Problem Set1

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January 24, 2018

Question1

p	q	$(p \Rightarrow q) \Rightarrow \neg q$
True	True	False
True	False	True
False	True	False
False	False	True

$(p \Rightarrow q) \Rightarrow \neg q$
$\equiv (\neg p \vee q) \Rightarrow \neg q$
$\equiv \neg(\neg p \vee q) \vee \neg q$
$\equiv \neg\neg p \wedge \neg q \vee \neg q$
$\equiv p \wedge \neg q \vee \neg q$
$\equiv \neg q$

p	q	$(p \Rightarrow \neg r) \wedge (\neg p \Rightarrow q)$
True	True	False
True	False	True
False	True	False
False	False	True

$(p \Rightarrow \neg r) \wedge (\neg p \Rightarrow q)$
$\equiv (\neg p \vee \neg r) \wedge (\neg p \Rightarrow q)$
$\equiv (\neg p \vee \neg r) \wedge (\neg\neg p \vee q)$
$\equiv (\neg p \vee \neg r) \wedge (p \vee q)$

Question2

(a) $\exists x \in \mathbb{N}, f(x) = x$
(b) $\exists x \in \mathbb{N}, \forall y \in \mathbb{N}, x = f(x) \wedge (x = f(x) \Rightarrow x < y \vee x = y)$
(c) $\exists x \in \mathbb{N}, \forall y \in \mathbb{N}, x = f(x) \wedge (x = f(x) \Rightarrow y < x \vee x = y)$
(d)
The fixed points of f are: 0,1,2,3,4,5,6
The least fixed point of f is: 0
The greatest fixed point of is: 6

Question3

(a) $R(x, y)$: "$\forall i \in \mathbb{N}, R(i, i) =$ True and all other values are False", where $x \in \mathbb{N}$, $y \in \mathbb{N}$
(b) $R(x, y)$: "$R(a, a) = R(b, b) = R(c, c) = R(d, d) =$ True and all other values are False", where $x \in \{a, b, c, d\}$, $y \in \{a, b, c, d\}$
(c) $R(x, y)$: "$R(a, a) = R(b, b) = R(c, c) = R(d, d) =$ True and all other values are False", where $x \in \{a, b, c, d\}$, $y \in \{a, b, c, d\}$
Justify:
When $d' = a, a = d' \vee \neg R(a, d') \equiv$True
When $d' = b, a = d' \equiv$ False, $\neg R(a, d') \equiv$ True, therefore $a = d' \vee \neg R(a, d') \equiv$ True
When $d' = c, a = d' \equiv$ False, $\neg R(a, d') \equiv$ True, therefore $a = d' \vee \neg R(a, d') \equiv$ True
When $d' = d, a = d' \equiv$ False, $\neg R(a, d') \equiv$ True, therefore $a = d' \vee \neg R(a, d') \equiv$ True
Hence, when $a \in D, \forall d' \in D, a = d' \vee \neg R(d, d')$ is always True, $a \in D$ is maximal
When $d' = b, R(b, a) \equiv$ False, therefore $\exists d' \in D, \neg R(d', d)$
Hence, a is not a greatest element

Question4

(a) $4 * 4 * 4 = 64$
(b) $A_4^3 = 24$
(c) $C_4^2 * A_3^3 = 36$
(d) $Function(R)$: "$[\forall x \in \mathbb{N}, \exists y \in \mathbb{N}, \forall z \in \mathbb{N}, R(x, y)] \wedge [R(x, z) \Rightarrow y = z]$", where R is a binary predicate with domain $\mathbb{N} \times \mathbb{N}$
(e) $OntoFunction(R)$: "$Function(R) \wedge [\forall y \in \mathbb{N}, \exists x \in \mathbb{N}, R(x, y)]$", where R is a binary predicate with domain $\mathbb{N} \times \mathbb{N}$
(f) $OneToOneFunction(R)$: "$Function(R) \wedge [\forall x_1 \in \mathbb{N}, \forall x_2 \in \mathbb{N}, \exists y \in \mathbb{N}, [R(x_1, y) \wedge R(x_2, y)] \Rightarrow x_1 = x_2]$", where R is a binary predicate with domain $\mathbb{N} \times \mathbb{N}$
(g) $InfiniteElementFunction(R)$: "$Function(R) \wedge [\forall y_1 \in \mathbb{N}, \exists y_2 \in \mathbb{N}, y_2 > y_1 \wedge [\exists x_2 \in \mathbb{N}, R(x_2, y_2)]]$", where R is a binary predicate with domain $\mathbb{N} \times \mathbb{N}$
(h) $AllButFiniteElementFunction(R)$: "$Function(R) \wedge [\exists y_1 \in \mathbb{N}, \forall y_2 \in \mathbb{N}, y_2 > y_1 \Rightarrow \exists x_2 \in \mathbb{N}, R(x_2, y_2)]$", where R is a binary predicate with domain $\mathbb{N} \times \mathbb{N}$