# Problem Set Week 5

GV4C8 Game Theory for Political Science

Kevin Li

## Question 4

**The Department of Energy and Climate Change needs to decide on a carbon tax to companies with large energy consumption (the tax rate can be any number between 0 and 100). The current rate is 7 per cent of their profits. The Department's preferred rate is 9 percent. Once the Department proposes a bill, the legislature is free to amend it before taking a final vote. When the preferred rate of a player is $r_i$, the utility they derive from rate $r$ is $u_i(r) = -|r - ri|$.**

- **a) What is the Department doing when the legislature's preferred policy is 10%? What about 6%?**
- **b) Identify the range of values of the preferred policy of the median member of the legislature for which no bill is proposed.**

Let us define the parameters and notation of this game:

1. Department $D$ is the "gatekeeper", with preferences at $r_D = 9$. They can propose some rate $\tilde{r} \in [0, 100]$, or not propose legislation.
2. Legislature $L$ has preferences at $r_L$. Status quo policy is $r_0 = 7$.

I will use $\succ$ to indicate strict preference, $\succsim$ to indicate weak preference, $\sim$ to indicate indifference, and $s.t.$ to indicate "such that".

What if $r_L = 10$? Let us solve by backwards induction.

Since it is open rule, the legislature will always amend the department proposal $\tilde{r}$ to $r_L = 10$.

Now, look at the department's choice of strategy. Department will only propose legislation $\tilde{r}$ if for themselves, $r_L \succ r_0$ (or $r_L \succsim r_0$ if indifference means acceptance, but this is not specified in the problem, and does not change the solution). Is $r_L \succ r_0$ for the department true for this scenario?

- $u_D(r_L) = -|r_L - r_D| = -|10 - 9| = -1$.
- $u_D(r_0) = -|r_0 - r_D| = -|7 - 9| = -2$.

My Answer!

Since $u_D(r_L) > u_D(r_0)$, then $r_L \succ r_0$ for the department. Thus, **department will propose legislation** when $r_L = 10$.

What if $r_L = 6$? Let us solve by backwards induction

Since it is open rule, the legislature will always amend the department proposal $\tilde{r}$ to $r_L = 6$.

Now, look at the department's choice of strategy. Department will only propose legislation $\tilde{r}$ if for themselves, $r_L \succ r_0$ (or or $r_L \succsim r_0$ if indifference means acceptance, but this is not specified in the problem, and does not change the solution). Is $r_L \succ r_0$ for the department true in this scenario?

- $u_D(r_L) = -|r_L - r_D| = -|6-9| = -3$.
- $u_D(r_0) = -|r_0 - r_D| = -|7-9| = -2$.

My Answer!

Since $u_D(r_L) < u_D(r_0)$, then the department will $r_0 \succ r_L$. Thus, Department likes the status quo more, and **will not propose any legislation** when $r_L = 6$.

What is the range of values of the preferred policy of the median member of the legislature, $r_L$, such that no bill will be proposed by the department?

We know that the department will not propose legislation when $r_0 \succsim r_L$. Thus, we must find the set of all $r_L$ s.t. $r_0 \succsim r_L$.

- This assumes indifference (of the department) means the department will not propose. If that is not the case (indifference means the department proposes), then condition is $r_0 \succ r_L$.

$r_0 \succsim r_L$ if and only if $u_D(r_0) \geq u_D(r_L)$. So, let us solve for what $r_L$ makes that previous statement true: (pardon my bad math skills)

$$\begin{aligned}
u_D(r_0) &\geq u_D(r_L) \text{ given } r_0 = 7, r_D = 9 \\
-|r_0 - r_D| &\geq -|r_L - r_D| \\
-|7-9| &\geq -|r_L - 9| \\
-2 &\geq -|r_L - 9| \\
2 &\leq |r_L - 9| \\
r_L &\leq 7, \ r_L \geq 11
\end{aligned}$$

My Answer!

So $r_0 \succsim r_L$ when $r_L \leq 7$ or $r_L \geq 11$, where the department will not propose anything.

More formally, the range of values of $r_L$, for which no bill is proposed by the department, are all $\{r_L \in \mathbb{R} | 0 \leq r_L \leq 7 \lor 11 \leq r_L \leq 100\}$ - the set of $r_L$ in all real numbers, such that $0 \leq r_L \leq 7$ **or** $11 \leq r_L \leq 100$.

This assumes indifference (of the department) means the department will not propose. If that is not the case (indifference means the department will propose), then the condition is $r_0 \succ r_L$.

- This would essentially mean changing all   to $<$.