

Computer Problems

2.1. (a) Show that the matrix

$$A = \begin{bmatrix} 0.1 & 0.2 & 0.3 \\ 0.4 & 0.5 & 0.6 \\ 0.7 & 0.8 & 0.9 \end{bmatrix}$$

is singular. Describe the set of solutions to the system $Ax = b$ if

$$b = \begin{bmatrix} 0.1 \\ 0.3 \\ 0.5 \end{bmatrix}.$$

(b) If we were to use Gaussian elimination with partial pivoting to solve this system using exact arithmetic, at what point would the process fail?

(c) Because some of the entries of A are not exactly representable in a binary floating-point system, the matrix is no longer exactly singular when entered into a computer; thus, solving the system by Gaussian elimination will not necessarily fail. Solve this system on a computer using a library routine for Gaussian elimination. Compare the computed solution with your description of the solution set in part a. If your software includes a condition estimator, what is the estimated value for $\text{cond}(A)$? How many digits of accuracy in the solution would this lead you to expect?

2.2. (a) Use a library routine for Gaussian elimination to solve the system $Ax = b$, where

$$A = \begin{bmatrix} 2 & 4 & -2 \\ 4 & 9 & -3 \\ -2 & -1 & 7 \end{bmatrix}, \quad b = \begin{bmatrix} 2 \\ 8 \\ 10 \end{bmatrix}.$$

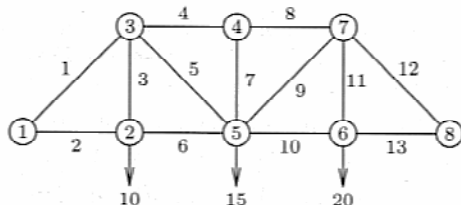
(b) Use the LU factorization of A already computed to solve the system $Ay = c$, where

$$c = \begin{bmatrix} 4 \\ 8 \\ -6 \end{bmatrix},$$

without refactoring the matrix.

(c) If the matrix A changes so that $a_{1,2} = 2$, use the Sherman-Morrison updating technique to compute the new solution x without refactoring the matrix, using the original right-hand-side vector b .

2.3. The following diagram depicts a plane truss having 13 members (the numbered lines) connected by 10 joints (the numbered circles). The indicated loads, in tons, are applied at joints 2, 5, and 6, and we wish to determine the resulting force on each member of the truss.



For the truss to be in static equilibrium, there must be no net force, horizontally or vertically, at any joint. Thus, we can determine the member forces by equating the horizontal forces to the left and right at each joint, and similarly equating the vertical forces upward and downward at each joint. For the eight joints, this would give 16 equations, which is more than the 13 unknown forces to be determined. For the truss to be statically determinate, that is, for there to be a unique solution, we assume that joint 1 is rigidly fixed both horizontally and vertically, and that joint 8 is fixed vertically. Resolving the member forces into horizontal and vertical components and defining $\alpha = \sqrt{2}/2$, we obtain the following system of equations for the member forces f_i :

$$\begin{aligned} \text{Joint 2: } & \begin{cases} f_2 = f_6 \\ f_3 = 10 \end{cases} \\ \text{Joint 3: } & \begin{cases} \alpha f_1 = f_4 + \alpha f_5 \\ \alpha f_1 + f_3 + \alpha f_5 = 0 \end{cases} \\ \text{Joint 4: } & \begin{cases} f_4 = f_8 \\ f_7 = 0 \end{cases} \\ \text{Joint 5: } & \begin{cases} \alpha f_5 + f_6 = \alpha f_9 + f_{10} \\ \alpha f_5 + f_7 + \alpha f_9 = 15 \end{cases} \\ \text{Joint 6: } & \begin{cases} f_{10} = f_{13} \\ f_{11} = 20 \end{cases} \\ \text{Joint 7: } & \begin{cases} f_8 + \alpha f_9 = \alpha f_{12} \\ \alpha f_9 + f_{11} + \alpha f_{12} = 0 \end{cases} \\ \text{Joint 8: } & \begin{cases} f_{13} + \alpha f_{12} = 0 \end{cases} \end{aligned}$$

Use a library routine to solve this system of linear equations for the vector f of member forces. Note that the matrix of this system is quite sparse, so