exclusive events $A_1, A_2, ..., A_k$, then the probability of those events all occurring is the sum of their probabilities. Mathematically, $Pr\left(\bigcup A_i\right) = \sum Pr(A_i)$
   - Note: mutually exclusive events are events that cannot occur at the same time together.

Other important properties to note include:

- $Pr(A') = 1 - Pr(A)$ - the probability of the complement of A, is equal to 1 minus the probability of A

- $Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$ - this is because of a property of unions, as shown in section 2.2

2.4 Joint and Conditional Probability

Joint Probability is the probability of two or more events occurring simultaneously. The joint probability of events A and B is notated $Pr(A \cap B)$.

For example, in a deck of cards, A could be the event of drawing an ace, and B could be the event of drawing a spade. Thus, $Pr(A \cap B)$ would be the probability of drawing a card that was both an ace and a spade.

Conditional Probability is the probability of one event occurring, given another has already occurred. Probability of event A, given event B has occurred, is notated as $Pr(A|B)$

To calculate the conditional probability, we use the following formula:

$$Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)}$$

2.5 Bayes' Theorem

Bayes' theorem is arguable the most important theorem in all of probability and statistics. Thus, instead of just telling you Bayes' theorem, we will actually derive it.

We start with the definition of conditional probability, as seen in the last lesson

$$Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)}$$

Let us rearrange the equation by solving for $Pr(A \cap B)$. We will get:

$$Pr(A \cap B) = Pr(A|B) \times P(B)$$

Now, let us consider the conditional probability of $B|A$ (the opposite way around). We use the same conditional probability formula, but switch the B and A. We get:

$$Pr(B|A) = \frac{Pr(B \cap A)}{Pr(A)}$$

Let us rearrange the equation by solving for $Pr(B \cap A)$. We will get:

$$Pr(B \cap A) = P(B|A) \times P(A)$$

Now we have two different equations, one for $Pr(A \cap B)$, and one for $Pr(B \cap A)$. Based on the commutative property of sets, we know that $Pr(A \cap B) = Pr(B \cap A)$. Thus, the other parts of the equation must also be equal to each other:

$$Pr(A|B) \times Pr(B) = Pr(B|A) \times Pr(A)$$

Now, let us solve for $Pr(A|B)$. After we do this, we will get the final form of **Bayes' Theorem**:

$$Pr(A|B) = \frac{Pr(B|A) \times Pr(A)}{Pr(B)}$$

Each part of Bayes' Theorem has a name. They are commonly referenced, so it is useful to know their names:

- $Pr(A|B)$ is the conditional probability
- $Pr(B|A)$ is the posterior probability
- $Pr(A)$ is the prior probability
- $Pr(B)$ is the marginal probability

Part II

Formal Mathematical Models

Chapter 3

Introduction to Game Theory

3.1 The Importance of Modelling

Game Theory, also called Formal Theory, is a way to use models to represent the world around us.

A model is a simplified version of reality to help us understand the complex world around us. This is especially useful in the exceedingly complex world of the social sciences.

To simplify reality, models make **assumptions**. These assumptions should be similar to reality, so we can draw conclusions from our model to the real world. However, these assumptions should also make the model simple, and only focus on the relevant features and details.

For example, a driving map is a model. It does not contain all the information in the world - it does not record the height of buildings, the colour of the terrain, and so on. It only contains streets, street names, and addresses. It even simplifies the spherical earth into a 2-dimensional flat map.

However, despite its simplifications of reality, a driving map is amazing for navigation. If you are trying to get from point A to point B by car, you do not care about the spherical nature of the globe. The simplifying assumptions on a driving map allow us to navigate the world around us, without being a perfect copy of reality.

There are several different approaches to modelling:

- Decision theory focuses on one individual, who aims to take actions to maximise their objective.
- Competitive equilibrium focuses on prices and quantities, unaffected by individual actions, such that a market clears and hits equilibrium
- Game theory focuses on social interactions that are characterised by strategic interdependence - one person's actions affect another's outcomes

3.2 Game Theory

Game Theory is the study of mathematical models of conflict and cooperation between rational players. The key property of game theory is that one player's actions, affect another player's outcomes/gains/payoffs.

Other key properties of Game Theory include:

- There is **common knowledge** of the rules of the game. This basically means that every player knows that every other player knows the rules of the game.
- A player will predict what their opponents will do, and will react to their opponents.
- The optimal decision of a player, depends on their beliefs on what the other players will do.

3.3 Properties of Games

Games are the environment in which strategic interaction between players occurs. For each game, it must consist of:

1. A set of **players**
2. For each player, a set of **actions** and **strategies**
3. For each player, **preferences** over the outcomes associated with the actions, such that they have an opinion on every possible outcome of the game

Players will take actions, and they yield some outcomes. The pathway between actions and outcomes is the **environment** - essentially the function that takes your action input, and outputs your outcome.

A player's belief on the environment - which includes the rules of the game, the other players' strategies, and the other players' beliefs, influence a player's choice of actions.

A player's preferences over the outcomes of games, in game theory, must be both **complete** and **transitive**.

- Complete preferences means that given any 2 outcomes, you can always compare them. Or in more intuitive terms, the player always has an opinion on the outcome - they are never "not sure" about how they feel about one outcome compared to another (however, they can be indifferent).
- Transtive preferences means that if you prefer a over b, and b over c, then you prefer a over c. It is basically that your preferences make sense and are consistent.

Preferences over outcomes can either be **ordinal** or **cardinal**

- Ordinal preferences means that you can rank your preferences of the potential outcomes, but you cannot give a specific value to that preference. For example, you can say you like a more than b, but you cannot say exactly how much you like a over b
- Cardinal preferences means that not only can you rank your preferences, you can also describe the magnitude of your preferences. For example, if you say you like a 3 times better than b, then you have cardinal preferences

We can assign numbers to both cardinal and ordinal preferences. Just be aware, if preferences are ordinal (and they often are), the numbers represent relative rankings, not specific magnitudes of preference.

- For example, if preferences are ordinal, the "distance" between preference values 1 and 2, may not be equal to the distance between 2 and 3

3.4 Types of Games

Games in Game Theory can be categorised according to two key properties: the **timing** of a game, and the **availability of information** in the game.

The timing of the game determines how players move relative to each other:

- A **static game** is when all players move "simultaneously". This does not actually mean they have to move at the exact same moment - it simply means that when one player moves, they cannot yet observe how the other player has moved. Thus, the players must anticipate the other players moves.
- A **dynamic game** is when players move sequentially, in an order. These are also called extensive form games. This means that players can see

how their opponents moved, before they choose their move. For example, Chess is a dynamic game.

The availability of information is another distinction between types of games:

- A game of **complete information** is one where all payoffs are known to all players.
- A game of **incomplete information** is when some payoffs or some actions are not common knowledge. These games are quite a bit more complex.

We will start with static games of complete information - the simplest form and the best type of game to introduce core concepts.

3.5 Game Theory Notation

Before we start discussing games, we need to discuss notation. Notation is key, because it allows us to be very precise and concise in defining our games.

Players are denoted $i = 1, 2, ...N$.

- When we say player i, we are referring to any specific player in the game.
- When we say player 1, or player 2..., we are referring to a specific player in the game.
- N represents the total number of players in a game.

The actions of a player i are denoted a_i

- So, the actions of player 1 are denoted a_1, and so on

The strategies of a player i are denoted s_i

- A player's selected strategy, if applicable, is denoted with a star: s_i^*
- All other strategies are denoted with an apostrophe (more specifically, a set complement sign): s_i'

The action profile (of all players' actions) is a vector $a = (a_1, a_2, ..., a_N)$

- We can split this vector into 2 different parts. a_i denotes player i's actions, while a_{-i} denotes the action profiles of all other players not player i

Preferences are represented by a payoff function $u_i(a_i, a_{-i})$

- Essentially, u_i is a function with the inputs of player i's actions a_i, and the actions of everyone else a_{-i}

Chapter 4

Dominant Strategy Equilibrium

4.1 Dominant Strategies

As we have established, game theory is when one person's actions affect another's payoffs. Thus, players have to anticipate what other players will do, and act accordingly.

However, sometimes, one of the actions available to a player i, is always better than any of the other actions available to them, no matter what their opponent decides to do. This strategy that is always better than the alternatives, no matter what the other player does, is called a **dominant strategy**.

Formally, a dominant strategy, notated s_i^D, is a dominant strategy, if it is player i's best strategy, no matter the strategies the other players pick. No matter what the opponent does, the payoff is highest for player i when they play s_i^D

Mathematically, we can represent that statement as following:

$$u_i(s_i^D, s{-}i) > u_i(s_i', s_{-i})$$

Which translates into words as - the utility of player i choosing their dominant strategy s_i^D, is always greater than the utility of player i choosing another strategy s_i', no matter the strategies chosen by the opponents s_{-i}

If player i has a dominant strategy s_i^D, then their other strategies s_i' are considered to be **dominated**.

4.2 Dominant Strategies in Prisoner's Dilemma

An example of a dominant strategy is the famous game: prisoner's dilemma. The story is as follows:

Two suspects are arrested. The police holds the suspects in separate cells and explains the consequences of their actions. If they both remain quiet (cooperate with each other), then both will be convicted for minor offences and sentenced to 1 month in jail. If both rat each other out (defect), then both will be sentenced to jail for 8 months. Finally, if one defects but the other does not, then the confessor is immediately released and the other is sentenced to 10 months, 8 for the crime and 2 for obstructing justice.

We can represent this game in a matrix, which is often called the normal form of the game. Note, the first value in each cell is the payoff of player 1 (p1), and the second value in each cell is the payoff of player 2 (p2).

	Cooperate (p2)	Defect (p2)
Cooperate (p1)	-1, -1	-10, 0
Defect (p1)	0, -10	-8, -8

Let us look at player 1. What is the best move for player 1?

Pretend player 2 plays cooperate. Thus, we can hide the defect column:

	Cooperate (p2)
Cooperate (p1)	-1, -1
Defect (p1)	0, -10

What is player 1's best strategy? Well, it is to defect, since a payoff of 0 is greater than a payoff of 1. <u>So, when player 2 picks cooperate, player 1 should pick defect</u>.

Now, pretend player 2 plays defect. Thus, we can hide the cooperate column:

	Defect (p2)
Cooperate (p1)	-10, 0
Defect (p1)	-8, -8

What is player 1's best strategy? Well, it is to defect, since a payoff of -8 is greater than a pyoff of 1. So, when player 2 picks defect, player 1 should pick defect.

So, we have established that player 1 will pick strategy defect, for both the scenarios that player 2 picks cooperate or player 2 picks defect. So, player 1 should always pick defect.

Thus, player 1's strategy of defect is the strategy that maximises their payoff, regardless of the strategy of player 2. Thus, player 1's strategy of defect is a dominant strategy.

We can do the same thing for player 2. We first pretend player 1 will pick cooperate, and we find player 2 should pick defect as their best strategy. Then we pretend player 1 will play defect, and we find player 2 should also pick defect as their best strategy. Thus, player 2 also has a dominant strategy of defect.

The Prisoner's Dilemma thus has a dominant strategy for both players - defect.

4.3 Solution Concepts

One of the key things about modelling situations is that, well, we want to find a likely outcome. An equilibrium solution concept is a concept that shows a likely solution to a game, by restricting potential strategies being played to the most reasonable one.

Any equilibrium solution concept has a few assumptions:

1. Players are rational - they choose actions that maximise their own payoffs. Without this assumption, it is very difficult to find a likely outcome
2. Structure of the game and player's rationality is common knowledge. If players do not know about the structure of the game, they well, cannot make best response decisions to their opponents
3. Any predictions must be self-enforcing - essentially, if we end up in a solution outcome, the players should not have any reason to want to leave that outcome, or else it would not be a final solution.

4.4 Dominant Strategy Equilibrium

A dominant strategy equilibrium is a solution concept where all players play dominant strategies.

- After all, if everyone has a best response to all other strategies of other players, and everyone is playing their best response, that is a very likely outcome of the game.

Formally, a strategy profile of all players $S^D = (S_1^D, S_2^D, ..., S_N^D)$ is a dominant strategy equilibrium, if for all players i, $u_i(S_i^D, s_{-i}) > u_i(s_i', s_{-i})$ for all s_{-i}

- Thie second part is the definition of dominant strategy. So, this basically says that a a set of strategies (strategy profile) is a dominant strategy equilibrium, given every player is playing a dominant strategy.

For example, take the **prisoner's dilemma**. We have already established that for both players, defect is a dominant strategy. Thus, if both players play defect, that is a dominant strategy equilibrium.

There, at most, can only be one dominant strategy equilibrium in a game. This makes dominant strategy equilibrium a great solution concept for predicting - as there will always only be one solution.

However, many games do not have any dominant strategies, and thus, many games do not have a dominant strategy equilibrium. As a result, this solution concept is not possible on a vast majority of games.

- After all, if all games had one strategy that was better than the rest, no matter what the opponents did, game theory would be trivially easy, and a pointless exercise.

Chapter 5

Nash Equilibrium

5.1 Best Response

We previously discussed that dominant strategies do not appear in most games, and thus, we need an alternate way of determining a solution.

Thus, we need to introduce the concept of a **best response**. We already introduced the intuition of a best response in the section on the prisoner's dilemma.

5.2 Finding Best Responses

5.3 Nash Equilibrium

5.4 Finding Nash Equilibrium

5.5 Multiple Nash Equilibrium and Focal Points

5.6 Trembling Hand Perfect Equilibrium

Chapter 6

Downsian Spatial Models

Chapter 7

Mixed Strategies

Chapter 8

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Chapter 9

Elections as Incentive Devices

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Chapter 11

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Part III

Topics in Political Economy