Problem Set Week 7

GV4C8 Game Theory for Political Science

Kevin Li

Question 3

Distributive politics (bargaining over the division of a pie with more than **2** players). Three legislators need to distribute a budget of 100. There are two rounds of negotiations.

- In the first round player 1 proposes a distribution of the budget among the **3** legislators. If a majority accepts the budget the game ends and the budget distributed accordingly. If the budget is rejected, the game continues to the second round.

- In the second round, one of the **3** players is randomly selected to propose a distribution. If a majority accepts the budget the game ends and the budget distributed accordingly. If the budget is rejected, the status quo budget (**25, 25, 50**) is implemented.

Assume that players discount the future by   and that the payoff of the players in each period is equal to the money they get.

- a) Model this situation as an extensive form game

- b) Solve the game by backwards induction. What is the SPNE of the game? (Assume that legislators vote in favour of a budget when they are indifferent)

- c) Assume now that unanimity is required for a proposal to be accepted (i.e. all legislators need to vote in favour). How would your responses to a and b change?

Part a) Model
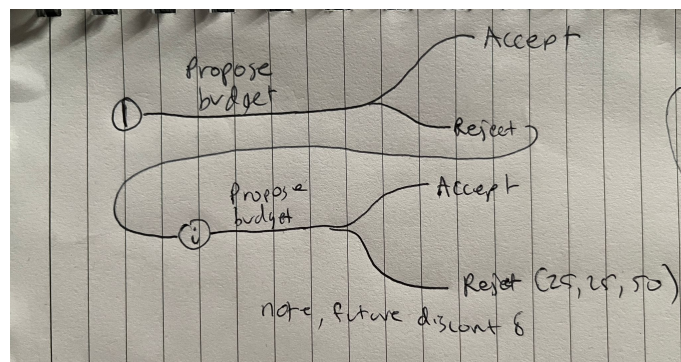
Part b) Majority Rule

2nd round: What budget would player i propose here?

- i must propose some budget that themselves, and one other player want to accept (since majority rule).

- i also wants to maximise their own payoff. The other players will only accept if the proposal gives them equal or more payoff.

- Thus, i will select a coalition partner of either player 1 or player 2 (since it makes no sense to partner with player 3, since you would need to give them 50 or more payoff).

Thus, player i will propose a budget such that they get 75, one other player (player 1 or 2) gets 25, and the third player always gets 0.

Now, let us calculate the expected utilities of each player in this second round. Note that all of second round is discounted by δ.

Thus, expected utilities of the second round are:

1. $\mathbb{E}[u_1] = \left[\frac{1}{3}(75) + \frac{1}{3}(25) + \frac{1}{3}(\frac{1}{2})(25)\right]\delta = (\frac{100}{3} + \frac{25}{6})\delta = \frac{225}{6} = \frac{75}{2}\delta$
   - Why? if P1 is selected as proposer (1/3 chance), she get 75
   - If P2 is selected as proposer (1/3 chance), P1 is in coalition, get 25
   - If P3 is selected as proposer (1/3 chance), there is 1/2 chance they are in coalition.
2. $\mathbb{E}[U_2] = \left[\frac{1}{3}(25) + \frac{1}{3}(75) + \frac{1}{3}(\frac{1}{2})(25)\right]\delta = \frac{75}{2}\delta$
   - If P1 is selected as proposer (1/3 chance), P2 is in coalition, get 25
   - If P2 is selected as proposer (1/3 chance), she gets 75
   - If P3 is selected as proposer (1/3 chance), there is 1/2 chance they are in coalition.
3. $\mathbb{E}[u_3] = \left[\frac{1}{3}(0) + \frac{1}{3}(0) + \frac{1}{3}(75)\right]\delta = 25\delta$
   - If P1 is selected as proposer (1/3 chance), they never choose P3 as coalition partner
   - If P2 is selected as proposer (1/3 chance), they never choose P3 as coalition partner
   - If P3 is selected as proposer (1/3 chance), they get 75

1st round:

Player 1 is proposing. They must get one other player on board with their proposal. Which player should they form the coalition with?

- Well, since $25\delta < \frac{75}{2}\delta$, then player 1 should include player 3 in the coalition since they are less costly to convince.

Thus, player 1 (maximising their own payoff) would propose: $(100 - 25\delta,\ 0,\ 25\delta)$.

> 💡 My Answer
>
> Thus SPNE outcome is:
> - P1 proposes budget $(100 - 25\delta,\ 0,\ 25\delta)$
> - P3 and P1 accepts that budget, P2 rejects. That is a majority. Thus that is final budget.
>
> Formal SPNE depends on δ value (since round 2 is off equilibrium path), but i'm too lazy.

Part c) Unanimity

2nd round: What budget would player i propose here?

- If they want proposal to be accepted, they must propose $(25, 25, 50)$, since that is the only way to not make anyone else worse off (compared to status quo) given the limited budget.

Thus, expected utilities are (doesn't matter proposer or not, since status quo and proposed budget are the same):

- $\mathbb{E}[u_1] = 25\delta$
- $\mathbb{E}[u_2] = 25\delta$
- $\mathbb{E}[u_3] = 50\delta$

1st round: player 1 is proposing. Must make all players better off than the expected 2nd round utilities in order to pass budget.

Thus, P1 will give P2 and P3 their expected 2nd round utilities (since indifference means acceptance), and P1 will take the remainder - $100 - 25\delta - 25\delta = 100 - 75\delta$.

Thus, proposal will be: $(100 - 75\delta, \ 25\delta, \ 50\delta)$.

> **💡 My Answer**
>
> Thus SPNE outcome is:
> - Player 1 proposes budget $(100 - 75\delta, \ 25\delta, \ 50\delta)$$
> - All 3 players agree, budget is passed.
>
> Formally, SPNE is (given 1st strategy is 1st round, 2nd strategy is 2nd round):
>
> $$(AA, AA, AA)$$
>
> Assuming players rationally propose budget proposals (i.e. give themselves the most possible budget while ensuring enough are on board to pass the budget).